CORRELATION FUNCTIONS

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CORRELATION FUNCTIONS

Dependencies

- A time series is *typically* analyzed with an assumption that observations have a potential relationship across time.
  - Ex: Weight

- Same approach can be taken with space as well as time.
  - Ex: Temperature

Autocorrelation Function

- *Autocorrelation* is the correlation between two sets of observations, from the same series, that are separated by *k* points in time.
- The autocorrelation function (ACF) is the function of all autocorrelations (between two **sets of observations** Y_t and Y_{t-k}) across time (for all values of *k*).

$$\rho_k = \mathrm{Corr}(Y_t, Y_{t-k})$$

Autocorrelation Function

t	Y_t	Y_{t-1}	Y_{t-2}
1	20	.	.
2	2	20	.
3	16	2	20
4	-3	16	2
5	-14	-3	16
6	-28	-14	-3
…	…	…	
999	0	29	17
1000	-19	0	29

Autocorrelation Function

t	Y_t	Y_{t-1}	Y_{t-2}
1	20	.	.
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$$\hat{\rho}_1 = 0.46$$

Autocorrelation Function

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1	20	.	.
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…	…	…	
999	0	29	17
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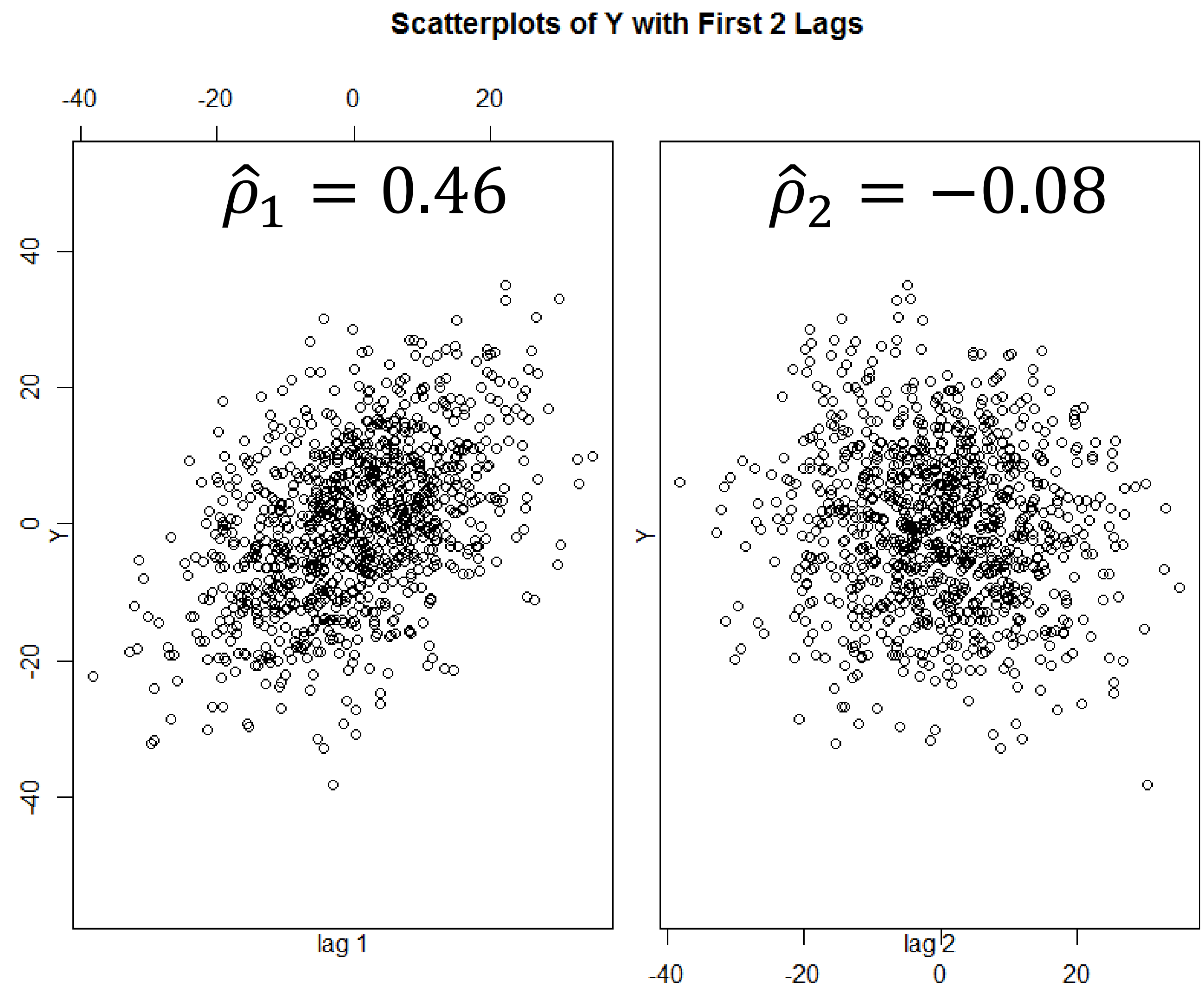
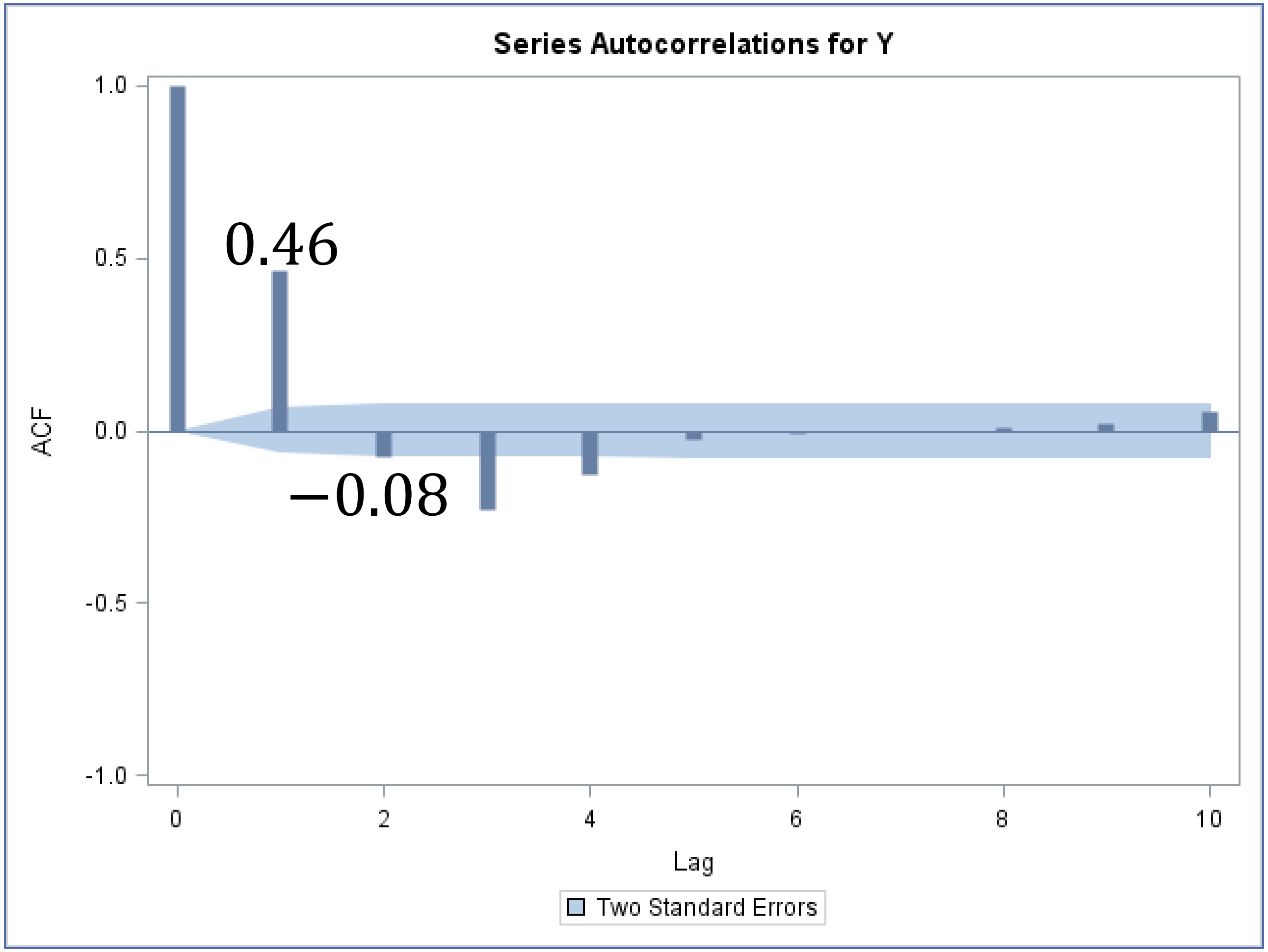
$\hat{\rho}_1 = 0.46$

$\hat{\rho}_2 = -0.08$

Autocorrelation Function



Autocorrelation Function



Autocorrelation Function

- Suppose that the first autocorrelation value (ACF(1)) is significant.
- This implies that two consecutive time points are related to each other.
  - March is related to April, April is related to May, etc.
  - Monday is related to Tuesday, Tuesday is related to Wednesday, etc.

Autocorrelation Function

- This relationship can be both in a positive and negative direction:
  - Positive – High Mondays imply high Tuesdays
  - Negative – High Mondays imply low Tuesdays

- This same relationship goes for all lags of the autocorrelation function.

Partial Autocorrelation Function

- *Partial autocorrelation* is the correlation between two sets of observations, from the same series, that are separated by *k* points in time, **after adjusting for all previous (1, 2, …, *k*-1) autocorrelations**.
- Partial autocorrelations are conditional correlations.
- The partial autocorrelation function (PACF) is the function of all partial autocorrelations (between two **sets of observations** Y_t and Y_{t-k}) across time (for all values of *k*).

$$\phi_k = \mathrm{Corr}(Y_t, Y_{t-k} \mid Y_{t-1}, Y_{t-2}, \ldots, Y_{t-k-1})$$

Partial Autocorrelation Function

t	Y_t	Y_{t-1}	Y_{t-2}
1	20	.	.
2	2	20	.
3	16	2	20
4	-3	16	2
5	-14	-3	16
6	-28	-14	-3
…	…	…	
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Partial Autocorrelation Function

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$$\hat{\rho}_1 = 0.46$$

$$\hat{\phi}_1 = 0.46$$

No time points in between to influence results!

Partial Autocorrelation Function

t	Y_t	Y_{t-1}	Y_{t-2}
1	20	.	.
2	2	20	.
3	16	2	20
4	-3	16	2
5	-14	-3	16
6	-28	-14	-3
…	…	…	
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$\hat{\rho}_1 = 0.46$

$\hat{\phi}_1 = 0.46$

$\hat{\rho}_2 = -0.08$

$\hat{\phi}_2 = -0.37$

Must remove influence of time point in between!

Partial Autocorrelation Function

- The partial autocorrelation for the ***k^th^*** **lag** is calculated from the following regression:

$$Y_t = \beta_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_k Y_{t-k} + e_t$$

Partial Autocorrelation Function

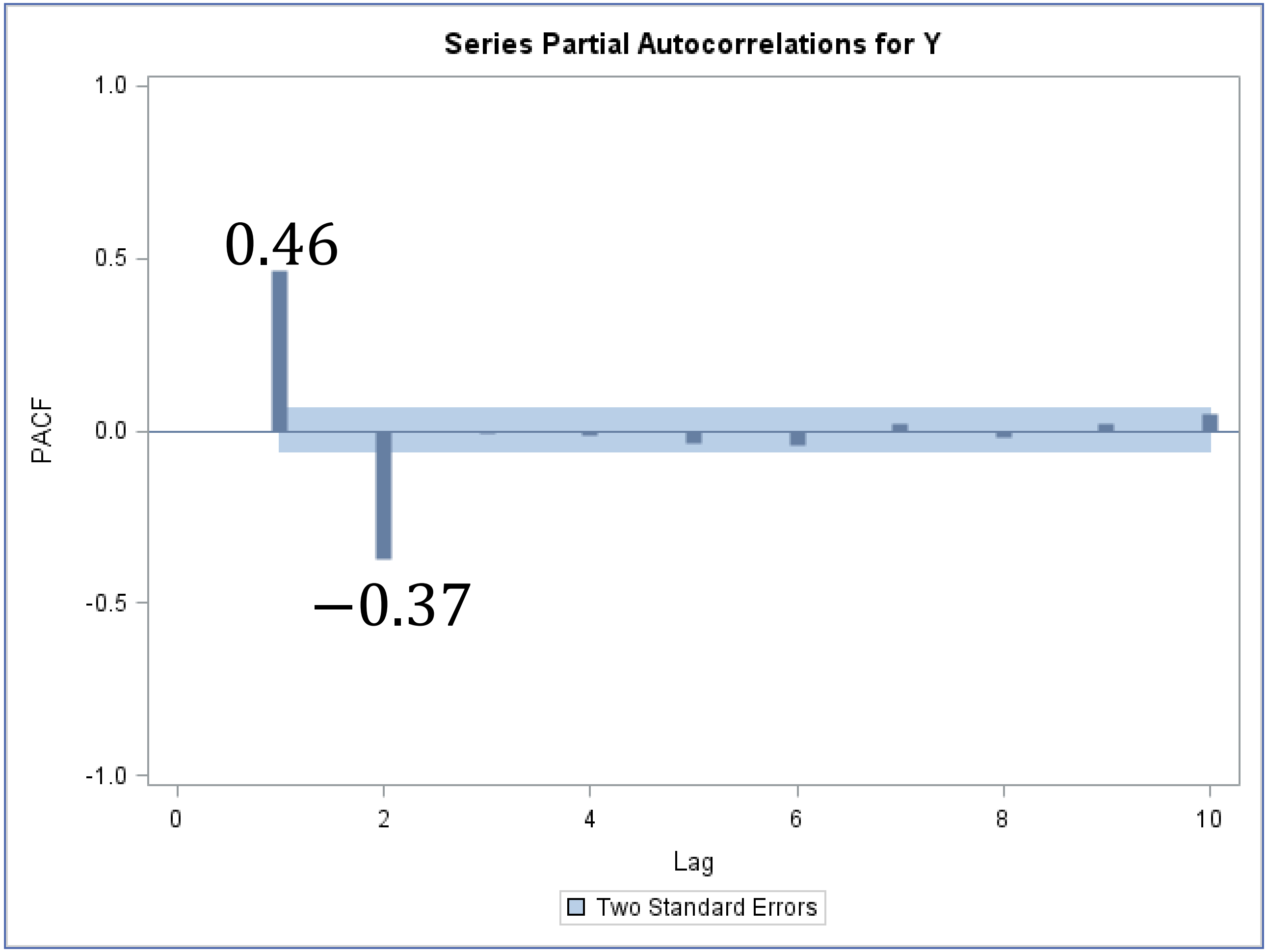
- The partial autocorrelation for the ***k**^th^* **lag** is calculated from the following regression:

$$Y_t = \beta_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_k Y_{t-k} + e_t$$

- For example, the 2nd partial autocorrelation (ϕ_2) is estimated from:

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\phi}_1 Y_{t-1} + \hat{\phi}_2 Y_{t-2}$$

Partial Autocorrelation Function



Partial Autocorrelation Function

- The partial autocorrelation functions tries to measure the direct relationship between two sets of observations, without the influence of other sets of time in between.

Correlation Functions – R

```
acf1=Acf(Y, lag=10)$acf
pacf1=Pacf(Y, lag=10)$acf

index1=seq(1,length(pacf1))

all.dat=data.frame(cbind(acf1[2:11],pacf1,index1))
colnames(all.dat)=c("acf","pacf","index")

ggplot(all.dat,aes(x=factor(index),y=acf))+geom_col+labs
(x="Lags")
```

ACF for Y