TO 313 Cheat Sheet - Vincent Wei

Facts

Order winners are **competitive advantages** that cause a firm's customers to select that company's products or services. Includes

- Price
- Quality
- Speed
- Flexibility or variety

Process Analysis

Definitions

- **Inventory**: The number of flow units contained in a process
- **Flow time**: The time it takes for a flow unit to get through the process
- **Throughput rate**: The rate that the process is delivering output
- **Cycle time**: The time between two consecutive units leaving the process. **The inverse of the throughput rate**.
- **Capacity**: The maximum **throughput rate** when working non-stop.
- **Bottleneck resource**: slowest resource (spends the most time on each unit), or the resource with the lowest capacity.
- **Process capacity**: The capacity of the bottleneck resource.

$$\text{Throughput rate} = \min(\text{Input rate}, \text{Process capacity}) \tag{1}$$

Input rate also known as **demand**.

Little's Law

$$I = R \cdot T \tag{2}$$

- I: Inventory
- R: Throughput rate
- T: Flow time

Units of R and T need to match.

Inventory Turnover

$$\text{Turnover} = \frac{1}{\text{Days of Inventory}} = \frac{1}{T} = \frac{R}{I} = \frac{\text{COGS}}{\text{Inventory}} \tag{3}$$

Utilization

$$\text{Actual Utilization} = \frac{R}{C} \tag{4}$$

$$\text{Implicit Utilization} = \frac{D}{C} \tag{5}$$

- R: Throughput rate
- D: Demand
- C: Capacity

The bottleneck is the resource with the **highest implied utilization**.

Build Up

Whenever implied utilization $> 100\%$, there will be build-up! Let buffer inventory be f. There are three cases:

- $D < C$: $R = D \implies$ No build up
- $D > C$: $R = C \implies$ Build up rate: $\frac{\partial f}{\partial t} = D - C$
- $D < C \wedge \exists$ Build-up: $R = C \implies$ Deplete rate: $\frac{\partial f}{\partial t} = C - D$

$$\text{Total wait time} = \int f \mathrm{d}t \tag{6}$$

Note that the unit is in buffer-unit $\times$ time-unit.

Variability and Queuing

Wait time depends on the variability and utilization of a process. The **higher** the variability, the **higher** the wait time.

- T_q: Avg. wait (queue) Time
- p: Avg. service (processing) Time
- a: Inter-arrival time, **inverse** of **arrival rate**
- u: Avg. system utilization, **inverse of serving rate**
- m: Number of identical workers

To solve a VUT problem, we need to first compute

$$u = \frac{\frac{1}{a}}{m\frac{1}{p}} = \frac{p}{ma} \tag{7}$$

$$CV_a = \frac{\sigma_a}{a} \tag{8}$$

$$CV_p = \frac{\sigma_p}{p} \tag{9}$$

By which we arrive at the following

$$T_q = VUT = \frac{CV_a^2 + CV_p^2}{2} \cdot \frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)} \cdot p \tag{10}$$

Here, V is **variability effect**, U is **utilization effect**, T is **time scale**. When $m = 1$, this is essentially

$$T_q = VUT = \frac{CV_a^2 + CV_p^2}{2} \cdot \frac{u}{1-u} \cdot p \tag{11}$$

VUT Assumptions and Insights

- VUT yields long-term, steady-state **average waiting time**.
- Waiting happens even when average capacity exceeds average demand
- Only applies when $u < 1$
- Assumes **infinite buffer** size and is good approximation when buffer size is finite but large
- Exact approximation when $m = 1$ and $A \sim \text{Poisson}(a)$
- Pooling is good: $\frac{\partial T_q}{\partial m} = -\frac{1}{m^2}$
- Lower utilization is good: $\frac{\partial T_q}{\partial u} = C\frac{1}{(1-u)^2}$

In addition, using previous concepts we have that

$$I_q + I_p = I_{sys} \tag{12}$$

$$T_q + p = T_{sys} \tag{13}$$

where I is the avg. number of people in a process (Inventory) and T is the avg. waiting time in a process (Flow time).

Using Little's Law, we have

$$I_q = R_q \cdot T_q = \frac{T_q}{\frac{1}{R_q}} = \frac{T_q}{a} \tag{14}$$

$$I_p = R_p \cdot T_p = \frac{p}{\frac{1}{R_p}} = \frac{p}{a} \tag{15}$$

$$I_{sys} = \frac{T_{sys}}{a} \tag{16}$$

Note that $R_q = R_p = R_{sys}$ because $u < 1 \implies D < C \implies R = D = \frac{1}{a}$.

Newsvendor

We have a newsvendor problem when

- Demand is uncertain
- We must stock before knowing the demand
- Customers won't wait! And unmet demand will be lost
- Leftover inventory loses value

We compute the optimal service level and quantity using

$$SL^\star = \frac{C_u}{C_u + C_o} \tag{17}$$

$$Q^\star = \mu + z\sigma \text{ if } D \sim N(\mu, \sigma) \tag{18}$$

$$Q^\star = \text{argmin}_q \left(P(Q = q) - P(Q = SL^\star)\right) \text{ if } D \text{ is discrete} \tag{19}$$

where C_u is cost of **understock** and C_o is the cost of **overstock**. z is computed using `NORM.S.INV(SL)`. Then, **round up** $Q^\star$.

Continuous Review Model

We use the continuous review model when
- Uncertain demand
- Multi-period
  - Customers are willing to wait, backorders
  - Excess inventory can be held & sold later
- Constant setup cost (S) per order
- Constant annual holding cost per unit ($H = iC$)
- Constant lead time (L days)

Simple Rule: **Event-triggered** order system. If inventory drops to ROP, order EOQ. **Exposure time**(EP) is L.

EOQ determines the order size (and frequency) that balances setup costs and holding costs. ROP is chosen to meet a desired service level for the lead time. demand.

Finding EOQ

$$EOQ = Q^\star = \sqrt{\frac{2DS}{H}} \tag{20}$$

Note that the unit of D and H **must match**. E.g. if H is in annual terms, then D has to be annual demand as well.

Finding ROP

Set ROP so the in-stock probability equals the desired SL during the lead time of L days (exposure period).
If D_L is normally distributed,

$$z = \texttt{NORM.S.INV}(SL) \tag{21}$$
$$SS = z\sigma_L = z\sigma_d\sqrt{L} \tag{22}$$
$$ROP = d_L + SS \tag{23}$$

If D_L has a discrete distribution,

$$ROP = \text{argmin}_q \left(P(D_L \leq ROP) - SL\right) \tag{24}$$
$$d_L = E(D_L) = \sum_i p_i d_i \tag{25}$$
$$SS = ROP - d_L \tag{26}$$

Average Inventory Profile

$$\mu_I = \frac{Q^\star}{2} + SS \text{ if firms pay upon receiving the order} \tag{27}$$
$$\mu_I = \frac{Q^\star}{2} + SS + dL \text{ if firms pay upon placing the order} \tag{28}$$

where $\frac{Q}{2}$ is the avg. cycle inventory and dL is the pipeline inventory (Demand for L days).

Inventory Pooling

Centralizing inventory (e.g., keeping all inventory in a central storage) reduces cost. Assume there are k stores each with a lead time demand of d and standard deviation of σ_L. Then, in a **decentralized** environment

$$SS = z\sum_i^k \sqrt{\sigma_L^2} = kz\sigma_L \tag{29}$$

Whereas in a centralized distribution system

$$SS = z\sqrt{\sum_i^k \sqrt{\sigma_L^2}} = \sqrt{k}z\sigma_L \tag{30}$$

Delayed Differentiation

Also reduces inventory cost. Assume k products with independent demands, and each with a lead time demand of d and standard deviation of σ_L. Then, if we keep inventory of final products

$$SS = kz\sigma_L \tag{31}$$

whereas if we keep inventory of base model, then postpone final differentiation

$$SS = \sqrt{k}z\sigma_L \tag{32}$$

Reasonings are alike inventory pooling.

Periodic review model

We use the continuous review model when
- Random demand
- If stockouts occur, demand is backordered until the next shipment arrives
- Excess inventory can still be sold in future periods
- Constant lead time (L days)
- Place orders every review period (T days)

Simple Rule: Periodic review system means **time-triggered order**. When it's time to order, order up to a TSL (target stock level). EP is $L + T$ (lead time + review period). EP is the length of time that the firm is exposed to the risk of stockouts.

$$\text{Order quantity} = TSL - \text{Net Inventory} \tag{33}$$
$$= TSL - (\text{On-hand inventory} + \text{Open orders} - \text{Backorders}) \tag{34}$$

Finding EP and TSL

Once an order is placed at time t, no other later orders can be received until time. Thus

$$EP = L + T \tag{35}$$

From here we can set the TSL. If D_{EP} is normally distributed,

$$z = \texttt{NORM.S.INV}(SL) \tag{36}$$
$$SS = z\sigma_{EP} = z\sigma_d\sqrt{L+T} \tag{37}$$
$$TSL = d_{EP} + SS \tag{38}$$

If D_{EP} is has a discrete distribution,

$$TSL = \text{argmin}_q \left(P(D_{EP} \leq TSL) - SL\right) \tag{39}$$
$$d_{EP} = E(D_{EP}) = \sum_i p_i d_i \tag{40}$$
$$SS = TSL - d_{EP} \tag{41}$$

Finding Optimal SL

We use the newsvendor logic here.
- C_o: Cost of overstocking a unit, which is Cost of **holding one unit** over an **order cycle of length** T
- C_u: Cost of understocking a unit, which is **backorder cost, goodwill cost, etc.**

Similarly,

$$SL = \frac{C_u}{C_u + C_o} \tag{42}$$

Average Inventory Profile

$$\mu_I = \frac{d_T}{2} + SS \text{ if firms pay upon receiving the order} \tag{43}$$
$$\mu_I = \frac{d_T}{2} + SS + dL \text{ if firms pay upon placing the order} \tag{44}$$

Summary

Continuous Review is more **flexible**, **shorter** EP – a firm would generally prefer to use it if possible.
Periodic review may be necessary if
- Too difficult/expensive to track current inventory
- Supplier has bargaining power and/or capacity constraints
- Complex/rigid Shipping and Logistics
- Coordinating orders across multiple products from the same supplier

In continuous review, **demand** affects **order timing**. In periodic review, **demand** affects **order quantity**.