Week 12

Things to Note ...

- Pre-exam consultation Fri 3 Nov, 2-4pm

In This Lecture ...

- Text processing algorithms
  - Pattern matching (slides)
  - Tries ([S] Ch.15.2)
  - Text compression (slides)

Coming Up ...

- Exams, Holidays ...

Nerds You Should Know

Our last in a series on famous computer scientists ...

She developed the first compiler ... and bugs!

Nerds You Should Know (cont)

Grace Hopper

- PhD in Mathematics from Yale (1934)
- Professor of Maths at Vassar (1930's)
- Joined U.S. Navy (research) (1943)
- coined the term "bug" (1940's, apocryphal)
- developed the **first compiler** (1950's)
- designer of the COBOL language (1950's)
- worked for Sperry/Rand/Univac (1960's)
- promoted to Admiral in US Navy (1980's)
- "retired" 1971 ... still consulting until 1992

Nerds You Should Know (cont)

The original "bug" ...

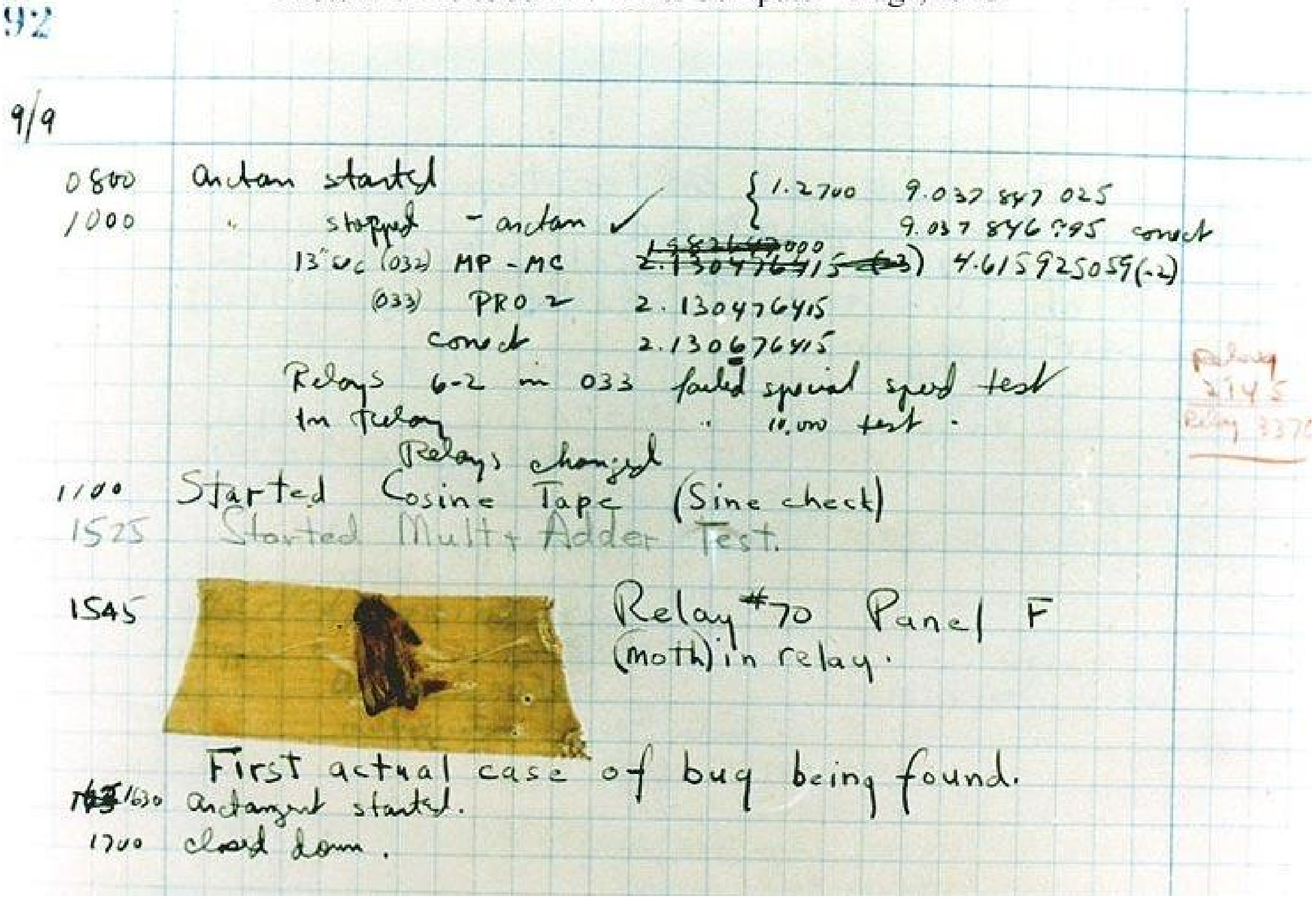


Photo # NH 96566-KN First Computer "Bug", 1945

Strings

Strings

A string is a sequence of characters.

An alphabet Σ is the set of possible characters in strings.

Examples of strings:

- C program
- HTML document
- DNA sequence
- Digitized image

Examples of alphabets:

- ASCII
- Unicode
- {0,1}
- {A,C,G,T}

Strings (cont)

Notation:

- *length(P)* … #characters in *P*
- λ … *empty* string   (*length*(λ) = 0)
- Σ^m … set of all strings of length *m* over alphabet Σ
- Σ^* … set of all strings over alphabet Σ

$v\omega$ denotes the concatenation of strings v and ω

Note: length(vω) = length(v)+length(ω) $\lambda\omega = \omega = \omega\lambda$

Strings (cont)

Notation:

- substring of P … any string Q such that $P = vQ\omega$, for some $v,\omega \in \Sigma^*$
- prefix of P … any string Q such that $P = Q\omega$, for some $\omega \in \Sigma^*$
- suffix of P … any string Q such that $P = \omega Q$, for some $\omega \in \Sigma^*$

Exercise #1: Strings

The string `a/a` of length 3 over the ASCII alphabet has

- how many prefixes?
- how many suffixes?
- how many substrings?

- 4 prefixes: `""` `"a"` `"a/"` `"a/a"`
- 4 suffixes: `"a/a"` `"/a"` `"a"` `""`
- 6 substrings: `""` `"a"` `"/"` `"a/"` `"/a"` `"a/a"`

Note:
`""` means the same as λ (= empty string)

Strings (cont)

ASCII (American Standard Code for Information Interchange)

- Specifies mapping of 128 characters to integers 0..127
- The characters encoded include:
  - upper and lower case English letters: A-Z and a-z
  - digits: 0-9
  - common punctuation symbols
  - special non-printing characters: e.g. *newline* and *space*

Ascii	Char	Ascii	Char	Ascii	Char	Ascii	Char
0	Null	32	Space	64	@	96	`
1	Start of heading	33	!	65	A	97	a
2	Start of text	34	"	66	B	98	b
3	End of text	35	#	67	C	99	c
4	End of transmit	36	$	68	D	100	d
5	Enquiry	37	%	69	E	101	e
6	Acknowledge	38	&	70	F	102	f
7	Audible bell	39	'	71	G	103	g
8	Backspace	40	(	72	H	104	h
9	Horizontal tab	41	)	73	I	105	i
10	Line feed	42	*	74	J	106	j
11	Vertical tab	43	+	75	K	107	k
12	Form feed	44	,	76	L	108	l
13	Carriage return	45	-	77	M	109	m
14	Shift in	46	.	78	N	110	n
15	Shift out	47	/	79	O	111	o
16	Data link escape	48	0	80	P	112	p
17	Device control 1	49	1	81	Q	113	q
18	Device control 2	50	2	82	R	114	r
19	Device control 3	51	3	83	S	115	s
20	Device control 4	52	4	84	T	116	t
21	Neg. acknowledge	53	5	85	U	117	u
22	Synchronous idle	54	6	86	V	118	v
23	End trans. block	55	7	87	W	119	w
24	Cancel	56	8	88	X	120	x
25	End of medium	57	9	89	Y	121	y
26	Substitution	58	:	90	Z	122	z
27	Escape	59	;	91	[	123	{
28	File separator	60	<	92	\	124	|
29	Group separator	61	=	93	]	125	}
30	Record separator	62	>	94	^	126	~
31	Unit separator	63	?	95	_	127	Forward del.

Strings (cont)

Reminder:

In C a string is an array of **`char`**s containing ASCII codes

- these arrays have an extra element containing a 0
- the extra 0 can also be written **`'\0'`** (*null character* or *null-terminator*)
- convenient because don't have to track the length of the string

Because strings are so common, C provides convenient syntax:

```
char str[] = "hello";  // same as char str[] = {'h', 'e', 'l', 'l', 'o', '\0'};
```

Note: **`str[]`** will have 6 elements

Strings (cont)

C provides a number of string manipulation functions via **`#include <string.h>`**, e.g.

```
strlen()    // length of string
strncpy()   // copy one string to another
strncat()   // concatenate two strings
strstr()    // find substring inside string
```

Example:

```
char *strncat(char *dest, char *src, int n)
```

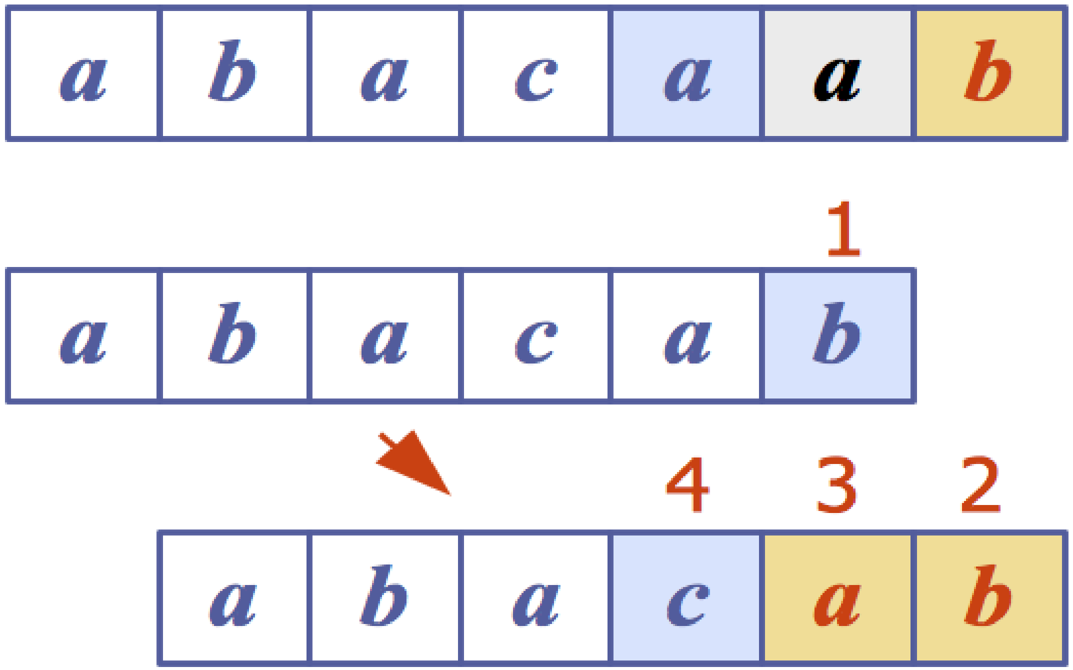
- appends string **`src`** to the end of **`dest`** overwriting the **`'\0'`** at the end of **`dest`** and adds terminating **`'\0'`**
- returns start of string **`dest`**
- will never add more than **`n`** characters

  (If **`src`** is less than **`n`** characters long, the remainder of **`dest`** is filled with **`'\0'`** characters. Otherwise, **`dest`** is not null-terminated.)

Pattern Matching

Pattern Matching

Example (pattern checked *backwards*):



- Text … **`abacaab`**
- Pattern … **`abacab`**

Pattern Matching (cont)

Given two strings T (text) and P (pattern), the pattern matching problem consists of finding a substring of T equal to P

Applications:

- Text editors
- Search engines
- Biological research

Pattern Matching (cont)

Brute-force pattern matching algorithm

- checks for each possible shift of *P* relative to *T*
  - untile a match is found, or
  - all placements of the pattern have been tried

```
BruteForceMatch(T,P):
   Input  text T of length n, pattern P of length m
   Output starting index of a substring of T equal to P
          -1 if no such substring exists

   for all i=0..n-m do
      j=0                                   // check from left to right
      while j<m ∧ T[i+j]=P[j] do            // test ith shift of pattern
         j=j+1
         if j=m then
            return i                        // entire pattern checked
         end if
      end while
   end for
   return -1                                // no match found
```

Analysis of Brute-force Pattern Matching

Brute-force pattern matching runs in O(n·m)

Examples of worst case (forward checking):

- *T* = **`aaa…ah`**
- *P* = **`aaah`**
- may occur in DNA sequences
- unlikely in English text

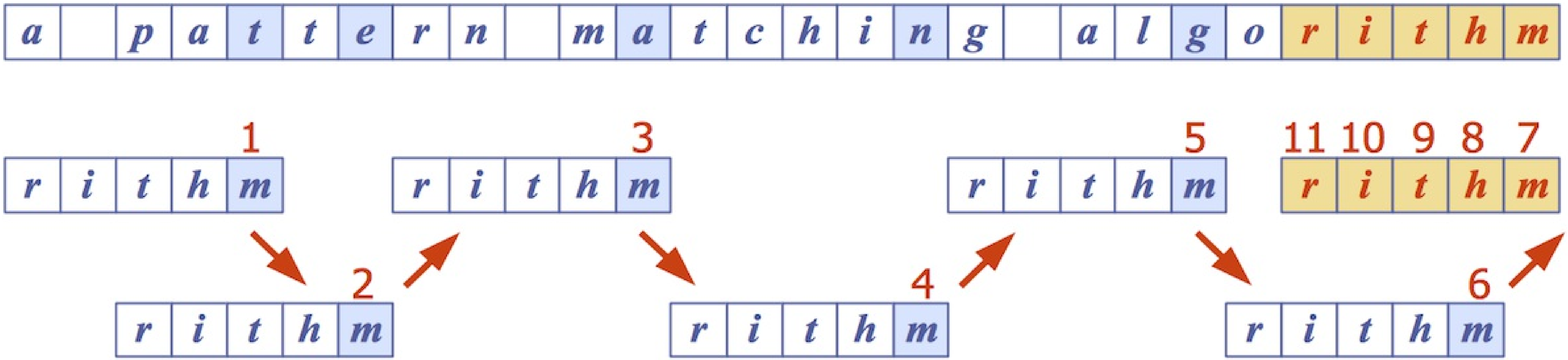
Boyer-Moore Algorithm

The Boyer-Moore pattern matching algorithm is based on two heuristics:

- Looking-glass heuristic: Compare *P* with subsequence of *T* moving *backwards*
- Character-jump heuristic: When a mismatch occurs at *T[i]*=**c**
  - if *P* contains **c** ⇒ shift *P* so as to align the last occurrence of **c** in *P* with *T[i]*
  - otherwise ⇒ shift *P* so as to align *P[0]* with *T[i+1]* (a.k.a. "big jump")

Boyer-Moore Algorithm (cont)

Example:



Boyer-Moore Algorithm (cont)

Boyer-Moore algorithm preprocesses pattern *P* and alphabet Σ to build

- last-occurrence function *L*
  - *L* maps Σ to integers such that *L(c)* is defined as
    - the largest index *i* such that *P[i]=c*, or
    - -1 if no such index exists

Example: Σ = {**a,b,c,d**}, *P* = **acab**

c	**a**	**b**	**c**	**d**
L(c)	2	3	1	-1

- *L* can be represented by an array indexed by the numeric codes of the characters
- *L* can be computed in $O(m+s)$ time (*m* … length of pattern, *s* … size of Σ)

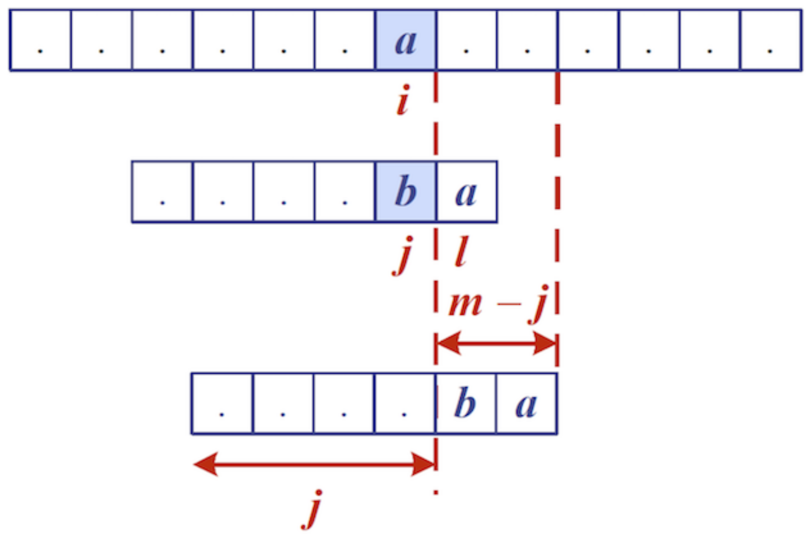
Boyer-Moore Algorithm (cont)

```
BoyerMooreMatch(T,P,Σ):
    Input  text T of length n, pattern P of length m, alphabet Σ
    Output starting index of a substring of T equal to P
           -1 if no such substring exists

    L=lastOccurenceFunction(P,Σ)
    i=m-1, j=m-1                          // start at end of pattern
    repeat
        if T[i]=P[j] then
            if j=0 then
                return i                  // match found at i
            else
                i=i-1, j=j-1
            end if
        else                              // character-jump
            i=i+mâ€“min(j,1+L[T[i]])
            j=m-1
        end if
    until i≥n
    return -1                             // no match
```

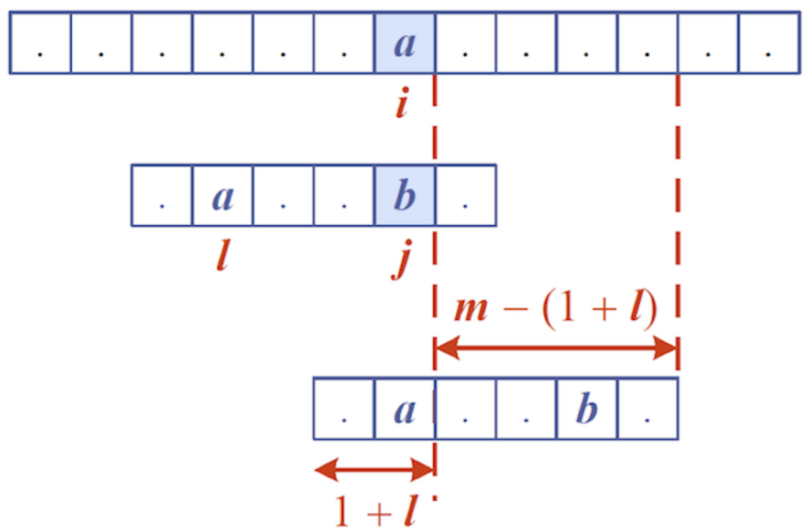
- Biggest jump (m characters ahead) occurs when *L[T[i]]* = -1

Boyer-Moore Algorithm (cont)

Case 1: $j \leq 1+L[c]$



Case 2: $1+L[c] < j$

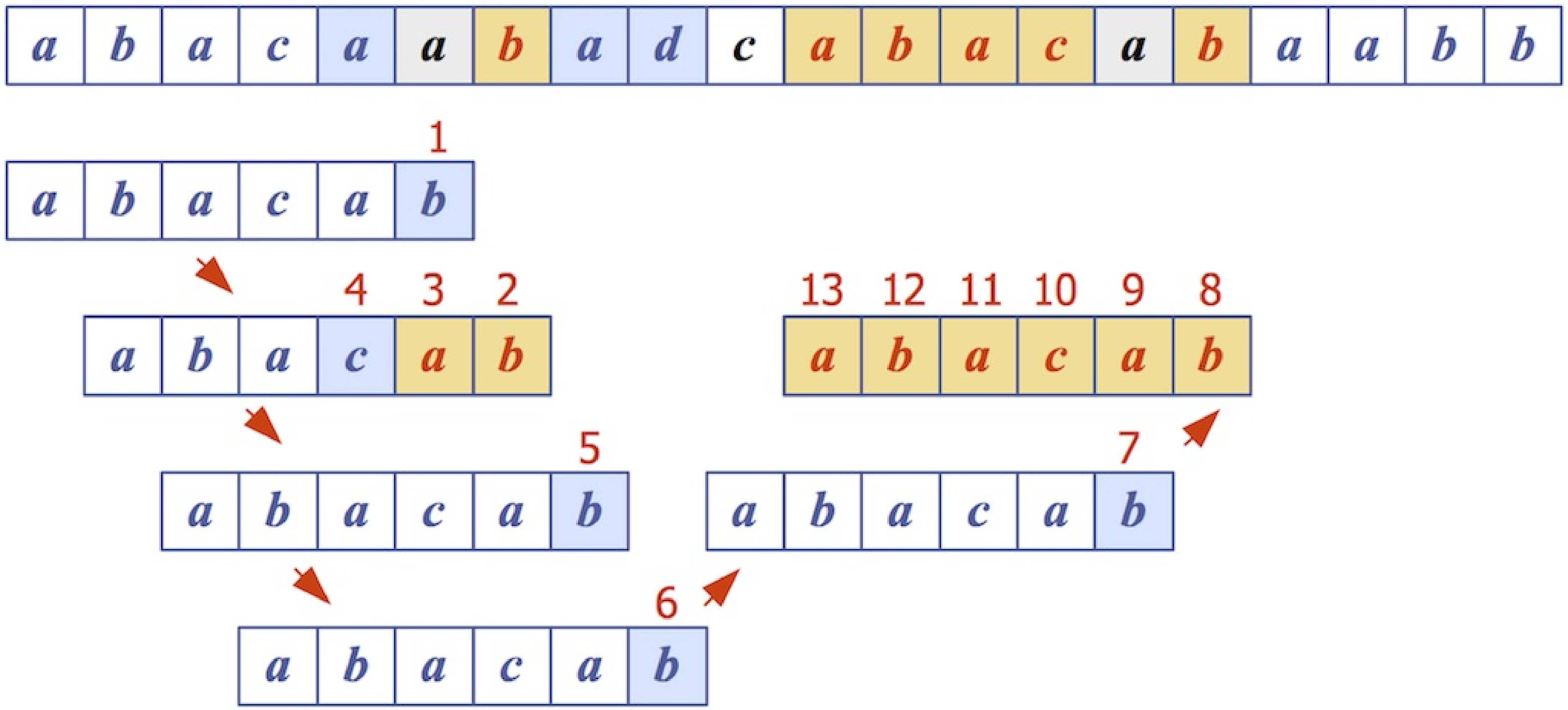


Exercise #2: Boyer-Moore algorithm

For the alphabet Σ = {**`a,b,c,d`**}

1. compute last-occurrence function *L* for pattern *P* = **`abacab`**
2. trace Boyer-More on *P* and text *T* = **`abacaabadcabacabaabb`**
    - how many comparisons are needed?

c	a	b	c	d
$L(c)$	4	5	3	-1



13 comparisons in total

Boyer-Moore Algorithm (cont)

Analysis of Boyer-Moore algorithm:

- Runs in $O(nm+s)$ time
  - m … length of pattern n … length of text s … size of alphabet
- Example of worst case:
  - T = **`aaa … a`**
  - P = **`baaa`**
- Worst case may occur in images and DNA sequences but unlikely in English texts
  ⇒ Boyer-Moore significantly faster than brute-force on English text

Knuth-Morris-Pratt Algorithm

The Knuth-Morris-Pratt algorithm …

- compares the pattern to the text *left-to-right*
- but shifts the pattern more intelligently than the brute-force algorithm

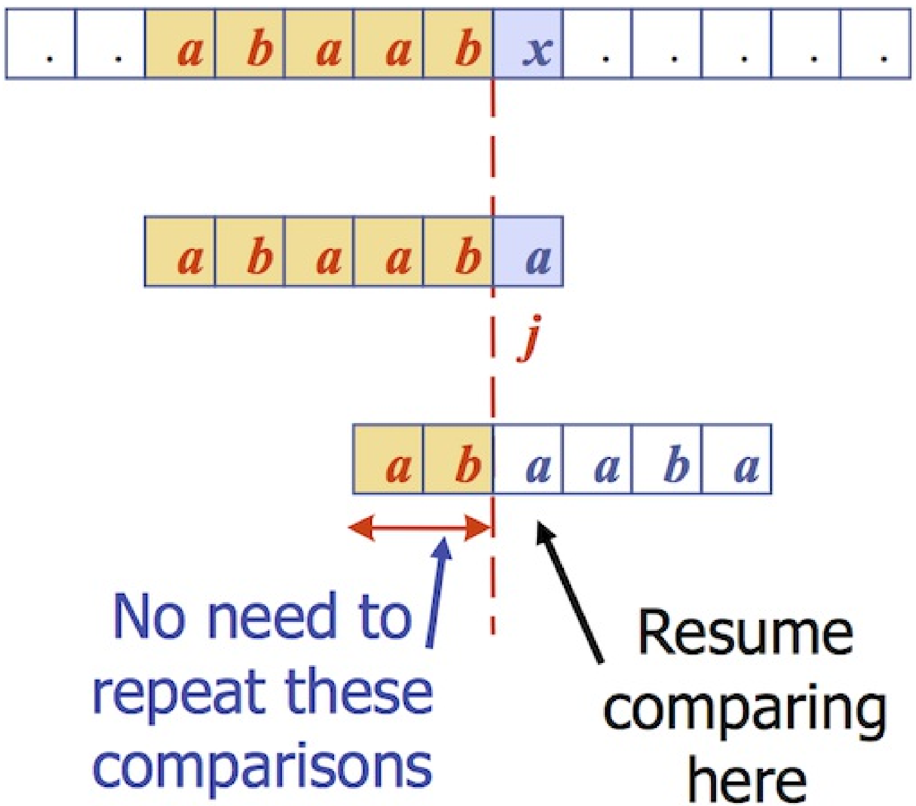
Reminder:

- Q is a *prefix* of P … $P = Q\omega$, for some $\omega \in \Sigma^*$
- Q is a *suffix* of P … $P = \omega Q$, for some $\omega \in \Sigma^*$

Knuth-Morris-Pratt Algorithm (cont)

When a mismatch occurs …

- what is the most we can shift the pattern to avoid redundant comparisons?
- Answer: the largest *prefix* of *P[0..j]* that is a *suffix* of *P[1..j]*



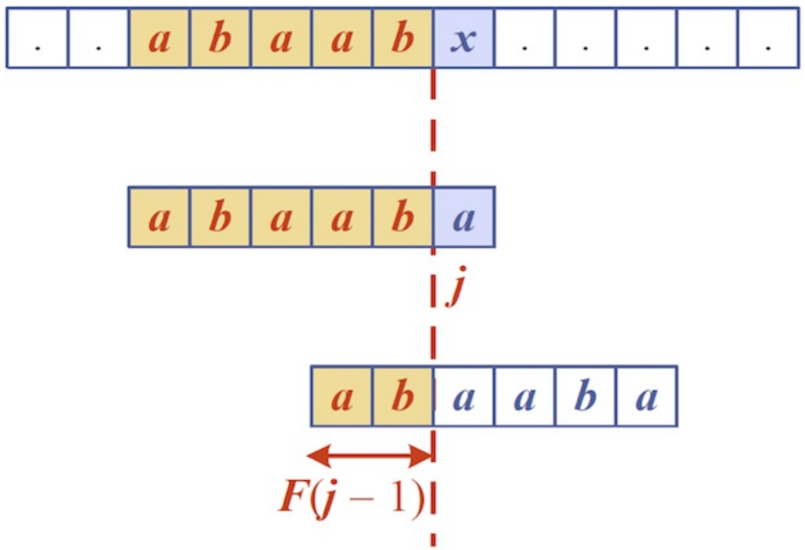
Knuth-Morris-Pratt Algorithm (cont)

KMP preprocesses the pattern to find matches of its prefixes with itself

- Failure function *F(j)* defined as
  - the size of the *largest prefix* of *P[0..j]* that is also a *suffix* of *P[1..j]*
- if mismatch occurs at P_j ⇒ advance *j* to *F(j-1)*

Example: *P* = **`abaaba`**

j	0	1	2	3	4	5
P_j	a	b	a	a	b	a
F(j)	0	0	1	1	2	3



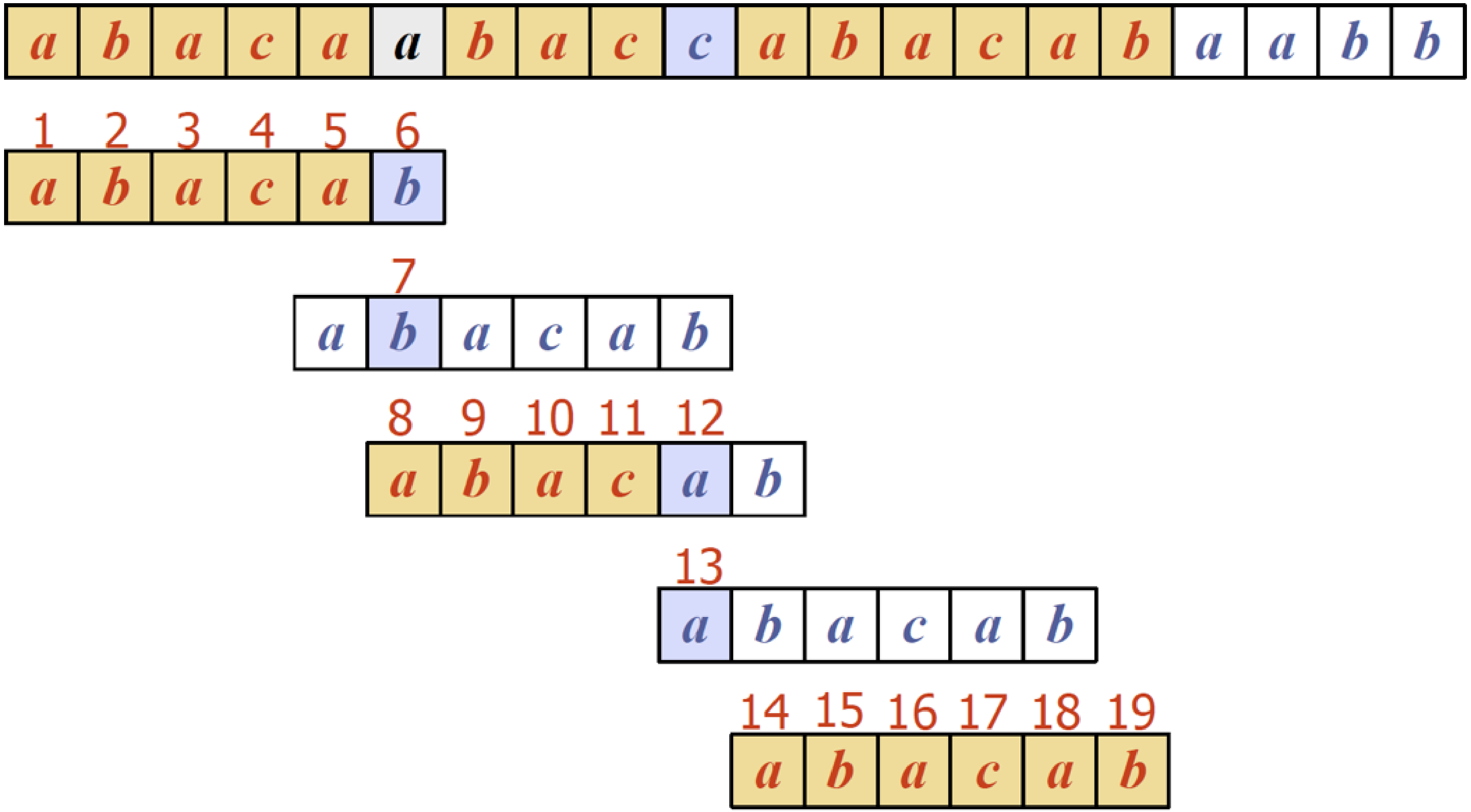
Knuth-Morris-Pratt Algorithm (cont)

```
KMPMatch(T,P):
   Input  text T of length n, pattern P of length m
   Output starting index of a substring of T equal to P
          -1 if no such substring exists

   F=failureFunction(P)
   i=0, j=0                            // start from left
   while i<n do
      if T[i]=P[j] then
         if j=m-1 then
            return i-j                 // match found at i-j
         else
            i=i+1, j=j+1
         end if
      else                             // mismatch at P[j]
         if j>0 then
            j=F[j-1]                   // resume comparing P at F[j-1]
         else
            i=i+1
         end if
      end if
   end while
   return -1                           // no match
```

KMP-Algorithm

1. compute failur function *F* for pattern *P* = **`abacab`**
2. trace Knuth-Morris-Pratt on *P* and text *T* = **`abacaabadcabacabaabb`**

j	0	1	2	3	4	5
P_j	a	b	a	c	a	b
$F(j)$	0	0	1	0	1	2



Knuth-Morris-Pratt Algorithm (cont)

Construction of the failure function is similar to the KMP algorithm itself:

```
failureFunction(P):
   Input  pattern P of length m
   Output failure function for P

   F[0]=0
   i=1, j=0
   while i<m do
      if P[i]=P[j] then       // we have matched j+1 characters
         F[i]=j+1
         i=i+1, j=j+1
      else if j>0 then        // use failure function to shift P
         j=F[j-1]
      else
         F[i]=0               // no match
         i=i+1
      end if
   end while
   return F
```

Knuth-Morris-Pratt Algorithm (cont)

Analysis of failure function computation:

- At each iteration of the while-loop, either
  - i increases by one, or
  - the "shift amount" i-j increases by at least one (observe that F(j-1) <j)
- Hence, there are no more than $2 \cdot m$ iterations of the while-loop

⇒ failure function can be computed in $O(m)$ time

Knuth-Morris-Pratt Algorithm (cont)

Analysis of Knuth-Morris-Pratt algorithm:

- Failure function can be computed in $O(m)$ time
- At each iteration of the while-loop, either
  - i increases by one, or
  - the "shift amount" i-j increases by at least one (observe that $F(j-1) < j$)
- Hence, there are no more than $2 \cdot n$ iterations of the while-loop

⇒ KMP's algorithm runs in optimal time $O(m+n)$

Boyer-Moore vs KMP

Boyer-Moore algorithm

- decides how far to jump ahead based on the mismatched character in the text
- works best on large alphabets and natural language texts (e.g. English)

Knuth-Morris-Pratt algorithm

- uses information embodied in the pattern to determine where the next match could begin
- works best on small alphabets (e.g. **`A,C,G,T`**)

Tries

Preprocessing Strings

Preprocessing the *pattern* speeds up pattern matching queries

- After preprocessing *P*, KMP algorithm performs pattern matching in time proportional to the text length

If the text is large, immutable and searched for often (e.g., works by Shakespeare)

- we can preprocess the *text* instead of the pattern

Preprocessing Strings (cont)

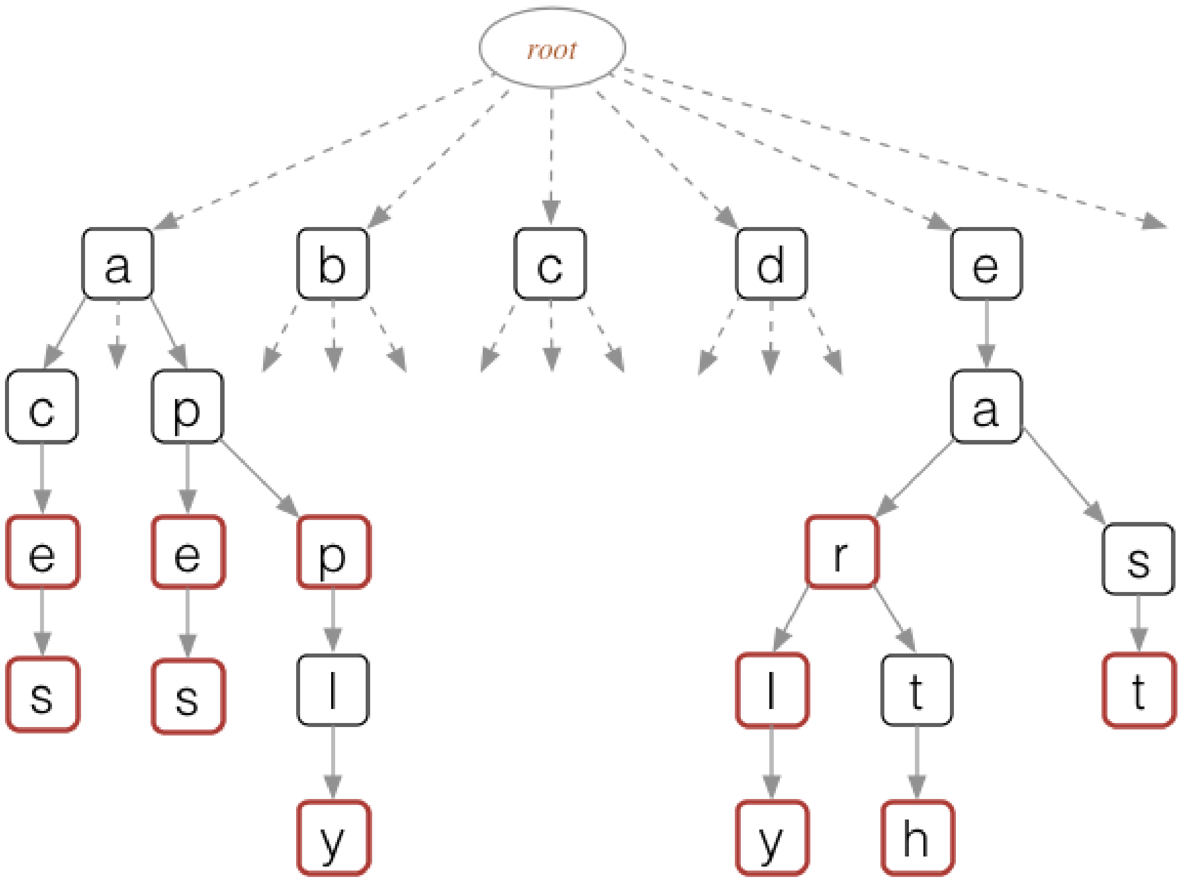
A trie …

- is a compact data structure for representing a set of strings
  - e.g. all the words in a text, a dictionary etc.
- supports pattern matching queries in time proportional to the pattern size

Note: Trie comes from *retrieval*, but is pronounced like "try" to distinguish it from "tree"

Tries

Tries are trees organised using parts of keys (rather than whole keys)



Tries (cont)

Each node in a trie …

- contains one part of a key (typically one character)
- may have up to 26 children
- may be tagged as a "finishing" node
- but even "finishing" nodes may have children

Depth *d* of trie = length of longest key value

Cost of searching *O(d)* (independent of *n*)

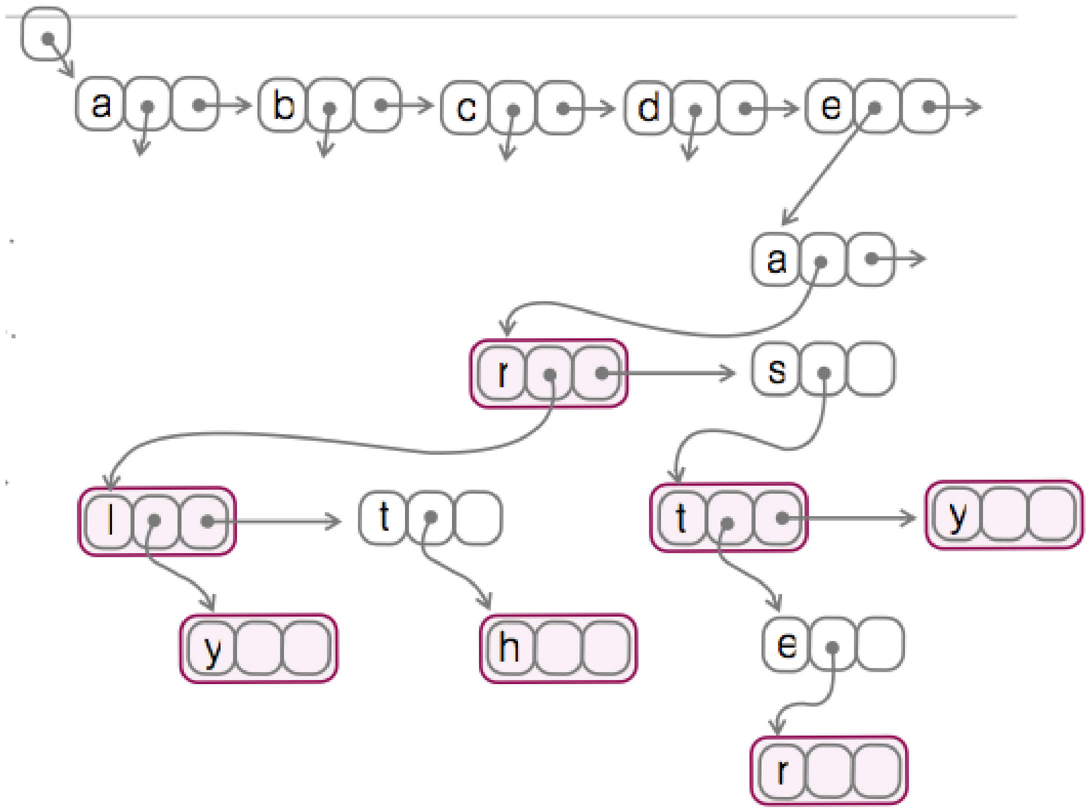
Tries (cont)

Possible trie representation:

```
#define ALPHABET_SIZE 26

typedef struct Node *Trie;

typedef struct Node {
   bool finish;       // last char in key?
   Item data;         // no Item if !finish
   Trie child[ALPHABET_SIZE];
} Node;

typedef char *Key;
```

Tries (cont)

Note: Can also use BST-like nodes for more space-efficient implementation of tries



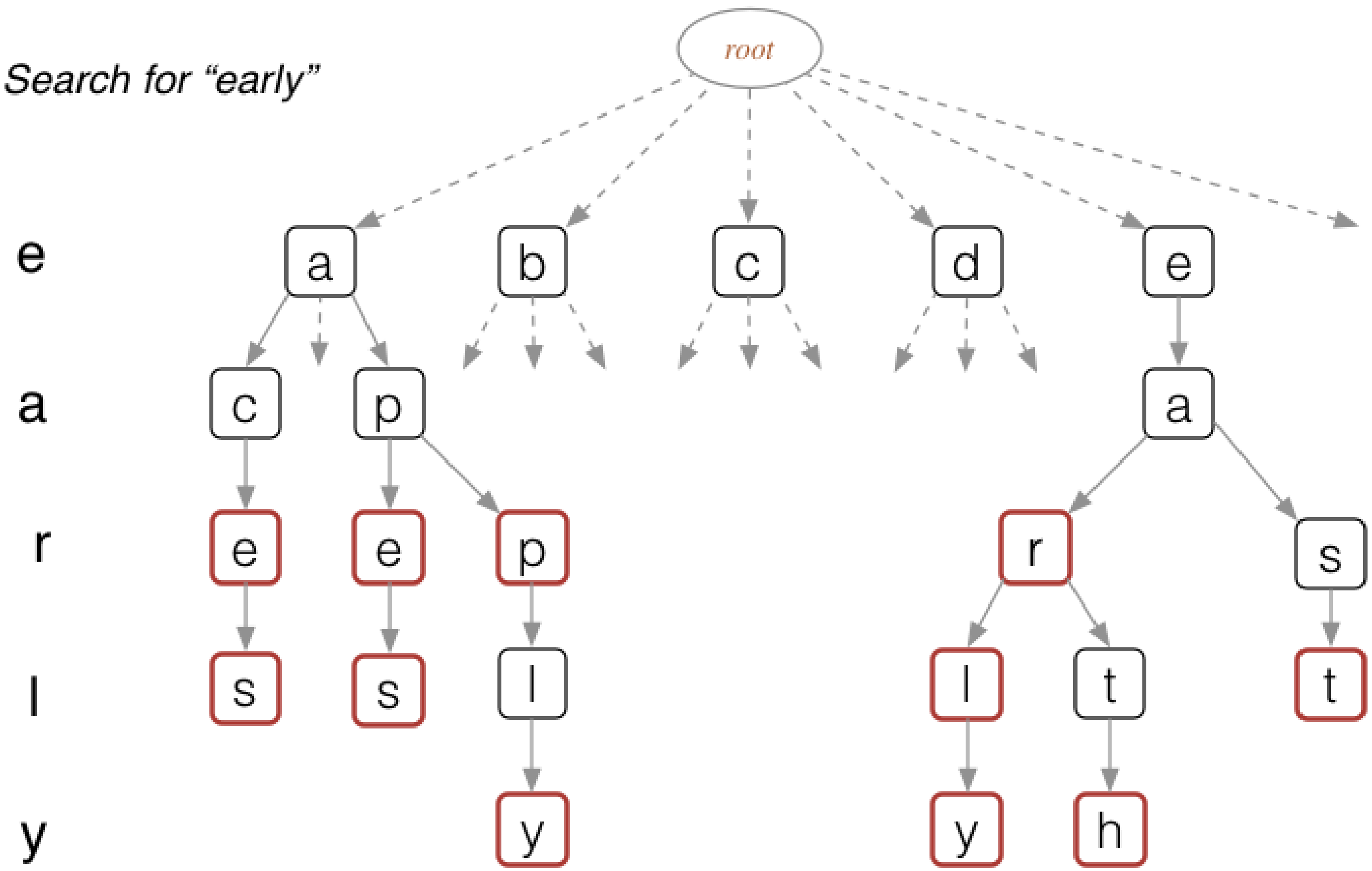
Trie Operations

Basic operations on tries:

1. search for a key
2. insert a key

Trie Operations

Search for "early"



Trie Operations (cont)

Traversing a path, using char-by-char from Key:

```
find(trie,key):
   Input  trie, key
   Output pointer to element in trie if key found
          NULL otherwise

   node=trie
   for each char in key do
      if node.child[char] exists then
         node=node.child[char]     // move down one level
      else
         return NULL
      end if
   end for
   if node.finish then           // "finishing" node reached?
      return node
   else
      return NULL
   end if
```

Trie Operations (cont)

Insertion into Trie:

```
insert(trie,item,key):
   Input  trie, item with key of length m
   Output trie with item inserted

   if trie is empty then
      t=new trie node
   end if
   if m=0 then
      t.finish=true, t.data=item
   else
      t.child[key[0]]=insert(trie,item,key[1..m-1])
   end if
   return t
```

Trie Operations (cont)

Analysis of standard tries:

- $O(n)$ space
- insertion and search in time $O(d \cdot m)$
  - n … total size of text (e.g. sum of lengths of all strings in a given dictionary)
  - m … size of the string parameter of the operation (the "key")
  - d … size of the underlying alphabet (e.g. 26)

Word Matching With Tries

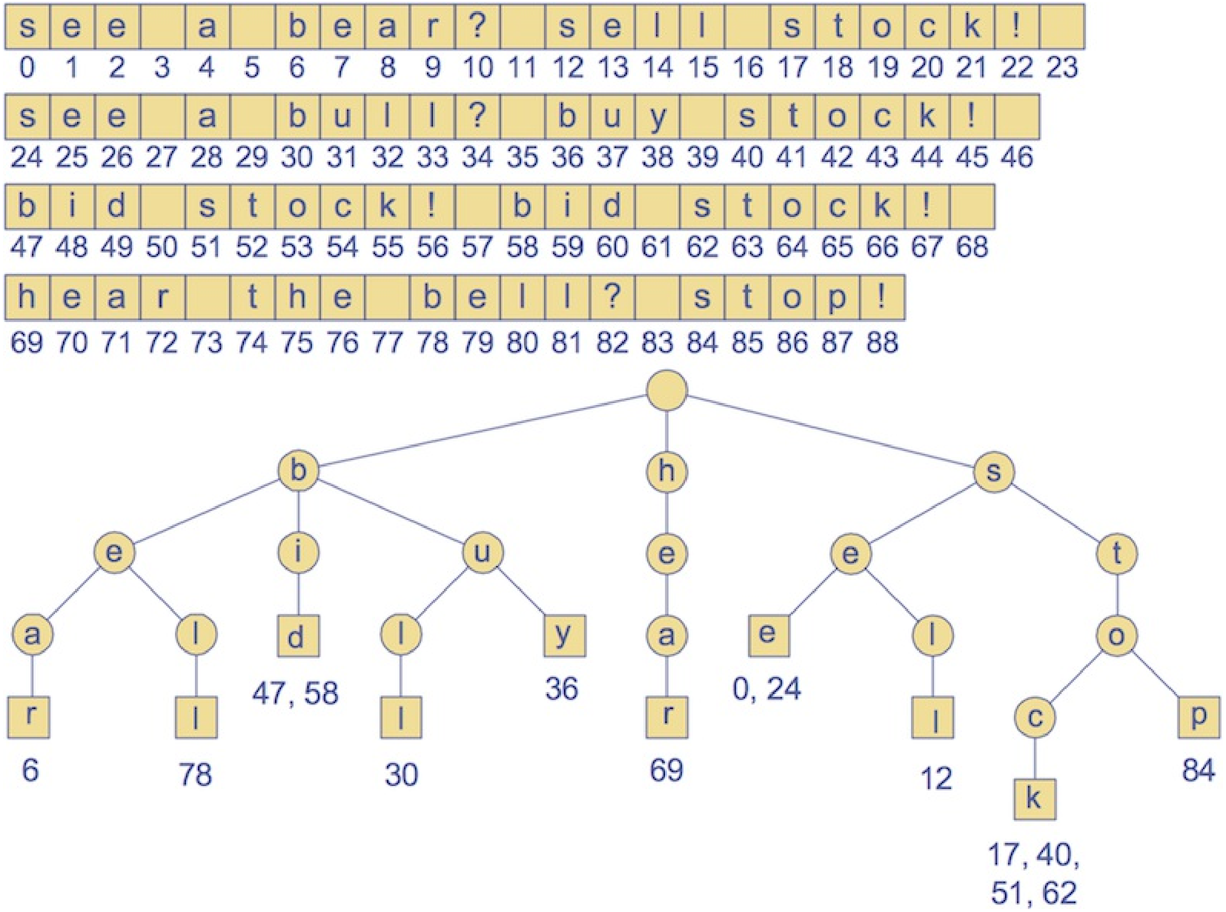
Word Matching with Tries

Preprocessing the text:

1. Insert all searchable words of a text into a trie
2. Each leaf stores the occurrence(s) of the associated word in the text

Word Matching with Tries (cont)

Example text and corresponding trie of searchable words:

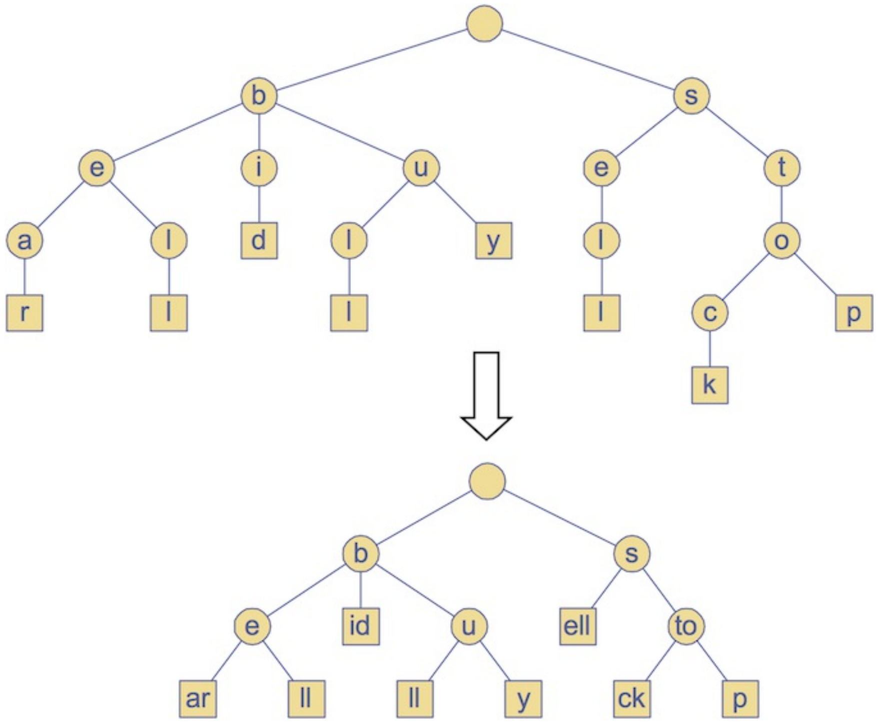


Compressed Tries

Compressed tries …

- have internal nodes of degree ≥ 2
- are obtained from standard tries by compressing "redundant" chains of nodes

Example:

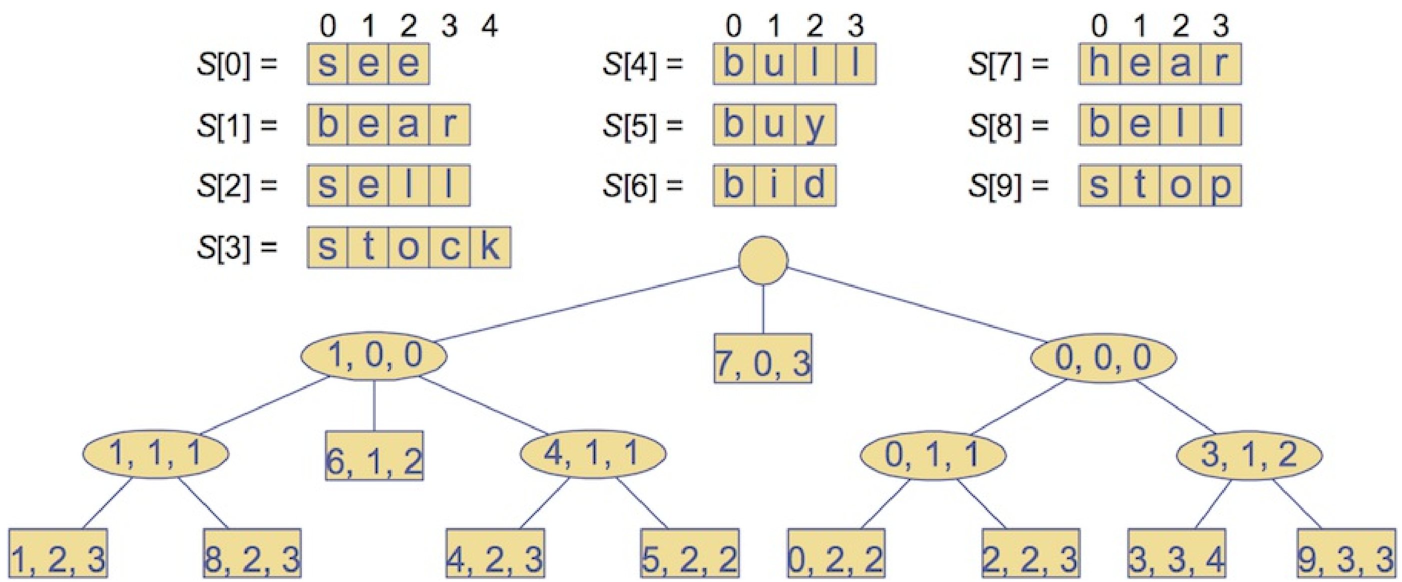


Compressed Tries (cont)

Possible compact representation of a compressed trie to encode an array *S* of strings:

- nodes store ranges of indices instead of substrings
  - use triple *(i,j,k)* to represente substring *S[i][j..k]*
- requires *O(s)* space (*s* = #strings in array *S*)

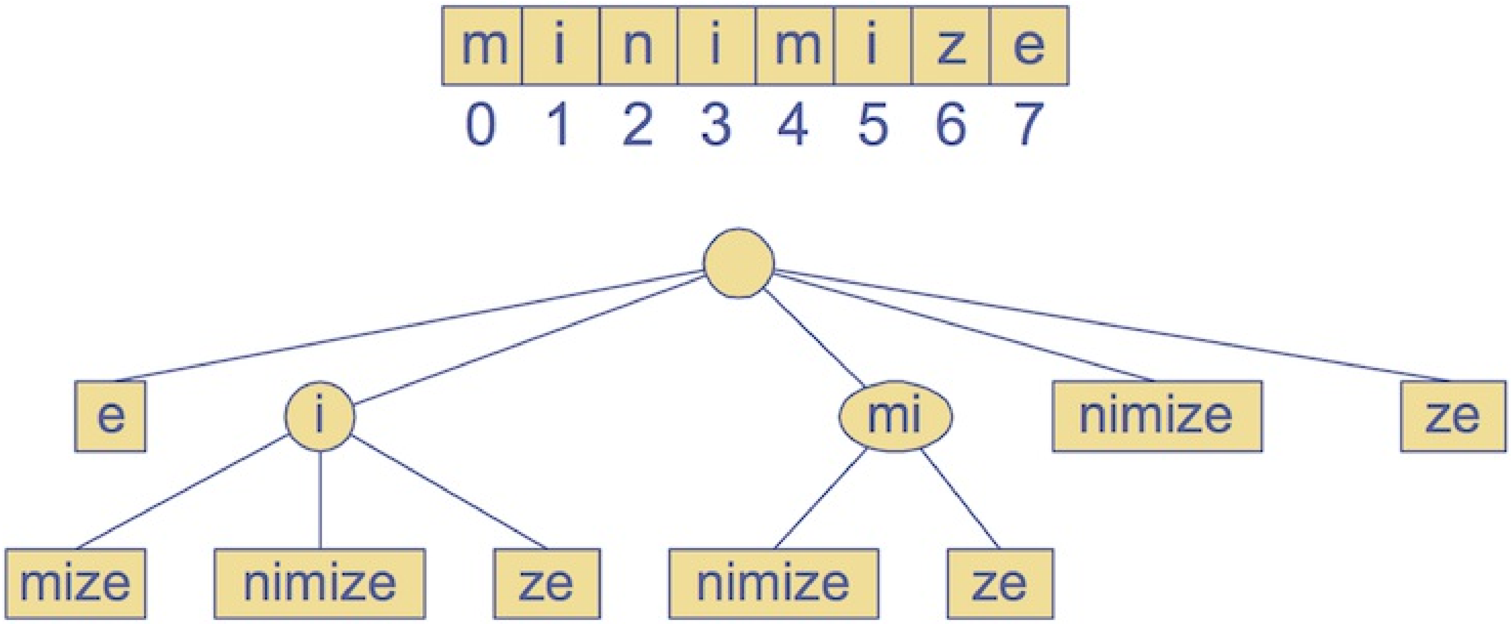
Example:



Pattern Matching With Suffix Tries

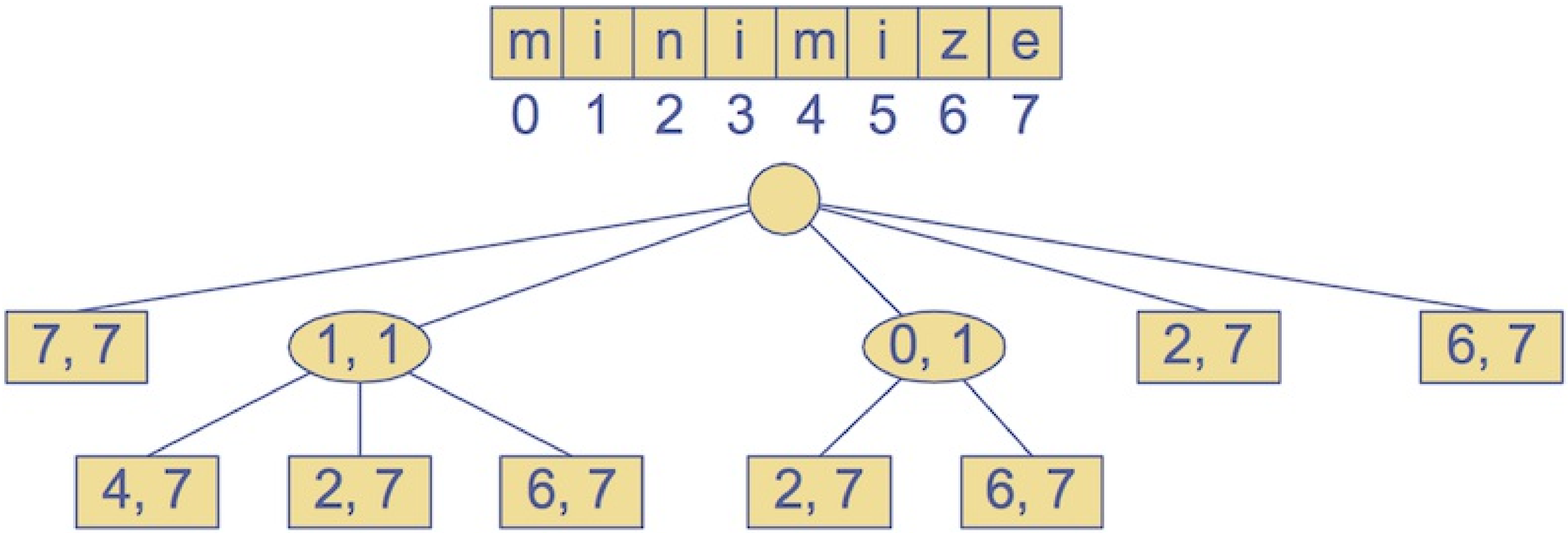
The suffix trie of a text *T* is the compressed trie of all the suffixes of *T*

Example:



Pattern Matching With Suffix Tries (cont)

Compact representation:



Pattern Matching With Suffix Tries (cont)

Input:

- compact suffix trie for text T
- pattern P

Goal:

- find starting index of a substring of T equal to P

Pattern Matching With Suffix Tries (cont)

```
suffixTrieMatch(trie,P):
   Input  compact suffix trie for text T, pattern P of length m
   Output starting index of a substring of T equal to P
          -1 if no such substring exists

   j=0, v=root of trie
   repeat
      // we have matched j+1 characters
      if ∃w∈children(v) such that P[j]=T[start(w)] then
         i=start(w)                    // start(w) is the start index of w
         x=end(w)-i+1                  // end(w) is the end index of w
         if m≤x then    // length of suffix ≤ length of the node label?
            if P[j..j+m-1]=T[i..i+m-1] then
               return i-j              // match at i-j
            else
               return -1               // no match
         else if P[j..j+x-1]=T[i..i+x-1] then
            j=j+x, m=m-x               // update suffix start index and length
            v=w                        // move down one level
         else return -1                // no match
         end if
      else
         return -1
      end if
   until v is leaf node
   return -1                           // no match
```

Pattern Matching With Suffix Tries (cont)

Analysis of pattern matching using suffix tries:

Suffix trie for a text of size n …

- can be constructed in $O(n)$ time
- uses $O(n)$ space
- supports pattern matching queries in $O(s \cdot m)$ time
    - m … length of the pattern
    - s … size of the alphabet

Text Compression

Text Compression

Problem: Efficiently encode a given string *X* by a smaller string *Y*

Applications:

- Save memory and/or bandwidth

Huffman's algorithm

- computes frequency *f(c)* for each character *c*
- encodes high-frequency characters with short code
- no code word is a prefix of another code word
- uses optimal encoding tree to determine the code words

Text Compression (cont)

Code … mapping of each character to a binary code word

Prefix code … binary code such that no code word is prefix of another code word

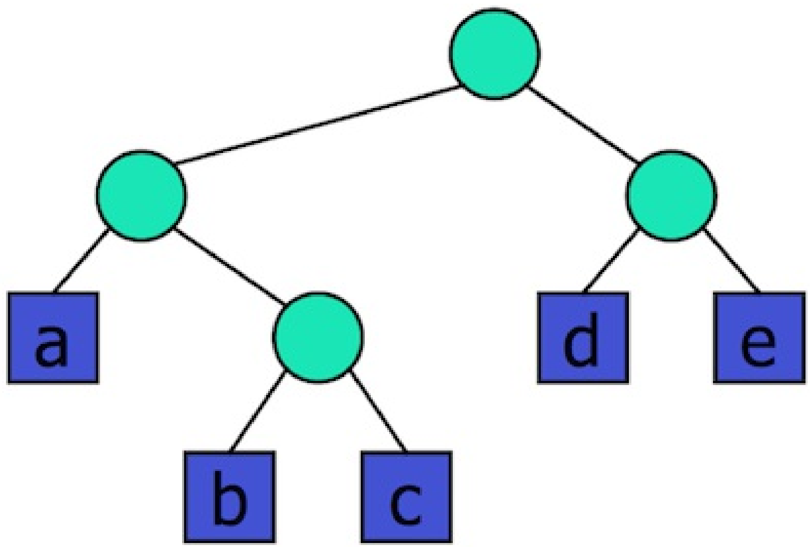
Encoding tree …

- represents a prefix code
- each leaf stores a character
- code word given by the path from the root to the leaf (0 for left child, 1 for right child)

Text Compression (cont)

Example:

00	010	011	10	11
a	b	c	d	e



Text Compression (cont)

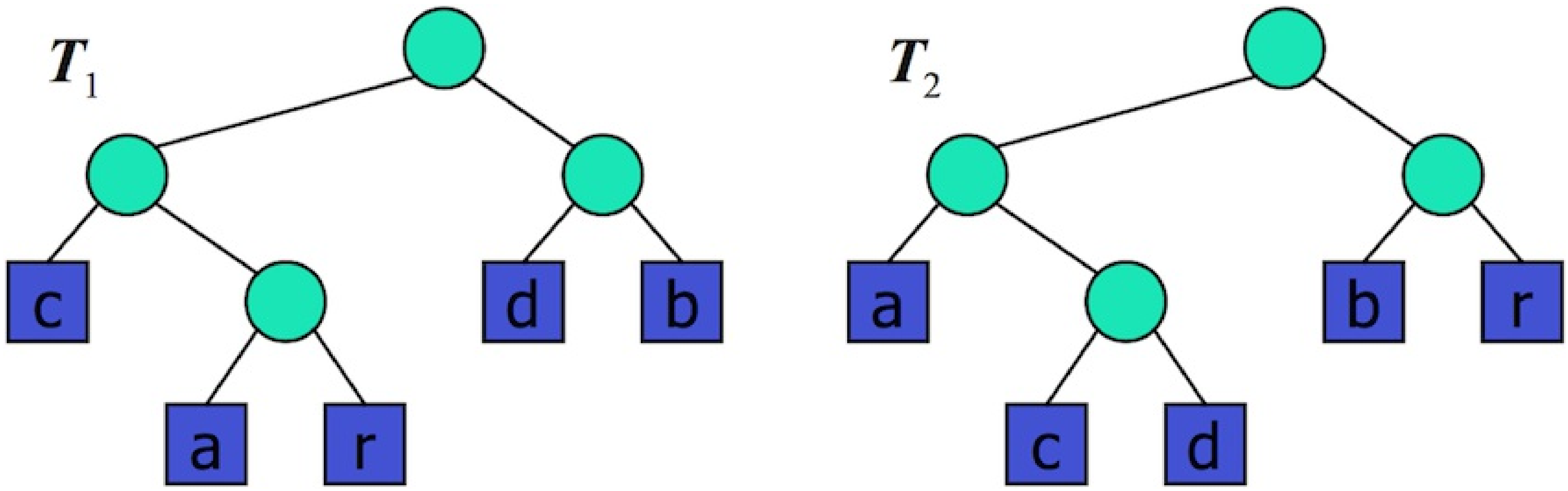
Text compression problem

Given a text T, find a prefix code that yields the shortest encoding of T

- short codewords for frequent characters
- long code words for rare characters

Text Compression (cont)

Example: T = **`abracadabra`**



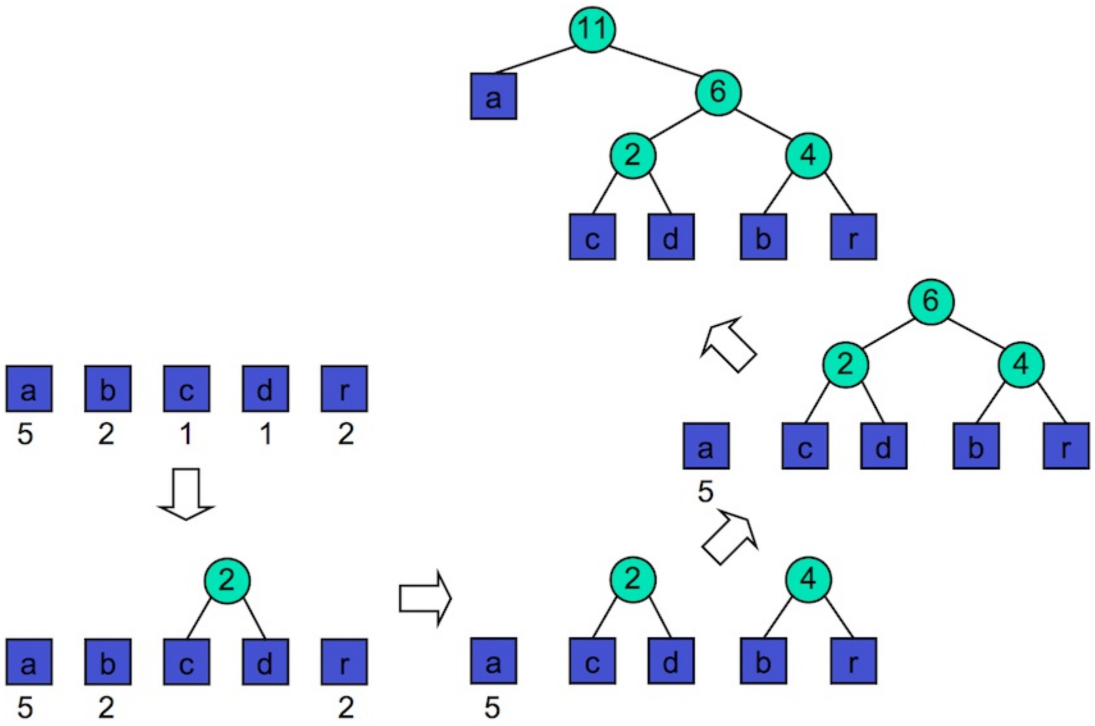
T_1 requires 29 bits to encode text T,
T_2 requires 24 bits

Text Compression (cont)

Huffman's algorithm

- computes frequency *f(c)* for each character
- successively combines pairs of lowest-frequency characters to build encoding tree "bottom-up"

Example: **`abracadabra`**



Huffman Code

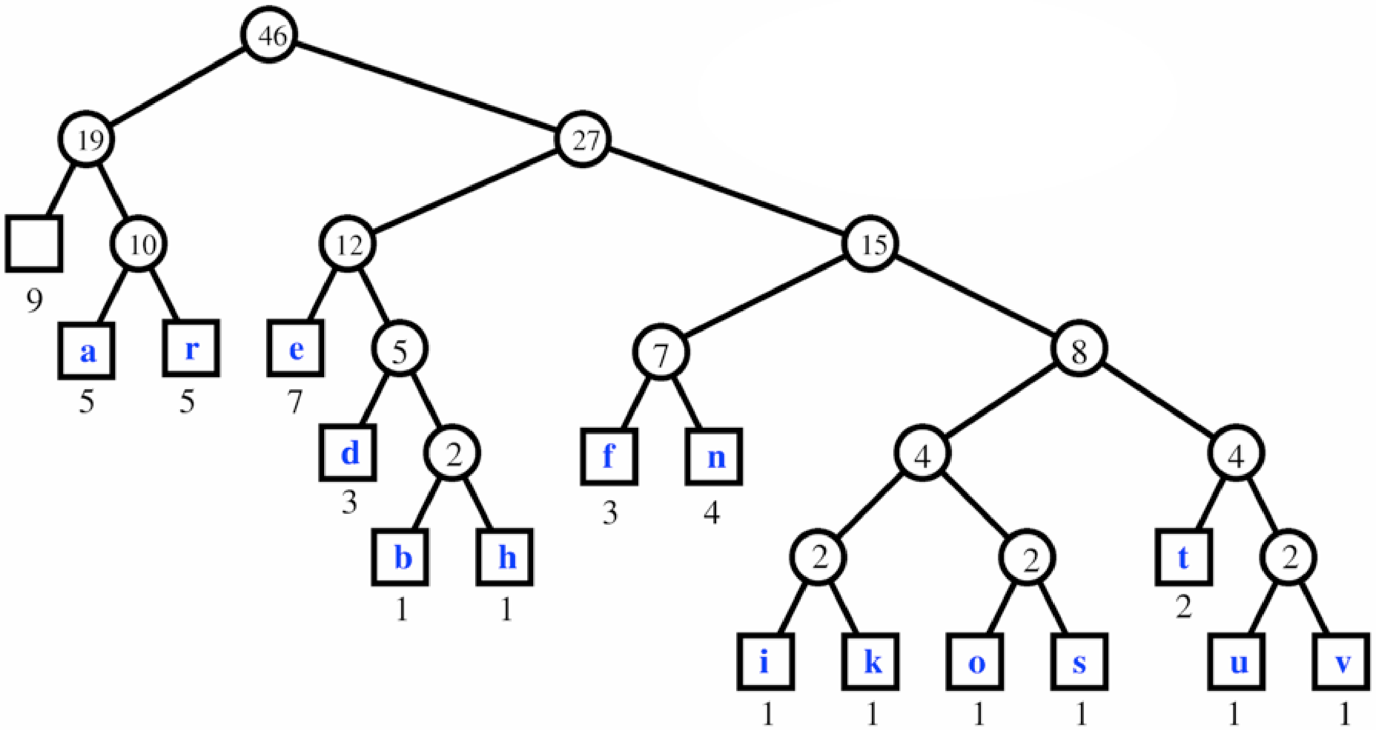
Huffman's algorithm using priority queue:

```
HuffmanCode(T):
|  Input  string T of size n
|  Output optimal encoding tree for T
|
|  compute frequency array
|  Q=new priority queue
|  for all characters c do
|     T=new single-node tree storing c
|     join(Q,T) with frequency(c) as key
|  end for
|  while |Q|≥2 do
|     f1=Q.minKey(), T1=leave(Q)
|     f2=Q.minKey(), T2=leave(Q)
|     T=new tree node with subtrees T1 and T2
|     join(Q,T) with f1+f2 as key
|  end while
|  return leave(Q)
```

Huffman Code (cont)

Larger example: a fast runner need never be afraid of the dark

Character		a	b	d	e	f	h	i	k	n	o	r	s	t	u	v
Frequency	9	5	1	3	7	3	1	1	1	4	1	5	1	2	1	1



Huffman Code (cont)

Analysis of Huffman's algorithm:

- $O(n+d \cdot \log d)$ time
  - n … length of the input text T
  - s … number of distinct characters in T

Summary

- Alphabets and words
- Pattern matching
  - Boyer-Moore, Knuth-Morris-Pratt
- Tries
- Text compression
  - Huffman code

- Suggested reading:
  - Tries … Sedgewick, Ch.15.2