Markov Chain Monte Carlo Diagnostics

For ignoring the MCMC diagnostics, he was sentenced to the Markov chain gang.

Bayesian Structural Time Series

(BSTS)

Outline

BSTS

Uses the Bayesian setting (distributions on parameters) and state space (think Exponential Smoothing models) together.

Was developed by Google around 2013

Easy to implement and run in R

R documentation: https://cran.r-project.org/web/packages/bsts/bsts.pdf

State Space models: Recall Holt-Winters Additive ESM

$$\hat{Y}_{t+h} = L_t + hT_t + S_{t-p+h}$$

$$L_t = \theta\left(Y_t - S_{t-p}\right) + (1-\theta)(L_{t-1} + T_{t-1})$$
$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1}$$
$$S_t = \delta(Y_t - L_{t-1} - T_{t-1}) + (1-\delta)S_{t-p}$$

Recall Holt-Winters Additive ESM

$$\hat{Y}_{t+h} = L_t + hT_t + S_{t-p+h}$$

$$L_t = \theta\left(Y_t - S_{t-p}\right) + (1-\theta)(L_{t-1} + T_{t-1})$$
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$$S_t = \delta(Y_t - L_{t-1} - T_{t-1}) + (1-\delta)S_{t-p}$$

Each of these components evolve over time….

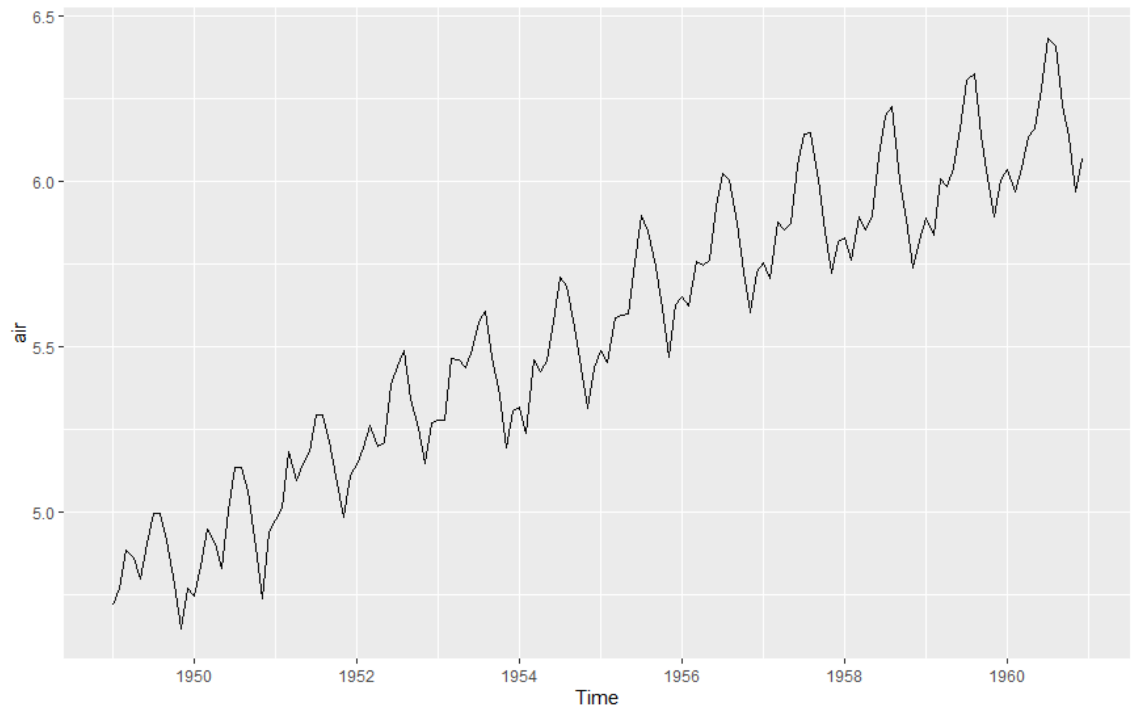
Simple “level” model

$$y_t = \underbrace{\mu_t}_{\text{Level}} + \epsilon_t$$

$\mu_{t+1} = \mu_t + u_t$ Level (u_t is error….assumes Normal distribution)

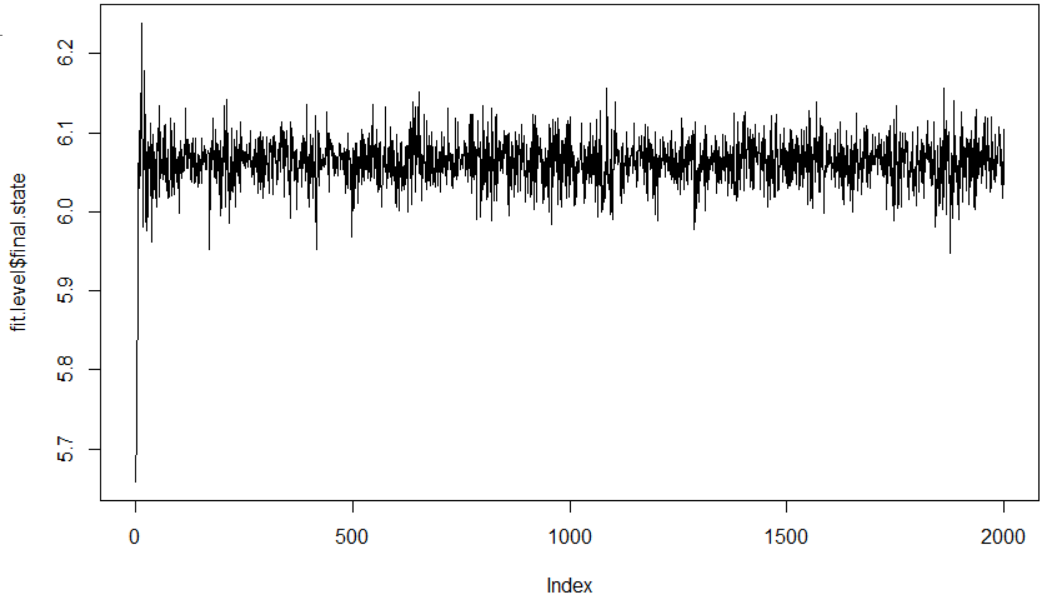
Example (airline passengers…older data)

Use the older airline passenger data (we will use the Log(passenger) as our time series)



Fitting a level BSTS

```
library(bsts)
library(tidyverse)
air.bsts=airline$LogPsngr
model_components=list()
model_components <- AddLocalLevel(model_components,
y = air.bsts)
fit.level=bsts(air.bsts, model_components, niter = 2000)
plot(fit.level$final.state,type='l')
```

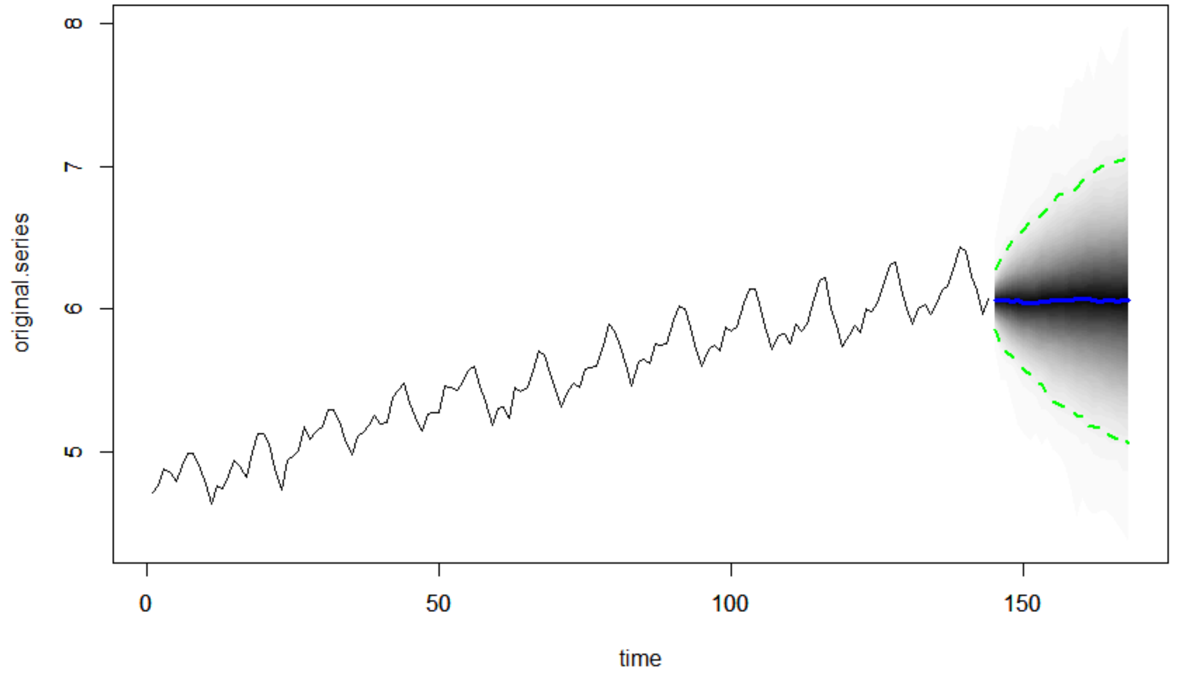


Making forecasts

```
pred.level<-predict(fit.level,burn = 200,horizon = 24)
plot(pred.level)
```

```
> pred.level$mean
 6.060904 6.058905 6.059396 6.057654
6.057066 6.052285 6.053708 6.053631
6.057384 6.059317 6.059745 6.061799
6.063273 6.064737 6.064800 6.067155
6.070143 6.066728 6.066637 6.062760
6.060922 6.056811 6.056758 6.058400
```

Can also ask for median and interval



Trend BSTS

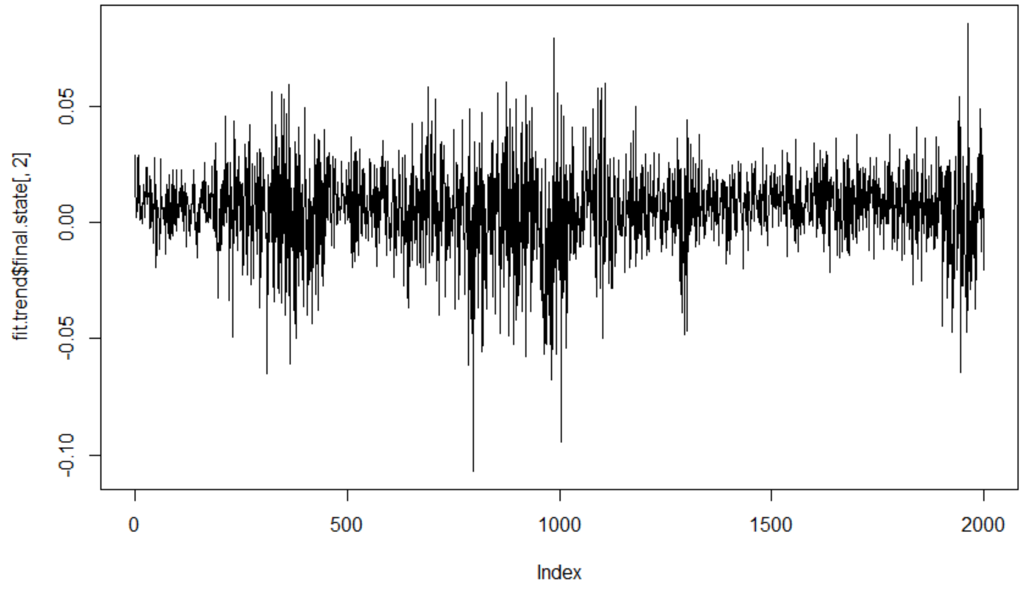
$$y_t = \underbrace{\mu_t}_{\text{Level/Trend}} + \epsilon_t$$

$$\left.\begin{aligned} \mu_{t+1} &= \mu_t + \delta_t + u_t \\ \delta_{t+1} &= \delta_t + v_t \end{aligned}\right\} \text{Level and Trend (} u_t \text{ and } v_t \text{ are error)}$$

Fit a trend BSTS

```
model_components=list()
model_components=AddLocalLinearTrend(model_components,
                                     y = air.bsts)
fit.trend=bsts(air.bsts, model_components, niter = 2000)
plot(fit.trend$final.state[,2],type='l')
```

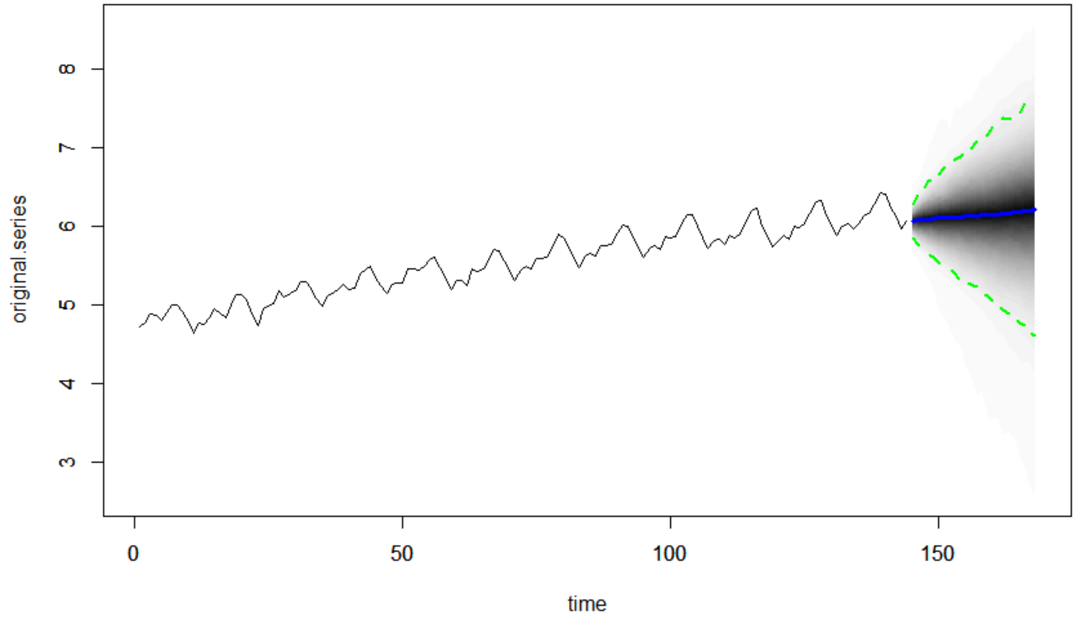
```
> head(fit.trend$final.state)
         [,1]        [,2]
[1,] 6.270559 0.028903721
[2,] 6.188468 0.015266010
[3,] 6.176237 0.006683758
[4,] 6.144252 0.002293991
[5,] 6.260150 0.005248778
[6,] 6.229388 0.005921211
```



Plot forecasts

```
pred.trend<-predict(fit.trend,burn = 500,horizon = 24)
plot(pred.trend)
```

```
> pred.trend$median
 6.069794 6.080965 6.084830 6.088169 6.094706
6.107259 6.112713 6.111001 6.114586 6.120687
6.129326 6.123488 6.131942 6.145842 6.141558
6.147235 6.145968 6.165901 6.164742 6.176605
6.187887 6.196770 6.202536 6.215285
```



Seasonal BSTS

$$y_t = \underbrace{\mu_t}_{\text{Level}} + \underbrace{\tau_t}_{\text{Season}} + \epsilon_t$$

$$\left.\begin{aligned} \mu_{t+1} &= \mu_t + \delta_t + u_t \\ \delta_{t+1} &= \delta_t + v_t \end{aligned}\right\}$$ Level and Trend (u_t and v_t are error)

$$\tau_{t+1} = -\sum \tau_t + w_t$$ Seasonality (w_t is the error term)

How to model Seasonality

Dummy variables

Trigonometric Functions

Dummy variables

Basically, you are doing a linear regression!!! Let's say we have monthly data (i.e. we need 11 dummy variables)…

$$x_1 = \begin{cases} 1 & if\ January \\ 0 & otherwise \end{cases}$$

$$x_2 = \begin{cases} 1 & if\ February \\ 0 & otherwise \end{cases}$$

$$\vdots$$

$$x_{11} = \begin{cases} 1 & if\ November \\ 0 & otherwise \end{cases}$$

Dummy variables

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$$x_2 = \begin{cases} 1 & if\ February \\ 0 & otherwise \end{cases}$$

•

•

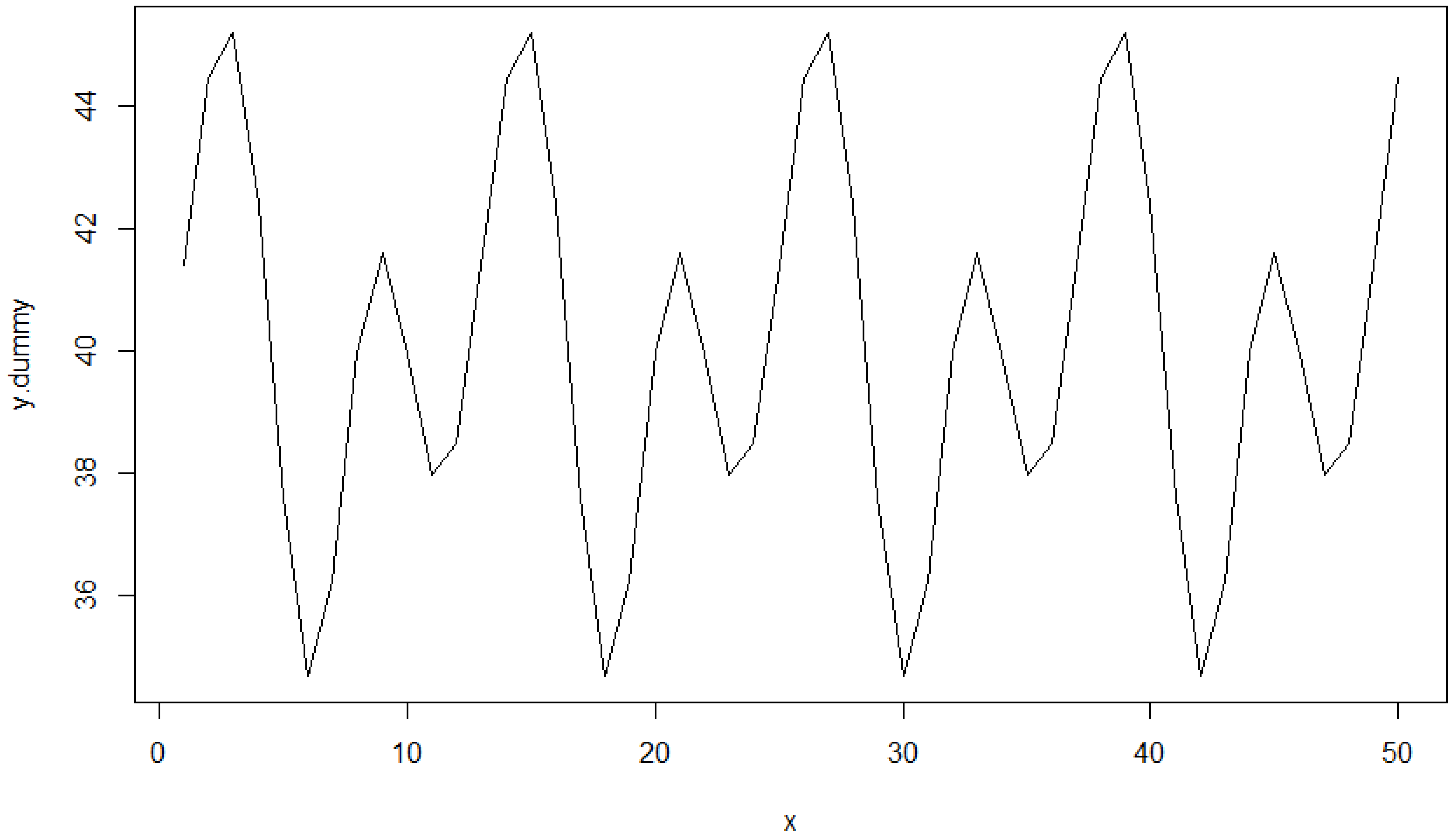
•

$$x_{11} = \begin{cases} 1 & if\ November \\ 0 & otherwise \end{cases}$$

$$Y_t = 38.5 + 2.9X_1 + 5.95X_2 + 6.7X_3 + 3.9124356X_4 - 0.91X_5 - 3.8X_6 - 2.27X$$
$$+1.49X_8 + 3.1X_9 + 1.4X_{10} - 0.53X_{11}$$

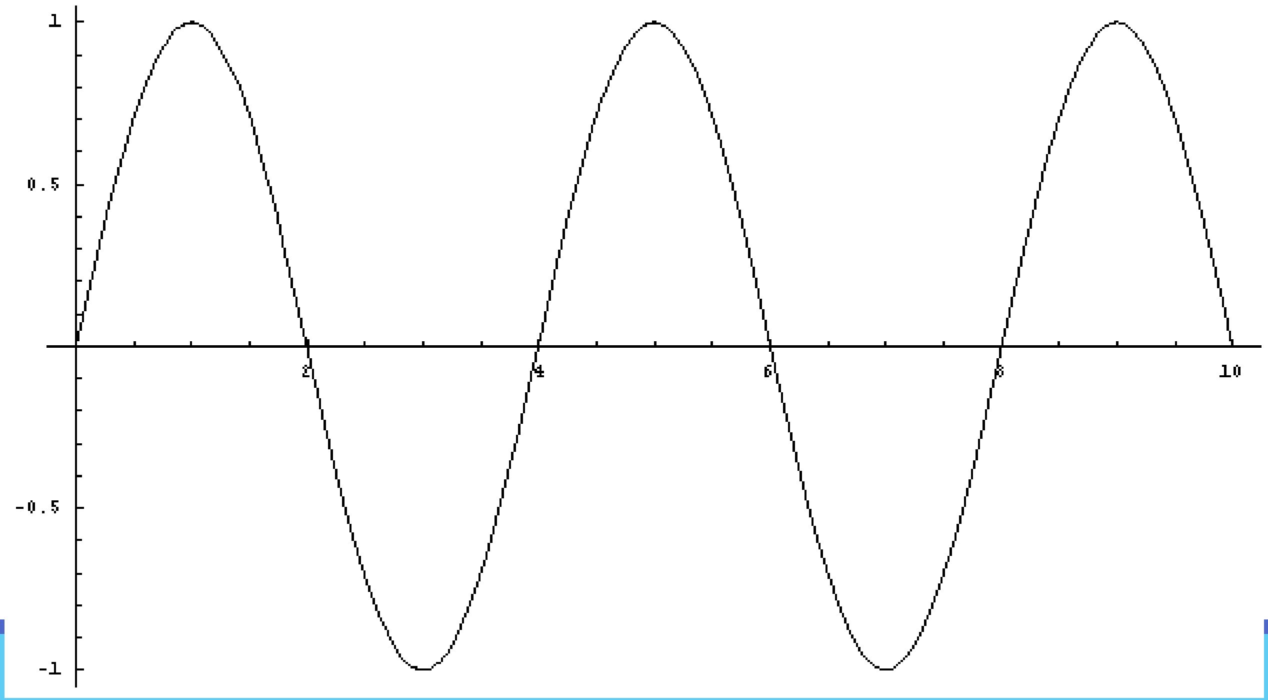
```
x.dummy[1:3,]
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
[1,]    1    0    0    0    0    0    0    0    0     0     0
[2,]    0    1    0    0    0    0    0    0    0     0     0
[3,]    0    0    1    0    0    0    0    0    0     0     0
```

```
y.dummy=38.5+x.dummy%*%c(2.9052559,5.9516660,6.7000000,
3.9124356,-0.9052559,-3.8000000,-
2.2660254,1.4875644,3.1000000 ,1.4483340,-0.5339746)
```


y.dummy
36
38
40
42
44
0
10
20
30
40
50
x

Trigonometric Functions

- Trigonometric functions in mathematics have a cyclical pattern.
- Use trigonometric functions, such as sine and cosine, to model seasonality.



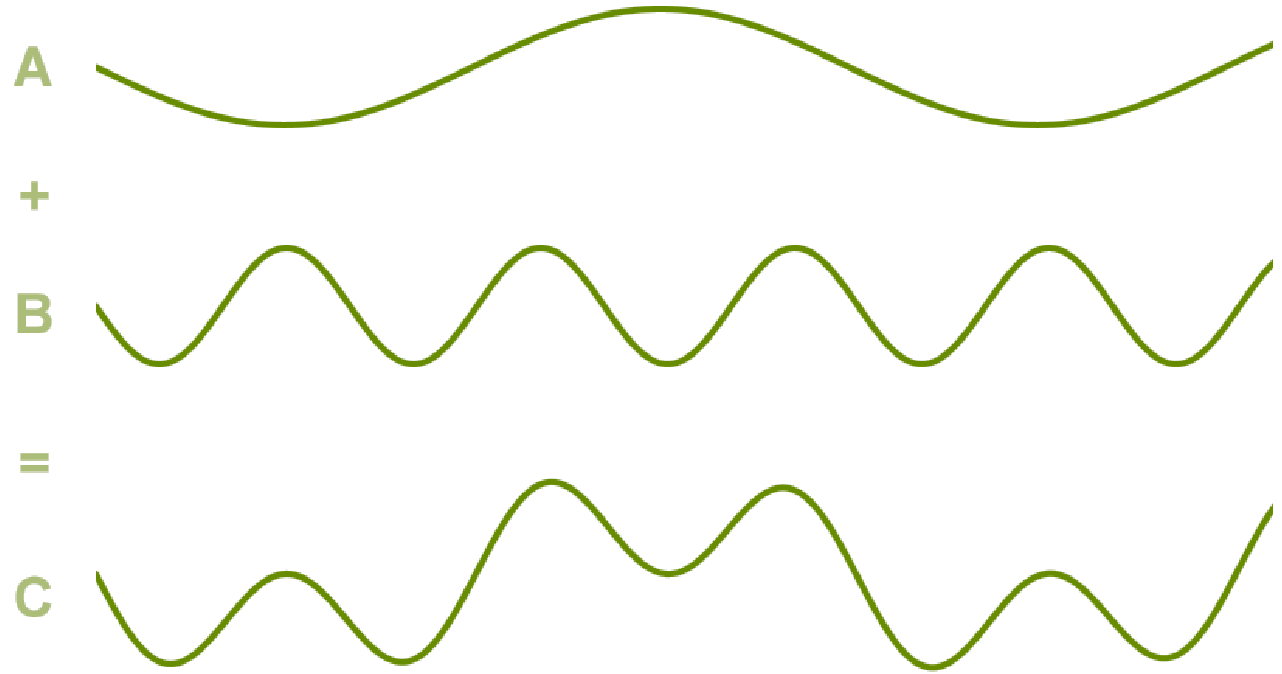
Trigonometric Regression

$$X_t = \sin\left(\frac{2\pi t}{S}\right)$$

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t$$

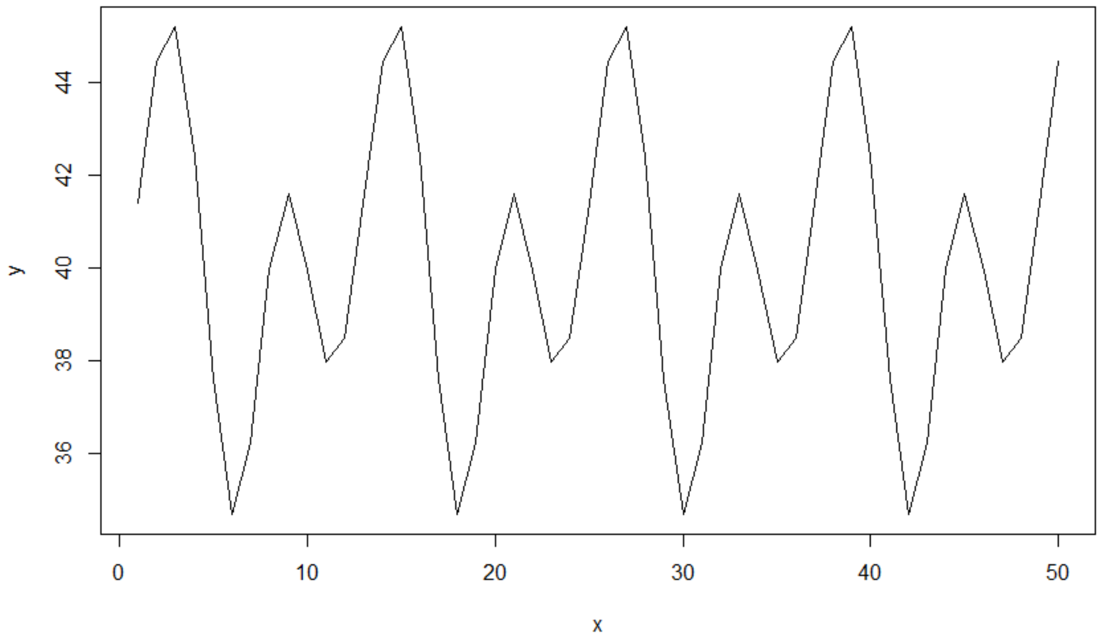
Trigonometric Regression

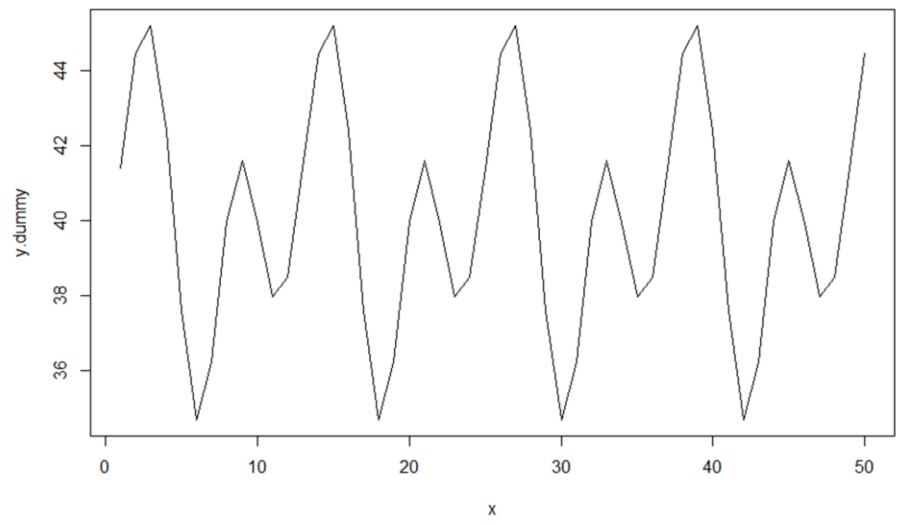
- You don’t have to limit yourselves to only one sine or cosine variable.
- Mixing sine and cosine functions might better fit your data (Fourier analysis).

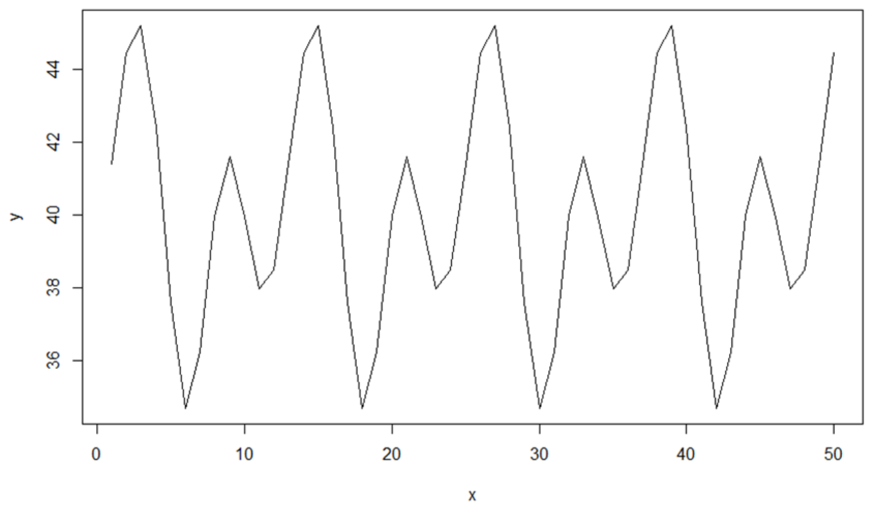


For example:

```
> x=seq(1,50,by=1)
> sin1=sin(1*2*pi*x/12)
> cos1=cos(1*2*pi*x/12)
> sin2=sin(2*2*pi*x/12)
> cos2=cos(2*2*pi*x/12)
> sin3=sin(3*2*pi*x/12)
> cos3=cos(3*2*pi*x/12)
> y=40+2*sin1+1.6*cos1+0.6*sin2-3.4*cos2+0.2*sin3+0.3*cos3
> plot(x,y,type='l')
```

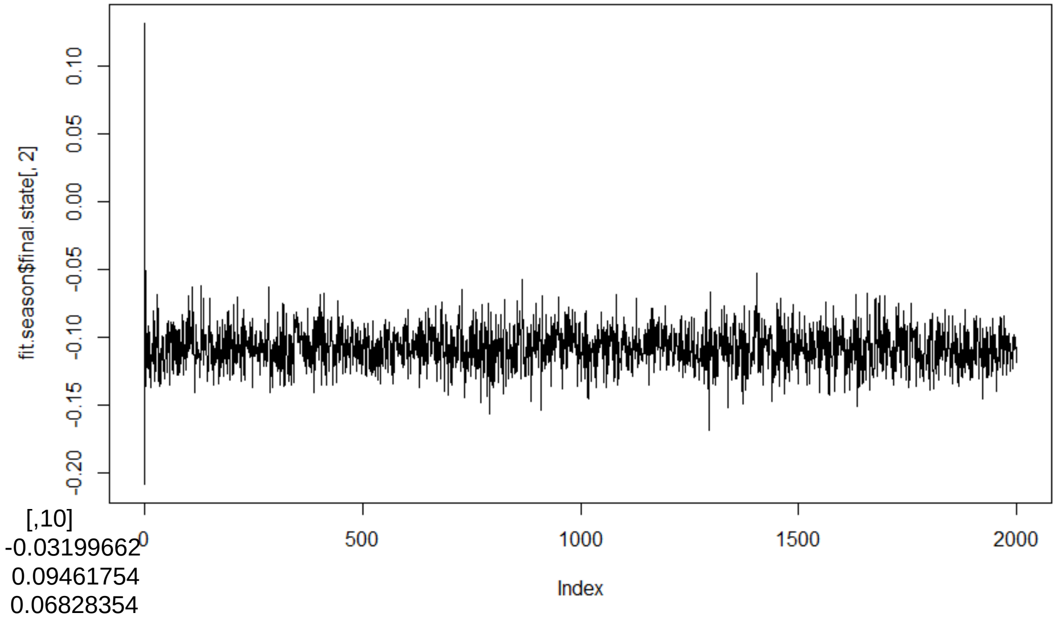



y.dummy
44
42
40
38
36
0
10
20
30
40
50
x


y
44
42
40
38
36
0
10
20
30
40
50
x

Seasonal (with Dummy and Level)

```
model_components=list()
model_components = AddLocalLevel(model_components,
                                 y = air.bsts)
model_components=AddSeasonal(model_components, y
= air.bsts, nseasons   = 12)

fit.season=bsts(air.bsts, model_components, niter = 2000)
plot(fit.season$final.state[,2],type='l')
```

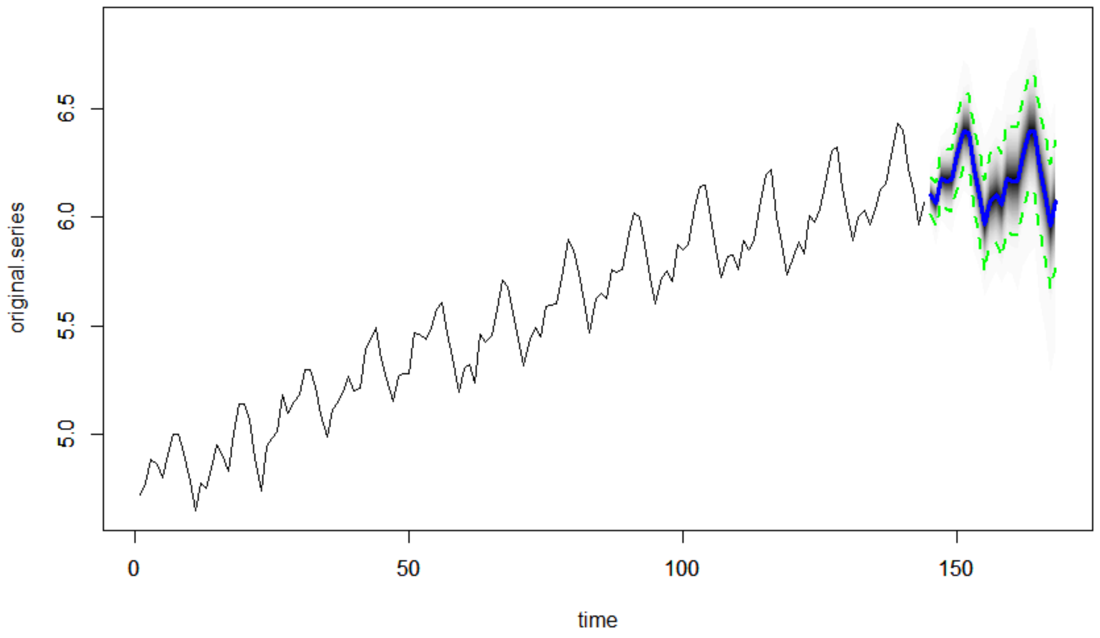


```
> head(fit.season$final.state)
         [,1]        [,2]       [,3]        [,4]         [,5]       [,6]       [,7]       [,8]         [,9]       [,10]
[1,] 5.611838  0.13122061 -0.2187225 -0.11985584 -0.009119432 0.18958485 0.11092930 0.20312464 -0.073750644 -0.03199662
[2,] 5.775567 -0.20784084 -0.2606871 -0.01023421  0.039516655 0.08946901 0.28148379 0.21886119 -0.072926289  0.09461754
[3,] 5.839541 -0.05099446 -0.1414026 -0.19565411  0.081198015 0.29500666 0.04275373 0.08099343  0.034689610  0.06828354
[4,] 5.847991 -0.08382377 -0.2623133 -0.20962298  0.230671767 0.20365406 0.18841063 0.12364207 -0.073111699 -0.02250975
[5,] 6.029962 -0.13583900 -0.1897727 -0.04038770  0.146914790 0.20151903 0.35220462 0.01275095  0.001114029 -0.03964137
[6,] 6.095697 -0.10922017 -0.2577005 -0.06908177  0.077977186 0.18769086 0.26883741 0.14330611 -0.023051989  0.04296503
             [,11]       [,12]
[1,] -0.1377430382 -0.20602724
[2,]  0.0093251931 -0.05982110
[3,]  0.0063418976 -0.05214867
[4,]  0.0804886616 -0.15250747
[5,] -0.0253000053 -0.05157723
[6,] -0.0004721743 -0.15021407
```

1 Level + 11 Dummy variables

Forecast(with dummy)

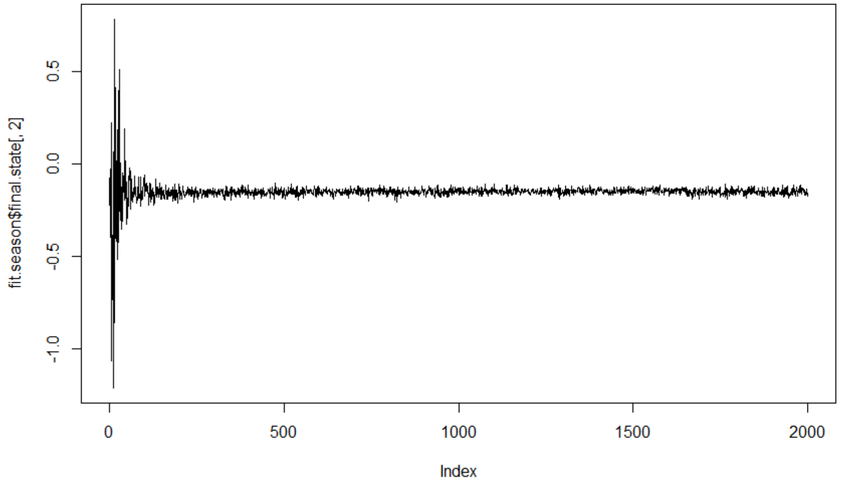
```
pred.season<-predict(fit.season,burn = 500,horizon = 24)
plot(pred.season)
```



```
> pred.season$mean
 [1] 6.104505 6.061527 6.182944 6.172317 6.173189 6.286979 6.401651 6.394572
6.226572 6.108766 5.964014 6.070397 6.105952  6.060050 6.179451 6.169520 6.167948
6.283719 6.400189 6.392691 6.224080 6.106716 5.961663 6.069726
```

Seasonal (Trig with level)

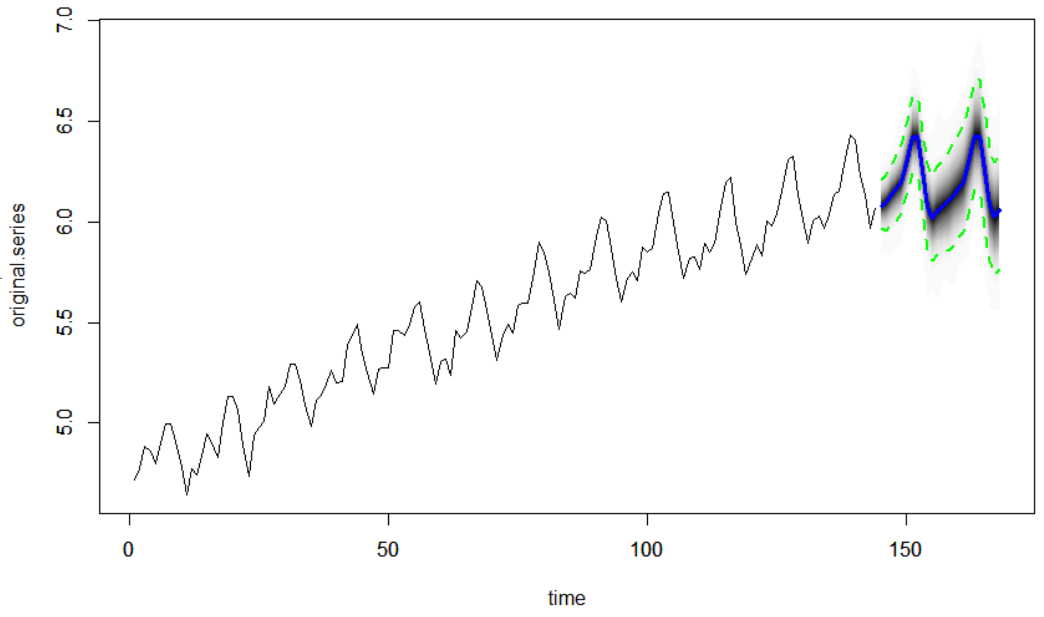
```
model_components=list()
model_components = AddLocalLevel(model_components,
                                  y = air.bsts)
model_components=AddTrig(model_components, y =
air.bsts,
                          period  = 12,frequencies = 1:3)

fit.season=bsts(air.bsts, model_components, niter = 2000)
plot(fit.season$final.state[,2],type='l')
```



```
> head(fit.season$final.state)
         [,1]        [,2]       [,3]        [,4]        [,5]      [,6]       [,7]
[1,] 5.579081 -0.14760642 -0.5717763  0.01409946 -0.82855883 0.3028314  0.1562200
[2,] 5.601328 -0.07600871 -0.4598823  0.14894884  0.20325190 1.0879606  0.1213216
[3,] 5.574211 -0.37114738 -0.2070764 -0.35487942 -1.35175170 0.5059916 -0.2237004
[4,] 5.614885 -0.39524983 -1.1895666  0.04371342  0.09413661 0.5763212  0.3782019
```

Forecast (Trig)

```
pred.season<-predict(fit.season,burn = 500,horizon = 24)
plot(pred.season)
```



```
> pred.season$interval
          [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]     [,8]     [,9]    [,10]    [,11]    [,12]
2.5%  5.965759 5.958060 5.984099 6.002435 6.041751 6.126399 6.241519 6.235748 6.056018 5.864842 5.808555 5.838206
97.5% 6.208353 6.235114 6.278911 6.324366 6.380711 6.484603 6.621303 6.623532 6.464349 6.287322 6.231317 6.276204
         [,13]    [,14]    [,15]    [,16]    [,17]    [,18]    [,19]    [,20]    [,21]    [,22]    [,23]    [,24]
2.5%  5.854577 5.859481 5.883375 5.926380 5.952490 6.034250 6.161035 6.149160 5.988632 5.806161 5.740073 5.762272
97.5% 6.306913 6.345058 6.393916 6.430722 6.483931 6.569421 6.701481 6.709872 6.553636 6.353762 6.293718 6.340721
```

Can also fit other X variables…

$$y_t = \underbrace{\mu_t}_{\text{Level}} + \underbrace{\tau_t}_{\text{Season}} + \underbrace{\beta^T x_t}_{\text{“X” Variables}} + \epsilon_t$$

$$\left.\begin{aligned} \mu_{t+1} &= \mu_t + \delta_t + u_t \\ \delta_{t+1} &= \delta_t + v_t \end{aligned}\right\}$$

Level and Trend (u_t and v_t are error)

$$\tau_{t+1} = -\sum \tau_t + w_t$$

Seasonality (w_t is the error term)

Questions?