

$tnvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T, F$	
$index, i$	indices

$relflag, \rho$	$::=$ $ $ $+$ $ $ $-$ $ $ app_rho ν S $ $ (ρ) S	relevance flag
$appflag, \nu$	$::=$ $ $ R $ $ ρ	applicative flag
$role, R$	$::=$ $ $ Nom $ $ Rep $ $ $R_1 \cap R_2$ S $ $ param $R_1 R_2$ S $ $ app_role ν S $ $ (R) S	Role
$constraint, \phi$	$::=$ $ $ $a \sim_{A/R} b$ $ $ (ϕ) S $ $ $\phi\{b/x\}$ S $ $ $ \phi $ S $ $ $a \sim_R b$ S	props
$tm, a, b, p, v, w, A, B, C$	$::=$ $ $ \star $ $ x $ $ $\lambda^\rho x:A.b$ bind x in b $ $ $\lambda^\rho x.b$ bind x in b $ $ $a \ b^\nu$ $ $ $\Pi^\rho x:A \rightarrow B$ bind x in B $ $ $\Lambda c:\phi.b$ bind c in b $ $ $\Lambda c.b$ bind c in b $ $ $a[\gamma]$ $ $ $\forall c:\phi.B$ bind c in B $ $ $a \triangleright_R \gamma$ $ $ F $ $ \square $ $ case _{R} a of $F \rightarrow b_1 \parallel - \rightarrow b_2$ $ $ K $ $ match a with brs $ $ sub $R a$ $ $ $a\{b/x\}$ S $ $ $a\{\gamma/c\}$ S $ $ $a\{b/x\}$ S $ $ $a\{\gamma/c\}$ S	types and kinds

		a	S	
		a	S	
		(a)	S	
		a	S	parsing precedence is hard
		$ a _R$	S	
		Int	S	
		Bool	S	
		Nat	S	
		Vec	S	
		0	S	
		S	S	
		True	S	
		Fix	S	
		Age	S	
		$a \rightarrow b$	S	
		$\phi \Rightarrow A$	S	
		$a \ b$	S	
		$\lambda x. a$	S	
		$\lambda x : A. a$	S	
		$\forall x : A \rightarrow B$	S	
		if ϕ then a else b	S	
brs	$::=$			case branches
		none		
		$K \Rightarrow a; brs$		
		$brs\{a/x\}$	S	
		$brs\{\gamma/c\}$	S	
		(brs)	S	
co, γ	$::=$			explicit coercions
		•		
		c		
		red $a \ b$		
		refl a		
		$(a \models_\gamma b)$		
		sym γ		
		$\gamma_1; \gamma_2$		
		sub γ		
		$\Pi^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\lambda^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\gamma_1 \ \gamma_2^{R,\rho}$		
		piFst γ		
		cpiFst γ		
		isoSnd γ		
		$\gamma_1 @ \gamma_2$		
		$\forall c : \gamma_1. \gamma_3$	bind c in γ_3	

			\emptyset	
			$\Sigma \cup \{F : sig_sort\}$	
			Σ_0	M
			Σ_1	M
			$ \Sigma $	M
$available_props, \Delta$	$::=$		\emptyset	
			Δ, x	
			Δ, c	
			$fv a$	M
			Δ, Δ'	M
			$\tilde{\Gamma}$	M
			$\tilde{\Omega}$	M
			(Δ)	M
Nat, \mathbb{N}	$::=$		$ a $	S
$terminals$	$::=$		\leftrightarrow	
			\Leftrightarrow	
			\longrightarrow	
			min	
			\equiv	
			\forall	
			\in	
			\notin	
			\Leftarrow	
			\Rightarrow	
			\Rightarrow^*	
			\rightarrow	
			Λ	
			\square	
			\vdash	
			\dashv	
			\models	
			\vDash	
			\neq	
			\triangleright	
			ok	
			$-$	
			\rightsquigarrow	
			\rightsquigarrow^*	
			\rightsquigarrow	
			\emptyset	

	\circ fv dom \sim \succ $ $ \bullet fst snd as $ \Rightarrow $ $\vdash =$ refl_2 $++$ $\{$ $\}$
<i>formula, ψ</i>	$::=$ <i>judgement</i> $x : A \in \Gamma$ $x : R \in \Omega$ $c : \phi \in \Gamma$ $F : \text{sig_sort} \in \Sigma$ $x \in \Delta$ $c \in \Delta$ $c \text{ not relevant} \in \gamma$ $x \notin \Delta$ $\text{uniq } \Gamma$ $\text{uniq}(\Omega)$ $c \notin \Delta$ $T \notin \text{dom } \Sigma$ $F \notin \text{dom } \Sigma$ $\mathbb{N}_1 < \mathbb{N}_2$ $\mathbb{N}_1 \leq \mathbb{N}_2$ $R_1 = R_2$ $a = b$ $\phi_1 = \phi_2$ $\Gamma_1 = \Gamma_2$ $\gamma_1 = \gamma_2$ $\neg \psi$ $\psi_1 \wedge \psi_2$ $\psi_1 \vee \psi_2$ $\psi_1 \Rightarrow \psi_2$ (ψ) ψ $c : (a : A \sim b : B) \in \Gamma$

		suppress lc hypothesis gen
$JSubRole$	$::=$ $R_1 \leq R_2$	Subroling judgement
$JPath$	$::=$ $Path\ a = F@Rs$	Type headed by constant
$JPatCtx$	$::=$ $\Omega; \Gamma \vdash p :_F B \Rightarrow A$	Contexts generated by a p
$JRename$	$::=$ $rename\ p \rightarrow a\ to\ p' \rightarrow a' \text{ excluding } \Delta \text{ and } \Delta'$	rename with fresh variable
$JMatchSubst$	$::=$ $match\ a_1 \text{ with } p \rightarrow b_1 = b_2$	match and substitute
$JTmPatternAgree$	$::=$ $a \leftrightarrow p$	term and pattern agree
$JTmSubPatternAgree$	$::=$ $a^+ = p$	sub-pattern agrees with te
$JSubTmPatternAgree$	$::=$ $a = p^+$	sub-term agrees with patt
$JValuePath$	$::=$ $ValuePath\ a = F$	Type headed by constant
$JCasePath$	$::=$ $CasePath_R\ a = F$	Type headed by constant
$JApplyArgs$	$::=$ $apply\ args\ a\ to\ b \mapsto b'$	apply arguments of a (hea
$JValue$	$::=$ $Value_R\ A$	values
$JValueType$	$::=$ $ValueType_R\ A$	Types with head forms (er
$Jconsistent$	$::=$ $consistent_R\ a\ b$	(erased) types do not diffe
$Jroleing$	$::=$ $\Omega \vdash a : R$	Roleing judgment

$JChk$	$::=$ $ \quad (\rho = +) \vee (x \notin \text{fv } A)$	irrelevant argument check
$Jpar$	$::=$ $ \quad \Omega \models a \Rightarrow_R b$ $ \quad \Omega \models a \Rightarrow_R^* b$ $ \quad \Omega \models a \Leftrightarrow_R b$	parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term
$Jbeta$	$::=$ $ \quad \models a > b/R$ $ \quad \models a \rightsquigarrow b/R$ $ \quad \models a \rightsquigarrow^* b/R$	primitive reductions on erased terms single-step head reduction for implicit language multistep reduction
$JBranchTyping$	$::=$ $ \quad \Gamma \models \text{case}_R a : A \text{ of } b : B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
$Jett$	$::=$ $ \quad \Gamma \models \phi \text{ ok}$ $ \quad \Gamma \models a : A$ $ \quad \Gamma; \Delta \models \phi_1 \equiv \phi_2$ $ \quad \Gamma; \Delta \models a \equiv b : A/R$ $ \quad \models \Gamma$	Prop wellformedness typing prop equality definitional equality context wellformedness
$Jsig$	$::=$ $ \quad \models \Sigma$	signature wellformedness
$Jann$	$::=$ $ \quad \Gamma \vdash \phi \text{ ok}$ $ \quad \Gamma \vdash a : A/R$ $ \quad \Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$ $ \quad \Gamma; \Delta \vdash \gamma : A \sim_R B$ $ \quad \vdash \Gamma$	prop wellformedness typing coercion between props coercion between types context wellformedness
$Jred$	$::=$ $ \quad \Gamma \vdash a \rightsquigarrow b/R$	single-step, weak head reduction to values for
$judgement$	$::=$ $ \quad JSubRole$ $ \quad JPath$ $ \quad JPatCtx$ $ \quad JRename$ $ \quad JMatchSubst$ $ \quad JTmPatternAgree$ $ \quad JTmSubPatternAgree$ $ \quad JSubTmPatternAgree$ $ \quad JValuePath$ $ \quad JCasePath$	

		<i>JApplyArgs</i>
		<i>JValue</i>
		<i>JValueType</i>
		<i>Jconsistent</i>
		<i>Jroleing</i>
		<i>Jchk</i>
		<i>Jpar</i>
		<i>Jbeta</i>
		<i>JBranchTyping</i>
		<i>Jett</i>
		<i>Jsig</i>
		<i>Jann</i>
		<i>Jred</i>
<i>user_syntax</i>	::=	
		<i>tmvar</i>
		<i>covar</i>
		<i>datacon</i>
		<i>const</i>
		<i>index</i>
		<i>relflag</i>
		<i>appflag</i>
		<i>role</i>
		<i>constraint</i>
		<i>tm</i>
		<i>brs</i>
		<i>co</i>
		<i>role_context</i>
		<i>roles</i>
		<i>sig_sort</i>
		<i>sort</i>
		<i>context</i>
		<i>sig</i>
		<i>available_props</i>
		<i>Nat</i>
		<i>terminals</i>
		<i>formula</i>

$\boxed{R_1 \leq R_2}$ Subroling judgement

$\overline{\mathbf{Nom} \leq R}$	NOMBOT
$\overline{R \leq \mathbf{Rep}}$	REPTOP
$\overline{R \leq R}$	REFL
$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3}$	TRANS

$\boxed{\text{Path } a = F@Rs}$ Type headed by constant (partial function)

$$\begin{array}{c}
\frac{F : A@Rs \in \Sigma_0}{\text{Path } F = F@Rs} \quad \text{PATH_ABSCONST} \\
\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0}{\text{Path } F = F@Rs} \quad \text{PATH_CONST} \\
\frac{\text{Path } a = F@R_1, Rs}{\text{Path } (a \ b'^{R_1}) = F@Rs} \quad \text{PATH_APP} \\
\frac{\text{Path } a = F@Rs}{\text{Path } (a \ b^-) = F@Rs} \quad \text{PATH_IAPP} \\
\frac{\text{Path } a = F@Rs}{\text{Path } (a[\bullet]) = F@Rs} \quad \text{PATH_CAPP}
\end{array}$$

$\boxed{\Omega; \Gamma \models p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\begin{array}{c}
\frac{}{\emptyset; \emptyset \models F :_F A \Rightarrow A} \quad \text{PATCTX_CONST} \\
\frac{\Omega; \Gamma \models p :_F \Pi^+ x : A' \rightarrow A \Rightarrow B}{\Omega, x : R; \Gamma, x : A' \models p \ x^R :_F A \Rightarrow B} \quad \text{PATCTX_PIREL} \\
\frac{\Omega; \Gamma \models p :_F \Pi^- x : A' \rightarrow A \Rightarrow B}{\Omega; \Gamma, x : A' \models p \ \square^- :_F A \Rightarrow B} \quad \text{PATCTX_PIIRR} \\
\frac{\Omega; \Gamma \models p :_F \forall c : \phi. A \Rightarrow B}{\Omega; \Gamma, c : \phi \models p[\bullet] :_F A \Rightarrow B} \quad \text{PATCTX_CPI}
\end{array}$$

$\boxed{\text{rename } p \rightarrow a \text{ to } p' \rightarrow a' \text{ excluding } \Delta \text{ and } \Delta'}$ rename with fresh variables

$$\begin{array}{c}
\frac{}{\text{rename } F \rightarrow a \text{ to } F \rightarrow a \text{ excluding } \Delta \text{ and } \emptyset} \quad \text{RENAME_BASE} \\
\frac{\text{rename } p_1 \rightarrow a_1 \text{ to } p_2 \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta' \quad y \notin (\Delta, \Delta')}{\text{rename } (p_1 \ x^R) \rightarrow a_1 \text{ to } (p_2 \ y^R) \rightarrow (a_2\{y/x\}) \text{ excluding } \Delta \text{ and } (\Delta', y)} \quad \text{RENAME_APPREL} \\
\frac{\text{rename } p_1 \rightarrow a_1 \text{ to } p_2 \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta'}{\text{rename } (p_1 \ \square^-) \rightarrow a_1 \text{ to } (p_2 \ \square^-) \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta'} \quad \text{RENAME_APPIRR} \\
\frac{\text{rename } p_1 \rightarrow a_1 \text{ to } p_2 \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta'}{\text{rename } (p_1[\bullet]) \rightarrow a_1 \text{ to } (p_2[\bullet]) \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta'} \quad \text{RENAME_CAPP}
\end{array}$$

$\boxed{\text{match } a_1 \text{ with } p \rightarrow b_1 = b_2}$ match and substitute

$$\begin{array}{c}
\frac{}{\text{match } F \text{ with } F \rightarrow b = b} \quad \text{MATCHSUBST_CONST} \\
\frac{\text{match } a_1 \text{ with } p_1 \rightarrow b_1 = b_2}{\text{match } (a_1 \ a^R) \text{ with } (p_1 \ x^R) \rightarrow b_1 = (b_2\{a/x\})} \quad \text{MATCHSUBST_APPREL} \\
\frac{\text{match } a_1 \text{ with } p_1 \rightarrow b_1 = b_2}{\text{match } (a_1 \ a^-) \text{ with } (p_1 \ \square^-) \rightarrow b_1 = b_2} \quad \text{MATCHSUBST_APPIRR} \\
\frac{\text{match } a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match } (a_1[\bullet]) \text{ with } (a_2[\bullet]) \rightarrow b_1 = b_2} \quad \text{MATCHSUBST_CAPP}
\end{array}$$

$\boxed{a \leftrightarrow p}$ term and pattern agree

$$\begin{array}{c}
\frac{}{F \leftrightarrow F} \quad \text{TM_PATTERN_AGREE_CONST} \\
\\
\frac{a_1 \leftrightarrow p_1}{(a_1 \ a_2^R) \leftrightarrow (p_1 \ x^R)} \quad \text{TM_PATTERN_AGREE_APPREL R} \\
\\
\frac{a_1 \leftrightarrow p_1}{(a_1 \ a^-) \leftrightarrow (p_1 \ \Box^-)} \quad \text{TM_PATTERN_AGREE_APP IrREL} \\
\\
\frac{a_1 \leftrightarrow p_1}{(a_1[\bullet]) \leftrightarrow (p_1[\bullet])} \quad \text{TM_PATTERN_AGREE_CAPP}
\end{array}$$

$\boxed{a^+ = p}$ sub-pattern agrees with term

$$\begin{array}{c}
\frac{a \leftrightarrow p}{a^+ = p} \quad \text{TM_SUBPATTERN_AGREE_BASE} \\
\\
\frac{a^+ = p}{a^+ = (p \ x^R)} \quad \text{TM_SUBPATTERN_AGREE_APPREL R} \\
\\
\frac{a^+ = p}{a^+ = (p \ \Box^-)} \quad \text{TM_SUBPATTERN_AGREE_APP IrREL} \\
\\
\frac{a^+ = p}{a^+ = (p[\bullet])} \quad \text{TM_SUBPATTERN_AGREE_CAPP}
\end{array}$$

$\boxed{a = p^+}$ sub-term agrees with pattern

$$\begin{array}{c}
\frac{a \leftrightarrow p}{a = p^+} \quad \text{SUBTM_PATTERN_AGREE_BASE} \\
\\
\frac{a = p^+}{a \ a_2^\nu = p^+} \quad \text{SUBTM_PATTERN_AGREE_APP} \\
\\
\frac{a = p^+}{a[\bullet] = p^+} \quad \text{SUBTM_PATTERN_AGREE_CAPP}
\end{array}$$

$\boxed{\text{ValuePath } a = F}$ Type headed by constant (role-sensitive partial function used in value)

$$\begin{array}{c}
\frac{F : A @ Rs \in \Sigma_0}{\text{ValuePath } F = F} \quad \text{VALUEPATH_ABSCONST} \\
\\
\frac{F : p \sim a : A / R_1 @ Rs \in \Sigma_0}{\text{ValuePath } F = F} \quad \text{VALUEPATH_CONST} \\
\\
\frac{\text{ValuePath } a = F}{\text{ValuePath } (a \ b^\nu) = F} \quad \text{VALUEPATH_APP} \\
\\
\frac{\text{ValuePath } a = F}{\text{ValuePath } (a[\bullet]) = F} \quad \text{VALUEPATH_CAPP}
\end{array}$$

$\boxed{\text{CasePath}_R a = F}$ Type headed by constant (role-sensitive partial function used in case)

$$\frac{\text{ValuePath } a = F \quad F : A @ Rs \in \Sigma_0}{\text{CasePath}_R a = F} \quad \text{CASEPATH_ABSCONST}$$

$$\frac{\text{ValuePath } a = F \quad F : p \sim b : A/R_1 @ Rs \in \Sigma_0 \quad \neg(R_1 \leq R)}{\text{CasePath}_R a = F} \quad \text{CASEPATH_CONST}$$

$$\frac{\text{ValuePath } a = F \quad F : p \sim b : A/R_1 @ Rs \in \Sigma_0 \quad \neg(a = p^+)}{\text{CasePath}_R a = F} \quad \text{CASEPATH_UNMATCH}$$

$\boxed{\text{apply args } a \text{ to } b \mapsto b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\text{apply args } F \text{ to } b \mapsto b} \quad \text{APPLYARGS_CONST}$$

$$\frac{\text{apply args } a \text{ to } b \mapsto b'}{\text{apply args } a \ a''^\nu \text{ to } b \mapsto b' \ a''^\nu} \quad \text{APPLYARGS_APP}$$

$$\frac{\text{apply args } a \text{ to } b \mapsto b'}{\text{apply args } a[\bullet] \text{ to } b \mapsto b'[\bullet]} \quad \text{APPLYARGS_CAPP}$$

$\boxed{\text{Value}_R A}$ values

$$\frac{}{\text{Value}_R \star} \quad \text{VALUE_STAR}$$

$$\frac{}{\text{Value}_R \Pi^\rho x : A \rightarrow B} \quad \text{VALUE_PI}$$

$$\frac{}{\text{Value}_R \forall c : \phi. B} \quad \text{VALUE_CPI}$$

$$\frac{}{\text{Value}_R \lambda^+ x : A. a} \quad \text{VALUE_ABSREL}$$

$$\frac{}{\text{Value}_R \lambda^+ x. a} \quad \text{VALUE_UABSREL}$$

$$\frac{\text{Value}_R a}{\text{Value}_R \lambda^- x. a} \quad \text{VALUE_UABSIRREL}$$

$$\frac{}{\text{Value}_R \Lambda c : \phi. a} \quad \text{VALUE_CABS}$$

$$\frac{}{\text{Value}_R \Lambda c. a} \quad \text{VALUE_UCABS}$$

$$\frac{\text{CasePath}_R a = F}{\text{Value}_R a} \quad \text{VALUE_PATH}$$

$\boxed{\text{ValueType}_R A}$ Types with head forms (erased language)

$$\frac{}{\text{ValueType}_R \star} \quad \text{VALUE_TYPE_STAR}$$

$$\frac{}{\text{ValueType}_R \Pi^\rho x : A \rightarrow B} \quad \text{VALUE_TYPE_PI}$$

$$\frac{}{\text{ValueType}_R \forall c : \phi. B} \quad \text{VALUE_TYPE_CPI}$$

$$\frac{\text{CasePath}_R a = F}{\text{ValueType}_R a} \quad \text{VALUE_TYPE_VALUEPATH}$$

$\boxed{\text{consistent}_R a \ b}$ (erased) types do not differ in their heads

$$\begin{array}{c}
\frac{}{\text{consistent}_R \star \star} \text{CONSISTENT_A_STAR} \\
\\
\frac{}{\text{consistent}_{R'} (\Pi^\rho x_1 : A_1 \rightarrow B_1) (\Pi^\rho x_2 : A_2 \rightarrow B_2)} \text{CONSISTENT_A_PI} \\
\\
\frac{}{\text{consistent}_R (\forall c_1 : \phi_1. A_1) (\forall c_2 : \phi_2. A_2)} \text{CONSISTENT_A_CPI} \\
\\
\frac{\text{CasePath}_R a_1 = F \quad \text{CasePath}_R a_2 = F}{\text{consistent}_R a_1 a_2} \text{CONSISTENT_A_CASEPATH} \\
\\
\frac{\neg \text{ValueType}_R b}{\text{consistent}_R a b} \text{CONSISTENT_A_STEP_R} \\
\\
\frac{\neg \text{ValueType}_R a}{\text{consistent}_R a b} \text{CONSISTENT_A_STEP_L}
\end{array}$$

$\boxed{\Omega \models a : R}$ Roleing judgment

$$\begin{array}{c}
\frac{\text{uniq}(\Omega)}{\Omega \models \square : R} \text{ROLE_A_BULLET} \\
\\
\frac{\text{uniq}(\Omega)}{\Omega \models \star : R} \text{ROLE_A_STAR} \\
\\
\frac{\text{uniq}(\Omega) \quad x : R \in \Omega \quad R \leq R_1}{\Omega \models x : R_1} \text{ROLE_A_VAR} \\
\\
\frac{\Omega, x : \mathbf{Nom} \models a : R}{\Omega \models (\lambda^\rho x. a) : R} \text{ROLE_A_ABS} \\
\\
\frac{\Omega \models a : R \quad \Omega \models b : \mathbf{Nom}}{\Omega \models (a \ b^\rho) : R} \text{ROLE_A_APP} \\
\\
\frac{\Omega \models a : R \quad \text{Path } a = F @ R_1, Rs \quad \Omega \models b : R_1}{\Omega \models a \ b^{R_1} : R} \text{ROLE_A_TAPP} \\
\\
\frac{\Omega \models A : R \quad \Omega, x : \mathbf{Nom} \models B : R}{\Omega \models (\Pi^\rho x : A \rightarrow B) : R} \text{ROLE_A_PI} \\
\\
\frac{\Omega \models a : R_1 \quad \Omega \models b : R_1 \quad \Omega \models A : R_0 \quad \Omega \models B : R}{\Omega \models (\forall c : a \sim_{A/R_1} b. B) : R} \text{ROLE_A_CPI} \\
\\
\frac{\Omega \models b : R}{\Omega \models (\Lambda c. b) : R} \text{ROLE_A_CABS} \\
\\
\frac{\Omega \models a : R}{\Omega \models (a[\bullet]) : R} \text{ROLE_A_CAPP}
\end{array}$$

$$\begin{array}{c}
\frac{\text{uniq}(\Omega) \quad F : A @ R_s \in \Sigma_0}{\Omega \models F : R} \text{ROLE_A_CONST} \\
\\
\frac{\text{uniq}(\Omega) \quad F : p \sim a : A / R @ R_s \in \Sigma_0}{\Omega \models F : R_1} \text{ROLE_A_FAM} \\
\\
\frac{\Omega \models a : R \quad \Omega \models b_1 : R_1 \quad \Omega \models b_2 : R_1}{\Omega \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2 : R_1} \text{ROLE_A_PATTERN} \\
\\
\boxed{(\rho = +) \vee (x \notin \text{fv } A)} \quad \text{irrelevant argument check} \\
\\
\frac{}{(+ = +) \vee (x \notin \text{fv } A)} \text{RHO_REL} \\
\\
\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \text{RHO_IRRREL} \\
\\
\boxed{\Omega \models a \Rightarrow_R b} \quad \text{parallel reduction (implicit language)} \\
\\
\frac{\Omega \models a : R}{\Omega \models a \Rightarrow_R a} \text{PAR_REFL} \\
\\
\frac{\Omega \models a \Rightarrow_R (\lambda^\rho x. a') \quad \Omega \models b \Rightarrow_{\mathbf{Nom}} b'}{\Omega \models a \ b^\rho \Rightarrow_R a' \{b'/x\}} \text{PAR_BETA} \\
\\
\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b \Rightarrow_{(\mathbf{app.role} \nu)} b'}{\Omega \models a \ b^\nu \Rightarrow_R a' \ b'^\nu} \text{PAR_APP} \\
\\
\frac{\Omega \models a \Rightarrow_R (\Lambda c. a')}{\Omega \models a[\bullet] \Rightarrow_R a' \{\bullet/c\}} \text{PAR_CBETA} \\
\\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models a[\bullet] \Rightarrow_R a'[\bullet]} \text{PAR_CAPP} \\
\\
\frac{\Omega, x : \mathbf{Nom} \models a \Rightarrow_R a'}{\Omega \models \lambda^\rho x. a \Rightarrow_R \lambda^\rho x. a'} \text{PAR_ABS} \\
\\
\frac{\Omega \models A \Rightarrow_R A' \quad \Omega, x : \mathbf{Nom} \models B \Rightarrow_R B'}{\Omega \models \Pi^\rho x : A \rightarrow B \Rightarrow_R \Pi^\rho x : A' \rightarrow B'} \text{PAR_PI} \\
\\
\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models \Lambda c. a \Rightarrow_R \Lambda c. a'} \text{PAR_CABS} \\
\\
\frac{\Omega \models A \Rightarrow_{R_0} A' \quad \Omega \models a \Rightarrow_{R_1} a' \quad \Omega \models b \Rightarrow_{R_1} b' \quad \Omega \models B \Rightarrow_R B'}{\Omega \models \forall c : a \sim_{A/R_1} b. B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'. B'} \text{PAR_CPI}
\end{array}$$

$$\begin{array}{c}
\frac{
\begin{array}{l}
F : F \sim b : A/R_1 @ R_s \in \Sigma_0 \\
R_1 \leq R \\
\text{uniq}(\Omega)
\end{array}
}{\Omega \models F \Rightarrow_R b} \quad \text{PAR_AXIOMBASE}
\\[10pt]
\frac{
\begin{array}{l}
F : p \sim b : A/R_1 @ R_s \in \Sigma_0 \\
a^+ = p \wedge \neg(a \leftrightarrow p) \\
\Omega \models a \Rightarrow_R a' \\
\Omega \models a_1 \Rightarrow_{(\text{app.role } \nu)} a'_1 \\
\text{rename } p \rightarrow b \text{ to } p' \rightarrow b' \text{ excluding } (\tilde{\Omega}, \text{fv} p) \text{ and } \Delta' \\
\text{match } (a' a_1^{\nu}) \text{ with } p' \rightarrow b' = a_2 \\
R_1 \leq R
\end{array}
}{\Omega \models a a_1^{\nu} \Rightarrow_R a_2} \quad \text{PAR_AXIOMAPP}
\\[10pt]
\frac{
\begin{array}{l}
F : p \sim b : A/R_1 @ R_s \in \Sigma_0 \\
a^+ = p \wedge \neg(a \leftrightarrow p) \\
\Omega \models a \Rightarrow_R a' \\
\text{rename } p \rightarrow b \text{ to } p' \rightarrow b' \text{ excluding } (\tilde{\Omega}, \text{fv} p) \text{ and } \Delta' \\
\text{match } (a'[\bullet]) \text{ with } p' \rightarrow b' = a_2 \\
R_1 \leq R
\end{array}
}{\Omega \models a[\bullet] \Rightarrow_R a_2} \quad \text{PAR_AXIOMCAPP}
\\[10pt]
\frac{
\begin{array}{l}
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2
\end{array}
}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} (\text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel - \rightarrow b'_2)} \quad \text{PAR_PATTERN}
\\[10pt]
\frac{
\begin{array}{l}
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2 \\
\text{CasePath}_R a' = F \\
\text{apply args } a' \text{ to } b'_1 \mapsto b
\end{array}
}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} b[\bullet]} \quad \text{PAR_PATTERNTRUE}
\\[10pt]
\frac{
\begin{array}{l}
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2 \\
\text{Value}_R a' \\
\neg(\text{CasePath}_R a' = F)
\end{array}
}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} b'_2} \quad \text{PAR_PATTERNFALSE}
\\[10pt]
\boxed{\Omega \models a \Rightarrow_R^* b} \quad \text{multistep parallel reduction}
\\[10pt]
\frac{}{\Omega \models a \Rightarrow_R^* a} \quad \text{MP_REFL}
\\[10pt]
\frac{
\begin{array}{l}
\Omega \models a \Rightarrow_R b \\
\Omega \models b \Rightarrow_R^* a'
\end{array}
}{\Omega \models a \Rightarrow_R^* a'} \quad \text{MP_STEP}
\\[10pt]
\boxed{\Omega \models a \Leftrightarrow_R b} \quad \text{parallel reduction to a common term}
\\[10pt]
\frac{
\begin{array}{l}
\Omega \models a_1 \Rightarrow_R^* b \\
\Omega \models a_2 \Rightarrow_R^* b
\end{array}
}{\Omega \models a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}
\end{array}$$

$\boxed{\models a > b/R}$ primitive reductions on erased terms

$$\frac{\text{Value}_{R_1} (\lambda^\rho x.v)}{\models (\lambda^\rho x.v) \ b^\rho > v\{b/x\}/R_1} \quad \text{BETA_APPABS}$$

$$\frac{}{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{BETA_CAPPCABS}$$

$F : p \sim b : A/R_1 @ Rs \in \Sigma_0$

rename $p \rightarrow b$ to $p_1 \rightarrow b_1$ excluding (fva, fvp) and Δ'

match a with $p_1 \rightarrow b_1 = b'$

$R_1 \leq R$

$$\frac{}{\models a > b'/R} \quad \text{BETA_AXIOM}$$

$$\frac{\begin{array}{l} \text{CasePath}_R \ a = F \\ \text{apply args } a \text{ to } b_1 \mapsto b'_1 \end{array}}{\models \text{case}_R \ a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2 > b'_1[\bullet]/R_0} \quad \text{BETA_PATTERNTRUE}$$

$$\frac{\begin{array}{l} \text{Value}_R \ a \\ \neg(\text{CasePath}_R \ a = F) \end{array}}{\models \text{case}_R \ a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2 > b_2/R_0} \quad \text{BETA_PATTERNFALSE}$$

$\boxed{\models a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^- x.a \rightsquigarrow \lambda^- x.a'/R_1} \quad \text{E_ABSTERM}$$

$$\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^\nu \rightsquigarrow a' \ b^\nu/R_1} \quad \text{E_APPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E_CAPPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models \text{case}_R \ a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2 \rightsquigarrow \text{case}_R \ a' \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2/R_0} \quad \text{E_PATTERN}$$

$$\frac{\models a > b/R}{\models a \rightsquigarrow b/R} \quad \text{E_PRIM}$$

$\boxed{\models a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\models a \rightsquigarrow^* a/R} \quad \text{EQUAL}$$

$$\frac{\begin{array}{l} \models a \rightsquigarrow b/R \\ \models b \rightsquigarrow^* a'/R \end{array}}{\models a \rightsquigarrow^* a'/R} \quad \text{STEP}$$

$\boxed{\Gamma \models \text{case}_R \ a : A \text{ of } b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{l} \text{uniq } \Gamma \\ \text{lc_tm } C \end{array}}{\Gamma \models \text{case}_R \ a : A \text{ of } b : A \Rightarrow \forall c : (a \sim_{A/R} b).C \mid C} \quad \text{BRANCHTYPING_BASE}$$

$$\frac{\Gamma, x : A \models \text{case}_R \ a : A_1 \text{ of } b \ x^+ : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R \ a : A_1 \text{ of } b : \Pi^+ x : A \rightarrow B \Rightarrow \Pi^+ x : A \rightarrow C \mid C'} \quad \text{BRANCHTYPING_PIREL}$$

$$\frac{\Gamma, x : A \models \text{case}_R \ a : A_1 \text{ of } b \ \square^- : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R \ a : A_1 \text{ of } b : \Pi^- x : A \rightarrow B \Rightarrow \Pi^- x : A \rightarrow C \mid C'} \quad \text{BRANCHTYPING_PIRREL}$$

$$\frac{\Gamma, c : \phi \models \text{case}_R a : A \text{ of } b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R a : A \text{ of } b : \forall c : \phi. B \Rightarrow \forall c : \phi. C \mid C'} \quad \text{BRANCHTypING_CPI}$$

$$\boxed{\Gamma \models \phi \text{ ok}} \quad \text{Prop wellformedness}$$

$$\frac{\begin{array}{c} \Gamma \models a : A \\ \Gamma \models b : A \\ \Gamma \models A : \star \end{array}}{\Gamma \models a \sim_{A/R} b \text{ ok}} \quad \text{E_WFF}$$

$$\boxed{\Gamma \models a : A} \quad \text{typing}$$

$$\begin{array}{c} \frac{\vdash \Gamma}{\Gamma \vdash \star : \star} \quad \text{E_STAR} \\ \\ \frac{\begin{array}{c} \vdash \Gamma \\ x : A \in \Gamma \end{array}}{\Gamma \vdash x : A} \quad \text{E_VAR} \\ \\ \frac{\begin{array}{c} \Gamma, x : A \vdash B : \star \\ \Gamma \vdash A : \star \end{array}}{\Gamma \vdash \Pi^\rho x : A \rightarrow B : \star} \quad \text{E_PI} \\ \\ \frac{\begin{array}{c} \Gamma, x : A \vdash a : B \\ \Gamma \vdash A : \star \\ (\rho = +) \vee (x \notin \text{fv } a) \end{array}}{\Gamma \vdash \lambda^\rho x. a : (\Pi^\rho x : A \rightarrow B)} \quad \text{E_ABS} \\ \\ \frac{\begin{array}{c} \Gamma \vdash b : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash a : A \end{array}}{\Gamma \vdash b \ a^+ : B\{a/x\}} \quad \text{E_APP} \\ \\ \frac{\begin{array}{c} \Gamma \vdash b : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash a : A \\ \text{Path } b = F @ R, Rs \end{array}}{\Gamma \vdash b \ a^R : B\{a/x\}} \quad \text{E_TAPP} \\ \\ \frac{\begin{array}{c} \Gamma \vdash b : \Pi^- x : A \rightarrow B \\ \Gamma \vdash a : A \end{array}}{\Gamma \vdash b \ \Box^- : B\{a/x\}} \quad \text{E_IAPP} \\ \\ \frac{\begin{array}{c} \Gamma \vdash a : A \\ \Gamma; \tilde{\Gamma} \vdash A \equiv B : \star / \mathbf{Rep} \\ \Gamma \vdash B : \star \end{array}}{\Gamma \vdash a : B} \quad \text{E_CONV} \\ \\ \frac{\begin{array}{c} \Gamma, c : \phi \vdash B : \star \\ \Gamma \vdash \phi \text{ ok} \end{array}}{\Gamma \vdash \forall c : \phi. B : \star} \quad \text{E_CPI} \\ \\ \frac{\begin{array}{c} \Gamma, c : \phi \vdash a : B \\ \Gamma \vdash \phi \text{ ok} \end{array}}{\Gamma \vdash \Lambda c. a : \forall c : \phi. B} \quad \text{E_CABS} \\ \\ \frac{\begin{array}{c} \Gamma \vdash a_1 : \forall c : (a \sim_{A/R} b). B_1 \\ \Gamma; \tilde{\Gamma} \vdash a \equiv b : A/R \end{array}}{\Gamma \vdash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E_CAPP} \end{array}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ F : A @ R s \in \Sigma_0 \\ \emptyset \vdash A : \star \end{array}}{\Gamma \vdash F : A} \quad \text{E_CONST}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ F : p \sim a : A / R_1 @ R s \in \Sigma_0 \\ \emptyset \vdash A : \star \end{array}}{\Gamma \vdash F : A} \quad \text{E_FAM}$$

$$\frac{\begin{array}{c} \Gamma \vdash a : A \\ \Gamma \vdash F : A_1 \\ \Gamma \vdash b_1 : B \\ \Gamma \vdash b_2 : C \\ \Gamma \vdash \text{case}_R a : A \text{ of } F : A_1 \Rightarrow B \mid C \end{array}}{\Gamma \vdash \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 : C} \quad \text{E_CASE}$$

$$\boxed{\Gamma; \Delta \vdash \phi_1 \equiv \phi_2} \quad \text{prop equality}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vdash A_1 \equiv A_2 : A / R \\ \Gamma; \Delta \vdash B_1 \equiv B_2 : A / R \end{array}}{\Gamma; \Delta \vdash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E_PROPCONG}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vdash A \equiv B : \star / R_0 \\ \Gamma \vdash A_1 \sim_{A/R} A_2 \text{ ok} \\ \Gamma \vdash A_1 \sim_{B/R} A_2 \text{ ok} \end{array}}{\Gamma; \Delta \vdash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E_ISOCONV}$$

$$\frac{\Gamma; \Delta \vdash \forall c : (a_1 \sim_{A/R_1} a_2). B_1 \equiv \forall c : (b_1 \sim_{B/R_2} b_2). B_2 : \star / R'}{\Gamma; \Delta \vdash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E_CPIFST}$$

$$\boxed{\Gamma; \Delta \vdash a \equiv b : A / R} \quad \text{definitional equality}$$

$$\frac{\begin{array}{c} \vdash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vdash a \equiv b : A / R} \quad \text{E_ASSN}$$

$$\frac{\Gamma \vdash a : A}{\Gamma; \Delta \vdash a \equiv a : A / R} \quad \text{E_REFL}$$

$$\frac{\Gamma; \Delta \vdash b \equiv a : A / R}{\Gamma; \Delta \vdash a \equiv b : A / R} \quad \text{E_SYM}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vdash a \equiv a_1 : A / R \\ \Gamma; \Delta \vdash a_1 \equiv b : A / R \end{array}}{\Gamma; \Delta \vdash a \equiv b : A / R} \quad \text{E_TRANS}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vdash a \equiv b : A / R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \vdash a \equiv b : A / R_2} \quad \text{E_SUB}$$

$$\frac{\begin{array}{c} \Gamma \vdash a_1 : B \\ \Gamma \vdash a_2 : B \\ \vdash a_1 > a_2 / R \end{array}}{\Gamma; \Delta \vdash a_1 \equiv a_2 : B / R} \quad \text{E_BETA}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \models A_1 \equiv A_2 : \star / R' \quad \Gamma, x : A_1; \Delta \models B_1 \equiv B_2 : \star / R' \quad \Gamma \models A_1 : \star \quad \Gamma \models \Pi^\rho x : A_1 \rightarrow B_1 : \star \quad \Gamma \models \Pi^\rho x : A_2 \rightarrow B_2 : \star}{\Gamma; \Delta \models (\Pi^\rho x : A_1 \rightarrow B_1) \equiv (\Pi^\rho x : A_2 \rightarrow B_2) : \star / R'} \quad \text{E_PiCONG} \\
\\
\frac{\Gamma, x : A_1; \Delta \models b_1 \equiv b_2 : B / R' \quad \Gamma \models A_1 : \star \quad (\rho = +) \vee (x \notin \text{fv } b_1) \quad (\rho = +) \vee (x \notin \text{fv } b_2)}{\Gamma; \Delta \models (\lambda^\rho x. b_1) \equiv (\lambda^\rho x. b_2) : (\Pi^\rho x : A_1 \rightarrow B) / R'} \quad \text{E_ABSCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A \rightarrow B) / R' \quad \Gamma; \Delta \models a_2 \equiv b_2 : A / \mathbf{Nom}}{\Gamma; \Delta \models a_1 \ a_2^+ \equiv b_1 \ b_2^+ : (B\{a_2/x\}) / R'} \quad \text{E_APPCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A \rightarrow B) / R' \quad \Gamma; \Delta \models a_2 \equiv b_2 : A / R \quad \text{Path } a_1 = F @ R, Rs \quad \text{Path } b_1 = F' @ R, Rs'}{\Gamma; \Delta \models a_1 \ a_2^R \equiv b_1 \ b_2^R : (B\{a_2/x\}) / R'} \quad \text{E_TAPPCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^- x : A \rightarrow B) / R' \quad \Gamma \models a : A}{\Gamma; \Delta \models a_1 \ \Box^- \equiv b_1 \ \Box^- : (B\{a/x\}) / R'} \quad \text{E_IAPPCONG} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star / R'}{\Gamma; \Delta \models A_1 \equiv A_2 : \star / R'} \quad \text{E_PiFST} \\
\\
\frac{\Gamma; \Delta \models \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star / R' \quad \Gamma; \Delta \models a_1 \equiv a_2 : A_1 / \mathbf{Nom}}{\Gamma; \Delta \models B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star / R'} \quad \text{E_PiSND} \\
\\
\frac{\Gamma; \Delta \models a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \quad \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \models A \equiv B : \star / R' \quad \Gamma \models a_1 \sim_{A_1/R} b_1 \text{ ok} \quad \Gamma \models \forall c : a_1 \sim_{A_1/R} b_1. A : \star \quad \Gamma \models \forall c : a_2 \sim_{A_2/R} b_2. B : \star}{\Gamma; \Delta \models \forall c : a_1 \sim_{A_1/R} b_1. A \equiv \forall c : a_2 \sim_{A_2/R} b_2. B : \star / R'} \quad \text{E_CPiCONG} \\
\\
\frac{\Gamma, c : \phi_1; \Delta \models a \equiv b : B / R \quad \Gamma \models \phi_1 \text{ ok}}{\Gamma; \Delta \models (\Lambda c. a) \equiv (\Lambda c. b) : \forall c : \phi_1. B / R} \quad \text{E_CABSCONG} \\
\\
\frac{\Gamma; \Delta \models a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b). B) / R' \quad \Gamma; \tilde{\Gamma} \models a \equiv b : A / \mathbf{param } R R'}{\Gamma; \Delta \models a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\}) / R'} \quad \text{E_CAPPiCONG} \\
\\
\frac{\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R} a_2). B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2). B_2 : \star / R_0 \quad \Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A / \mathbf{param } R R_0 \quad \Gamma; \tilde{\Gamma} \models a'_1 \equiv a'_2 : A' / \mathbf{param } R' R_0}{\Gamma; \Delta \models B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star / R_0} \quad \text{E_CPiSND}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \vdash a \equiv b : A/R \quad \Gamma; \Delta \vdash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'}{\Gamma; \Delta \vdash a' \equiv b' : A'/R'} \quad \text{E_CAST} \\
\\
\frac{\Gamma; \Delta \vdash a \equiv b : A/R \quad \Gamma; \tilde{\Gamma} \vdash A \equiv B : \star/\mathbf{Rep} \quad \Gamma \vdash B : \star}{\Gamma; \Delta \vdash a \equiv b : B/R} \quad \text{E_EQCONV} \\
\\
\frac{\Gamma; \Delta \vdash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \vdash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E_ISOSND} \\
\\
\frac{\Gamma; \Delta \vdash a \equiv a' : A/R \quad \Gamma; \Delta \vdash b_1 \equiv b'_1 : B/R_0 \quad \Gamma; \Delta \vdash b_2 \equiv b'_2 : B/R_0}{\Gamma; \Delta \vdash \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \rightarrow b_2 \equiv \text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel \rightarrow b'_2 : B/R_0} \quad \text{E_PATCONG} \\
\\
\frac{\text{ValuePath } a = F \quad \text{ValuePath } a' = F \quad \Gamma \vdash a : \Pi^+ x : A \rightarrow B \quad \Gamma \vdash b : A \quad \Gamma \vdash a' : \Pi^+ x : A \rightarrow B \quad \Gamma \vdash b' : A \quad \Gamma; \Delta \vdash a \ b^{R_1} \equiv a' \ b'^{R_1} : B\{b/x\}/R' \quad \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R'}{\Gamma; \Delta \vdash a \equiv a' : \Pi^+ x : A \rightarrow B/R'} \quad \text{E_LEFTREL} \\
\\
\frac{\text{ValuePath } a = F \quad \text{ValuePath } a' = F \quad \Gamma \vdash a : \Pi^- x : A \rightarrow B \quad \Gamma \vdash b : A \quad \Gamma \vdash a' : \Pi^- x : A \rightarrow B \quad \Gamma \vdash b' : A \quad \Gamma; \Delta \vdash a \ \square^- \equiv a' \ \square^- : B\{b/x\}/R' \quad \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0}{\Gamma; \Delta \vdash a \equiv a' : \Pi^- x : A \rightarrow B/R'} \quad \text{E_LEFTIRREL} \\
\\
\frac{\text{ValuePath } a = F \quad \text{ValuePath } a' = F \quad \Gamma \vdash a : \Pi^+ x : A \rightarrow B \quad \Gamma \vdash b : A \quad \Gamma \vdash a' : \Pi^+ x : A \rightarrow B \quad \Gamma \vdash b' : A \quad \Gamma; \Delta \vdash a \ b^+ \equiv a' \ b'^+ : B\{b/x\}/R' \quad \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0}{\Gamma; \Delta \vdash b \equiv b' : A/\mathbf{param} \ R_1 \ R'} \quad \text{E_RIGHT} \\
\\
\frac{\text{ValuePath } a = F \quad \text{ValuePath } a' = F \quad \Gamma \vdash a : \forall c : (a_1 \sim_{A/R_1} a_2). B \quad \Gamma \vdash a' : \forall c : (a_1 \sim_{A/R_1} a_2). B \quad \Gamma; \tilde{\Gamma} \vdash a_1 \equiv a_2 : A/R' \quad \Gamma; \Delta \vdash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'}{\Gamma; \Delta \vdash a \equiv a' : \forall c : (a_1 \sim_{A/R_1} a_2). B/R'} \quad \text{E_CLEFT}
\end{array}$$

$\boxed{\models \Gamma}$ context wellformedness

$$\begin{array}{c}
\overline{\models \emptyset} \quad \text{E_EMPTY} \\
\\
\begin{array}{c}
\models \Gamma \\
\Gamma \models A : \star \\
x \notin \tilde{\Gamma}
\end{array}
\overline{\models \Gamma, x : A} \quad \text{E_CONSTM} \\
\\
\begin{array}{c}
\models \Gamma \\
\Gamma \models \phi \text{ ok} \\
c \notin \tilde{\Gamma}
\end{array}
\overline{\models \Gamma, c : \phi} \quad \text{E_CONSCo}
\end{array}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\begin{array}{c}
\overline{\models \emptyset} \quad \text{SIG_EMPTY} \\
\\
\begin{array}{c}
\models \Sigma \\
\emptyset \models A : \star \\
F \notin \text{dom } \Sigma
\end{array}
\overline{\models \Sigma \cup \{F : A @ Rs\}} \quad \text{SIG_CONSTCONST} \\
\\
\begin{array}{c}
\models \Sigma \\
F \notin \text{dom } \Sigma \\
\emptyset \models A : \star \\
\Omega; \Gamma \models p :_F B \Rightarrow A \\
\Gamma \models a : B \\
\Omega \models a : R
\end{array}
\overline{\models \Sigma \cup \{F : p \sim a : A / R @ \text{range } \Omega\}} \quad \text{SIG_CONSAx}
\end{array}$$

$\boxed{\Gamma \vdash \phi \text{ ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A / R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \rightsquigarrow b / R}$ single-step, weak head reduction to values for annotated language

Definition rules: 159 good 0 bad
Definition rule clauses: 446 good 0 bad