

# HIERARCHICAL AND GROUPED TIME SERIES

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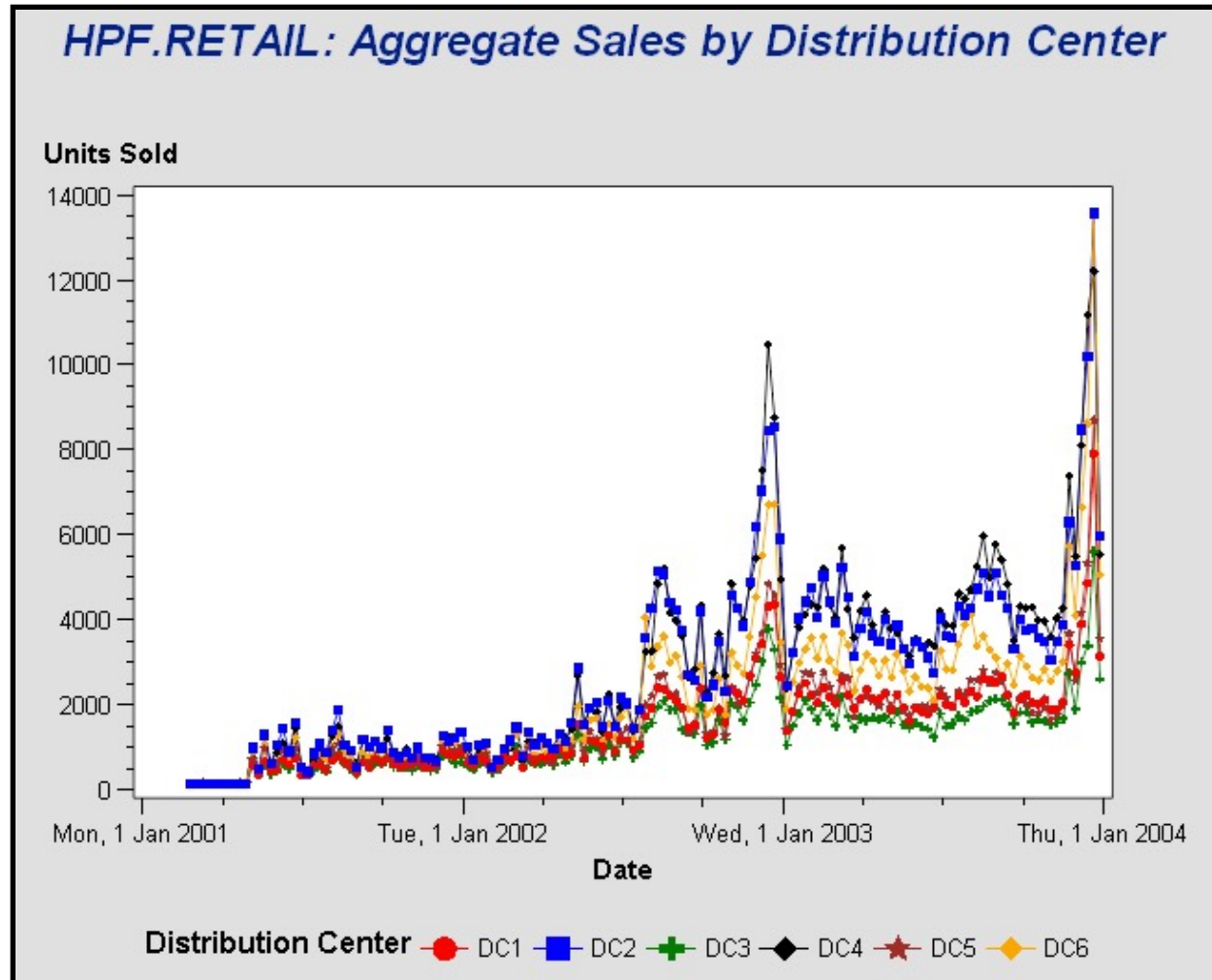
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# LARGE SCALE FORECASTING

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# Motivation



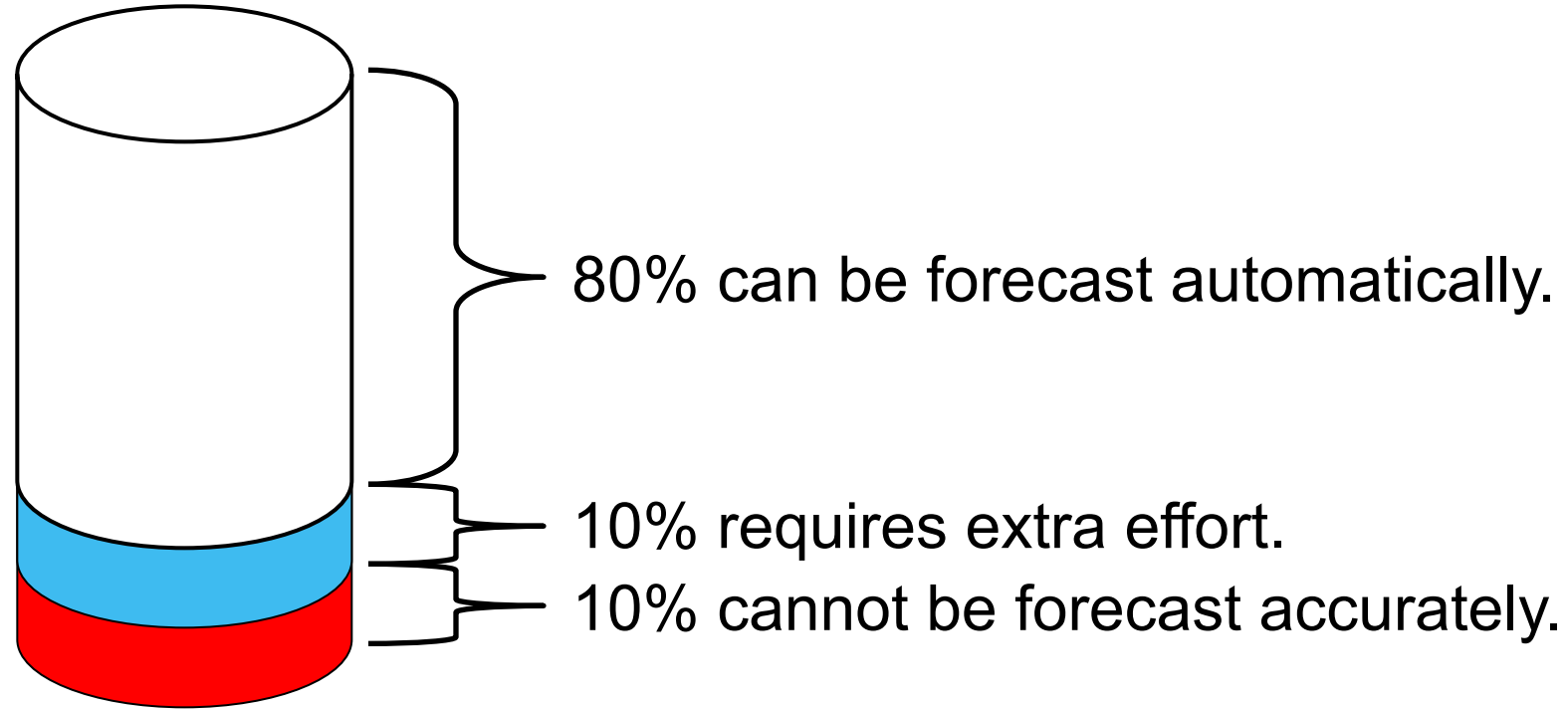
# Forecasting a Single Time Series

- Skilled analysts can forecast an individual time series by one of the following:
  - Applying good judgement based on knowledge and experience
  - Use proven time series analysis techniques

# Large-Scale Forecasting

- Modern businesses require efficient, reliable forecasts for many series, not just one.
- Not sufficient resources to apply the same individualized approaches to all the series that need to be forecasted.
- Series might have hierarchically arranged elements and require reconciliation of forecasts at different levels.

# Large-Scale Forecasting



Time Series Data

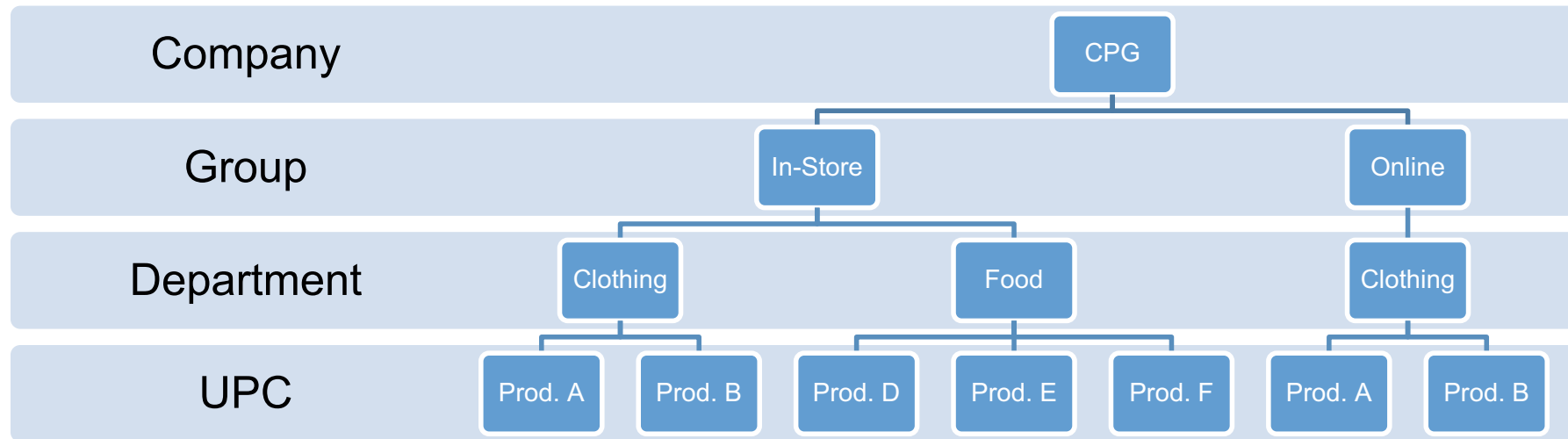


# HIERARCHICAL FORECASTING

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# Data Hierarchies



# Automatic Model Building

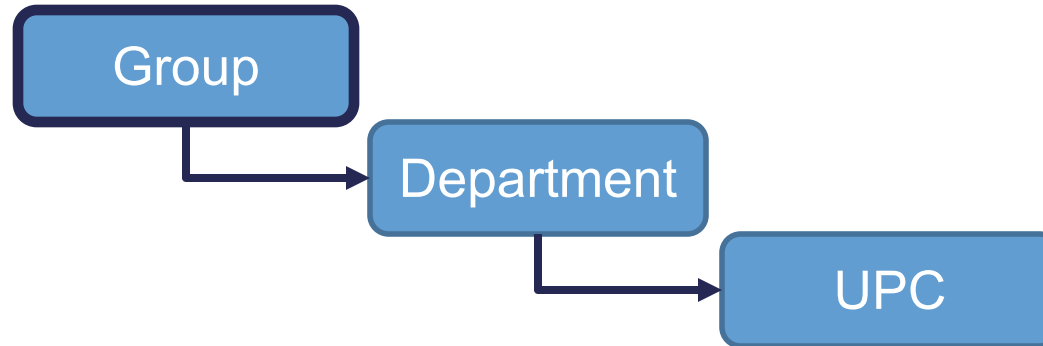
- Each model inside a certain hierarchy could easily be different.
- Most software can do this “automatically”:
  - SAS Forecast Studio uses the model that “fits the data the best.”
    - Builds models based on MAPE using automatic selection techniques.
  - R hts package
    - Builds models based on BIC

# Model Reconciliation

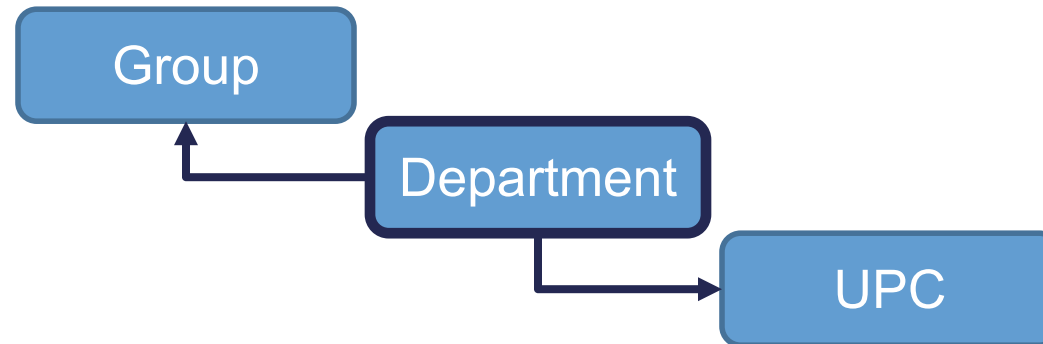
- If we were to model each series in each hierarchy, the statistical forecasts typically wouldn't add up to between hierarchies as we would want.
- Example:
  - If we sell 20 products in a region, we want our region forecast to be the sum of the 20 product forecasts in that region.
- **Reconciliation** is the process of making the statistical forecasts add up for each time interval in the data.

# Reconciliation Approaches

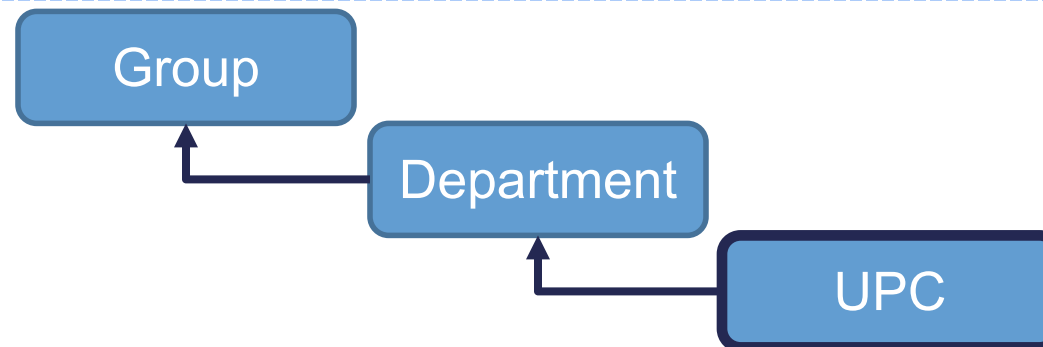
Top-Down



Middle-Out

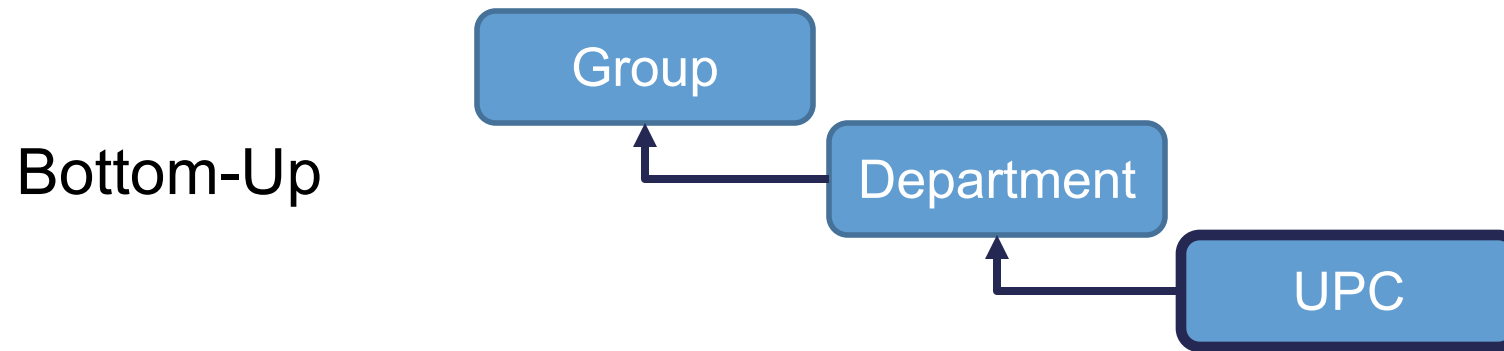


Bottom-Up



# Bottom-up Approach

- Most common approach to hierarchical forecasting.
- Build a model for each series in the very bottom of the hierarchy structure.
- Add up the individual forecasts to build the hierarchy above it.

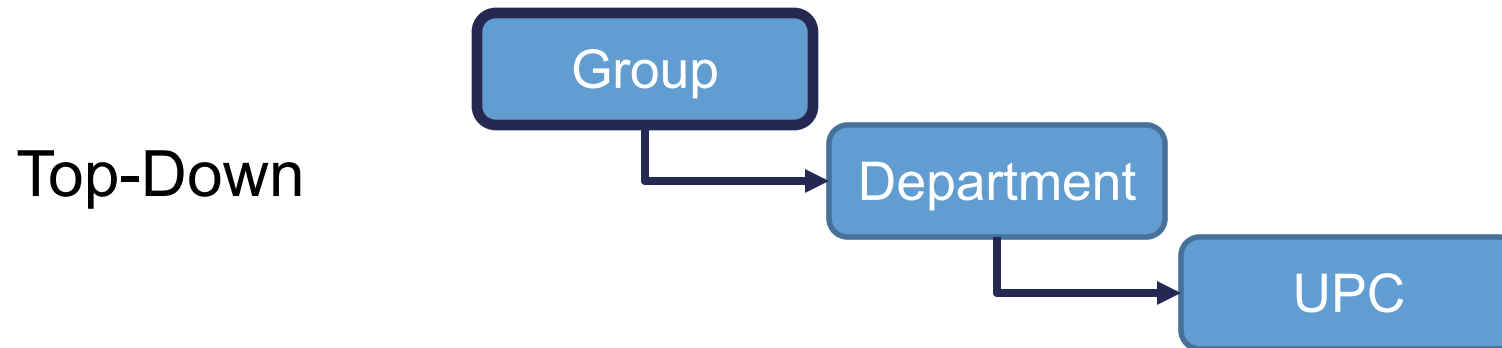


# Bottom-up Approach

- Advantage:
  - NO LOSS OF INFORMATION!
- Disadvantage:
  - Very noisy data typically lies in the lowest hierarchy.
  - Potentially **MANY** models to build.
- All the detail is saved, but that means that you potentially have a lot of unneeded noise you are trying to model away.

# Top-down Approach

- Simplest technique across all approaches.
- Build a forecast for the overall process and disaggregate this forecast down the tiers of the hierarchy.



# Disaggregation Techniques

- There are a variety of different ways to disaggregate the forecasts in one tier down to the hierarchical tier below it.
- 3 Common Techniques:
  1. Average Historical Proportions
  2. Proportion of Historical Averages
  3. Forecasted Proportions



# Average Historical Proportions

- Each proportion reflects the average of the historical proportions of the series a tier below relative to the total.

$$p_j = \frac{1}{T} \sum_{t=1}^T \frac{y_{j,t}}{y_t}$$

- Example:
  - On average, Department 1 historically makes up 34.8% of the total sales.

# Example

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|              | Year 1   | Year 2   | Year 3  | Year 4   |
|--------------|----------|----------|---------|----------|
| Department 1 | \$3,000  | \$5,000  | \$2,900 | \$4,000  |
| Company      | \$10,000 | \$11,000 | \$9,500 | \$12,000 |
| Proportion   | 0.3      | 0.45     | 0.31    | 0.33     |

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Average = 0.348

# Proportion of Historical Averages

- Each proportion reflects the proportion of the historical averages of the series a tier below relative to the average total series.

$$p_j = \sum_{t=1}^T \frac{y_{j,t}}{T} / \sum_{t=1}^T \frac{y_t}{T}$$

- Example:
  - The proportion of Department 1's historical average makes up 35.1% of the average total sales.

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|              | Year 1   | Year 2   | Year 3  | Year 4   | Avg.     |
|--------------|----------|----------|---------|----------|----------|
| Department 1 | \$3,000  | \$5,000  | \$2,900 | \$4,000  | \$3,725  |
| Company      | \$10,000 | \$11,000 | \$9,500 | \$12,000 | \$10,625 |

# Example

- Example:
  - The proportion of Warehouse 1's historical average makes up 35.1% of the average total sales.

|              | Year 1   | Year 2   | Year 3  | Year 4   | Avg.     |
|--------------|----------|----------|---------|----------|----------|
| Department 1 | \$3,000  | \$5,000  | \$2,900 | \$4,000  | \$3,725  |
| Company      | \$10,000 | \$11,000 | \$9,500 | \$12,000 | \$10,625 |

Proportion = 0.351



# Disaggregation Techniques

- There are a variety of different ways to disaggregate the forecasts in one tier down to the hierarchical tier below it.
- 3 Common Techniques:
  1. Average Historical Proportions
  2. Proportion of Historical Averages
  3. Forecasted Proportions

Use time series models to  
forecast values of proportions  
over time!

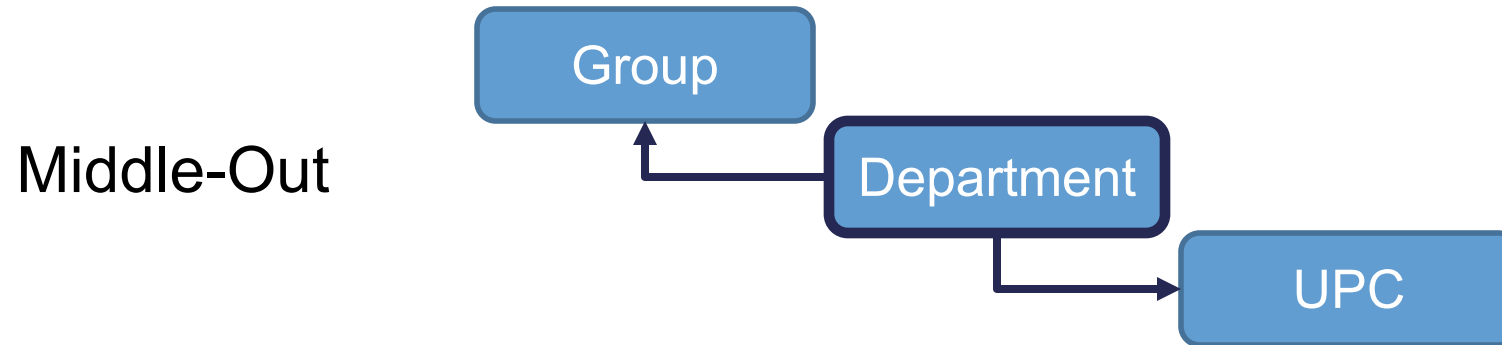
# Top-down Approach

- Advantage:
  - Simple to build as there are lower number of models needed.
- Disadvantage:
  - **LOSS OF POTENTIAL VALUABLE INFORMATION!**
- Potential loss of valuable information in the lower levels as our lower level forecasts are just proportions of upper level models that were easier to build.



# Middle-out Approach

- Combines attributes from both bottom-up and top-down approaches.
- Aggregates up the tiers above with bottom-up approach.
- Disaggregates down the tiers with top-down approach.



# Middle-out Approach

- Tries to balance both approaches.
- Advantage:
  - Uses some of the detailed information, but not so detailed that it is too noisy.
- Disadvantage:
  - Doesn't use ALL the information as disaggregate still takes place.

