

Week 02: Abstract Data Types

Abstract Data Objects and Types

Abstract Data Types

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A *data type* is ...

- a set of *values* (atomic or structured values) e.g. *integer stacks*
- a collection of *operations* on those values e.g. *push, pop, isEmpty?*

An *abstract data type* is ...

- an approach to implementing data types
- separates *interface* from *implementation*
- users of the ADT see only the interface
- builders of the ADT provide an implementation

... Abstract Data Types

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ADT *interface* provides

- a user-view of the data structure
- function signatures (prototypes) for all operations
- semantics of operations (via documentation)
- \Rightarrow a "contract" between ADT and its clients

ADT *implementation* gives

- concrete definition of the data structures
- function implementations for all operations

... Abstract Data Types

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ADT interfaces are *opaque*

- clients *cannot* see the implementation via the interface

ADTs are important because ...

- facilitate decomposition of complex programs
- make implementation changes invisible to clients
- improve readability and structuring of software

... Abstract Data Types

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Typical operations with ADTs

- *create* a value of the type
- *modify* one variable of the type
- *combine* two values of the type

Collections

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Common ADTs ...

- consist of a *collection* of *items*
- where each item may be a simple type or an ADT
- and items often have a *key* (to identify them)

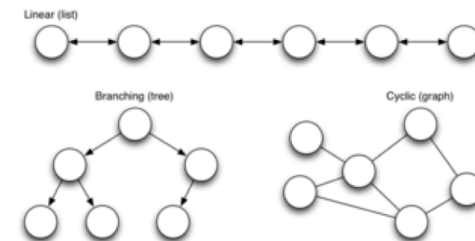
Collections may be categorised by ...

- *structure*:
 - linear (array, linked list), branching (tree), cyclic (graph)
- *usage*:
 - matrix, stack, queue, set, search-tree, dictionary, map, ...

... Collections

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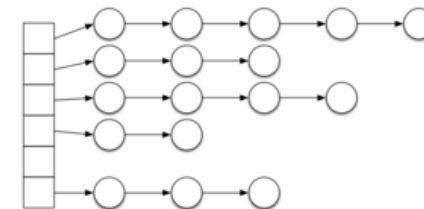
Collection structures:



... Collections

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Or even a hybrid structure like:



... Collections

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For a given collection type

- many different data representations are possible

For a given operation and data representation

- several different algorithms are possible
- efficiency of algorithms may vary widely

Generally,

- there is no overall "best" representation/implementation
- cost depends on the mix of operations (e.g. proportion of inserts, searches, deletions, ...)

ADOs and ADTs

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We want to distinguish ...

- ADO = *abstract data object*
- ADT = *abstract data type*

Warning: Sedgewick's first few examples are ADOs, not ADTs.

Example: Abstract Stack Data Object

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Stack, aka *pushdown stack* or *LIFO data structure*

Assume (for the time being) stacks of `char` values

Operations:

- *create* an empty stack
- insert (*push*) an item onto stack
- remove (*pop*) most recently pushed item
- check whether stack *is empty*

... Example: Abstract Stack Data Object

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Example of use:

Stack	Operation	Return value
?	create	-
-	push a	-
a	push b	-
a b	push c	-

a b c	pop	c
a b	isempty	false

Exercise #1: Stack vs Queue

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Consider the previous example but with a queue instead of a stack.

Which element would have been taken out ("dequeued") first?

a

Stack as ADO

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Interface (a file named `Stack.h`)

```
// Stack ADO header file

void StackInit();           // set up empty stack
int  StackIsEmpty();        // check whether stack is empty
void StackPush(char);       // insert char on top of stack
char StackPop();            // remove char from top of stack
```

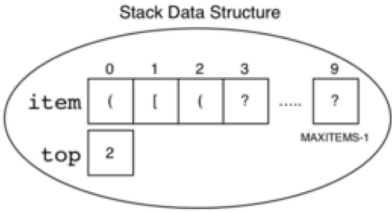
Note:

- no explicit reference to Stack object
- this makes it an *Abstract Data Object (ADO)*

... Stack as ADO

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Implementation may use the following data structure:



... Stack as ADO

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Sidetrack: Character I/O Functions in C (requires `<stdio.h>`)

```
int getchar(void);
```

- returns character read from standard input as an `int`, or returns **EOF** on end of file

```
int putchar(int ch);
```

- writes the character `ch` to standard output
- returns the character written, or EOF on error

Both functions do automatic *type conversion*

- `putchar('A')` has the same effect as `putchar((int) 'A')` (explicit type conversion)

... Stack as ADO

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Implementation (in a file named `Stack.c`):

```
#include "Stack.h"
#include <assert.h>

#define MAXITEMS 10
static struct {
    char item[MAXITEMS];
    int top;
} stackObject; // defines the Data Object

// set up empty stack
void StackInit() {
    stackObject.top = -1;
}

// check whether stack is empty
int StackIsEmpty() {
    return (stackObject.top < 0);
}

// insert char on top of stack
void StackPush(char ch) {
    assert(stackObject.top < MAXITEMS-1);
    stackObject.top++;
    int i = stackObject.top;
    stackObject.item[i] = ch;
}

// remove char from top of stack
char StackPop() {
    assert(stackObject.top > -1);
    int i = stackObject.top;
    char ch = stackObject.item[i];
    stackObject.top--;
    return ch;
}
```

assert(test) terminates program with error message if *test* fails.

Exercise #2: Bracket Matching

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Bracket matching ... check whether all opening brackets such as '(', '[', '{' have matching closing brackets ')', ']', '}'

Which of the following expressions are balanced?

- `(a+b) * c`
- `a[i]+b[j]*c[k]`
- `(a[i]+b[j])*c[k]`
- `a(a+b)*c`
- `void f(char a[], int n) {int i; for(i=0;i<n;i++) { a[i] = (a[i]*a[i])*(i+1); }}`
- `a(a+b * c`

- balanced
- not balanced (case 1: an opening bracket is missing)
- balanced
- not balanced (case 2: closing bracket doesn't match opening bracket)
- balanced
- not balanced (case 3: missing closing bracket)

... Stack as ADO

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Bracket matching algorithm, to be implemented as a *client* for Stack ADO:

```
#include "Stack.h"

bracketMatching(s):
    Input  stream s of characters
    Output TRUE if parentheses in s balanced, FALSE otherwise

    for each ch in s do
        if ch = open bracket then
            push ch onto stack
        else if ch = closing bracket then
            if stack is empty then
                return FALSE // opening bracket missing (case 1)
            else
                pop top of stack
                if brackets do not match then
                    return FALSE // wrong closing bracket (case 2)
                end if
            end if
        end if
    end for
    if stack is not empty then return FALSE // some brackets unmatched (case 3)
    else return TRUE
```

... Stack as ADO

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Execution trace of client on sample input:

([{ }])

Next char	Stack	Check
-	empty	-
((-
[((-
{	(({	-
}	(({ vs } ✓
]	([vs] ✓
)	empty	(vs) ✓
eof	empty	-

Exercise #3: Bracket Matching Algorithm

Trace the algorithm on the input

```
void f(char a[], int n) {
    int i;
    for(i=0;i<n;i++) { a[i] = a[i]*a[i]}*(i+1); }
}
```

Next bracket	Stack	Check
start	empty	-
((-
[((-
]	((✓
)	empty	✓
{	{	-
({{	-
)	{	✓
{	{{{	-
[{{[-
]	{{	✓
[{{[-
]	{{	✓
[{{[-
]	{{	✓
)	{	FALSE

Compilation and Makefiles

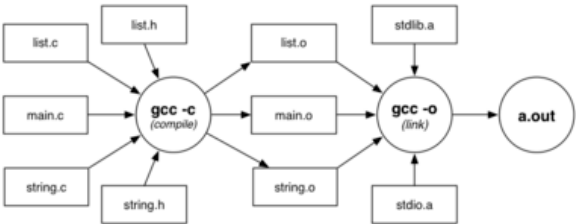
Compilers

Compilers are programs that

- convert program source code to executable form
- "executable" might be machine code or bytecode

The Gnu C compiler (**gcc**)

- applies source-to-source transformation (pre-processor)
- compiles *source code* to produce *object files*
- links object files and *libraries* to produce *executables*



... Compilers

Compilation/linking with gcc

```
gcc -c Stack.c
produces Stack.o, from Stack.c and Stack.h
gcc -c bracket.c
produces bracket.o, from bracket.c and Stack.h
gcc -o rbt bracket.o Stack.o
links bracket.o, Stack.o and libraries
producing executable program called rbt
```

Note that `stdio`, `assert` included implicitly.

gcc is a multi-purpose tool

- compiles (`-c`), links, makes executables (`-o`)

Make/Makefiles

Compilation process is complex for large systems.

How much to compile?

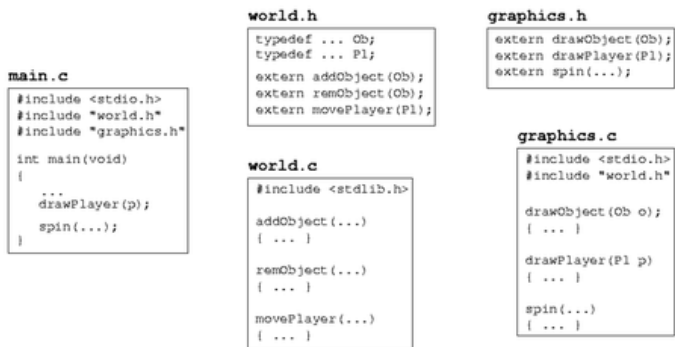
- ideally, what's changed since last compile
- practically, recompile everything, to be sure

The **make** command assists by allowing

- programmers to document *dependencies* in code
- minimal re-compilation, based on dependencies

... Make/Makefiles

Example multi-module program ...



... Make/Makefiles

make is driven by dependencies given in a **Makefile**

A *dependency* specifies

target : *source₁* *source₂* ...
 commands to build target from sources

e.g.

```
game : main.o graphics.o world.o
      gcc -o game main.o graphics.o world.o
```

Rule: *target* is rebuilt if older than any *source_i*

... Make/Makefiles

A **Makefile** for the example program:

```
game : main.o graphics.o world.o
      gcc -o game main.o graphics.o world.o
```

```
main.o : main.c graphics.h world.h
      gcc -Wall -Werror -c main.c
```

```
graphics.o : graphics.c world.h
      gcc -Wall -Werror -c graphics.c
```

```
world.o : world.c
      gcc -Wall -Werror -c world.c
```

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Things to note:

- A *target* (`game`, `main.o`, ...) is on a newline
 - followed by a **:**
 - then followed by the files that the target is dependent on
- The *action* (`gcc ...`) is always on a newline
 - and must be indented with a *TAB*

... Make/Makefiles

If make arguments are targets, build just those targets:

```
prompt$ make world.o
gcc -Wall -Werror -c world.c
```

If no args, build first target in the Makefile.

```
prompt$ make
gcc -Wall -Werror -c main.c
gcc -Wall -Werror -c graphics.c
gcc -Wall -Werror -c world.c
gcc -o game main.o graphics.o world.o
```

Exercise #4: Makefile

Write a Makefile for the bracket matching program.

From ADOs to ADTs

Abstract Data *Objects*

- `Stack.c` provides a single abstract object **stackObject**

Abstract Data *Types*

- allow clients to create and manipulate arbitrarily many data objects of an abstract type
- ... without revealing the implementation to a client

In C, ADTs are implemented using *pointers* and *dynamic memory allocation*

Pointers

Sidetrack: Numeral Systems

Numeral system ... system for representing numbers using digits or other symbols.

- Most cultures have developed a *decimal* system (based on 10)

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- For computers it is convenient to use a *binary* (base 2) or a *hexadecimal* (base 16) system

... Sidetrack: Numeral Systems

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Decimal representation

- The **base** is 10; digits 0 - 9
- Example: decimal number 4705 can be interpreted as
$$4 \cdot 10^3 + 7 \cdot 10^2 + 0 \cdot 10^1 + 5 \cdot 10^0$$
- Place values:

...	1000	100	10	1
...	10^3	10^2	10^1	10^0
- Write number as 4705_{10}
 - Note use of subscript to denote base

... Sidetrack: Numeral Systems

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Binary representation

- The **base** is 2; digits 0 and 1
- Example: binary number 1011 can be interpreted as
$$1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$
- Place values:

...	8	4	2	1
...	2^3	2^2	2^1	2^0
- Write number as 1011_2 (= 11_{10})

... Sidetrack: Numeral Systems

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Hexadecimal representation

- The **base** is 16; digits 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- Example: hexadecimal number 3AF1 can be interpreted as
$$3 \cdot 16^3 + 10 \cdot 16^2 + 15 \cdot 16^1 + 1 \cdot 16^0$$
- Place values:

...	4096	256	16	1
...	16^3	16^2	16^1	16^0
- Write number as $3AF1_{16}$ (= 15089_{10})

Exercise #5: Conversion Between Different Numeral Systems

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1. Convert 101011_2 to base 10
2. Convert 74_{10} to base 2
3. Convert $2D_{16}$ to base 10
4. Convert 273_{10} to base 16

1. 43_{10}
2. 1001010_2
3. 45_{10}
4. 111_{16}

... Sidetrack: Numeral Systems

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Conversion between binary and hexadecimal

0	1	2	3	4	5	6	7
0000	0001	0010	0011	0100	0101	0110	0111
8	9	A	B	C	D	E	F
1000	1001	1010	1011	1100	1101	1110	1111

- Binary to hexadecimal
 - Collect bits into groups of four starting from right to left
 - "pad" out left-hand side with 0's if necessary
 - Convert each group of four bits into its equivalent hexadecimal representation (given in table above)
- Hexadecimal to binary
 - Reverse the previous process
 - Convert each hex digit into equivalent 4-bit binary representation

Exercise #6: Conversion Between Binary and Hexadecimal

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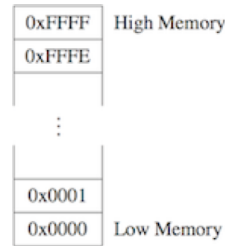
1. Convert 1011111000101001_2 to base 16
 - Hint: 1011111000101001
2. Convert 10111101011100_2 to base 16
 - Hint: 10111101011100
3. Convert $12D_{16}$ to base 2

1. $BE29_{16}$
2. $2F5C_{16}$
3. 100101101_2

Memory

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Computer memory ... large array of consecutive data cells or bytes



- char ... 1 byte int, float ... 4 bytes double ... 8 bytes

When a variable is declared, the operating system finds a place in memory to store the appropriate number of bytes.

If we declare a variable called k ...

- the place where k is stored is denoted by `&k`
- also called the **address** of k

It is convenient to print memory addresses in Hexadecimal notation

... Memory

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Example:

```
int k;
int m;

printf("address of k is %p\n", &k);
printf("address of m is %p\n", &m);
```

address of k is BFFFFB80
address of m is BFFFFB84

This means that

- k occupies the four bytes from BFFFFB80 to BFFFFB83
- m occupies the four bytes from BFFFFB84 to BFFFFB87

Note the use of `%p` as placeholder for an address ("pointer" value)

... Memory

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When an array is declared, the elements of the array are guaranteed to be stored in consecutive memory locations:

```
int array[5];

for (i = 0; i < 5; i++) {
    printf("address of array[%d] is %p\n", i, &array[i]);
}
```

address of array[0] is BFFFFB60

address of array[1] is BFFFFB64
address of array[2] is BFFFFB68
address of array[3] is BFFFFB6C
address of array[4] is BFFFFB70

Application: Input Using `scanf()`

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Standard I/O function `scanf()` requires the *address* of a variable as argument

- `scanf()` uses a format string like `printf()`
- use `%d` to read an integer value

```
#include <stdio.h>
...
int answer;
printf("Enter your answer: ");
scanf("%d", &answer);
```

- use `%f` to read a floating point value (`%lf` for double)

```
float e;
printf("Enter e: ");
scanf("%f", &e);
```

- `scanf()` returns a value — the number of items read
 - use this value to determine if `scanf()` successfully read a number
 - `scanf()` could fail e.g. if the user enters letters

Exercise #7: Using `scanf`

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Write a program that

- asks the user for a number
- checks that it is positive
- applies Collatz's process (Exercise 4, Problem Set 1) to the number

Pointers

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A *pointer* ...

- is a special type of variable
- storing the **address** (memory location) of another variable

A pointer occupies space in memory, just like any other variable of a certain type

The number of memory cells needed for a pointer depends on the computer's architecture:

- Old computer, or hand-held device with only 64KB of addressable memory:
 - 2 memory cells (i.e. 16 bits) to hold any address from 0x0000 to 0xFFFF (= 65535)
- Desktop machine with 4GB of addressable memory

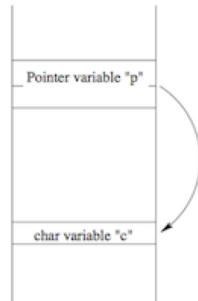
- 4 memory cells (i.e. 32 bits) to hold any address from 0x00000000 to 0xFFFFFFFF (= 4294967295)
- Modern 64-bit computer
 - 8 memory cells (can address 2^{64} bytes, but in practice the amount of memory is limited by the CPU)

... Pointers

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Suppose we have a pointer **p** that "points to" a char variable **c**.

Assuming that the pointer **p** requires 2 bytes to store the address of **c**, here is what the memory map might look like:



... Pointers

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Now that we have assigned to **p** the address of variable **c** ...

- need to be able to reference the data in that memory location

Operator ***** is used to access the object the pointer points to

- e.g. to change the value of **c** using the pointer **p**:

```
*p = 'T'; // sets the value of c to 'T'
```

The ***** operator is sometimes described as "*dereferencing*" the pointer, to access the underlying variable

... Pointers

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Things to note:

- all pointers constrained to point to a particular type of object

```
// a potential pointer to any object of type char
char *s;
```

```
// a potential pointer to any object of type int
int *p;
```

- if pointer **p** is pointing to an integer variable **x**
 ⇒ ***p** can occur in any context that **x** could

Examples of Pointers

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```
int *p; int *q; // this is how pointers are declared
int a[5];
int x = 10, y;

p = &x;        // p now points to x
*p = 20;       // whatever p points to is now equal to 20
y = *p;        // y is now equal to whatever p points to
p = &a[2];     // p points to an element of array a[]
q = p;         // q and p now point to the same thing
```

Exercise #8: Pointers

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What is the output of the following program?

```
1 #include <stdio.h>
2
3 int main(void) {
4     int *ptr1, *ptr2;
5     int i = 10, j = 20;
6
7     ptr1 = &i;
8     ptr2 = &j;
9
10    *ptr1 = *ptr1 + *ptr2;
11    ptr2 = ptr1;
12    *ptr2 = 2 * (*ptr2);
13    printf("Val = %d\n", *ptr1 + *ptr2);
14    return 0;
15 }
```

Val = 120

... Examples of Pointers

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Can we write a function to "swap" two variables?

The *wrong* way:

```
void swap(int a, int b) {
    int temp = a;           // only local "copies" of a and b will swap
    a = b;
    b = temp;
}
```



```

    b = temp;
}

int main(void) {
    int a = 5, b = 7;
    swap(a, b);
    printf("a = %d, b = %d\n", a, b); // a and b still have their original values
    return 0;
}

```

... Examples of Pointers

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In C, parameters are "call-by-value"

- changes made to the value of a parameter do not affect the original
- function `swap()` tries to swap the values of `a` and `b`, but fails because it only swaps the copies, not the "real" variables in `main()`

We can achieve "simulated call-by-reference" by passing pointers as parameters

- this allows the function to change the "actual" value of the variables

... Examples of Pointers

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Can we write a function to "swap" two variables?

The *right* way:

```

void swap(int *p, int *q) {
    int temp = *p;           // change the actual values of a and b
    *p = *q;
    *q = temp;
}

int main(void) {
    int a = 5, b = 7;
    swap(&a, &b);
    printf("a = %d, b = %d\n", a, b); // a and b now successfully swapped
    return 0;
}

```

Pointers and Arrays

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An alternative approach to iteration through an array:

- determine the **address of the first element** in the array
- determine the **address of the last element** in the array
- set a pointer variable to refer to the first element
- use **pointer arithmetic** to move from element to element
- terminate loop when address exceeds that of last element

Example:

```

int a[6];
int *p = &a[0];
while (p <= &a[5]) {
    printf("%2d ", *p);
    p++;
}

```

... Pointers and Arrays

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Pointer-based scan written in more typical style

```

int *p;
int a[6];
for (p = &a[0]; p < &a[6]; p++)
    printf("%2d ", *p);

```

Annotations:

- address of first element (points to `&a[0]`)
- address of last element + 1 (points to `&a[6]`)
- access current element (points to `*p`)
- pointer arithmetic (move to next element) (points to `p++`)

Note: because of pointer/array connection `a[i] == *(a+i)`

Sidetrack: Pointer Arithmetic

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A *pointer* variable holds a value which is an *address*.

C knows what type of object is being pointed to

- it knows the `sizeof` that object
- it can compute where the next/previous object is located

Example:

```

int a[6]; // address 0x1000
int *p;
p = &a[0]; // p contains 0x1000
p = p + 1; // p now contains 0x1004

```

... Sidetrack: Pointer Arithmetic

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For a pointer declared as `T *p;` (where `T` is a type)

- if the pointer initially contains address `A`
 - executing `p = p + k;` (where `k` is a constant)
 - changes the value in `p` to `A + k*sizeof(T)`

The value of `k` can be positive or negative.

Example:

```
int a[6];      (addr 0x1000)      char s[10];    (addr 0x2000)
int *p;        (p == ?)          char *q;        (q == ?)
p = &a[0];      (p == 0x1000)      q = &s[0];      (q == 0x2000)
p = p + 2;      (p == 0x1008)      q++;              (q == 0x2001)
```

Arrays of Strings

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One common type of pointer/array combination are the *command line arguments*

- These are 0 or more strings specified when program is run
- If you run this command in a terminal:

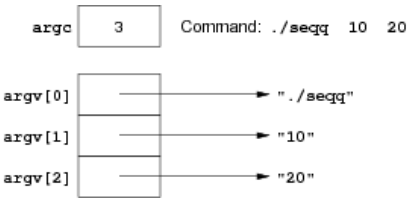
```
prompt$ ./seqq 10 20
```

then seqq will be given 2 command-line arguments: "10", "20"

... Arrays of Strings

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```
prompt$ ./seqq 10 20
```



Each element of `argv[]` is

- a pointer to the start of a character array (`char *`)
 - containing a `\0`-terminated string

... Arrays of Strings

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More detail on how `argv` is represented:

```
prompt$ ./seqq 5 20
```



... Arrays of Strings

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`main()` needs different prototype if you want to access command-line arguments:

```
int main(int argc, char *argv[]) { ...
```

- `argc` ... stores the number of command-line arguments + 1
 - `argc == 1` if no command-line arguments
- `argv[]` ... stores program name + command-line arguments
 - `argv[0]` always contains the program name
 - `argv[1], argv[2], ...` are the command-line arguments if supplied

`<stdlib.h>` defines useful functions to convert strings:

- `atoi(char *s)` converts string to int
- `atof(char *s)` converts string to double (can also be assigned to float variable)

Exercise #9: Command Line Arguments

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Write a program that

- checks for a single command line argument
 - if not, outputs a usage message and exits with failure
- converts this argument to a number and checks that it is positive
- applies Collatz's process (Exercise 4, Problem Set 1) to the number

```
#include <stdio.h>
#include <stdlib.h>
```

```
void collatz(int n) {
    printf("%d\n", n);
    while (n != 1) {
        if (n % 2 == 0)
            n = n / 2;
        else
            n = 3*n + 1;
        printf("%d\n", n);
    }
}
```

```
int main(int argc, char *argv[]) {
    if (argc != 2) {
        printf("Usage: %s [number]\n", argv[0]);
        return 1;
    }
    int n = atoi(argv[1]);
    if (n > 0)
        collatz(n);
    return 0;
}
```

argv can also be viewed as *double pointer* (a pointer to a pointer)

⇒ Alternative prototype for main():

```
int main(int argc, char **argv) { ...
```

Can still use argv[0], argv[1], ...

Pointers and Structures

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Like any object, we can get the address of a struct via &.

```
typedef char Date[11]; // e.g. "03-08-2017"
typedef struct {
    char name[60];
    Date birthday;
    int status; // e.g. 1 (≡ full time)
    float salary;
} WorkerT;
```

```
WorkerT w; WorkerT *wp;
wp = &w;
// a problem ...
*wp.salary = 125000.00;
// does not have the same effect as
w.salary = 125000.00;
// because it is interpreted as
*(wp.salary) = 125000.00;
```

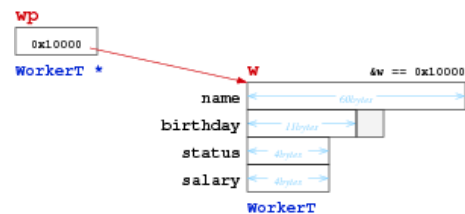
```
// to achieve the correct effect, we need
(*wp).salary = 125000.00;
// a simpler alternative is normally used in C
wp->salary = 125000.00;
```

Learn this well; we will frequently use it in this course.

... Pointers and Structures

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Diagram of scenario from program above:



... Pointers and Structures

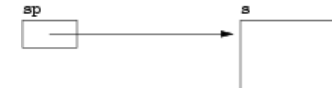
General principle ...

If we have:

```
SomeStructType s, *sp = &s;
```

then the following are all equivalent:

```
s.SomeElem    sp->SomeElem    (*sp).SomeElem
```



Summary

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- Introduction to ADOs and ADTs
- Compilation and **Makefiles**
- Numeral systems
- Pointers
- Suggested reading:
 - introduction to ADTs ... Sedgewick, Ch.4.1-4.3
 - pointers ... Moffat, Ch.6.6-6.7

Produced: 1 Aug 2017