1	2	3	4	Σ
4	5	5	0	14

Abgabe Ananlysis IIa, Blatt 01

Studierende(r): Weerts, Steffen, steffen.weerts@uni-oldenburg.de
Tutorium: T8

Aufgabe 1

Berechnen Sie folgende unbestimmte Integrale:

(a) Beh.: $\displaystyle\int (x-1)^2 e^{-x} dx \; = \; -e^{-x}(x^2+1) + C.$
Es gilt:

$$\begin{aligned}
&\int (x-1)^2 e^{-x} dx \\
= \; & -e^{-x}(x-1)^2 - \int 2(x-1)(-e^{-x})dx \\
= \; & -e^{-x}(x-1)^2 - \int (-2)(x-1)e^{-x}dx \\
= \; & -e^{-x}(x-1)^2 + 2\cdot \int (x-1)e^{-x}dx \\
= \; & -e^{-x}(x-1)^2 + \left((x-1)(-e^{-x}) - \int (-e^{-x})dx\right) \\
= \; & -e^{-x}(x-1)^2 - 2(x-1)e^{-x} + 2\cdot \int e^{-x}dx \\
= \; & -e^{-x}(x-1)^2 - 2(x-1)e^{-x} + 2(-e^{-x}) + C \\
= \; & -e^{-x}(x-1)^2 - 2xe^{-x} + 2e^{-x} - 2e^{-x} + C \\
= \; & -e^{-x}(x^2-2x+1) - 2xe^{-x} + C \\
= \; & -e^{-x}x^2 + 2e^{-x}x - e^{-x} - 2xe^{-x} + C \\
= \; & -e^{-x}x^2 - e^{-x} + C \\
= \; & -e^{-x}(x^2+1) + C.
\end{aligned}$$

2/2

□

(b) Fehlt. 0/2

(c) Beh.: $\displaystyle\int (x+1)^2 \cos(2x) dx \; = \; \frac{1}{2}\left(\sin(2x)\left((x+1)^2 - \frac{1}{2}\right) + \cos(2x)(x+1)\right) + C.$
Es gilt:

$$\int (x+1)^2 \cos(2x) dx$$

$$= \frac{1}{2}\sin(2x)(x+1)^2 - \int 2(x+1)\cdot\frac{1}{2}\sin(2x)dx$$
$$= \frac{1}{2}\sin(2x)(x+1)^2 - \int (x+1)\cdot\sin(2x)dx$$

$$= \frac{1}{2}\sin(2x)(x+1)^2 - (x+1)(-\cos(2x)\cdot\frac{1}{2}) + \int(-\cos(2x)\cdot\frac{1}{2})dx$$
$$= \frac{1}{2}\sin(2x)(x+1)^2 + \frac{1}{2}(x+1)\cos(2x) - \frac{1}{2}\cdot\int\cos(2x)dx$$
$$= \frac{1}{2}\sin(2x)(x+1)^2 + \frac{1}{2}\cos(2x)(x+1) - \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C$$
$$= \frac{1}{2}\left(\sin(2x)\left((x+1)^2 - \frac{1}{2}\right) + \cos(2x)(x+1)\right) + C.$$

2/2

□

Aufgabe 2

Berechnen Sie folgende unbestimmte Integrale:

(a) Beh.: $\int \sqrt{x}\sin\sqrt{x}dx \;=\; 4\sqrt{x}\cdot\sin\sqrt{x} - 2(x-2)\cos\sqrt{x} + C.$
Es gilt:

$$\int \sqrt{x}\sin\sqrt{x}dx$$
$$= \int \sqrt{x}\cdot\sin\sqrt{x}\cdot\frac{\sqrt{x}}{\sqrt{x}}dx$$
$$= 2\cdot\int(\sqrt{x})^2\cdot\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}dx$$
$$= 2\cdot\int\varphi(x)^2\cdot\sin\varphi(x)\cdot\varphi'(x)dx$$
$$= 2\cdot\int\varphi^2\cdot\sin\varphi d\varphi$$
$$= 2\cdot\left(\varphi^2\cdot(-\cos\varphi) - \int 2\varphi(-\cos\varphi)d\varphi\right)$$
$$= 2\cdot\left(\varphi^2\cdot(-\cos\varphi) - 2\varphi(-\sin\varphi) + \int 2(-\sin\varphi)d\varphi\right)$$
$$= 2\cdot\left(-\cos\varphi\cdot\varphi^2 + 2\varphi\cdot\sin\varphi - 2(-\cos\varphi)\right) + C$$
$$= 2\cdot\left(-\cos\varphi\cdot\varphi^2 + 2\varphi\cdot\sin\varphi + 2\cos\varphi\right) + C$$

Was ist φ?

$$
\begin{aligned}
&= -2x \cdot \cos\sqrt{x} + 4\sqrt{x} \cdot \sin\sqrt{x} + 4\cos\sqrt{x} + C \\
&= 4\sqrt{x} \cdot \sin\sqrt{x} + (4 - 2x) \cdot \cos\sqrt{x} + C \\
&= 4\sqrt{x} \cdot \sin\sqrt{x} - 2(x - 2) \cdot \cos\sqrt{x} + C.
\end{aligned}
$$

2/2

$\square$

(b) Beh.: $\displaystyle\int \frac{\sin\ln(x)}{x}dx = -\cos\ln(x) + C.$
Es gilt:

$$
\begin{aligned}
&\int \frac{\sin\ln(x)}{x}dx \\
&= \int \sin\ln(x) \cdot \frac{1}{x}dx \\
&= \int \sin\varphi(x) \cdot \varphi'(x)dx \\
&= \int \sin\varphi d\varphi \\
&= -\cos\varphi + C \\
&= -\cos\ln(x) + C.
\end{aligned}
$$

1/1

$\square$

(c) Beh.: $\displaystyle\int \frac{dx}{e^x + 4e^{-x}} = -\frac{1}{2}\arctan(\frac{2}{e^x}) + C.$
Es gilt:

$\varphi(x) = 2e^{-x}$

$$
\begin{aligned}
&\int \frac{dx}{e^x + 4e^{-x}} \\
&= \int \frac{dx}{e^x(1 + 4e^{-2x})} \\
&= \int \frac{e^{-x}}{1 + 4e^{-2x}}dx \\
&= -\frac{1}{2} \cdot \int \frac{2(-e^{-x})}{1 + (2e^{-x})^2}dx \\
&= -\frac{1}{2} \cdot \int \frac{\varphi'(x)}{1 + (\varphi(x))^2}dx \\
&= -\frac{1}{2} \cdot \int \frac{1}{1 + \varphi^2}d\varphi \\
&= -\frac{1}{2} \cdot \arctan(\varphi) + C
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{2}\cdot\arctan(2e^{-x})+C\\
&= -\frac{1}{2}\cdot\arctan\left(\frac{2}{e^{x}}\right)+C.
\end{aligned}
$$

2/2

$\square$

Aufgabe 3

Berechnen Sie $\int \frac{x}{x^3+8}$.

Es gilt:

$$
\begin{aligned}
&\int \frac{x}{x^3+8}dx\\
&= \int\left(\frac{x}{x^3+\frac{288}{36}}\right)dx\\
&= \int\left(\frac{36x}{36x^3-72x^2+144x+72x^2-144x+240+48}\right)dx\\
&= \int\left(\frac{6x^2+12x+12x+24}{(6x^2-12x+24)(6x+12)}-\frac{6x^2-12x+24}{(6x^2-12x+24)(6x+12)}\right)dx\\
&= \int\left(\frac{(x+2)(6x+12)}{(6x^2-12x+24)(6x+12)}-\frac{6x^2-12x+24}{(6x^2-12x+24)(6x+12)}\right)dx\\
&= \int\left(\frac{x+2}{6x^2-12x+24}-\frac{1}{6x+12}\right)dx\\
&= \frac{1}{6}\left(\int\frac{x+2}{x^2-2x+4}dx-\int\frac{1}{x+2}dx\right)\\
&= \frac{1}{6}\left(\int\left(\frac{x-1}{x^2-2x+4}+\frac{3}{x^2-2x+4}\right)dx-\int\frac{1}{x+2}dx\right)\\
&= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{2x-2}{x^2-2x+4}dx+3\cdot\int\frac{1}{x^2-2x+4}dx-\int\frac{1}{x+2}dx\right)\\
&= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{u'(x)}{u(x)}dx+3\cdot\int\frac{1}{x^2-2x+4}dx-\int\frac{1}{x+2}dx\right)\\
&= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du+3\cdot\int\frac{1}{x^2-2x+4}dx-\int\frac{1}{x+2}dx\right)\\
&= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du+3\cdot\int\frac{1}{(x-1)^2+3}dx-\int\frac{1}{x+2}dx\right)\\
&= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du+3\cdot\int\frac{1}{v(x)^2+3}v'(x)dx-\int\frac{1}{x+2}dx\right)
\end{aligned}
$$

u = ?

$$= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du + 3\cdot\int\frac{1}{v^2+3}dv - \int\frac{1}{x+2}dx\right)$$

$$= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du + 3\cdot\int\frac{1}{3(\frac{v^2}{3}+1)}dv - \int\frac{1}{x+2}dx\right)$$

$$= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du + \int\frac{1}{(\frac{v}{\sqrt{3}})^2+1}dv - \int\frac{1}{x+2}dx\right)$$

$$= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du + \sqrt{3}\cdot\int\frac{1}{(\frac{v}{\sqrt{3}})^2+1}\cdot\frac{1}{\sqrt{3}}dv - \int\frac{1}{x+2}dx\right)$$

$$= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du + \sqrt{3}\cdot\int\frac{1}{(w(v))^2+1}\cdot w'(v)dv - \int\frac{1}{x+2}dx\right)$$

$$= \frac{1}{6}\left(\frac{1}{2}\cdot\int\frac{1}{u}du + \sqrt{3}\cdot\int\frac{1}{w^2+1}dw - \int\frac{1}{x+2}dx\right)$$

$$= \frac{1}{6}\left(\frac{1}{2}\cdot\ln\mid u\mid + \sqrt{3}\cdot\arctan(w) - \ln\mid x+2\mid\right) + C$$

$$= \frac{1}{6}\left(\frac{1}{2}\cdot ln\mid x^2-2x+4\mid + \sqrt{3}\cdot\arctan(\frac{v}{\sqrt{3}}) - \ln\mid x+2\mid\right) + C$$

$$= \frac{1}{6}\left(\frac{1}{2}\cdot\ln\mid x^2-2x+4\mid + \sqrt{3}\cdot\arctan\left(\frac{x-1}{\sqrt{3}}\right) - ln\mid x+2\mid\right) + C.$$

$w = ?$

5/5

□

Aufgabe 4

Fehlt.

0/1

In Zukunft die Substitution konkret angeben!