Abgabe Ananlysis IIa, Blatt 01

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Tutorium: T8

Aufgabe 1

Berechnen Sie folgende unbestimmte Integrale:

(a) Beh.: $\int (x-1)^2 e^{-x} dx = -e^{-x}(x^2+1) + C.$
Es gilt:

$$\int (x-1)^2 e^{-x} dx$$

$$= -e^{-x}(x-1)^2 - \int 2(x-1)(-e^{-x})dx$$

$$= -e^{-x}(x-1)^2 - \int (-2)(x-1)e^{-x}dx$$

$$= -e^{-x}(x-1)^2 + 2 \cdot \int (x-1)e^{-x}dx$$

$$= -e^{-x}(x-1)^2 + \left((x-1)(-e^{-x}) - \int (-e^{-x})dx \right)$$

$$= -e^{-x}(x-1)^2 - 2(x-1)e^{-x} + 2 \cdot \int e^{-x}dx$$

$$= -e^{-x}(x-1)^2 - 2(x-1)e^{-x} + 2(-e^{-x}) + C$$
$$= -e^{-x}(x-1)^2 - 2xe^{-x} + 2e^{-x} - 2e^{-x} + C$$
$$= -e^{-x}(x^2 - 2x + 1) - 2xe^{-x} + C$$
$$= -e^{-x}x^2 + 2e^{-x}x - e^{-x} - 2xe^{-x} + C$$
$$= -e^{-x}x^2 - e^{-x} + C$$
$$= -e^{-x}(x^2+1) + C.$$

$\square$

(b) Fehlt.

(c) Beh.: $\int (x+1)^2 \cos(2x)dx = \frac{1}{2}\left(\sin(2x)\left((x+1)^2 - \frac{1}{2} \right) + \cos(2x)(x+1) \right) + C.$
Es gilt:

$$\int (x+1)^2 \cos(2x)dx$$

$$= \frac{1}{2}\sin(2x)(x+1)^2 - \int 2(x+1) \cdot \frac{1}{2}\sin(2x)dx$$

$$= \frac{1}{2}\sin(2x)(x+1)^2 - \int (x+1) \cdot \sin(2x)dx$$

$$= \frac{1}{2}\sin(2x)(x+1)^2 - (x+1)(-\cos(2x) \cdot \frac{1}{2}) + \int (-\cos(2x) \cdot \frac{1}{2})dx$$

$$= \frac{1}{2}\sin(2x)(x+1)^2 + \frac{1}{2}(x+1)\cos(2x) - \frac{1}{2} \cdot \int \cos(2x)dx$$

$$= \frac{1}{2}\sin(2x)(x+1)^2 + \frac{1}{2}\cos(2x)(x+1) - \frac{1}{2} \cdot \frac{1}{2}\sin(2x) + C$$

$$= \frac{1}{2}\left(\sin(2x)\left((x+1)^2 - \frac{1}{2}\right) + \cos(2x)(x+1)\right) + C.$$

$\square$

Aufgabe 2

Berechnen Sie folgende unbestimmte Integrale:

(a) Beh.: $\int \sqrt{x}\sin\sqrt{x}dx = 4\sqrt{x} \cdot \sin\sqrt{x} - 2(x-2)\cos\sqrt{x} + C.$
Es gilt:

$$\int \sqrt{x}\sin\sqrt{x}dx$$

$$= \int \sqrt{x} \cdot \sin\sqrt{x} \cdot \frac{\sqrt{x}}{\sqrt{x}}dx$$

$$= 2 \cdot \int (\sqrt{x})^2 \cdot \sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}}dx$$

$$= 2 \cdot \int \varphi(x)^2 \cdot \sin\varphi(x) \cdot \varphi'(x)dx$$

$$= 2 \cdot \int \varphi^2 \cdot \sin\varphi d\varphi$$

$$= 2 \cdot \left(\varphi^2 \cdot (-\cos\varphi) - \int 2\varphi(-\cos\varphi)d\varphi\right)$$

$$= 2 \cdot \left(\varphi^2 \cdot (-\cos\varphi) - 2\varphi(-\sin\varphi) + \int 2(-\sin\varphi)d\varphi\right)$$

$$= 2 \cdot (-\cos\varphi \cdot \varphi^2 + 2\varphi \cdot \sin\varphi - 2(-\cos\varphi)) + C$$

$$= 2 \cdot (-\cos\varphi \cdot \varphi^2 + 2\varphi \cdot \sin\varphi + 2\cos\varphi) + C$$

$$= -2x \cdot \cos\sqrt{x} + 4\sqrt{x} \cdot \sin\sqrt{x} + 4\cos\sqrt{x} + C$$
$$= 4\sqrt{x} \cdot \sin\sqrt{x} + (4 - 2x) \cdot \cos\sqrt{x} + C$$
$$= 4\sqrt{x} \cdot \sin\sqrt{x} - 2(x - 2) \cdot \cos\sqrt{x} + C.$$

$\square$

(b) Beh.: $\displaystyle\int \frac{\sin\ln(x)}{x} dx = -\cos\ln(x) + C.$
Es gilt:

$$\int \frac{\sin\ln(x)}{x} dx$$
$$= \int \sin\ln(x) \cdot \frac{1}{x} dx$$
$$= \int \sin\varphi(x) \cdot \varphi'(x) dx$$
$$= \int \sin\varphi \, d\varphi$$
$$= -\cos\varphi + C$$
$$= -\cos\ln(x) + C.$$

$\square$

(c) Beh.: $\displaystyle\int \frac{dx}{e^x + 4e^{-x}} = -\frac{1}{2}\arctan(\frac{2}{e^x}) + C.$
Es gilt:

$$\int \frac{dx}{e^x + 4e^{-x}}$$
$$= \int \frac{dx}{e^x(1 + 4e^{-2x})}$$
$$= \int \frac{e^{-x}}{1 + 4e^{-2x}} dx$$
$$= -\frac{1}{2} \cdot \int \frac{2(-e^{-x})}{1 + (2e^{-x})^2} dx$$
$$= -\frac{1}{2} \cdot \int \frac{\varphi'(x)}{1 + (\varphi(x))^2} dx$$
$$= -\frac{1}{2} \cdot \int \frac{1}{1 + \varphi^2} d\varphi$$
$$= -\frac{1}{2} \cdot \arctan(\varphi) + C$$

$$= -\frac{1}{2} \cdot \arctan(2e^{-x}) + C$$

$$= -\frac{1}{2} \cdot \arctan\left(\frac{2}{e^x}\right) + C.$$

$\square$

Aufgabe 3

Berechnen Sie $\displaystyle\int \frac{x}{x^3 + 8}$.

Es gilt:

$$\int \frac{x}{x^3 + 8} dx$$

$$= \int \left(\frac{x}{x^3 + \frac{288}{36}}\right) dx$$

$$= \int \left(\frac{36x}{36x^3 - 72x^2 + 144x + 72x^2 - 144x + 240 + 48}\right) dx$$

$$= \int \left(\frac{6x^2 + 12x + 12x + 24}{(6x^2 - 12x + 24)(6x + 12)} - \frac{6x^2 - 12x + 24}{(6x^2 - 12x + 24)(6x + 12)}\right) dx$$

$$= \int \left(\frac{(x + 2)(6x + 12)}{(6x^2 - 12x + 24)(6x + 12)} - \frac{6x^2 - 12x + 24}{(6x^2 - 12x + 24)(6x + 12)}\right) dx$$

$$= \int \left(\frac{x + 2}{6x^2 - 12x + 24} - \frac{1}{6x + 12}\right) dx$$

$$= \frac{1}{6} \left(\int \frac{x + 2}{x^2 - 2x + 4} dx - \int \frac{1}{x + 2} dx\right)$$

$$= \frac{1}{6} \left(\int \left(\frac{x - 1}{x^2 - 2x + 4} + \frac{3}{x^2 - 2x + 4}\right) dx - \int \frac{1}{x + 2} dx\right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{2x - 2}{x^2 - 2x + 4} dx + 3 \cdot \int \frac{1}{x^2 - 2x + 4} dx - \int \frac{1}{x + 2} dx\right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{u'(x)}{u(x)} dx + 3 \cdot \int \frac{1}{x^2 - 2x + 4} dx - \int \frac{1}{x + 2} dx\right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + 3 \cdot \int \frac{1}{x^2 - 2x + 4} dx - \int \frac{1}{x + 2} dx\right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + 3 \cdot \int \frac{1}{(x - 1)^2 + 3} dx - \int \frac{1}{x + 2} dx\right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + 3 \cdot \int \frac{1}{v(x)^2 + 3} v'(x) dx - \int \frac{1}{x + 2} dx\right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + 3 \cdot \int \frac{1}{v^2 + 3} dv - \int \frac{1}{x+2} dx \right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + 3 \cdot \int \frac{1}{3(\frac{v^2}{3} + 1)} dv - \int \frac{1}{x+2} dx \right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + \int \frac{1}{(\frac{v}{\sqrt{3}})^2 + 1} dv - \int \frac{1}{x+2} dx \right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + \sqrt{3} \cdot \int \frac{1}{(\frac{v}{\sqrt{3}})^2 + 1} \cdot \frac{1}{\sqrt{3}} dv - \int \frac{1}{x+2} dx \right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + \sqrt{3} \cdot \int \frac{1}{(w(v))^2 + 1} \cdot w'(v) dv - \int \frac{1}{x+2} dx \right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \int \frac{1}{u} du + \sqrt{3} \cdot \int \frac{1}{w^2 + 1} dw - \int \frac{1}{x+2} dx \right)$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \ln | u | + \sqrt{3} \cdot \arctan(w) - \ln | x + 2 | \right) + C$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot ln | x^2 - 2x + 4 | + \sqrt{3} \cdot \arctan(\frac{v}{\sqrt{3}}) - \ln | x + 2 | \right) + C$$

$$= \frac{1}{6} \left(\frac{1}{2} \cdot \ln | x^2 - 2x + 4 | + \sqrt{3} \cdot \arctan \left(\frac{x-1}{\sqrt{3}} \right) - ln | x + 2 | \right) + C.$$

□

Aufgabe 4

Fehlt.