$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	$::=$			relevance flag
	$\vert$	$+$		
	$\vert$	$-$		
	$\vert$	$\mathbf{app\_rho}\,\nu$	S	
	$\vert$	(ρ)	S	
appflag, ν	$::=$			applicative flag
	$\vert$	R		
	$\vert$	ρ		
role, R	$::=$			Role
	$\vert$	$\mathbf{Nom}$		
	$\vert$	$\mathbf{Rep}$		
	$\vert$	$R_1 \cap R_2$	S	
	$\vert$	$\mathbf{param}\, R_1\ R_2$	S	
	$\vert$	$\mathbf{app\_role}\,\nu$	S	
	$\vert$	(R)	S	
constraint, ϕ	$::=$			props
	$\vert$	$a \sim_{A/R} b$		
	$\vert$	(ϕ)	S	
	$\vert$	$\phi\{b/x\}$	S	
	$\vert$	$\vert\phi\vert$	S	
	$\vert$	$a \sim_R b$	S	
tm, a, b, p, v, w, A, B, C	$::=$			types and kinds
	$\vert$	$\star$		
	$\vert$	x		
	$\vert$	$\lambda^{\rho} x\!:\!A.b$	bind x in b	
	$\vert$	$\lambda^{\rho} x.b$	bind x in b	
	$\vert$	$a\ b^{\nu}$		
	$\vert$	$\Pi^{\rho} x\!:\!A \to B$	bind x in B	
	$\vert$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\vert$	$\Lambda c.b$	bind c in b	
	$\vert$	$a[\gamma]$		
	$\vert$	$\forall c\!:\!\phi.B$	bind c in B	
	$\vert$	$a \triangleright_R \gamma$		
	$\vert$	F		
	$\vert$	$\square$		
	$\vert$	$\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \Vert_{\_} \to b_2$		
	$\vert$	K		
	$\vert$	$\mathbf{match}\, a\, \mathbf{with}\, brs$		
	$\vert$	$\mathbf{sub}\, R\ a$		
	$\vert$	$a\{b/x\}$	S	
	$\vert$	$a\{\gamma/c\}$	S	
	$\vert$	$a\{b/x\}$	S	
	$\vert$	$a\{\gamma/c\}$	S	

	|	a	S	
	|	a	S	
	|	(a)	S	
	|	a	S	parsing precedence is hard
	|	$\|a\|_R$	S	
	|	**Int**	S	
	|	**Bool**	S	
	|	Nat	S	
	|	**Vec**	S	
	|	0	S	
	|	S	S	
	|	**True**	S	
	|	**Fix**	S	
	|	**Age**	S	
	|	$a \to b$	S	
	|	$\phi \Rightarrow A$	S	
	|	$a\ b$	S	
	|	$\lambda x.a$	S	
	|	$\lambda x : A.a$	S	
	|	$\forall\, x : A \to B$	S	
	|	**if** ϕ **then** a **else** b	S	
brs	$::=$			case branches
	|	**none**		
	|	$K \Rightarrow a; brs$		
	|	$brs\{a/x\}$	S	
	|	$brs\{\gamma/c\}$	S	
	|	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	|	$\bullet$		
	|	c		
	|	**red** $a\ b$		
	|	**refl** a		
	|	$(a \mid=\mid_\gamma b)$		
	|	**sym** γ		
	|	$\gamma_1; \gamma_2$		
	|	**sub** γ		
	|	$\Pi^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2	
	|	$\lambda^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2	
	|	$\gamma_1\ \gamma_2^{R,\rho}$		
	|	**piFst** γ		
	|	**cpiFst** γ		
	|	**isoSnd** γ		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c : \gamma_1.\gamma_3$	bind c in γ_3	

	|	$\lambda c{:}\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1, \gamma_2)$		
	|	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright_R \gamma_2$		
	|	$\gamma_1 \sim_A \gamma_2$		
	|	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	|	$\mathbf{eta}\, a$		
	|	$\mathbf{left}\, \gamma\, \gamma'$		
	|	$\mathbf{right}\, \gamma\, \gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
role_context, Ω	::=			role$_c$*ontexts*
	|	$\varnothing$		
	|	$x : R$		
	|	$\Omega, x : R$		
	|	Ω, Ω'	M	
	|	$\mathbf{var\_pat}\, p$	M	
	|	(Ω)	M	
	|	Ω	M	
roles, Rs	::=			
	|	$\mathbf{nilR}$		
	|	R, Rs		
	|	$\mathbf{range}\, \Omega$	S	
sig_sort	::=			signature classifier
	|	$A @ Rs$		
	|	$p \sim a : A / R @ Rs$		
sort	::=			binding classifier
	|	$\mathbf{Tm}\, A$		
	|	$\mathbf{Co}\, \phi$		
context, Γ	::=			contexts
	|	$\varnothing$		
	|	$\Gamma, x : A$		
	|	$\Gamma, c : \phi$		
	|	$\Gamma\{b/x\}$	M	
	|	$\Gamma\{\gamma/c\}$	M	
	|	Γ, Γ'	M	
	|	$\|\Gamma\|$	M	
	|	(Γ)	M	
	|	Γ	M	
sig, Σ	::=			signatures

	$\mid$	$\varnothing$	
	$\mid$	$\Sigma \cup \{F : \mathit{sig\_sort}\}$	
	$\mid$	Σ_0	M
	$\mid$	Σ_1	M
	$\mid$	$\lvert\Sigma\rvert$	M
$\mathit{available\_props},\ \Delta$	$::=$		
	$\mid$	$\varnothing$	
	$\mid$	Δ, x	
	$\mid$	Δ, c	
	$\mid$	$\mathsf{fv}\, a$	M
	$\mid$	Δ, Δ'	M
	$\mid$	$\widetilde{\Gamma}$	M
	$\mid$	$\widetilde{\Omega}$	M
	$\mid$	(Δ)	M
$\mathit{Nat},\ \mathbb{N}$	$::=$		
	$\mid$	$\lvert a\rvert$	S
$\mathit{terminals}$	$::=$		
	$\mid$	$\leftrightarrow$	
	$\mid$	$\Leftrightarrow$	
	$\mid$	$\longrightarrow$	
	$\mid$	min	
	$\mid$	$\equiv$	
	$\mid$	$\forall$	
	$\mid$	$\in$	
	$\mid$	$\not\in$	
	$\mid$	$\Leftarrow$	
	$\mid$	$\Rightarrow$	
	$\mid$	$\Rightarrow^*$	
	$\mid$	$\rightarrow$	
	$\mid$	Λ	
	$\mid$		
	$\mid$	$\square$	
	$\mid$	$\vdash$	
	$\mid$	$\dashv$	
	$\mid$	$\models$	
	$\mid$	$\models$	
	$\mid$	$\neq$	
	$\mid$	$\triangleright$	
	$\mid$	ok	
	$\mid$	$\_$	
	$\mid$	$\leadsto$	
	$\mid$	$\leadsto^*$	
	$\mid$	$\leadsto$	
	$\mid$	$\varnothing$	

| $\circ$
| fv
| dom
| $\sim$
| $\asymp$
| $|$
| $\bullet$
| **fst**
| **snd**
| **as**
| $|\Rightarrow|$
| $\vdash_{=}$
| $\mathbf{refl_2}$
| $++$
| $\{$
| $\}$

$formula,\ \psi$ ::=
| $judgement$
| $x : A \in \Gamma$
| $x : R \in \Omega$
| $c : \phi \in \Gamma$
| $F : sig\_sort \in \Sigma$
| $x \in \Delta$
| $c \in \Delta$
| $c\,\textbf{not relevant} \in \gamma$
| $x \notin \Delta$
| $uniq\,\Gamma$
| $uniq(\Omega)$
| $c \notin \Delta$
| $T \notin \mathsf{dom}\,\Sigma$
| $F \notin \mathsf{dom}\,\Sigma$
| $\mathbb{N}_1 < \mathbb{N}_2$
| $\mathbb{N}_1 \leq \mathbb{N}_2$
| $R_1 = R_2$
| $a = b$
| $\phi_1 = \phi_2$
| $\Gamma_1 = \Gamma_2$
| $\gamma_1 = \gamma_2$
| $\neg\psi$
| $\psi_1 \wedge \psi_2$
| $\psi_1 \vee \psi_2$
| $\psi_1 \Rightarrow \psi_2$
| (ψ)
| ψ
| $c : (a : A \sim b : B) \in \Gamma$

	$\vert$		suppress lc hypothesis gen
JSubRole	$::=$		
	$\vert$	$R_1 \leq R_2$	Subroling judgement
JPath	$::=$		
	$\vert$	$\mathsf{Path}\ a = F@Rs$	Type headed by constant
JPatCtx	$::=$		
	$\vert$	$\Omega; \Gamma \vDash p :_F B \Rightarrow A$	Contexts generated by a p
JRename	$::=$		
	$\vert$	$\mathsf{rename}\ p \rightarrow a\ \mathsf{to}\ p' \rightarrow a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'$	rename with fresh variable
JMatchSubst	$::=$		
	$\vert$	$\mathsf{match}\ a_1\ \mathsf{with}\ p \rightarrow b_1 = b_2$	match and substitute
JTmPatternAgree	$::=$		
	$\vert$	$a \leftrightarrow p$	term and pattern agree
JTmSubPatternAgree	$::=$		
	$\vert$	$a^+ = p$	sub-pattern agrees with te
JSubTmPatternAgree	$::=$		
	$\vert$	$a = p^+$	sub-term agrees with patt
JValuePath	$::=$		
	$\vert$	$\mathsf{ValuePath}\ a = F$	Type headed by constant
JCasePath	$::=$		
	$\vert$	$\mathsf{CasePath}_R\ a = F$	Type headed by constant
JApplyArgs	$::=$		
	$\vert$	$\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'$	apply arguments of a (hea
JValue	$::=$		
	$\vert$	$\mathsf{Value}_R\ A$	values
JValueType	$::=$		
	$\vert$	$\mathsf{ValueType}_R\ A$	Types with head forms (er
Jconsistent	$::=$		
	$\vert$	$\mathsf{consistent}_R\ a\ b$	(erased) types do not diffe
Jroleing	$::=$		
	$\vert$	$\Omega \vDash a : R$	Roleing judgment

JChk	::=		
	|	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	::=		
	|	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	|	$\Omega \vDash a \Rightarrow^*_R b$	multistep parallel reduction
	|	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	::=		
	|	$\vDash a > b/R$	primitive reductions on erased terms
	|	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit langu
	|	$\vDash a \rightsquigarrow^* b/R$	multistep reduction
JBranchTyping	::=		
	|	$\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
Jett	::=		
	|	$\Gamma \vDash \phi\ \ \mathsf{ok}$	Prop wellformedness
	|	$\Gamma \vDash a : A$	typing
	|	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	|	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness
Jsig	::=		
	|	$\vDash \Sigma$	signature wellformedness
Jann	::=		
	|	$\Gamma \vdash \phi\ \ \mathsf{ok}$	prop wellformedness
	|	$\Gamma \vdash a : A/R$	typing
	|	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	|	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	|	$\vdash \Gamma$	context wellformedness
Jred	::=		
	|	$\Gamma \vdash a \rightsquigarrow b/R$	single-step, weak head reduction to values fo
judgement	::=		
	|	*JSubRole*	
	|	*JPath*	
	|	*JPatCtx*	
	|	*JRename*	
	|	*JMatchSubst*	
	|	*JTmPatternAgree*	
	|	*JTmSubPatternAgree*	
	|	*JSubTmPatternAgree*	
	|	*JValuePath*	
	|	*JCasePath*	

$$
\begin{array}{lcl}
 & | & \mathit{JApplyArgs} \\
 & | & \mathit{JValue} \\
 & | & \mathit{JValueType} \\
 & | & \mathit{Jconsistent} \\
 & | & \mathit{Jroleing} \\
 & | & \mathit{JChk} \\
 & | & \mathit{Jpar} \\
 & | & \mathit{Jbeta} \\
 & | & \mathit{JBranchTyping} \\
 & | & \mathit{Jett} \\
 & | & \mathit{Jsig} \\
 & | & \mathit{Jann} \\
 & | & \mathit{Jred}
\end{array}
$$

$$
\begin{array}{lcl}
\mathit{user\_syntax} & ::= & \\
 & | & \mathit{tmvar} \\
 & | & \mathit{covar} \\
 & | & \mathit{datacon} \\
 & | & \mathit{const} \\
 & | & \mathit{index} \\
 & | & \mathit{relflag} \\
 & | & \mathit{appflag} \\
 & | & \mathit{role} \\
 & | & \mathit{constraint} \\
 & | & \mathit{tm} \\
 & | & \mathit{brs} \\
 & | & \mathit{co} \\
 & | & \mathit{role\_context} \\
 & | & \mathit{roles} \\
 & | & \mathit{sig\_sort} \\
 & | & \mathit{sort} \\
 & | & \mathit{context} \\
 & | & \mathit{sig} \\
 & | & \mathit{available\_props} \\
 & | & \mathit{Nat} \\
 & | & \mathit{terminals} \\
 & | & \mathit{formula}
\end{array}
$$

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}\ a = F @ Rs}$ Type headed by constant (partial function)

$$\frac{F : A@Rs \in \Sigma_0}{\mathsf{Path}\ F = F@Rs} \quad \textsc{Path\_AbsConst}$$

$$\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0}{\mathsf{Path}\ F = F@Rs} \quad \textsc{Path\_Const}$$

$$\frac{\mathsf{Path}\ a = F@R_1, Rs}{\mathsf{Path}\ (a\ b'^{R_1}) = F@Rs} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}\ a = F@Rs}{\mathsf{Path}\ (a\ b^-) = F@Rs} \quad \textsc{Path\_IApp}$$

$$\frac{\mathsf{Path}\ a = F@Rs}{\mathsf{Path}\ (a[\bullet]) = F@Rs} \quad \textsc{Path\_CApp}$$

$\boxed{\Omega; \Gamma \vDash p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\varnothing; \varnothing \vDash F :_F A \Rightarrow A} \quad \textsc{PatCtx\_Const}$$

$$\frac{\Omega; \Gamma \vDash p :_F \Pi^+ x\!:\!A' \to A \Rightarrow B}{\Omega, x : R; \Gamma, x : A' \vDash p\ x^R :_F A \Rightarrow B} \quad \textsc{PatCtx\_PiRel}$$

$$\frac{\Omega; \Gamma \vDash p :_F \Pi^- x\!:\!A' \to A \Rightarrow B}{\Omega; \Gamma, x : A' \vDash p\ \Box^- :_F A \Rightarrow B} \quad \textsc{PatCtx\_PiIrr}$$

$$\frac{\Omega; \Gamma \vDash p :_F \forall c\!:\!\phi.A \Rightarrow B}{\Omega; \Gamma, c : \phi \vDash p[\bullet] :_F A \Rightarrow B} \quad \textsc{PatCtx\_CPi}$$

$\boxed{\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}$ rename with fresh variables

$$\frac{}{\mathsf{rename}\ F \to a\ \mathsf{to}\ F \to a\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \varnothing} \quad \textsc{Rename\_Base}$$

$$\frac{\begin{array}{l}\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta' \\ y \notin (\Delta, \Delta')\end{array}}{\mathsf{rename}\ (p_1\ x^R) \to a_1\ \mathsf{to}\ (p_2\ y^R) \to (a_2\{y/x\})\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ (\Delta', y)} \quad \textsc{Rename\_AppRel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}{\mathsf{rename}\ (p_1\ \Box^-) \to a_1\ \mathsf{to}\ (p_2\ \Box^-) \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'} \quad \textsc{Rename\_AppIrrel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}{\mathsf{rename}\ (p_1[\bullet]) \to a_1\ \mathsf{to}\ (p_2[\bullet]) \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'} \quad \textsc{Rename\_CApp}$$

$\boxed{\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 = b_2}$ match and substitute

$$\frac{}{\mathsf{match}\ F\ \mathsf{with}\ F \to b = b} \quad \textsc{MatchSubst\_Const}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ p_1 \to b_1 = b_2}{\mathsf{match}\ (a_1\ a^R)\ \mathsf{with}\ (p_1\ x^R) \to b_1 = (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRelR}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ p_1 \to b_1 = b_2}{\mathsf{match}\ (a_1\ a^-)\ \mathsf{with}\ (p_1\ \Box^-) \to b_1 = b_2} \quad \textsc{MatchSubst\_AppIrrel}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 = b_2}{\mathsf{match}\ (a_1[\bullet])\ \mathsf{with}\ (a_2[\bullet]) \to b_1 = b_2} \quad \textsc{MatchSubst\_CApp}$$

$\boxed{a \leftrightarrow p}$ term and pattern agree

$$\frac{}{F \leftrightarrow F} \quad \textsc{tm\_pattern\_agree\_Const}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \ a_2{}^R) \leftrightarrow (p_1\ \ x^R)} \quad \textsc{tm\_pattern\_agree\_AppRelR}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \ a^-) \leftrightarrow (p_1\ \Box^-)} \quad \textsc{tm\_pattern\_agree\_AppIrrel}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1[\bullet]) \leftrightarrow (p_1[\bullet])} \quad \textsc{tm\_pattern\_agree\_CApp}$$

$\boxed{a^+ = p}$ sub-pattern agrees with term

$$\frac{a \leftrightarrow p}{a^+ = p} \quad \textsc{tm\_subpattern\_agree\_Base}$$

$$\frac{a^+ = p}{a^+ = (p\ \ x^R)} \quad \textsc{tm\_subpattern\_agree\_AppRelR}$$

$$\frac{a^+ = p}{a^+ = (p\ \Box^-)} \quad \textsc{tm\_subpattern\_agree\_AppIrrel}$$

$$\frac{a^+ = p}{a^+ = (p[\bullet])} \quad \textsc{tm\_subpattern\_agree\_CAppp}$$

$\boxed{a = p^+}$ sub-term agrees with pattern

$$\frac{a \leftrightarrow p}{a = p^+} \quad \textsc{subtm\_pattern\_agree\_Base}$$

$$\frac{a = p^+}{a\ \ a_2{}^{\nu} = p^+} \quad \textsc{subtm\_pattern\_agree\_App}$$

$$\frac{a = p^+}{a[\bullet] = p^+} \quad \textsc{subtm\_pattern\_agree\_CAppp}$$

$\boxed{\mathsf{ValuePath}\ \ a = F}$ Type headed by constant (role-sensitive partial function used in value)

$$\frac{F : A@Rs\ \in\ \Sigma_0}{\mathsf{ValuePath}\ \ F = F} \quad \textsc{ValuePath\_AbsConst}$$

$$\frac{F :\ p \sim a : A/R_1@Rs\ \in\ \Sigma_0}{\mathsf{ValuePath}\ \ F = F} \quad \textsc{ValuePath\_Const}$$

$$\frac{\mathsf{ValuePath}\ \ a = F}{\mathsf{ValuePath}\ \ (a\ \ b'^{\nu}) = F} \quad \textsc{ValuePath\_App}$$

$$\frac{\mathsf{ValuePath}\ \ a = F}{\mathsf{ValuePath}\ \ (a[\bullet]) = F} \quad \textsc{ValuePath\_CApp}$$

$\boxed{\mathsf{CasePath}_R\ \ a = F}$ Type headed by constant (role-sensitive partial function used in case)

$$\frac{\begin{array}{c}\mathsf{ValuePath}\ \ a = F \\ F : A@Rs\ \in\ \Sigma_0\end{array}}{\mathsf{CasePath}_R\ \ a = F} \quad \textsc{CasePath\_AbsConst}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F\\ F : p \sim b : A/R_1@Rs \in \Sigma_0\\ \neg(R_1 \leq R)\end{array}}{\mathsf{CasePath}_R\ a = F}\ \textsc{CasePath\_Const}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F\\ F : p \sim b : A/R_1@Rs \in \Sigma_0\\ \neg(a = p^+)\end{array}}{\mathsf{CasePath}_R\ a = F}\ \textsc{CasePath\_UnMatch}$$

$\boxed{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\mathsf{apply\ args}\ F\ \mathsf{to}\ b \mapsto b}\ \textsc{ApplyArgs\_Const}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}{\mathsf{apply\ args}\ a\ a'^{\nu}\ \mathsf{to}\ b \mapsto b'\ a'^{\nu}}\ \textsc{ApplyArgs\_App}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}{\mathsf{apply\ args}\ a[\bullet]\ \mathsf{to}\ b \mapsto b'[\bullet]}\ \textsc{ApplyArgs\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star}\ \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^{\rho} x\!:\!A \to B}\ \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B}\ \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x\!:\!A.a}\ \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x.a}\ \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^{-} x.a}\ \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a}\ \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a}\ \textsc{Value\_UCAbs}$$

$$\frac{\mathsf{CasePath}_R\ a = F}{\mathsf{Value}_R\ a}\ \textsc{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\ \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^{\rho} x\!:\!A \to B}\ \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B}\ \textsc{value\_type\_CPi}$$

$$\frac{\mathsf{CasePath}_R\ a = F}{\mathsf{ValueType}_R\ a}\ \textsc{value\_type\_ValuePath}$$

$\boxed{\mathsf{consistent}_R\ a\ b}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\ \star}\quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1 : A_1 \to B_1)\ (\Pi^\rho x_2 : A_2 \to B_2)}\quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1 : \phi_1.A_1)\ (\forall c_2 : \phi_2.A_2)}\quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{c}\mathsf{CasePath}_R\ a_1 = F \\ \mathsf{CasePath}_R\ a_2 = F\end{array}}{\mathsf{consistent}_R\ a_1\ a_2}\quad \textsc{consistent\_a\_CasePath}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ a\ b}\quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ a\ b}\quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \square : R}\quad \textsc{role\_a\_Bullet}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R}\quad \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega) \\ x : R \in \Omega \\ R \leq R_1\end{array}}{\Omega \vDash x : R_1}\quad \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R}\quad \textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R \\ \Omega \vDash b : \mathbf{Nom}\end{array}}{\Omega \vDash (a\ b^\rho) : R}\quad \textsc{role\_a\_App}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R \\ \mathsf{Path}\ a = F@R_1, Rs \\ \Omega \vDash b : R_1\end{array}}{\Omega \vDash a\ b^{R_1} : R}\quad \textsc{role\_a\_TApp}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R \\ \Omega, x : \mathbf{Nom} \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x : A \to B) : R}\quad \textsc{role\_a\_Pi}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R_1 \\ \Omega \vDash b : R_1 \\ \Omega \vDash A : R_0 \\ \Omega \vDash B : R\end{array}}{\Omega \vDash (\forall c : a \sim_{A/R_1} b.B) : R}\quad \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R}\quad \textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R}\quad \textsc{role\_a\_CApp}$$

$$\frac{\begin{array}{c} \mathit{uniq}(\Omega) \\ F : A@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R} \quad \textsc{role\_a\_Const}$$

$$\frac{\begin{array}{c} \mathit{uniq}(\Omega) \\ F : p \sim a : A/R@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R_1} \quad \textsc{role\_a\_Fam}$$

$$\frac{\begin{array}{c} \Omega \vDash a : R \\ \Omega \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1 \end{array}}{\Omega \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : R_1} \quad \textsc{role\_a\_Pattern}$$

$\boxed{(\rho = +) \lor (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \lor (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \lor (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b' \end{array}}{\Omega \vDash a\ b^\rho \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{(\mathbf{app\_role}\,\nu)} b' \end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\ b'^\nu} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{c} \Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \Pi^\rho x\!:\!A \to B \Rightarrow_R \Pi^\rho x\!:\!A' \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{c} \Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l}F : F \sim b : A/R_1@Rs \in \Sigma_0\\ R_1 \leq R\\ uniq(\Omega)\end{array}}{\Omega \vDash F \Rightarrow_R b} \quad \text{PAR\_AXIOMBASE}$$

$$\frac{\begin{array}{l}F : p \sim b : A/R_1@Rs \in \Sigma_0\\ a^+ = p \wedge \neg(a \leftrightarrow p)\\ \Omega \vDash a \Rightarrow_R a'\\ \Omega \vDash a_1 \Rightarrow_{(\mathbf{app\_role}\,\nu)} a_1'\\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p' \to b'\ \mathsf{excluding}\ (\widetilde{\Omega}, \mathsf{fv}p)\ \mathsf{and}\ \Delta'\\ \mathsf{match}\ (a'\ {a_1'}^{\nu})\ \mathsf{with}\ p' \to b' = a_2\\ R_1 \leq R\end{array}}{\Omega \vDash a\ {a_1}^{\nu} \Rightarrow_R a_2} \quad \text{PAR\_AXIOMAPP}$$

$$\frac{\begin{array}{l}F : p \sim b : A/R_1@Rs \in \Sigma_0\\ a^+ = p \wedge \neg(a \leftrightarrow p)\\ \Omega \vDash a \Rightarrow_R a'\\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p' \to b'\ \mathsf{excluding}\ (\widetilde{\Omega}, \mathsf{fv}p)\ \mathsf{and}\ \Delta'\\ \mathsf{match}\ (a'[\bullet])\ \mathsf{with}\ p' \to b' = a_2\\ R_1 \leq R\end{array}}{\Omega \vDash a[\bullet] \Rightarrow_R a_2} \quad \text{PAR\_AXIOMCAPP}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a'\\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1'\\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2) \Rightarrow_{R_0} (\mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|\_ \to b_2')} \quad \text{PAR\_PATTERN}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a'\\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1'\\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\\ \mathsf{CasePath}_R\ a' = F\\ \mathsf{apply\ args}\ a'\ \mathsf{to}\ b_1' \mapsto b\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2) \Rightarrow_{R_0} b[\bullet]} \quad \text{PAR\_PATTERNTRUE}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a'\\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1'\\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\\ \mathsf{Value}_R\ a'\\ \neg(\mathsf{CasePath}_R\ a' = F)\end{array}}{\Omega \vDash (\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2) \Rightarrow_{R_0} b_2'} \quad \text{PAR\_PATTERNFALSE}$$

$\boxed{\Omega \vDash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow_R^* a} \quad \text{MP\_REFL}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R b\\ \Omega \vDash b \Rightarrow_R^* a'\end{array}}{\Omega \vDash a \Rightarrow_R^* a'} \quad \text{MP\_STEP}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{l}\Omega \vDash a_1 \Rightarrow_R^* b\\ \Omega \vDash a_2 \Rightarrow_R^* b\end{array}}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ b^\rho > v\{b/x\}/R_1}\ \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R}\ \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l} F : p \sim b : A/R_1@Rs \in \Sigma_0 \\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p_1 \to b_1\ \mathsf{excluding}\ (\mathsf{fv}a, \mathsf{fv}p)\ \mathsf{and}\ \Delta' \\ \mathsf{match}\ a\ \mathsf{with}\ p_1 \to b_1 = b' \\ R_1 \leq R \end{array}}{\vDash a > b'/R}\ \textsc{Beta\_Axiom}$$

$$\frac{\begin{array}{l} \mathsf{CasePath}_R\ a = F \\ \mathsf{apply\ args}\ a\ \mathsf{to}\ b_1 \mapsto b_1' \end{array}}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 > b_1'[\bullet]/R_0}\ \textsc{Beta\_PatternTrue}$$

$$\frac{\begin{array}{l} \mathsf{Value}_R\ a \\ \neg(\mathsf{CasePath}_R\ a = F) \end{array}}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 > b_2/R_0}\ \textsc{Beta\_PatternFalse}$$

$\boxed{\vDash a \leadsto b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'/R_1}{\vDash \lambda^- x.a \leadsto \lambda^- x.a'/R_1}\ \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'/R_1}{\vDash a\ b^\nu \leadsto a'\ b^\nu/R_1}\ \textsc{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash a[\bullet] \leadsto a'[\bullet]/R}\ \textsc{E\_CAppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 \leadsto \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1 \| \_ \to b_2/R_0}\ \textsc{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \leadsto b/R}\ \textsc{E\_Prim}$$

$\boxed{\vDash a \leadsto^* b/R}$ multistep reduction

$$\frac{}{\vDash a \leadsto^* a/R}\ \textsc{Equal}$$

$$\frac{\begin{array}{l} \vDash a \leadsto b/R \\ \vDash b \leadsto^* a'/R \end{array}}{\vDash a \leadsto^* a'/R}\ \textsc{Step}$$

$\boxed{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{c} \mathit{uniq}\ \Gamma \\ \mathtt{lc\_tm}\ C \end{array}}{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : A \Rightarrow \forall c{:}(a \sim_{A/R} b).C \mid C}\ \textsc{BranchTyping\_Base}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b\ x^+ : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b : \Pi^+ x{:}A \to B \Rightarrow \Pi^+ x{:}A \to C \mid C'}\ \textsc{BranchTyping\_PiRel}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b\ \Box^- : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b : \Pi^- x{:}A \to B \Rightarrow \Pi^- x{:}A \to C \mid C'}\ \textsc{BranchTyping\_PiIrrel}$$

$$\frac{\Gamma, c : \phi \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : \forall c{:}\phi.B \Rightarrow \forall c{:}\phi.C \mid C'} \quad \textsc{BranchTyping\_CPi}$$

$\boxed{\Gamma \vDash \phi\ \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash b : A \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash a \sim_{A/R} b\ \ \mathsf{ok}} \quad \textsc{E\_Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \textsc{E\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A\ \in\ \Gamma\end{array}}{\Gamma \vDash x : A} \quad \textsc{E\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash B : \star \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash \Pi^\rho x{:}A \to B : \star} \quad \textsc{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash a : B \\ \Gamma \vDash A : \star \\ (\rho = +) \lor (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x{:}A \to B)} \quad \textsc{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x{:}A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \ a^+ : B\{a/x\}} \quad \textsc{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x{:}A \to B \\ \Gamma \vDash a : A \\ \mathsf{Path}\ \ b = F@R, Rs\end{array}}{\Gamma \vDash b\ \ a^R : B\{a/x\}} \quad \textsc{E\_TApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x{:}A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \Box^- : B\{a/x\}} \quad \textsc{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma \vDash a : B} \quad \textsc{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star \\ \Gamma \vDash \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c{:}\phi.B : \star} \quad \textsc{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B \\ \Gamma \vDash \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B} \quad \textsc{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \textsc{E\_CApp}$$

$$\frac{\models \Gamma \quad F : A@Rs \in \Sigma_0 \quad \varnothing \models A : \star}{\Gamma \models F : A} \quad \text{E\_CONST}$$

$$\frac{\models \Gamma \quad F : p \sim a : A/R_1@Rs \in \Sigma_0}{\Gamma \models F : A} \quad \text{E\_FAM}$$

$$\frac{\Gamma \models a : A \quad \Gamma \models F : A_1 \quad \Gamma \models b_1 : B \quad \Gamma \models b_2 : C \quad \Gamma \models \mathsf{case}_R\ a : A\ \mathsf{of}\ F : A_1 \Rightarrow B \mid C}{\Gamma \models \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : C} \quad \text{E\_CASE}$$

$\boxed{\Gamma; \Delta \models \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\Gamma; \Delta \models A_1 \equiv A_2 : A/R \quad \Gamma; \Delta \models B_1 \equiv B_2 : A/R}{\Gamma; \Delta \models A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PROPCONG}$$

$$\frac{\Gamma; \Delta \models A \equiv B : \star/R_0 \quad \Gamma \models A_1 \sim_{A/R} A_2\ \mathsf{ok} \quad \Gamma \models A_1 \sim_{B/R} A_2\ \mathsf{ok}}{\Gamma; \Delta \models A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_ISOCONV}$$

$$\frac{\Gamma; \Delta \models \forall c{:}(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c{:}(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \models a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPIFST}$$

$\boxed{\Gamma; \Delta \models a \equiv b : A/R}$ definitional equality

$$\frac{\models \Gamma \quad c : (a \sim_{A/R} b) \in \Gamma \quad c \in \Delta}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_ASSN}$$

$$\frac{\Gamma \models a : A}{\Gamma; \Delta \models a \equiv a : A/R} \quad \text{E\_REFL}$$

$$\frac{\Gamma; \Delta \models b \equiv a : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_SYM}$$

$$\frac{\Gamma; \Delta \models a \equiv a_1 : A/R \quad \Gamma; \Delta \models a_1 \equiv b : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_TRANS}$$

$$\frac{\Gamma; \Delta \models a \equiv b : A/R_1 \quad R_1 \leq R_2}{\Gamma; \Delta \models a \equiv b : A/R_2} \quad \text{E\_SUB}$$

$$\frac{\Gamma \models a_1 : B \quad \Gamma \models a_2 : B \quad \models a_1 > a_2/R}{\Gamma; \Delta \models a_1 \equiv a_2 : B/R} \quad \text{E\_BETA}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv A_2 : \star / R' \\ \Gamma, x : A_1; \Delta \vDash B_1 \equiv B_2 : \star / R' \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x{:}A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x{:}A_2 \to B_2 : \star\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x{:}A_1 \to B_1) \equiv (\Pi^\rho x{:}A_2 \to B_2) : \star / R'} \quad \textsc{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1; \Delta \vDash b_1 \equiv b_2 : B / R' \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x{:}A_1 \to B) / R'} \quad \textsc{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x{:}A \to B) / R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A / \mathbf{Nom}\end{array}}{\Gamma; \Delta \vDash a_1\ \ {a_2}^+ \equiv b_1\ \ {b_2}^+ : (B\{a_2/x\}) / R'} \quad \textsc{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x{:}A \to B) / R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A / R \\ \mathsf{Path}\ \ a_1 = F @ R, Rs \\ \mathsf{Path}\ \ b_1 = F' @ R, Rs'\end{array}}{\Gamma; \Delta \vDash a_1\ \ {a_2}^R \equiv b_1\ \ {b_2}^R : (B\{a_2/x\}) / R'} \quad \textsc{E\_TAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x{:}A \to B) / R' \\ \Gamma \vDash a : A\end{array}}{\Gamma; \Delta \vDash a_1\ \Box^- \equiv b_1\ \Box^- : (B\{a/x\}) / R'} \quad \textsc{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x{:}A_1 \to B_1 \equiv \Pi^\rho x{:}A_2 \to B_2 : \star / R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star / R'} \quad \textsc{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x{:}A_1 \to B_1 \equiv \Pi^\rho x{:}A_2 \to B_2 : \star / R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1 / \mathbf{Nom}\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star / R'} \quad \textsc{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star / R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c{:}a_1 \sim_{A_1/R} b_1.A : \star \\ \Gamma \vDash \forall c{:}a_2 \sim_{A_2/R} b_2.B : \star\end{array}}{\Gamma; \Delta \vDash \forall c{:}a_1 \sim_{A_1/R} b_1.A \equiv \forall c{:}a_2 \sim_{A_2/R} b_2.B : \star / R'} \quad \textsc{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv b : B / R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c{:}\phi_1.B / R} \quad \textsc{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c{:}(a \sim_{A/R} b).B) / R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A / \mathbf{param}\, R\, R'\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\}) / R'} \quad \textsc{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \forall c{:}(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c{:}(a_1' \sim_{A'/R'} a_2').B_2 : \star / R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A / \mathbf{param}\, R\, R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A' / \mathbf{param}\, R'\, R_0\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star / R_0} \quad \textsc{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R \\ \Gamma;\Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma;\Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R \\ \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma;\Delta \vDash a \equiv b : B/R} \quad \text{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv a' : A/R \\ \Gamma;\Delta \vDash b_1 \equiv b_1' : B/R_0 \\ \Gamma;\Delta \vDash b_2 \equiv b_2' : B/R_0\end{array}}{\Gamma;\Delta \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 \equiv \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|_{\_} \to b_2' : B/R_0} \quad \text{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^{R_1} \equiv a'\ b'^{R_1} : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^+ x\!:\!A \to B/R'} \quad \text{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ \Box^- \equiv a'\ \Box^- : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^- x\!:\!A \to B/R'} \quad \text{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^+ \equiv a'\ b'^+ : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash b \equiv b' : A/\mathbf{param}\,R_1\ R'} \quad \text{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R' \\ \Gamma;\Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'} \quad \text{E\_CLeft}$$

$\boxed{\models \Gamma}$ context wellformedness

$$\frac{}{\models \varnothing} \quad \text{E\_EMPTY}$$

$$\frac{\begin{array}{l} \models \Gamma \\ \Gamma \models A : \star \\ x \notin \widetilde{\Gamma} \end{array}}{\models \Gamma, x : A} \quad \text{E\_CONSTM}$$

$$\frac{\begin{array}{l} \models \Gamma \\ \Gamma \models \phi \ \mathsf{ok} \\ c \notin \widetilde{\Gamma} \end{array}}{\models \Gamma, c : \phi} \quad \text{E\_CONSCO}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\frac{}{\models \varnothing} \quad \text{SIG\_EMPTY}$$

$$\frac{\begin{array}{l} \models \Sigma \\ \varnothing \models A : \star \\ F \notin \mathsf{dom}\,\Sigma \end{array}}{\models \Sigma \cup \{F : A @ Rs\}} \quad \text{SIG\_CONSCONST}$$

$$\frac{\begin{array}{l} \models \Sigma \\ F \notin \mathsf{dom}\,\Sigma \\ \varnothing \models A : \star \\ \Omega; \Gamma \models p :_F B \Rightarrow A \\ \Gamma \models a : B \\ \Omega \models a : R \end{array}}{\models \Sigma \cup \{F :\ p \sim a : A/R @ \mathbf{range}\,\Omega\}} \quad \text{SIG\_CONSAX}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A/R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

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Definition rules:        159 good    0 bad
Definition rule clauses: 445 good    0 bad
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