Mathematics 2

Name	
Adm. No	
Tutorial group	
Question No.	**Marks**
1	
2	
3	
Penalty (Late)	
Penalty (Missing cover sheet)	
Total	/60

Instructions:

- **This is an individual assignment!** Copying from another student constitutes cheating, and will result in academic disciplinary proceedings, if detected.
- The Assignment consists of 3 questions each worth 20 marks. Attempt all 3 questions.
- The Assignment is due: **3pm, Monday 26 January.**
- If late: -10% every day for the first 5 working days. If > 5 days late, then 0%.
- Submit a hardcopy of this assignment in the Math 2 assignment drop-box on level 5, SIDM building. (On the canteen, not the car park side.)
- Fill out your details above and staple this page to the front of your assignment. If this is not done, a penalty of 3 marks will be applied.
- Show your working out, not just the solution.
- If you have a reasonable excuse to be late (e.g. MC), please contact me at michael_lucht@nyp.edu.sg or x1844 to negotiate an alternative due date. (Ideally, *before* the Assignment is due.)

Question 1

Consider the following two lines:

(1) $\overrightarrow{r_1} = \begin{bmatrix}10\\10\end{bmatrix} + \lambda_1 \begin{bmatrix}-1\\-2\end{bmatrix}$

(2) $\overrightarrow{r_2} = \lambda_2 \begin{bmatrix}3\\1\end{bmatrix}$

(a) Write equations (1) and (2) as parametric equations. **[2 marks]**

(b) At what point do the lines intersect? Hint: they intersect when $\overrightarrow{r_1} = \overrightarrow{r_2}$. **[5 marks]**

Missile Command (1980) Atari

Now, imagine that you are working on a Gulf War themed computer game. Let *t* be the variable standing for time, and let $t = \lambda_1 = \lambda_2$ in the two above equations. For a given time *t*, $\overrightarrow{r_1}$ is the position vector of an incoming hostile Scud missile, and $\overrightarrow{r_2}$ is the position vector of a Patriot anti-missile missile.

(c) Sketch the trajectories of the two missiles. Assume that ground level is at x=0. Clearly label your diagram and indicate the position of each of the missiles at two different times. **[5 marks]**

(d) Will the Patriot shoot down the Scud or will it miss? Note that the missiles need to be at the same position, at the same time in order to collide. Explain your answer. **[4 marks]**

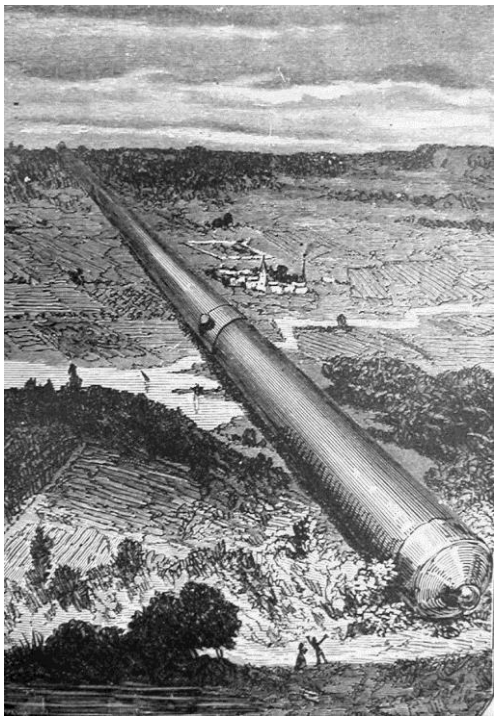
(e) Assume that besides its initial velocity, the Patriot missile (missile 2) accelerates at $2ms^{-2}$ along the same direction as before. i.e. direction vector is: $\begin{bmatrix}3\\1\end{bmatrix}$. Write down the parametric equations of the accelerating Patriot missile. **[4 marks]**

Hint:

- The parametric equation of the x-component of a straight line is given by: $x = x_0 + nt$ where *n* is the x-component of the direction vector, and *t* is the parameter.
- The 3rd Kinematic equation is: $s = s_0 + ut + \frac{1}{2}at^2$ (Slightly modified. s_0 is an initial displacement that we normally take to be 0.)
- Notice the similarity?

Question 2

A space gun is a gun so large that it can shoot objects into space. Because Jules Verne described it in his 1865 novel, "From the Earth to the Moon", a space gun is also called a Verne gun. Quoting from Wikipedia: "Space guns could thus potentially provide a method of non-rocket space launch. It has been conjectured that space guns could place satellites into Earth's orbit (although after-launch propulsion of the satellite would be necessary to achieve a stable orbit) , and could also launch spacecraft beyond Earth's gravitational pull and into other parts of the Solar System by exceeding Earth's escape velocity of about 11.2 km/s". (**Note**: per second, not per hour!)



Jerry Woodfill (1886) Illustrated Edition of "From the Earth to the Moon"

This question concerns the potential use of a Verne gun for human space travel – which is what happens in Jules Verne's novel. The maximum acceleration that human being can survive for a minute or so is about $200ms^{-2}$.

(a) How long does the barrel of a Verne gun have to be to achieve escape velocity? Assume that initially the space capsule is at rest, and that the gun gives it a constant acceleration of $200ms^{-2}$. **[5 marks]**

(b) How long will the space travelers experience the $200ms^{-2}$ acceleration? (i.e. for how long will they be inside the barrel?) **[4 marks]**

(c) Given that the space capsule with equipment and astronauts weights 5000 kilograms, how much work does the Verne Gun do? **[5 marks]**

(d) Given that a 51 gram Mars Bar contains $1.888 \times 10^6 J$ of chemical energy, and that a Mars Bar costs $1, how much will it cost (assuming that the Verne gun is powered by Mars bars and is 100% efficient) to launch the space capsule into space? Do you think it's affordable? **[4 marks]**

(e) Put the length of the Verne gun into context by comparing it with similar sized man-made objects or geographic features. (e.g. how many Eiffel Towers is it equal to? If placed horizontally, could it be build within Singapore?) **[2 marks]**

Question 3

Homer has just eaten a large donut (containing 302.8 kilocalories of chemical energy[1]) 1 kilocalorie is equivalent to 4184 Joules. Homer's mass is 110 kilograms. He wants to maintain his impressive physique by exercising off the donut.


Simpsons, Fox

(a) One Raffles Place, one of the three tallest buildings in Singapore, is 280 meters tall. How often must Homer climb it to the top to work off his donut? (Base your calculation purely on the potential energy that Homer gains by climbing up. Ignore any other ways that he is likely to use up energy, such as by excessive wailing.) Use $g = 9.8ms^{-2}$. **[10 marks]**

(b) Thinking that there must be an easier way, Homer decided to try to work off the donut by pushing his car. The mass of the car is 2 metric tons. (1 metric ton = 1000 kg). What speed must the car reach in order for Homer to work off his donut? Assume the car is on level ground. This time base your calculation entirely on the Kinetic Energy of the car. e.g. ignore friction and the energy Homer expends on cursing. **[10 marks]**

[1] http://calorielab.com/foods/doughnuts/5