Categorical Data Analysis

Institute for Advanced Analytics
MSA Class of 2022

Qualitative Data Types

- **Categorical Variables**:
  - Data whose measurement scale is inherently categorical.
  - **Nominal** – categories with no logical ordering
  - **Ordinal** – categories with a logical order / only two ways to order the categories (binary IS ordinal)

Ames Housing Data

```
train <- train %>%
           mutate(Bonus = ifelse(Sale_Price > 175000, 1, 0))
```

Examining Categorical Variables

- By examining the distributions of categorical variables, you can do the following:
  1. Determine the frequencies of data values
  2. Recognize possible associations among variables

Categorical Variables Association

- An association exists between two categorical variables if the distribution of one variable changes when the level (or value) of the other variable changes.
- If there is no association, the distribution of the first variable is the same regardless of the level of the other variable.

No Association

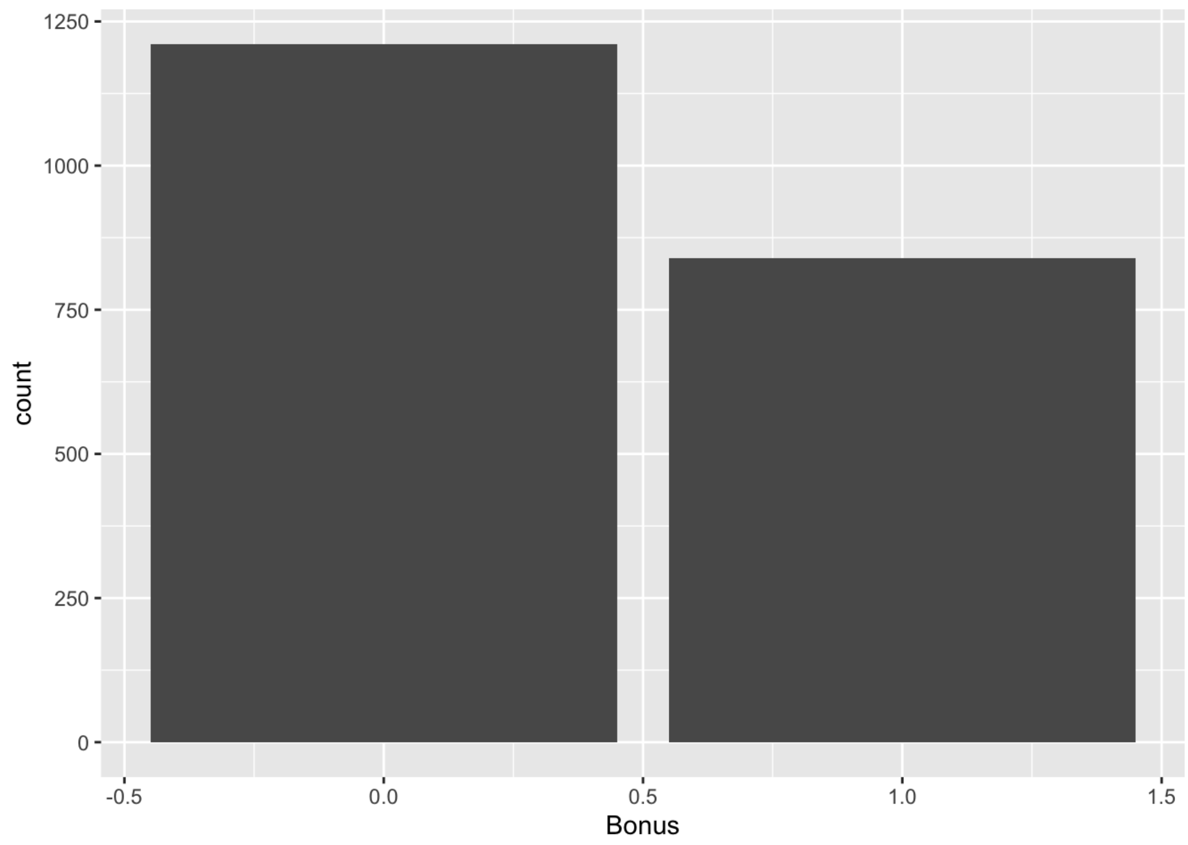
	Bonus Eligible	
	Yes	**No**
Central Air	42%	58%
No Central Air	42%	58%

Association

	Bonus Eligible	
	Yes	No
Central Air	44%	56%
No Central Air	3%	97%

Exploring the Data

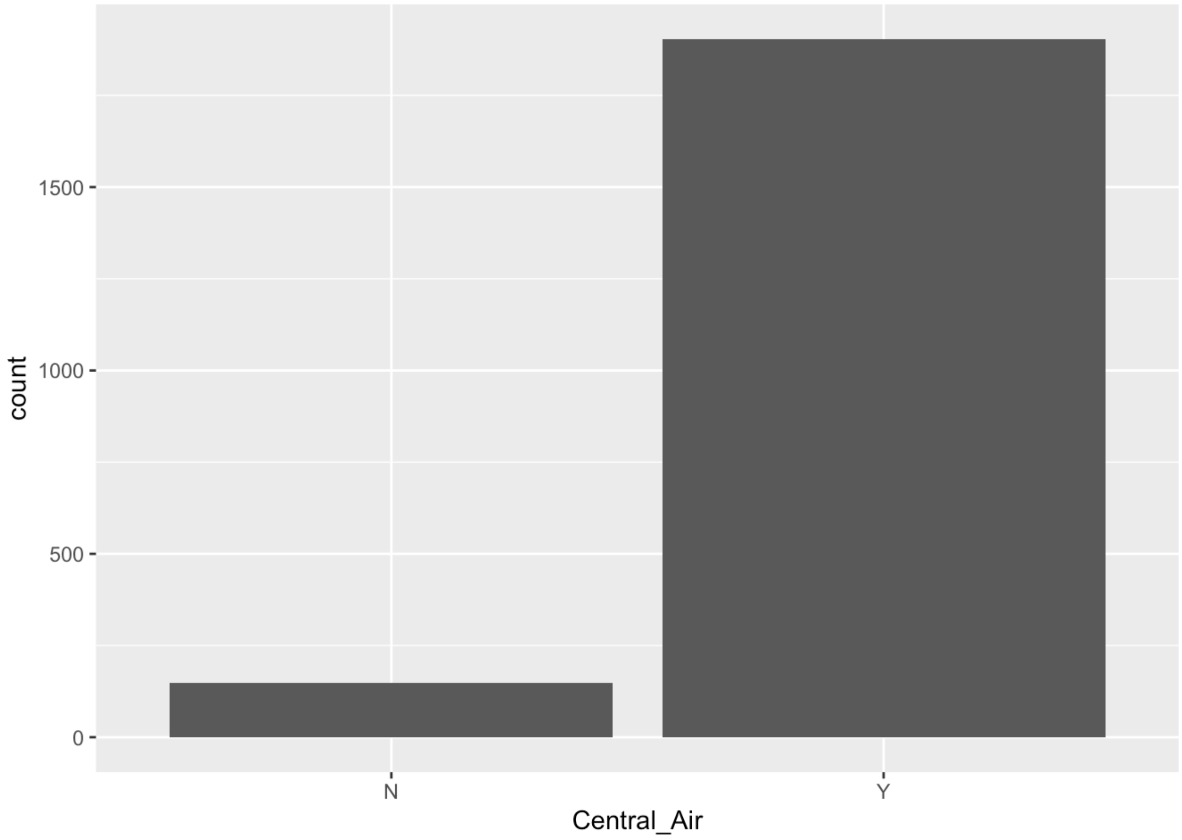
```r
table(train$Bonus)
##
##    0   1
## 1211 840

ggplot(data = train) +
  geom_bar(mapping =
             aes(x = Bonus))
```



Exploring the Data

```
table(train$Central_Air)
##
##    N    Y
##  147 1904
```

```
ggplot(data = train) +
  geom_bar(mapping =
             aes(x = Central_Air))
```



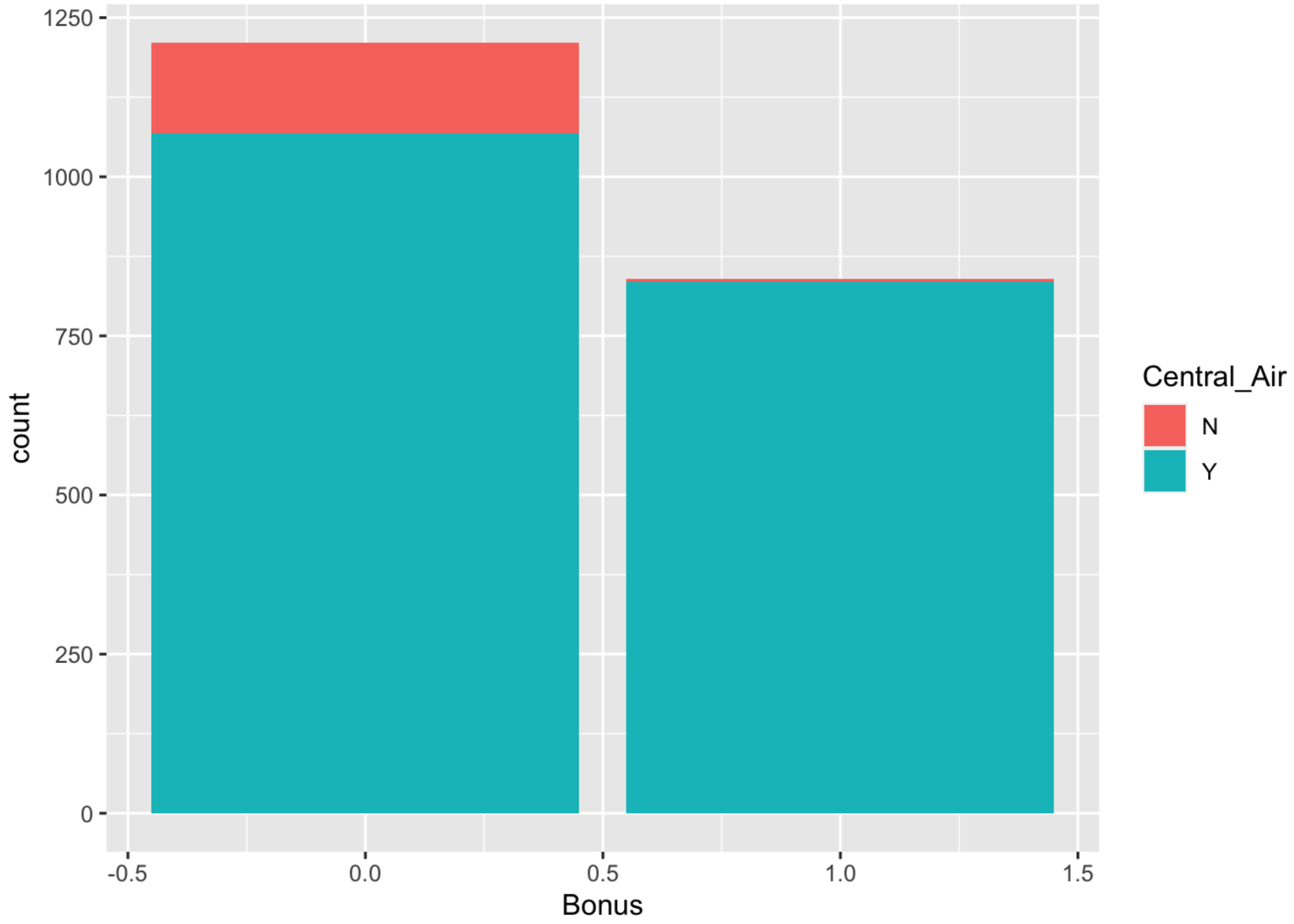
Cross-Tabulation Table

- Explore two variables with **cross-tabulation** tables.
- Shows the number of observations for each combination of the row and column variables.

Exploring the Data

```
table(train$Central_Air, train$Bonus)

##
##        0   1
## N    142   5
## Y   1069 840
```

```
ggplot(data = train) +
  geom_bar(mapping =
           aes(x = Bonus,
               fill = Central_Air))
```



Exploring the Data

```
library(gmodels)

CrossTable(train$Central_Air, train$Bonus)
```

Exploring the Data

```
## Cell Contents
## |-------------------------|
## |                       N |
## | Chi-square contribution |
## |           N / Row Total |
## |           N / Col Total |
## |         N / Table Total |
## |-------------------------|
```

```
##                   | train$Bonus
## train$Central_Air |          0 |          1 |  Row Total |
## ------------------|-----------|-----------|-----------|
##                 N |       142 |         5 |       147 |
##                   |    35.112 |    50.620 |           |
##                   |     0.966 |     0.034 |     0.072 |
##                   |     0.117 |     0.006 |           |
##                   |     0.069 |     0.002 |           |
## ------------------|-----------|-----------|-----------|
##                 Y |      1069 |       835 |      1904 |
##                   |     2.711 |     3.908 |           |
##                   |     0.561 |     0.439 |     0.928 |
##                   |     0.883 |     0.994 |           |
##                   |     0.521 |     0.407 |           |
## ------------------|-----------|-----------|-----------|
##      Column Total |      1211 |       840 |      2051 |
##                   |     0.590 |     0.410 |           |
## ------------------|-----------|-----------|-----------|
```

?

Tests of Association

Association

	Bonus Eligible	
	Yes	**No**
Central Air	44%	56%
No Central Air	3%	97%

How **much of a change** is required

to believe there actually is a difference?

Tests of Association - Hypotheses

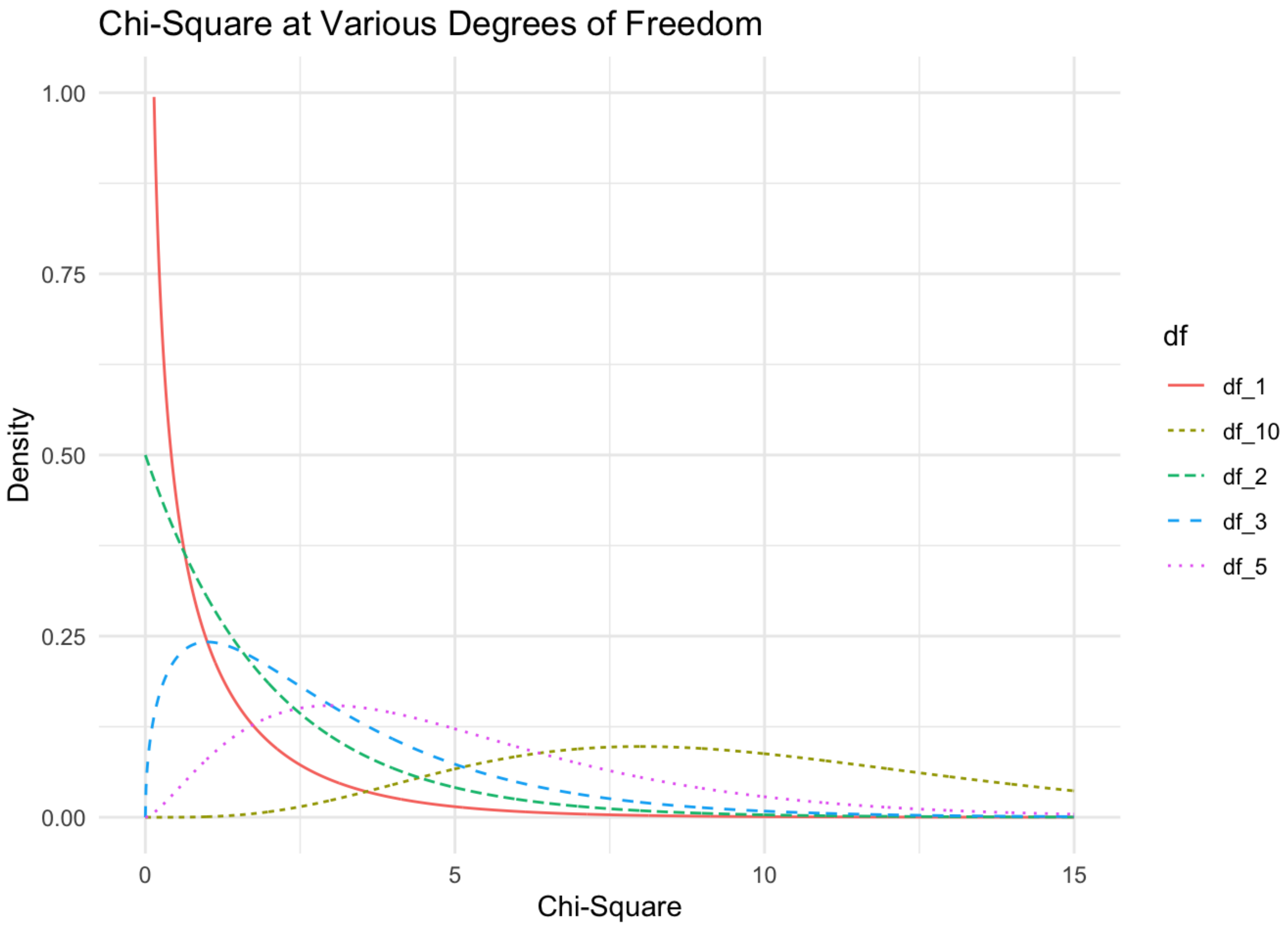
- **Null Hypothesis**
  - There is no association.
  - The distribution of one variable does not change across levels of another variable.

- **Alternative Hypothesis**
  - There ***is*** an association.
  - The distribution of one variable changes across levels of another variable.

χ^2-Distribution

- The Chi-Square test comes from the $\boldsymbol{\chi^2}$**-distribution**.
- Characteristics of the χ^2-distribution:
  1. Bounded Below By Zero
  2. Right Skewed
  3. One set of Degrees of Freedom

χ^2-Distribution



Pearson Chi-Square Test

- The Pearson χ^2 test works for comparing any two categorical variables.

$$\chi_P^2 = \sum_{i=1}^{R} \sum_{j=1}^{C} \frac{\left(Obs_{i,j} - Exp_{i,j}\right)^2}{Exp_{i,j}}$$

D.F. = (# Rows – 1)(# Columns – 1)

Expected Cell Counts

```
##                   | train$Bonus 
## train$Central_Air |           0 |           1 | Row Total | 
## ------------------|-------------|-------------|-----------|
##                 N |         142 |           5 |       147 | 
##                   |             |             |           | 
## ------------------|-------------|-------------|-----------|
##                 Y |        1069 |         835 |      1904 | 
##                   |             |             |           | 
## ------------------|-------------|-------------|-----------|
##      Column Total |        1211 |         840 |      2051 | 
##                   |       0.590 |       0.410 |           | 
## ------------------|-------------|-------------|-----------|
```

Expected Cell Counts

```
##                   | train$Bonus
## train$Central_Air |           0 |           1 | Row Total |
## ------------------|-------------|-------------|-----------|
##                 N |         142 |           5 |       147 |
##                   |             |             |           |
## ------------------|-------------|-------------|-----------|
##                 Y |        1069 |         835 |      1904 |
##                   |             |             |           |
## ------------------|-------------|-------------|-----------|
##      Column Total |        1211 |         840 |      2051 |
##                   |       0.590 |       0.410 |           |
## ------------------|-------------|-------------|-----------|
```

Expected Cell Counts

```
##                   | train$Bonus
## train$Central_Air |           0 |           1 |   Row Total |
## ------------------|-------------|-------------|-------------|
##                 N |         142 |           5 |         147 |
##                   |       86.73 |       60.27 |             |
## ------------------|-------------|-------------|-------------|
##                 Y |        1069 |         835 |        1904 |
##                   |             |             |             |
## ------------------|-------------|-------------|-------------|
##      Column Total |        1211 |         840 |        2051 |
##                   |       0.590 |       0.410 |             |
## ------------------|-------------|-------------|-------------|
```

Expected Cell Counts

```
##                  | train$Bonus
## train$Central_Air |         0 |         1 | Row Total |
## -----------------|-----------|-----------|-----------|
##                N |       142 |         5 |       147 |
##                  |     86.73 |     60.27 |           |
## -----------------|-----------|-----------|-----------|
##                Y |      1069 |       835 |      1904 |
##                  |   1123.36 |    780.64 |           |
## -----------------|-----------|-----------|-----------|
##     Column Total |      1211 |       840 |      2051 |
##                  |     0.590 |     0.410 |           |
## -----------------|-----------|-----------|-----------|
```

Pearson Chi-Square Test

```r
chisq.test(table(train$Central_Air, train$Bonus))
```

```
##
## Pearson's Chi-squared test with Yates' continuity correction
##
## data:  table(train$Central_Air, train$Bonus)
## X-squared = 90.686, df = 1, p-value < 2.2e-16
```

Likelihood Ratio Chi-Square Test

- The Likelihood Ratio χ^2 test works for comparing any two categorical variables.

$$\chi^2_{LR} = 2 \times \sum_{i=1}^{R} \sum_{j=1}^{C} Obs_{i,j} \times \log\left(\frac{Obs_{i,j}}{Exp_{i,j}}\right)$$

D.F. = (# Rows – 1)(# Columns – 1)

Assumptions

- Both of the above tests have a sample size requirement.
- The sample size requirement is 80% or more of the cells in the cross-tabulation table need **expected** count larger than 5.

```
##                   | train$Bonus
## train$Central_Air |           0 |           1 |   Row Total |
## ------------------|-------------|-------------|-------------|
##                 N |         142 |           5 |         147 |
##                   |       86.73 |       60.27 |             |
## ------------------|-------------|-------------|-------------|
##                 Y |        1069 |         835 |        1904 |
##                   |     1123.36 |      780.64 |             |
## ------------------|-------------|-------------|-------------|
##      Column Total |        1211 |         840 |        2051 |
##                   |       0.590 |       0.410 |             |
## ------------------|-------------|-------------|-------------|
```

Fisher’s Exact Test

- When we **don’t** meet the assumption we can use the **Fisher’s exact test** that calculates all possible permutations of data.

```
fisher.test(table(train$Central_Air, train$Bonus))
```

```
##
##  Fisher's Exact Test for Count Data
##
## data:  table(train$Central_Air, train$Bonus)
## p-value < 2.2e-16
```

Ordinal Compared to Nominal Tests

- Both the Pearson and Likelihood Ratio Chi-Square tests can handle any type of categorical variable – either ordinal, nominal, or both.
- However, ordinal variables provide us extra information since the order of the categories actually matters compared to nominal.
- We can test for even more with ordinal variables against other ordinal variables – whether two ordinal variables have a **linear relationship** as compared to just a general one.

Mantel-Haenszel Chi-Square Test

- The Mantel-Haenszel χ^2 test works for comparing any two **ordinal** variables.

$$\chi^2_{MH} = (\mathrm{n} - 1)\mathrm{r}^2$$

D.F. = 1

Mantel-Haenszel Chi-Square Test

```
library(vcdExtra)

CMHtest(table(train$Central_Air, train$Bonus))$table[1,]
```

```
##         Chisq            Df          Prob
## 9.230619e+01 1.000000e+00 7.425180e-22
```

?

Measures of Association

Chi-Square Tests

- Determines whether an association exists
- **MAY NOT** measure the strength of the association
  - Can compare when sample size similar
  - Can **NOT** compare when sample size **different**

Measures of Association

- Measures the **strength of the association**

- There are many different measures of association.
- Two common measures of association are the following:
  1. Odds Ratios (Only for 2x2 tables – binary vs. binary)
  2. Cramer's V (Any size table)
  3. Spearman's Correlation (ordinal vs. ordinal)

Odds Ratios

- An *odds ratio* indicates how much more likely, with respect to **odds**, a certain event occurs in one group relative to its occurrence in another group.
- The **odds** of an event occurring is NOT the same as the probability that an event occurs.

Odds Ratios

- An *odds ratio* indicates how much more likely, with respect to **odds**, a certain event occurs in one group relative to its occurrence in another group.
- The **odds** of an event occurring is NOT the same as the probability that an event occurs.

$$Odds = \frac{p}{1-p}$$

Probability versus Odds of an Outcome

```
##                    | train$Bonus
## train$Central_Air |           0 |           1 | Row Total |
## -------------------|-------------|-------------|-----------|
##                  N |         142 |           5 |       147 |
##                    |       0.966 |       0.034 |           |
## -------------------|-------------|-------------|-----------|
##                  Y |        1069 |         835 |      1904 |
##                    |       0.561 |       0.439 |           |
## -------------------|-------------|-------------|-----------|
##       Column Total |        1211 |         840 |      2051 |
##                    |       0.590 |       0.410 |           |
## -------------------|-------------|-------------|-----------|
```

Probability of **NOT** bonus eligible without central air = 0.966

Probability versus Odds of an Outcome

```
##                  | train$Bonus
## train$Central_Air |          0 |          1 | Row Total |
## -----------------|-----------|-----------|-----------|
##                N |        142 |          5 |       147 |
##                  |      0.966 |      0.034 |           |
## -----------------|-----------|-----------|-----------|
##                Y |       1069 |        835 |      1904 |
##                  |      0.561 |      0.439 |           |
## -----------------|-----------|-----------|-----------|
##     Column Total |       1211 |        840 |      2051 |
##                  |      0.590 |      0.410 |           |
## -----------------|-----------|-----------|-----------|
```

Odds of **NOT** bonus eligible without central air

Probability of **NOT** bonus eligible without central air = 0.966

Probability of bonus eligible without central air = 0.034

$$= \frac{0.966}{0.034} = 28.41$$

Odds Ratio

```
##                  | train$Bonus
## train$Central_Air |           0 |           1 | Row Total |
## -------------------|-------------|-------------|-----------|
##                  N |         142 |           5 |       147 |
##                    |       0.966 |       0.034 |           |
## -------------------|-------------|-------------|-----------|
##                  Y |        1069 |         835 |      1904 |
##                    |       0.561 |       0.439 |           |
## -------------------|-------------|-------------|-----------|
##       Column Total |        1211 |         840 |      2051 |
##                    |       0.590 |       0.410 |           |
## -------------------|-------------|-------------|-----------|
```

Odds ratio

Odds of **NOT** bonus eligible without central air = 28.41

Odds of **NOT** bonus eligible with central air = 1.28

$$= \frac{28.41}{1.28} = 22.2$$

Odds Ratio

Odds ratio

Odds of **NOT** bonus eligible without central air = 28.41

Odds of **NOT** bonus eligible with central air = 1.28

$$= \frac{28.41}{1.28} = 22.2$$

- Homes without central air have **22.2 times the odds** (22.2 times more likely) to not be bonus eligible as compared to homes with central air.

Odds Ratio

Odds ratio

Odds of **NOT** bonus eligible without central air = 28.41

Odds of **NOT** bonus eligible with central air = 1.28

$$= \frac{28.41}{1.28} = 22.2$$

- Homes without central air have **22.2 times the odds** (22.2 times more likely) to not be bonus eligible as compared to homes with central air.
- Reverse is also true!
- Homes with central air are 22.2 times more likely to be bonus eligible as compared to homes without central air.

Odds Ratio

```
library(DescTools)

OddsRatio(table(train$Central_Air, train$Bonus))


## [1] 22.18335
```

Cramer's V

- Odds ratios provide value for binary vs. binary relationships, but when you have more than two categories in one or both variables use **Cramer's V**.

$$V = \sqrt{\frac{\left(\frac{\chi_P^2}{n}\right)}{\min(\#\text{Rows} - 1, \#\text{Columns} - 1)}}$$

- Bounded between 0 and 1 (-1 and 1 for 2x2 scenario) where closer to 0 the weaker the relationship.

Cramer’s V

```
assocstats(table(train$Central_Air, train$Bonus))
```

```
##                       X^2 df P(> X^2)
## Likelihood Ratio 121.499  1        0
## Pearson           92.351  1        0
##
## Phi-Coefficient   : 0.212
## Contingency Coeff.: 0.208
## Cramer's V        : 0.212
```

Spearman’s Correlation

- Spearman’s correlation measures the strength of association between two ordinal variables.
- Calculated with the Pearson’s correlation on the ranks of the observations instead of the values of the observations.

Spearman's Correlation

```r
cor.test(x = as.numeric(ordered(train$Central_Air)),
         y = as.numeric(ordered(train$Bonus)),
         method = "spearman")
```

```
##  Spearman's rank correlation rho
##
## data:  x and y
## S = 1132826666, p-value < 2.2e-16
## alternative hypothesis: true rho is not equal to 0
## sample estimates:
##       rho
## 0.2121966
```

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Introduction to Logistic Regression

Modeling Categorical Data

Continuous Target Variable → **Linear Regression**, **ANOVA**, **Regularized Regression**

Categorical Target Variable → **Logistic Regression**

Modeling Categorical Data

Categorical Target Variable	→	**Logistic Regression**
Binary	→	Binary
Ordinal	→	Ordinal
Nominal	→	Nominal

Introduction to Logistic Regression

LINEAR PROBABILITY MODEL

Why Not Least Squares Regression?

$$y_i = \beta_0 + \beta_1 x_{1,i} + \varepsilon_i$$

- If the response variable is categorical, then how do you code the response numerically?
- If the response is coded (1=Yes and 0=No) and your regression equation predicts 0.5 or 1.1 or -0.4, what does that mean practically?
- If there are only two (or a few) possible response levels, is it reasonable to assume constant variance and normality?

Linear Probability Model

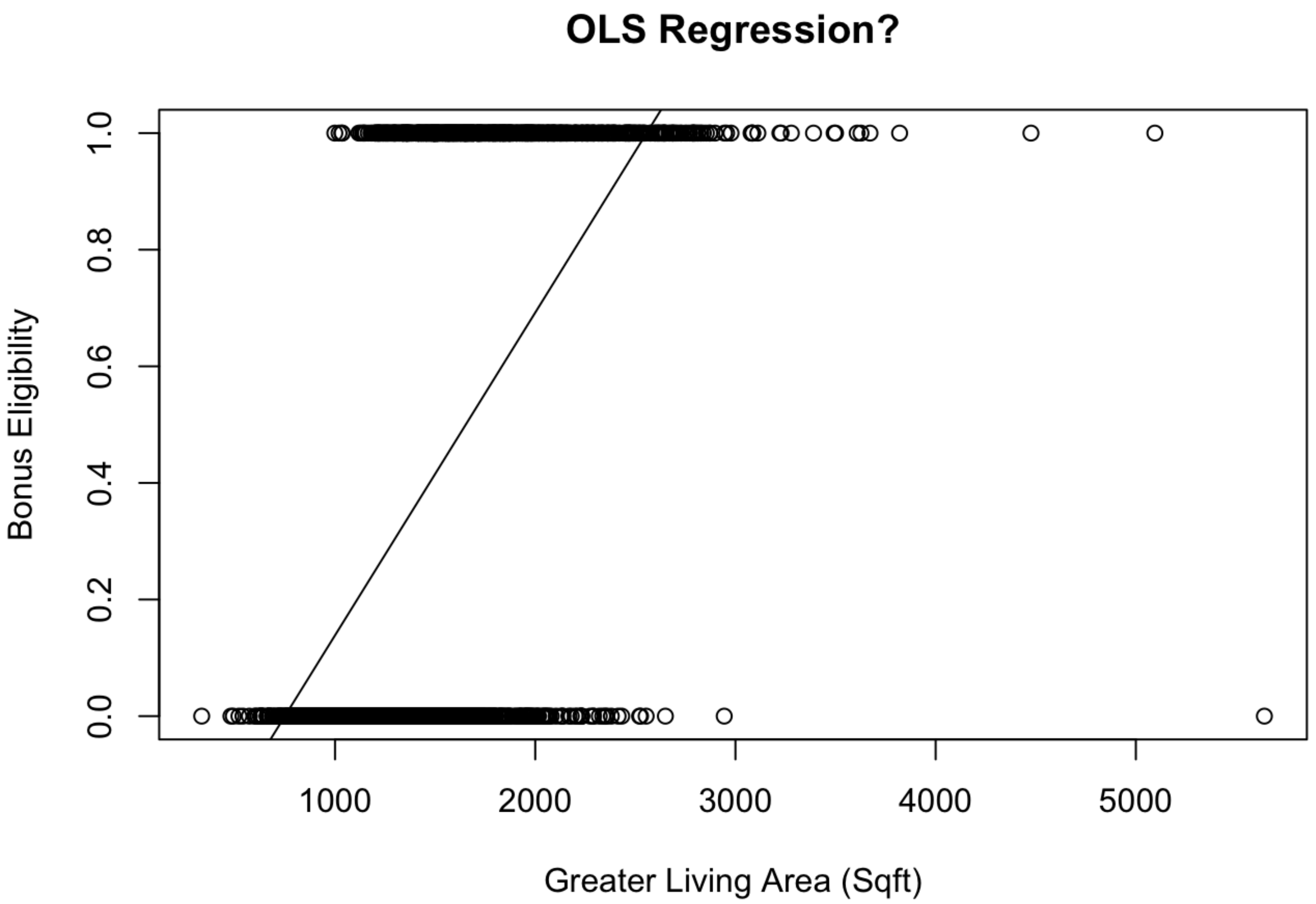
$$p_i = \beta_0 + \beta_1 x_{1,i}$$

- Probabilities are bounded, but linear functions can take on any value. (Once again, how do you interpret a predicted value of -0.4 or 1.1?)
- Given the bounded nature of probabilities, can you assume a linear relationship between X and *p* throughout the possible range of X?
- Can you assume a random error with constant variance?
- What is the observed probability for an observation?

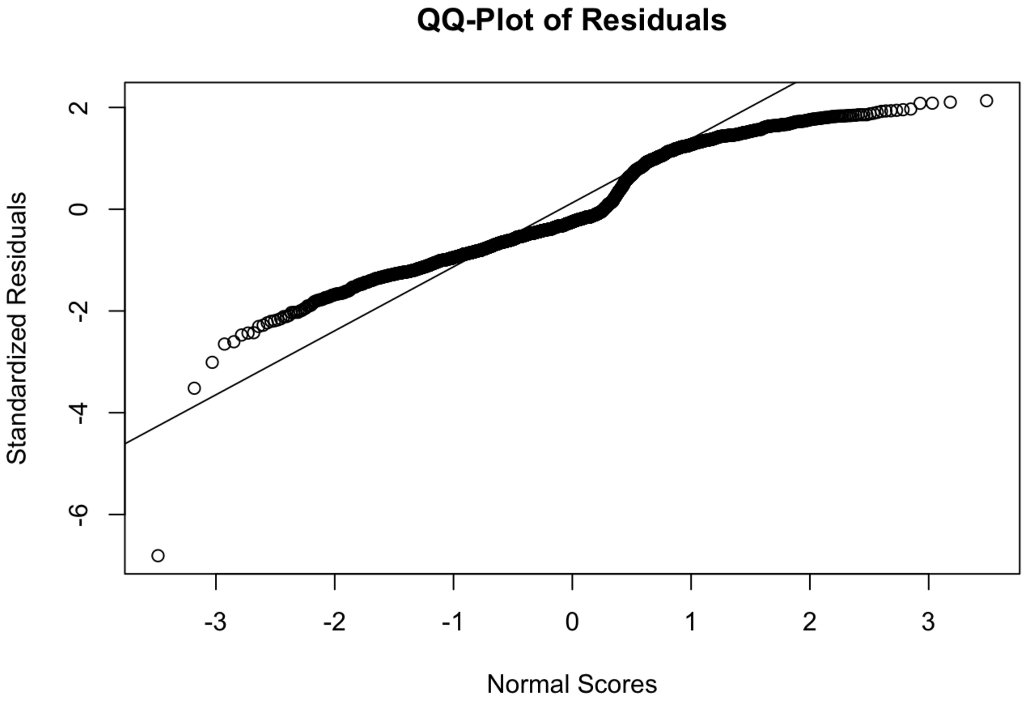
Linear Probability Model

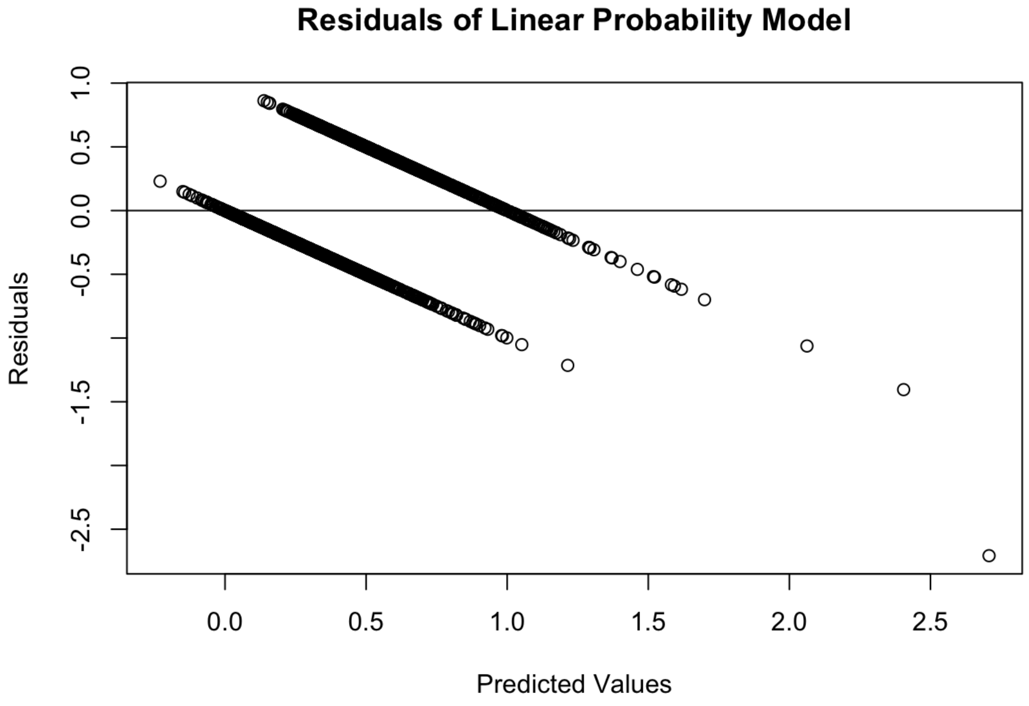
```
lp.model <- lm(Bonus ~ Gr_Liv_Area, data = train)
with(train, plot(x = Gr_Liv_Area, y = Bonus,
                 main = 'OLS Regression?',
                 xlab = 'Greater Living Area (Sqft)',
                 ylab = 'Bonus Eligibility'))
abline(lp.model)
```

Linear Probability Model



Linear Probability Model





?

Introduction to Logistic Regression

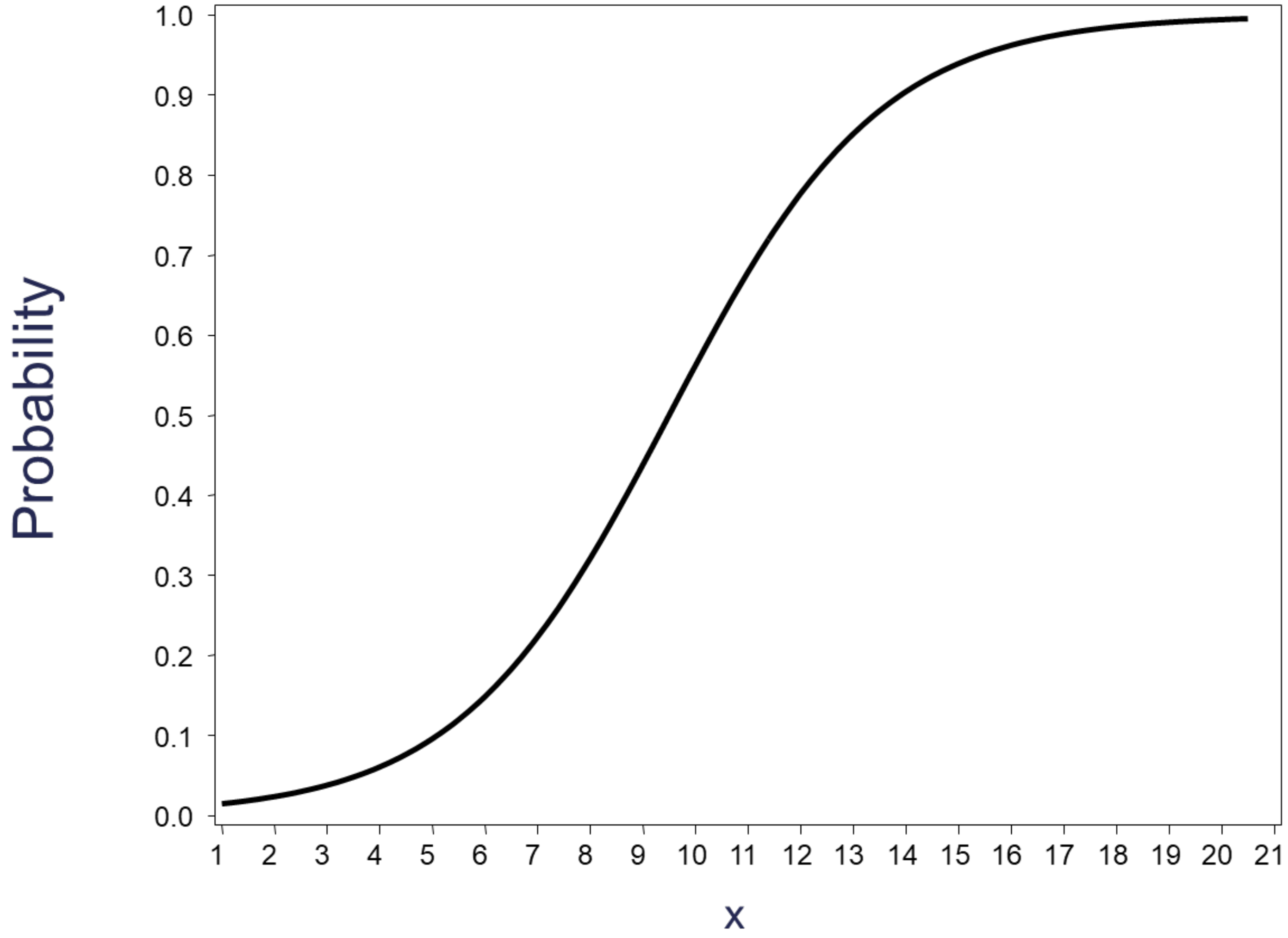
BINARY LOGISTIC REGRESSION

Logistic Regression Model

$$p_i = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_{1,i} + \cdots \beta_k x_{k,i})}}$$

- Has desired properties:
  - The predicted probability will always be between 0 and 1.
  - The parameter estimates do not enter the model equation linearly.
  - The rate of change of the probability varies as the X's vary.

Logistic Regression Model



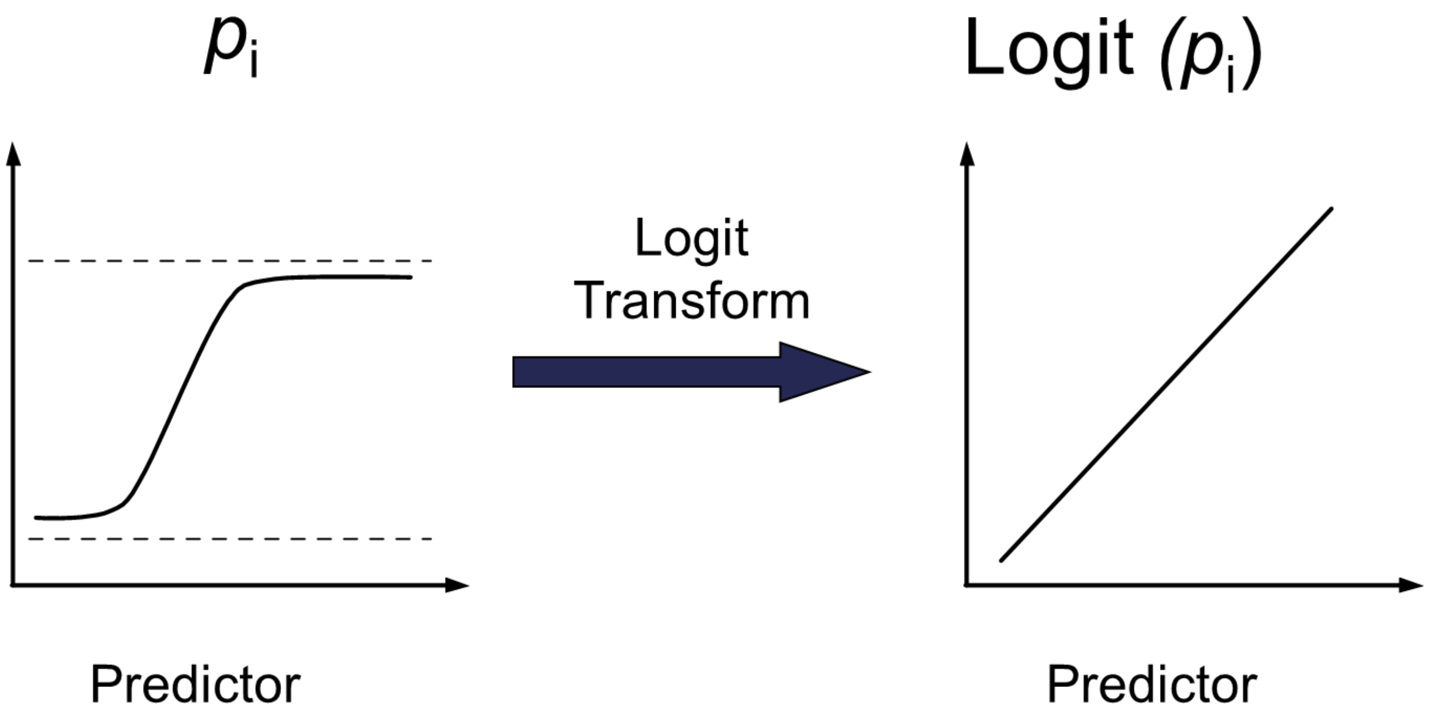
Logit Link Function

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 x_{1,i} + \cdots \beta_k x_{k,i}$$

- To create a linear model, a link function (logit) is applied to the probabilities.
- The relationship between the parameters and the **logits** are linear.
- Logits unbounded.

Assumptions

1. Independence of observations
2. Logit is linearity related to variables



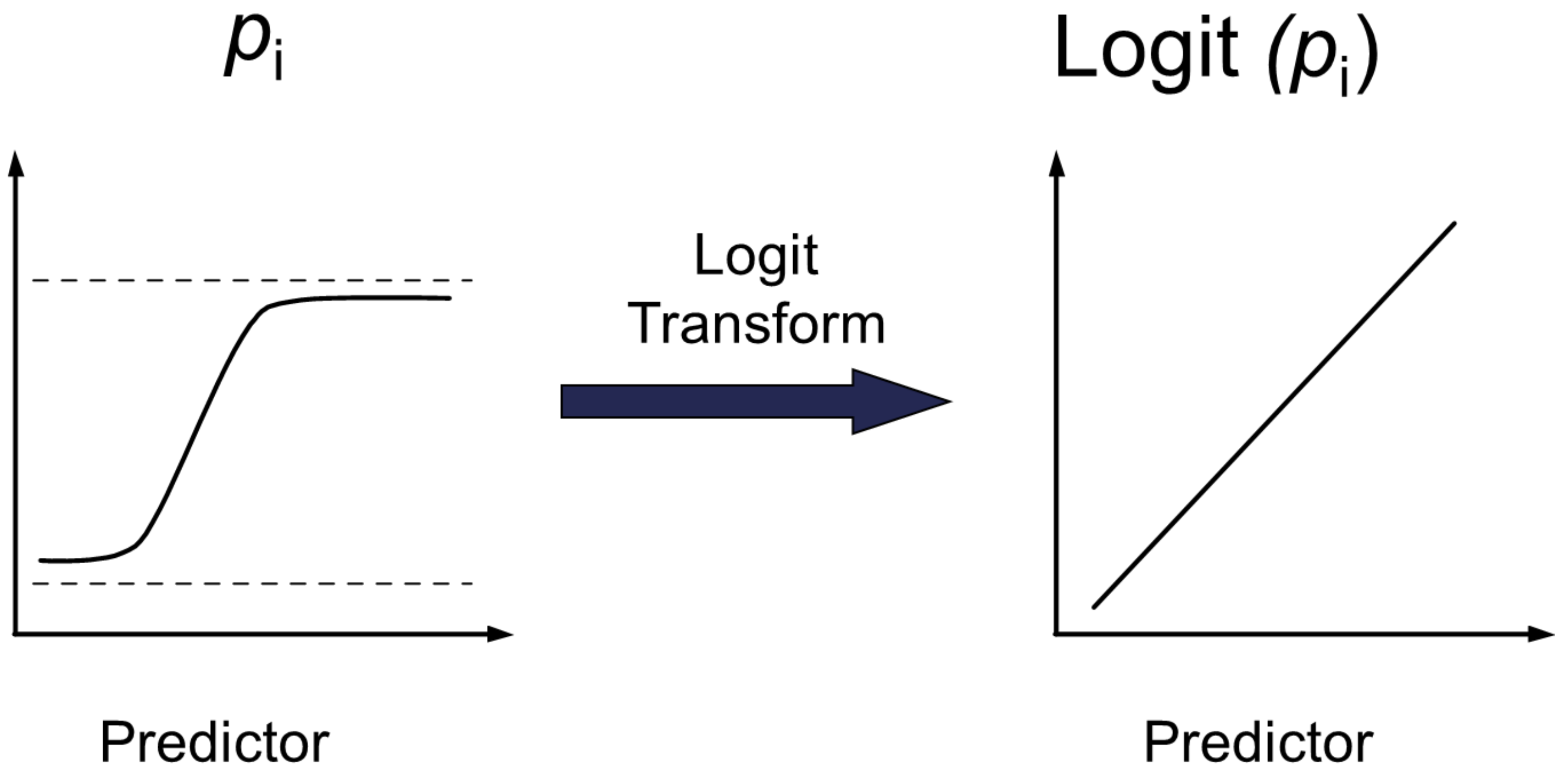
Binary Logistic Regression

```
ames_logit <- glm(Bonus ~ Gr_Liv_Area, data = train,
                  family = binomial(link = "logit"))
summary(ames_logit)
```

Binary Logistic Regression

```
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -5.5796  -0.6942  -0.3647   0.8060   2.1857
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -6.1348858  0.2757473  -22.25   <2e-16 ***
## Gr_Liv_Area  0.0038463  0.0001799   21.38   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 2775.8  on 2050  degrees of freedom
## Residual deviance: 1926.4  on 2049  degrees of freedom
## AIC: 1930.4
##
## Number of Fisher Scoring iterations: 5
```

Unit Change in Predictor does…?



$100 * \left(e^{\hat{\beta}} - 1\right)\%$ change in **Odds**

$\hat{\beta}$ change in **Logit**

Odds Ratio from a Logistic Regression

- Estimated logistic regression model:

$$\text{logit}(p_i) = -6.13 + 0.0038 * \text{Gr\_Liv\_Area} + \cdots$$

- Estimated odds ratio (Additional SF of Greater Living Area):

$$\text{OR} = \frac{e^{-6.13\, +\, 0.0038(GLA+1)+\cdots}}{e^{-6.13\, +\, 0.0038(GLA)+\cdots}} = e^{0.0038} = 1.0038$$

$$100 \times (1.0038 - 1)\% = 0.38\%$$

- Every additional square foot of greater living area **increases the expected odds of being bonus eligible by 0.38%**.

Odds Ratios

```
100*(exp(cbind(coef(ames_logit), confint(ames_logit)))-1)
```

```
##                                    2.5 %       97.5 %
## (Intercept) -99.7834027 -99.8755103 -99.6328865
## Gr_Liv_Area   0.3853699   0.3508132   0.4216532
```

Odds Ratio from a Logistic Regression

- Estimated logistic regression model:

$$\text{logit}(p_i) = -6.13 + 0.0038 * \text{Gr\_Liv\_Area} + \cdots$$

- Estimated odds ratio (Additional 100 SF of Greater Living Area):

$$\text{OR} = \frac{e^{-6.13\, +\, 0.0038(GLA+100)+\cdots}}{e^{-6.13\, +\, 0.0038(GLA)+\cdots}} = e^{0.0038\times 100} = 1.46$$

$$100 \times (1.46 - 1)\% = 46\%$$

- Every additional 100 square foot of greater living area **increases the expected odds of being bonus eligible by 46%**.

Odds Ratio for a Categorical Variable

- Estimated logistic regression model:

$$\text{logit}(p_i) = -9.97 + 3.56 * \text{Central\_AirY} + \cdots$$

- Estimated odds ratio (Additional 100 SF of Greater Living Area):

$$\text{OR} = \frac{e^{-9.97 + 3.56(CA\times 1)+\cdots}}{e^{-9.97 + 3.56(CA\times 0)+\cdots}} = e^{3.56} = 35.16$$

$$100 \times (35.16 - 1)\% = 3416\%$$

- Homes with central air **increases the expected odds of being bonus eligible by 3416%** compared to those without central air.

Odds Ratio for a Categorical Variable

- Estimated logistic regression model:

$$\text{logit}(p_i) = -9.97 + 3.56 * \text{Central\_AirY} + \cdots$$

- Estimated odds ratio (Additional 100 SF of Greater Living Area):

$$\text{OR} = \frac{e^{-9.97\, +\, 3.56(CA\times 1)+\cdots}}{e^{-9.97\, +\, 3.56(CA\times 0)+\cdots}} = e^{3.56} = 35.16$$

$$100 \times (35.16 - 1)\% = 3416\%$$

- Homes with central air **are 35.16 times more likely to be bonus eligible** then compared to those without central air.

Odds Ratios

```r
ames_logit2 <- glm(Bonus ~ Gr_Liv_Area + Central_Air + factor(Fireplaces),
                   data = train, family = binomial(link = "logit"))
100*(exp(cbind(coef(ames_logit2), confint(ames_logit2)))-1)
```

```
## Coefficients:
##                       Estimate Std. Error z value Pr(>|z|)
## (Intercept)         -9.970e+00  6.549e-01 -15.223  < 2e-16 ***
## Gr_Liv_Area          3.759e-03  2.031e-04  18.506  < 2e-16 ***
## Central_AirY         3.564e+00  5.310e-01   6.711 1.93e-11 ***
## factor(Fireplaces)1  9.822e-01  1.253e-01   7.837 4.60e-15 ***
## factor(Fireplaces)2  6.734e-01  2.406e-01   2.799  0.00513 **
## factor(Fireplaces)3 -3.993e-02  8.711e-01  -0.046  0.96344
## factor(Fireplaces)4  9.025e+00  3.247e+02   0.028  0.97783
```

?

Introduction to Logistic Regression

MODEL ASSESSMENT

Assessment of Logistic Regression

- Many different ways to assess logistic regression models
- One foundational way to evaluate models are comparing every pair of 0’s and 1’s in the target variable.
- These pairs are either considered **concordant, discordant, or tied**.

Concordant Pair

- A **concordant** pair is a 0 and 1 pair where the bonus eligible home (the 1 in our model) has a higher predicted probability than the non-bonus eligible home (the 0 in our model).
- Model successfully ordered these two observations by probability.
- It does not matter what the actual predicted probability values are as long as the bonus eligible home has a higher predicted probability than the non-bonus eligible home.

Discordant Pair

- A **discordant** pair is a 0 and 1 pair where the bonus eligible home (the 1 in our model) has a lower predicted probability than the non-bonus eligible home (the 0 in our model).
- Model unsuccessfully ordered the homes.
- It does not matter what the actual predicted probability values are as long as the bonus eligible home has a lower predicted probability than the non-bonus eligible home.

Tied Pair

- A **tied** pair is a 0 and 1 pair where the bonus eligible home has the same predicted probability as the non-bonus eligible home.
- Model is confused and sees these two different things as the same.
- In general, you want a high percentage of concordant pairs and low percentages of discordant and tied pairs.

Concordance

```
library(InformationValue)

Concordance(train$Bonus, predict(ames_logit, type = "response"))
```

```
## $Concordance
## [1] 0.862933
##
## $Discordance
## [1] 0.137067
##
## $Tied
## [1] -5.551115e-17
##
## $Pairs
## [1] 1017240
```

Our mode correctly ranks bonus eligible homes ahead of non-bonus eligible homes 86.3% of the time!

OR

Our model correctly assigns higher probability to bonus eligible homes 86.3% of the time.

Concordance

```
library(InformationValue)

Concordance(train$Bonus, predict(ames_logit, type = "response"))
```

```
## $Concordance
## [1] 0.862933
##
## $Discordance
## [1] 0.137067
##
## $Tied
## [1] -5.551115e-17
##
## $Pairs
## [1] 1017240
```

Our mode correctly ranks bonus eligible homes ahead of non-bonus eligible homes 86.3% of the time!

NOT

Our model is accurate 86.3% of the time.

Introduction to Logistic Regression

VARIABLE SELECTION AND REGULARIZED REGRESSION

Variable Selection

- All of the same approaches to variable selection available in linear regression are available in logistic regression.
- Forward, backward, stepwise, LASSO, etc.

Forward and Backward Selection

```
train_sel_log <- train %>%
                   dplyr::select(Bonus, Lot_Area, Street, Bldg_Type, House_Style,
                                 Overall_Qual, Roof_Style, Central_Air,
                                 First_Flr_SF, Second_Flr_SF, Full_Bath, Half_Bath,
                                 Fireplaces, Garage_Area, Gr_Liv_Area,
                                 TotRms_AbvGrd) %>%
                   replace(is.na(.), 0)
full.model <- glm(Bonus ~ . , data = train_sel_log)
empty.model <- glm(Bonus ~ 1, data = train_sel_log)
```

Forward and Backward Selection

```r
for.model <- step(empty.model,
                  scope = list(lower = formula(empty.model),
                               upper = formula(full.model)),
                  direction = "forward",
                  k = log(dim(train_sel_log)[1]))
back.model <- step(full.model,
                   scope = list(lower = formula(empty.model),
                                upper = formula(full.model)),
                   direction = "backward",
                   k = log(dim(train_sel_log)[1]))
```

Results

```
## Step:  AIC=947.31
## Bonus ~ Lot_Area + Bldg_Type + Overall_Qual + First_Flr_SF +
##     Full_Bath + Half_Bath + Fireplaces + Garage_Area
```

```
## $Concordance
## [1] 0.9612668
##
## $Discordance
## [1] 0.03873324
##
## $Tied
## [1] -2.081668e-17
```

Regularized Regression

- Although not shown here, regularized regression can use the same link function to obtain logistic regression
- Ridge, LASSO, Elastic Net

?