Ratio and Proportion

- Course Overview

Presented By

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About me

- M.Tech. from IIT Kharagpur.
- Over 18 years of Corporate experience.
- Teaching from 10+ years
- Core Areas : Mathematics, Aptitude, English.
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Maths is Easy

Introduction

Ratio and proportion is very easy but important topic.

This is quite useful for various competitive exams. You can it as an all-rounder as it's utility is vast; whether its arithmetic or algebra, Geometry or Trigonometry, Mensuration or even data interpretation; Its required to solve problems.

So it would be a very good value add, if Ratio and proportion can be mastered.

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Objective

- Students shall be able to develop strong hold and solve problems in various areas of quantitative aptitude.
- Easily understand the core concepts and methods to solve problems with ease.
- Its an easy topic so it can be scored heavily and not get successful in competitive exams but get top rank.

Maths is Easy

Course Plan

This course is planned as follows:

- What is ratio and proportion and how they are related?
- Core concepts explanation with examples and its usage.
- Equivalent ratio meaning and usage.
- Solve various ratio Problems with ease.
- Proportion meaning and its concept.
- Various types of proportion

Course Plan

- Componendo, Dividendo, Alternendo, Componendo-Dividendo
- Solving algebraic equations with Ratios
- Difficult problems involving multiple ratios.
- Concept of partnership and its usage.
- Partner's share and profit.
- Solving Complex problems on partnership

Maths is Easy

Course Plan

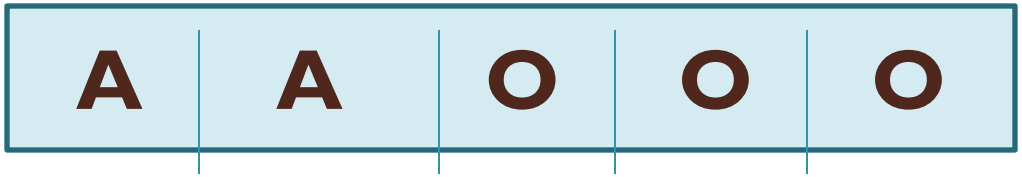
- Partnership problems when some partners joins late or leave in between.
- Miscellaneous problems involving ratio, proportion, partnership.
- Chain Rule
- Quiz for practice with speed.
- Strategies for super-fast solutions.
- Commonly asked question types.
- Previous years Questions and solutions with easy method.

Ratio and Proportion : Concept

What is the significance and meaning of RATIO

Ratio specifies relationship between two quantities or numbers.

Say Apple and Oranges are in ratio of 2:3 in a box means :



Ratio is denoted by :

Ratio can be written as fraction → 2/3

If ratio is x : y it means x / y

Ratio : Explanation

Suppose if we say there are 100 students in a class and ratio of Boys and Girls is 2:3 resp.

Need to find : How many Boys ? How many Girls ?

Explanation :

Ratio 2:3 means → If there are 5 students in class, Boy's share would be 2/5 and

Girl's share would be 3/5

Ratio : Example

Suppose if we say there are 100 students in a class and ratio of Boys and Girls is 2:3 resp.

Need to find : How many Boys ? How many Girls ?

Explanation :

So If there are 100 students in class :

Boy's share = (2/5) X 100 = 40

Girl's share = (3/5) X 100 = 60

Cross check : 40 + 60 = 100

40 : 60 = 2 : 3

Ratio : Method to Solve problems

Suppose if Rs. 5000 needs to be shared among A, B and C in ratio 2: 3 : 5 respectively.

How to do ?

Given A:B:C is 2 : 3 : 5 Total parts = 2 + 3 + 5 = 10

A's share : 2/10, B's share : 3/10, C's share : 5/10

Hence 5000 would be divided among A,B,C as

A shall get (2/10) x 5000 = 1000

B shall get (3/10) x 5000 = 1500

C shall get (5/10) x 5000 = 2500

Concept 2 : Equivalent Ratio

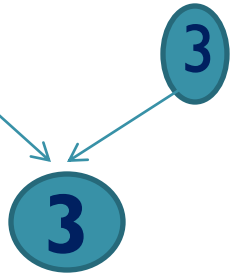
Suppose A: B is 2:3 and B:C is 3 : 5 then

What is A:B:C ? ← Equivalent Ratios

How to do ?

A : B B : C

2 : 3 3 : 5



A : B : C = 2 : 3 : 5

Equivalent Ratio : Solving Method

Suppose A: B is 2:3 and B:C is 4 : 5 then

What is A:B:C ? How to do ?

A	:	B		B	:	C
2	:	3		4	:	5
		x 4		x 3		
8	:	12		12	:	15

LCM = 12

A : B : C

8 : 12 : 15

Proportion meaning

What is Proportion and is it related with Ratio ?

Let's see.

- When 2 ratios are equal they are in proportion.

Example If A : B and C : D are equal then

A, B, C, D are said to be in proportion.

- Proportion is denoted by ::

- So A : B :: C : D means

Ratio A : B is proportional to ratio C : D

Proportion insights

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Say if A, B, C and D are in proportion

- A : B = C : D
- A / B = C / D
- Rule of proportion :

A : B :: C : D

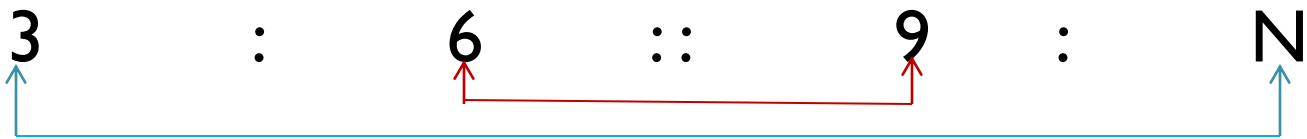
Product of Extremes = Product of means

A x D = B x C

Proportion Problem solving

Say if numbers 3, 6, 9, and another number are in proportion, find fourth proportion.

- Let 4th number is N
- Using Rule of proportion :



Product of Extremes = Product of means

3 x N = 6 x 9

So N = 6 x 9 / 3 ***Fourth number = N is 18***

Concept : Third Proportion

Q : What is third proportion of 5 and 10 ?

- If A : B = B : C then C is called Third proportion.
- Method : Using Rule of third proportion :

A : B :: B : C

5 x C = 10 x 10

So C = 10 x 10 / 5

Third proportion = C is 20

Concept : Mean Proportion

Q : What is mean proportion ?

Concept : Mean proportion between A and B is given by sqrt (AB)

Q : Find mean proportion of 25 and 36

mean proportion = square root of (25 x 36)

mean proportion = 5 x 6 = 30

Q : Find mean proportion of 0.49 and 0.81

mean proportion = square root of (0.49 x 0.81)

mean proportion = 0.7 x 0.9 = 0.63

Ratio with Percentage

Q: Two numbers are respectively 30% and 50% of third number. What is the ratio of the two numbers?

Easy Method: Let third number is x

Then first number is 30% of x means $0.3x$

And second number is 50% of x means $0.5x$

Ratio of 1st and 2nd number is $0.3x : 0.5x$

Just Simplify → $0.3\cancel{x} : 0.5\cancel{x}$ → $0.3 : 0.5$

Remove the decimals by multiplying with 10 → 3:5

Answer : 3:5

Ratio with mixtures

Q: The proportion of milk and water in 3 samples is 2:1, 4: 3 and 5:4. A mixture comprising of equal quantities of all 3 samples is made. What would be proportion of milk and water in the mixture?

Concept and Method: If two quantities A and B are in ratio x : y it means share of A is x/(x+y) and share of B is y/(x+y). Right?

First Sample		Second Sample		Third Sample	
Milk	Water	Milk	Water	Milk	Water
2/3	*1/3*	*4/7*	*3/7*	*5/9*	*4/9*

In Mixture, amount of milk and water would be sum of quantity in each sample. Right??

Ratio with mixtures

In Mixture, amount of milk and water would be sum of quantity in each sample. Right??

So in mixture →

Quantity of Milk would be 2/3 + 4/7 + 5/9

➔ Solve by taking LCM, you will get 113/ 63

Quantity of Water would be 1/3 + 3/7 + 4/9

➔ Solve by taking LCM, you will get 76/ 63

Hence ratio of Milk and Water mixture would be

113 / 63 : 76 / 63

On simplifying, required ratio is → 113 : 76

Answer : Milk: Water is 113 : 76

Ratio : with division among group

Q : If Rs. 1066 is divided among A, B, C and D such that A:B =3:4, B:C = 5:6, and C: D = 7:5, who will get the minimum ?

Concept : Suppose X :Y : Z = 2 : 3 : 4 then we can easily know that value of X would be minimum and value of Z would be maximum.

Method : So if we can find equivalent ratio of A:B:C:D then we can find who will get minimum. Right?

Solution : Given the ratios A : B , B : C, C : D

Need to find equivalent ratio A : B : C : D

So if we can find equivalent ratio then irrespective of Total amount, finding minimum is possible.

Ratio : with division among group

Q : If Rs. 1066 is divided among A, B, C and D such that A:B =3:4, B:C = 5:6, and C: D = 7:5, who will get the minimum ?

Given the ratios

A : B	B : C	C : D
3 : 4	5 : 6	7 : 5

First we will find A:B:C which would be A : B : C = 15 : 20 : 24

Now find A : B : C : D from A : B : C and C : D

15 : 20 : 24 7 : 5

Hence A : B : C : D = 105 : 140 : 168 : 120

Clearly A would be minimum as A has smallest part of ratio.

Ratio : with problem on ages

Q : The difference between present age of Rohan and Sumit is 14 years. Seven years ago, the ratio of their ages was 5 :7 respectively. What is Sumit's present age ?

Concept: Since ratio is given this problem can be solved using 1 variable only.

Method : Let x is that variable

Suppose,

Age 7 years ago ➔ Rohan = 5x and Sumit = 7x

Then

Present Age would be Rohan = 5x + 7 and Sumit = 7x + 7

Ratio : with problem on ages

Now difference of present age is given as 14 years.

Hence, Present Age of Sumit - Present Age of Rohan = 14

- ➔ $(7x + 7) – (5x + 7) = 14$
- ➔ $7x – 5x + 7 - 7 = 14$
- ➔ $2x = 14$
- ➔ $x = 7$

Hence

Sumit's present age = $7x + 7$ (On substituting value of x)

= $7 * 7 + 7$

Sumit's present age = 56 Years

Ratio : with problem on numbers

Q : When 1 is added to each of the two given numbers their ratio becomes 3 : 4 and when 5 is subtracted from each, the ratio becomes 7 : 10. Find the numbers.

Concept: Since 2 scenarios and ratios are given two equations can be formed and solved.

Method : Let x be first number and y be second number.

As per first statement,

$(x + 1) : (y + 1) = 3 : 4$ (equation 1)

As per second statement,

$(x - 5) : (y - 5) = 7 : 10$ (equation 2)

Ratio : with problem on numbers

Q : When 1 is added to each of the two given numbers their ratio becomes 3 : 4 and when 5 is subtracted from each, the ratio becomes 7 : 10. Find the numbers.

Cross multiplying equation 1, $4(x + 1) = 3(y + 1)$

$\rightarrow 4x - 3y = -1$ ….. equation 3

Cross multiplying equation 2, $10(x - 5) = 7(y - 5)$

$\rightarrow 10x - 7y = 15$ ….. equation 4

Solving equation 3 and equation 4

We get, $x = 26$ and $y = 30$

So the required numbers are 26 and 30

What is proportion ?

Proportion = Equality of two ratios

Example:

Let's say ratio A : B = C : D

Then it means A, B, C and D are in proportion.

Or simply speaking

Product of extremes = Product of means

$$A \times D = B \times C$$

Usage of proportion

Let's understand its usage with an example

Example:

If 25 pens costs Rs. 450 then how much will 85 pens cost ?

This problem can be solved by various methods like

- Unitary method
- Equations
- Proportions

We will use proportion method. See how simple it is.

Usage of proportion

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Method of proportion

Let's say P denotes Pens and C denotes Cost.

P1 : C1 :: P2 : C2

Points to Note : Here order is very important.

If relationship is directly proportional or inversely proportional.

Here as number of pens increases ⬆ cost will also increase ⬆

hence relationship is directly proportional.

On substituting, we get ➔ 25 : 450 :: 85 : C2

Using rule of proportion ➔ Product of extremes = Product of means

➔ 25 x C2 = 450 x 85

➔ C2 = 450 x 85 / 25 = Rs 1530 ***85 pens will cost Rs 1530***

Usage of proportion – example 2

Let's take same example to understand importance of order.

Q : If 25 pens costs Rs. 450 then how many pens will cost Rs. 1584 ?

Again say P denotes Pens and C denotes Cost.

P1 : C1 :: P2 : C2

Points to Note : Carefully note the order here.

On substituting, we get ➔ 25 : 450 :: **P2** : 1584

Using rule of proportion ➔ Product of extremes = Product of means

➔ 450 x P2 = 25 x 1584

➔ P2 = 25 x 1584 / 450

➔ = 88 *In Rs 1585, you can get 88 pens.*

Properties of proportion

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Inverse:

If $a / b = c / d$

then $b / a = d / c$

Example:

Say $\frac{9a}{4b} = \frac{5}{7}$ *then* $\frac{4b}{9a} = \frac{7}{5}$

By property of inverse

Properties of proportion

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Alternendo:

If $a / b = c / d$

then $a / c = b / d$

Example:

Say $\frac{9a}{4b} = \frac{5}{7}$ *then* $\frac{9a}{5} = \frac{4b}{7}$

By property of alternendo

Properties of proportion

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Componendo:

If $a / b = c / d$

then $(a+b) / b = (c+d) / d$

Example:

Say $\frac{9a}{4b} = \frac{5}{7}$ *then* $\frac{9a + 4b}{4b} = \frac{5+7}{7}$

By property of componendo

Properties of proportion

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Dividendo:

If $a / b = c / d$

then $(a- b) / b = (c- d) / d$

Example:

Say $\frac{9a}{4b} = \frac{5}{7}$ *then* $\frac{9a - 4b}{4b} = \frac{5 - 7}{7}$

By property of dividendo

Properties of proportion

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Componendo - Dividendo:

If $a / b = c / d$

then $(a + b) / (a - b) = (c + d) / (c - d)$

Example:

Say $\frac{9a}{4b} = \frac{5}{7}$ *then* $\frac{9a + 4b}{9a - 4b} = \frac{5 + 7}{5 - 7}$

By property of componendo - dividendo

Properties of proportion

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Solving problem using Componendo – Dividendo

If $x : y = 3 : 4$ then, find $(2x + 3y) / (2x - 3y)$

Method:

Given $\frac{x}{y} = \frac{3}{4}$ *then* $\frac{2x}{3y} = \frac{6}{12}$ *(Multiplied Numerator by 2)*

(Multiplied Denominator by 3)

By using property of componendo – dividendo

Given $\frac{2x + 3y}{2x - 3y} = \frac{1 + 2}{1 - 2} = \frac{3}{-1} = -3$

What is partnership ?

When two or more persons run business jointly ➔ PARTNERSHIP

- **Simple partnership :** Capitals of all partners are invested for same time period.
- **Compound partnership :** Capitals of the partners are invested for different time periods.

- **Working Partner :** Who manages business.
- **Sleeping Partner :** Who simply invests the money.

Rules of partnership

Basic rules of PARTNERSHIP

Unless otherwise mentioned specifically,

- **Sharing of profit :** Always be as per ratio of equivalent Capitals of all partners for same time period.
- **Bearing of loss:** Always be as per ratio of equivalent Capitals of all partners for same time period.

Core concept : *Equivalent ratio of Capitals*

Partnerships examples

Let's understand PARTNERSHIP with an example:

Q : Three partners A, B, C started a business by investing Rs 50000, Rs 80000 and Rs 70000 respectively. Find the share of profit of each partner if annual profit was Rs 15000.

Concept : Profit must be divided as per ratio of capitals.

Solution:

Ratio of capitals of A : B : C = 50000 : 80000 : 70000

➔ Equivalent ratio = 5 : 8 : 7

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Partnerships examples

Ratio of capitals of A : B : C = 5 : 8 : 7

Profit must also be shared in the ratio 5 : 8 : 7

Using the method to find individual's share from ratio :

A's share of profit = (5/20) x 15000 = Rs 3750

B's share of profit = (8/20) x 15000 = Rs 6000

C's share of profit = (7/20) x 15000 = Rs 5250

Cross check ➔ Total profit = share of profit of each

= 3750 + 6000 + 5250 = Rs 15000

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Partnership : When investment changes in between

Let's take an example when investment changes.

Q: A, B , C started a business with each partner invested Rs 40000. After 5 months, A withdrew Rs 10000, B withdrew Rs 8000 and C invests Rs 12000 more. Total profit of Rs 55920 was recorded at the end of the year. How the profit would be shared?

Concept : Profit must be divided as per ratio of capitals.

Solution: Note that investment was changed in between period.

Equivalent Ratio of capitals of A : B : C needs to be find out by equating the periods because investment period and amount is different.

Let's see how to do this !!!

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Partnership : When investment changes in between

Equivalent Ratio of capitals of A : B : C needs to be find out by equating the periods because investment period and amount is different.

	Investment period & Invested amount	Equivalent Amount
A	*Rs 40000 for 5 months and*	= *40000 x 5 + 30000 x 7*
	Rs 30000 for 7 months	= ***Rs 410000***
B	*Rs 40000 for 5 months and*	= *40000 x 5 + 32000 x 7*
	Rs 32000 for 7 months	= ***Rs 424000***
C	*Rs 40000 for 5 months and*	= *40000 x 5 + 52000 x 7*
	Rs 52000 for 7 months	= ***Rs 564000***

Equivalent ratio of capitals → A:B:C = 410000:424000:564000

= 410 : 424 : 564 = 205 : 212 : 282

Partnership : When investment changes in between

Equivalent ratio of capitals ➔ A:B:C = **205 : 212 : 282**

Concept : Profit must be divided as per ratio of capitals.

A's share of profit = (205 / 699) x 55920 = Rs 16400

B's share of profit = (212 / 699) x 55920 = Rs 16960

C's share of profit = (282 / 699) x 55920 = Rs 22560

Cross check ➔ Total profit = share of profit of each

= 16400 + 16960 + 22560 = Rs 55920

Important point : Equivalent ratio of capitals is MUST

Partnership : When new investor joins in between

Let's take an example

Q: Rakesh invested Rs 76000 in a business. After few months Pooja joined him with Rs 57000. Total profit at the end of year was divided in ratio 2:1 between them. After how many months did Pooja join ?

Concept : Profit must be divided as per ratio of capitals.

Solution: Let's say Pooja joined after T months.

Rakesh's investment period is 1 year.

Pooja's investment period is different from Rakesh's investment period.

Equivalent Ratio of capitals of A : B : C needs to be find out because investment period and amount is different.

Let's see how to do this !!!

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Partnership : When new investor joins in between

Rakesh's investment period = 1 year i.e. 12 months

Pooja's investment period = (12 – T) months

Since Profit must be divided as per ratio of capitals.

Profit Ratio is 2 : 1 hence investment ratio would also be 2 :1

Investment amount of Rakesh = 76000 x 12

Investment amount of Pooja = 57000 x (12 – T)

Hence,

$$\frac{76000 \times 12}{57000 \times (12 - T)} = \frac{2}{1}$$

Solving for T, we get T = 4

Hence Pooja joined after 4 months.

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What is partnership ?

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Rules of partnership

Basic rules of PARTNERSHIP

Unless otherwise mentioned specifically,

- **Sharing of profit :** Always be as per ratio of equivalent Capitals of all partners for same time period.
- **Bearing of loss:** Always be as per ratio of equivalent Capitals of all partners for same time period.

Core concept : *Equivalent ratio of Capitals*

Partnerships problem on sharing rent

Q : Abha, Babita and Chinu took a house on rent for 1 year for Rs 13824. They remained together for 4 months and then Chinu left the house. After 5 more months, Babita also left the house. How much rent should each pay?

Concept : Rent must be divided as per ratio of stay.

Yearly rent = 13824 so monthly rent ➔ 13824/12 = **Rs 1152**

Method : First 4 month all stayed together so rent for first 4 months shall be divided equally.

Rent for first 4 months = 1152 x 4 = Rs 4608

Share of each ➔ 4608 /3 = **Rs 1536**

Partnerships problem on sharing rent

- Abha, Babita and Chinu stayed for first 4 months.

Share of each ➔ 4608 /3 = **Rs 1536**

Rent for next 5 months = 1152 x 5 = Rs 5760

- Only Abha and Babita stayed for next 5 months.

Share of Abha and Babita each ➔ 5760 /2 = **Rs 2880**

Rent for last 3 months = 1152 x 3 = Rs 3456

- Only Abha stayed for last 3 months. Abha to pay **Rs 2880**

Total rent paid by Abha = Rs (1536 + 2880 + 3456) = **Rs 7872**

Total rent paid by Babita = Rs (1536 + 2880) = **Rs 4416**

Total rent paid by Chinu = **Rs 1536**

Partnerships problem with remuneration

Q : A and B invest Rs 3000 and Rs 4000 in a business. A receives Rs10 per month out of profit as a remuneration for running the business and rest of profit is divided in proportion to the investments. If in a year A receives totally Rs 390, what does B receives ?

Solution :

Note here that remuneration is mentioned out of profit.

Hence Balance profit = Total profit – remuneration

Now ***Balance Profit*** must be divided as per ratio of capitals.

Balance profit of A = 390 – 12 x 10

➔ Balance profit of A = Rs 270

Partnerships problem with remuneration

Now ***Balance Profit*** must be divided as per ratio of capitals.

➔ Balance profit of A = Rs 270

$$\frac{\text{Balance profit of A}}{\text{Balance profit of B}} = \frac{\text{Investment of A}}{\text{Investment of B}}$$

Substituting,

$$\frac{270}{\text{Balance profit of B}} = \frac{3000}{4000}$$

Hence, Balance profit of B = 270 x 4 / 3

Balance profit of B = Rs 360

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Partnerships problem with combined capital

Q : A, B and C are partner in business. B's capital is 1/6 of the total and A's is equal to that of B and C together. How much does C receive out of a total profit of Rs 2400 ?

Here the capitals are given in A's, B's, C's and total's reference.

Let's say total investment is T

Investment is given as :

A = B + C and B = 1/6 of T

This can be solved by forming equation and solving.

Let's see how.

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Partnerships problem with combined capital

Investment is given as :

A = B + C (equation 1) B = 1/6 of T (equation 2)

Inherently we know that A + B + C = T (equation 3)

Substituting (equation 1) into (equation 3),

We get, 2(B+C) = T (equation 4)

Substituting (equation 2) into (equation 4),

We get, 2(T/6 +C) = T

Solving we get, C = T/3

Now profit sharing would be proportional to investment ratio.

Hence profit of C = Total profit / 3

= 2400 / 3 = Rs 800 **C will get Rs 800**

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- ❑ Art of speed math
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- ❑ Ratio, proportion and partnership simplified
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