§9.4 简谐振动的合成

一、同方向同频率简谐振动的合成

- 同方向，假定两个简谐振动都沿 x 方向振动
- 同频率，所以圆频率相同，假设两个简谐振动的圆频率都为 ω_0
- 如果一个质点同时参与两个同方向同频率的简谐振动

$$x_1 = A_1\cos(\omega_0 t + \varphi_{10})$$
$$x_2 = A_2\cos(\omega_0 t + \varphi_{20})$$

上式表明，t 时刻，第一个简谐振动使质点离开平衡位置 x_1，第二个简谐振动使质点离开平衡位置 x_2，则质点总的位移

$$\begin{aligned} x &= x_1 + x_2 \\ &= A_1\cos(\omega_0 t + \varphi_{10}) + A_2\cos(\omega_0 t + \varphi_{20}) \end{aligned}$$

- 三角函数的解析运算
  - 三角函数的两角和公式

$$\begin{aligned}
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos(\omega_0 t+\varphi_{10}) &= \cos(\omega_0 t)\cos\varphi_{10} - \sin(\omega_0 t)\sin\varphi_{10} \\
\cos(\omega_0 t+\varphi_{20}) &= \cos(\omega_0 t)\cos\varphi_{20} - \sin(\omega_0 t)\sin\varphi_{20}
\end{aligned}$$

  - 代入、合并同类项

$$\begin{aligned}
x &= A_1\cos(\omega_0 t+\varphi_{10}) + A_2\cos(\omega_0 t+\varphi_{20}) \\
&= A_1[\cos(\omega_0 t)\cos\varphi_{10} - \sin(\omega_0 t)\sin\varphi_{10}] + A_2[\cos(\omega_0 t)\cos\varphi_{20} - \sin(\omega_0 t)\sin\varphi_{20}] \\
&= \cos(\omega_0 t)[A_1\cos\varphi_{10} + A_2\cos\varphi_{20}] - \sin(\omega_0 t)[A_1\sin\varphi_{10} + A_2\sin\varphi_{20}]
\end{aligned}$$

- 三角函数的解析运算
  - 令

$$\begin{aligned}A\cos\varphi_0 &= A_1\cos\varphi_{10} + A_2\cos\varphi_{20}\\ A\sin\varphi_0 &= A_1\sin\varphi_{10} + A_2\sin\varphi_{20}\end{aligned}$$

代入可得

$$\begin{aligned}x &= \cos(\omega_0 t)[A_1\cos\varphi_{10} + A_2\cos\varphi_{20}] - \sin(\omega_0 t)[A_1\sin\varphi_{10} + A_2\sin\varphi_{20}]\\ &= A[\cos(\omega_0 t)\cos\varphi_0 - \sin(\omega_0 t)\sin\varphi_0]\\ &= A\cos(\omega_0 t + \varphi_0)\end{aligned}$$

  - 两个同方向同频率简谐振动的合运动仍然是一个简谐振动，而且合运动的频率和分振动的频率相同

$$\begin{aligned}x &= A_1\cos(\omega_0 t + \varphi_{10}) + A_2\cos(\omega_0 t + \varphi_{20})\\ &= A\cos(\omega_0 t + \varphi_0)\end{aligned}$$

- 三角函数的解析运算
  - 合振动的振幅

$$
\begin{aligned}
A\cos\varphi_0 &= A_1\cos\varphi_{10} + A_2\cos\varphi_{20} \\
A\sin\varphi_0 &= A_1\sin\varphi_{10} + A_2\sin\varphi_{20} \\
A &= \sqrt{(A\cos\varphi_0)^2 + (A\sin\varphi_0)^2} \\
&= \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\varphi)} \\
\Delta\varphi &= \varphi_{20} - \varphi_{10}
\end{aligned}
$$

$$
\begin{aligned}
A\cos\varphi_0 &= A_1\cos\varphi_{10} + A_2\cos\varphi_{20} \\
A\sin\varphi_0 &= A_1\sin\varphi_{10} + A_2\sin\varphi_{20} \\
(A\cos\varphi_0)^2 &= A_1^2\cos^2\varphi_{10} + A_2^2\cos\varphi_{20} + 2A_1A_2\cos\varphi_{10}\cos\varphi_{20} \\
(A\sin\varphi_0)^2 &= A_1^2\sin^2\varphi_{10} + A_2^2\sin\varphi_{20} + 2A_1A_2\sin\varphi_{10}\sin\varphi_{20} \\
A^2 &= A_1^2 + A_2^2 + 2A_1A_2(\cos\varphi_{10}\cos\varphi_{20} + \sin\varphi_{10}\sin\varphi_{20}) \\
&= A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\varphi) \\
A &= \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\varphi)}
\end{aligned}
$$

这里用到了三角函数的两角和公式

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos\varphi_{10}\cos\varphi_{20} + \sin\varphi_{10}\sin\varphi_{20} = \cos(\Delta\varphi)$$

$$\Delta\varphi = \varphi_{10} - \varphi_{20} \quad \text{or} \quad \Delta\varphi = \varphi_{20} - \varphi_{10}$$

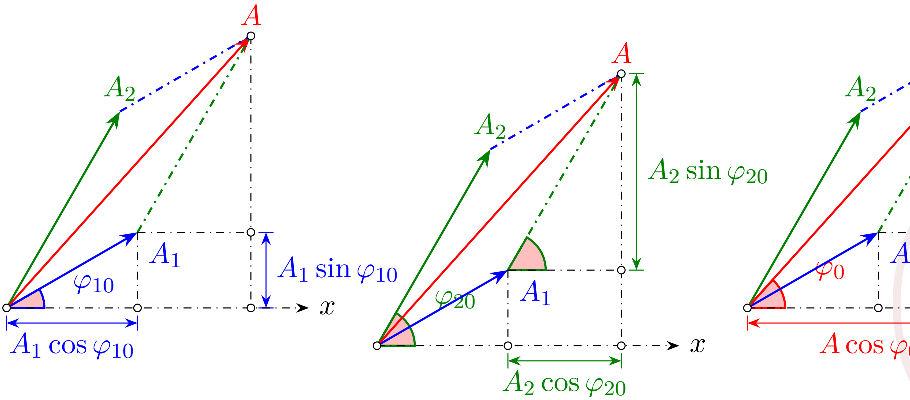
- 三角函数的解析运算
  - 合振动的初位相

$$
\begin{aligned}
A\cos\varphi_0 &= A_1\cos\varphi_{10} + A_2\cos\varphi_{20} \\
A\sin\varphi_0 &= A_1\sin\varphi_{10} + A_2\sin\varphi_{20} \\
\tan\varphi_0 &= \frac{A\sin\varphi_0}{A\cos\varphi_0} \\
&= \frac{A_1\sin\varphi_{10} + A_2\sin\varphi_{20}}{A_1\cos\varphi_{10} + A_2\cos\varphi_{20}} \\
\varphi_{01} &= \arctan\frac{A_1\sin\varphi_{10} + A_2\sin\varphi_{20}}{A_1\cos\varphi_{10} + A_2\cos\varphi_{20}} \\
\varphi_{02} &= \arctan\frac{A_1\sin\varphi_{10} + A_2\sin\varphi_{20}}{A_1\cos\varphi_{10} + A_2\cos\varphi_{20}} + \pi
\end{aligned}
$$

- 从 $\tan\varphi_0$ 的值在 $0 \to 2\pi$ 或 $-\pi \to \pi$ 范围内可以得到两个 φ_0，其中只有一个是正确的解，另一个要舍弃掉
- $\arctan$ 函数的取值范围在 $-\frac{\pi}{2} \to \frac{\pi}{2}$ 之间，所以要根据 $\sin\varphi_0$ 和 $\cos\varphi_0$ 的正负号来判断 φ_0 到底在哪个象限以决定具体取哪个解

$\sin\varphi_0$	$+$	$+$	$-$	$-$
$\cos\varphi_0$	$+$	$-$	$-$	$+$
象限	一	二	三	四
φ_0	φ_{01}	φ_{02}	φ_{02}	φ_{01}

- 旋转矢量法
  - 三角函数的解析运算求同方向同频率简谐振动合成的合振幅和初位相，原理比较清楚，但计算复杂
  - 旋转矢量法求同方向同频率简谐振动合成的合振幅和初位相，结果相同，但过程直观

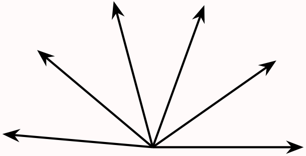


$$A\cos\varphi_0 = A_1\cos\varphi_{10} + A_2\cos\varphi_{20}$$

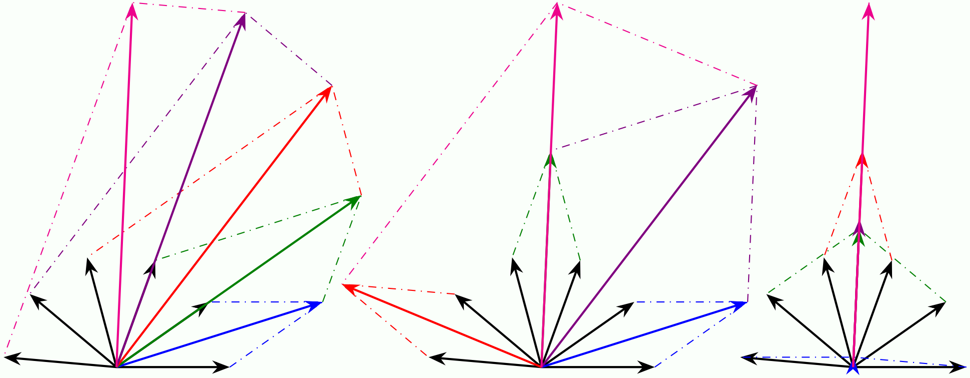
$$A\sin\varphi_0 = A_1\sin\varphi_{10} + A_2\sin\varphi_{20}$$

例题

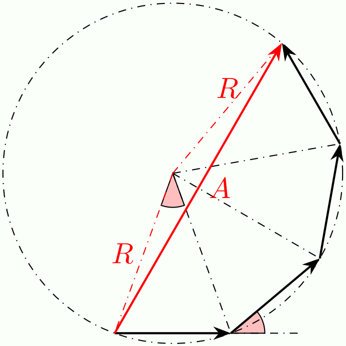
设有 N 个同方向、同频率的简谐振动，它们的振幅均为 a，初相分别 0、α、2α、3α、$\cdots$，依次相差 α 角 (设 $N\alpha < 2\pi$)，求它们的合振动。



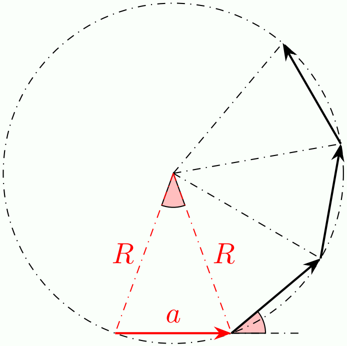
解答



解答



$$A = 2R\sin\frac{N\alpha}{2}$$



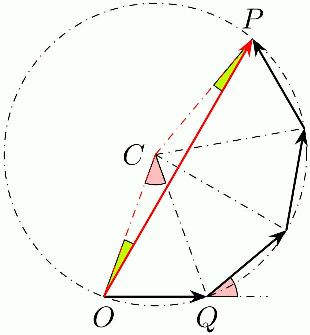
$$a = 2R\sin\frac{\alpha}{2}$$

$$A = 2R\sin\frac{N\alpha}{2}$$

$$a = 2R\sin\frac{\alpha}{2}$$

$$A = a\frac{\sin\frac{N\alpha}{2}}{\sin\frac{\alpha}{2}}$$

解答

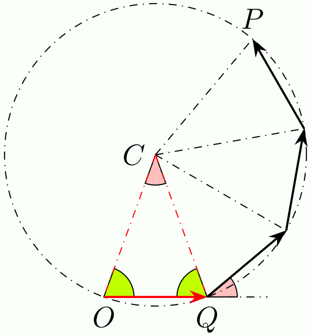


$$\angle OCP = N\alpha$$

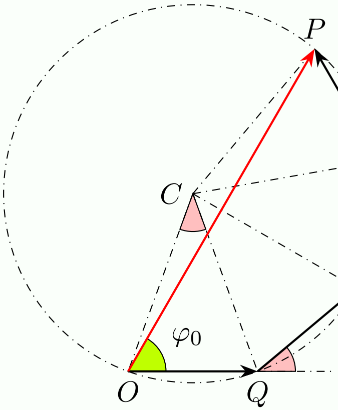
$$\angle COP = \angle CPO = \frac{1}{2}(\pi - \angle OCP) = \frac{\pi - N\alpha}{2}$$

$$\angle OCQ = \alpha$$

$$\angle COQ = \angle CQO = \frac{1}{2}(\pi - \angle OCQ) = \frac{\pi - \alpha}{2}$$



解答



$$\angle COP = \frac{\pi - N\alpha}{2}$$

$$\angle COQ = \frac{\pi - \alpha}{2}$$

$$\begin{aligned}\varphi_0 &= \angle POQ \\ &= \angle COQ - \angle COP \\ &= \frac{(N-1)\alpha}{2}\end{aligned}$$

二、同方向不同频率简谐振动的合成

- 同方向，假定两个简谐振动都沿 x 方向振动
- 不同频率，所以圆频率不同，假设两个简谐振动的圆频率分别为 ω_{10} 和 ω_{20}
- 两个一般的同方向不同频率的简谐振动的合成比较复杂

$$x_1 = A_1\cos(\omega_{10}t+\varphi_{10})$$
$$x_2 = A_2\cos(\omega_{20}t+\varphi_{20})$$
$$\begin{aligned} x &= x_1 + x_2 \\ &= A_1\cos(\omega_{10}t+\varphi_{10}) + A_2\cos(\omega_{20}t+\varphi_{20}) \end{aligned}$$

- 考虑一种特殊情况

$$A_1 = A_2 = A_0$$
$$\varphi_{10} = \varphi_{20} = \varphi_0$$
$$\omega_{10} = \omega_0 + \Delta\omega$$
$$\omega_{20} = \omega_0 - \Delta\omega$$
$$\omega_0 \gg \Delta\omega$$

即两个简谐振动的振幅相同、初位相相同，频率相差很小，则质点的合运动

$$\begin{aligned} x &= A_0\cos(\omega_{10}t+\varphi_0) + A_0\cos(\omega_{20}t+\varphi_0) \\ &= A_0[\cos(\omega_{10}t+\varphi_0) + \cos(\omega_{20}t+\varphi_0)] \end{aligned}$$

$$\alpha = \omega_{10}t + \varphi_0$$
$$\beta = \omega_{20}t + \varphi_0$$
$$\omega_{10} = \omega_0 + \Delta\omega$$
$$\omega_{20} = \omega_0 - \Delta\omega$$
$$\frac{\alpha+\beta}{2} = \frac{(\omega_{10}t+\varphi_0)+(\omega_{20}t+\varphi_0)}{2} = \frac{\omega_{10}+\omega_{20}}{2}t + \varphi_0 = \omega_0 t + \varphi_0$$
$$\frac{\alpha-\beta}{2} = \frac{(\omega_{10}t+\varphi_0)-(\omega_{20}t+\varphi_0)}{2} = \frac{\omega_{10}-\omega_{20}}{2}t = (\Delta\omega)t$$

三角函数的和差化积公式

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos(\omega_{10}t+\varphi_0) + \cos(\omega_{20}t+\varphi_0) = 2\cos(\omega_0 t+\varphi_0)\cos[(\Delta\omega)t]$$

- 质点的合运动

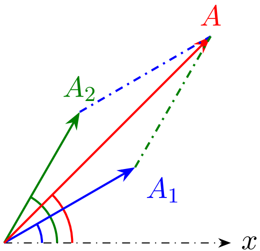
$$
\begin{aligned}
x &= A_0\cos(\omega_{10}t+\varphi_0)+A_0\cos(\omega_{20}t+\varphi_0) \\
&= A_0[\cos(\omega_{10}t+\varphi_0)+\cos(\omega_{20}t+\varphi_0)] \\
&= 2A_0\cos[(\Delta\omega)t]\cos(\omega_0 t+\varphi_0)
\end{aligned}
$$

- 由于 $\omega_0 \gg \Delta\omega$，所以 $\cos[(\Delta\omega)t]$ 变化的周期比 $\cos(\omega_0 t+\varphi_0)$ 大得多，即与 $\cos(\omega_0 t+\varphi_0)$ 相比较，$\cos[(\Delta\omega)t]$ 是一个变化缓慢的函数，所以可以将质点合运动看成一个振幅 $|2A_0\cos[(\Delta\omega)t]|$ 随时间变化的简谐振动
- 通常将因子 $2A_0\cos[(\Delta\omega)t]$ 称为合运动的振幅项【振幅是其绝对值】
- 因子 $\cos(\omega_0 t+\varphi_0)$ 称为合运动的简谐项，当振幅项为负时，简谐项的相位要加 π

- 旋转矢量

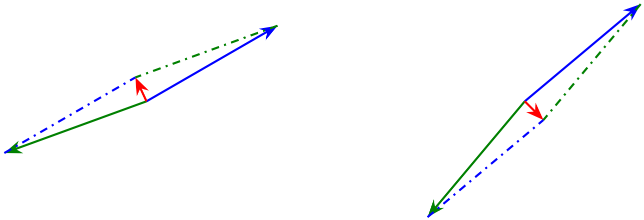
任意 t 时刻

$$A_1 = A_2 = A_0$$
$$\varphi_1 = \omega_{10}t + \varphi_0$$
$$\varphi_2 = \omega_{20}t + \varphi_0$$
$$\omega_{10} = \omega_0 + \Delta\omega$$
$$\omega_{20} = \omega_0 - \Delta\omega$$
$$\varphi = \frac{\varphi_1 + \varphi_2}{2} = \frac{\omega_{10} + \omega_{20}}{2}t + \varphi_0 = \omega_0 t + \varphi_0$$

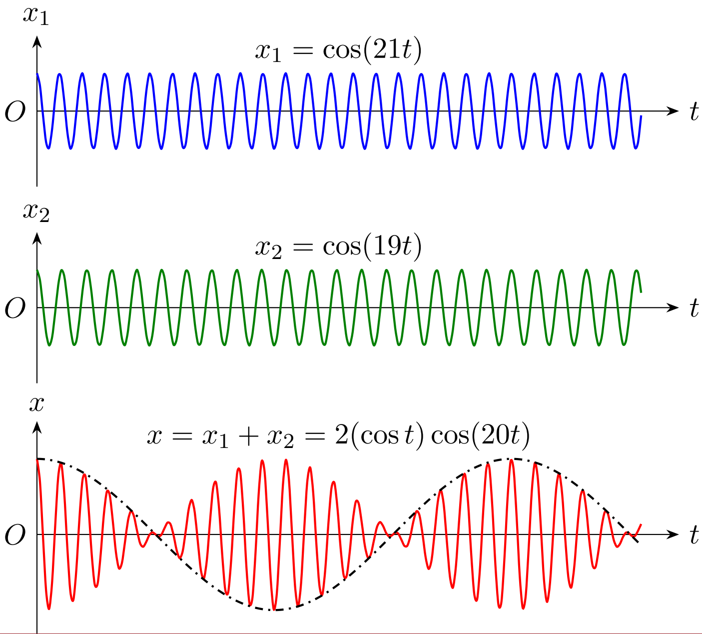


$$\begin{aligned} A &= \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)} \\ &= A_0\sqrt{2 + 2\cos(\varphi_2 - \varphi_1)} \\ &= 2A_0\sqrt{\cos^2\frac{\varphi_2 - \varphi_1}{2}} \\ &= 2A_0\left|\cos\frac{\varphi_2 - \varphi_1}{2}\right| \\ &= 2A_0\left|\cos\frac{(\omega_{20} - \omega_{10})t}{2}\right| \\ &= 2A_0|\cos[(-\Delta\omega)t]| \\ &= 2A_0|\cos[(\Delta\omega)t]| \end{aligned}$$

- 旋转矢量



由于 $\omega_{10} \neq \omega_{20}$，所以两个矢量之间的夹角慢慢增大，在夹角为 π 前后，合矢量突然反向，因此合振动的相位 φ 有一个 π 的突变



- 这种振幅随时间做周期性变化的现象称为“拍”

$$|2A_0 \cos[(\Delta\omega)t]|$$

- 当 $(\Delta\omega)t$ 变化 π 时，振幅变化完成了一个周期
- 拍的圆频率为 $2\Delta\omega = |\omega_{10} - \omega_{20}|$
- 振幅随时间变化的频率称为拍频

$$f = |f_1 - f_2|$$

三、互相垂直同频率简谐振动的合成

- 如果两个分振动分别沿 x 轴和 y 轴，但振动圆频率相同，均为 ω_0，即

$$
\begin{aligned}
x &= A_1\cos(\omega_0 t+\varphi_{10}) \\
y &= A_2\cos(\omega_0 t+\varphi_{20})
\end{aligned}
$$

质点在一个二维平面上运动，运动的轨迹方程可以由以上质点的运动方程消去时间参数而得到

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\frac{x}{A_1} = \cos(\omega_0 t + \varphi_{10}) = \cos(\omega_0 t)\cos\varphi_{10} - \sin(\omega_0 t)\sin\varphi_{10}$$

$$\frac{y}{A_2} = \cos(\omega_0 t + \varphi_{20}) = \cos(\omega_0 t)\cos\varphi_{20} - \sin(\omega_0 t)\sin\varphi_{20}$$

$$\frac{x}{A_1}\sin\varphi_{20} - \frac{y}{A_2}\sin\varphi_{10} = \cos(\omega_0 t)(\cos\varphi_{10}\sin\varphi_{20} - \sin\varphi_{10}\cos\varphi_{20}) = \cos(\omega_0 t)\sin(\varphi_{20}-\varphi_{10})$$

$$\frac{x}{A_1}\cos\varphi_{20} - \frac{y}{A_2}\cos\varphi_{10} = \sin(\omega_0 t)(\cos\varphi_{10}\sin\varphi_{20} - \sin\varphi_{10}\cos\varphi_{20}) = \sin(\omega_0 t)\sin(\varphi_{20}-\varphi_{10})$$

$$\begin{aligned}\sin^2(\varphi_{20}-\varphi_{10}) &= \left[\frac{x}{A_1}\sin\varphi_{20} - \frac{y}{A_2}\sin\varphi_{10}\right]^2 + \left[\frac{x}{A_1}\cos\varphi_{20} - \frac{y}{A_2}\cos\varphi_{10}\right]^2 \\ &= \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1A_2}(\sin\varphi_{20}\sin\varphi_{10} + \cos\varphi_{20}\cos\varphi_{10}) \\ &= \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1A_2}\cos(\varphi_{20}-\varphi_{10})\end{aligned}$$

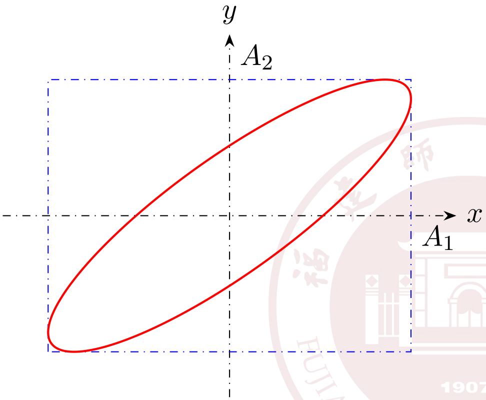
- 质点运动的轨迹方程

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\Delta\varphi_0)=\sin^2(\Delta\varphi_0)$$

$$\Delta\varphi_0=\varphi_{20}-\varphi_{10}$$

这是一个椭圆

$$Ax^2+By^2+Cxy+Dx+Ey+F=0$$



- 质点运动的轨迹方程

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\Delta\varphi_0)=\sin^2(\Delta\varphi_0)$$

- $\Delta\varphi_0=2n\pi$

$$\frac{x}{A_1}-\frac{y}{A_2}=0$$

$$y=\frac{A_2}{A_1}x$$

过原点的、在 1、3 象限上的直线段

- $\Delta\varphi_0=(2n+1)\pi$

$$\frac{x}{A_1}+\frac{y}{A_2}=0$$

$$y=-\frac{A_2}{A_1}x$$

过原点的、在 2、4 象限上的直线段

- $\Delta\varphi_0=\left(n+\frac{1}{2}\right)\pi$

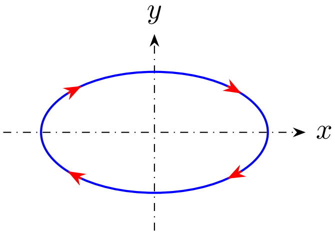
$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}=1$$

长短轴分别在 x、y 轴上的正椭圆

$$x = A_1 \cos(\omega_0 t + \varphi_{10}), y = A_2 \cos(\omega_0 t + \varphi_{20})$$

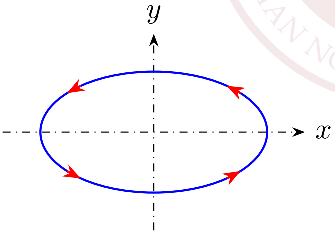
$$\Delta\varphi_0 = \varphi_{20} - \varphi_{10} = \frac{1}{2}\pi$$

$\omega_0 t + \varphi_{10}$	$\omega_0 t + \varphi_{20}$	(x, y)
0	$\frac{1}{2}\pi$	$(A_1, 0)$
$\frac{1}{2}\pi$	π	$(0, -A_2)$
π	$\frac{3}{2}\pi$	$(-A_1, 0)$
$\frac{3}{2}\pi$	2π	$(0, A_2)$
2π	$\frac{5}{2}\pi$	$(A_1, 0)$



$$\Delta\varphi_0 = \varphi_{20} - \varphi_{10} = -\frac{1}{2}\pi$$

$\omega_0 t + \varphi_{10}$	$\omega_0 t + \varphi_{20}$	(x, y)
0	$-\frac{1}{2}\pi$	$(A_1, 0)$
$\frac{1}{2}\pi$	0	$(0, A_2)$
π	$\frac{1}{2}\pi$	$(-A_1, 0)$
$\frac{3}{2}\pi$	π	$(0, -A_2)$
2π	$\frac{3}{2}\pi$	$(A_1, 0)$



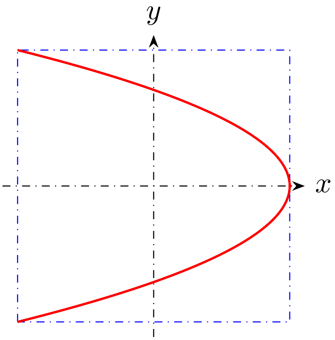
四、互相垂直不同频率简谐振动的合成 · 李萨如图形

- 设两个分别沿 x 轴和 y 轴振动的简谐振动分别为

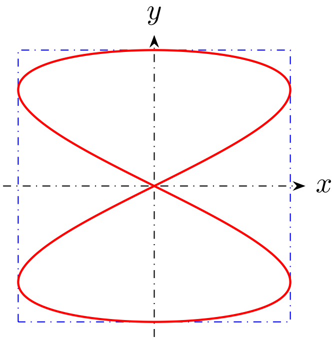
$$x = A_1 \cos(\omega_{10} t + \varphi_{10})$$
$$y = A_2 \cos(\omega_{20} t + \varphi_{20})$$

  则它们的合运动一般较复杂，运动轨迹也不一定是封闭曲线，即合运动不一定是周期性的运动

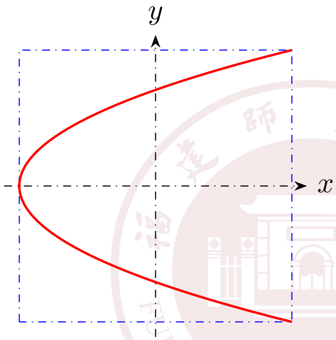
- 如果两个互相垂直的振动频率成整数比，则合运动的轨迹是封闭曲线，合运动也具有周期。这种运动轨迹的图形称为李萨如 (Lissajou) 图形



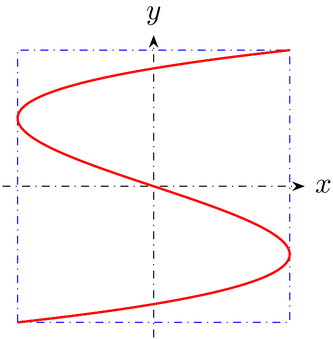
$$\frac{\omega_{20}}{\omega_{10}} = \frac{1}{2}$$
$$\varphi_{10} = 0$$
$$\varphi_{20} = -\frac{\pi}{2}$$



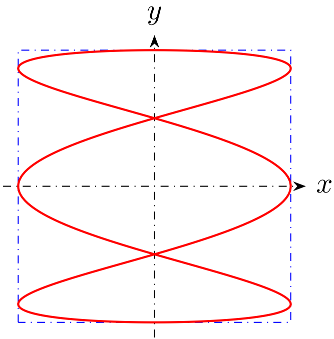
$$\frac{\omega_{20}}{\omega_{10}} = \frac{1}{2}$$
$$\varphi_{10} = -\frac{\pi}{2}$$
$$\varphi_{20} = -\frac{\pi}{2}$$



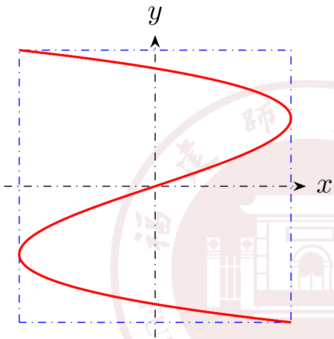
$$\frac{\omega_{20}}{\omega_{10}} = \frac{1}{2}$$
$$\varphi_{10} = 0$$
$$\varphi_{20} = 0$$



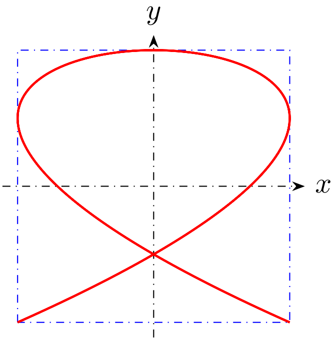
$$\frac{\omega_{20}}{\omega_{10}} = \frac{1}{3}$$
$$\varphi_{10} = 0$$
$$\varphi_{20} = 0$$



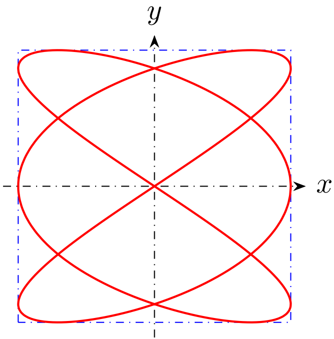
$$\frac{\omega_{20}}{\omega_{10}} = \frac{1}{3}$$
$$\varphi_{10} = 0$$
$$\varphi_{20} = -\frac{\pi}{2}$$



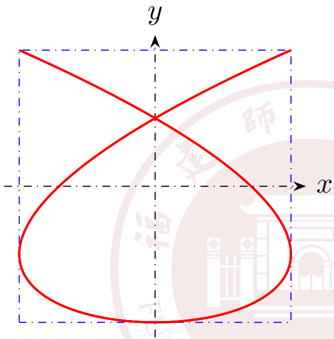
$$\frac{\omega_{20}}{\omega_{10}} = \frac{1}{3}$$
$$\varphi_{10} = -\frac{\pi}{2}$$
$$\varphi_{20} = -\frac{\pi}{2}$$



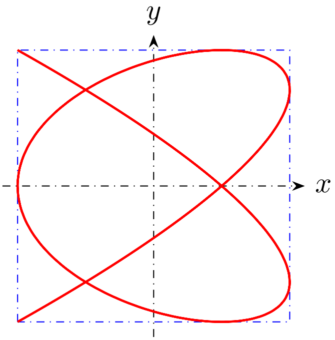
$$\frac{\omega_{20}}{\omega_{10}} = \frac{2}{3}$$
$$\varphi_{10} = \frac{\pi}{2}$$
$$\varphi_{20} = 0$$



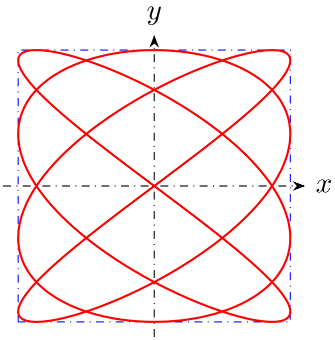
$$\frac{\omega_{20}}{\omega_{10}} = \frac{2}{3}$$
$$\varphi_{10} = 0$$
$$\varphi_{20} = -\frac{\pi}{2}$$



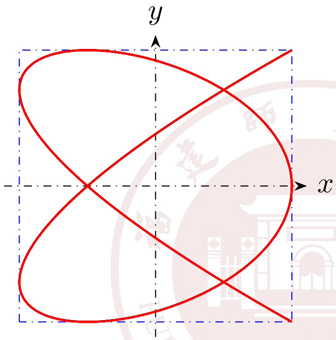
$$\frac{\omega_{20}}{\omega_{10}} = \frac{2}{3}$$
$$\varphi_{10} = 0$$
$$\varphi_{20} = 0$$



$$\frac{\omega_{20}}{\omega_{10}} = \frac{3}{4}$$
$$\varphi_{10} = \pi$$
$$\varphi_{20} = 0$$



$$\frac{\omega_{20}}{\omega_{10}} = \frac{3}{4}$$
$$\varphi_{10} = \frac{\pi}{2}$$
$$\varphi_{20} = -\frac{\pi}{2}$$



$$\frac{\omega_{20}}{\omega_{10}} = \frac{3}{4}$$
$$\varphi_{10} = 0$$
$$\varphi_{20} = 0$$

习题 9.4.1

在电子示波器中，由于互相垂直的电场的作用，使电子在荧光屏上的位移为

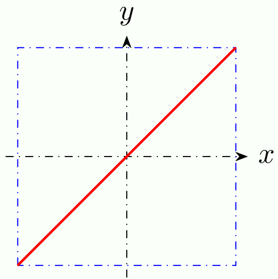
$$x = A\cos(\omega t)$$
$$y = A\cos(\omega t + \alpha)$$

求出 $\alpha = 0, \frac{\pi}{3}, \frac{\pi}{2}$ 时的轨迹方程并画图表示。

解答

$\alpha = 0$

$$x = A\cos(\omega t)$$
$$y = A\cos(\omega t)$$
$$y = x$$



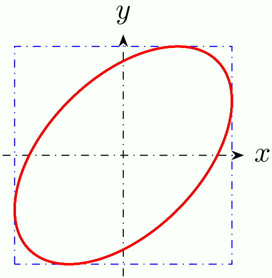
解答

$\alpha = \frac{\pi}{3}$

$$
\begin{aligned}
x &= A\cos(\omega t) \\
y &= A\cos\left(\omega t + \frac{\pi}{3}\right) \\
&= A\left[\cos(\omega t)\cos\frac{\pi}{3} - \sin(\omega t)\sin\frac{\pi}{3}\right] \\
&= \frac{1}{2}A\cos(\omega t) - \frac{\sqrt{3}}{2}A\sin(\omega t) \\
&= \frac{1}{2}x - \frac{\sqrt{3}}{2}A\sin(\omega t) \\
A\sin(\omega t) &= \frac{2}{\sqrt{3}}\left(y - \frac{1}{2}x\right)
\end{aligned}
$$

$$
A^2 = x^2 + \frac{4}{3}\left(y^2 - xy + \frac{x^2}{4}\right)
$$

$$
x^2 + y^2 - xy = \frac{3}{4}A^2
$$



解答

$\alpha = \frac{\pi}{2}$

$$x = A\cos(\omega t)$$

$$y = A\cos\left(\omega t + \frac{\pi}{2}\right) = -A\sin(\omega t)$$

$$x^2 + y^2 = A^2$$