Fourier Algorithm for ASC Analysis: Part I

- Background
- Motivation
- Method
- Beam propagation
  - Reproduce results obtained using ABCD matrix method
- Control
  - Mirror tilt
  - Accuracy
  - Resolution
  - Displacement in x-k space at OMC waist
  - Actuation orthogonality between mirrors
- Conclusion – so far

Background

- Mode-matching-calculator
  - Beam propagation using ABCD matrix method
  - Analysis in terms of beam-waist, W, and defocus, S.
  - Calculation of lens/mirror strengths for exact mode-matching.
  - Calculation of optimum mirror placement for orthogonal control of mode shape (Automatic Wave-front Control, AWC)
  - Propagation on-axis

Motivation

- Alignment Sensing & Control (ASC).
- Requirement to sense alignment error
  - Sense orthogonal degrees of freedom: position, direction
  - Optimal placement of sensors with respect to OMC (target) waist
- Requirement for orthogonal beam control
  - Correction for errors
  - Orthogonal control of beam: position and direction
  - Two mirrors separated in Gouy-phase by 90˚
- Modelling
  - Analyse effect of errors and changes in beam position and displacement
  - Have tool to analyse beam propagation on-axis
  - Need tool to analyse beam propagation off-axis

Fourier algorithm

- Sziklas & Siegman, ‘Mode calculation in unstable resonators with flowing saturable gain. 2: Fast Fourier transform method,’ *App.Opt*, **14**, 1874 (1975).
- Johannes Courtial, Optics group
    - WaveTrace (LabView)
    - Young TIM (Java)
    - Leavey, & Courtial, ‘Young TIM User Guide’ (2017)

Fourier algorithm

- Input: complex wave amplitude at plane, z = 0.
    - Gaussian beam profile. Phase includes information about curvature.
    - Can be arbitrary phase and amplitude distribution.
- Amplitude can be expressed as integral over spatial frequency distribution
- Likewise, spatial frequency distribution can be expressed as integral over amplitude distribution
- Amplitude and Spatial Frequency related by Fourier and inverse Fourier transforms

Fourier algorithm

- Propagation through distance, Δz:
  - Input array, $U(x)$, at plane $z = 0$.
  - Fourier transform to give Spatial frequency distribution, $P(k_x)$, at plane $z = 0$.
    - Deconstruct input beam into its constituent plane waves having spatial frequencies, k_x.
  - Multiply each plane wave element by phase-factor, $\exp(ik_z\Delta z)$
    - Represents propagation through distance, Δz
    - k_z derived from monochromaticity condition: $(2\pi/\lambda)^2 = k_0^2 = k_z^2 + k_x^2$.
  - Inverse Fourier transform to reconstruct amplitude array at plane $z = \Delta z$.

Fourier algorithm

- Insert elements: lenses(focus), mirrors(focus, tilt) etc.
  - At element plane, multiply amplitude by phase-factor, exp(-i$\Delta\Phi$)
- Lens, focal length, f:
  - $\Delta\Phi = kx^2/2f$
- Concave mirror, ROC, R:
  - $\Delta\Phi = kx^2/R$
- Mirror, tilted by angle, α
  - $\Delta\Phi = kx \sin(\alpha)$

Fourier algorithm

- Image beam in plane, z:
  - Plot absolute value of amplitude, $|U(x)|$
  - Gaussian fit gives offset and width
- Fourier transform:
  - Plot absolute value of spatial frequency distribution $|P(k_x)|$
  - Gaussian fit gives offset and width
- Beam profile:
  - Plot waist as a function of z

Fourier algorithm

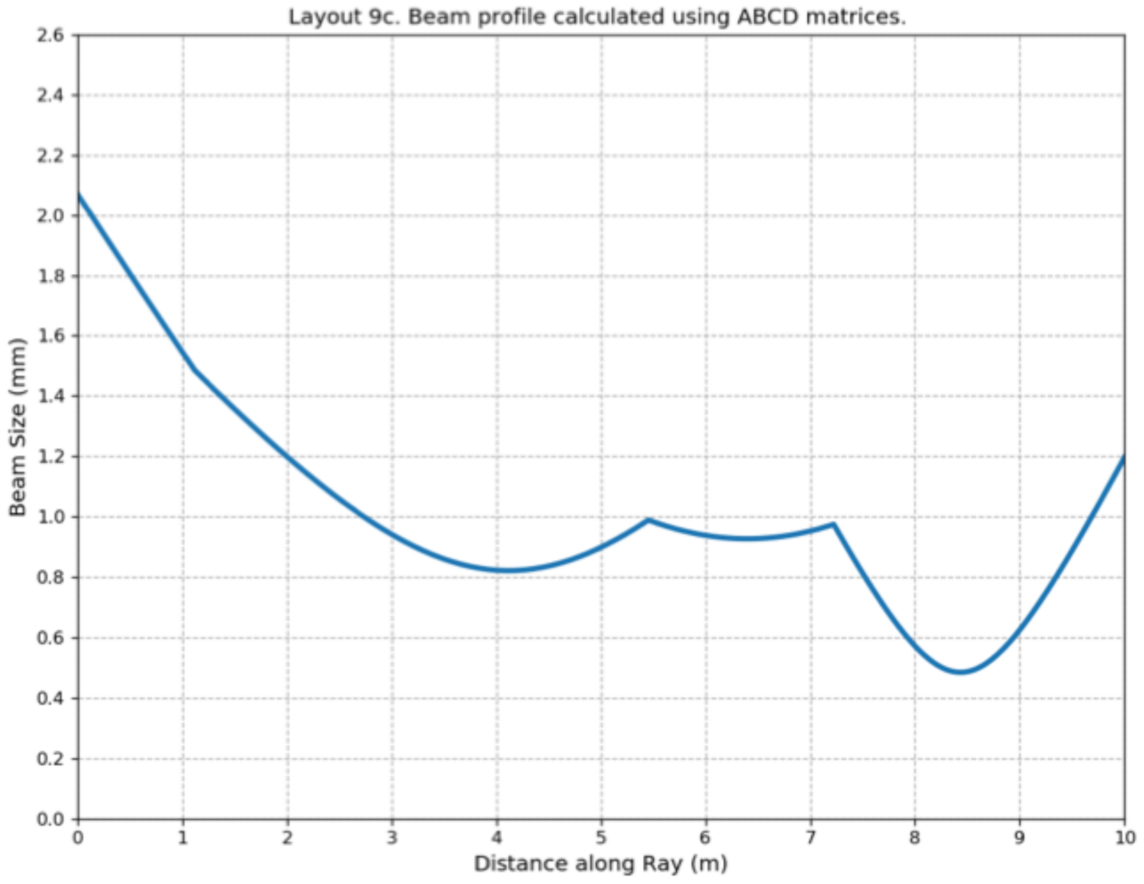
- Implement in Python
  - Define global variables:
    - Input parameters (wavelength, spacings, focal lengths, etc.)
    - Window and resolution in x. Resolution in z.
    - Array in x-space; corresponding arrays in kx and kz.
  - Define Amplitude Class representing complex amplitude distribution
  - Methods:
    - Propagation
    - Focusing by lens
    - Focusing with concave mirror
    - Tilt with mirror
    - Fitting and plotting amplitude and spatial frequency distributions
  - Functions:
    - Calculation of initial amplitude distribution
    - Plotting beam profile
    - Composite beam path with multiple elements and propagation between them
    - Analysis of effect of mirror tilt at OMC waist
    - Calculation of phase-separation (ASC-phase) with change in mirror separation
    - Analysis of effect of input error (position, direction)
    - Calculation of offset in position, x, and direction, k_x, as function of distance from OMC waist.

Beam propagation

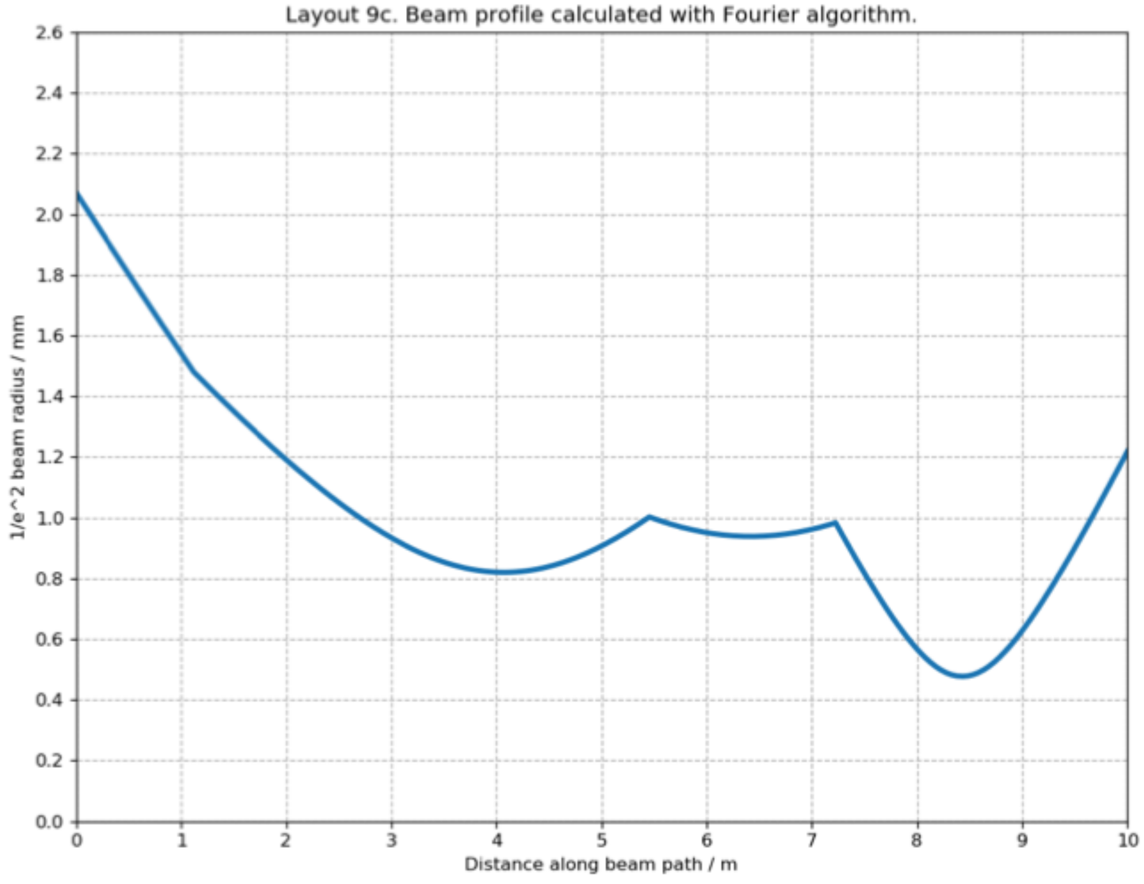
- Gaussian beam with $q = -3.55 - 1.09i$.
- Initial amplitude array: $U = (1/q) \exp(ikx^2/2q)$
- Include spacings and focusing elements for Layout 9c, path B:
  - OFI = -9.9 m
  - R1 = 5.5 m
  - R2 = 2.7 m
- Propagate beam over 10 m with resolution of 50 mm.
- Plot beam profile ($1/e^2$ beam radius) as a function of z.
- Compare with result from propagation with ABCD matrices (mode-matching calculator).

Beam propagation

ABCD



Fourier



Beam propagation

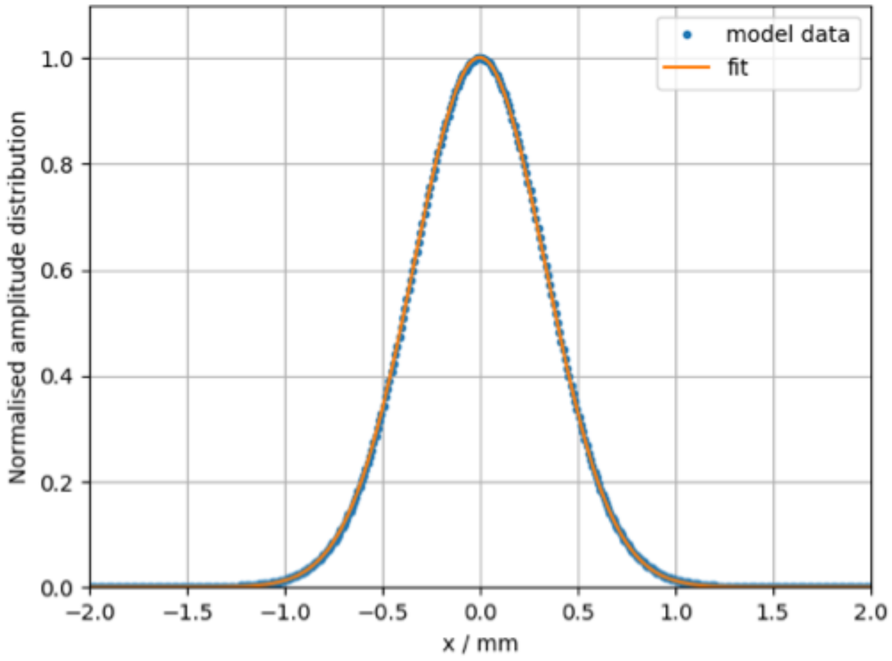
- Good agreement after propagation through multiple spacings and elements.
- In one case, beam profile calculated using ABCD matrix method:
  - transformation of complex beam parameter, q.
- In other case, beam profile calculated using Fourier algorithm:
  - deconstruction of complex amplitude into plane-wave constituents, multiplication by phase-factors, re-construction after propagation.
- Agreement gives confidence in method and its implementation in Python.

ASC Analysis: Control

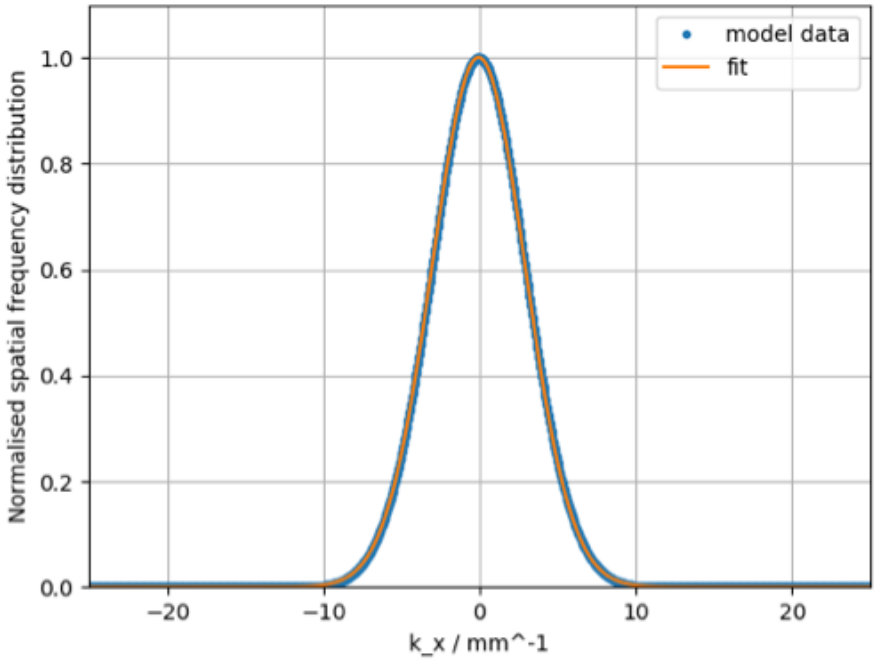
- Include tilt of mirrors, OMB3 (M1) and OMB2 (M2).
- M1 and M2 separated by 4,335 mm, as determined by mode-matching calculator.
- Gouy phase-separation of 91˚.
- Apply tilt of ± 1 mrad.
- Analyse effect at OMC beam waist.
- Plot and fit to Gaussian distribution in x- and k_x-space.
- Determine offset in x, k_x.
- Repeat for various window sizes and resolutions.

ASC Analysis: Control

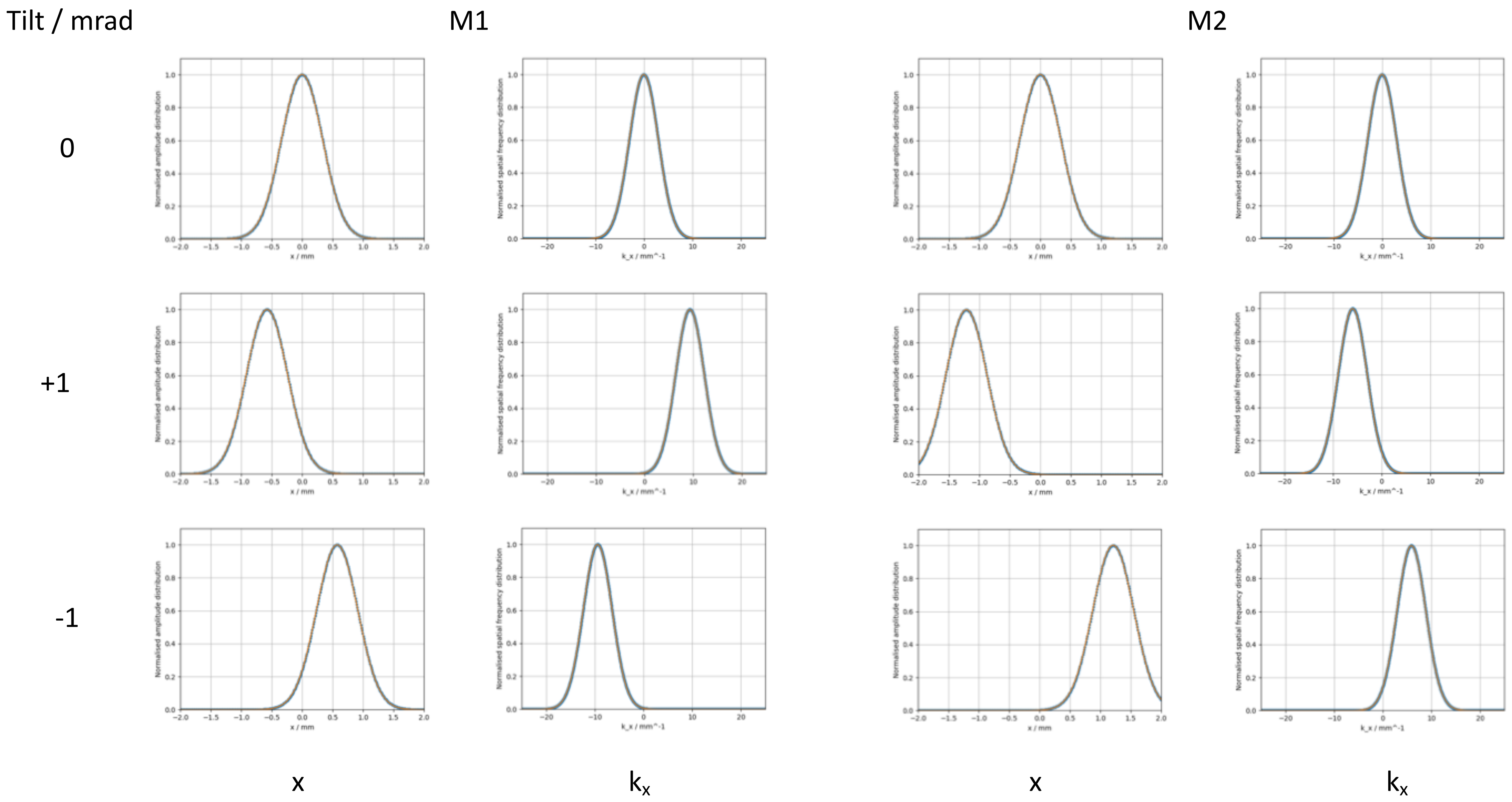
Gaussian distributions in x- and k_x-space at OMC waist (no tilt).



x



k_x


Tilt / mrad
M1
M2
0
+1
-1
Normalised amplitude distribution
Normalised spatial frequency distribution
x / mm
k_x / mm^-1
x
k_x
x
k_x

ASC Analysis: Control

Change width and resolution of array in x-space.

x-array / mm		Tilt / mrad		Offset / mm		Error / %
width	resolution	M1	M2	x	k	
300	0.003	0	0	-0.0016	-0.043	
300	0.003	1	0	-0.5796	9.298	
300	0.003	-1	0	0.5765	-9.384	~ 0.5
300	0.003	0	1	-1.2137	-5.959	
300	0.003	0	-1	1.2106	5.872	
100	0.01	0	0	-0.0051	-0.131	
100	0.01	1	0	-0.5756	9.284	
100	0.01	-1	0	0.5663	-9.545	~ 2
100	0.01	0	1	-1.2215	-6.067	
100	0.01	0	-1	1.2118	5.804	
30	0.03	0	0	-0.0218	-0.445	
30	0.03	1	0	-0.56	9.244	
30	0.03	-1	0	0.5257	-10.132	~ 5
30	0.03	0	1	-1.2523	-6.456	
30	0.03	0	-1	1.2151	5.55	

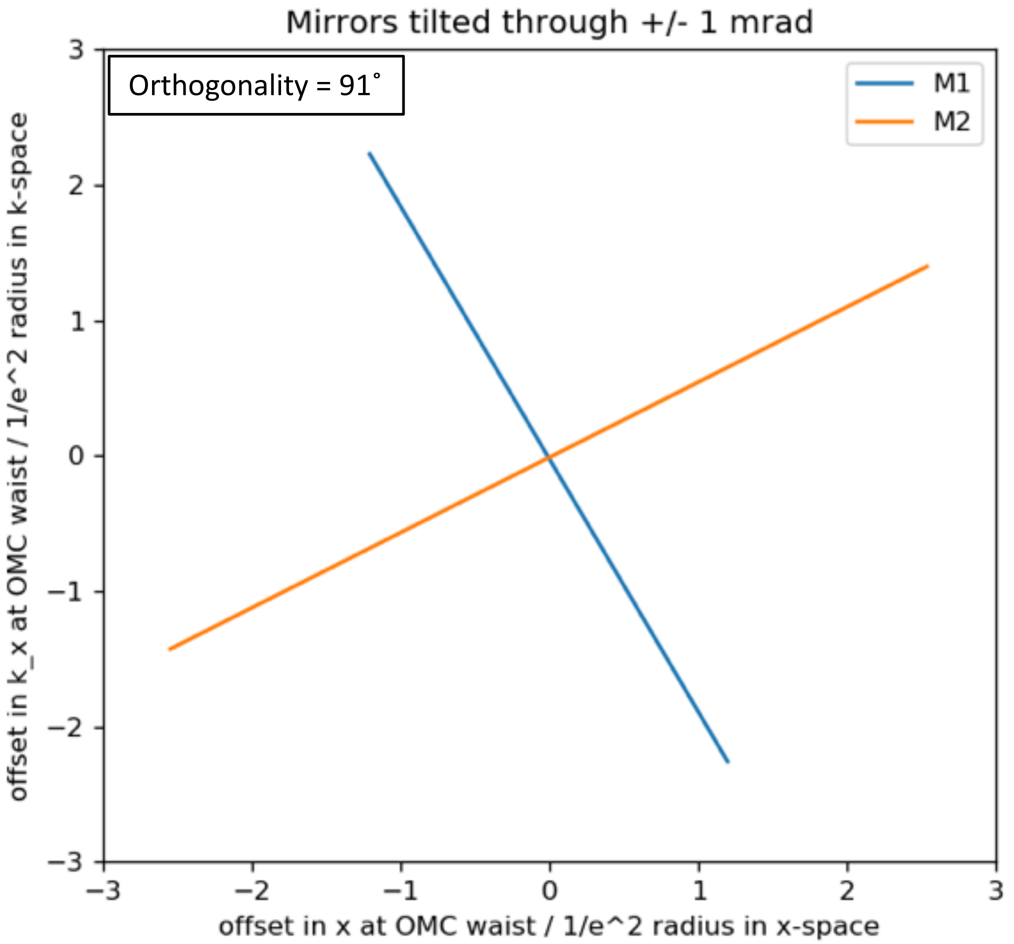
ASC Analysis: Control

- Estimate error from size of offset for 0 and comparing magnitude of offset for ± 1 mrad tilt.
- Error depends on window size, W, and resolution of x-array.
  - Resolution in k-space determined by inverse of window size (= 1/W)
- Large number of points (window/resolution) needed for accuracy.
- Choose values which give similar resolution in x- and k-space relative to the width of the Gaussian distribution.
- W = 200 mm, res = 0.02: gives 0.5 – 1 % error.

ASC Analysis: Control

- Plot effect of tilting mirrors, M1 and M2 in x-k space.
- Width of Gaussian distribution is the natural scale.
- Plot dimensionless quantity: offset / width.
- Calculate angle between actuation effected by mirrors (orthogonality):
  - dot product between unit vectors corresponding mirror tilts in x-k space.
- Compare with Gouy-phase-separation as determined by mode-matching calculator.

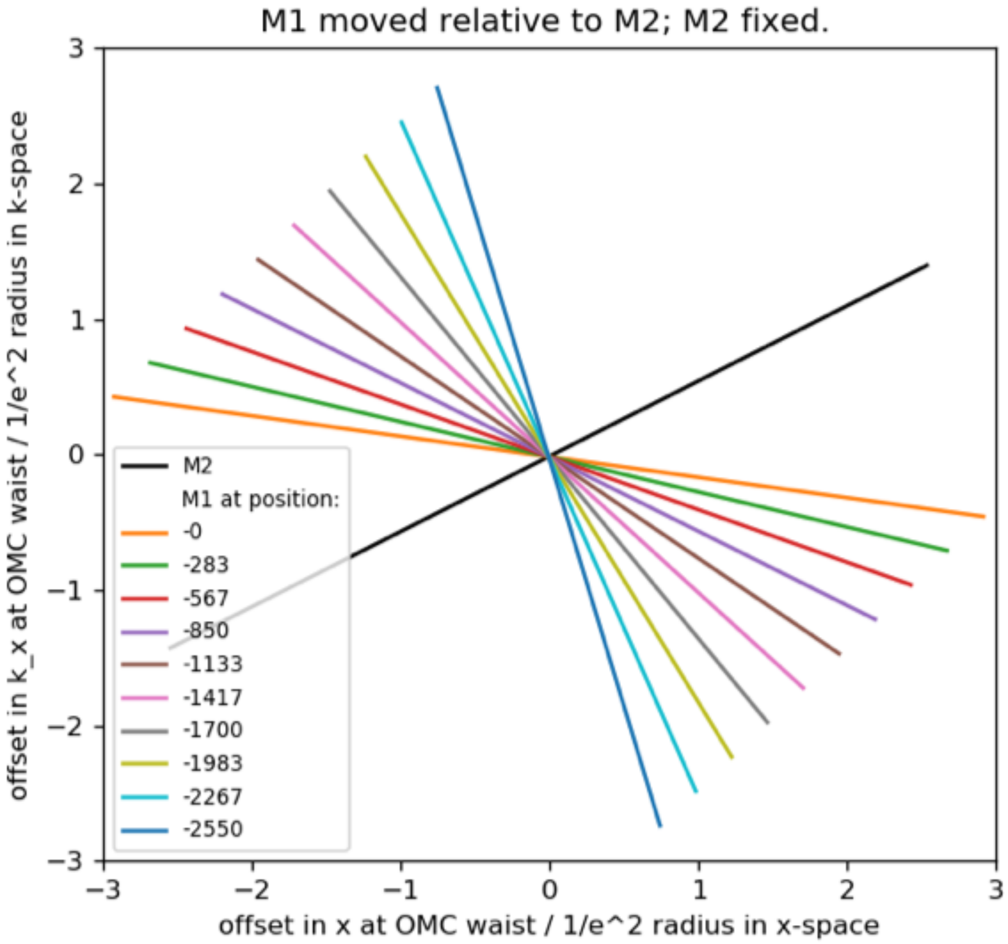
ASC Analysis: Control



ASC Analysis: Control

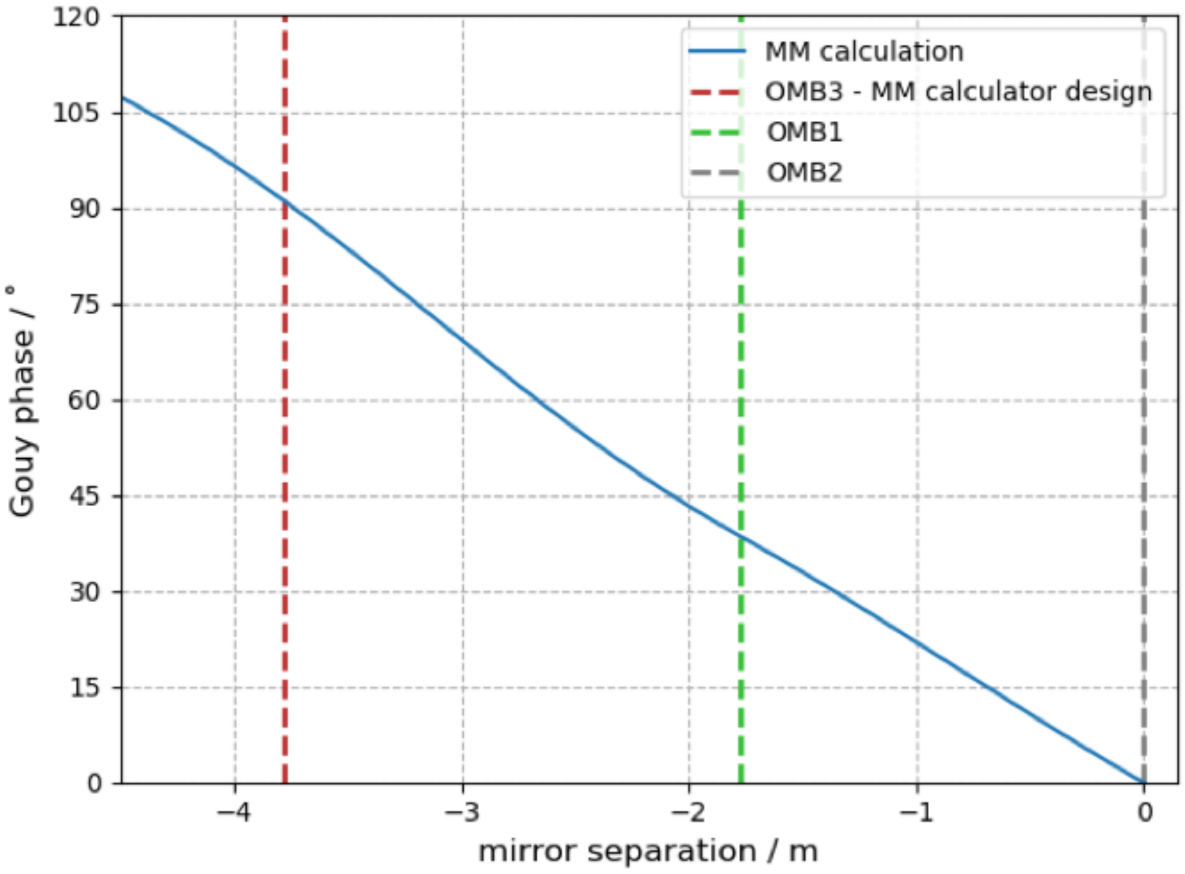
- Orthogonality = 91˚ (angle between unit vectors in x-k space).
- Good agreement with Gouy-phase-separation from mode-matching calculator – also 91˚!
- Calculate and plot orthogonality as a function of mirror separation.

ASC Analysis: Control

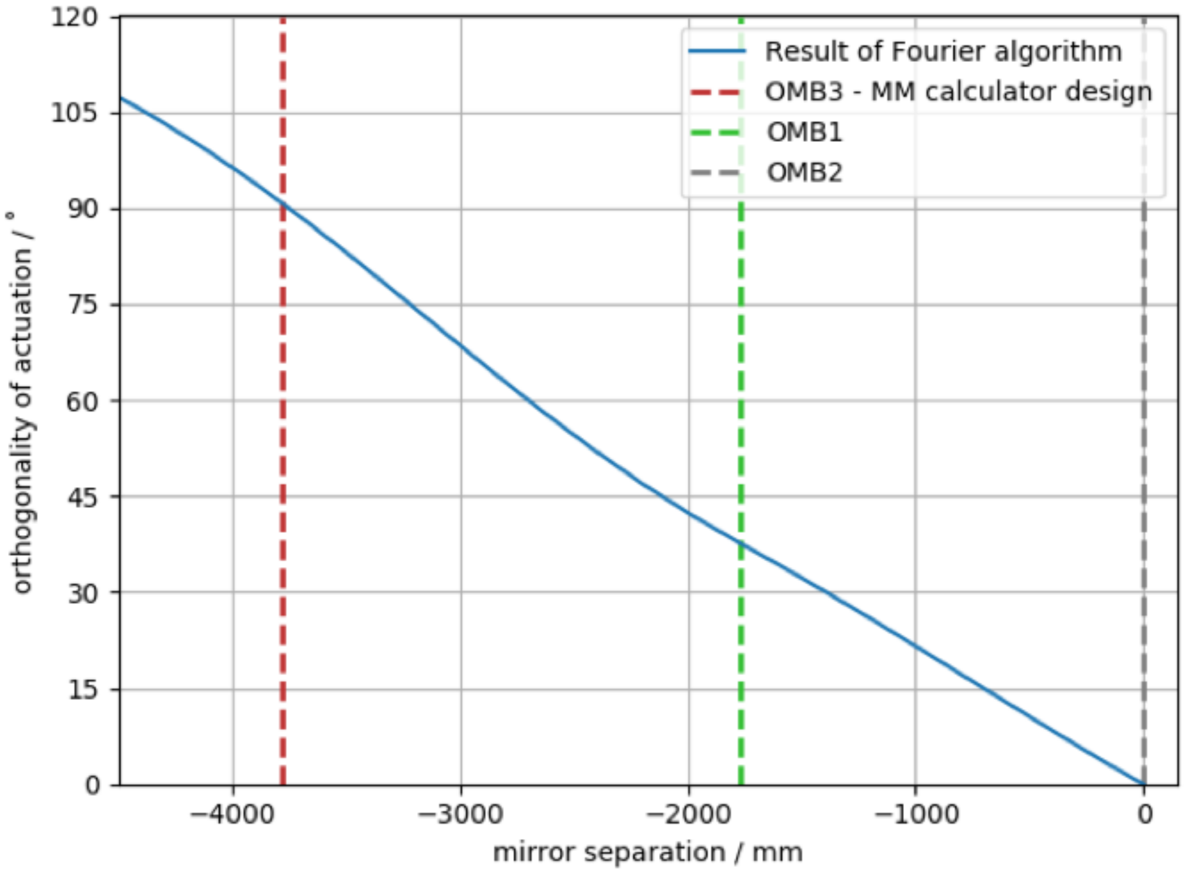


ASC Analysis: Control

ABCD



Fourier



ASC Analysis: Control

- Variation as expected
  - Orthogonality / phase-separation decreases as mirrors move closer together.
  - Orthogonality of actuation reaches 91˚ at separation for which the mode-matching calculator predicts a Gouy-phase separation of 91˚.

Conclusion – so far

- Able to reproduce findings of beam propagation analysis using ABCD matrices.
  - Beam profile
  - Orthogonality of actuation: Gouy-phase-separation
- Gives confidence in proceding to investigate positioning of sensors.

Fourier Algorithm for ASC Analysis: Part II

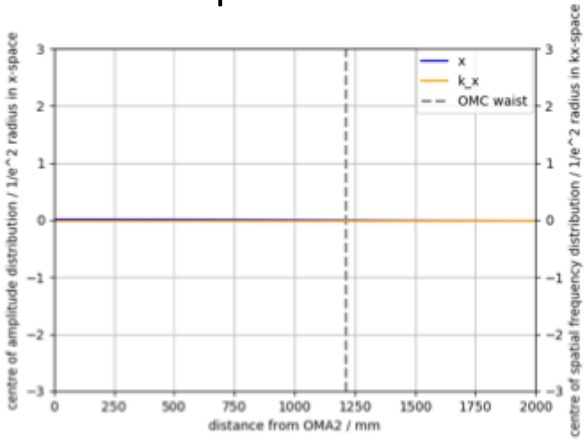
- Sensing
  - Input error
  - Displacement in x-k space
  - Dependence on distance from OMC waist

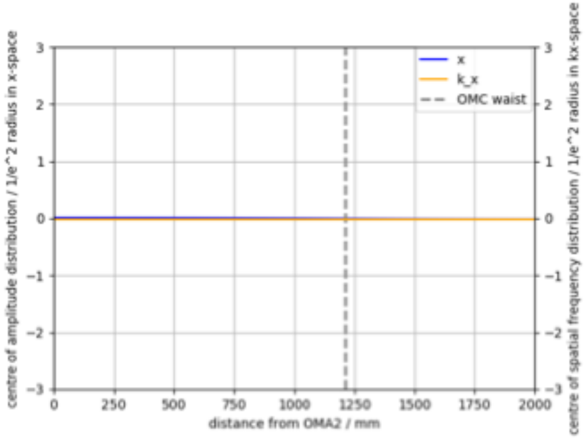
ASC Analysis: Sensing

- Input Error
  - Model as error in position and direction of beam at SRM.
  - Calculate displacement in x-k space as a function of distance from OMB2.

Position error: | position | direction | Direction error:

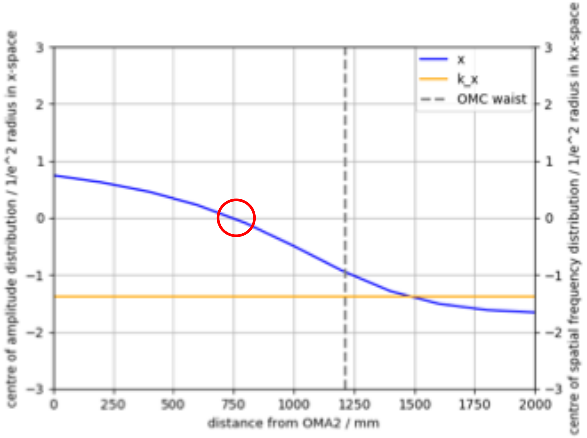
0

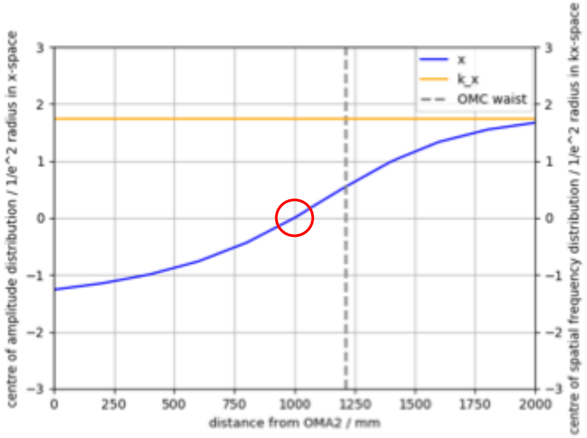




0

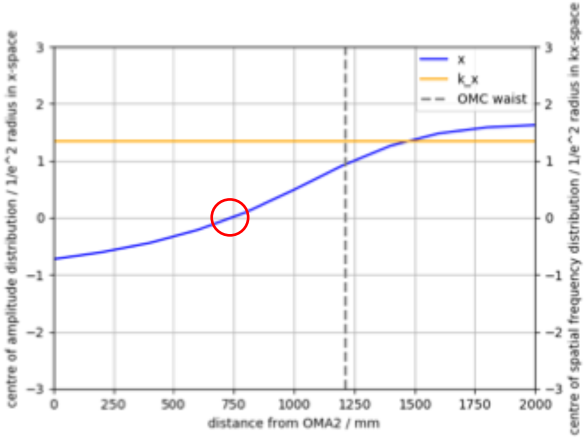
+1 mm

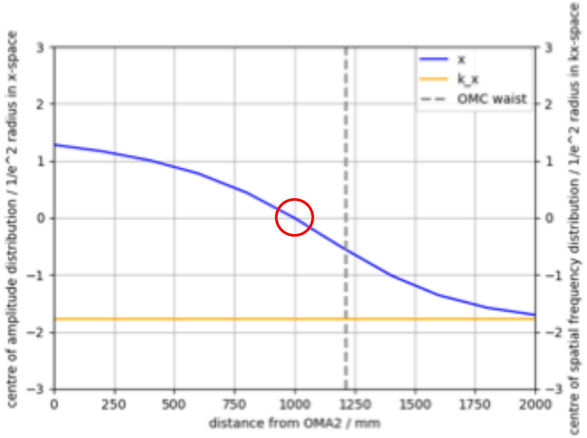




+300 µrad

-1 mm





-300 µrad

ASC Analysis: Sensing

- x-offset varies with distance from OMC waist.
- k-offset independent of distance from OMC waist, i.e. beam has a certain direction.
- Note: positions where variation in x-offset has a null.
    - Different depending on whether input error is in position or direction.