$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
	$\mid$	$\mathbf{app\_rho}\,\nu$	S	
	$\mid$	(ρ)	S	
appflag, ν	$::=$			applicative flag
	$\mid$	R		
	$\mid$	ρ		
role, R	$::=$			Role
	$\mid$	$\mathbf{Nom}$		
	$\mid$	$\mathbf{Rep}$		
	$\mid$	$R_1 \cap R_2$	S	
	$\mid$	$R_1 \wedge R_2$	S	
	$\mid$	$\mathbf{app\_role}\,\nu\,R$	S	
	$\mid$	(R)	S	
constraint, ϕ	$::=$			props
	$\mid$	$a \sim_R b : A$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
	$\mid$	$a \sim_R b$	S	
	$\mid$	$a \sim b$	S	
tm, a, b, p, v, w, A, B, C	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^{\rho} x : A.b$	bind x in b	
	$\mid$	$\lambda^{\rho} x.b$	bind x in b	
	$\mid$	$a\ b^{\nu}$		
	$\mid$	$\Pi^{\rho} x : A \to B$	bind x in B	
	$\mid$	$\Lambda c : \phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a\ \gamma$		
	$\mid$	$\forall c : \phi.B$	bind c in B	
	$\mid$	$a \triangleright_R \gamma$		
	$\mid$	F		
	$\mid$	$\square$		
	$\mid$	case a of $F\ \overline{v} \to b_1 \Vert_{\_} \to b_2$		
	$\mid$	case a of $F\ \overline{\mu} \to b_1 \Vert_{\_} \to b_2$	M	
	$\mid$	$a\ \overline{\mu}$	M	
	$\mid$	K		
	$\mid$	$\mathbf{match}\ a\ \mathbf{with}\ \mathit{brs}$		
	$\mid$	$\mathbf{sub}\ R\ a$		
	$\mid$	$\mathbf{coerce}$		

	$\vert$	$a\ b$		
	$\vert$	$a\{b/x\}$	S	
	$\vert$	$a\{\gamma/c\}$	S	
	$\vert$	$a\{b/x\}$	S	
	$\vert$	$a\{\gamma/c\}$	S	
	$\vert$	a	S	
	$\vert$	a	S	
	$\vert$	$\theta\ a$	S	
	$\vert$	(a)	S	
	$\vert$	a	S	parsing precedence is hard
	$\vert$	$\vert a\vert_R$	S	
	$\vert$	**Int**	S	
	$\vert$	**Age**	S	
	$\vert$	**Bool**	S	
	$\vert$	Nat	S	
	$\vert$	**Vec**	S	
	$\vert$	0	S	
	$\vert$	S	S	
	$\vert$	**True**	S	
	$\vert$	**Fix**	S	
	$\vert$	**Maybe**	S	
	$\vert$	**Just**	S	
	$\vert$	**Nothing**	S	
	$\vert$	$a \to b$	S	
	$\vert$	$\phi \Rightarrow A$	S	
	$\vert$	$\lambda x.a$	S	
	$\vert$	$\lambda x\!:\!A.a$	S	
	$\vert$	$\forall\, x : A \to B$	S	
	$\vert$	**if** ϕ **then** a **else** b	S	
brs	$::=$			case branches
	$\vert$	**none**		
	$\vert$	$K \Rightarrow a; brs$		
	$\vert$	$brs\{a/x\}$	S	
	$\vert$	$brs\{\gamma/c\}$	S	
	$\vert$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\vert$	$\bullet$		
	$\vert$	c		
	$\vert$	**red** $a\ b$		
	$\vert$	**refl** a		
	$\vert$	$(a\ \vert{=}\vert_{\gamma}\ b)$		
	$\vert$	**sym** γ		
	$\vert$	$\gamma_1; \gamma_2$		
	$\vert$	**sub** γ		

	|	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2
	|	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2
	|	$\gamma_1\ \gamma_2^{R,\rho}$	
	|	$\mathbf{piFst}\,\gamma$	
	|	$\mathbf{cpiFst}\,\gamma$	
	|	$\mathbf{isoSnd}\,\gamma$	
	|	$\gamma_1 @ \gamma_2$	
	|	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3
	|	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3
	|	$\gamma(\gamma_1, \gamma_2)$	
	|	$\gamma @ (\gamma_1 \sim \gamma_2)$	
	|	$\gamma_1 \triangleright_R \gamma_2$	
	|	$\gamma_1 \sim_A \gamma_2$	
	|	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$	
	|	$\mathbf{eta}\,a$	
	|	$\mathbf{left}\,\gamma\,\gamma'$	
	|	$\mathbf{right}\,\gamma\,\gamma'$	
	|	(γ)	S
	|	γ	S
	|	$\gamma\{a/x\}$	S
υ	$::=$		
	|	ν	
	|	$\bullet$	
$\overline{\upsilon}$	$::=$		
	|		
	|	$\overline{\upsilon}\ \upsilon$	M
	|	$\upsilon\ \overline{\upsilon}$	
	|	$(\overline{\upsilon})$	M
θ	$::=$		
	|		
	|	$x \mapsto \mu, \theta$	
$role\_context,\ \Omega$	$::=$		$\mathrm{role}_c ontexts$
	|	$\varnothing$	
	|	$x : R$	
	|	$\Omega, x : R$	
	|	Ω, Ω'	M
	|	$\mathbf{var\_pat}\,p$	M
	|	(Ω)	M
	|	Ω	M
$roles,\ \overline{R}$	$::=$		
	|	$.$	
	|	$R, \overline{R}$	

	$\vert$	$\textbf{range}\,\Omega$	S	
	$\vert$	$(\overline{R})$	S	
	$\vert$	$\overline{R} \mathbin{++} \overline{R}'$	S	
	$\vert$		S	
	$\vert$	R_1	S	
	$\vert$	R_1, R_2	S	
	$\vert$	R_1, R_2, R_3	S	
sig_sort	$::=$			signature classifier
	$\vert$	$A\ @\ \overline{R}$		
	$\vert$	$p \sim_R a : A\ @\ \overline{R}$		
$sort$	$::=$			binding classifier
	$\vert$	$\mathbf{Tm}\,A$		
	$\vert$	$\mathbf{Co}\,\phi$		
$context$, Γ	$::=$			contexts
	$\vert$	$\varnothing$		
	$\vert$	$\Gamma, x : A$		
	$\vert$	$\Gamma, c : \phi$		
	$\vert$	$\Gamma\{b/x\}$	M	
	$\vert$	$\Gamma\{\gamma/c\}$	M	
	$\vert$	Γ, Γ'	M	
	$\vert$	$\vert\Gamma\vert$	M	
	$\vert$	(Γ)	M	
	$\vert$	Γ	M	
sig, Σ	$::=$			signatures
	$\vert$	$\varnothing$		
	$\vert$	$\Sigma \cup \{F : sig\_sort\}$		
	$\vert$	Σ_0	M	
	$\vert$	Σ_1	M	
	$\vert$	$\vert\Sigma\vert$	M	
	$\vert$	$\theta\,\Gamma$	M	
$available\_props$, Δ	$::=$			
	$\vert$	$\varnothing$		
	$\vert$	Δ, x		
	$\vert$	Δ, c		
	$\vert$	$\mathsf{fv}\,a$	M	
	$\vert$	Δ, Δ'	M	
	$\vert$	$\widetilde{\Gamma}$	M	
	$\vert$	$\widetilde{\Omega}$	M	
	$\vert$	(Δ)	M	
Nat, $\mathbb{N}$	$::=$			
	$\vert$	0	M	

	$\vert$	$\Sigma\,\mathbb{N}$	M	
	$\vert$	$(\mathbb{N})$	M	
	$\vert$	$\vert a\vert$	S	
μ	$::=$			Pattern arguments
	$\vert$	a^{ν}		
	$\vert$	γ		
$\overline{\mu},\ PA$	$::=$			
	$\vert$			
	$\vert$	$\mu\ PA$		
	$\vert$	$PA\ \mu$		
$terminals$	$::=$			
	$\vert$	$\leftrightarrow$		
	$\vert$	$\Leftrightarrow$		
	$\vert$	$\longrightarrow$		
	$\vert$	min		
	$\vert$	$\equiv$		
	$\vert$	$\forall$		
	$\vert$	$\in$		
	$\vert$	$\notin$		
	$\vert$	$\Leftarrow$		
	$\vert$	$\Rightarrow$		
	$\vert$	$\Rightarrow^{*}$		
	$\vert$	$\rightarrow$		
	$\vert$	Λ		
	$\vert$			
	$\vert$	$\square$		
	$\vert$	$\vdash$		
	$\vert$	$\dashv$		
	$\vert$	$\models$		
	$\vert$	$\models$		
	$\vert$	$\models_{\mathsf{src}}$		
	$\vert$	$\neq$		
	$\vert$	$\triangleright$		
	$\vert$	ok		
	$\vert$	$\_$		
	$\vert$	$\rightsquigarrow$		
	$\vert$	$\rightsquigarrow^{*}$		
	$\vert$	$\rightsquigarrow$		
	$\vert$	$\varnothing$		
	$\vert$	$\circ$		
	$\vert$	fv		
	$\vert$	dom		
	$\vert$	$\sim$		
	$\vert$	$\asymp$		

$|$ $|$
$|$ $\bullet$
$|$ **fst**
$|$ **snd**
$|$ **as**
$|$ $| \Rightarrow |$
$|$ $\vdash_{=}$
$|$ $\mathbf{refl_2}$
$|$ $++$
$|$ $\{$
$|$ $\}$
$|$ $\nrightarrow$

$\mathit{formula},\ \psi$ ::=
$|$ $\mathit{judgement}$
$|$ $x : A \ \in\ \Gamma$
$|$ $x : R \ \in\ \Omega$
$|$ $c : \phi \ \in\ \Gamma$
$|$ $F : \mathit{sig\_sort} \ \in\ \Sigma$
$|$ $x \ \in\ \Delta$
$|$ $c \ \in\ \Delta$
$|$ $c\,\textbf{not relevant} \ \in\ \gamma$
$|$ $x \ \notin\ \Delta$
$|$ $\mathit{uniq}\,\Gamma$
$|$ $\mathit{uniq}(\Omega)$
$|$ $c \ \notin\ \Delta$
$|$ $T \ \notin\ \mathsf{dom}\,\Sigma$
$|$ $F \ \notin\ \mathsf{dom}\,\Sigma$
$|$ $\mathbb{N}_1 < \mathbb{N}_2$
$|$ $\mathbb{N}_1 \leq \mathbb{N}_2$
$|$ $\nu = \rho$
$|$ $R_1 = R_2$
$|$ $a = b$
$|$ $\phi_1 = \phi_2$
$|$ $\Gamma_1 = \Gamma_2$
$|$ $\gamma_1 = \gamma_2$
$|$ $\neg\psi$
$|$ $\psi_1 \wedge \psi_2$
$|$ $\psi_1 \vee \psi_2$
$|$ $\psi_1 \Rightarrow \psi_2$
$|$ (ψ)
$|$ ψ
$|$ $c : (a : A \sim b : B) \ \in\ \Gamma$
$|$ suppress lc hypothesis generated by Ott
$|$ $\Delta \# \Delta_2$

JSubRole	::=		
	|	$R_1 \leq R_2$	Subroling judgement
JRolePath	::=		
	|	$\mathsf{RolePath}\ a \not\leftrightarrow F@\overline{R}$	Type headed by constant
JAppsPath	::=		
	|	$\mathsf{AppsPath}\ a \not\leftrightarrow F\ \overline{v}$	Type headed by constant
JSat	::=		
	|	$\overline{v} \leftrightarrow \overline{R}$	
	|	$\mathbf{Sat}\, F\, \overline{v}$	
JPatCtx	::=		
	|	$\Omega; \Gamma; \Delta \vDash p :_F B \Rightarrow A$	Contexts generated by a p
JRename	::=		
	|	$\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'$	rename with fresh variable
JMatchSubst	::=		
	|	$\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 \not\leftrightarrow b_2$	match and substitute
JIsPattern	::=		
	|	$\mathbf{pattern}\, p$	
JSubPat	::=		
	|	$\mathbf{subpattern}\, p'\, p$	Subpattern
JTmPatternAgree	::=		
	|	$a \leftrightarrow p$	term and pattern agree
JTmSubPatternAgree	::=		
	|	$a \sqsubseteq p$	sub-pattern agrees with te
JSubTmPatternAgree	::=		
	|	$a \sqsupseteq p$	sub-term agrees with patt
JValuePath	::=		
	|	$\mathsf{ValidPath}\ a \not\leftrightarrow F$	Path headed by valid cons
JCasePath	::=		
	|	$\mathsf{ValuePath}_R\ a \not\leftrightarrow F$	Path that is a value
JApplyArgs	::=		
	|	$\mathsf{apply\ args}\ a\ \mathsf{to}\ b \not\leftrightarrow b'$	apply arguments of a (hea
JValue	::=		

	$\vert$	$\mathsf{Value}_R\ A$	values
JValueType	$::=$		
	$\vert$	$\mathsf{ValueType}_R\ A$	Types with head forms (erased lang
Jconsistent	$::=$		
	$\vert$	$\mathsf{consistent}_R\ a\ b$	Types do not differ in their heads
Jroleing	$::=$		
	$\vert$	$\Omega \vDash a : R$	Roleing judgment
JChk	$::=$		
	$\vert$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	$::=$		
	$\vert$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction
	$\vert$	$\Omega \vDash a \Rightarrow_R^* b$	multistep parallel reduction
	$\vert$	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common ter
Jbeta	$::=$		
	$\vert$	$\vDash a \to_R^\beta b$	primitive reductions
	$\vert$	$\vDash a \leadsto_R b$	single-step head reduction for impli
	$\vert$	$\vDash a \leadsto^* b/R$	multistep reduction
JBranchTyping	$::=$		
	$\vert$	$\Gamma \vDash \mathsf{case}\,(a \sim b\ \overline{\mu} : A)\,\mathsf{of}\ F\ \overline{v} : B \Rightarrow C \mid C'$	Branch Typing (aligning the types
Jett	$::=$		
	$\vert$	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	$\vert$	$\Gamma \vDash a : A$	typing
	$\vert$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	$\vert$	$\Gamma; \Delta \vDash a \equiv_R b : A$	definitional equality
	$\vert$	$\vDash \Gamma$	context wellformedness
Jsig	$::=$		
	$\vert$	$\vDash \Sigma$	signature wellformedness
Jhiding	$::=$		
	$\vert$	$\overline{R}_1 \Leftarrow \overline{R}_2$	
	$\vert$	$\Sigma_1 \Leftarrow \Sigma_2$	
JSrc	$::=$		
	$\vert$	$\Gamma \vDash_{\mathsf{src}} a : A$	source typing
	$\vert$	$\Gamma \vDash_{\mathsf{src}} a \leadsto a' : A$	source translation
	$\vert$	$\Gamma \vDash_{\mathsf{src}} \phi \leadsto \phi'\ \mathsf{ok}$	Prop wellformedness
Jann	$::=$		

	$\mid$	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	$\mid$	$\Gamma \vdash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	$\mid$	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	$\mid$	$\vdash \Gamma$	context wellformedness
Jred	$::=$		
	$\mid$	$\Gamma \vdash a \leadsto b/R$	single-step, weak head reduction to values for annotated la
judgement	$::=$		
	$\mid$	*JSubRole*	
	$\mid$	*JRolePath*	
	$\mid$	*JAppsPath*	
	$\mid$	*JSat*	
	$\mid$	*JPatCtx*	
	$\mid$	*JRename*	
	$\mid$	*JMatchSubst*	
	$\mid$	*JIsPattern*	
	$\mid$	*JSubPat*	
	$\mid$	*JTmPatternAgree*	
	$\mid$	*JTmSubPatternAgree*	
	$\mid$	*JSubTmPatternAgree*	
	$\mid$	*JValuePath*	
	$\mid$	*JCasePath*	
	$\mid$	*JApplyArgs*	
	$\mid$	*JValue*	
	$\mid$	*JValueType*	
	$\mid$	*Jconsistent*	
	$\mid$	*Jroleing*	
	$\mid$	*JChk*	
	$\mid$	*Jpar*	
	$\mid$	*Jbeta*	
	$\mid$	*JBranchTyping*	
	$\mid$	*Jett*	
	$\mid$	*Jsig*	
	$\mid$	*Jhiding*	
	$\mid$	*JSrc*	
	$\mid$	*Jann*	
	$\mid$	*Jred*	
user_syntax	$::=$		
	$\mid$	*tmvar*	
	$\mid$	*covar*	
	$\mid$	*datacon*	
	$\mid$	*const*	
	$\mid$	*index*	
	$\mid$	*relflag*	

| *appflag*
| *role*
| *constraint*
| *tm*
| *brs*
| *co*
| υ
| $\overline{\upsilon}$
| θ
| *role_context*
| *roles*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *Nat*
| μ
| $\overline{\mu}$
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{RolePath}\ a \nrightarrow F@\overline{R}}$ Type headed by constant (partial function)

$$\frac{F : A\ @\ \overline{R} \in \Sigma_0}{\mathsf{RolePath}\ F \nrightarrow F@\overline{R}} \quad \textsc{RolePath\_AbsConst}$$

$$\frac{F : p \sim_{R_1} a : A\ @\ \overline{R} \in \Sigma_0}{\mathsf{RolePath}\ F \nrightarrow F@\overline{R}} \quad \textsc{RolePath\_Const}$$

$$\frac{\mathsf{RolePath}\ a \nrightarrow F@R_1, \overline{R}}{\mathsf{RolePath}\ (a\ b'^{R_1}) \nrightarrow F@\overline{R}} \quad \textsc{RolePath\_TApp}$$

$$\frac{\mathsf{RolePath}\ a \nrightarrow F@\overline{R}}{\mathsf{RolePath}\ (a\ b'^{+}) \nrightarrow F@\overline{R}} \quad \textsc{RolePath\_App}$$

$$\frac{\mathsf{RolePath}\ a \nrightarrow F@\overline{R}}{\mathsf{RolePath}\ (a\ \Box^{-}) \nrightarrow F@\overline{R}} \quad \textsc{RolePath\_IApp}$$

$$\frac{\mathsf{RolePath}\ a \nrightarrow F@\overline{R}}{\mathsf{RolePath}\ (a\ \bullet) \nrightarrow F@\overline{R}} \quad \textsc{RolePath\_CApp}$$

$\boxed{\mathsf{AppsPath}\ a \nrightarrow F\ \overline{v}}$ Type headed by constant (partial function)

$$\frac{F : A \ @\ \overline{R} \in \Sigma_0}{\mathsf{AppsPath}\ F \nrightarrow F} \quad \textsc{AppsPath\_AbsConst}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} a : A \ @\ \overline{R} \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{AppsPath}\ F \nrightarrow F} \quad \textsc{AppsPath\_Const}$$

$$\frac{\mathsf{AppsPath}\ a \nrightarrow F\ \overline{v}}{\mathsf{AppsPath}\ (a\ \ b'^{R_1}) \nrightarrow F\ (\overline{v}\ R_1)} \quad \textsc{AppsPath\_App}$$

$$\frac{\mathsf{AppsPath}\ a \nrightarrow F\ \overline{v}}{\mathsf{AppsPath}\ (a\ \ b^{-}) \nrightarrow F\ (\overline{v}\ -)} \quad \textsc{AppsPath\_IApp}$$

$$\frac{\mathsf{AppsPath}\ a \nrightarrow F\ \overline{v}}{\mathsf{AppsPath}\ (a\ \bullet) \nrightarrow F\ (\overline{v}\ \bullet)} \quad \textsc{AppsPath\_CApp}$$

$\boxed{\overline{v} \leftrightarrow \overline{R}}$

$$\frac{}{\leftrightarrow \cdot} \quad \textsc{AR\_nil}$$

$$\frac{\overline{v} \leftrightarrow \overline{R}}{R_1\ \overline{v} \leftrightarrow R_1, \overline{R}} \quad \textsc{AR\_consTApp}$$

$$\frac{\overline{v} \leftrightarrow \overline{R}}{+\ \overline{v} \leftrightarrow \overline{R}} \quad \textsc{AR\_consApp}$$

$$\frac{\overline{v} \leftrightarrow \overline{R}}{-\ \overline{v} \leftrightarrow \overline{R}} \quad \textsc{AR\_consIApp}$$

$$\frac{\overline{v} \leftrightarrow \overline{R}}{\bullet\ \overline{v} \leftrightarrow \overline{R}} \quad \textsc{AR\_consCApp}$$

$\boxed{\mathbf{Sat}\ F\ \overline{v}}$

$$\frac{\begin{array}{l} F : A \ @\ \overline{R} \in \Sigma_0 \\ \overline{v} \leftrightarrow \overline{R} \end{array}}{\mathbf{Sat}\ F\ \overline{v}} \quad \textsc{Sat\_Const}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} a_0 : A_1 \ @\ \overline{R} \in \Sigma_0 \\ \neg(R_1 \leq \mathbf{Nom}) \\ \overline{v} \leftrightarrow \overline{R} \end{array}}{\mathbf{Sat}\ F\ \overline{v}} \quad \textsc{Sat\_Axiom}$$

$\boxed{\Omega; \Gamma; \Delta \vDash p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\varnothing; \varnothing; \varnothing \vDash F :_F A \Rightarrow A} \quad \textsc{PatCtx\_Const}$$

$$\frac{\Omega; \Gamma; \Delta \vDash p :_F \Pi^{+} x\!:\!A' \to A \Rightarrow B}{\Omega, x : R; \Gamma, x : A'; \Delta \vDash p\ x^{R} :_F A \Rightarrow B} \quad \textsc{PatCtx\_PiRel}$$

$$\frac{\Omega; \Gamma; \Delta \vDash p :_F \Pi^{-} x\!:\!A' \to A \Rightarrow B}{\Omega; \Gamma, x : A'; \Delta, x \vDash p\ \Box^{-} :_F A \Rightarrow B} \quad \textsc{PatCtx\_PiIrr}$$

$$\frac{\Omega; \Gamma; \Delta \vDash p :_F \forall c\!:\!\phi.A \Rightarrow B}{\Omega; \Gamma, c : \phi; \Delta \vDash p \bullet :_F A \Rightarrow B} \quad \textsc{PatCtx\_CPi}$$

$\boxed{\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}$ rename with fresh variables

$$\frac{}{\mathsf{rename}\ F \to a\ \mathsf{to}\ F \to a\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \varnothing}\quad \textsc{Rename\_Base}$$

$$\frac{\begin{array}{l}\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta' \\ y \notin (\Delta, \Delta')\end{array}}{\mathsf{rename}\ (p_1\ x^R) \to a_1\ \mathsf{to}\ (p_2\ y^R) \to (a_2\{y/x\})\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ (\Delta', y)}\quad \textsc{Rename\_AppRel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}{\mathsf{rename}\ (p_1\ \Box^-) \to a_1\ \mathsf{to}\ (p_2\ \Box^-) \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}\quad \textsc{Rename\_AppIrrel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}{\mathsf{rename}\ (p_1\ \bullet) \to a_1\ \mathsf{to}\ (p_2\ \bullet) \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}\quad \textsc{Rename\_CApp}$$

$\boxed{\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 \not\leftrightarrow b_2}$ match and substitute

$$\frac{}{\mathsf{match}\ F\ \mathsf{with}\ F \to b \not\leftrightarrow b}\quad \textsc{MatchSubst\_Const}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ p_1 \to b_1 \not\leftrightarrow b_2}{\mathsf{match}\ (a_1\ a^R)\ \mathsf{with}\ (p_1\ x^R) \to b_1 \not\leftrightarrow (b_2\{a/x\})}\quad \textsc{MatchSubst\_AppRelR}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ p_1 \to b_1 \not\leftrightarrow b_2}{\mathsf{match}\ (a_1\ a^-)\ \mathsf{with}\ (p_1\ \Box^-) \to b_1 \not\leftrightarrow b_2}\quad \textsc{MatchSubst\_AppIrrel}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 \not\leftrightarrow b_2}{\mathsf{match}\ (a_1\ \bullet)\ \mathsf{with}\ (a_2\ \bullet) \to b_1 \not\leftrightarrow b_2}\quad \textsc{MatchSubst\_CApp}$$

$\boxed{\mathbf{pattern}\, p}$

$$\frac{}{\mathbf{pattern}\, F}\quad \textsc{Pattern\_Head}$$

$$\frac{\mathbf{pattern}\, p}{\mathbf{pattern}\, (p\ a^R)}\quad \textsc{Pattern\_Rel}$$

$$\frac{\mathbf{pattern}\, p}{\mathbf{pattern}\, (p\ a^-)}\quad \textsc{Pattern\_Irrel}$$

$$\frac{\mathbf{pattern}\, p}{\mathbf{pattern}\, (p\ \gamma)}\quad \textsc{Pattern\_Co}$$

$\boxed{\mathbf{subpattern}\, p'\, p}$ Subpattern

$$\frac{\mathbf{pattern}\, p}{\mathbf{subpattern}\, p\, p}\quad \textsc{SubPat\_Refl}$$

$$\frac{\mathbf{subpattern}\, p'\, p}{\mathbf{subpattern}\, p'\, (p\ x^R)}\quad \textsc{SubPat\_Rel}$$

$$\frac{\mathbf{subpattern}\, p'\, p}{\mathbf{subpattern}\, p'\, (p\ \Box^-)}\quad \textsc{SubPat\_Irrel}$$

$$\frac{\mathbf{subpattern}\, p'\, p}{\mathbf{subpattern}\, p'\, (p\ \bullet)}\quad \textsc{SubPat\_Co}$$

$\boxed{a \leftrightarrow p}$ term and pattern agree

$$\frac{}{F \leftrightarrow F}\quad \textsc{tm\_pattern\_agree\_Const}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \ a_2{}^R) \leftrightarrow (p_1\ \ x^R)} \quad \textsc{tm\_pattern\_agree\_AppRelR}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \ a^-) \leftrightarrow (p_1\ \ \square^-)} \quad \textsc{tm\_pattern\_agree\_AppIrrel}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \bullet) \leftrightarrow (p_1\ \bullet)} \quad \textsc{tm\_pattern\_agree\_CApp}$$

$\boxed{a \sqsubseteq p}$ sub-pattern agrees with term

$$\frac{a \leftrightarrow p}{a \sqsubseteq p} \quad \textsc{tm\_subpattern\_agree\_Base}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ \ x^R)} \quad \textsc{tm\_subpattern\_agree\_AppRelR}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ \ \square^-)} \quad \textsc{tm\_subpattern\_agree\_AppIrrel}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ \bullet)} \quad \textsc{tm\_subpattern\_agree\_CApp}$$

$\boxed{a \sqsupseteq p}$ sub-term agrees with pattern

$$\frac{a \leftrightarrow p}{a \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_Base}$$

$$\frac{a \sqsupseteq p}{a\ \ a_2{}^\nu \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_App}$$

$$\frac{a \sqsupseteq p}{a\ \bullet \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_CApp}$$

$\boxed{\mathsf{ValidPath}\ \ a \nleftrightarrow F}$ Path headed by valid constructor

$$\frac{F : A\ @\ \overline{R} \in \Sigma_0}{\mathsf{ValidPath}\ \ F \nleftrightarrow F} \quad \textsc{ValuePath\_AbsConst}$$

$$\frac{F : p \sim_{R_1} a : A\ @\ \overline{R} \in \Sigma_0}{\mathsf{ValidPath}\ \ F \nleftrightarrow F} \quad \textsc{ValuePath\_Const}$$

$$\frac{\mathsf{ValidPath}\ \ a \nleftrightarrow F}{\mathsf{ValidPath}\ \ (a\ \ b'^\nu) \nleftrightarrow F} \quad \textsc{ValuePath\_App}$$

$$\frac{\mathsf{ValidPath}\ \ a \nleftrightarrow F}{\mathsf{ValidPath}\ \ (a\ \bullet) \nleftrightarrow F} \quad \textsc{ValuePath\_CApp}$$

$\boxed{\mathsf{ValuePath}_R\ \ a \nleftrightarrow F}$ Path that is a value

$$\frac{\begin{array}{c}\mathsf{ValidPath}\ \ a \nleftrightarrow F \\ F : A\ @\ \overline{R} \in \Sigma_0\end{array}}{\mathsf{ValuePath}_R\ \ a \nleftrightarrow F} \quad \textsc{CasePath\_AbsConst}$$

$$\frac{\begin{array}{l}\mathsf{ValidPath}\ \ a \nleftrightarrow F \\ F : p \sim_{R_1} b : A\ @\ \overline{R} \in \Sigma_0 \\ \neg(R_1 \leq R)\end{array}}{\mathsf{ValuePath}_R\ \ a \nleftrightarrow F} \quad \textsc{CasePath\_Const}$$

$$\frac{\begin{array}{l}\mathsf{ValidPath}\ a \nrightarrow F \\ F : p \sim_{R_1} b : A\ @\ \overline{R} \in \Sigma_0 \\ \neg(a \sqsupseteq p)\end{array}}{\mathsf{ValuePath}_R\ a \nrightarrow F} \quad \textsc{CasePath\_UnMatch}$$

$\boxed{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nrightarrow b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\mathsf{apply\ args}\ F\ \mathsf{to}\ b \nrightarrow b} \quad \textsc{ApplyArgs\_Const}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nrightarrow b'}{\mathsf{apply\ args}\ (a\ a'^{R})\ \mathsf{to}\ b \nrightarrow (b'\ a'^{+})} \quad \textsc{ApplyArgs\_AppRole}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nrightarrow b'}{\mathsf{apply\ args}\ (a\ a'^{\rho})\ \mathsf{to}\ b \nrightarrow (b'\ a'^{\rho})} \quad \textsc{ApplyArgs\_AppRho}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nrightarrow b'}{\mathsf{apply\ args}\ a\ \bullet\ \mathsf{to}\ b \nrightarrow b'\ \bullet} \quad \textsc{ApplyArgs\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^{\rho} x\!:\!A \to B} \quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x\!:\!A.a} \quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x.a} \quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^{-} x.a} \quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a} \quad \textsc{Value\_UCAbs}$$

$$\frac{\mathsf{ValuePath}_R\ a \nrightarrow F}{\mathsf{Value}_R\ a} \quad \textsc{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star} \quad \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^{\rho} x\!:\!A \to B} \quad \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B} \quad \textsc{value\_type\_CPi}$$

$$\frac{\mathsf{ValuePath}_R\ a \nrightarrow F}{\mathsf{ValueType}_R\ a} \quad \textsc{value\_type\_ValuePath}$$

$\boxed{\mathsf{consistent}_R\ a\ b}$ Types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\ \star} \quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1 \!:\! A_1 \to B_1)\ (\Pi^\rho x_2 \!:\! A_2 \to B_2)} \quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_{R}\ (\forall c_1 \!:\! \phi_1 . A_1)\ (\forall c_2 \!:\! \phi_2 . A_2)} \quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}_R\ a_1 \nrightarrow F \\ \mathsf{ValuePath}_R\ a_2 \nrightarrow F\end{array}}{\mathsf{consistent}_{R}\ a_1\ a_2} \quad \textsc{consistent\_a\_CasePath}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_{R}\ a\ b} \quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_{R}\ a\ b} \quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \Box : R} \quad \textsc{role\_a\_Bullet}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R} \quad \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega) \\ x : R \in \Omega \\ R \leq R_1\end{array}}{\Omega \vDash x : R_1} \quad \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x . a) : R} \quad \textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R \\ \Omega \vDash b : \mathbf{Nom}\end{array}}{\Omega \vDash (a\ b^\rho) : R} \quad \textsc{role\_a\_App}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R \\ \Omega \vDash b : R_1 \wedge R\end{array}}{\Omega \vDash a\ b^{R_1} : R} \quad \textsc{role\_a\_TApp}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R \\ \Omega, x : \mathbf{Nom} \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x \!:\! A \to B) : R} \quad \textsc{role\_a\_Pi}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R_1 \\ \Omega \vDash b : R_1 \\ \Omega \vDash A : \mathbf{Rep} \\ \Omega \vDash B : R\end{array}}{\Omega \vDash (\forall c \!:\! a \sim_{R_1} b : A . B) : R} \quad \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c . b) : R} \quad \textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a\ \bullet) : R} \quad \textsc{role\_a\_CApp}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega) \\ F : A\ @\ \overline{R} \in \Sigma_0\end{array}}{\Omega \vDash F : R} \quad \textsc{role\_a\_Const}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F : p \sim_R a : A \ @\ \overline{R} \in \Sigma_0 \end{array}}{\Omega \vDash F : R_1} \quad \textsc{role\_a\_Fam}$$

$$\frac{\begin{array}{l} \Omega \vDash a : \mathbf{Nom} \\ \Omega \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1 \end{array}}{\Omega \vDash \mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \|_{\_} \to b_2 : R_1} \quad \textsc{role\_a\_Pattern}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b' \end{array}}{\Omega \vDash a\ b^\rho \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{(\mathbf{app\_role}\,\nu\,R)} b' \end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\ b'^\nu} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a \bullet \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a \bullet \Rightarrow_R a' \bullet} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{l} \Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \Pi^\rho x\!:\!A \to B \Rightarrow_R \Pi^\rho x\!:\!A' \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{l} \Omega \vDash A \Rightarrow_{\mathbf{Rep}} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \forall c\!:\!a \sim_{R_1} b : A.B \Rightarrow_R \forall c\!:\!a' \sim_{R_1} b' : A'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l} F : F \sim_{R_1} b : A \ @\ \overline{R} \in \Sigma_0 \\ R_1 \leq R \\ uniq(\Omega) \end{array}}{\Omega \vDash F \Rightarrow_R b} \quad \textsc{Par\_AxiomBase}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} b : A\ @\ \overline{R} \in \Sigma_0 \\ a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash a_1 \Rightarrow_{(\mathbf{app\_role}\,\nu\,R)} a_1' \\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p' \to b'\ \mathsf{excluding}\ (\widetilde{\Omega}, \mathsf{fv}p)\ \mathsf{and}\ \Delta' \\ \mathsf{match}\ (a'\ {a_1'}^{\nu})\ \mathsf{with}\ p' \to b' \nrightarrow a_2 \\ R_1 \leq R \end{array}}{\Omega \vDash a\ {a_1}^{\nu} \Rightarrow_R a_2} \quad \textsc{Par\_AxiomApp}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} b : A\ @\ \overline{R} \in \Sigma_0 \\ a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\ \Omega \vDash a \Rightarrow_R a' \\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p' \to b'\ \mathsf{excluding}\ (\widetilde{\Omega}, \mathsf{fv}p)\ \mathsf{and}\ \Delta' \\ \mathsf{match}\ (a'\ \bullet)\ \mathsf{with}\ p' \to b' \nrightarrow a_2 \\ R_1 \leq R \end{array}}{\Omega \vDash a\ \bullet \Rightarrow_R a_2} \quad \textsc{Par\_AxiomCApp}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_{\mathbf{Nom}} a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \end{array}}{\Omega \vDash (\mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} (\mathsf{case}\ a'\ \mathsf{of}\ F\ \overline{v} \to b_1' \| \_ \to b_2')} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_{\mathbf{Nom}} a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{AppsPath}\ a' \nrightarrow F\ \overline{v} \\ \mathsf{apply\ args}\ a'\ \mathsf{to}\ b_1' \nrightarrow b \\ \mathbf{Sat}\, F\, \overline{v}' \end{array}}{\Omega \vDash (\mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} b\ \bullet} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_{\mathbf{Nom}} a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_{\mathbf{Nom}}\ a' \\ \neg(\mathsf{AppsPath}\ a' \nrightarrow F\ \overline{v}) \end{array}}{\Omega \vDash (\mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2) \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vDash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow^*_R a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_R b \\ \Omega \vDash b \Rightarrow^*_R a' \end{array}}{\Omega \vDash a \Rightarrow^*_R a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{c} \Omega \vDash a_1 \Rightarrow^*_R b \\ \Omega \vDash a_2 \Rightarrow^*_R b \end{array}}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \textsc{join}$$

$\boxed{\vDash a \to^{\beta}_R b}$ primitive reductions

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{\rho} x.v)}{\vDash (\lambda^{\rho} x.v)\ b^{\rho} \to^{\beta}_{R_1} v\{b/x\}} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a') \bullet \to^{\beta}_{R} a'\{\bullet/c\}} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} b : A \mathbin{@} \overline{R} \in \Sigma_0 \\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p_1 \to b_1\ \mathsf{excluding}\ (\mathsf{fv}a, \mathsf{fv}p)\ \mathsf{and}\ \Delta' \\ \mathsf{match}\ a\ \mathsf{with}\ p_1 \to b_1 \not\to b' \\ R_1 \leq R \end{array}}{\vDash a \to^{\beta}_{R} b'} \quad \textsc{Beta\_Axiom}$$

$$\frac{\begin{array}{c} \mathsf{AppsPath}\ a \not\to F\ \overline{v} \\ \mathsf{apply\ args}\ a\ \mathsf{to}\ b_1 \not\to b_1' \\ \mathbf{Sat}\, F\, \overline{v}' \end{array}}{\vDash \mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2 \to^{\beta}_{R_0} b_1' \bullet} \quad \textsc{Beta\_PatternTrue}$$

$$\frac{\begin{array}{c} \mathsf{Value}_{\mathbf{Nom}}\ a \\ \neg(\mathsf{AppsPath}\ a \not\to F\ \overline{v}) \end{array}}{\vDash \mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2 \to^{\beta}_{R_0} b_2} \quad \textsc{Beta\_PatternFalse}$$

$\boxed{\vDash a \rightsquigarrow_R b}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow_{R_1} a'}{\vDash \lambda^{-} x.a \rightsquigarrow_{R_1} \lambda^{-} x.a'} \quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \rightsquigarrow_{R_1} a'}{\vDash a\ b^{\nu} \rightsquigarrow_{R_1} a'\ b^{\nu}} \quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \rightsquigarrow_{R} a'}{\vDash a \bullet \rightsquigarrow_{R} a' \bullet} \quad \textsc{E\_CAppLeft}$$

$$\frac{\vDash a \rightsquigarrow_{\mathbf{Nom}} a'}{\vDash \mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2 \rightsquigarrow_{R_0} \mathsf{case}\ a'\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2} \quad \textsc{E\_Pattern}$$

$$\frac{\vDash a \to^{\beta}_{R} b}{\vDash a \rightsquigarrow_{R} b} \quad \textsc{E\_Prim}$$

$\boxed{\vDash a \rightsquigarrow^{*} b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^{*} a/R} \quad \textsc{Equal}$$

$$\frac{\begin{array}{c} \vDash a \rightsquigarrow_{R} b \\ \vDash b \rightsquigarrow^{*} a'/R \end{array}}{\vDash a \rightsquigarrow^{*} a'/R} \quad \textsc{Step}$$

$\boxed{\Gamma \vDash \mathsf{case}\,(a \sim b\ \overline{\mu} : A)\,\mathsf{of}\ F\ \overline{v} : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{c} \mathit{uniq}\ \Gamma \\ C_1\{\bullet/c\} = C_2 \end{array}}{\Gamma \vDash \mathsf{case}\,(a \sim b\ \overline{\mu} : A)\,\mathsf{of}\ F\ \ : A \Rightarrow \forall c\!:\!(a \sim_{\mathbf{Nom}} b\ \overline{\mu} : A).C_1 \mid C_2} \quad \textsc{BranchTyping\_Base}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}\,(a \sim b\ \overline{\mu}\ x^{R} : A_1)\,\mathsf{of}\ F\ \overline{v} : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}\,(a \sim b\ \overline{\mu} : A_1)\,\mathsf{of}\ F\ (R\ \overline{v}) : \Pi^{+} x\!:\!A \to B \Rightarrow \Pi^{+} x\!:\!A \to C \mid C'} \quad \textsc{BranchTyping\_PiRole}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}\,(a \sim b\ \overline{\mu}\ x^{+} : A_1)\,\mathsf{of}\ F\ \overline{v} : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}\,(a \sim b\ \overline{\mu} : A_1)\,\mathsf{of}\ F\ (+\ \overline{v}) : \Pi^{+} x\!:\!A \to B \Rightarrow \Pi^{+} x\!:\!A \to C \mid C'} \quad \textsc{BranchTyping\_PiRel}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}\,(a \sim b\ \overline{\mu}\ \Box^{-} : A_1)\,\mathsf{of}\ F\ \overline{v} : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}\,(a \sim b\ \overline{\mu} : A_1)\,\mathsf{of}\ F\ (-\ \overline{v}) : \Pi^{-} x\!:\!A \to B \Rightarrow \Pi^{-} x\!:\!A \to C \mid C'} \quad \textsc{BranchTyping\_PiIrrel}$$

$$\frac{\Gamma, c:\phi \vDash \mathsf{case}\,(a \sim b\; \overline{\mu}\; \bullet : A)\,\mathsf{of}\; F\; \overline{v} : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}\,(a \sim b\; \overline{\mu} : A)\,\mathsf{of}\; F\; (\bullet\; \overline{v}) : \forall c{:}\phi.B \Rightarrow \forall c{:}\phi.C \mid C'} \quad \textsc{BranchTyping\_CPi}$$

$\boxed{\Gamma \vDash \phi\;\; \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vDash a : A\\ \Gamma \vDash b : A\\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash a \sim_R b : A\;\; \mathsf{ok}} \quad \textsc{E\_Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \textsc{E\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ x : A \in \Gamma\end{array}}{\Gamma \vDash x : A} \quad \textsc{E\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash B : \star\\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash \Pi^{\rho} x{:}A \to B : \star} \quad \textsc{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash a : B\\ \Gamma \vDash A : \star\\ (\rho = +) \vee (x \notin \mathsf{fv}\; a)\end{array}}{\Gamma \vDash \lambda^{\rho} x.a : (\Pi^{\rho} x{:}A \to B)} \quad \textsc{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^{+} x{:}A \to B\\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\;\; a^{+} : B\{a/x\}} \quad \textsc{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^{+} x{:}A \to B\\ \Gamma \vDash a : A\\ \mathsf{RolePath}\;\; b \not\rightarrow F@R, \overline{R}\end{array}}{\Gamma \vDash b\;\; a^{R} : B\{a/x\}} \quad \textsc{E\_TApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^{-} x{:}A \to B\\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\; \Box^{-} : B\{a/x\}} \quad \textsc{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A\\ \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Rep}} B : \star\\ \Gamma \vDash B : \star\end{array}}{\Gamma \vDash a : B} \quad \textsc{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star\\ \Gamma \vDash \phi\;\; \mathsf{ok}\end{array}}{\Gamma \vDash \forall c{:}\phi.B : \star} \quad \textsc{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B\\ \Gamma \vDash \phi\;\; \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B} \quad \textsc{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c{:}(a \sim_R b : A).B_1\\ \Gamma; \widetilde{\Gamma} \vDash a \equiv_R b : A\end{array}}{\Gamma \vDash a_1\; \bullet : B_1\{\bullet/c\}} \quad \textsc{E\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F : A \mathbin{@} \overline{R} \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_Const}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F : p \sim_{R_1} a : A \mathbin{@} \overline{R} \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash b_1 : B \\ \Gamma \vDash b_2 : C \\ \Gamma \vDash \mathsf{case}\,(a \sim F\ \ : A)\ \mathsf{of}\ F\,\overline{v} : A_1 \Rightarrow B \mid C \\ \Gamma \vDash F : A_1 \\ \textbf{Sat}\ F\,\overline{v}\end{array}}{\Gamma \vDash \mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2 : C} \quad \text{E\_Case}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv_R A_2 : A \\ \Gamma; \Delta \vDash B_1 \equiv_R B_2 : A\end{array}}{\Gamma; \Delta \vDash A_1 \sim_R B_1 : A \equiv A_2 \sim_R B_2 : A} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A \equiv_{R_0} B : \star \\ \Gamma \vDash A_1 \sim_R A_2 : A\ \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_R A_2 : B\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_R A_2 : A \equiv A_1 \sim_R A_2 : B} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}(a_1 \sim_{R_1} a_2 : A).B_1 \equiv_{R'} \forall c{:}(b_1 \sim_{R_2} b_2 : B).B_2 : \star}{\Gamma; \Delta \vDash a_1 \sim_{R_1} a_2 : A \equiv b_1 \sim_{R_2} b_2 : B} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv_R b : A}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_R b : A) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv_R b : A} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv_R a : A} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv_R a : A}{\Gamma; \Delta \vDash a \equiv_R b : A} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv_R a_1 : A \\ \Gamma; \Delta \vDash a_1 \equiv_R b : A\end{array}}{\Gamma; \Delta \vDash a \equiv_R b : A} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv_{R_1} b : A \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv_{R_2} b : A} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B \\ \Gamma \vDash a_2 : B \\ \vDash a_1 \to^{\beta}_{R} a_2\end{array}}{\Gamma; \Delta \vDash a_1 \equiv_R a_2 : B} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv_{R'} A_2 : \star \\ \Gamma, x : A_1; \Delta \vDash B_1 \equiv_{R'} B_2 : \star \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_2 \to B_2 : \star\end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x\!:\!A_1 \to B_1) \equiv_{R'} (\Pi^\rho x\!:\!A_2 \to B_2) : \star} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1; \Delta \vDash b_1 \equiv_{R'} b_2 : B \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^\rho x.b_1) \equiv_{R'} (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv_{R'} b_1 : (\Pi^+ x\!:\!A \to B) \\ \Gamma; \Delta \vDash a_2 \equiv_{\mathbf{Nom}} b_2 : A\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^+ \equiv_{R'} b_1\ {b_2}^+ : (B\{a_2/x\})} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv_{R'} b_1 : (\Pi^+ x\!:\!A \to B) \\ \Gamma; \Delta \vDash a_2 \equiv_{R \wedge R'} b_2 : A \\ \mathsf{RolePath}\ a_1 \nrightarrow F@R, \overline{R} \\ \mathsf{RolePath}\ b_1 \nrightarrow F@R, \overline{R} \\ \Gamma \vDash b_1\ {b_2}^R : B\{a_2/x\}\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^R \equiv_{R'} b_1\ {b_2}^R : (B\{a_2/x\})} \quad \text{E\_TAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv_{R'} b_1 : (\Pi^- x\!:\!A \to B) \\ \Gamma \vDash a : A\end{array}}{\Gamma; \Delta \vDash a_1\ \Box^- \equiv_{R'} b_1\ \Box^- : (B\{a/x\})} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv_{R'} \Pi^\rho x\!:\!A_2 \to B_2 : \star}{\Gamma; \Delta \vDash A_1 \equiv_{R'} A_2 : \star} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv_{R'} \Pi^\rho x\!:\!A_2 \to B_2 : \star \\ \Gamma; \Delta \vDash a_1 \equiv_{\mathbf{Nom}} a_2 : A_1\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv_{R'} B_2\{a_2/x\} : \star} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_R b_1 : A_1 \equiv a_2 \sim_R b_2 : A_2 \\ \Gamma, c : a_1 \sim_R b_1 : A_1; \Delta \vDash A \equiv_{R'} B : \star \\ \Gamma \vDash a_1 \sim_R b_1 : A_1\ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_R b_1 : A_1.A : \star \\ \Gamma \vDash \forall c\!:\!a_2 \sim_R b_2 : A_2.B : \star\end{array}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_R b_1 : A_1.A \equiv_{R'} \forall c\!:\!a_2 \sim_R b_2 : A_2.B : \star} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv_R b : B \\ \Gamma \vDash \phi_1\ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv_R (\Lambda c.b) : \forall c\!:\!\phi_1.B} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv_{R'} b_1 : (\forall c\!:\!(a \sim_R b : A).B) \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv_R b : A\end{array}}{\Gamma; \Delta \vDash a_1\ \bullet \equiv_{R'} b_1\ \bullet : (B\{\bullet/c\})} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_R a_2 : A).B_1 \equiv_{R_0} \forall c\!:\!(a_1' \sim_{R'} a_2' : A').B_2 : \star \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv_R a_2 : A \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv_{R'} a_2' : A'\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv_{R_0} B_2\{\bullet/c\} : \star} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vDash a \equiv_R b : A \\ \Gamma;\Delta \vDash a \sim_R b : A \equiv a' \sim_{R'} b' : A'\end{array}}{\Gamma;\Delta \vDash a' \equiv_{R'} b' : A'} \quad \text{E\_CAST}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv_R b : A \\ \Gamma;\widetilde{\Gamma} \vDash A \equiv_{\mathbf{Rep}} B : \star \\ \Gamma \vDash B : \star\end{array}}{\Gamma;\Delta \vDash a \equiv_R b : B} \quad \text{E\_EQCONV}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{R_1} b : A \equiv a' \sim_{R_1} b' : A'}{\Gamma;\Delta \vDash A \equiv_{\mathbf{Rep}} A' : \star} \quad \text{E\_ISOSND}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv_{\mathbf{Nom}} a' : A \\ \Gamma;\Delta \vDash b_1 \equiv_{R_0} b_1' : B \\ \Gamma;\Delta \vDash b_2 \equiv_{R_0} b_2' : C \\ \Gamma \vDash \mathsf{case}\,(a \sim F \ : A)\,\mathsf{of}\,F\,\overline{v} : A_1 \Rightarrow B \mid C \\ \Gamma \vDash \mathsf{case}\,(a' \sim F \ : A)\,\mathsf{of}\,F\,\overline{v} : A_1 \Rightarrow B' \mid C \\ \Gamma;\Delta \vDash B \equiv_{\mathbf{Rep}} B' : \star \\ \mathbf{Sat}\,F\,\overline{v} \\ \Gamma \vDash F : A_1\end{array}}{\Gamma;\Delta \vDash \mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \|\_ \to b_2 \equiv_{R_0} \mathsf{case}\ a'\ \mathsf{of}\ F\ \overline{v} \to b_1' \|\_ \to b_2' : C} \quad \text{E\_PATCONG}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}_{R'}\ (a\ b^{R_1}) \nrightarrow F \\ \mathsf{ValuePath}_{R'}\ (a'\ {b'}^{R_1}) \nrightarrow F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^{R_1} \equiv_{R'} a'\ {b'}^{R_1} : B\{b/x\} \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv_{\mathbf{Rep}} B\{b'/x\} : \star\end{array}}{\Gamma;\Delta \vDash a \equiv_{R'} a' : \Pi^+ x\!:\!A \to B} \quad \text{E\_LEFTREL}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}_{R'}\ (a\ \Box^-) \nrightarrow F \\ \mathsf{ValuePath}_{R'}\ (a'\ \Box^-) \nrightarrow F \\ \Gamma \vDash a : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ \Box^- \equiv_{R'} a'\ \Box^- : B\{b/x\} \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv_{\mathbf{Rep}} B\{b'/x\} : \star\end{array}}{\Gamma;\Delta \vDash a \equiv_{R'} a' : \Pi^- x\!:\!A \to B} \quad \text{E\_LEFTIRREL}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}_{R'}\ (a\ b^{R_1}) \nrightarrow F \\ \mathsf{ValuePath}_{R'}\ (a'\ {b'}^{R_1}) \nrightarrow F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^{R_1} \equiv_{R'} a'\ {b'}^{R_1} : B\{b/x\} \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv_{\mathbf{Rep}} B\{b'/x\} : \star\end{array}}{\Gamma;\Delta \vDash b \equiv_{R_1 \wedge R'} b' : A} \quad \text{E\_RIGHT}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}_{R'}\ (a\ \bullet) \nrightarrow F\\ \mathsf{ValuePath}_{R'}\ (a'\ \bullet) \nrightarrow F\\ \Gamma \vDash a : \forall c{:}(a_1 \sim_{R_1} a_2 : A).B\\ \Gamma \vDash a' : \forall c{:}(a_1 \sim_{R_1} a_2 : A).B\\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv_{R_1 \wedge R'} a_2 : A\\ \Gamma; \Delta \vDash a\ \bullet \equiv_{R'} a'\ \bullet : B\{\bullet/c\}\end{array}}{\Gamma; \Delta \vDash a \equiv_{R'} a' : \forall c{:}(a_1 \sim_{R_1} a_2 : A).B} \quad \text{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash A : \star\\ x \notin \widetilde{\Gamma}\end{array}}{\vDash \Gamma, x : A} \quad \text{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash \phi\ \mathsf{ok}\\ c \notin \widetilde{\Gamma}\end{array}}{\vDash \Gamma, c : \phi} \quad \text{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma\\ \varnothing \vDash A : \star\\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vDash \Sigma \cup \{F : A\ @\ \overline{R}\}} \quad \text{Sig\_ConsConst}$$

$$\frac{\begin{array}{l}\vDash \Sigma\\ F \notin \mathsf{dom}\,\Sigma\\ \varnothing \vDash A : \star\\ \Omega; \Gamma; \Delta \vDash p :_F B \Rightarrow A\\ \Gamma \vDash a : B\\ \Delta \# \mathsf{fv}\, a\\ \Omega \vDash a : R\end{array}}{\vDash \Sigma \cup \{F : p \sim_R a : A\ @\ \mathbf{range}\,\Omega\}} \quad \text{Sig\_ConsAx}$$

$\boxed{\overline{R}_1 \Leftarrow \overline{R}_2}$

$$\frac{}{\cdot \Leftarrow \cdot} \quad \text{R\_Nil}$$

$$\frac{\begin{array}{c}R_2 \leq R_1\\ \overline{R}_1 \Leftarrow \overline{R}_2\end{array}}{R_1, \overline{R}_1 \Leftarrow R_2, \overline{R}_2} \quad \text{R\_Cons}$$

$\boxed{\Sigma_1 \Leftarrow \Sigma_2}$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2}{\Sigma_1 \Leftarrow \Sigma_2 \cup \{F : \mathit{sig\_sort}\}} \quad \text{S\_Forget}$$

$$\frac{\begin{array}{c}\Sigma_1 \Leftarrow \Sigma_2 \\ \overline{R}_1 \Leftarrow \overline{R}_2\end{array}}{\Sigma_1 \cup \{F : A \mathbin{@} \overline{R}_1\} \Leftarrow \Sigma_2 \cup \{F : p \sim_R a : A \mathbin{@} \overline{R}_2\}} \quad \text{S\_HIDE}$$

$$\frac{\begin{array}{c}\Sigma_1 \Leftarrow \Sigma_2 \\ \overline{R}_1 \Leftarrow \overline{R}_2\end{array}}{\Sigma_1 \cup \{F : A \mathbin{@} \overline{R}_1\} \Leftarrow \Sigma_2 \cup \{F : A \mathbin{@} \overline{R}_2\}} \quad \text{S\_WEAKENCONST}$$

$$\frac{\begin{array}{c}\Sigma_1 \Leftarrow \Sigma_2 \\ \overline{R}_1 \Leftarrow \overline{R}_2\end{array}}{\Sigma_1 \cup \{F : p' \sim_R a : A \mathbin{@} \overline{R}_1\} \Leftarrow \Sigma_2 \cup \{F : p \sim_R a : A \mathbin{@} \overline{R}_2\}} \quad \text{S\_WEAKENAXIOM}$$

$$\frac{}{\varnothing \Leftarrow \varnothing} \quad \text{S\_EMPTY}$$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2}{\Sigma_1 \cup \{F : \mathit{sig\_sort}\} \Leftarrow \Sigma_2 \cup \{F : \mathit{sig\_sort}\}} \quad \text{S\_SAME}$$

$\boxed{\Gamma \vDash_{\mathsf{src}} a : A}$ source typing

$$\frac{\vDash \Gamma}{\Gamma \vDash_{\mathsf{src}} \star : \star} \quad \text{S\_STAR}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A \in \Gamma\end{array}}{\Gamma \vDash_{\mathsf{src}} x : A} \quad \text{S\_VAR}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} A : \star \\ \Gamma \vDash_{\mathsf{src}} A \leadsto A' : \star \\ \Gamma, x : A' \vDash_{\mathsf{src}} B : \star\end{array}}{\Gamma \vDash_{\mathsf{src}} (\Pi^{\rho} x{:}A \to B) : \star} \quad \text{S\_PI}$$

$$\frac{\Gamma, x : A \vDash_{\mathsf{src}} a : B}{\Gamma \vDash_{\mathsf{src}} \lambda x.a : (\Pi^{+} x{:}A \to B)} \quad \text{S\_ABS}$$

$$\frac{\Gamma, x : A \vDash_{\mathsf{src}} a : B}{\Gamma \vDash_{\mathsf{src}} a : (\Pi^{-} x{:}A \to B)} \quad \text{S\_IABS}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Nom}} B : \star\end{array}}{\Gamma \vDash_{\mathsf{src}} a : B} \quad \text{S\_CONV}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Rep}} B : \star\end{array}}{\Gamma \vDash_{\mathsf{src}} \mathbf{coerce}\ a : B} \quad \text{S\_COERCE}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} b : \Pi^{+} x{:}A \to B \\ \Gamma \vDash_{\mathsf{src}} a \leadsto a' : A\end{array}}{\Gamma \vDash_{\mathsf{src}} b\ \ a : B\{a'/x\}} \quad \text{S\_APP}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} b : \Pi^{-} x{:}A \to B \\ \Gamma \vDash a' : A\end{array}}{\Gamma \vDash_{\mathsf{src}} b : B\{a'/x\}} \quad \text{S\_IAPP}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} \phi \leadsto \phi' \ \mathsf{ok} \\ \Gamma, c : \phi' \vDash_{\mathsf{src}} B : \star\end{array}}{\Gamma \vDash_{\mathsf{src}} \forall c{:}\phi.B : \star} \quad \text{S\_CPI}$$

$$\frac{\Gamma, c : \phi' \vDash_{\mathsf{src}} a : B}{\Gamma \vDash_{\mathsf{src}} a : \forall c{:}\phi.B} \quad \text{S\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} a_1 : \forall c{:}(a \sim_R b : A).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv_R b : A\end{array}}{\Gamma \vDash_{\mathsf{src}} a_1 : B_1\{\bullet/c\}} \quad \text{S\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F : A \mathbin{@} \overline{R} \in \Sigma_0\end{array}}{\Gamma \vDash_{\mathsf{src}} F : A} \quad \text{S\_Const}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash_{\mathsf{src}} F : A} \quad \text{S\_Fam}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} a : A \\ \Gamma \vDash_{\mathsf{src}} b_1' : B \\ \Gamma \vDash_{\mathsf{src}} b_2' : C\end{array}}{\Gamma \vDash_{\mathsf{src}} \mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \|_{\_} \to b_2 : C} \quad \text{S\_Case}$$

$\boxed{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A}$ source translation

$$\frac{\vDash \Gamma}{\Gamma \vDash_{\mathsf{src}} \star \rightsquigarrow \star : \star} \quad \text{ST\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A \in \Gamma\end{array}}{\Gamma \vDash_{\mathsf{src}} x \rightsquigarrow x : A} \quad \text{ST\_Var}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} A \rightsquigarrow A' : \star \\ \Gamma, x : A' \vDash_{\mathsf{src}} B \rightsquigarrow B' : \star\end{array}}{\Gamma \vDash_{\mathsf{src}} (\Pi^{\rho} x{:}A \to B) \rightsquigarrow (\Pi^{\rho} x{:}A' \to B') : \star} \quad \text{ST\_Pi}$$

$$\frac{\Gamma, x : A \vDash_{\mathsf{src}} a \rightsquigarrow a' : B}{\Gamma \vDash_{\mathsf{src}} \lambda x.a \rightsquigarrow \lambda^{+} x.a' : (\Pi^{+} x{:}A \to B)} \quad \text{ST\_Abs}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash_{\mathsf{src}} a \rightsquigarrow a' : B \\ x \notin \mathsf{fv} a\end{array}}{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow \lambda^{-} x.a : (\Pi^{-} x{:}A \to B)} \quad \text{ST\_IAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} b \rightsquigarrow b' : \Pi^{+} x{:}A \to B \\ \Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A\end{array}}{\Gamma \vDash_{\mathsf{src}} b\ a \rightsquigarrow b'\ {a'}^{+} : B\{a'/x\}} \quad \text{ST\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} b \rightsquigarrow b' : \Pi^{+} x{:}A \to B \\ \Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A \\ \mathsf{RolePath}\ b \not\mapsto F @ R, \overline{R}\end{array}}{\Gamma \vDash_{\mathsf{src}} b\ a \rightsquigarrow b'\ {a'}^{R} : B\{a/x\}} \quad \text{ST\_TApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} b \rightsquigarrow b' : \Pi^{-} x{:}A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash_{\mathsf{src}} b \rightsquigarrow b'\ \square^{-} : B\{a/x\}} \quad \text{ST\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Nom}} B : \star\end{array}}{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : B} \quad \text{ST\_Conv}$$

$$\frac{\begin{array}{c}\Gamma \vDash_{\mathsf{src}} a \leadsto a' : A \\ \Gamma ; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Rep}} B : \star\end{array}}{\Gamma \vDash_{\mathsf{src}} \mathbf{coerce}\ a \leadsto a' : B} \quad \text{ST\_COERCE}$$

$$\frac{\begin{array}{c}\Gamma \vDash_{\mathsf{src}} \phi \leadsto \phi' \ \mathsf{ok} \\ \Gamma, c : \phi' \vDash_{\mathsf{src}} B \leadsto B' : \star\end{array}}{\Gamma \vDash_{\mathsf{src}} \forall c{:}\phi.B \leadsto \forall c{:}\phi.B' : \star} \quad \text{ST\_CPI}$$

$$\frac{\Gamma, c : \phi \vDash_{\mathsf{src}} a \leadsto a' : B}{\Gamma \vDash_{\mathsf{src}} a \leadsto \Lambda c.a' : \forall c{:}\phi.B} \quad \text{ST\_CABS}$$

$$\frac{\begin{array}{c}\Gamma \vDash_{\mathsf{src}} a_1 \leadsto a_1' : \forall c{:}(a \sim_R b : A).B_1 \\ \Gamma ; \widetilde{\Gamma} \vDash a \equiv_R b : A\end{array}}{\Gamma \vDash_{\mathsf{src}} a_1 \leadsto a_1' \bullet : B_1\{\bullet/c\}} \quad \text{ST\_CAPP}$$

$$\frac{\begin{array}{c}\vDash \Gamma \\ F : A\ @\ \overline{R} \in \Sigma_0\end{array}}{\Gamma \vDash_{\mathsf{src}} F \leadsto F : A} \quad \text{ST\_CONST}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash_{\mathsf{src}} F \leadsto F : A} \quad \text{ST\_FAM}$$

$$\frac{\begin{array}{c}\Gamma \vDash_{\mathsf{src}} a \leadsto a' : A \\ \Gamma \vDash_{\mathsf{src}} b_1 \leadsto b_1' : B \\ \Gamma \vDash_{\mathsf{src}} b_2 \leadsto b_2' : C\end{array}}{\Gamma \vDash_{\mathsf{src}} \mathsf{case}\ a\ \mathsf{of}\ F\ \overline{v} \to b_1 \| \_ \to b_2 \leadsto \mathsf{case}\ a'\ \mathsf{of}\ F\ \overline{v} \to b_1' \| \_ \to b_2' : C} \quad \text{ST\_CASE}$$

$\boxed{\Gamma \vDash_{\mathsf{src}} \phi \leadsto \phi' \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{c}\Gamma \vDash_{\mathsf{src}} a \leadsto a' : A \\ \Gamma \vDash_{\mathsf{src}} b \leadsto b' : A\end{array}}{\Gamma \vDash_{\mathsf{src}} (a \sim_{\mathbf{Nom}} b : A) \leadsto (a' \sim_{\mathbf{Nom}} b' : A)\ \mathsf{ok}} \quad \text{S\_WFF}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A/R}$ typing

$\boxed{\Gamma ; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma ; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \leadsto b/R}$ single-step, weak head reduction to values for annotated language

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Definition rules:        222 good    0 bad
Definition rule clauses: 612 good    0 bad
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