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COSC 320 --- 001 Homework 2 2/20/2025

**Use the master method to give tight asymptotic bound for the following recurrences. Please give detailed steps and reasoning.**

**1. T(n)=2T(n/4)+1**

Using the formula for standard recurrence: $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, we have that:

$$f(n) = 1 \text{ , and } a = 2, b = 4$$

Therefore, comparing $f(n)$ to $n^{log_b a}$,

$$n^{log_b a} = n^{log_4 2} = n^{\frac{1}{2}} = \sqrt{n}$$

Considering that $f(n) = 1$ grows slower than $\sqrt{n}$, we should use Case 1 of Master Theorem:

$$T(n) = \theta(n^{log_b a}) = \theta(n^{log_4 2}) = \theta(\sqrt{n})$$
$$T(n) = \theta(\sqrt{n})$$

**2. T(n)= 2T(n/4)+$\sqrt{n}$**

Using the formula for standard recurrence: $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, we have that:

$$f(n) = \sqrt{n} \text{ , and } a = 2, b = 4$$

Therefore, comparing $f(n)$ to $n^{log_b a}$,

$$n^{log_b a} = n^{log_4 2} = n^{\frac{1}{2}} = \sqrt{n}$$

Considering that $f(n) = \sqrt{n}$ are asymptotically equal to $\sqrt{n}$, we should use Case 2 of Master Theorem:

$$T(n) = \theta(n^{log_b a}\lg(n)) = \theta(n^{log_4 2}\lg(n)) = \theta(\sqrt{n}(\lg(n)))$$
$$T(n) = \theta(\sqrt{n}(\lg(n)))$$

**3. T(n)= 2T(n/4)+n**

Using the formula for standard recurrence: $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, we have that:

$$f(n) = n \text{ , and } a = 2, b = 4$$

Therefore, comparing $f(n)$ to $n^{log_b a}$,

$$n^{log_b a} = n^{log_4 2} = n^{\frac{1}{2}} = \sqrt{n}$$

Considering that $f(n) = n$ grows faster than $\sqrt{n}$, we should validate whether $f(n)$ satisfies the condition for Case 3:

$$f(n) = \Omega(n^{log_b a+\varepsilon}) = \Omega(n^{\frac{1}{2}+\varepsilon})$$

Which satisfies the condition for some $\mathcal{E} > 0$ where $\mathcal{E} = \frac{1}{4}$ therefore, we use Case 3 of Master Theorem:

$$T(n) = \theta(f(n)) = \theta(n)$$
$$T(n) = \theta(n)$$

**4. T(n)= 2T(n/4)+n2**

Using the formula for standard recurrence: $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, we have that:

$$f(n) = n^2 \text{ , and } a = 2, b = 4$$

Therefore, comparing $f(n)$ to $n^{log_b a}$,

$$n^{log_b a} = n^{log_4 2} = n^{\frac{1}{2}} = \sqrt{n}$$

Considering that $f(n) = n^2$ grows faster than $\sqrt{n}$, we should validate whether $f(n)$ satisfies the condition for Case 3:

$$f(n) = \Omega(n^{log_b a + \mathcal{E}}) = \Omega(n^{\frac{1}{2}+\mathcal{E}})$$

Which satisfies the condition for some $\mathcal{E} > 0$ where $\mathcal{E} = 1$ therefore, we use Case 3 of Master Theorem:

$$T(n) = \theta(f(n)) = \theta(n^2)$$
$$T(n) = \theta(n^2)$$