$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
	$\mid$	$\mathbf{app\_rho}\,\nu$	S	
	$\mid$	(ρ)	S	
appflag, ν	$::=$			applicative flag
	$\mid$	R		
	$\mid$	ρ		
role, R	$::=$			Role
	$\mid$	$\mathbf{Nom}$		
	$\mid$	$\mathbf{Rep}$		
	$\mid$	$R_1 \cap R_2$	S	
	$\mid$	$\mathbf{param}\,R_1\,R_2$	S	
	$\mid$	$\mathbf{app\_role}\,\nu$	S	
	$\mid$	(R)	S	
constraint, ϕ	$::=$			props
	$\mid$	$a \sim_{A/R} b$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
	$\mid$	$a \sim_R b$	S	
tm, a, b, p, v, w, A, B, C	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^{\rho} x\!:\!A.b$	bind x in b	
	$\mid$	$\lambda^{\rho} x.b$	bind x in b	
	$\mid$	$a\ b^{\nu}$		
	$\mid$	$\Pi^{\rho} x\!:\!A \to B$	bind x in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$a \triangleright_R \gamma$		
	$\mid$	F		
	$\mid$	$\square$		
	$\mid$	$\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \Vert_{\_} \to b_2$		
	$\mid$	K		
	$\mid$	$\mathbf{match}\,a\,\mathbf{with}\,brs$		
	$\mid$	$\mathbf{sub}\,R\,a$		
	$\mid$	$a\{b/x\}$	S	
	$\mid$	$a\{\gamma/c\}$	S	
	$\mid$	$a\{b/x\}$	S	
	$\mid$	$a\{\gamma/c\}$	S	

	$\mid$	a	S	
	$\mid$	a	S	
	$\mid$	(a)	S	
	$\mid$	a	S	parsing precedence is hard
	$\mid$	$\lvert a\rvert_R$	S	
	$\mid$	**Int**	S	
	$\mid$	**Bool**	S	
	$\mid$	Nat	S	
	$\mid$	**Vec**	S	
	$\mid$	0	S	
	$\mid$	S	S	
	$\mid$	**True**	S	
	$\mid$	**Fix**	S	
	$\mid$	**Age**	S	
	$\mid$	$a \to b$	S	
	$\mid$	$\phi \Rightarrow A$	S	
	$\mid$	$a\ b$	S	
	$\mid$	$\lambda x.a$	S	
	$\mid$	$\lambda x{:}A.a$	S	
	$\mid$	$\forall\, x : A \to B$	S	
	$\mid$	**if** ϕ **then** a **else** b	S	
brs	$::=$			case branches
	$\mid$	**none**		
	$\mid$	$K \Rightarrow a; brs$		
	$\mid$	$brs\{a/x\}$	S	
	$\mid$	$brs\{\gamma/c\}$	S	
	$\mid$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\mid$	$\bullet$		
	$\mid$	c		
	$\mid$	**red** $a\ b$		
	$\mid$	**refl** a		
	$\mid$	$(a \mid=\mid_{\gamma}\ b)$		
	$\mid$	**sym** γ		
	$\mid$	$\gamma_1 ; \gamma_2$		
	$\mid$	**sub** γ		
	$\mid$	$\Pi^{R,\rho} x{:}\gamma_1.\gamma_2$	bind x in γ_2	
	$\mid$	$\lambda^{R,\rho} x{:}\gamma_1.\gamma_2$	bind x in γ_2	
	$\mid$	$\gamma_1\ \gamma_2^{R,\rho}$		
	$\mid$	**piFst** γ		
	$\mid$	**cpiFst** γ		
	$\mid$	**isoSnd** γ		
	$\mid$	$\gamma_1 @ \gamma_2$		
	$\mid$	$\forall c{:}\gamma_1.\gamma_3$	bind c in γ_3	

	$\mid$	$\lambda c\!:\!\gamma_1.\gamma_3@\gamma_4$	bind c in γ_3	
	$\mid$	$\gamma(\gamma_1, \gamma_2)$		
	$\mid$	$\gamma@(\gamma_1 \sim \gamma_2)$		
	$\mid$	$\gamma_1 \triangleright_R \gamma_2$		
	$\mid$	$\gamma_1 \sim_A \gamma_2$		
	$\mid$	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	$\mid$	$\mathbf{eta}\, a$		
	$\mid$	$\mathbf{left}\, \gamma\, \gamma'$		
	$\mid$	$\mathbf{right}\, \gamma\, \gamma'$		
	$\mid$	(γ)	S	
	$\mid$	γ	S	
	$\mid$	$\gamma\{a/x\}$	S	
$role\_context,\ \Omega$	$::=$			role$_c$$ontexts$
	$\mid$	$\varnothing$		
	$\mid$	$x : R$		
	$\mid$	$\Omega, x : R$		
	$\mid$	Ω, Ω'	M	
	$\mid$	$\mathbf{var\_pat}\, p$	M	
	$\mid$	(Ω)	M	
	$\mid$	Ω	M	
$roles,\ Rs$	$::=$			
	$\mid$	$\mathbf{nilR}$		
	$\mid$	R, Rs		
	$\mid$	$\mathbf{range}\, \Omega$	S	
sig_sort	$::=$			signature classifier
	$\mid$	$A@Rs$		
	$\mid$	$p \sim a : A/R@Rs$		
$sort$	$::=$			binding classifier
	$\mid$	$\mathbf{Tm}\, A$		
	$\mid$	$\mathbf{Co}\, \phi$		
$context,\ \Gamma$	$::=$			contexts
	$\mid$	$\varnothing$		
	$\mid$	$\Gamma, x : A$		
	$\mid$	$\Gamma, c : \phi$		
	$\mid$	$\Gamma\{b/x\}$	M	
	$\mid$	$\Gamma\{\gamma/c\}$	M	
	$\mid$	Γ, Γ'	M	
	$\mid$	$\|\Gamma\|$	M	
	$\mid$	(Γ)	M	
	$\mid$	Γ	M	
$sig,\ \Sigma$	$::=$			signatures

	$\|$	$\varnothing$		
	$\|$	$\Sigma \cup \{F : sig\_sort\}$		
	$\|$	Σ_0	M	
	$\|$	Σ_1	M	
	$\|$	$\|\Sigma\|$	M	
$available\_props,\ \Delta$	$::=$			
	$\|$	$\varnothing$		
	$\|$	Δ, x		
	$\|$	Δ, c		
	$\|$	$\mathsf{fv}\, a$	M	
	$\|$	Δ, Δ'	M	
	$\|$	$\widetilde{\Gamma}$	M	
	$\|$	$\widetilde{\Omega}$	M	
	$\|$	(Δ)	M	
$Nat,\ \mathbb{N}$	$::=$			
	$\|$	$\|a\|$	S	
$pattern\_arg$	$::=$			Pattern arguments
	$\|$	$\backslash Rel\, a\, R$		
	$\|$	$\backslash Irr\, a$		
	$\|$	$\backslash Coe \gamma$		
$pattern\_args,\ PA$	$::=$			
	$\|$	**none**		
	$\|$	$PA, pattern\_arg$		
$terminals$	$::=$			
	$\|$	$\leftrightarrow$		
	$\|$	$\Leftrightarrow$		
	$\|$	$\longrightarrow$		
	$\|$	min		
	$\|$	$\equiv$		
	$\|$	$\forall$		
	$\|$	$\in$		
	$\|$	$\notin$		
	$\|$	$\Leftarrow$		
	$\|$	$\Rightarrow$		
	$\|$	$\Rightarrow^*$		
	$\|$	$\rightarrow$		
	$\|$	Λ		
	$\|$			
	$\|$	$\Box$		
	$\|$	$\vdash$		
	$\|$	$\dashv$		
	$\|$	$\models$		

| $\vDash$
| $\neq$
| $\triangleright$
| ok
| _
| $\leadsto$
| $\leadsto^*$
| $\leadsto$
| $\varnothing$
| $\circ$
| fv
| dom
| $\sim$
| $\asymp$
| $|$
| $\bullet$
| **fst**
| **snd**
| **as**
| $| \Rightarrow |$
| $\vdash_=$
| $\mathbf{refl_2}$
| $++$
| $\{$
| $\}$

$\mathit{formula},\ \psi$ ::=
| $\mathit{judgement}$
| $x : A \ \in \ \Gamma$
| $x : R \ \in \ \Omega$
| $c : \phi \ \in \ \Gamma$
| $F : \mathit{sig\_sort} \ \in \ \Sigma$
| $x \ \in \ \Delta$
| $c \ \in \ \Delta$
| $c\,\mathbf{not\,relevant} \ \in \ \gamma$
| $x \ \notin \ \Delta$
| $\mathit{uniq}\,\Gamma$
| $\mathit{uniq}(\Omega)$
| $c \ \notin \ \Delta$
| $T \ \notin \ \mathsf{dom}\,\Sigma$
| $F \ \notin \ \mathsf{dom}\,\Sigma$
| $\mathbb{N}_1 < \mathbb{N}_2$
| $\mathbb{N}_1 \leq \mathbb{N}_2$
| $R_1 = R_2$
| $a = b$
| $\phi_1 = \phi_2$

	|	$\Gamma_1 = \Gamma_2$	
	|	$\gamma_1 = \gamma_2$	
	|	$\neg\psi$	
	|	$\psi_1 \wedge \psi_2$	
	|	$\psi_1 \vee \psi_2$	
	|	$\psi_1 \Rightarrow \psi_2$	
	|	(ψ)	
	|	ψ	
	|	$c : (a : A \sim b : B) \in \Gamma$	
	|		suppress lc hypothesis gen
$JSubRole$	::=		
	|	$R_1 \leq R_2$	Subroling judgement
$JPath$	::=		
	|	$\mathsf{Path}\ a = F@Rs$	Type headed by constant
$JPatCtx$	::=		
	|	$\Omega; \Gamma \vDash p :_F B \Rightarrow A$	Contexts generated by a p
$JRename$	::=		
	|	$\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'$	rename with fresh variable
$JMatchSubst$	::=		
	|	$\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 = b_2$	match and substitute
$JPatData$	::=		
	|	$\mathbf{uncurry}\, p = F@PA$	Pattern data (head argun
$JIsPattern$	::=		
	|	$\mathbf{pattern}\, p$	
$JSubPat$	::=		
	|	$\mathbf{subpattern}\, p'\, p$	Subpattern
$JTmPatternAgree$	::=		
	|	$a \leftrightarrow p$	term and pattern agree
$JTmSubPatternAgree$	::=		
	|	$a \sqsubseteq p$	sub-pattern agrees with te
$JSubTmPatternAgree$	::=		
	|	$a \sqsupseteq p$	sub-term agrees with patt
$JValuePath$	::=		
	|	$\mathsf{ValuePath}\ a = F$	Type headed by constant

JCasePath	::=		
	|	$\mathsf{CasePath}_R\ a = F$	Type headed by constant (role-sensitive part
JApplyArgs	::=		
	|	$\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'$	apply arguments of a (headed by a constant
JValue	::=		
	|	$\mathsf{Value}_R\ A$	values
JValueType	::=		
	|	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	::=		
	|	$\mathsf{consistent}_R\ a\ b$	(erased) types do not differ in their heads
Jroleing	::=		
	|	$\Omega \vDash a : R$	Roleing judgment
JChk	::=		
	|	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	::=		
	|	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	|	$\Omega \vDash a \Rightarrow^*_R b$	multistep parallel reduction
	|	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	::=		
	|	$\vDash a > b/R$	primitive reductions on erased terms
	|	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit langu
	|	$\vDash a \rightsquigarrow^* b/R$	multistep reduction
JBranchTyping	::=		
	|	$\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
Jett	::=		
	|	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	|	$\Gamma \vDash a : A$	typing
	|	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	|	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness
Jsig	::=		
	|	$\vDash \Sigma$	signature wellformedness
Jann	::=		
	|	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	|	$\Gamma \vdash a : A/R$	typing

	|	$\Gamma ; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	|	$\Gamma ; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	|	$\vdash \Gamma$	context wellformedness
Jred	::=		
	|	$\Gamma \vdash a \rightsquigarrow b / R$	single-step, weak head reduction to values for annotated la
judgement	::=		
	|	*JSubRole*	
	|	*JPath*	
	|	*JPatCtx*	
	|	*JRename*	
	|	*JMatchSubst*	
	|	*JPatData*	
	|	*JIsPattern*	
	|	*JSubPat*	
	|	*JTmPatternAgree*	
	|	*JTmSubPatternAgree*	
	|	*JSubTmPatternAgree*	
	|	*JValuePath*	
	|	*JCasePath*	
	|	*JApplyArgs*	
	|	*JValue*	
	|	*JValueType*	
	|	*Jconsistent*	
	|	*Jroleing*	
	|	*JChk*	
	|	*Jpar*	
	|	*Jbeta*	
	|	*JBranchTyping*	
	|	*Jett*	
	|	*Jsig*	
	|	*Jann*	
	|	*Jred*	
user_syntax	::=		
	|	*tmvar*	
	|	*covar*	
	|	*datacon*	
	|	*const*	
	|	*index*	
	|	*relflag*	
	|	*appflag*	
	|	*role*	
	|	*constraint*	
	|	*tm*	
	|	*brs*	

| *co*
| *role_context*
| *roles*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *Nat*
| *pattern_arg*
| *pattern_args*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{Path}\ a = F@Rs}$ Type headed by constant (partial function)

$$\frac{F : A@Rs \in \Sigma_0}{\mathsf{Path}\ F = F@Rs} \quad \textsc{Path\_AbsConst}$$

$$\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0}{\mathsf{Path}\ F = F@Rs} \quad \textsc{Path\_Const}$$

$$\frac{\mathsf{Path}\ a = F@R_1, Rs}{\mathsf{Path}\ (a\ b'^{R_1}) = F@Rs} \quad \textsc{Path\_App}$$

$$\frac{\mathsf{Path}\ a = F@Rs}{\mathsf{Path}\ (a\ b^{-}) = F@Rs} \quad \textsc{Path\_IApp}$$

$$\frac{\mathsf{Path}\ a = F@Rs}{\mathsf{Path}\ (a[\bullet]) = F@Rs} \quad \textsc{Path\_CApp}$$

$\boxed{\Omega; \Gamma \vDash p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\varnothing; \varnothing \vDash F :_F A \Rightarrow A} \quad \textsc{PatCtx\_Const}$$

$$\frac{\Omega; \Gamma \vDash p :_F \Pi^{+} x : A' \to A \Rightarrow B}{\Omega, x : R; \Gamma, x : A' \vDash p\ x^{R} :_F A \Rightarrow B} \quad \textsc{PatCtx\_PiRel}$$

$$\frac{\Omega; \Gamma \vDash p :_F \Pi^{-} x : A' \to A \Rightarrow B}{\Omega; \Gamma, x : A' \vDash p\ \square^{-} :_F A \Rightarrow B} \quad \textsc{PatCtx\_PiIrr}$$

$$\frac{\Omega; \Gamma \vDash p :_F \forall c : \phi . A \Rightarrow B}{\Omega; \Gamma, c : \phi \vDash p[\bullet] :_F A \Rightarrow B} \quad \textsc{PatCtx\_CPi}$$

$\boxed{\textsf{rename } p \to a \textsf{ to } p' \to a' \textsf{ excluding } \Delta \textsf{ and } \Delta'}$ rename with fresh variables

$$\frac{}{\textsf{rename } F \to a \textsf{ to } F \to a \textsf{ excluding } \Delta \textsf{ and } \varnothing} \quad \textsc{Rename\_Base}$$

$$\frac{\begin{array}{l}\textsf{rename } p_1 \to a_1 \textsf{ to } p_2 \to a_2 \textsf{ excluding } \Delta \textsf{ and } \Delta' \\ y \notin (\Delta, \Delta')\end{array}}{\textsf{rename } (p_1\ x^R) \to a_1 \textsf{ to } (p_2\ y^R) \to (a_2\{y/x\}) \textsf{ excluding } \Delta \textsf{ and } (\Delta', y)} \quad \textsc{Rename\_AppRel}$$

$$\frac{\textsf{rename } p_1 \to a_1 \textsf{ to } p_2 \to a_2 \textsf{ excluding } \Delta \textsf{ and } \Delta'}{\textsf{rename } (p_1\ \Box^-) \to a_1 \textsf{ to } (p_2\ \Box^-) \to a_2 \textsf{ excluding } \Delta \textsf{ and } \Delta'} \quad \textsc{Rename\_AppIrrel}$$

$$\frac{\textsf{rename } p_1 \to a_1 \textsf{ to } p_2 \to a_2 \textsf{ excluding } \Delta \textsf{ and } \Delta'}{\textsf{rename } (p_1[\bullet]) \to a_1 \textsf{ to } (p_2[\bullet]) \to a_2 \textsf{ excluding } \Delta \textsf{ and } \Delta'} \quad \textsc{Rename\_CApp}$$

$\boxed{\textsf{match } a_1 \textsf{ with } p \to b_1 = b_2}$ match and substitute

$$\frac{}{\textsf{match } F \textsf{ with } F \to b = b} \quad \textsc{MatchSubst\_Const}$$

$$\frac{\textsf{match } a_1 \textsf{ with } p_1 \to b_1 = b_2}{\textsf{match } (a_1\ a^R) \textsf{ with } (p_1\ x^R) \to b_1 = (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRelR}$$

$$\frac{\textsf{match } a_1 \textsf{ with } p_1 \to b_1 = b_2}{\textsf{match } (a_1\ a^-) \textsf{ with } (p_1\ \Box^-) \to b_1 = b_2} \quad \textsc{MatchSubst\_AppIrrel}$$

$$\frac{\textsf{match } a_1 \textsf{ with } a_2 \to b_1 = b_2}{\textsf{match } (a_1[\bullet]) \textsf{ with } (a_2[\bullet]) \to b_1 = b_2} \quad \textsc{MatchSubst\_CApp}$$

$\boxed{\textbf{uncurry}\, p = F@PA}$ Pattern data (head arguments)

$$\frac{}{\textbf{uncurry}\, F = F@\textbf{none}} \quad \textsc{PatData\_Head}$$

$$\frac{\textbf{uncurry}\, p = F@PA}{\textbf{uncurry}\, (p\ a^R) = F@PA, \backslash Rel a\, R} \quad \textsc{PatData\_Rel}$$

$\boxed{\textbf{pattern}\, p}$

$$\frac{}{\textbf{pattern}\, F} \quad \textsc{Pattern\_Head}$$

$$\frac{\textbf{pattern}\, p}{\textbf{pattern}\, (p\ a^R)} \quad \textsc{Pattern\_Rel}$$

$\boxed{\textbf{subpattern}\, p'\, p}$ Subpattern

$$\frac{\textbf{pattern}\, p}{\textbf{subpattern}\, p\, p} \quad \textsc{SubPat\_Refl}$$

$$\frac{\textbf{subpattern}\, p'\, p}{\textbf{subpattern}\, p'\, (p\ x^R)} \quad \textsc{SubPat\_Rel}$$

$$\frac{\textbf{subpattern}\, p'\, p}{\textbf{subpattern}\, p'\, (p\ \Box^-)} \quad \textsc{SubPat\_Irr}$$

$$\frac{\textbf{subpattern}\, p'\, p}{\textbf{subpattern}\, p'\, (p[\bullet])} \quad \textsc{SubPat\_Coe}$$

$\boxed{a \leftrightarrow p}$ term and pattern agree

$$\frac{}{F \leftrightarrow F} \quad \textsc{tm\_pattern\_agree\_Const}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \ a_2{}^R) \leftrightarrow (p_1\ \ x^R)} \quad \textsc{tm\_pattern\_agree\_AppRelR}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \ a^-) \leftrightarrow (p_1\ \ \Box^-)} \quad \textsc{tm\_pattern\_agree\_AppIrrel}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1[\bullet]) \leftrightarrow (p_1[\bullet])} \quad \textsc{tm\_pattern\_agree\_CApp}$$

$\boxed{a \sqsubseteq p}$ sub-pattern agrees with term

$$\frac{a \leftrightarrow p}{a \sqsubseteq p} \quad \textsc{tm\_subpattern\_agree\_Base}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ \ x^R)} \quad \textsc{tm\_subpattern\_agree\_AppRelR}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ \ \Box^-)} \quad \textsc{tm\_subpattern\_agree\_AppIrrel}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p[\bullet])} \quad \textsc{tm\_subpattern\_agree\_CAppp}$$

$\boxed{a \sqsupseteq p}$ sub-term agrees with pattern

$$\frac{a \leftrightarrow p}{a \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_Base}$$

$$\frac{a \sqsupseteq p}{a\ \ a_2{}^\nu \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_App}$$

$$\frac{a \sqsupseteq p}{a[\bullet] \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_CAppp}$$

$\boxed{\mathsf{ValuePath}\ \ a = F}$ Type headed by constant (role-sensitive partial function used in value)

$$\frac{F : A@Rs\ \in\ \Sigma_0}{\mathsf{ValuePath}\ \ F = F} \quad \textsc{ValuePath\_AbsConst}$$

$$\frac{F :\ p \sim a : A/R_1@Rs\ \in\ \Sigma_0}{\mathsf{ValuePath}\ \ F = F} \quad \textsc{ValuePath\_Const}$$

$$\frac{\mathsf{ValuePath}\ \ a = F}{\mathsf{ValuePath}\ \ (a\ \ b'^\nu) = F} \quad \textsc{ValuePath\_App}$$

$$\frac{\mathsf{ValuePath}\ \ a = F}{\mathsf{ValuePath}\ \ (a[\bullet]) = F} \quad \textsc{ValuePath\_CApp}$$

$\boxed{\mathsf{CasePath}_R\ \ a = F}$ Type headed by constant (role-sensitive partial function used in case)

$$\frac{\begin{array}{c}\mathsf{ValuePath}\ \ a = F \\ F : A@Rs\ \in\ \Sigma_0\end{array}}{\mathsf{CasePath}_R\ \ a = F} \quad \textsc{CasePath\_AbsConst}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F\\ F : p \sim b : A/R_1@Rs \in \Sigma_0\\ \neg(R_1 \leq R)\end{array}}{\mathsf{CasePath}_R\ a = F}\quad \textsc{CasePath\_Const}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F\\ F : p \sim b : A/R_1@Rs \in \Sigma_0\\ \neg(a \sqsupseteq p)\end{array}}{\mathsf{CasePath}_R\ a = F}\quad \textsc{CasePath\_UnMatch}$$

$\boxed{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\mathsf{apply\ args}\ F\ \mathsf{to}\ b \mapsto b}\quad \textsc{ApplyArgs\_Const}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}{\mathsf{apply\ args}\ a\ a'^{\nu}\ \mathsf{to}\ b \mapsto b'\ a'^{\nu}}\quad \textsc{ApplyArgs\_App}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}{\mathsf{apply\ args}\ a[\bullet]\ \mathsf{to}\ b \mapsto b'[\bullet]}\quad \textsc{ApplyArgs\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star}\quad \textsc{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^{\rho} x\!:\!A \to B}\quad \textsc{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B}\quad \textsc{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x\!:\!A.a}\quad \textsc{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x.a}\quad \textsc{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^{-} x.a}\quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a}\quad \textsc{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a}\quad \textsc{Value\_UCAbs}$$

$$\frac{\mathsf{CasePath}_R\ a = F}{\mathsf{Value}_R\ a}\quad \textsc{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\quad \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^{\rho} x\!:\!A \to B}\quad \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B}\quad \textsc{value\_type\_CPi}$$

$$\frac{\mathsf{CasePath}_R\ a = F}{\mathsf{ValueType}_R\ a}\quad \textsc{value\_type\_ValuePath}$$

$\boxed{\mathsf{consistent}_R\ a\ b}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\ \star}\quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1 \!:\! A_1 \to B_1)\ (\Pi^\rho x_2 \!:\! A_2 \to B_2)}\quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1 \!:\! \phi_1 . A_1)\ (\forall c_2 \!:\! \phi_2 . A_2)}\quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{c}\mathsf{CasePath}_R\ a_1 = F\\ \mathsf{CasePath}_R\ a_2 = F\end{array}}{\mathsf{consistent}_R\ a_1\ a_2}\quad \textsc{consistent\_a\_CasePath}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ a\ b}\quad \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ a\ b}\quad \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \Box : R}\quad \textsc{role\_a\_Bullet}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R}\quad \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ x : R \in \Omega\\ R \leq R_1\end{array}}{\Omega \vDash x : R_1}\quad \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x . a) : R}\quad \textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{c}\Omega \vDash a : R\\ \Omega \vDash b : \mathbf{Nom}\end{array}}{\Omega \vDash (a\ b^\rho) : R}\quad \textsc{role\_a\_App}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \mathsf{Path}\ a = F@R_1, Rs\\ \Omega \vDash b : R_1\end{array}}{\Omega \vDash a\ b^{R_1} : R}\quad \textsc{role\_a\_TApp}$$

$$\frac{\begin{array}{c}\Omega \vDash A : R\\ \Omega, x : \mathbf{Nom} \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x \!:\! A \to B) : R}\quad \textsc{role\_a\_Pi}$$

$$\frac{\begin{array}{c}\Omega \vDash a : R_1\\ \Omega \vDash b : R_1\\ \Omega \vDash A : R_0\\ \Omega \vDash B : R\end{array}}{\Omega \vDash (\forall c \!:\! a \sim_{A/R_1} b . B) : R}\quad \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c . b) : R}\quad \textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R}\quad \textsc{role\_a\_CApp}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F : A@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R} \quad \textsc{role\_a\_Const}$$

$$\frac{\begin{array}{l} uniq(\Omega) \\ F : p \sim a : A/R@Rs \in \Sigma_0 \end{array}}{\Omega \vDash F : R_1} \quad \textsc{role\_a\_Fam}$$

$$\frac{\begin{array}{l} \Omega \vDash a : R \\ \Omega \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1 \end{array}}{\Omega \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : R_1} \quad \textsc{role\_a\_Pattern}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b' \end{array}}{\Omega \vDash a\ b^\rho \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{(\mathbf{app\_role}\,\nu)} b' \end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\ b'^\nu} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{l} \Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \Pi^\rho x\!:\!A \to B \Rightarrow_R \Pi^\rho x\!:\!A' \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{l} \Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \forall c\!:\!a \sim_{A/R_1} b.B \Rightarrow_R \forall c\!:\!a' \sim_{A'/R_1} b'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l} F : F \sim b : A/R_1 @ Rs \in \Sigma_0 \\ R_1 \leq R \\ uniq(\Omega) \end{array}}{\Omega \vDash F \Rightarrow_R b} \quad \textsc{Par\_AxiomBase}$$

$$\frac{\begin{array}{l} F : p \sim b : A/R_1 @ Rs \in \Sigma_0 \\ a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash a_1 \Rightarrow_{(\mathbf{app\_role}\,\nu)} a_1' \\ \textsf{rename } p \rightarrow b \textsf{ to } p' \rightarrow b' \textsf{ excluding } (\widetilde{\Omega}, \textsf{fv} p) \textsf{ and } \Delta' \\ \textsf{match } (a' \ {a_1'}^{\nu}) \textsf{ with } p' \rightarrow b' = a_2 \\ R_1 \leq R \end{array}}{\Omega \vDash a \ {a_1}^{\nu} \Rightarrow_R a_2} \quad \textsc{Par\_AxiomApp}$$

$$\frac{\begin{array}{l} F : p \sim b : A/R_1 @ Rs \in \Sigma_0 \\ a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\ \Omega \vDash a \Rightarrow_R a' \\ \textsf{rename } p \rightarrow b \textsf{ to } p' \rightarrow b' \textsf{ excluding } (\widetilde{\Omega}, \textsf{fv} p) \textsf{ and } \Delta' \\ \textsf{match } (a'[\bullet]) \textsf{ with } p' \rightarrow b' = a_2 \\ R_1 \leq R \end{array}}{\Omega \vDash a[\bullet] \Rightarrow_R a_2} \quad \textsc{Par\_AxiomCApp}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \end{array}}{\Omega \vDash (\textsf{case}_R \ a \textsf{ of } F \rightarrow b_1 \|_{\_} \rightarrow b_2) \Rightarrow_{R_0} (\textsf{case}_R \ a' \textsf{ of } F \rightarrow b_1' \|_{\_} \rightarrow b_2')} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \textsf{CasePath}_R \ a' = F \\ \textsf{apply args } a' \textsf{ to } b_1' \mapsto b \end{array}}{\Omega \vDash (\textsf{case}_R \ a \textsf{ of } F \rightarrow b_1 \|_{\_} \rightarrow b_2) \Rightarrow_{R_0} b[\bullet]} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \textsf{Value}_R \ a' \\ \neg(\textsf{CasePath}_R \ a' = F) \end{array}}{\Omega \vDash (\textsf{case}_R \ a \textsf{ of } F \rightarrow b_1 \|_{\_} \rightarrow b_2) \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vDash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow_R^* a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R b \\ \Omega \vDash b \Rightarrow_R^* a' \end{array}}{\Omega \vDash a \Rightarrow_R^* a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{l} \Omega \vDash a_1 \Rightarrow_R^* b \\ \Omega \vDash a_2 \Rightarrow_R^* b \end{array}}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \textsc{join}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^{\rho}x.v)}{\vDash (\lambda^{\rho}x.v)\ b^{\rho} > v\{b/x\}/R_1}\quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R}\quad \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l} F : p \sim b : A/R_1 @ Rs \in \Sigma_0 \\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p_1 \to b_1\ \mathsf{excluding}\ (\mathsf{fv}a, \mathsf{fv}p)\ \mathsf{and}\ \Delta' \\ \mathsf{match}\ a\ \mathsf{with}\ p_1 \to b_1 = b' \\ R_1 \leq R \end{array}}{\vDash a > b'/R}\quad \textsc{Beta\_Axiom}$$

$$\frac{\begin{array}{l} \mathsf{CasePath}_R\ a = F \\ \mathsf{apply\ args}\ a\ \mathsf{to}\ b_1 \mapsto b_1' \end{array}}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 > b_1'[\bullet]/R_0}\quad \textsc{Beta\_PatternTrue}$$

$$\frac{\begin{array}{l} \mathsf{Value}_R\ a \\ \neg(\mathsf{CasePath}_R\ a = F) \end{array}}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 > b_2/R_0}\quad \textsc{Beta\_PatternFalse}$$

$\boxed{\vDash a \leadsto b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \leadsto a'/R_1}{\vDash \lambda^{-}x.a \leadsto \lambda^{-}x.a'/R_1}\quad \textsc{E\_AbsTerm}$$

$$\frac{\vDash a \leadsto a'/R_1}{\vDash a\ b^{\nu} \leadsto a'\ b^{\nu}/R_1}\quad \textsc{E\_AppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash a[\bullet] \leadsto a'[\bullet]/R}\quad \textsc{E\_CAppLeft}$$

$$\frac{\vDash a \leadsto a'/R}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 \leadsto \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1 \| \_ \to b_2/R_0}\quad \textsc{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \leadsto b/R}\quad \textsc{E\_Prim}$$

$\boxed{\vDash a \leadsto^{*} b/R}$ multistep reduction

$$\frac{}{\vDash a \leadsto^{*} a/R}\quad \textsc{Equal}$$

$$\frac{\begin{array}{c} \vDash a \leadsto b/R \\ \vDash b \leadsto^{*} a'/R \end{array}}{\vDash a \leadsto^{*} a'/R}\quad \textsc{Step}$$

$\boxed{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{c} \mathit{uniq}\ \Gamma \\ \mathtt{lc\_tm}\ C \end{array}}{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : A \Rightarrow \forall c{:}(a \sim_{A/R} b).C \mid C}\quad \textsc{BranchTyping\_Base}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b\ x^{+} : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b : \Pi^{+}x{:}A \to B \Rightarrow \Pi^{+}x{:}A \to C \mid C'}\quad \textsc{BranchTyping\_PiRel}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b\ \square^{-} : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b : \Pi^{-}x{:}A \to B \Rightarrow \Pi^{-}x{:}A \to C \mid C'}\quad \textsc{BranchTyping\_PiIrrel}$$

$$\frac{\Gamma, c : \phi \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : \forall c{:}\phi.B \Rightarrow \forall c{:}\phi.C \mid C'} \quad \textsc{BranchTyping\_CPi}$$

$\boxed{\Gamma \vDash \phi\ \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash b : A \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash a \sim_{A/R} b\ \ \mathsf{ok}} \quad \textsc{E\_Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \textsc{E\_Star}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ x : A\ \in\ \Gamma\end{array}}{\Gamma \vDash x : A} \quad \textsc{E\_Var}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash B : \star \\ \Gamma \vDash A : \star\end{array}}{\Gamma \vDash \Pi^\rho x{:}A \to B : \star} \quad \textsc{E\_Pi}$$

$$\frac{\begin{array}{l}\Gamma, x : A \vDash a : B \\ \Gamma \vDash A : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x{:}A \to B)} \quad \textsc{E\_Abs}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x{:}A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \ a^+ : B\{a/x\}} \quad \textsc{E\_App}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^+ x{:}A \to B \\ \Gamma \vDash a : A \\ \mathsf{Path}\ \ b = F@R, Rs\end{array}}{\Gamma \vDash b\ \ a^R : B\{a/x\}} \quad \textsc{E\_TApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash b : \Pi^- x{:}A \to B \\ \Gamma \vDash a : A\end{array}}{\Gamma \vDash b\ \square^- : B\{a/x\}} \quad \textsc{E\_IApp}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma \vDash a : B} \quad \textsc{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star \\ \Gamma \vDash \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c{:}\phi.B : \star} \quad \textsc{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B \\ \Gamma \vDash \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B} \quad \textsc{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c{:}(a \sim_{A/R} b).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \textsc{E\_CApp}$$

$$\frac{\models \Gamma \quad F : A@Rs \in \Sigma_0 \quad \varnothing \models A : \star}{\Gamma \models F : A} \quad \text{E\_CONST}$$

$$\frac{\models \Gamma \quad F : p \sim a : A/R_1@Rs \in \Sigma_0 \quad \varnothing \models A : \star}{\Gamma \models F : A} \quad \text{E\_FAM}$$

$$\frac{\Gamma \models a : A \quad \Gamma \models F : A_1 \quad \Gamma \models b_1 : B \quad \Gamma \models b_2 : C \quad \Gamma \models \mathsf{case}_R\ a : A\ \mathsf{of}\ F : A_1 \Rightarrow B \mid C}{\Gamma \models \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_\_ \to b_2 : C} \quad \text{E\_CASE}$$

$\boxed{\Gamma; \Delta \models \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\Gamma; \Delta \models A_1 \equiv A_2 : A/R \quad \Gamma; \Delta \models B_1 \equiv B_2 : A/R}{\Gamma; \Delta \models A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PROPCONG}$$

$$\frac{\Gamma; \Delta \models A \equiv B : \star/R_0 \quad \Gamma \models A_1 \sim_{A/R} A_2\ \mathsf{ok} \quad \Gamma \models A_1 \sim_{B/R} A_2\ \mathsf{ok}}{\Gamma; \Delta \models A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_ISOCONV}$$

$$\frac{\Gamma; \Delta \models \forall c{:}(a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c{:}(b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \models a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPIFST}$$

$\boxed{\Gamma; \Delta \models a \equiv b : A/R}$ definitional equality

$$\frac{\models \Gamma \quad c : (a \sim_{A/R} b) \in \Gamma \quad c \in \Delta}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_ASSN}$$

$$\frac{\Gamma \models a : A}{\Gamma; \Delta \models a \equiv a : A/R} \quad \text{E\_REFL}$$

$$\frac{\Gamma; \Delta \models b \equiv a : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_SYM}$$

$$\frac{\Gamma; \Delta \models a \equiv a_1 : A/R \quad \Gamma; \Delta \models a_1 \equiv b : A/R}{\Gamma; \Delta \models a \equiv b : A/R} \quad \text{E\_TRANS}$$

$$\frac{\Gamma; \Delta \models a \equiv b : A/R_1 \quad R_1 \leq R_2}{\Gamma; \Delta \models a \equiv b : A/R_2} \quad \text{E\_SUB}$$

$$\frac{\Gamma \models a_1 : B \quad \Gamma \models a_2 : B \quad \models a_1 > a_2/R}{\Gamma; \Delta \models a_1 \equiv a_2 : B/R} \quad \text{E\_BETA}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1;\Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_2 \to B_2 : \star\end{array}}{\Gamma;\Delta \vDash (\Pi^\rho x\!:\!A_1 \to B_1) \equiv (\Pi^\rho x\!:\!A_2 \to B_2) : \star/R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1;\Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma;\Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma;\Delta \vDash a_2 \equiv b_2 : A/\mathbf{Nom}\end{array}}{\Gamma;\Delta \vDash a_1\ \ {a_2}^+ \equiv b_1\ \ {b_2}^+ : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma;\Delta \vDash a_2 \equiv b_2 : A/R \\ \mathsf{Path}\ a_1 = F@R, Rs \\ \mathsf{Path}\ b_1 = F'@R, Rs'\end{array}}{\Gamma;\Delta \vDash a_1\ \ {a_2}^R \equiv b_1\ \ {b_2}^R : (B\{a_2/x\})/R'} \quad \text{E\_TAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A \to B)/R' \\ \Gamma \vDash a : A\end{array}}{\Gamma;\Delta \vDash a_1\ \Box^- \equiv b_1\ \Box^- : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R'}{\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R' \\ \Gamma;\Delta \vDash a_1 \equiv a_2 : A_1/\mathbf{Nom}\end{array}}{\Gamma;\Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1;\Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A : \star \\ \Gamma \vDash \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star\end{array}}{\Gamma;\Delta \vDash \forall c\!:\!a_1 \sim_{A_1/R} b_1.A \equiv \forall c\!:\!a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1;\Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma;\Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_{A/R} b).B)/R' \\ \Gamma;\widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\, R\, R'\end{array}}{\Gamma;\Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \forall c\!:\!(a_1 \sim_{A/R} a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\, R\, R_0 \\ \Gamma;\widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\, R'\, R_0\end{array}}{\Gamma;\Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R \\ \Gamma;\Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'\end{array}}{\Gamma;\Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_CAST}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R \\ \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star\end{array}}{\Gamma;\Delta \vDash a \equiv b : B/R} \quad \text{E\_EQCONV}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma;\Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_ISOSND}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv a' : A/R \\ \Gamma;\Delta \vDash b_1 \equiv b_1' : B/R_0 \\ \Gamma;\Delta \vDash b_2 \equiv b_2' : B/R_0\end{array}}{\Gamma;\Delta \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_- \to b_2 \equiv \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|_- \to b_2' : B/R_0} \quad \text{E\_PATCONG}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^{R_1} \equiv a'\ b'^{R_1} : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^+ x\!:\!A \to B/R'} \quad \text{E\_LEFTREL}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ \Box^- \equiv a'\ \Box^- : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^- x\!:\!A \to B/R'} \quad \text{E\_LEFTIRREL}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma;\Delta \vDash a\ b^+ \equiv a'\ b'^+ : B\{b/x\}/R' \\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0\end{array}}{\Gamma;\Delta \vDash b \equiv b' : A/\mathbf{param}\,R_1\ R'} \quad \text{E\_RIGHT}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a = F \\ \mathsf{ValuePath}\ a' = F \\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R' \\ \Gamma;\Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'} \quad \text{E\_CLEFT}$$

$\boxed{\models \Gamma}$ context wellformedness

$$\frac{}{\models \varnothing} \quad \text{E\_EMPTY}$$

$$\frac{\begin{array}{l}\models \Gamma \\ \Gamma \models A : \star \\ x \notin \widetilde{\Gamma}\end{array}}{\models \Gamma, x : A} \quad \text{E\_CONSTM}$$

$$\frac{\begin{array}{l}\models \Gamma \\ \Gamma \models \phi \ \mathsf{ok} \\ c \notin \widetilde{\Gamma}\end{array}}{\models \Gamma, c : \phi} \quad \text{E\_CONSCO}$$

$\boxed{\models \Sigma}$ signature wellformedness

$$\frac{}{\models \varnothing} \quad \text{SIG\_EMPTY}$$

$$\frac{\begin{array}{l}\models \Sigma \\ \varnothing \models A : \star \\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\models \Sigma \cup \{F : A@Rs\}} \quad \text{SIG\_CONSCONST}$$

$$\frac{\begin{array}{l}\models \Sigma \\ F \notin \mathsf{dom}\,\Sigma \\ \varnothing \models A : \star \\ \Omega; \Gamma \models p :_F B \Rightarrow A \\ \Gamma \models a : B \\ \Omega \models a : R\end{array}}{\models \Sigma \cup \{F :\ p \sim a : A/R@\mathbf{range}\,\Omega\}} \quad \text{SIG\_CONSAX}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A/R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

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Definition rules:        167 good    0 bad
Definition rule clauses: 460 good    0 bad
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