

$tnvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T, F$	
$index, i$	indices

$relflag, \rho$	$::=$ $ $ $+$ $ $ $-$ $ $ app_rho ν S $ $ (ρ) S	relevance flag
$appflag, \nu$	$::=$ $ $ R $ $ ρ	applicative flag
$role, R$	$::=$ $ $ Nom $ $ Rep $ $ $R_1 \cap R_2$ S $ $ param $R_1 R_2$ S $ $ app_role ν S $ $ (R) S	Role
$constraint, \phi$	$::=$ $ $ $a \sim_{A/R} b$ $ $ (ϕ) S $ $ $\phi\{b/x\}$ S $ $ $ \phi $ S $ $ $a \sim_R b$ S	props
$tm, a, b, p, v, w, A, B, C$	$::=$ $ $ \star $ $ x $ $ $\lambda^\rho x:A.b$ bind x in b $ $ $\lambda^\rho x.b$ bind x in b $ $ $a \ b^\nu$ $ $ $\Pi^\rho x:A \rightarrow B$ bind x in B $ $ $\Lambda c:\phi.b$ bind c in b $ $ $\Lambda c.b$ bind c in b $ $ $a[\gamma]$ $ $ $\forall c:\phi.B$ bind c in B $ $ $a \triangleright_R \gamma$ $ $ F $ $ \square $ $ case $_R a$ of $F \rightarrow b_1 \parallel - \rightarrow b_2$ $ $ K $ $ match a with brs $ $ sub $R a$ $ $ $a\{b/x\}$ S $ $ $a\{\gamma/c\}$ S $ $ $a\{b/x\}$ S $ $ $a\{\gamma/c\}$ S	types and kinds

		a	S	
		a	S	
		(a)	S	
		a	S	parsing precedence is hard
		$ a _R$	S	
		Int	S	
		Bool	S	
		Nat	S	
		Vec	S	
		0	S	
		S	S	
		True	S	
		Fix	S	
		Age	S	
		$a \rightarrow b$	S	
		$\phi \Rightarrow A$	S	
		$a \ b$	S	
		$\lambda x. a$	S	
		$\lambda x : A. a$	S	
		$\forall x : A \rightarrow B$	S	
		if ϕ then a else b	S	
brs	$::=$			case branches
		none		
		$K \Rightarrow a; brs$		
		$brs\{a/x\}$	S	
		$brs\{\gamma/c\}$	S	
		(brs)	S	
co, γ	$::=$			explicit coercions
		•		
		c		
		red $a \ b$		
		refl a		
		$(a \models_{\gamma} b)$		
		sym γ		
		$\gamma_1; \gamma_2$		
		sub γ		
		$\Pi^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\lambda^{R,\rho} x : \gamma_1. \gamma_2$	bind x in γ_2	
		$\gamma_1 \ \gamma_2^{R,\rho}$		
		piFst γ		
		cpiFst γ		
		isoSnd γ		
		$\gamma_1 @ \gamma_2$		
		$\forall c : \gamma_1. \gamma_3$	bind c in γ_3	

		\emptyset	
		$\Sigma \cup \{F : sig_sort\}$	
		Σ_0	M
		Σ_1	M
		$ \Sigma $	M
$available_props, \Delta$::=		
		\emptyset	
		Δ, x	
		Δ, c	
		fva	M
		Δ, Δ'	M
		$\tilde{\Gamma}$	M
		$\tilde{\Omega}$	M
		(Δ)	M
Nat, \mathbb{N}	::=		
		$ a $	S
$pattern_arg$::=		Pattern arguments
		$\backslash Rel a R$	
		$\backslash Irr a$	
		$\backslash Coe \gamma$	
$pattern_args, PA$::=		
		none	
		$PA, pattern_arg$	
$terminals$::=		
		\leftrightarrow	
		\Leftrightarrow	
		\longrightarrow	
		min	
		\equiv	
		\forall	
		\in	
		\notin	
		\Leftarrow	
		\Rightarrow	
		\Rightarrow^*	
		\rightarrow	
		Λ	
		\square	
		\vdash	
		\dashv	
		\models	

	\models
	\neq
	\triangleright
	ok
	-
	\rightsquigarrow
	\rightsquigarrow^*
	\rightsquigarrow
	\emptyset
	\circ
	fv
	dom
	\sim
	\succ
	$ $
	\bullet
	fst
	snd
	as
	$ \Rightarrow $
	$\vdash_{=}$
	refl₂
	$++$
	{
	}
<i>formula, ψ</i>	$::=$
	<i>judgement</i>
	$x : A \in \Gamma$
	$x : R \in \Omega$
	$c : \phi \in \Gamma$
	$F : sig_sort \in \Sigma$
	$x \in \Delta$
	$c \in \Delta$
	c not relevant $\in \gamma$
	$x \notin \Delta$
	<i>uniq</i> Γ
	<i>uniq</i> (Ω)
	$c \notin \Delta$
	$T \notin \text{dom } \Sigma$
	$F \notin \text{dom } \Sigma$
	$\mathbb{N}_1 < \mathbb{N}_2$
	$\mathbb{N}_1 \leq \mathbb{N}_2$
	$\nu = \rho$
	$R_1 = R_2$
	$a = b$

	$ \begin{array}{l} \quad \phi_1 = \phi_2 \\ \quad \Gamma_1 = \Gamma_2 \\ \quad \gamma_1 = \gamma_2 \\ \quad \neg\psi \\ \quad \psi_1 \wedge \psi_2 \\ \quad \psi_1 \vee \psi_2 \\ \quad \psi_1 \Rightarrow \psi_2 \\ \quad (\psi) \\ \quad \psi \\ \quad c : (a : A \sim b : B) \in \Gamma \\ \end{array} $	suppress lc hypothesis gen
$JSubRole$	$ \begin{array}{l} ::= \\ \quad R_1 \leq R_2 \end{array} $	Subroling judgement
$JPath$	$ \begin{array}{l} ::= \\ \quad \text{Path } a = F@Rs \end{array} $	Type headed by constant
$JPatCtx$	$ \begin{array}{l} ::= \\ \quad \Omega; \Gamma \models p :_F B \Rightarrow A \end{array} $	Contexts generated by a p
$JRename$	$ \begin{array}{l} ::= \\ \quad \text{rename } p \rightarrow a \text{ to } p' \rightarrow a' \text{ excluding } \Delta \text{ and } \Delta' \end{array} $	rename with fresh variable
$JMatchSubst$	$ \begin{array}{l} ::= \\ \quad \text{match } a_1 \text{ with } p \rightarrow b_1 = b_2 \end{array} $	match and substitute
$JPatData$	$ \begin{array}{l} ::= \\ \quad \mathbf{uncurry } p = F@PA \end{array} $	Pattern data (head argum
$JIsPattern$	$ \begin{array}{l} ::= \\ \quad \mathbf{pattern } p \end{array} $	
$JSubPat$	$ \begin{array}{l} ::= \\ \quad \mathbf{subpattern } p' p \end{array} $	Subpattern
$JTmPatternAgree$	$ \begin{array}{l} ::= \\ \quad a \leftrightarrow p \end{array} $	term and pattern agree
$JTmSubPatternAgree$	$ \begin{array}{l} ::= \\ \quad a \sqsubseteq p \end{array} $	sub-pattern agrees with te
$JSubTmPatternAgree$	$ \begin{array}{l} ::= \\ \quad a \sqsupseteq p \end{array} $	sub-term agrees with patt
$JValuePath$	$ \begin{array}{l} ::= \\ \quad \text{ValuePath } a = F \end{array} $	Type headed by constant

$JCasePath$	$::=$ $ \quad \text{CasePath}_R \ a = F$	Type headed by constant (role-sensitive part)
$JApplyArgs$	$::=$ $ \quad \text{apply args } a \text{ to } b \mapsto b'$	apply arguments of a (headed by a constant)
$JValue$	$::=$ $ \quad \text{Value}_R \ A$	values
$JValueType$	$::=$ $ \quad \text{ValueType}_R \ A$	Types with head forms (erased language)
$Jconsistent$	$::=$ $ \quad \text{consistent}_R \ a \ b$	(erased) types do not differ in their heads
$Jroleing$	$::=$ $ \quad \Omega \models a : R$	Roleing judgment
$JChk$	$::=$ $ \quad (\rho = +) \vee (x \notin \text{fv } A)$	irrelevant argument check
$Jpar$	$::=$ $ \quad \Omega \models a \Rightarrow_R b$ $ \quad \Omega \models a \Rightarrow_R^* b$ $ \quad \Omega \models a \Leftrightarrow_R b$	parallel reduction (implicit language) multistep parallel reduction parallel reduction to a common term
$Jbeta$	$::=$ $ \quad \models a > b/R$ $ \quad \models a \rightsquigarrow b/R$ $ \quad \models a \rightsquigarrow^* b/R$	primitive reductions on erased terms single-step head reduction for implicit language multistep reduction
$JBranchTyping$	$::=$ $ \quad \Gamma \models \text{case}_R \ a : A \text{ of } b : B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
$Jett$	$::=$ $ \quad \Gamma \models \phi \text{ ok}$ $ \quad \Gamma \models a : A$ $ \quad \Gamma; \Delta \models \phi_1 \equiv \phi_2$ $ \quad \Gamma; \Delta \models a \equiv b : A/R$ $ \quad \models \Gamma$	Prop wellformedness typing prop equality definitional equality context wellformedness
$Jsig$	$::=$ $ \quad \models \Sigma$	signature wellformedness
$Jann$	$::=$ $ \quad \Gamma \vdash \phi \text{ ok}$ $ \quad \Gamma \vdash a : A/R$	prop wellformedness typing

	$\begin{array}{l} \quad \Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2 \\ \quad \Gamma; \Delta \vdash \gamma : A \sim_R B \\ \quad \vdash \Gamma \end{array}$	coercion between props coercion between types context wellformedness
<i>Jred</i>	$\begin{array}{l} ::= \\ \quad \Gamma \vdash a \rightsquigarrow b/R \end{array}$	single-step, weak head reduction to values for annotated la
<i>judgement</i>	$\begin{array}{l} ::= \\ \quad JSubRole \\ \quad JPath \\ \quad JPatCtx \\ \quad JRename \\ \quad JMatchSubst \\ \quad JPatData \\ \quad JIsPattern \\ \quad JSubPat \\ \quad JTmPatternAgree \\ \quad JTmSubPatternAgree \\ \quad JSubTmPatternAgree \\ \quad JValuePath \\ \quad JCasePath \\ \quad JApplyArgs \\ \quad JValue \\ \quad JValueType \\ \quad Jconsistent \\ \quad Jroleing \\ \quad JChk \\ \quad Jpar \\ \quad Jbeta \\ \quad JBranchTyping \\ \quad Jett \\ \quad Jsig \\ \quad Jann \\ \quad Jred \end{array}$	
<i>user_syntax</i>	$\begin{array}{l} ::= \\ \quad tmvar \\ \quad covar \\ \quad datacon \\ \quad const \\ \quad index \\ \quad relflag \\ \quad appflag \\ \quad role \\ \quad constraint \\ \quad tm \\ \quad brs \end{array}$	

co
 $role_context$
 $roles$
 sig_sort
 $sort$
 $context$
 sig
 $available_props$
 Nat
 $pattern_arg$
 $pattern_args$
 $terminals$
 $formula$

$R_1 \leq R_2$ Subrolling judgement

$$\begin{array}{c}
\overline{\mathbf{Nom} \leq R} \quad \text{NOMBOT} \\
\overline{R \leq \mathbf{Rep}} \quad \text{REPTOP} \\
\overline{R \leq R} \quad \text{REFL} \\
\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{TRANS}
\end{array}$$

$\text{Path } a = F@Rs$ Type headed by constant (partial function)

$$\begin{array}{c}
\frac{F : A@Rs \in \Sigma_0}{\text{Path } F = F@Rs} \quad \text{PATH_ABSCONST} \\
\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0}{\text{Path } F = F@Rs} \quad \text{PATH_CONST} \\
\frac{\text{Path } a = F@R_1, Rs}{\text{Path } (a \ b'^{R_1}) = F@Rs} \quad \text{PATH_APP} \\
\frac{\text{Path } a = F@Rs}{\text{Path } (a \ \Box^-) = F@Rs} \quad \text{PATH_IAPP} \\
\frac{\text{Path } a = F@Rs}{\text{Path } (a[\bullet]) = F@Rs} \quad \text{PATH_CAPP}
\end{array}$$

$\Omega; \Gamma \models p :_F B \Rightarrow A$ Contexts generated by a pattern (variables bound by the pattern)

$$\begin{array}{c}
\overline{\emptyset; \emptyset \models F :_F A \Rightarrow A} \quad \text{PATCTX_CONST} \\
\frac{\Omega; \Gamma \models p :_F \Pi^+ x : A' \rightarrow A \Rightarrow B}{\Omega, x : R; \Gamma, x : A' \models p \ x^R :_F A \Rightarrow B} \quad \text{PATCTX_PIREL} \\
\frac{\Omega; \Gamma \models p :_F \Pi^- x : A' \rightarrow A \Rightarrow B}{\Omega; \Gamma, x : A' \models p \ \Box^- :_F A \Rightarrow B} \quad \text{PATCTX_PIIRR} \\
\frac{\Omega; \Gamma \models p :_F \forall c : \phi. A \Rightarrow B}{\Omega; \Gamma, c : \phi \models p[\bullet] :_F A \Rightarrow B} \quad \text{PATCTX_CPI}
\end{array}$$

$\boxed{\text{rename } p \rightarrow a \text{ to } p' \rightarrow a' \text{ excluding } \Delta \text{ and } \Delta'}$ rename with fresh variables

$\frac{}{\text{rename } F \rightarrow a \text{ to } F \rightarrow a \text{ excluding } \Delta \text{ and } \emptyset}$ RENAME_BASE

$\frac{\text{rename } p_1 \rightarrow a_1 \text{ to } p_2 \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta' \quad y \notin (\Delta, \Delta')}{}$

$\frac{}{\text{rename } (p_1 \ x^R) \rightarrow a_1 \text{ to } (p_2 \ y^R) \rightarrow (a_2\{y/x\}) \text{ excluding } \Delta \text{ and } (\Delta', y)}$ RENAME_APPREL

$\frac{\text{rename } p_1 \rightarrow a_1 \text{ to } p_2 \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta'}{\text{rename } (p_1 \ \Box^-) \rightarrow a_1 \text{ to } (p_2 \ \Box^-) \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta'}$ RENAME_APPIRREL

$\frac{\text{rename } p_1 \rightarrow a_1 \text{ to } p_2 \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta'}{\text{rename } (p_1[\bullet]) \rightarrow a_1 \text{ to } (p_2[\bullet]) \rightarrow a_2 \text{ excluding } \Delta \text{ and } \Delta'}$ RENAME_CAPP

$\boxed{\text{match } a_1 \text{ with } p \rightarrow b_1 = b_2}$ match and substitute

$\frac{}{\text{match } F \text{ with } F \rightarrow b = b}$ MATCHSUBST_CONST

$\frac{\text{match } a_1 \text{ with } p_1 \rightarrow b_1 = b_2}{\text{match } (a_1 \ a^R) \text{ with } (p_1 \ x^R) \rightarrow b_1 = (b_2\{a/x\})}$ MATCHSUBST_APPREL

$\frac{\text{match } a_1 \text{ with } p_1 \rightarrow b_1 = b_2}{\text{match } (a_1 \ a^-) \text{ with } (p_1 \ \Box^-) \rightarrow b_1 = b_2}$ MATCHSUBST_APPIRREL

$\frac{\text{match } a_1 \text{ with } a_2 \rightarrow b_1 = b_2}{\text{match } (a_1[\bullet]) \text{ with } (a_2[\bullet]) \rightarrow b_1 = b_2}$ MATCHSUBST_CAPP

$\boxed{\text{uncurry } p = F@PA}$ Pattern data (head arguments)

$\frac{}{\text{uncurry } F = F@\text{none}}$ PATDATA_HEAD

$\frac{\text{uncurry } p = F@PA}{\text{uncurry } (p \ a^R) = F@PA, \backslash Rel \ a \ R}$ PATDATA_REL

$\boxed{\text{pattern } p}$

$\frac{}{\text{pattern } F}$ PATTERN_HEAD

$\frac{\text{pattern } p}{\text{pattern } (p \ a^R)}$ PATTERN_REL

$\frac{\text{pattern } p}{\text{pattern } (p \ a^-)}$ PATTERN_IRR

$\frac{\text{pattern } p}{\text{pattern } (p[\gamma])}$ PATTERN_COE

$\boxed{\text{subpattern } p' \ p}$ Subpattern

$\frac{\text{pattern } p}{\text{subpattern } p \ p}$ SUBPAT_REFL

$\frac{\text{subpattern } p' \ p}{\text{subpattern } p' \ (p \ x^R)}$ SUBPAT_REL

$\frac{\text{subpattern } p' \ p}{\text{subpattern } p' \ (p \ \Box^-)}$ SUBPAT_IRR

$$\frac{\text{subpattern } p' p}{\text{subpattern } p' (p[\bullet])} \quad \text{SUBPAT_COE}$$

$a \leftrightarrow p$ term and pattern agree

$$\frac{}{F \leftrightarrow F} \quad \text{TM_PATTERN_AGREE_CONST}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1 \ a_2^R) \leftrightarrow (p_1 \ x^R)} \quad \text{TM_PATTERN_AGREE_APPREL R}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1 \ a^-) \leftrightarrow (p_1 \ \Box^-)} \quad \text{TM_PATTERN_AGREE_APP IrREL}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1[\bullet]) \leftrightarrow (p_1[\bullet])} \quad \text{TM_PATTERN_AGREE_CAPP}$$

$a \sqsubseteq p$ sub-pattern agrees with term

$$\frac{a \leftrightarrow p}{a \sqsubseteq p} \quad \text{TM_SUBPATTERN_AGREE_BASE}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p \ x^R)} \quad \text{TM_SUBPATTERN_AGREE_APPREL R}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p \ \Box^-)} \quad \text{TM_SUBPATTERN_AGREE_APP IrREL}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p[\bullet])} \quad \text{TM_SUBPATTERN_AGREE_CAPP}$$

$a \sqsupseteq p$ sub-term agrees with pattern

$$\frac{a \leftrightarrow p}{a \sqsupseteq p} \quad \text{SUBTM_PATTERN_AGREE_BASE}$$

$$\frac{a \sqsupseteq p}{a \ a_2^\nu \sqsupseteq p} \quad \text{SUBTM_PATTERN_AGREE_APP}$$

$$\frac{a \sqsupseteq p}{a[\bullet] \sqsupseteq p} \quad \text{SUBTM_PATTERN_AGREE_CAPP}$$

$\text{ValuePath } a = F$ Type headed by constant (role-sensitive partial function used in value)

$$\frac{F : A @ Rs \in \Sigma_0}{\text{ValuePath } F = F} \quad \text{VALUEPATH_ABSCONST}$$

$$\frac{F : p \sim a : A / R_1 @ Rs \in \Sigma_0}{\text{ValuePath } F = F} \quad \text{VALUEPATH_CONST}$$

$$\frac{\text{ValuePath } a = F}{\text{ValuePath } (a \ b^\nu) = F} \quad \text{VALUEPATH_APP}$$

$$\frac{\text{ValuePath } a = F}{\text{ValuePath } (a[\bullet]) = F} \quad \text{VALUEPATH_CAPP}$$

$\text{CasePath}_R a = F$ Type headed by constant (role-sensitive partial function used in case)

$$\frac{\text{ValuePath } a = F \quad F : A @ Rs \in \Sigma_0}{\text{CasePath}_R a = F} \quad \text{CASEPATH_ABSCONST}$$

$$\frac{\text{ValuePath } a = F \quad F : p \sim b : A/R_1 @ Rs \in \Sigma_0 \quad \neg(R_1 \leq R)}{\text{CasePath}_R a = F} \quad \text{CASEPATH_CONST}$$

$$\frac{\text{ValuePath } a = F \quad F : p \sim b : A/R_1 @ Rs \in \Sigma_0 \quad \neg(a \sqsupseteq p)}{\text{CasePath}_R a = F} \quad \text{CASEPATH_UNMATCH}$$

$\boxed{\text{apply args } a \text{ to } b \mapsto b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\text{apply args } F \text{ to } b \mapsto b} \quad \text{APPLYARGS_CONST}$$

$$\frac{\text{apply args } a \text{ to } b \mapsto b'}{\text{apply args } a \ a''^\nu \text{ to } b \mapsto b' \ a''^\nu} \quad \text{APPLYARGS_APP}$$

$$\frac{\text{apply args } a \text{ to } b \mapsto b'}{\text{apply args } a[\bullet] \text{ to } b \mapsto b'[\bullet]} \quad \text{APPLYARGS_CAPP}$$

$\boxed{\text{Value}_R A}$ values

$$\frac{}{\text{Value}_R \star} \quad \text{VALUE_STAR}$$

$$\frac{}{\text{Value}_R \Pi^\rho x : A \rightarrow B} \quad \text{VALUE_PI}$$

$$\frac{}{\text{Value}_R \forall c : \phi. B} \quad \text{VALUE_CPI}$$

$$\frac{}{\text{Value}_R \lambda^+ x : A. a} \quad \text{VALUE_ABSR}$$

$$\frac{}{\text{Value}_R \lambda^+ x. a} \quad \text{VALUE_UABSR}$$

$$\frac{\text{Value}_R a}{\text{Value}_R \lambda^- x. a} \quad \text{VALUE_UABSI}$$

$$\frac{}{\text{Value}_R \Lambda c : \phi. a} \quad \text{VALUE_CABS}$$

$$\frac{}{\text{Value}_R \Lambda c. a} \quad \text{VALUE_UCABS}$$

$$\frac{\text{CasePath}_R a = F}{\text{Value}_R a} \quad \text{VALUE_PATH}$$

$\boxed{\text{ValueType}_R A}$ Types with head forms (erased language)

$$\frac{}{\text{ValueType}_R \star} \quad \text{VALUE_TYPE_STAR}$$

$$\frac{}{\text{ValueType}_R \Pi^\rho x : A \rightarrow B} \quad \text{VALUE_TYPE_PI}$$

$$\frac{}{\text{ValueType}_R \forall c : \phi. B} \quad \text{VALUE_TYPE_CPI}$$

$$\frac{\text{CasePath}_R a = F}{\text{ValueType}_R a} \quad \text{VALUE_TYPE_VALUEPATH}$$

$\boxed{\text{consistent}_R a \ b}$ (erased) types do not differ in their heads

$$\begin{array}{c}
\frac{}{\text{consistent}_R \star \star} \text{CONSISTENT_A_STAR} \\
\\
\frac{}{\text{consistent}_{R'} (\Pi^\rho x_1 : A_1 \rightarrow B_1) (\Pi^\rho x_2 : A_2 \rightarrow B_2)} \text{CONSISTENT_A_PI} \\
\\
\frac{}{\text{consistent}_R (\forall c_1 : \phi_1. A_1) (\forall c_2 : \phi_2. A_2)} \text{CONSISTENT_A_CPI} \\
\\
\frac{\text{CasePath}_R a_1 = F \quad \text{CasePath}_R a_2 = F}{\text{consistent}_R a_1 a_2} \text{CONSISTENT_A_CASEPATH} \\
\\
\frac{\neg \text{ValueType}_R b}{\text{consistent}_R a b} \text{CONSISTENT_A_STEP_R} \\
\\
\frac{\neg \text{ValueType}_R a}{\text{consistent}_R a b} \text{CONSISTENT_A_STEP_L}
\end{array}$$

$\boxed{\Omega \models a : R}$ Roleing judgment

$$\begin{array}{c}
\frac{\text{uniq}(\Omega)}{\Omega \models \square : R} \text{ROLE_A_BULLET} \\
\\
\frac{\text{uniq}(\Omega)}{\Omega \models \star : R} \text{ROLE_A_STAR} \\
\\
\frac{\text{uniq}(\Omega) \quad x : R \in \Omega \quad R \leq R_1}{\Omega \models x : R_1} \text{ROLE_A_VAR} \\
\\
\frac{\Omega, x : \mathbf{Nom} \models a : R}{\Omega \models (\lambda^\rho x. a) : R} \text{ROLE_A_ABS} \\
\\
\frac{\Omega \models a : R \quad \Omega \models b : \mathbf{Nom}}{\Omega \models (a \ b^\rho) : R} \text{ROLE_A_APP} \\
\\
\frac{\Omega \models a : R \quad \Omega \models b : R_1}{\Omega \models a \ b^{R_1} : R} \text{ROLE_A_TAPP} \\
\\
\frac{\Omega \models A : R \quad \Omega, x : \mathbf{Nom} \models B : R}{\Omega \models (\Pi^\rho x : A \rightarrow B) : R} \text{ROLE_A_PI} \\
\\
\frac{\Omega \models a : R_1 \quad \Omega \models b : R_1 \quad \Omega \models A : R_0 \quad \Omega \models B : R}{\Omega \models (\forall c : a \sim_{A/R_1} b. B) : R} \text{ROLE_A_CPI} \\
\\
\frac{\Omega \models b : R}{\Omega \models (\Lambda c. b) : R} \text{ROLE_A_CABS} \\
\\
\frac{\Omega \models a : R}{\Omega \models (a[\bullet]) : R} \text{ROLE_A_CAPP}
\end{array}$$

$$\frac{\text{uniq}(\Omega) \quad F : A @ R_s \in \Sigma_0}{\Omega \models F : R} \quad \text{ROLE_A_CONST}$$

$$\frac{\text{uniq}(\Omega) \quad F : p \sim a : A / R @ R_s \in \Sigma_0}{\Omega \models F : R_1} \quad \text{ROLE_A_FAM}$$

$$\frac{\Omega \models a : R \quad \Omega \models b_1 : R_1 \quad \Omega \models b_2 : R_1}{\Omega \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2 : R_1} \quad \text{ROLE_A_PATTERN}$$

$$\boxed{(\rho = +) \vee (x \notin \text{fv } A)} \quad \text{irrelevant argument check}$$

$$\overline{(+ = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_REL}$$

$$\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)} \quad \text{RHO_IRRREL}$$

$$\boxed{\Omega \models a \Rightarrow_R b} \quad \text{parallel reduction (implicit language)}$$

$$\frac{\Omega \models a : R}{\Omega \models a \Rightarrow_R a} \quad \text{PAR_REFL}$$

$$\frac{\Omega \models a \Rightarrow_R (\lambda^\rho x. a') \quad \Omega \models b \Rightarrow_{\mathbf{Nom}} b'}{\Omega \models a \ b^\rho \Rightarrow_R a' \{b'/x\}} \quad \text{PAR_BETA}$$

$$\frac{\Omega \models a \Rightarrow_R a' \quad \Omega \models b \Rightarrow_{(\mathbf{app.role} \ \nu)} b'}{\Omega \models a \ b^\nu \Rightarrow_R a' \ b'^\nu} \quad \text{PAR_APP}$$

$$\frac{\Omega \models a \Rightarrow_R (\Lambda c. a')}{\Omega \models a[\bullet] \Rightarrow_R a' \{\bullet/c\}} \quad \text{PAR_CBETA}$$

$$\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR_CAPP}$$

$$\frac{\Omega, x : \mathbf{Nom} \models a \Rightarrow_R a'}{\Omega \models \lambda^\rho x. a \Rightarrow_R \lambda^\rho x. a'} \quad \text{PAR_ABS}$$

$$\frac{\Omega \models A \Rightarrow_R A' \quad \Omega, x : \mathbf{Nom} \models B \Rightarrow_R B'}{\Omega \models \Pi^\rho x : A \rightarrow B \Rightarrow_R \Pi^\rho x : A' \rightarrow B'} \quad \text{PAR_PI}$$

$$\frac{\Omega \models a \Rightarrow_R a'}{\Omega \models \Lambda c. a \Rightarrow_R \Lambda c. a'} \quad \text{PAR_CABS}$$

$$\frac{\Omega \models A \Rightarrow_{R_0} A' \quad \Omega \models a \Rightarrow_{R_1} a' \quad \Omega \models b \Rightarrow_{R_1} b' \quad \Omega \models B \Rightarrow_R B'}{\Omega \models \forall c : a \sim_{A/R_1} b. B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'. B'} \quad \text{PAR_CPI}$$

$$\begin{array}{c}
\frac{F : F \sim b : A/R_1 @ R_s \in \Sigma_0 \quad R_1 \leq R \quad \text{uniq}(\Omega)}{\Omega \models F \Rightarrow_R b} \quad \text{PAR_AXIOMBASE} \\
\\
\frac{
\begin{array}{l}
F : p \sim b : A/R_1 @ R_s \in \Sigma_0 \\
a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\
\Omega \models a \Rightarrow_R a' \\
\Omega \models a_1 \Rightarrow_{(\text{app.role } \nu)} a'_1 \\
\text{rename } p \rightarrow b \text{ to } p' \rightarrow b' \text{ excluding } (\tilde{\Omega}, \text{fv} p) \text{ and } \Delta' \\
\text{match } (a' \ a'_1{}^\nu) \text{ with } p' \rightarrow b' = a_2 \\
R_1 \leq R
\end{array}
}{\Omega \models a \ a_1{}^\nu \Rightarrow_R a_2} \quad \text{PAR_AXIOMAPP} \\
\\
\frac{
\begin{array}{l}
F : p \sim b : A/R_1 @ R_s \in \Sigma_0 \\
a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\
\Omega \models a \Rightarrow_R a' \\
\text{rename } p \rightarrow b \text{ to } p' \rightarrow b' \text{ excluding } (\tilde{\Omega}, \text{fv} p) \text{ and } \Delta' \\
\text{match } (a'[\bullet]) \text{ with } p' \rightarrow b' = a_2 \\
R_1 \leq R
\end{array}
}{\Omega \models a[\bullet] \Rightarrow_R a_2} \quad \text{PAR_AXIOMCAPP} \\
\\
\frac{
\begin{array}{l}
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2
\end{array}
}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} (\text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel - \rightarrow b'_2)} \quad \text{PAR_PATTERN} \\
\\
\frac{
\begin{array}{l}
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2 \\
\text{CasePath}_R a' = F \\
\text{apply args } a' \text{ to } b'_1 \mapsto b
\end{array}
}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} b[\bullet]} \quad \text{PAR_PATTERNTRUE} \\
\\
\frac{
\begin{array}{l}
\Omega \models a \Rightarrow_R a' \\
\Omega \models b_1 \Rightarrow_{R_0} b'_1 \\
\Omega \models b_2 \Rightarrow_{R_0} b'_2 \\
\text{Value}_R a' \\
\neg(\text{CasePath}_R a' = F)
\end{array}
}{\Omega \models (\text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2) \Rightarrow_{R_0} b'_2} \quad \text{PAR_PATTERNFALSE} \\
\\
\boxed{\Omega \models a \Rightarrow_R^* b} \quad \text{multistep parallel reduction} \\
\\
\frac{}{\Omega \models a \Rightarrow_R^* a} \quad \text{MP_REFL} \\
\\
\frac{
\begin{array}{l}
\Omega \models a \Rightarrow_R b \\
\Omega \models b \Rightarrow_R^* a'
\end{array}
}{\Omega \models a \Rightarrow_R^* a'} \quad \text{MP_STEP} \\
\\
\boxed{\Omega \models a \Leftrightarrow_R b} \quad \text{parallel reduction to a common term} \\
\\
\frac{
\begin{array}{l}
\Omega \models a_1 \Rightarrow_R^* b \\
\Omega \models a_2 \Rightarrow_R^* b
\end{array}
}{\Omega \models a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}
\end{array}$$

$\boxed{\models a > b/R}$ primitive reductions on erased terms

$$\frac{\text{Value}_{R_1} (\lambda^\rho x.v)}{\models (\lambda^\rho x.v) \ b^\rho > v\{b/x\}/R_1} \quad \text{BETA_APPABS}$$

$$\frac{}{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{BETA_CAPPCABS}$$

$F : p \sim b : A/R_1 @ R_s \in \Sigma_0$

rename $p \rightarrow b$ to $p_1 \rightarrow b_1$ excluding (fva, fvp) and Δ'

match a with $p_1 \rightarrow b_1 = b'$

$R_1 \leq R$

$$\frac{}{\models a > b'/R} \quad \text{BETA_AXIOM}$$

$$\frac{\begin{array}{l} \text{CasePath}_R \ a = F \\ \text{apply args } a \text{ to } b_1 \mapsto b'_1 \end{array}}{\models \text{case}_R \ a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2 > b'_1[\bullet]/R_0} \quad \text{BETA_PATTERNTRUE}$$

$$\frac{\begin{array}{l} \text{Value}_R \ a \\ \neg(\text{CasePath}_R \ a = F) \end{array}}{\models \text{case}_R \ a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2 > b_2/R_0} \quad \text{BETA_PATTERNFALSE}$$

$\boxed{\models a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\models a \rightsquigarrow a'/R_1}{\models \lambda^- x.a \rightsquigarrow \lambda^- x.a'/R_1} \quad \text{E_ABSTERM}$$

$$\frac{\models a \rightsquigarrow a'/R_1}{\models a \ b^\nu \rightsquigarrow a' \ b^\nu/R_1} \quad \text{E_APPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E_CAPPLEFT}$$

$$\frac{\models a \rightsquigarrow a'/R}{\models \text{case}_R \ a \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2 \rightsquigarrow \text{case}_R \ a' \text{ of } F \rightarrow b_1 \parallel_- \rightarrow b_2/R_0} \quad \text{E_PATTERN}$$

$$\frac{\models a > b/R}{\models a \rightsquigarrow b/R} \quad \text{E_PRIM}$$

$\boxed{\models a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\models a \rightsquigarrow^* a/R} \quad \text{EQUAL}$$

$$\frac{\begin{array}{l} \models a \rightsquigarrow b/R \\ \models b \rightsquigarrow^* a'/R \end{array}}{\models a \rightsquigarrow^* a'/R} \quad \text{STEP}$$

$\boxed{\Gamma \models \text{case}_R \ a : A \text{ of } b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{l} \text{uniq } \Gamma \\ \text{lc_tm } C \end{array}}{\Gamma \models \text{case}_R \ a : A \text{ of } b : A \Rightarrow \forall c : (a \sim_{A/R} b).C \mid C} \quad \text{BRANCHTYPING_BASE}$$

$$\frac{\Gamma, x : A \models \text{case}_R \ a : A_1 \text{ of } b \ x^+ : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R \ a : A_1 \text{ of } b : \Pi^+ x : A \rightarrow B \Rightarrow \Pi^+ x : A \rightarrow C \mid C'} \quad \text{BRANCHTYPING_PIREL}$$

$$\frac{\Gamma, x : A \models \text{case}_R \ a : A_1 \text{ of } b \ \square^- : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R \ a : A_1 \text{ of } b : \Pi^- x : A \rightarrow B \Rightarrow \Pi^- x : A \rightarrow C \mid C'} \quad \text{BRANCHTYPING_PIRREL}$$

$$\frac{\Gamma, c : \phi \models \text{case}_R a : A \text{ of } b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \models \text{case}_R a : A \text{ of } b : \forall c : \phi. B \Rightarrow \forall c : \phi. C \mid C'} \quad \text{BRANCHTypING_CPI}$$

$$\boxed{\Gamma \models \phi \text{ ok}} \quad \text{Prop wellformedness}$$

$$\frac{\begin{array}{c} \Gamma \models a : A \\ \Gamma \models b : A \\ \Gamma \models A : \star \end{array}}{\Gamma \models a \sim_{A/R} b \text{ ok}} \quad \text{E_WFF}$$

$$\boxed{\Gamma \models a : A} \quad \text{typing}$$

$$\begin{array}{c} \frac{\vdash \Gamma}{\Gamma \vdash \star : \star} \quad \text{E_STAR} \\ \\ \frac{\begin{array}{c} \vdash \Gamma \\ x : A \in \Gamma \end{array}}{\Gamma \vdash x : A} \quad \text{E_VAR} \\ \\ \frac{\begin{array}{c} \Gamma, x : A \vdash B : \star \\ \Gamma \vdash A : \star \end{array}}{\Gamma \vdash \Pi^\rho x : A \rightarrow B : \star} \quad \text{E_PI} \\ \\ \frac{\begin{array}{c} \Gamma, x : A \vdash a : B \\ \Gamma \vdash A : \star \\ (\rho = +) \vee (x \notin \text{fv } a) \end{array}}{\Gamma \vdash \lambda^\rho x. a : (\Pi^\rho x : A \rightarrow B)} \quad \text{E_ABS} \\ \\ \frac{\begin{array}{c} \Gamma \vdash b : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash a : A \end{array}}{\Gamma \vdash b \ a^+ : B\{a/x\}} \quad \text{E_APP} \\ \\ \frac{\begin{array}{c} \Gamma \vdash b : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash a : A \\ \text{Path } b = F @ R, Rs \end{array}}{\Gamma \vdash b \ a^R : B\{a/x\}} \quad \text{E_TAPP} \\ \\ \frac{\begin{array}{c} \Gamma \vdash b : \Pi^- x : A \rightarrow B \\ \Gamma \vdash a : A \end{array}}{\Gamma \vdash b \ \Box^- : B\{a/x\}} \quad \text{E_IAPP} \\ \\ \frac{\begin{array}{c} \Gamma \vdash a : A \\ \Gamma; \tilde{\Gamma} \vdash A \equiv B : \star / \mathbf{Rep} \\ \Gamma \vdash B : \star \end{array}}{\Gamma \vdash a : B} \quad \text{E_CONV} \\ \\ \frac{\begin{array}{c} \Gamma, c : \phi \vdash B : \star \\ \Gamma \vdash \phi \text{ ok} \end{array}}{\Gamma \vdash \forall c : \phi. B : \star} \quad \text{E_CPI} \\ \\ \frac{\begin{array}{c} \Gamma, c : \phi \vdash a : B \\ \Gamma \vdash \phi \text{ ok} \end{array}}{\Gamma \vdash \Lambda c. a : \forall c : \phi. B} \quad \text{E_CABS} \\ \\ \frac{\begin{array}{c} \Gamma \vdash a_1 : \forall c : (a \sim_{A/R} b). B_1 \\ \Gamma; \tilde{\Gamma} \vdash a \equiv b : A/R \end{array}}{\Gamma \vdash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E_CAPP} \end{array}$$

$$\frac{\begin{array}{c} \models \Gamma \\ F : A @ R s \in \Sigma_0 \\ \emptyset \models A : \star \end{array}}{\Gamma \models F : A} \quad \text{E_CONST}$$

$$\frac{\begin{array}{c} \models \Gamma \\ F : p \sim a : A / R_1 @ R s \in \Sigma_0 \\ \emptyset \models A : \star \end{array}}{\Gamma \models F : A} \quad \text{E_FAM}$$

$$\frac{\begin{array}{c} \Gamma \models a : A \\ \Gamma \models F : A_1 \\ \Gamma \models b_1 : B \\ \Gamma \models b_2 : C \\ \Gamma \models \text{case}_R a : A \text{ of } F : A_1 \Rightarrow B \mid C \end{array}}{\Gamma \models \text{case}_R a \text{ of } F \rightarrow b_1 \parallel - \rightarrow b_2 : C} \quad \text{E_CASE}$$

$$\boxed{\Gamma; \Delta \models \phi_1 \equiv \phi_2} \quad \text{prop equality}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models A_1 \equiv A_2 : A / R \\ \Gamma; \Delta \models B_1 \equiv B_2 : A / R \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E_PROPCONG}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models A \equiv B : \star / R_0 \\ \Gamma \models A_1 \sim_{A/R} A_2 \text{ ok} \\ \Gamma \models A_1 \sim_{B/R} A_2 \text{ ok} \end{array}}{\Gamma; \Delta \models A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E_ISOCONV}$$

$$\frac{\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R_1} a_2). B_1 \equiv \forall c : (b_1 \sim_{B/R_2} b_2). B_2 : \star / R'}{\Gamma; \Delta \models a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E_CPIFST}$$

$$\boxed{\Gamma; \Delta \models a \equiv b : A / R} \quad \text{definitional equality}$$

$$\frac{\begin{array}{c} \models \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \models a \equiv b : A / R} \quad \text{E_ASSN}$$

$$\frac{\Gamma \models a : A}{\Gamma; \Delta \models a \equiv a : A / R} \quad \text{E_REFL}$$

$$\frac{\Gamma; \Delta \models b \equiv a : A / R}{\Gamma; \Delta \models a \equiv b : A / R} \quad \text{E_SYM}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models a \equiv a_1 : A / R \\ \Gamma; \Delta \models a_1 \equiv b : A / R \end{array}}{\Gamma; \Delta \models a \equiv b : A / R} \quad \text{E_TRANS}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \models a \equiv b : A / R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \models a \equiv b : A / R_2} \quad \text{E_SUB}$$

$$\frac{\begin{array}{c} \Gamma \models a_1 : B \\ \Gamma \models a_2 : B \\ \models a_1 > a_2 / R \end{array}}{\Gamma; \Delta \models a_1 \equiv a_2 : B / R} \quad \text{E_BETA}$$

$$\begin{array}{c}
\frac{
\begin{array}{l}
\Gamma; \Delta \models A_1 \equiv A_2 : \star / R' \\
\Gamma, x : A_1; \Delta \models B_1 \equiv B_2 : \star / R' \\
\Gamma \models A_1 : \star \\
\Gamma \models \Pi^\rho x : A_1 \rightarrow B_1 : \star \\
\Gamma \models \Pi^\rho x : A_2 \rightarrow B_2 : \star
\end{array}
}{\Gamma; \Delta \models (\Pi^\rho x : A_1 \rightarrow B_1) \equiv (\Pi^\rho x : A_2 \rightarrow B_2) : \star / R'} \quad \text{E_PiCONG}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma, x : A_1; \Delta \models b_1 \equiv b_2 : B / R' \\
\Gamma \models A_1 : \star \\
(\rho = +) \vee (x \notin \text{fv } b_1) \\
(\rho = +) \vee (x \notin \text{fv } b_2)
\end{array}
}{\Gamma; \Delta \models (\lambda^\rho x. b_1) \equiv (\lambda^\rho x. b_2) : (\Pi^\rho x : A_1 \rightarrow B) / R'} \quad \text{E_ABSCONG}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A \rightarrow B) / R' \\
\Gamma; \Delta \models a_2 \equiv b_2 : A / \mathbf{Nom}
\end{array}
}{\Gamma; \Delta \models a_1 \ a_2^+ \equiv b_1 \ b_2^+ : (B\{a_2/x\}) / R'} \quad \text{E_APPCONG}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^+ x : A \rightarrow B) / R' \\
\Gamma; \Delta \models a_2 \equiv b_2 : A / R \\
\text{Path } a_1 = F @ R, Rs \\
\text{Path } b_1 = F' @ R, Rs'
\end{array}
}{\Gamma; \Delta \models a_1 \ a_2^R \equiv b_1 \ b_2^R : (B\{a_2/x\}) / R'} \quad \text{E_TAPPCONG}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma; \Delta \models a_1 \equiv b_1 : (\Pi^- x : A \rightarrow B) / R' \\
\Gamma \models a : A
\end{array}
}{\Gamma; \Delta \models a_1 \ \Box^- \equiv b_1 \ \Box^- : (B\{a/x\}) / R'} \quad \text{E_IAPPCONG}
\\[10pt]
\frac{
\Gamma; \Delta \models \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star / R'
}{\Gamma; \Delta \models A_1 \equiv A_2 : \star / R'} \quad \text{E_PiFST}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma; \Delta \models \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star / R' \\
\Gamma; \Delta \models a_1 \equiv a_2 : A_1 / \mathbf{Nom}
\end{array}
}{\Gamma; \Delta \models B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star / R'} \quad \text{E_PiSND}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma; \Delta \models a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\
\Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \models A \equiv B : \star / R' \\
\Gamma \models a_1 \sim_{A_1/R} b_1 \text{ ok} \\
\Gamma \models \forall c : a_1 \sim_{A_1/R} b_1. A : \star \\
\Gamma \models \forall c : a_2 \sim_{A_2/R} b_2. B : \star
\end{array}
}{\Gamma; \Delta \models \forall c : a_1 \sim_{A_1/R} b_1. A \equiv \forall c : a_2 \sim_{A_2/R} b_2. B : \star / R'} \quad \text{E_CPiCONG}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma, c : \phi_1; \Delta \models a \equiv b : B / R \\
\Gamma \models \phi_1 \text{ ok}
\end{array}
}{\Gamma; \Delta \models (\Lambda c. a) \equiv (\Lambda c. b) : \forall c : \phi_1. B / R} \quad \text{E_CABSCONG}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma; \Delta \models a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b). B) / R' \\
\Gamma; \tilde{\Gamma} \models a \equiv b : A / \mathbf{param } R R'
\end{array}
}{\Gamma; \Delta \models a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\}) / R'} \quad \text{E_CAPPiCONG}
\\[10pt]
\frac{
\begin{array}{l}
\Gamma; \Delta \models \forall c : (a_1 \sim_{A/R} a_2). B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2). B_2 : \star / R_0 \\
\Gamma; \tilde{\Gamma} \models a_1 \equiv a_2 : A / \mathbf{param } R R_0 \\
\Gamma; \tilde{\Gamma} \models a'_1 \equiv a'_2 : A' / \mathbf{param } R' R_0
\end{array}
}{\Gamma; \Delta \models B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star / R_0} \quad \text{E_CPiSND}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Delta \vdash a \equiv b : A/R \quad \Gamma; \Delta \vdash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'}{\Gamma; \Delta \vdash a' \equiv b' : A'/R'} \quad \text{E_CAST} \\
\\
\frac{\Gamma; \Delta \vdash a \equiv b : A/R \quad \Gamma; \tilde{\Gamma} \vdash A \equiv B : \star/\mathbf{Rep} \quad \Gamma \vdash B : \star}{\Gamma; \Delta \vdash a \equiv b : B/R} \quad \text{E_EQCONV} \\
\\
\frac{\Gamma; \Delta \vdash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \vdash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E_ISOSND} \\
\\
\frac{\Gamma; \Delta \vdash a \equiv a' : A/R \quad \Gamma; \Delta \vdash b_1 \equiv b'_1 : B/R_0 \quad \Gamma; \Delta \vdash b_2 \equiv b'_2 : B/R_0}{\Gamma; \Delta \vdash \text{case}_R a \text{ of } F \rightarrow b_1 \parallel \rightarrow b_2 \equiv \text{case}_R a' \text{ of } F \rightarrow b'_1 \parallel \rightarrow b'_2 : B/R_0} \quad \text{E_PATCONG} \\
\\
\frac{\begin{array}{l} \text{ValuePath } a = F \\ \text{ValuePath } a' = F \\ \Gamma \vdash a : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash b : A \\ \Gamma \vdash a' : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash b' : A \\ \Gamma; \Delta \vdash a \ b^{R_1} \equiv a' \ b'^{R_1} : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R' \end{array}}{\Gamma; \Delta \vdash a \equiv a' : \Pi^+ x : A \rightarrow B/R'} \quad \text{E_LEFTREL} \\
\\
\frac{\begin{array}{l} \text{ValuePath } a = F \\ \text{ValuePath } a' = F \\ \Gamma \vdash a : \Pi^- x : A \rightarrow B \\ \Gamma \vdash b : A \\ \Gamma \vdash a' : \Pi^- x : A \rightarrow B \\ \Gamma \vdash b' : A \\ \Gamma; \Delta \vdash a \ \square^- \equiv a' \ \square^- : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \end{array}}{\Gamma; \Delta \vdash a \equiv a' : \Pi^- x : A \rightarrow B/R'} \quad \text{E_LEFTIRREL} \\
\\
\frac{\begin{array}{l} \text{ValuePath } a = F \\ \text{ValuePath } a' = F \\ \Gamma \vdash a : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash b : A \\ \Gamma \vdash a' : \Pi^+ x : A \rightarrow B \\ \Gamma \vdash b' : A \\ \Gamma; \Delta \vdash a \ b^+ \equiv a' \ b'^+ : B\{b/x\}/R' \\ \Gamma; \tilde{\Gamma} \vdash B\{b/x\} \equiv B\{b'/x\} : \star/R_0 \end{array}}{\Gamma; \Delta \vdash b \equiv b' : A/\mathbf{param } R_1 \ R'} \quad \text{E_RIGHT} \\
\\
\frac{\begin{array}{l} \text{ValuePath } a = F \\ \text{ValuePath } a' = F \\ \Gamma \vdash a : \forall c : (a_1 \sim_{A/R_1} a_2). B \\ \Gamma \vdash a' : \forall c : (a_1 \sim_{A/R_1} a_2). B \\ \Gamma; \tilde{\Gamma} \vdash a_1 \equiv a_2 : A/R' \\ \Gamma; \Delta \vdash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R' \end{array}}{\Gamma; \Delta \vdash a \equiv a' : \forall c : (a_1 \sim_{A/R_1} a_2). B/R'} \quad \text{E_CLEFT}
\end{array}$$

$\boxed{\models \Gamma}$ context wellformedness

$$\begin{array}{c}
\overline{\models \emptyset} \quad \text{E_EMPTY} \\
\\
\begin{array}{c}
\models \Gamma \\
\Gamma \models A : \star \\
x \notin \tilde{\Gamma}
\end{array}
\overline{\models \Gamma, x : A} \quad \text{E_CONSTM} \\
\\
\begin{array}{c}
\models \Gamma \\
\Gamma \models \phi \text{ ok} \\
c \notin \tilde{\Gamma}
\end{array}
\overline{\models \Gamma, c : \phi} \quad \text{E_CONSCo}
\end{array}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\begin{array}{c}
\overline{\models \emptyset} \quad \text{SIG_EMPTY} \\
\\
\begin{array}{c}
\models \Sigma \\
\emptyset \models A : \star \\
F \notin \text{dom } \Sigma
\end{array}
\overline{\models \Sigma \cup \{F : A @ Rs\}} \quad \text{SIG_CONSTCONST} \\
\\
\begin{array}{c}
\models \Sigma \\
F \notin \text{dom } \Sigma \\
\emptyset \models A : \star \\
\Omega; \Gamma \models p :_F B \Rightarrow A \\
\Gamma \models a : B \\
\Omega \models a : R
\end{array}
\overline{\models \Sigma \cup \{F : p \sim a : A / R @ \text{range } \Omega\}} \quad \text{SIG_CONSAx}
\end{array}$$

$\boxed{\Gamma \vdash \phi \text{ ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A / R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \rightsquigarrow b / R}$ single-step, weak head reduction to values for annotated language

Definition rules: 169 good 0 bad
Definition rule clauses: 463 good 0 bad