The Nature of Chain Drift

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Abstract

A chained price index is said to suffer from chain drift bias if it indicates an overall price change, even though the prices and quantities in the current period have reverted back to their levels of the base period. The empirical relevance of this bias is well documented in studies that apply sub-annual chaining to scanner data. There it is shown that stockpiling can lead to *downward* chain drift bias. The present paper draws attention to the fact that smoothing consumption causes *upward* chain drift. In addition, this study introduces a stochastic simulation approach with a novel utility framework that is consistent with both, stockpiling and consumption smoothing. A "stress test" is conducted that examines whether rolling window variants of multilateral indices (GEKS, TPD, and GK) effectively curtail the chain drift problem.

Keywords: bias, habits, multilateral price index, sales, scanner data, simulation
JEL-Classification: C43, E31

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1 Introduction

The conventional approach to compute the price change between a base period 0 and a comparison period T uses only the prices and quantities of these two periods. Such index formulas are known as *bilateral price indices* (or direct price indices). In the following, they are denoted by $P^{T/0}$. As an alternative to the bilateral index, $P^{T/0}$, one may compute the price change between periods 0 and T from a sequence of overlapping adjacent bilateral price indices. The elements of this sequence can be linked by multiplication: $P^{1/0}P^{2/1}\ldots P^{T/T-1}$. Such products are known as *chain indices* and its factors as *chain links*.

When the item universe remains constant over time and all prices and quantities in the current period T revert back to their levels in the base period 0, a price index comparing the current period with the base period should indicate that no price change occurred. All reasonable bilateral price indices, $P^{T/0}$, satisfy this requirement. However, chained bilateral price indices usually violate it. In the present paper, this violation is denoted as *chain drift bias*. It is usually attributed to *sales* triggering non-standard substitution behaviour of consumers. Feenstra and Shapiro (2003, p. 135) examine scanner data on canned tuna and compile a weekly chained Törnqvist index that exhibits *upward* chain drift caused by sales, while de Haan (2008, p. 19), studying scanner data on detergents, finds that sales lead to *downward* chain drift, even though both studies apply the same index formula and frequency of chaining.

These contradictory results suggest analysing the causes of chain drift in a more systematic way. The present paper puts its focus on chained bilateral indices that include the quantities of both, the base and the current period (e.g., the indices of Törnqvist, Fisher, Marshall-Edgeworth, and Walsh). It is shown that the chain drift of such chained bilateral price indices is caused by quantity and price changes that are not perfectly synchronised in time.[2] Such asynchronous price and quantity changes can arise from different forms of intertemporal optimisation behaviour of consumers.

A well known example of such consumer behaviour is *stockpiling*. During a sale, the consumers increase both their consumption and their stocks. Therefore, the purchases "overshoot" consumption. As soon as the price returns to normal, the purchases overshoot consumption in the downward direction because the consumers first use up their extra stocks. The purchase of a durable increases the stock of consumption such that expenditure When in the subsequent periods the price remains on its normal level, purchased quantities gradually align with consumed quantities. For simplicity, these consequences of stockpiling are denoted here as *overshooting quantity* reactions to price changes. Also temporary price spikes trigger overshooting quantity reactions.

[2]Hill (2006, pp. 314-315) reaches a similar conclusion for the chained Laspeyres index and the chained Paasche index.

There are other forms of intertemporal optimisation behaviour leading to asynchronous price and quantity changes. Probably the most important one are delayed quantity adjustments to price changes. Often, this *smoothing* is caused by search and adjustment costs. Even so the price of a consumer's favourite product may have permanently increased or the price of a competing product may have permanently fallen, the consumer may show no or only a moderate immediate change in her usual purchasing behaviour. Only at some later point of time, after acquiring information about the qualitative features or the handling of alternative products, she may partly or completely switch over to such alternatives. Other reasons for delayed quantity reactions are consumption habits caused by harmful addictions (e.g., nicotine) or by past investments in increased enjoyment from consumed goods (e.g., ability to cook tasty and healthy food). For simplicity, all delayed quantity adjustments are denoted here as *sticky quantity* reactions to price changes.

Several studies of scanner data (e.g., de Haan, 2008, p. 18; de Haan and van der Grient, 2011, p. 43) demonstrate that overshooting quantities cause *downward* chain drift. The chain drift arising from sticky quantities, however, went largely unnoticed.[3] Therefore, the present paper's first contribution is to demonstrate that sticky quantities lead to *upward* chain drift.

As a solution to the chain drift problem, Ivancic *et al.* (2011) advocate a rolling window variant of the Gini-Éltető-Köves-Szulc (GEKS) approach. The GEKS index is free of chain drift. However, its rolling window variant (R-GEKS) involves a mechanism that links the current window to past windows. This special form of chaining is usually denoted as *splicing*. It cannot be ruled out that splicing generates chain drift.

By now, several variants of such R-GEKS indices have been developed.[4] Besides R-GEKS indices, many other multilateral approaches have been proposed to overcome the chain drift problem. These include rolling window variants of the time-product dummy method (R-TPD) and of the Geary-Khamis approach (R-GK).[5] Again, these methods require some form of splicing. Consequently, they might also suffer from chain drift bias.

How can one investigate this suspicion? A meaningful examination requires an unassailable benchmark such that the deviation of an index number from that benchmark represents chain drift bias. This benchmark directly follows from the definition of chain drift: when all prices and quantities revert back to their former levels a price index should indicate that no price change occurred. Unfortunately, the prices and quantities of real world data (e.g., scanner data) never return to their original levels. Therefore, an analysis of chain drift that uses real world data must do without this natural benchmark.

[3] A notable exception is Triplett (2003, p. 152) who points out that storage, search, and information cost may generate measurement problems for scanner data price indices.

[4] For compact surveys see, for example, Diewert and Fox (2020, pp. 3-5), Fox *et al.* (2022, pp. 9-12), Van Loon and Roels (2018, pp. 7-8), or Appendix B of the present paper.

[5] For recent surveys of the various methods see, for example, Chessa *et al.* (2017), de Haan and Krsinich (2014), or Diewert and Fox (2020).

By contrast, in a simulation approach the researcher can generate price-quantity scenarios where all prices and quantities return to their original levels. However, the simulation must ensure that the price-quantity scenarios reflect features of real world purchasing behaviour. This requires price scenarios with sales and ordinary price changes as well as quantities that reflect the intertemporal optimisation behaviour of households. Thus, the second contribution of the present study is a simple and transparent utility framework that, for the purpose of the simulation, transforms price scenarios into quantity scenarios. It captures overshooting and sticky quantities arising from consumer behaviour such as stockpiling and smoothing of consumption.

This study's third contribution is a quantitative analysis of the chain drift bias of the R-GEKS, R-TPD, and R-GK approaches. It turns out that the suspicion was correct: these index methods reduce chain drift bias, but they cannot eliminate it. Furthermore, some variants are more effective than others.

As a first step, Section 2 introduces the notion of chain drift and explains, why overshooting quantities and sticky quantities cause different directions of chain drift. Section 3 briefly discusses the implications of chain drift for price measurement relying on scanner data and explains why a simulation approach is appropriate for a systematic examination of the various index methods' resilience to chain drift bias. Section 4 introduces a new utility framework. Drawing on this utility function, Section 5 presents a simulation based "stress test" that examines whether the R-GEKS, R-TPD, and R-GK methods curtail chain drift and whether some variants are better suited than others. Section 6 concludes.

2 Chain Drift and Its Sources

Let the integers $i = 1, \ldots, N$, represent the N items of an economy. All items are available during the base period ($t = 0$) and the comparison period ($t = T$). The period t vector of prices is $(p_1^t, \ldots, p_N^t)$ and the corresponding vector of quantities is $(x_1^t, \ldots, x_N^t)$. It is customary to interpret a bilateral price index, P, as a mapping of the N-dimensional vectors $(p_1^0, \ldots, p_N^0)$, $(x_1^0, \ldots, x_N^0)$, $(p_1^T, \ldots, p_N^T)$, and $(x_1^T, \ldots, x_N^T)$ into a single positive number, $P^{T/0}$, that measures the "overall price change" between periods 0 and T. All bilateral price indices considered in this study are listed in Appendix A.

Consider some sequence of time periods, $t = 0, \ldots, T$.

Definition 1 *A chain index that compares period T to the base period 0 is defined by*

$$\widetilde{P}^{T/0} = P^{1/0} P^{2/1} \ldots P^{T-1/T-2} P^{T/T-1} \,. \tag{1}$$

When the prices and quantities in period T return to their levels of period 0, the chain index, $\widetilde{P}^{T/0}$, should give the same index number as the bilateral index, $P^{T/0}$, namely 1 (e.g., Ivancic *et al.*, 2011, p. 26). Accordingly, in this specific price-quantity-scenario, the deviation from unity is an appropriate measure of the extent of chain drift bias (e.g., Ribe, 2012, p. 3; Diewert, 2018, p. 25; Diewert and Fox, 2020, p. 3). This interpretation of chain drift bias can be formalised in the following way:

Definition 2 *The chain drift test of the bilateral price index, $P^{t/t-1}$, postulates that*

$$P^{1/0}P^{2/1}\ldots P^{T-1/T-2}P^{0/T-1} = 1\ . \tag{2}$$

Condition (2) can be found in Walsh (1901, p. 401).[6] Note that for $T = 1$ the chain drift test simplifies to the time reversal test: $P^{1/0} = 1/P^{0/1}$.[7]

In standard microeconomic consumer theory, consumers instantaneously adjust their current purchases to the current prices. The usual assumption is that consumers substitute away from products that have become relatively more expensive. The larger the price elasticity of demand, the more pronounced are these quantity reactions. Figure 1 translates this standard theory into a highly stylised example. The figure depicts the prices and quantities of some item during nine consecutive periods, $t = 0, \ldots, 8$. In the depicted *Scenario 1*, the prices of the item can take only the values "low", "normal", and "high", and the quantities sold can be "small", "normal", or "large".

Price changes are marked by the weight icons. The price starts at a normal level, drops in period 1 to the lower level, stays there for another period, returns to normal in period 3, and stays there also during period 4. In period 5, the price increases to high, stays there for another period, before it drops back to normal during period 7 and remains there during period 8. The quantities purchased move exactly inversely to the prices, that is, the consumers' quantity reactions are perfectly synchronous to the price changes. As soon as the price returns to normal, also the quantity returns to normal. In times of constant prices also the quantities remain constant. When the price elasticity of demand is less than unity, expenditure shares and prices are positively correlated.

[6] Walsh later calls condition (2) the circularity test (see Diewert, 1993, p. 40). In the current price index literature, this label is reserved for the condition $P^{2/0} = P^{2/1}P^{1/0}$. This condition is stricter than the alternative formalization of chain drift because it does not prescribe that the prices and quantities of periods 0 and 2 coincide. In the context of spatial price comparisons, circularity is usually denoted as transitivity. Diewert (1993, p. 40) coins the chain drift test as the "multiperiod identity test". Note, however, that the chain drift (or multiperiod identity) test considers the case where the quantities reverse to their base period values, while in the identity test the evolution of the quantities is completely irrelevant. In fact, a chain drift test without the quantity reversal postulate is considered in de Haan (2008, p. 10). He calls it the "invariance to price bouncing test".

[7] An anonymous referee pointed out that for bilateral price indices that satisfy the time reversal test, one gets $P^{T/T-1} = P^{0/T-1} = 1/P^{T-1/0}$ and condition (2) becomes $(P^{1/0}P^{2/1}\ldots P^{T-1/T-2})/(P^{T-1/0}) = 1$. According to this alternative formalization, a bilateral price index suffers from chain drift when the ratio of the chained index and the corresponding bilateral index, $\widetilde{P}^{T-1/0}/P^{T-1/0}$, deviates from unity. Several writers use this alternative formalization of chain drift (e.g., Forsyth and Fowler, 1981, p. 234; Frisch, 1936, p. 8; Hill, 2006, pp. 14-15; Lent, 2000, p. 314; Persons, 1921, p. 109; Persons, 1928, p. 101).

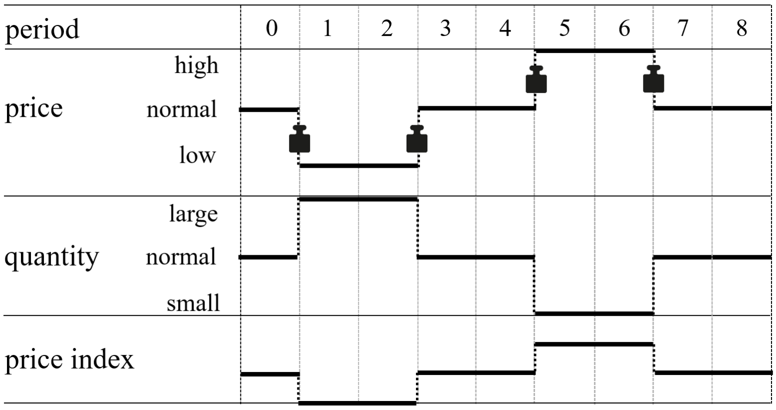

Figure 1: Synchronous Quantity Reactions to Price Changes (Scenario 1).

Many price indices are expenditure weighted averages of intertemporal price ratios (p_i^T/p_i^0) where the impact of the two periods 0 and T on the expenditure weights is symmetric (e.g., Törnqvist, Walsh, Marshall-Edgeworth, Theil, Sato-Vartia).[8] Due to this symmetric impact, Scenario 1 generates weights such that the price decline between periods 0 and 1 (its weight is related to the expenditures during periods 0 and 1) and the price increase between periods 2 and 3 (its weight is related to the expenditures during periods 2 and 3) exactly offset each other. Graphically, this balanced weighting is indicated by the equal sized weight icons located at the price decline between periods 0 and 1 and at the price increase between periods 2 and 3. The same is true for the price increase between periods 4 and 5 and the price decline between periods 6 and 7. As a consequence of this balanced weighting of price increases and price declines, symmetrically weighted indices are immune to chain drift from scenarios like Scenario 1 and one gets $P^{1/0} \cdot P^{2/1} \cdot \ldots \cdot P^{8/7} = 1$.[9]

Scenario 1 corresponds to standard consumer theory as presented in introductory microeconomics textbooks. Real world consumer behaviour, however, is more complex. One important aspect ignored by standard consumer theory is stockpiling. For example, sales usually lead to increased purchases, part of which are stored. If in the next period the price returns to its normal level, the purchased quantity falls *below* its normal level, because consumers first use up their extra stock. Only after the extra stock is depleted, the purchased quantity returns to its normal level. In the following, such a scenario is denoted as an *overshooting* quantity response to sales.[10] During the sales period,

[8]The Fisher index and the generalised unit value (GUV) indices of Banerjee, Davies, and Lehr (Auer, 2014, pp. 848-852) exhibit a similar type of "intertemporal symmetry". A more complete classification of "symmetric bilateral price indices" is provided by Auer and Shumskikh (2022, Section 2).

[9]Chain drift arises, however, for the Laspeyres and Paasche index. This issue is addressed in various studies including Forsyth and Fowler (1981, pp. 234-235), Szulc (1983, pp. 540-541), and Hill (2006, pp. 314-315).

[10]This type of scenario is described, for example, in Ivancic *et al.* (2009, p. 4), de Haan and van der

acquisitions exceed consumption (that is, stocks increase), while right after the sales period, consumption exceeds acquisitions (stocks decrease).

Periods 0 to 3 of *Scenario 2* (see Figure 2) depict this case in a highly stylised form. In such a scenario, symmetrically weighted indices exhibit chain drift. As a result of the overshooting quantity response, the weight attached to the price reduction between periods 0 and 1 is larger than the weight attached to the price increase between periods 1 and 2. Therefore, the price index level of period 2 is below that of period 0. At the end of period 2, the customers' inventory is back to its normal level. Therefore, the purchases in period 3 increase to their normal level, even though the price is constant. In the literature, there is a rather broad consensus that every valid bilateral index formula must satisfy the identity test. This test says that, in the absence of any price changes, the price index is unity, regardless of any quantity changes. All price indices listed in Appendix A satisfy the identity test. Therefore, the price index level of period 3 remains on the level of period 2 and, thus, below that of period 0. In other words, downward chain drift arises.

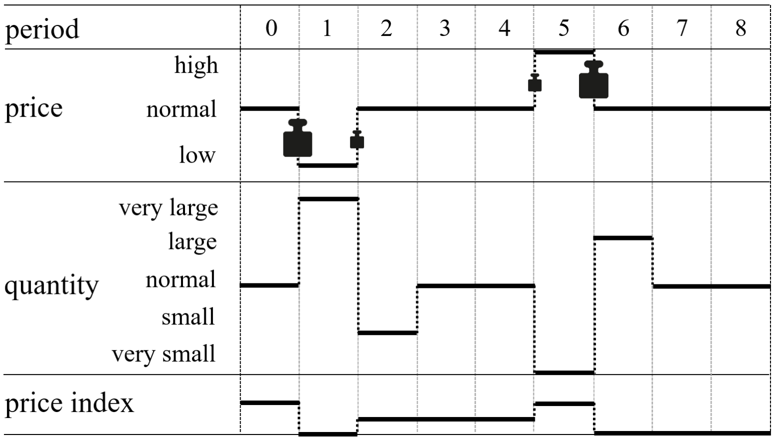

Figure 2: Overshooting Quantity Reactions Caused By Sales Or Price Spikes (Scenario 2).

Periods 0 to 3 of Scenario 2 describe a storable item that consumers keep in stock. In times of unusually low prices (sales) the consumers add to their ordinary stock some extra stock and they deplete that extra stock when the price reverts to its normal level. This is the standard narrative of stockpiling.

However, stockpiling is not only relevant in times of sales, but also in times of price spikes. During such spikes consumers can plunder their ordinary stock and restock it as soon as the price returns to normal. Again, overshooting quantity reactions arise. This is depicted by periods 4 to 8 of Scenario 2. The price increases in period 5. Many consumers switch to using up their stocks. This leads to a negative quantity reaction

Grient (2011, p. 39), and Ribe (2012, p. 3).

that is larger than for items that cannot be stored. In period 6, the price returns to its normal level. This gives the consumers the opportunity to refresh their inventories. At the same time they return to their normal consumption. Therefore, the total quantity purchased exceeds the normal quantity. As a result of this overshooting quantity response, the weight attached to the price increase between periods 4 and 5 is smaller than that attached to the price reduction between periods 5 and 6. This leads again to downward chain drift. At the end of period 6 the stock is back to its standard level, such that the purchases in period 7 return to their normal level even though no price change occurs between periods 6 and 7.

In sum, overshooting quantities triggered by sales or by price spikes work in the same direction. Both generate downward chain drift. Overshooting quantities, however, are only one driver of chain drift. Another driver operates in the opposite direction, that is, it causes upward chain drift.

In the field of industrial organisation there is extensive literature on search and adjustment costs and their implications for markets.[11] Also in the field of price measurement it is well known that search and adjustment costs are relevant in real world consumption decisions and that they create problems for price measurement purposes (e.g., Reinsdorf, 1994, p. 137; Triplett, 2003, p. 152). Such costs can delay the consumers' substitution behaviour, such that part of the quantity response or the complete quantity response happens in a later period than the underlying price change. This is particularly true when the length of a period is relatively short (e.g., one week or one month). Another cause of delayed quantity responses are harmful or beneficial consumption habits that the consumers have developed over time. Delayed quantity responses are denoted here as *sticky* quantities.

Figure 3 illustrates the consequences of sticky quantities. In the depicted *Scenario 3* demand is completely price inelastic in the short-run, but price elastic in the long-run. More specifically, the complete quantity reaction to each price change is delayed by one period.

In period 1 the price drops, while the quantity remains unchanged. Therefore, the observed expenditure during period 1 is smaller than it would be with the usual quantity reaction of an elastic demand. As a consequence, also the weight attached to the price decline is smaller than it would be with an elastic demand. Graphically, this diminished weight is indicated by the small weight icon at the price decline between periods 0 and 1.

The complete quantity reaction to the reduced price in period 1 occurs one period delayed, that is, in period 2. The price in period 2 remains on the level of period 1. All popular price indices satisfy the identity test. Therefore, their price index level does not change between periods 1 and 2.

Since demand is inelastic in the short-run, the price increase between periods 2 and 3

[11] A recent survey of this literature is Fisher Ellison (2016).

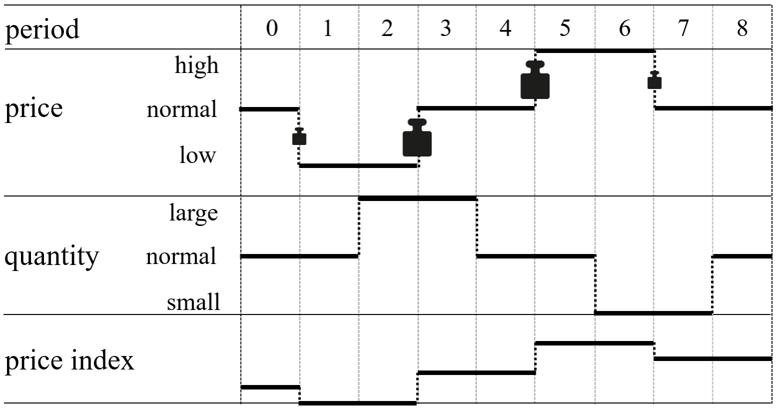

Figure 3: Sticky Quantity Reactions to Price Changes (Scenario 3).

occurs without a quantity reduction. Therefore, the weight attached to this price increase is larger than it would be with the usual quantity reduction. The large weight icon at the transition from period 2 to period 3 highlights this inflated weight. The price increase between periods 4 and 5 is analogous to that between periods 2 and 3. Again, the price increase receives an inflated weight. The price decline between periods 6 and 7 shows the same pattern as that between periods 0 and 1. This price decline receives a diminished weight. Overall, this unbalanced weighting of price declines and price increases leads to *upward* chain drift.

Theoretically, one can think of quantity reactions that are antedated by one period. This could be regarded as a "negative delay". The weighting effects are exactly opposite to those of Scenario 3. Price increases receive a diminished weight, whereas price declines receive an inflated weight. As a result, downward chain drift would arise. In reality, such anticipating consumption behaviour is unlikely, unless stockpiling is involved. Then stocks are depleted in anticipation of the sale. This tends to aggravate the overshooting quantities effect and the resulting downward chain drift. Anticipated price spikes have the same effect.

In summary, sticky quantities (the result of smoothing of consumption) lead to upward chain drift, whereas overshooting quantities (the result of stockpiling) lead to downward chain drift.[12] The underlying problem of both cases is the asymmetric weighting of price increases and price reductions.[13] This asymmetry is facilitated by quantity changes in times of constant prices. Since all popular bilateral price indices satisfy the identity test,

[12]Stockpiling and consumption smoothing do not themselves create chain drift bias. They become relevant when not all prices and quantities move exactly the same and/or when some items are less suitable for stockpiling (e.g., hair cut) than others or some items are less suitable for consumption smoothing (e.g., medical service) than others.

[13]A Cobb-Douglas index with weights that remain fixed over the complete time horizon would avoid such asymmetries.

they indicate in such periods no overall price change. This suggests that a solution to the chain drift problem may come from price indices that violate the identity test. The R-GEKS indices studied in Sections 4 and 5 are a prominent example. They violate the identity test.[14] This could be interpreted as a weakness of the R-GEKS indices. On the contrary, the fact that immunity to chain drift requires price indices that violate the identity test can be interpreted as a weakness of the identity test.[15]

Overshooting and sticky quantities generate diametrical chain drift bias. For example, Feenstra and Shapiro (2003, p. 133) identify delayed quantity responses in the context of sales.[16] In their data set, the immediate quantity response to a sale is modest, but substantially increased after, in a later period, the sale is advertised. Overall, they identify upward chain drift bias. This suggests that in their data set the upward bias generated by sticky quantities dominates the downward bias generated by overshooting quantities.[17]

3 Implications for a Quantitative Analysis of Chain Drift Bias

Storing one's favourite beer when it is on sale may reduce further purchases of beer for some weeks or months but probably not for years. Delayed quantity reactions are also rather a question of weeks than years. This implies that chaining of monthly price indices is more likely to cause chain drift bias than chaining of yearly price indices. Since scanner data allow for the compilation of monthly or even weekly price indices, the issue of chain drift becomes particularly relevant for this type of data source.

GEKS indices offer themselves as a solution because they are transitive and, therefore, free of chain drift.[18] However, when the time span covered by a GEKS index becomes

[14]This is noted also by Ribe (2012, p. 4).

[15]In ILO (2004, p. 293) it is reported that the identity test is somewhat controversial. A more elaborate critique of the identity test can be found in Auer (2008, pp. 2-7).

[16]Adapting an artificial scenario analysed in Persons (1928, p. 102), Diewert (2018, p. 27) discusses a situation that combines sales with somewhat delayed quantity responses. This scenario resembles the pattern that Feenstra and Shapiro (2003) identify in their scanner data set.

[17]In footnote 7, Hill (2006, p. 315) states two *sufficient* conditions for the Fisher index to exhibit downward chain drift bias. Put simply, the first condition says that the quantities of period t and the prices of period $t+1$ are negatively correlated. This condition corresponds to the case of antedated quantity reactions briefly mentioned in the present paper's exposition. The second condition says that the prices of period t and the quantities of period $t+1$ are positively correlated. This correlation corresponds to the stockpiling behaviour depicted in Figure 2 of the present study. Reversing the sign of the correlations, yields two sufficient conditions for the Fisher index to exhibit upward chain drift bias. The second of these conditions corresponds to consumption smoothing depicted in Figure 3 of the present study.

[18]The acronym GEKS honours the publications of Gini (1924), Éltető and Köves (1964), and Szulc (1964) who introduced this approach for interregional price comparisons. Balk (1981, pp. 73-74) adopts

too large, the measured price change of neighbouring periods is affected by very distant periods. This may reduce the reliability of the results. As a solution, Ivancic *et al.* (2011, p. 33) propose a rolling window variant of the GEKS approach. This rolling window variant is denoted here as R-GEKS index. In its original form, the GEKS index is based on the bilateral Fisher index, while Caves *et al.* (1982) propose to use the Törnqvist index. Diewert and Fox (2020, p. 1) denote the latter variant as the Caves-Cristensen-Diewert-Inklaar (CCDI) approach.

When a R-GEKS index is used, the price levels compiled from the most recent window must be linked to the price levels compiled from the earlier windows. Empirical studies demonstrate that the choice of the linking procedure (called "splicing") affects the results of R-GEKS indices (e.g., Fox *et al.*, 2022, pp. 17-22; Lamboray, 2021, pp. 13-17; Melser, 2018, pp. 518-521; Van Loon and Roels, 2018, pp. 9-14). The R-GEKS approach and five different splicing variants are explained in Appendix B. The five mentioned variants are the following: movement splice (Ivancic *et al.*, 2011, p. 33), window splice (Krsinich, 2016, pp. 383-87), half splice (de Haan, 2015, pp. 25-26), mean splice (Diewert and Fox, 2020, pp. 6-7), and mean movement splice (Melser, 2018, p. 518).[19] Splicing can be viewed as a special form of chaining. In fact, the splicing of chain drift free GEKS indices can cause chain drift bias.

R-GEKS indices are not the only rolling window indices that have been proposed to solve the issue of chain drift bias. The alternatives include rolling window variants of the time-product dummy method (R-TPD) and of the Geary-Khamis approach (R-GK).[20] The available splicing variants are the same as for R-GEKS indices and the suspicion of chain drift bias also applies to the R-TPD and R-GK approaches.

The diversity of index and splicing methods raises the question whether one method is more immune to chain drift than others. Does the relative performance of the methods depend on the relative strength of stockpiling (overshooting quantities) and smoothing of consumption (sticky quantities)? The cycle lengths of overshooting quantities and sticky quantities are likely to differ. Therefore, a window length that is suitable for addressing overshooting quantities is unlikely to be adequate for sticky quantities. In other words, a one-size-fits-all window may not exist. In such a situation, should one apply a shorter or rather a longer window length? Do the answers to these questions depend on the consumers' behavioural characteristics (e.g., price elasticity of demand)?

A quantitative analysis of these questions should use an unassailable benchmark for assessing the extent of chain drift bias inherent in the index methods to be compared. To

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this approach to the intertemporal price measurement of seasonal products. That multilateral price indices can curtail chain drift bias is pointed out also in Kokoski *et al.* (1999, p. 141).

[19] Diewert and Fox (2020, p. 7) advocate another splicing strategy that is not considered in the present study. Melser and Webster (2021, p. 765) point out that for theoretical purposes the linking stage could be replaced by a second round of GEKS aggregation.

[20] An exposition of these methods can be found in various studies (e.g., Diewert and Fox, 2020, pp. 5-7).

obtain this benchmark, the data set should be such that, after some periods, all prices and quantities return to their original values. Scanner data and other real world data cannot be expected to satisfy this postulate.[21] Therefore, Goolsbee and Klenow (2018) propose to add to the scanner data set a hypothetical observation for period $T+1$ where all prices and quantities return to their original values. However, when the prices and quantities move far away from their original values, a sudden return to those original values would constitute a rather unrealistic scenario.

An alternative to such semi-artificial price-quantity scenarios are fully artificial price-quantity scenarios. However, these scenarios should be sufficiently "realistic", that is, they should capture those patterns of real world data that can cause chain drift bias. To begin with, the price data should be generated by a transparent stochastic process that features ordinary price changes as well as sales. Furthermore, the transformation of the price scenario into a corresponding quantity scenario should mimic utility maximising households that smooth consumption and benefit from stockpiling. In other words, a model is required that combines both behavioural traits in one unifying utility framework.[22]

Once a price-quantity scenario is generated in this way, the index numbers of the various index methods could be computed and compared to the reference. As a result, some index methods may look superior to some other methods. However, this ranking may depend on the specific price-quantity scenario. Therefore, the complete process should be repeated many times and, for each index method, the deviations of the index numbers from the reference should be averaged over these repetitions.[23]

The ranking of the index methods may also depend on the parameter values of the utility framework transforming the price scenarios into quantity scenarios. Therefore, alternative parameter values must be examined. To this end, the complexities of real world intertemporal decision making should be condensed into a simple utility framework with few parameters. Each parameter should have a clear economic interpretation. For

[21]Estimating from real world data the parameters driving the households' intertemporal consumption paths is a difficult task because the observable date of a product's acquisition often deviates from its unobservable date of consumption and stockpiling. Various sophisticated stockpiling models attempt to solve this problem. For example, Osborne (2018) considers stockpiling households and estimates a dynamic structural utility model from real world household-level scanner data. The author also relates his approach to alternative stockpiling models, including those of Feenstra and Shapiro (2003) as well as Hendel and Nevo (2006). There is a separate but similarly sophisticated literature on empirical models of smoothing of consumption. For example, Hong and Shum (2006) show how the equilibrium conditions of standard search models can be exploited to estimate search cost distributions solely from observed prices. Honka et al. (2019) survey the econometric literature on consumer behaviour in the presence of search cost.

[22]In Section 4, the need for a unifying utility framework is further elaborated.

[23]Real world data can be interpreted as an experiment with only one sample. Repeated samples could generate results that differ from the observed one. This possibility weakens the reliability of the results obtained from the actually observed sample.

example, one parameter should represent the cost of stocking and some other parameter should represent the households' desire to smooth consumption. The following section introduces such a utility framework.

4 Utility Framework

Triplett (2003, p. 152) points out that the analysis of high-frequency data such as scanner data is an analysis of acquisitions and not of consumption: "To confront the household behaviour recorded in high-frequency data requires a theory that adequately describes search, storage, shopping, and other household activities that drive a wedge between acquisition periodicity and consumption periodicity." Households build up inventories during sales and afterwards gradually reduce these inventories to their standard levels. This stockpiling behaviour results in overshooting quantities. Households benefit from stockpiling in at least three ways. By exploiting sales, households can buy the product at a lower average price. Furthermore, as the Covid-19 pandemic and the Russian invasion of Ukraine has shown, stocks represent a hedge against supply interruptions. Finally, consumers usually have adjustment costs when they adapt their purchased quantities to price changes. Stocks represent buffers against unwanted adjustments and the associated cost. Thus, stockpiling and smoothing of consumption are not independent.

Neither are they fully dependent. Stockpiling would occur even without the desire to smooth consumption, simply as a hedge against future price increases. Conversely, due to habits and addiction, smoothing of consumption would occur even if the cost of stockpiling were infinite. These considerations imply that a quantitative analysis of the chain drift bias of different index methods should address the phenomena of stockpiling and smoothing of consumption in one unifying framework.

Regardless of which benefit of stockpiling prevails, sales lead to overshooting quantities. It would be desirable to have a utility framework that captures this effect. For the purpose of the intended simulation approach outlined in Section 3, a simple solution suffices. Therefore, the present paper proposes a utility framework with *myopic* households where stockpiling *directly* contributes to the household's utility. This considerably simplifies the analysis. The household optimises present consumption and stocks, given the prices, income, and stocking costs of the present period, and given the consumption and stocks of the past. The utility framework incorporates not only stockpiling behaviour but also delayed quantity adjustments. This second important dynamic feature of consumer behaviour is absent from existing models of households engaged in stockpiling.

The new utility framework is composed of two CES functions. One of them captures the subutility of consumption and the other one the subutility of stockpiling. The two CES functions are nested in a Cobb-Douglas utility function.

Consider some representative household endowed with income m and stocks $\bar{s}_i$ ($i =$

$1, \ldots, N$) inherited from the previous period. To avoid a cluttered notation, time subscripts are omitted. Instead, a bar on a variable indicates that the value is from the previous period. The budget constraint of the household in the present period is

$$m + \sum_{i=1}^{N} p_i \bar{s}_i = \sum_{i=1}^{N} p_i c_i + \sum_{i=1}^{N} p_i (1 + \mu) s_i \ , \tag{3}$$

where p_i is the price, c_i is the consumed quantity, s_i is the stock (retained for the next period), and $\mu p_i s_i$ ($\mu \geq 0$) is the storage cost of item i in the present period. For simplicity, it is assumed that, if the household wishes, it can sell during the present period units of the inherited stock $\bar{s}_i$ at the price p_i. The left-hand side of Equation (3) can be considered as the household's "net endowment", $\tilde{m}$. The purchased quantity, x_i, is the sum of the consumed quantity, c_i, and the change in stocks, $(s_i - \bar{s}_i)$:

$$x_i = c_i + s_i - \bar{s}_i \ . \tag{4}$$

The household's utility function is given by

$$U = C^{\alpha} S^{1-\alpha} \tag{5}$$

with

$$C = \left[\sum_{i=1}^{N} \left(\frac{c_i}{(\bar{c}_i)^{\gamma}} \right)^{\theta} \right]^{1/\theta} \qquad \text{and} \qquad S = \left[\sum_{i=1}^{N} (s_i)^{\phi} \right]^{1/\phi} \ , \tag{6}$$

where the term $c_i/(\bar{c}_i)^{\gamma}$ describes, for given past consumption, $\bar{c}_i$, the contribution of present consumption, c_i, to subutility C.[24] A positive value of γ would capture the phenomenon that past consumption produces "aspirations" that are harmful to the utility derived from present consumption. Besides the assumptions $\theta, \phi \neq 0$, the following parameter restrictions are imposed: $0 < \alpha \leq 1$ and $\theta, \phi < 1$.

The parameter α represents the household's preference for instantaneous consumption. If α were equal to 1, the household would never want to stockpile. Furthermore, Appendix C shows that $-1/(1-\theta)$ and $-1/(1-\phi)$ approximate the price elasticities of present consumption and stockpiling, respectively. Therefore, negative values of θ and ϕ imply that consumption and stockpiling are inelastic and households cannot easily substitute between the various items.

A price-induced change of c_i implies that in next period's optimisation a new value $\bar{c}_i$ is relevant, triggering another adjustment of c_i even without any further price changes. The term $-\gamma\theta/(1-\theta)$ approximates the elasticity of present consumption, c_i, with respect

[24] The framework can be easily generalized. For example, U can be interpreted as the subutility of the utility function $W = U^{\beta} V^{1-\beta}$, where V is the subutility from all items that cannot be stored (e.g., many services).

to past consumption, $\bar{c}_i$ (see Appendix C). A stable steady-state equilibrium requires that $|-\gamma\theta/(1-\theta)| < 1$. The sticky quantities of smoothing behaviour require that $0 < -\gamma\theta/(1-\theta) < 1$. For $-1 < -\gamma\theta/(1-\theta) < 0$, the adjustment path to the steady-state equilibrium would be oscillatory. Only when $\gamma\theta = 0$, a price change causes an instantaneous and complete quantity adjustment.

The maximisation of the utility function defined by (5) and (6), and subject to the constraint (3) yields the following purchasing behaviour (see Appendix C):

$$x_i^* = \underbrace{\alpha \tilde{m} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}}}_{c_i^*} + \underbrace{(1-\alpha)\, \tilde{m} \tilde{p}_i^{\frac{-1}{1-\phi}} P_S^{\frac{\phi}{1-\phi}}}_{s_i^*} - \bar{s}_i \;, \tag{7}$$

with $\tilde{p}_i = p_i\,(1+\mu)$ representing the price of stocks including storage costs. The "price indices" P_C and P_S are defined by

$$P_C = \left(\sum_{j=1}^{N} \bar{c}_j^{\frac{-\gamma\theta}{1-\theta}} p_j^{\frac{-\theta}{1-\theta}}\right)^{-\frac{(1-\theta)}{\theta}} \qquad \text{and} \qquad P_S = \left(\sum_{j=1}^{N} \tilde{p}_j^{\frac{-\phi}{1-\phi}}\right)^{-\frac{(1-\phi)}{\phi}} \;. \tag{8}$$

The variables x_i^*, c_i^*, and s_i^* in (7) are the optimal values of the variables x_i, c_i, and s_i in Equation (4).

The demand function (7) can be used to compute for any scenario (prices, p_i, income, m, inherited stocks, $\bar{s}_i$, former consumption levels, $\bar{c}_i$, as well as parameters α, θ, ϕ, μ, and γ) the corresponding quantities purchased by the utility maximising household. To capture a stockpiling household (leading to overshooting quantities) that does not smooth quantity adjustments to price changes (no sticky quantities), the parameters must be such that $0 < \alpha < 1$ and $\gamma = 0$. In this case, the demand function (7) simplifies to

$$x_i^* = \tilde{m}\left(\alpha p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}} + (1-\alpha)\, \tilde{p}_i^{\frac{-1}{1-\phi}} P_S^{\frac{\phi}{1-\phi}}\right) - \bar{s}_i \;.$$

Smoothing without stockpiling arises for $\alpha = 1$ and $\gamma \neq 0$. The resulting purchases are

$$x_i^* = \tilde{m} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}} \;,$$

with $\tilde{m} = m$. Setting $0 < \alpha < 1$ and $\gamma \neq 0$ gives a household that combines smoothing and stockpiling.

5 Simulation and Implications

Simulation studies in the context of price index theory are not new. A prominent example is Diewert and Fox (2020, pp. 8-10).[25] We elaborate their approach in various dimensions.

[25]Their simulation considers one price scenario covering four items and twelve periods. They allow for sales and derive the corresponding quantities from consumers with ordinary CES preferences. Since these CES preferences do not account for stockpiling, the quantities are manually adjusted afterwards.

Our simulation comprises $T = 120$ periods ($t = 1, 2, \ldots, 120$). In each period, the households can choose among the same $N = 40$ items ($i = 1, \ldots, 40$). Each item receives a randomly drawn "base price" between 2.00 and 5.00. The simulation starts with a phase-in interval of 10 periods during which all N items are sold at their base price. This interval is followed by a core interval of 100 periods in which some randomly drawn items exhibit price changes triggering quantity reactions. After the core interval, there is a phase-out interval of 10 periods in which all items are sold at their respective base prices. Therefore, not only the prices but also all quantities return to the levels that had prevailed at the start of the phase-in interval.

10 of the 40 items keep their base price throughout the time horizon, while 10 other items exhibit cyclical sales patterns during the core interval. Every cycle starts with a price reduction that randomly lasts for one or two periods. Afterwards the price returns to its basic level and stays there until the next cycle starts. Each of the sales items has its own fixed price reduction (randomly drawn between 10%, 20%, 30%, and 40%) and its own fixed cycle length (randomly drawn from 6 to 12 periods). Also the period for the start of the first cycle differs between the items. It occurs within the first twelve periods of the core interval and is randomly drawn. As an illustration, the upper part of Figure 4 depicts the cyclical sales prices of item 10 of the simulation's first iteration (out of 5000 iterations).

The remaining 20 items exhibit price changes that are not related to sales. On average, the price of such an item changes every fifth period. The periods of change are randomly drawn and the new price can deviate from the previous price by a percentage drawn from a normal distribution (with mean 0 and standard deviation 0.2). This random process applies to periods 11 to 60, that is, to the first half of the core interval. For the second half of the core interval the order of the prices is simply reversed. Thus, even prices that have drifted a long way from their original levels return to these levels in a gradual manner.[26] The upper part of Figure 5 depicts the price evolution of item 9 of the simulation's first iteration.

All households are confronted with the same price tableau (40 items during 120 periods). Two different types of households exist: stockpiling households ($\alpha < 1$ and $\gamma = 0$) and smoothing households ($\alpha = 1$ and $\gamma \neq 0$). The stockpiling households generate *overshooting* quantities, while the smoothing households generate *sticky* quantities.

The two types of households have many features in common. The purchased quantities are derived from the demand function (7). The parameters that govern the price elasticities are set to $\theta = -0.3$ and $\phi = -0.3$ (inelastic consumption and stocking). The values of the storage cost parameter and the endowment are $\mu = 0.1$ and $m = 1000$,

[26] This symmetric design was a suggestion of one of the anonymous referees. The design avoids unrealistic situations where prices and quantities that are far from their original values suddenly revert to these original values.

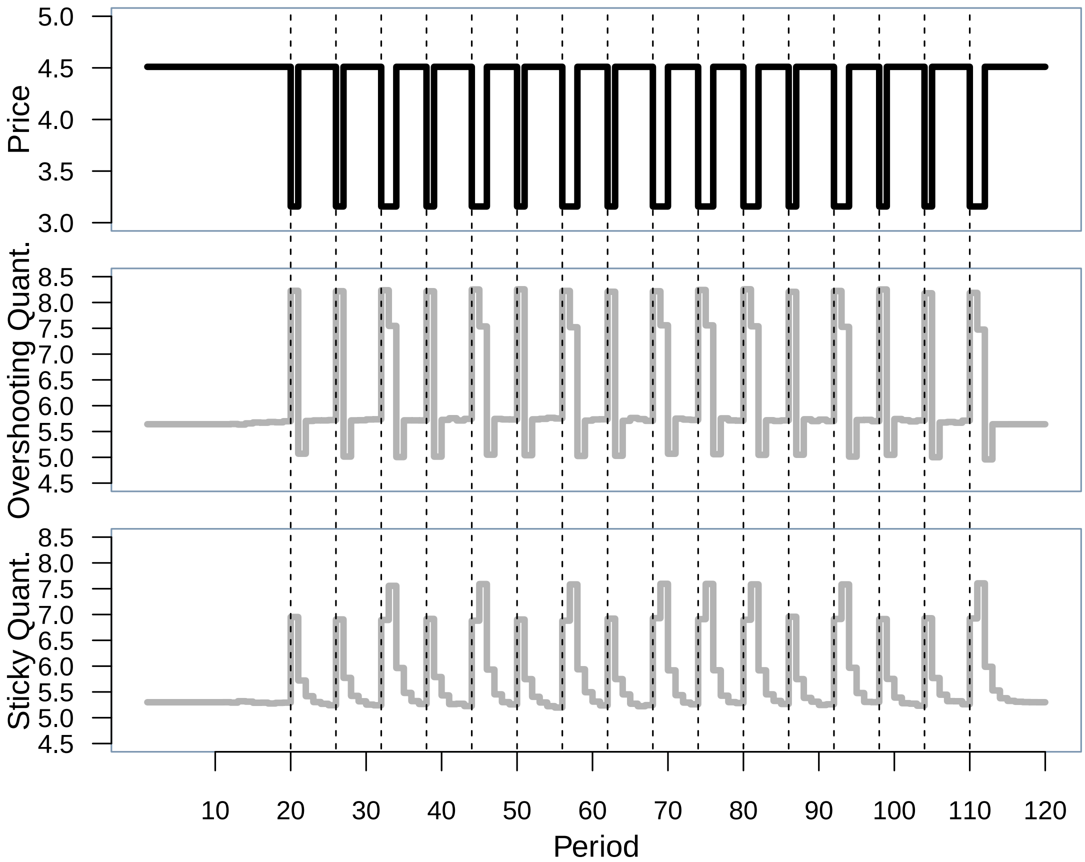

Figure 4: Sales (Upper Panel) and the Resulting Quantities of Stockpiling Households (Middle Panel) and Smoothing Households (Bottom Panel).

respectively. The $\bar{c}_i$-levels (former consumption) and $\bar{s}_i$-levels (stocks inherited from the preceding period) relevant for period $t = 1$ are set to the steady-state equilibrium values corresponding to the base prices.[27]

The differences between the two types of households are the following. For stockpiling households, the importance of consumption relative to stockpiling is set to $\alpha = 0.7$ and the smoothing parameter of consumption to $\gamma = 0$ (no smoothing). For smoothing households, the importance of consumption relative to stocks is set to $\alpha = 1$ (no stockpiling) and the smoothing parameter of consumption is $\gamma = 1.5$.

The upper panel of Figure 4 depicts the typical price pattern of regular sales. The middle panel shows the corresponding overshooting quantities purchased by stockpiling households. They are derived from demand function (7), with $\alpha = 0.7$ and $\gamma = 0$. The sticky quantities purchased by smoothing households can be seen in the lower panel. In their computation, the parameter values in demand function (7) are set to $\alpha = 1.0$ and

[27]These values are computed from the steady-state formulas derived in Appendix C.

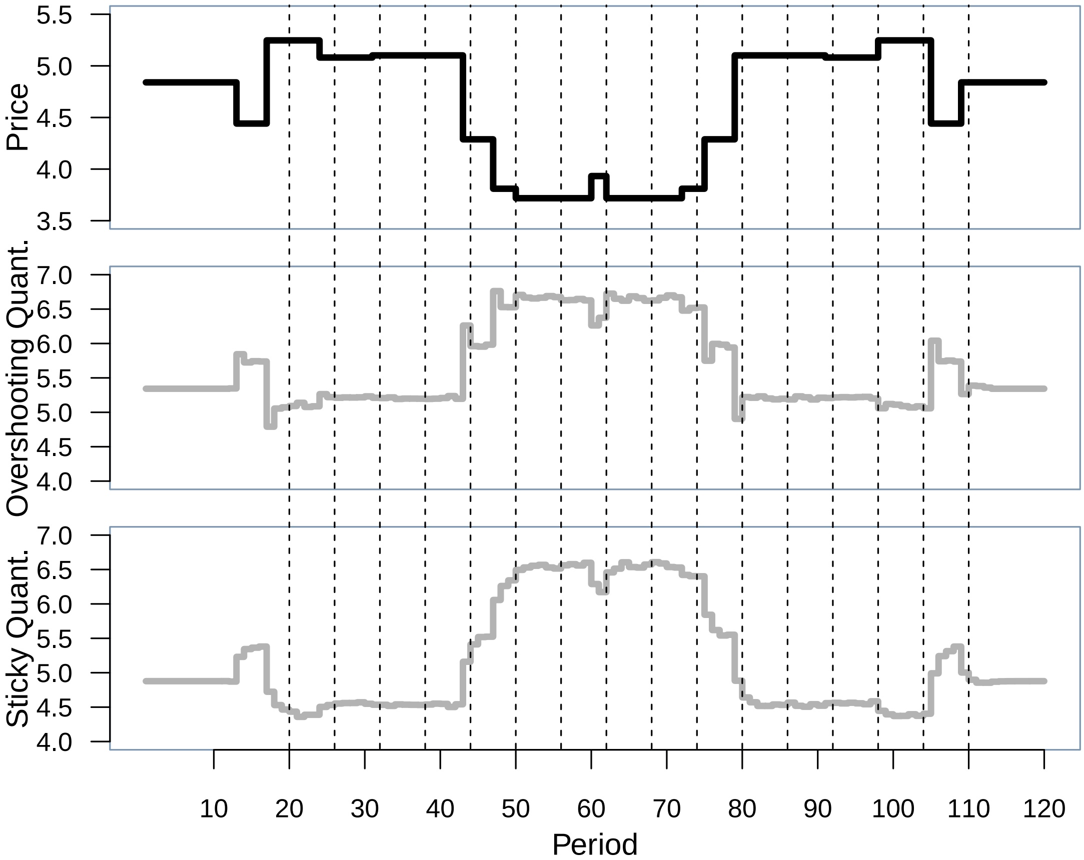

Figure 5: Ordinary Price Changes (Upper Panel) and the Resulting Quantities of Stockpiling Households (Middle Panel) and Smoothing Households (Bottom Panel).

$\gamma = 1.5$.

The upper panel of Figure 5 shows price changes not attributable to sales. The corresponding overshootig quantities purchased by stockpiling households are depicted in the middle panel, while the bottom panel shows the sticky quantities of smoothing households.

In the *overshooting quantities scenario*, all households are stockpiling households, while in the *sticky quantities scenario*, only smoothing households exist. In the *hybrid quantities scenario*, every fourth household is a stockpiling household and all other households are smoothing households. For all three scenarios (Overshooting, Sticky, Hybrid), the index numbers of various chained bilateral indices and of many different R-GEKS, R-TPD, and R-GK variants are compiled.

The price level of period $t = 1$ is set equal to 100. In period $t = 111$, all prices and quantities have returned to their initial values and stay there for all subsequent periods. Therefore, in period $t = 120$ the price index number generated by the various index methods should have returned to its initial level, that is, to 100.[28] The extent of chain

[28]This is a weaker condition than the condition of the chain drift test (Definition 2) of bilateral price

drift bias is the deviation of the price level in period $t = 120$ from 100.

For each of the three scenarios and each index formula, this simulation exercise is repeated 5000 times. For example, one receives 5000 different numbers for the chain drift bias of a chained Törnqvist index in the overshooting quantities scenario (only stockpiling households exist). These 5000 numbers are averaged. The result of this averaging is the top left number in Table 1. In the overshooting quantities scenario, the Törnqvist index reaches 8.51% *downward* chain drift. The other listed price indices exhibit a downward chain drift that varies between 7.96% and 8.68%. In the sticky quantities scenario (only smoothing households exist), the listed bilateral indices show an *upward* chain drift that varies between 9.13% and 9.63%. The results of the hybrid scenario (three quarters are smoothing households, one quarter are stockpiling households) lie between the other two scenarios.

Table 1: Chain Drift of Several Bilateral Price Indices With Symmetric Expenditure Weights (in %).

	Overshooting	Sticky	Hybrid
Törnqvist	-8.51	9.63	4.90
Marshall-Edgeworth	-7.96	9.13	4.69
Walsh	-8.68	9.46	4.86
Theil	-8.60	9.48	4.86
Sato-Vartia	-8.60	9.48	4.86

Table 2 documents the chain drift of the other chained bilateral price indices mentioned in the paper. Again, the overshooting quantities scenario generates a large downward chain drift that varies between the indices. Also the upward chain drift values arising from sticky quantities are similar to those listed in Table 1.

The tables show that chained bilateral indices generate the expected results. Overshooting quantities lead to considerable downward chain drift, while sticky quantities generate considerable upward chain drift. Overall, the bilateral price indices of Marshall-Edgeworth, Fisher, and Davies slightly outperform the other ones. This is true for overshooting quantities as well as for sticky quantities.

GEKS indices are transitive and, thus, immune to chain drift bias arising from overshooting quantities. Therefore, R-GEKS indices have been proposed as a remedy for chain drift bias. However, these indices involve some form of splicing which, in turn, can cause chain drift bias. The performance of the various R-GEKS indices may depend on the

indices. The latter postulates that already in period $t = 111$ the price index number must be 100.

Table 2: Chain Drift of Some Additional Bilateral Price Indices (in %).

	Overshooting	Sticky	Hybrid
Fisher	-7.95	9.12	4.68
Banerjee	-8.60	9.48	4.86
Davies	-7.96	9.13	4.69
Lehr	-9.19	9.60	4.96

choice of the splicing method and the window length (e.g., Melser, 2018, p. 517). Therefore, Table 3 presents the results for various splicing methods and for window lengths of 4, 8, 12, and 24 periods. These results are derived from R-GEKS indices that use the Törnqvist index as their bilateral base index.

The index numbers in the first two rows of Table 3 show that the mean movement splice and the mean splice produce virtually the same R-GEKS index numbers. Overall, the numbers reinforce the arguments of Diewert and Fox (2020, pp. 6-7) and de Haan (2015, pp. 25-26) in favour of a more "balanced" splicing approach than the movement splice or window splice. With a sufficiently large window, the R-GEKS approach in conjunction with the mean movement splice, mean splice, or half splice effectively curtails chain drift bias arising from overshooting quantities (upper left part of Table 3). The half splice with a window length of twelve months performs best. The upper middle part of Table 3 reveals that the previous findings carry over to the scenario of sticky quantities.

In the hybrid scenario, three quarters of the households are smoothing households. The resulting R-GEKS index numbers are listed in the upper right part of Table 3. The chain drift bias caused by the smoothing households seems to dominate the chain drift bias caused by the stockpiling households. However, this effect is driven by the parameters α and γ as well as by the share of stockpiling households. When that share is sufficiently increased or the relevance of consumption (α) and smoothing (γ) is sufficiently reduced, the dominance would be reversed (not shown in the table).

The other qualitative results are not affected by changes of α, γ, and the share of stockpiling households. The mean movement splice, mean splice, and half splice remain the least biased options when applied with a sufficiently large window length. Reducing the price elasticity of consumption (governed by the parameter θ) clearly increases the chain drift bias in the sticky quantities scenario, while the chain drift bias in the overshooting quantities scenario is not much affected. By contrast, reducing the price elasticity of stockpiling (governed by the parameter ϕ) clearly *reduces* the chain drift bias in the overshooting quantities scenario, while the chain drift bias in the sticky quantities

Table 3: Chain Drift Bias of R-GEKS, R-TPD, and R-GK Indices (in %) for Different Scenarios, Splicing Methods, and Window Lengths (4, 8, 12, and 24 Months).

	Overshooting				Sticky				Hybrid			
Window Length	4	8	12	24	4	8	12	24	4	8	12	24
R-GEKS												
Mean Move.	-2.41	-0.60	-0.35	-0.14	3.49	0.99	0.61	0.25	2.03	0.60	0.38	0.16
Mean	-2.41	-0.60	-0.34	-0.13	3.49	0.99	0.59	0.24	2.03	0.60	0.37	0.15
Movement	-2.56	-1.28	-0.80	-0.33	3.38	1.47	0.97	0.41	1.90	0.79	0.54	0.23
Half	-2.11	-0.23	-0.01	-0.15	3.72	0.59	0.20	0.28	2.27	0.39	0.15	0.18
Window	-2.56	-1.28	-0.78	-0.32	3.38	1.47	0.96	0.39	1.90	0.79	0.53	0.22
R-TPD												
Mean Move.	-1.65	-0.21	-0.01	0.22	3.14	0.65	0.33	-0.03	2.00	0.44	0.25	0.03
Mean	-2.44	-0.60	-0.33	-0.12	3.33	0.98	0.58	0.23	1.96	0.60	0.36	0.14
Movement	-0.16	0.09	0.46	0.68	2.70	0.60	0.17	-0.29	1.97	0.47	0.24	-0.06
Half	-2.21	-0.23	-0.01	-0.14	3.45	0.61	0.22	0.27	2.16	0.40	0.16	0.17
Window	-4.91	-2.62	-2.01	-1.29	3.86	2.30	1.71	1.04	1.76	1.10	0.81	0.47
R-GK												
Mean Move.	-4.03	-1.34	-1.24	-1.22	1.23	-0.45	-0.91	-1.46	-0.04	-0.67	-0.99	-1.41
Mean	-2.67	-0.64	-0.35	-0.13	3.63	1.03	0.60	0.23	2.13	0.62	0.37	0.14
Movement	-6.88	-4.18	-4.00	-3.35	-3.48	-3.58	-4.17	-4.23	-4.34	-3.74	-4.13	-4.02
Half	-2.30	-0.22	0.03	-0.16	3.79	0.60	0.19	0.27	2.38	0.40	0.14	0.16
Window	1.36	1.46	2.35	2.75	11.12	6.96	6.46	5.26	8.77	5.62	5.46	4.65

scenario is not much affected.

R-GEKS indices are not the only approach to curb the chain drift problem. Among the alternatives are the R-TPD and the R-GK approach. Therefore, the same stress test with the same splicing options and window lengths has been conducted for the R-TPD and the R-GK approach. Table 3 also presents these results. For the R-GK approach, the choice of window length and splicing method causes more variation than for the R-GEKS and the R-TPD approach.[29] When the window splice is avoided, the R-TPD approach slightly outperforms the R-GEKS approach which, in turn, outperforms the R-GK approach. When the mean splice or the half splice are applied, the choice between R-GEKS, R-TPD, and R-GK is of minor relevance.[30]

It should be kept in mind that in the applied simulation no item attrition occurs. With item attrition, large windows may generate assignment or assortment bias (e.g., Auer, 2017, p. 84). In a context of item attrition, Melser and Webster (2021, pp. 777-783) identify in their own simulations life cycle pricing and, in particular, run-out sales as an important driver of chain drift. Auer and Weinand (2022) study the issue of data gaps in the context of interregional price comparisons. Their results imply that, in an intertemporal context, the TPD approach possibly produces biased estimates when a correlation exists between a product's intertemporal price variation and its number of missing observations. The results in Weinand (2020) imply that the GEKS approach may suffer from the same problem. Whether these speculations are justified and whether splicing adds a new angle to those findings, can be studied in the context of the simulation approach outlined in the present study.To this end, it would be useful to identify the basic structure of product turnover in real world scanner data and to transfer this structure to the simulations.

Such a simulation may also address another important issue. To mitigate chain drift bias, one may try to aggregate the weekly prices into monthly or even quarterly unit values, say (e.g., Diewert, 2007, p. 3). In the simulation one could systematically study the effects of such a strategy.

6 Concluding Remarks

Sales and the associated stockpiling give rise to "overshooting quantity" movements. It is well known that overshooting quantities create problems for sub-annual chaining of bilateral price indices, because such quantities generate downward chain drift bias. The

[29]This is in line with findings in Fox *et al.* (2022, pp. 17-22), Lamboray (2021, pp. 13-17) and Van Loon and Roels (2018, pp. 9-13). Note that the former study uses household level scanner data while the latter two studies use point-of-sales scanner data. None of these studies differentiates between stockpiling and consumption smoothing.

[30]The comparable performance of the R-GEKS and R-GK approach is also reported in Fox *et al.* (2022).

present study argues that overshooting quantities are only part of the chain drift problem. Another important cause of chain drift bias are search and adjustment costs as well as habits. They imply that price changes lead to delayed quantity changes. The resulting "sticky quantities" generate upward chain drift.

In the literature, R-GEKS, R-TPD, and R-GK approaches have been proposed as a remedy for chain drift bias. However, it is unclear whether these approaches merely reduce chain drift bias or even eliminate it. Furthermore, some approaches may be more effective than others. The present paper answers all of these questions. To this end, it develops a novel utility framework that is consistent with households that have a desire for stockpiling (overshooting quantities) and/or for smoothing quantity adjustments over time (sticky quantities). This utility framework assumes that households are myopic and directly derive utility from stockpiling.

Building on this utility framework, a stress test is developed that examines the resilience of different price indices against chain drift. This stress test is applied to various bilateral price indices and to several splicing variants and window lengths of R-GEKS, R-TPD, and R-GK indices. The bilateral price indices show the expected results. Overshooting quantities generate downward chain drift bias, while sticky quantities generate upward chain drift bias. R-GEKS, R-TPD, and R-GK indices reduce this bias, but do not eliminate it. Shorter window lengths tend to generate more chain drift bias than longer ones. The window splice is clearly outperformed by the half splice and mean splice. When the latter two splicing variants are used, the choice between R-GEKS, R-TPD, or R-GK indices is of minor relevance.

Appendix A: Index Formulas

The paper considers several price indices that are symmetrically weighted averages of price ratios:

$$\begin{aligned}
\text{Törnqvist}:\ \ \ln P_{Tö} &= \sum \frac{1}{2}\left(\frac{p_i^0 x_i^0}{\sum p_j^0 x_j^0} + \frac{p_i^T x_i^T}{\sum p_j^T x_j^T}\right) \ln \frac{p_i^T}{p_i^0} \\
\text{Marshall-Edgeworth}:\ \ P_{ME} &= \frac{\sum p_i^T (x_i^0 + x_i^T)}{\sum p_i^0 (x_i^0 + x_i^T)} = \sum \frac{\left[p_i^0 x_i^0 + p_i^T x_i^T / \left(p_i^T / p_i^0\right)\right]}{\sum \left[p_j^0 x_j^0 + p_j^T x_j^T / \left(p_j^T / p_j^0\right)\right]} \frac{p_i^T}{p_i^0} \\
\text{Walsh}:\ \ P_W &= \frac{\sum p_i^T \sqrt{x_i^0 x_i^T}}{\sum p_i^0 \sqrt{x_i^0 x_i^T}} = \sum \frac{\sqrt{p_i^0 x_i^0 p_i^T x_i^T / \left(p_i^T / p_i^0\right)}}{\sum \sqrt{p_j^0 x_j^0 p_j^T x_j^T / \left(p_j^T / p_j^0\right)}} \frac{p_i^T}{p_i^0} \\
\text{Theil}:\ \ \ln P_{Th} &= \sum \left[\frac{\sqrt[3]{\frac{1}{2}\left(p_i^0 x_i^0 + p_i^T x_i^T\right) p_i^0 x_i^0 p_i^T x_i^T}}{\sum \sqrt[3]{\frac{1}{2}\left(p_j^0 x_j^0 + p_j^T x_j^T\right) p_j^0 x_j^0 p_j^T x_j^T}}\right] \ln \frac{p_i^T}{p_i^0} \\
\text{Sato-Vartia}:\ \ \ln P_{SV} &= \sum \frac{L\left(p_i^T x_i^T, p_i^0 x_i^0\right)}{L\left(\sum p_j^T x_j^T, \sum p_j^0 x_j^0\right)} \ln \frac{p_i^T}{p_i^0}\ , \\
\text{where } L(a,b) &= \frac{a-b}{\ln a - \ln b} \text{ for } a \neq b \text{ and } L(a,b) = a \text{ for } a = b.
\end{aligned}$$

Other popular price indices include

$$\begin{aligned}
\text{Laspeyres}:\ \ P_L &= \frac{\sum p_i^T x_i^0}{\sum p_i^0 x_i^0} \\
\text{Paasche}:\ \ P_P &= \frac{\sum p_i^T x_i^T}{\sum p_i^0 x_i^T} \\
\text{Fisher}:\ \ P_F &= \sqrt{P_L P_P}\ .
\end{aligned}$$

The GUV indices of Davies, Banerjee, and Lehr are defined by

$$P = \frac{\sum p_i^T x_i^T}{\sum p_i^0 x_i^0} \frac{\sum x_i^0 \hat{z}_i}{\sum x_i^T \hat{z}_i}\ ,$$

with

$$\begin{aligned}
\text{Davies}:\ \ \hat{z}_i &= \sqrt{p_i^0 p_i^T} \\
\text{Banerjee}:\ \ \hat{z}_i &= \left(p_i^0 + p_i^T\right)/2 \\
\text{Lehr}:\ \ \hat{z}_i &= \left(p_i^0 x_i^0 + p_i^T x_i^T\right) / \left(x_i^0 + x_i^T\right)\ .
\end{aligned}$$

Appendix B: Some Popular Splicing Methods

Figure 6 illustrates various splicing methods proposed in the literature. The illustration utilises a window length of four periods. Period t is the current period. The upper grey bar represents the current window, covering periods $t-3$ to t. The lower grey bar highlights the window associated with the previous period $(t-1)$. This old window covers the periods $t-4$ to $t-1$. The filled circle on the lower grey bar indicates the price level of period $t-1$: P^{t-1}. This is the reference for the price level to be computed in period t: P^t. The latter is represented by the circle on the upper grey bar.

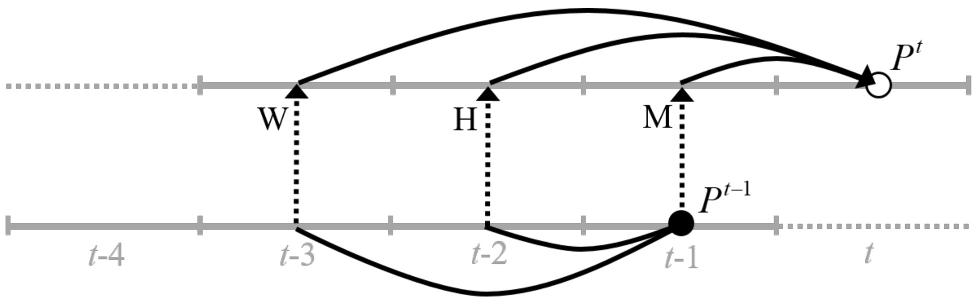

Figure 6: Four Different Methods of Splicing.

Applying a multilateral price index (GEKS, TPD, or GK) to the information contained in the current window, $[t-3, t]$, generates for all pairs of periods (t' and t'') included in this window a price ratio $P^{t'/t''}_{[t-3,t]}$, where the superscript indicates the periods that are compared (comparison period t', base period t''), while the subscript documents the window from which the result is computed. For example, the GEKS index computes the price ratio $P^{t'/t''}_{[t-3,t]}$ as the geometric average of all possible pairs of direct price indices that link the periods t'' and t' via some link period s included in the window:

$$P^{t'/t''}_{[t-3,t]} = \prod_{s=t-3}^{t} \left(P^{t'/s} P^{s/t''} \right)^{1/4} . \tag{9}$$

This averaging principle is applied for each pair of periods covered by the window $[t-3, t]$. The computed ratios are transitive, that is, $P^{t/(t-2)}_{[t-3,t]} = P^{(t-1)/(t-2)}_{[t-3,t]} P^{t/(t-1)}_{[t-3,t]}$.

To obtain the new price level P^t, the results of the current window must be linked to the price level P^{t-1} that was computed from the previous window (regardless of whether the values of that window were computed by the GEKS, TPD, or GK method). Several methods have been proposed for this linking procedure (denoted as "splicing"). They are illustrated in Figure 6.

Ivancic *et al.* (2011, p. 33) propose a "movement splice" (indicated by the "M" in Figure 6). This approach multiplies the price level P^{t-1} by the price change between periods t and $t-1$ computed from the current window:

$$P^t = P^{t-1} P_{[t-3,t]}^{t/(t-1)} \qquad \text{(movement splice)} . \tag{10}$$

As an alternative, Krsinich (2016, pp. 383-87) suggests the "window splice". This approach ("W" in Figure 6) uses the price ratio $P_{[t-4,t-1]}^{(t-3)/(t-1)}$ compiled by the old window to get backward from the price level P^{t-1} to the price level of period $t-3$, the first period of the current window. The result of the backward computation is multiplied by the price change between periods $t-3$ and t as compiled by the current window:

$$P^t = P^{t-1} P_{[t-4,t-1]}^{(t-3)/(t-1)} P_{[t-3,t]}^{t/(t-3)} \qquad \text{(window splice)} .$$

"Half splice" is another option ("H" in Figure 6). The idea and its justification is due to de Haan (2015, pp. 25-26).[31] Here, the period linking the current and the new window is period $t-2$ which is half way between the linking periods of movement splice (period $t-1$) and window splice (period $t-3$):

$$P^t = P^{t-1} P_{[t-4,t-1]}^{(t-2)/(t-1)} P_{[t-3,t]}^{t/(t-2)} \qquad \text{(half splice)} .$$

Diewert and Fox (2020, pp. 6-7) introduce yet another splicing variant which they call "mean splice". They compute the geometric average of the results obtained from the window, movement, and half splice:

$$P^t = P^{t-1} \left[P_{[t-3,t]}^{t/(t-1)} \left(P_{[t-4,t-1]}^{(t-3)/(t-1)} P_{[t-3,t]}^{t/(t-3)} \right) \left(P_{[t-4,t-1]}^{(t-2)/(t-1)} P_{[t-3,t]}^{t/(t-2)} \right) \right]^{1/3} \qquad \text{(mean splice)} .$$

When the window is longer than four periods, also the results compiled from all additional possible linking periods are included in this geometric average.

Melser (2018, p. 518) develops a "mean movement splice" that is illustrated in Figure 7. Like the mean splice it relies on several links.[32]

To obtain the current price level P^t, the mean movement approach links the results from the current window not only to the price level P^{t-1} computed from the previous window, but also to the price levels P^{t-2} and P^{t-3} that were derived from the two preceding windows (in Figure 7 the filled black circles).[33] The links to the formerly computed price levels P^{t-1}, P^{t-2}, and P^{t-3} follow the basic principle of the movement splice, that is, no backward computation from the current period to some previous period is involved. The price levels of these previous periods are directly taken from the results of the preceding windows. More specifically, the formerly computed price level P^{t-3} is multiplied by the

[31] He makes this proposal in the context of a time-product dummy index.

[32] Melser (2018, p. 517) also suggests replacing the GEKS index by a weighted variant of the GEKS index. We do not explore this issue in the present paper and focus on the linking procedures.

[33] Chessa (2016, p. 16) applies a similar idea in the context of a Geary-Khamis index. He denotes his index as the "revisionless QU-GK method".

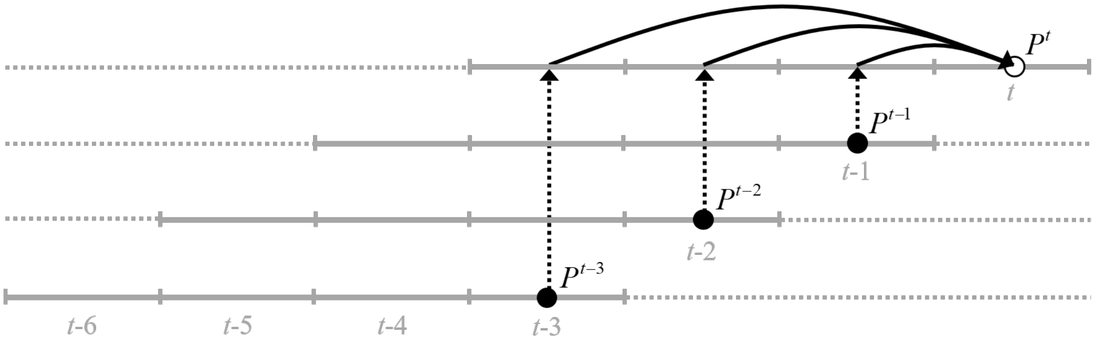

Figure 7: Mean Movement Splicing.

bilateral price index $P^{t/(t-3)}_{[t-3,t]}$ obtained from the current window. Analogously, P^{t-2} is multiplied by $P^{t/(t-2)}_{[t-3,t]}$, and P^{t-1} is multiplied by $P^{t/(t-1)}_{[t-3,t]}$. In a final step, a geometric average of these three products is computed:

$$P^t = \left[\left(P^{t-3} P^{t/(t-3)}_{[t-3,t]} \right) \left(P^{t-2} P^{t/(t-2)}_{[t-3,t]} \right) \left(P^{t-1} P^{t/(t-1)}_{[t-3,t]} \right) \right]^{1/3} .$$

Appendix C: Derivation of Demand Function (7)

A generalised form of the utility function (5) and (6) is

$$U = \Bigg[\underbrace{\left[\sum_{i=1}^{N} f_i^{\theta}\right]^{1/\theta}}_{=C}\Bigg]^{\alpha} \Bigg[\underbrace{\left[\sum_{i=1}^{N} g_i^{\phi}\right]^{1/\phi}}_{=S}\Bigg]^{1-\alpha} ,$$

with $f_i = f_i(c_i, \bar{c}_i)$ and $g_i = g_i(s_i, \bar{s}_i)$. The compilation of optimal consumption c_i^* and stocks s_i^* $(i = 1, \ldots, N)$ starts with the derivation of the cost minimising values of c_i for reaching a given subutility level C. The derivation of the cost minimising values of s_i for reaching a given subutility level S is perfectly analogous. Afterwards, the utility maximising values of C and S are determined, given the net endowment, $\tilde{m} = m + \sum_{i=1}^{N} p_i \bar{s}_i$, and past consumption, $\bar{c}_i$ $(i = 1, \ldots, N)$. Putting everything together yields consumption c_i^* and stocks s_i^* $(i = 1, \ldots, N)$.

The cost minimising values of c_i corresponding to some subutility level C are derived from the Lagrangian

$$\mathcal{L} = \sum_{i=1}^{N} p_i c_i + \lambda \left(\sum_{i=1}^{N} f_i^{\theta} - C^{\theta}\right) ,$$

with $f_i = f_i(c_i, \bar{c}_i)$ being a monotonic continuous function. The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial c_i} = p_i - \lambda \theta f_i^{\theta - 1} f_i' = 0 , \qquad (i = 1, \ldots, N) \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i=1}^{N} f_i^{\theta} - C^{\theta} = 0 . \tag{12}$$

Each pair of conditions (11) leads to the relationship

$$p_i f_i^{1-\theta} f_j' = p_j f_j^{1-\theta} f_i' .$$

Exponentiating by $\theta / (1 - \theta)$ and rearranging yields

$$f_i^{\theta} = p_i^{\frac{-\theta}{1-\theta}} p_j^{\frac{\theta}{1-\theta}} \left(f_j'\right)^{\frac{-\theta}{1-\theta}} \left(f_i'\right)^{\frac{\theta}{1-\theta}} f_j^{\theta} . \tag{13}$$

Inserting (13) in (12) gives

$$C^{\theta} = f_j^{\theta} \left(f_j'\right)^{\frac{-\theta}{1-\theta}} p_j^{\frac{\theta}{1-\theta}} \sum_{i=1}^{N} p_i^{\frac{-\theta}{1-\theta}} \left(f_i'\right)^{\frac{\theta}{1-\theta}} . \tag{14}$$

In the simulation, the specification $f_i = c_i / (\bar{c}_i^{\gamma})$ is used. This specification gives the derivative $f_i' = \bar{c}_i^{-\gamma}$. Then, exponentiating (14) by $1/\theta$, gives

$$C = c_j \bar{c}_j^{-\gamma} \left(\bar{c}_j^{-\gamma}\right)^{\frac{-1}{1-\theta}} p_j^{\frac{1}{1-\theta}} \left(\sum_{i=1}^{N} p_i^{\frac{-\theta}{1-\theta}} \left(\bar{c}_i^{-\gamma}\right)^{\frac{\theta}{1-\theta}}\right)^{\frac{1}{\theta}} .$$

Swapping the indices j and i and rearranging, yields

$$c_i = C\bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{1}{1-\theta}} \ , \tag{15}$$

with

$$P_C^{\frac{1}{1-\theta}} = \left(\sum_{j=1}^{N} p_j^{\frac{-\theta}{1-\theta}} \bar{c}_j^{\frac{-\gamma\theta}{1-\theta}} \right)^{-\frac{1}{\theta}} \ . \tag{16}$$

Therefore, the minimum cost of subutility C can be expressed as

$$\sum_{i=1}^{N} p_i c_i = C P_C^{\frac{1}{1-\theta}} \sum_{i=1}^{N} p_i^{\frac{-\theta}{1-\theta}} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} = C P_C \ . \tag{17}$$

From (16), differentiating $P_C^{\frac{\theta}{1-\theta}}$ with respect to p_i gives

$$\frac{\partial P_C^{\frac{\theta}{1-\theta}}}{\partial p_i} = -\left(\sum_{j=1}^{N} p_j^{\frac{-\theta}{1-\theta}} \bar{c}_j^{\frac{-\gamma\theta}{1-\theta}} \right)^{-2} p_i^{\frac{-\theta}{1-\theta}-1} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} = -P_C^{\frac{2\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} \ . \tag{18}$$

When the number of items, N, is sufficiently large, the derivative of P_C with respect to p_i is approximately 0:

$$\frac{\partial P_C}{\partial p_i} \approx 0 \ . \tag{19}$$

Then, differentiating (15) with respect to p_i gives

$$\frac{\partial c_i}{\partial p_i} \approx \frac{-1}{1-\theta} p_i^{-1} C\bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{1}{1-\theta}} = \frac{-1}{1-\theta} p_i^{-1} c_i \ .$$

Rearranging yields the approximated price elasticity of present consumption:

$$\frac{\partial c_i}{\partial p_i} \frac{p_i}{c_i} \approx \frac{-1}{1-\theta} \ .$$

An analogous derivation applies to subutility S. Choosing the specification $g_i = s_i/(\bar{s}_i^{\delta})$, the derivation of the optimal s_i-value, for a given subutility level S, is perfectly analogous to the derivation of (15) and (16). One obtains

$$s_i = S\bar{s}_i^{\frac{-\delta\phi}{1-\phi}} \tilde{p}_i^{\frac{-1}{1-\phi}} P_S^{\frac{1}{1-\phi}} \ , \tag{20}$$

where $\tilde{p}_i = p_i\,(1+\mu)$ and

$$P_S^{\frac{1}{1-\phi}} = \left(\sum_{j=1}^{N} \tilde{p}_j^{\frac{-\phi}{1-\phi}} \bar{s}_j^{\frac{-\delta\phi}{1-\phi}} \right)^{-\frac{1}{\phi}} \ . \tag{21}$$

Total expenditures on stockpiling are

$$\sum_{i=1}^{N} \tilde{p}_i s_i = S P_S \ . \tag{22}$$

Using (17) and (22), the budget constraint (3) can be expressed in the form

$$\tilde{m} = C P_C + S P_S \ . \tag{23}$$

Utility defined in (5) is maximised, subject to the budget constraint (23). The Lagrangian can be written in the form

$$\mathcal{L} = \alpha \ln C + (1-\alpha) \ln S - \lambda \left(\tilde{m} - C P_C - S P_S \right) \ .$$

From the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial C} = \frac{\alpha}{C} + \lambda P_C = 0 \qquad \text{and} \qquad \frac{\partial \mathcal{L}}{\partial S} = \frac{1-\alpha}{S} + \lambda P_S = 0$$

one obtains

$$-C P_C = \frac{\alpha}{\lambda} \tag{24}$$

$$-S P_S = \frac{1-\alpha}{\lambda} \ . \tag{25}$$

Inserting these expressions in (23), yields the third first-order condition:

$$-\tilde{m} = \frac{\alpha}{\lambda} + \frac{1-\alpha}{\lambda} = \frac{1}{\lambda} \ . \tag{26}$$

Inserting this result back in (24) and (25) gives

$$C = \alpha \frac{\tilde{m}}{P_C} \tag{27}$$

$$S = (1-\alpha) \frac{\tilde{m}}{P_S} \ . \tag{28}$$

Inserting these results in (15) and (20) and noting that in the simulations the parameter δ is set to 0, yields the optimal levels of consumption and stocks stated in (7):

$$c_i^* = \alpha \tilde{m} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}} \tag{29}$$

$$s_i^* = (1-\alpha)\, \tilde{m} \underbrace{\bar{s}_i^{\frac{-\delta\phi}{1-\phi}}}_{=1 \text{ for } \delta=0} \tilde{p}_i^{\frac{-1}{1-\phi}} P_S^{\frac{\phi}{1-\phi}} \ . \tag{30}$$

Differentiating (29) with respect to p_i and using (18) gives

$$
\begin{aligned}
\frac{\partial c_i^*}{\partial p_i} &= \alpha \tilde{m} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} \left[\frac{-1}{1-\theta} p_i^{\frac{-1}{1-\theta}-1} P_C^{\frac{\theta}{1-\theta}} - p_i^{\frac{-1}{1-\theta}} P_C^{\frac{2\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} \right] \\
&= \alpha \tilde{m} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} P_C^{\frac{\theta}{1-\theta}} p_i^{\frac{-2+\theta}{1-\theta}} \left[\frac{-1}{1-\theta} - p_i^{\frac{-\theta}{1-\theta}} P_C^{\frac{\theta}{1-\theta}} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} \right] < 0 \,. \qquad (31)
\end{aligned}
$$

Analogously, one gets

$$
\frac{\partial s_i^*}{\partial p_i} = \frac{\partial s_i^*}{\partial \tilde{p}_i} \frac{\partial \tilde{p}_i}{\partial p_i} < 0 \qquad \text{and} \qquad \frac{\partial s_i^*}{\partial \mu} = \frac{\partial s_i^*}{\partial \tilde{p}_i} \frac{\partial \tilde{p}_i}{\partial \mu} < 0 \,.
$$

Using (19), the derivative of (29) with respect to $\bar{c}_i$ yields

$$
\frac{\partial c_i^*}{\partial \bar{c}_i} \approx \frac{-\gamma\theta}{1-\theta} \bar{c}_i^{-1} \alpha \tilde{m} \bar{c}_i^{\frac{-\gamma\theta}{1-\theta}} p_i^{\frac{-1}{1-\theta}} P_C^{\frac{\theta}{1-\theta}} = \frac{-\gamma\theta}{1-\theta} \frac{c_i^*}{\bar{c}_i} \,.
$$

Rearranging gives the approximated elasticity of present consumption, c_i^*, with respect to past consumption, $\bar{c}_i$:

$$
\frac{\partial c_i^*}{\partial \bar{c}_i} \frac{\bar{c}_i}{c_i^*} \approx \frac{-\gamma\theta}{1-\theta} \,.
$$

For a non-oscillatory time path to a stable steady-state equilibrium, this elasticity must be non-negative but below 1: $0 \leq -\gamma\theta < 1-\theta$. Analogously, the approximated elasticity of stockpiling with respect to past stocks is

$$
\frac{\partial s_i^*}{\partial \bar{s}_i} \frac{\bar{s}_i}{s_i^*} \approx \frac{-\delta\phi}{1-\phi} \,.
$$

A non-oscillatory time path to a stable steady-state equilibrium requires that $0 \leq -\delta\phi < 1-\phi$. For $\delta = 0$ (as in the simulations), this stability condition is satisfied.

The final step is the derivation of the stable steady-state equilibrium. From (17), (22), (27), and (28) it can be seen that the expenditure *shares* of total consumption (CP_C) and total stockpiling (SP_S) are constants. However, delayed adjustments in stockpiling (this requires that $\delta \neq 0$) lead to a change in the net endowment, $\tilde{m}$. If the prices, p_i ($i = 1, \ldots, N$), and income, m, remain unchanged, the stocks defined by (30) and, therefore, the net endowment, $\tilde{m}$, eventually reach their unique and stable steady-state levels (s_i^{**} and $\tilde{m}^{**}$). Then, income m is used to pay for consumption and storage cost, but no longer for adjustments of the stocks:

$$
m = \alpha \tilde{m}^{**} + \mu \sum_{i=1}^{N} p_i s_i^{**} \,. \qquad (32)
$$

In a steady-state equilibrium, one can replace in (30) s_i^* and $\bar{s}_i$ by s_i^{**}:

$$
\left(s_i^{**}\right)^{\frac{1-\phi+\delta\phi}{1-\phi}} = (1-\alpha)\, \tilde{m}^{**} \tilde{p}^{\frac{-1}{1-\phi}} \left(P_S^{**}\right)^{\frac{\phi}{1-\phi}} \,, \qquad (33)
$$

with

$$\left(P_S^{**}\right)^{\frac{-\phi}{1-\phi}} = \sum_{j=1}^{N} \tilde{p}_j^{\frac{-\phi}{1-\phi}} \left(s_j^{**}\right)^{\frac{-\delta\phi}{1-\phi}} \ . \tag{34}$$

Exponentiating (33) by $-\delta\phi/\left(1-\phi+\delta\phi\right)$ and then multiplying by $\tilde{p}_i^{\frac{-\phi}{1-\phi}}$ gives

$$\tilde{p}_i^{\frac{-\phi}{1-\phi}} \left(s_i^{**}\right)^{\frac{-\delta\phi}{1-\phi}} = \left[\left(1-\alpha\right)\tilde{m}^{**}\right]^{\frac{-\delta\phi}{1-\phi+\delta\phi}} \tilde{p}_i^{\frac{-\phi(1-\delta)}{1-\phi(1-\delta)}} \left(P_S^{**}\right)^{\frac{\delta\phi^2}{(1-\phi+\delta\phi)(1-\phi)}} \ .$$

Summing over all N products and rearranging leads to

$$\left(P_S^{**}\right)^{\frac{\phi}{1-\phi}} = \left[\left(1-\alpha\right)\tilde{m}^{**}\right]^{\frac{\delta\phi}{1-\phi}} \left(\sum_{i=1}^{N} \tilde{p}_i^{\frac{-\phi(1-\delta)}{1-\phi(1-\delta)}}\right)^{-\frac{1-\phi(1-\delta)}{1-\phi}} \ . \tag{35}$$

Inserting this result in (33) and exponentiating by $\left(1-\phi\right)/\left(1-\phi+\delta\phi\right)$, yields

$$s_i^{**} = \left(1-\alpha\right)\tilde{m}^{**}\tilde{p}_i^{\frac{-1}{1-\phi(1-\delta)}} \left(\sum_{j=1}^{N} \tilde{p}_j^{\frac{-\phi(1-\delta)}{1-\phi(1-\delta)}}\right)^{-1} \ . \tag{36}$$

Inserting this expression in (32) gives

$$m = \alpha\tilde{m}^{**} + \left(1-\alpha\right)\tilde{m}^{**}\frac{\mu}{1+\mu}\sum_{i=1}^{N}\tilde{p}_i\tilde{p}_i^{\frac{-1}{1-\phi(1-\delta)}} \left(\sum_{j=1}^{N} \tilde{p}_j^{\frac{-\phi(1-\delta)}{1-\phi(1-\delta)}}\right)^{-1}$$

and, therefore,

$$\tilde{m}^{**} = m\frac{1+\mu}{\alpha+\mu} \ . \tag{37}$$

Inserting this result in (36) gives the steady-state stockpiling level:

$$s_i^{**} = \left(1-\alpha\right)\frac{1+\mu}{\alpha+\mu}m\tilde{p}_i^{\frac{-1}{1-\phi(1-\delta)}} \left(\sum_{j=1}^{N} \tilde{p}_j^{\frac{-\phi(1-\delta)}{1-\phi(1-\delta)}}\right)^{-1} \ .$$

Inserting (37) in (35) and exponentiating by $\left(1-\phi\right)/\phi$ yields the steady-state price index of stockpiling

$$P_S^{**} = \left[\left(1-\alpha\right)\frac{1+\mu}{\alpha+\mu}m\right]^{\delta} \left(\sum_{i=1}^{N} \tilde{p}_i^{\frac{-\phi(1-\delta)}{1-\phi(1-\delta)}}\right)^{-\frac{1-\phi(1-\delta)}{\phi}} \ . \tag{38}$$

Steady-state expenditures on stockpiling are

$$\tilde{p}_i s_i^{**} = \left(1-\alpha\right)\frac{1+\mu}{\alpha+\mu}m\tilde{p}_i^{\frac{-\phi(1-\delta)}{1-\phi(1-\delta)}} \left(\sum_{j=1}^{N} \tilde{p}_j^{\frac{-\phi(1-\delta)}{1-\phi(1-\delta)}}\right)^{-1} \ .$$

Adding over all N products yields total expenditures on stockpiling:

$$\sum_{i=1}^{N} \tilde{p}_i s_i^{**} = (1-\alpha)\,\frac{1+\mu}{\alpha+\mu} m \ . \tag{39}$$

Analogously, the steady-state consumption level is

$$c_i^{**} = \alpha \frac{1+\mu}{\alpha+\mu} m p_i^{\frac{-1}{1-\theta(1-\gamma)}} \left(\sum_{j=1}^{N} p_j^{\frac{-\theta(1-\gamma)}{1-\theta(1-\gamma)}} \right)^{-1} \ . \tag{40}$$

Steady-state expenditures on consumption are

$$p_i c_i^{**} = \alpha \frac{1+\mu}{\alpha+\mu} m p_i^{\frac{-\theta(1-\gamma)}{1-\theta(1-\gamma)}} \left(\sum_{j=1}^{N} p_j^{\frac{-\theta(1-\gamma)}{1-\theta(1-\gamma)}} \right)^{-1}$$

and total consumption expenditures are

$$\sum_{i=1}^{N} p_i c_i^{**} = \alpha \frac{1+\mu}{\alpha+\mu} m \ . \tag{41}$$

Furthermore, the steady-state price index of consumption is

$$P_C^{**} = \left(\alpha \frac{1+\mu}{\alpha+\mu} m \right)^{\gamma} \left(\sum_{i=1}^{N} p_i^{\frac{-\theta(1-\gamma)}{1-\theta(1-\gamma)}} \right)^{-\frac{1-\theta(1-\gamma)}{\theta}} \ . \tag{42}$$

Note that (27) and (28) hold also for the steady-state equilibrium. Therefore, the steady-state subutility levels, C^{**} and S^{**}, are defined by

$$C^{**} = \frac{\alpha \tilde{m}^{**}}{P_C^{**}} \qquad \text{and} \qquad S^{**} = \frac{(1-\alpha)\,\tilde{m}^{**}}{P_S^{**}} \ . \tag{43}$$

Inserting (37) in (43) and the results in utility function (5), gives

$$U = m \frac{1+\mu}{\alpha+\mu} \left(\frac{\alpha}{P_C^{**}} \right)^{\alpha} \left(\frac{1-\alpha}{P_S^{**}} \right)^{1-\alpha} ,$$

where P_S^{**} and P_C^{**} are defined by (38) and (42), respectively. Solving for m yields the long-run cost function, that is, the long-run minimum cost of achieving utility level U for given prices p_i ($i = 1, \dots, N$):

$$\text{Cost}_{\text{long-run}} = U \frac{\alpha+\mu}{1+\mu} \left(\frac{P_C^{**}}{\alpha} \right)^{\alpha} \left(\frac{P_S^{**}}{1-\alpha} \right)^{1-a} \ . \tag{44}$$

The short-run cost of achieving the same utility level, U, can be larger or smaller than the long-run cost because it depends on the inherited stocks, $\bar{s}_i$ $(i = 1, \ldots, N)$, and past consumption levels, $\bar{c}_i$ $(i = 1, \ldots, N)$. Exploiting (27) and (28), the utility function (5) gives

$$U = \tilde{m} \left(\frac{\alpha}{P_C} \right)^{\alpha} \left(\frac{1-\alpha}{P_S} \right)^{1-\alpha} ,$$

where P_C and P_S are defined by (16) and (21), respectively. Inserting (23) and solving for $\tilde{m}$ gives the short-run cost function:

$$\text{Cost}_{\text{short-run}} = U \left(\frac{P_C}{\alpha} \right)^{\alpha} \left(\frac{P_S}{1-\alpha} \right)^{1-\alpha} - \sum_{i=1}^{N} p_i \bar{s}_i \, .$$

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