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Give proof by induction for Theorem 3: If $\boldsymbol{T(n)}$ is a polynomial of degree $\boldsymbol{x}$, then $\boldsymbol{T(n) = O(n^x)}$.

$T(n)$ is a polynomial of degree x,
$T(n) = a_x n^x + a_{x-1} n^{x-1} + \cdots + a_1 n + a_0$.

Given the definition of asymptotic notation, $f(n) \in O\big(g(n)\big)$, There exist $c > 0$ and $n_0 > 0$ such that, $(\forall n > n_0)\big(0 \le f(n) \le cg(n)\big)$.

<u>Base Case</u>

$n = 1$, evaluate $T(1)$:
$T(1) = a_x 1^x + a_{x-1} 1^{x-1} + \cdots + a_1 1 + a_0$
Considering this is a sum of constants, we can assume there is a constant C such that:
$T(1) \le C(1^x)$ thus, the base case is true.

<u>Inductive Hypothesis</u>

We assume true for $n = k$, and therefore have our inductive hypothesis as the following:

$T(k) \le Ck^x$.

Our goal now is to show that $n = k + 1$, holds true for all n. Given our induction hypothesis, we can say that,

$T(k + 1) \le C(k + 1)^x$,

And since lower-order terms grow slower than k^x, we have proven that for any n,
$T(n) = O(n^x)$.