Specification of System D and System DC

tmvar, x, y, f, m, n	variables	
covar, c	coercion variables	
$\mathit{datacon}$, K		
const, T		
tyfam, F		
index, i	indices	

$\mathit{relflag},\ \rho$	$::=$		relevance flag
	$\mid$	$+$	
	$\mid$	$-$	
$\mathit{constraint},\ \phi$	$::=$		props
	$\mid$	$a \sim_A b$	
$\mathit{tm},\ a,\ b,\ v,\ w,\ A,\ B$	$::=$		types and kinds
	$\mid$	$\star$	
	$\mid$	x	
	$\mid$	$\lambda^\rho x\!:\!A.b$	annotated lambda
	$\mid$	$\lambda^\rho x.b$	erased lambda
	$\mid$	$a\ b^\rho$	
	$\mid$	F	definition
	$\mid$	T	constant
	$\mid$	$\Pi^\rho x\!:\!A \to B$	
	$\mid$	$a \triangleright \gamma$	coercion
	$\mid$	$\forall c\!:\!\phi.B$	
	$\mid$	$\Lambda c\!:\!\phi.b$	annotated coercion abstraction
	$\mid$	$\Lambda c.b$	erased coercion abstraction
	$\mid$	$a[\gamma]$	
	$\mid$	$\square$	erased term
	$\mid$	$\Sigma^\rho x\!:\!A.B$	
	$\mid$	$({}^\rho a, b)$	erased pair
	$\mid$	$({}^\rho a, b)\ \mathsf{as}\ A$	annotated pair
	$\mid$	$\mathbf{fst}\ a$	
	$\mid$	$\mathbf{snd}\ a$	

$co,\ \gamma$	$::=$		explicit coercions
	$\mid$	$\bullet$	erased coercion
	$\mid$	c	
	$\mid$	$\mathbf{red}\, a\ b$	
	$\mid$	$\mathbf{refl}\, a$	
	$\mid$	$(a \mid=\mid_{\gamma}\ b)$	
	$\mid$	$\mathbf{sym}\, \gamma$	
	$\mid$	$\gamma_1 ; \gamma_2$	
	$\mid$	$\Pi^{\rho} x : \gamma_1 . \gamma_2$	
	$\mid$	$\lambda^{\rho} x : \gamma_1 . \gamma_2$	
	$\mid$	$\gamma_1\ \gamma_2^{\rho}$	
	$\mid$	$\mathbf{piFst}\, \gamma$	
	$\mid$	$\mathbf{cpiFst}\, \gamma$	
	$\mid$	$\mathbf{isoSnd}\, \gamma$	
	$\mid$	$\gamma_1 @ \gamma_2$	
	$\mid$	$\forall c : \gamma_1 . \gamma_3$	
	$\mid$	$\lambda c : \gamma_1 . \gamma_3 @ \gamma_4$	
	$\mid$	$\gamma(\gamma_1, \gamma_2)$	
	$\mid$	$\gamma @ (\gamma_1 \sim \gamma_2)$	
	$\mid$	$\gamma_1 \triangleright \gamma_2$	
	$\mid$	$\gamma_1 \sim_A \gamma_2$	
	$\mid$	$\mathbf{conv}\ \phi_1 \sim_{\gamma} \phi_2$	
	$\mid$	$\mathbf{eta}\, a$	
	$\mid$	$\mathbf{left}\, \gamma\, \gamma'$	
	$\mid$	$\mathbf{right}\, \gamma\, \gamma'$	
	$\mid$	$\Sigma x :^{\rho} \gamma_1 . \gamma_2$	
	$\mid$	$(\rho\, \gamma_1, \gamma_2)\, \mathbf{as}\, \gamma_3$	
	$\mid$	$\mathbf{fst}\, \gamma_1$	
	$\mid$	$\mathbf{snd}\, \gamma_1$	
	$\mid$	$\mathbf{sigmaFst}\, \gamma$	

sig_sort	::=		signature classifier
	|	$\mathbf{Cs}\,A$	
	|	$\mathbf{Ax}\,a\,A$	
sort	::=		binding classifier
	|	$\mathbf{Tm}\,\rho\,A$	
	|	$\mathbf{Co}\,\phi$	
context, Γ	::=		contexts
	|	$\varnothing$	
	|	$\Gamma, x : \rho\, A$	
	|	$\Gamma, c : \phi$	
available_props, Δ	::=		
	|	$\varnothing$	
	|	Δ, c	
sig, Σ	::=		signatures
	|	$\varnothing$	
	|	$\Sigma \cup \{T : A\}$	
	|	$\Sigma \cup \{F \sim a : A\}$	

$\boxed{\mathbf{CoercedValue}\,A}$ *(Values with at most one coercion at the top)*

CV
$$\frac{[\mathsf{Value}\ a]}{\mathbf{CoercedValue}\,a}$$

CC
$$\frac{[\mathsf{Value}\ a]}{\mathbf{CoercedValue}\,(a \rhd \gamma)}$$

$\boxed{[\mathsf{Value}\ A]}$ *(values)*

Value-Star
$$\frac{}{[\mathsf{Value}\ \star]}$$

Value-Pi
$$\frac{}{[\mathsf{Value}\ \Pi^{\rho} x\!:\!A \to B]}$$

Value-CPi
$$\frac{}{[\mathsf{Value}\ \forall c\!:\!\phi.B]}$$

Value-AbsRel
$$\frac{}{[\mathsf{Value}\ \lambda^{+} x\!:\!A.a]}$$

Value-UAbsRel
$$\frac{}{[\mathsf{Value}\ \lambda^{+} x.a]}$$

Value-UAbsIrrel
$$\frac{[\mathsf{Value}\ a]}{[\mathsf{Value}\ \lambda^{-} x.a]}$$

Value-AbsIrrel
$$\frac{\mathbf{CoercedValue}\,a}{[\mathsf{Value}\ \lambda^{-} x\!:\!A.a]}$$

Value-CAbs
$$\frac{}{[\mathsf{Value}\ \Lambda c\!:\!\phi.a]}$$

Value-UCAbs
$$\frac{}{[\mathsf{Value}\ \Lambda c.a]}$$

Value-Sigma
$$\frac{}{[\mathsf{Value}\ \Sigma^{\rho} x\!:\!A.B]}$$

Value-UPair
$$\frac{}{[\mathsf{Value}\ ({}^{\rho}\,a, b)]}$$

Value-Pair
$$\frac{}{[\mathsf{Value}\ ({}^{\rho}\,a, b)\ \mathsf{as}\ A]}$$

$\boxed{\textbf{ValueType}\ A}$ *(Types with head forms (erased language))*

$$\frac{}{\textbf{ValueType}\ \star}\ \textsc{value-type-Star} \qquad \frac{}{\textbf{ValueType}\ \Pi^\rho x\!:\!A \to B}\ \textsc{value-type-Pi} \qquad \frac{}{\textbf{ValueType}\ \forall c\!:\!\phi.B}\ \textsc{value-type-CPi}$$

$$\frac{}{\textbf{ValueType}\ \Sigma^\rho x\!:\!A.B}\ \textsc{value-type-Sigma}$$

$\boxed{\textbf{consistent}\ a\ b}$ *((erased) types do not differ in their heads)*

$$\frac{}{\textbf{consistent}\ \star\ \star}\ \textsc{consistent-a-Star} \qquad \frac{}{\textbf{consistent}\ (\Pi^\rho x_1\!:\!A_1 \to B_1)\ (\Pi^\rho x_2\!:\!A_2 \to B_2)}\ \textsc{consistent-a-Pi}$$

$$\frac{}{\textbf{consistent}\ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)}\ \textsc{consistent-a-CPi}$$

$$\frac{}{\textbf{consistent}\ (\Sigma^\rho x\!:\!A_1.B_1)\ (\Sigma^\rho x\!:\!A_2.B_2)}\ \textsc{consistent-a-Sigma} \qquad \frac{\textbf{not ValueType}\ b}{\textbf{consistent}\ a\ b}\ \textsc{consistent-a-Step-R}$$

$$\frac{\textbf{not ValueType}\ a}{\textbf{consistent}\ a\ b}\ \textsc{consistent-a-Step-L}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ *(irrelevant argument check)*

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)}\ \textsc{Rho-Rel} \qquad \frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)}\ \textsc{Rho-IrrRel}$$

$\boxed{\textbf{erased\_tm}\ a}$ *()*

$$\frac{}{\textbf{erased\_tm}\ \Box}\ \textsc{erased-a-Bullet} \qquad \frac{}{\textbf{erased\_tm}\ \star}\ \textsc{erased-a-Star} \qquad \frac{}{\textbf{erased\_tm}\ x}\ \textsc{erased-a-Var}$$

$$\frac{\textbf{erased\_tm}\ a \quad (\rho = +) \vee (x \notin \mathsf{fv}\ a)}{\textbf{erased\_tm}\ (\lambda^\rho x.a)}\ \textsc{erased-a-Abs} \qquad \frac{\textbf{erased\_tm}\ a \quad \textbf{erased\_tm}\ b}{\textbf{erased\_tm}\ (a\ \ b^+)}\ \textsc{erased-a-App} \qquad \frac{\textbf{erased\_tm}\ a}{\textbf{erased\_tm}\ (a\ \Box^-)}\ \textsc{erased-a-AppIrrel}$$

$$\textsc{erased-a-Pi}\quad \frac{\textbf{erased\_tm}\ A \qquad \textbf{erased\_tm}\ B}{\textbf{erased\_tm}\ (\Pi^\rho x\!:\!A \to B)}$$

$$\textsc{erased-a-CPi}\quad \frac{\textbf{erased\_tm}\ a \qquad \textbf{erased\_tm}\ b \qquad \textbf{erased\_tm}\ A \qquad \textbf{erased\_tm}\ B}{\textbf{erased\_tm}\ (\forall c\!:\!a \sim_A b.B)}$$

$$\textsc{erased-a-CAbs}\quad \frac{\textbf{erased\_tm}\ b}{\textbf{erased\_tm}\ (\Lambda c.b)}$$

$$\textsc{erased-a-CApp}\quad \frac{\textbf{erased\_tm}\ a}{\textbf{erased\_tm}\ (a[\bullet])}$$

$$\textsc{erased-a-Fam}\quad \frac{}{\textbf{erased\_tm}\ F}$$

$$\textsc{erased-a-Sigma}\quad \frac{\textbf{erased\_tm}\ A \qquad \textbf{erased\_tm}\ B}{\textbf{erased\_tm}\ (\Sigma^\rho x\!:\!A.B)}$$

$$\textsc{erased-a-UPairRel}\quad \frac{\textbf{erased\_tm}\ a \qquad \textbf{erased\_tm}\ b}{\textbf{erased\_tm}\ ({}^+a, b)}$$

$$\textsc{erased-a-UPairIrrel}\quad \frac{\textbf{erased\_tm}\ b}{\textbf{erased\_tm}\ ({}^-\Box, b)}$$

$$\textsc{erased-a-Fst}\quad \frac{\textbf{erased\_tm}\ a}{\textbf{erased\_tm}\ (\textbf{fst}\ a)}$$

$$\textsc{erased-a-Snd}\quad \frac{\textbf{erased\_tm}\ a}{\textbf{erased\_tm}\ (\textbf{snd}\ a)}$$

$\boxed{\vDash a \Rightarrow b}$ *(parallel reduction (implicit language))*

$$\textsc{Par-Refl}\quad \frac{}{\vDash a \Rightarrow a}$$

$$\textsc{Par-Beta}\quad \frac{\vDash a \Rightarrow (\lambda^+ x.a') \qquad \vDash b \Rightarrow b'}{\vDash a\ b^+ \Rightarrow a'\{b'/x\}}$$

$$\textsc{Par-BetaIrrel}\quad \frac{\vDash a \Rightarrow (\lambda^- x.a')}{\vDash a\ \Box^- \Rightarrow a'\{\Box/x\}}$$

$$\textsc{Par-App}\quad \frac{\vDash a \Rightarrow a' \qquad \vDash b \Rightarrow b'}{\vDash a\ b^+ \Rightarrow a'\ b'^+}$$

$$\textsc{Par-AppIrrel}\quad \frac{\vDash a \Rightarrow a'}{\vDash a\ \Box^- \Rightarrow a'\ \Box^-}$$

$$\textsc{Par-CBeta}\quad \frac{\vDash a \Rightarrow (\Lambda c.a')}{\vDash a[\bullet] \Rightarrow a'\{\bullet/c\}}$$

$$\textsc{Par-CApp}\quad \frac{\vDash a \Rightarrow a'}{\vDash a[\bullet] \Rightarrow a'[\bullet]}$$

$$\textsc{Par-Abs}\quad \frac{\vDash a \Rightarrow a'}{\vDash \lambda^\rho x.a \Rightarrow \lambda^\rho x.a'}$$

$$\textsc{Par-Pi}\quad \frac{\vDash A \Rightarrow A' \qquad \vDash B \Rightarrow B'}{\vDash \Pi^\rho x\!:\!A \to B \Rightarrow \Pi^\rho x\!:\!A' \to B'}$$

$$\textsc{Par-CAbs}\quad \frac{\vDash a \Rightarrow a'}{\vDash \Lambda c.a \Rightarrow \Lambda c.a'}$$

$$\textsc{Par-CPi}\quad \frac{\vDash A \Rightarrow A' \qquad \vDash B \Rightarrow B' \qquad \vDash a \Rightarrow a' \qquad \vDash A_1 \Rightarrow A_1'}{\vDash \forall c\!:\!A \sim_{A_1} B.a \Rightarrow \forall c\!:\!A' \sim_{A_1'} B'.a'}$$

$$\textsc{Par-Axiom}\quad \frac{F \sim a : A \in \Sigma_0}{\vDash F \Rightarrow a}$$

$$\textsc{Par-Eta}\quad \frac{\vDash b \Rightarrow b' \qquad a = b\ x^+}{\vDash \lambda^+ x.a \Rightarrow b'}$$

$$\textsc{Par-EtaIrrel}\quad \frac{\vDash b \Rightarrow b' \qquad a = b\ \Box^-}{\vDash \lambda^- x.a \Rightarrow b'}$$

$$\textsc{Par-EtaC}\quad \frac{\vDash b \Rightarrow b' \qquad a = b[\bullet]}{\vDash \Lambda c.a \Rightarrow b'}$$

$$\textsc{Par-Sigma}\quad \frac{\vDash A \Rightarrow A' \qquad \vDash B \Rightarrow B'}{\vDash \Sigma^{\rho} x\!:\!A.B \Rightarrow \Sigma^{\rho} x\!:\!A'.B'}$$

$$\textsc{Par-Pair}\quad \frac{\vDash a \Rightarrow a' \qquad \vDash b \Rightarrow b'}{\vDash ({}^{+}a, b) \Rightarrow ({}^{+}a', b')}$$

$$\textsc{Par-PairIrrel}\quad \frac{\vDash b \Rightarrow b'}{\vDash ({}^{-}\Box, b) \Rightarrow ({}^{-}\Box, b')}$$

$$\textsc{Par-Fst}\quad \frac{\vDash a \Rightarrow ({}^{\rho}a_1, a_2)}{\vDash \mathbf{fst}\, a \Rightarrow a_1}$$

$$\textsc{Par-Snd}\quad \frac{\vDash a \Rightarrow ({}^{\rho}a_1, a_2)}{\vDash \mathbf{snd}\, a \Rightarrow a_2}$$

$\boxed{\vdash a \Rightarrow^* b}$ *(multistep parallel reduction)*

$$\text{MP-}\textsc{Refl}\quad \frac{}{\vdash a \Rightarrow^* a}$$

$$\text{MP-}\textsc{Step}\quad \frac{\vDash a \Rightarrow b \qquad \vdash b \Rightarrow^* a'}{\vdash a \Rightarrow^* a'}$$

$\boxed{\vdash a \Leftrightarrow b}$ *(parallel reduction to a common term)*

$$\textsc{join}\quad \frac{\vdash a_1 \Rightarrow^* b \qquad \vdash a_2 \Rightarrow^* b}{\vdash a_1 \Leftrightarrow a_2}$$

$\boxed{\vDash a > b}$ *(primitive reductions on erased terms)*

$$\textsc{Beta-AppAbs}\quad \frac{}{\vDash (\lambda^{+}x.v)\ b^{+} > v\{b/x\}}$$

$$\textsc{Beta-AppAbsIrrel}\quad \frac{[\mathsf{Value}\ (\lambda^{-}x.v)]}{\vDash (\lambda^{-}x.v)\ \Box^{-} > v\{\Box/x\}}$$

$$\textsc{Beta-CAppCAbs}\quad \frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}}$$

$$\textsc{Beta-Axiom}\quad \frac{F \sim a : A \in \Sigma_0}{\vDash F > a}$$

$$\textsc{Beta-Fst}\quad \frac{}{\vDash \mathbf{fst}\, ({}^{\rho}a, b) > a}$$

$$\textsc{Beta-Snd}\quad \frac{}{\vDash \mathbf{snd}\, ({}^{\rho}a, b) > b}$$

$\boxed{\vDash a \leadsto b}$ *(single-step head reduction for implicit language)*

$$\textsc{E-AbsTerm}\quad \frac{\vDash a \leadsto a'}{\vDash \lambda^- x.a \leadsto \lambda^- x.a'}$$

$$\textsc{E-AppLeft}\quad \frac{\vDash a \leadsto a'}{\vDash a\ b^+ \leadsto a'\ b^+}$$

$$\textsc{E-AppLeftIrrel}\quad \frac{\vDash a \leadsto a'}{\vDash a\ \Box^- \leadsto a'\ \Box^-}$$

$$\textsc{E-CAppLeft}\quad \frac{\vDash a \leadsto a'}{\vDash a[\bullet] \leadsto a'[\bullet]}$$

$$\textsc{E-AppAbs}\quad \frac{}{\vDash (\lambda^+ x.v)\ a^+ \leadsto v\{a/x\}}$$

$$\textsc{E-AppAbsIrrel}\quad \frac{[\mathsf{Value}\ (\lambda^- x.v)]}{\vDash (\lambda^- x.v)\ \Box^- \leadsto v\{\Box/x\}}$$

$$\textsc{E-CAppCAbs}\quad \frac{}{\vDash (\Lambda c.b)[\bullet] \leadsto b\{\bullet/c\}}$$

$$\textsc{E-Axiom}\quad \frac{F \sim a : A \in \Sigma_0}{\vDash F \leadsto a}$$

$$\textsc{E-FstRed}\quad \frac{}{\vDash \mathbf{fst}\,({}^\rho a, b) \leadsto a}$$

$$\textsc{E-SndRed}\quad \frac{}{\vDash \mathbf{snd}\,({}^\rho a, b) \leadsto b}$$

$$\textsc{E-FstCongRed}\quad \frac{\vDash a \leadsto a'}{\vDash \mathbf{fst}\ a \leadsto \mathbf{fst}\ a'}$$

$$\textsc{E-SndCongRed}\quad \frac{\vDash a \leadsto a'}{\vDash \mathbf{snd}\ a \leadsto \mathbf{snd}\ a'}$$

$\boxed{\vDash a \leadsto^* b}$ *(multistep reduction)*

$$\textsc{Equal}\quad \frac{}{\vDash a \leadsto^* a}$$

$$\textsc{Step}\quad \frac{\begin{array}{c}\vDash a \leadsto b\\ \vDash b \leadsto^* a'\end{array}}{\vDash a \leadsto^* a'}$$

$\boxed{\Gamma \vDash \rho\,\phi\ \mathsf{ok}}$ *(Prop wellformedness)*

$$\textsc{E-Wff}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, a : A\\ \Gamma \vDash \rho\, b : A\\ [\Gamma \vDash -\, A : \star]\end{array}}{\Gamma \vDash \rho\, a \sim_A b\ \ \mathsf{ok}}$$

$\boxed{\Gamma \vDash \rho\, a : A}$ *(typing)*

$$\textsc{E-Star}\quad \frac{\vDash \Gamma}{\Gamma \vDash \rho \star : \star}$$

$$\textsc{E-Var}\quad \frac{\vDash \Gamma \qquad x : \rho\, A \in \Gamma}{\Gamma \vDash \rho\, x : A}$$

$$\textsc{E-Pi}\quad \frac{\begin{array}{c}\Gamma, x : \rho_1\, A \vDash \rho_1\, B : \star\\ \Gamma \vDash \rho_1\, A : \star\end{array}}{\Gamma \vDash \rho_1\, \Pi^\rho x\!:\!A \to B : \star}$$

$$\textsc{E-Abs}\quad \frac{\begin{array}{c}\Gamma, x : \rho\, A \vDash \rho_1\, a : B\\ [\Gamma \vDash -\, A : \star]\\ (\rho = +) \vee (x \notin \mathsf{fv}\ a)\end{array}}{\Gamma \vDash \rho_1\, \lambda^\rho x.a : \Pi^\rho x\!:\!A \to B}$$

$$\textsc{E-App}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, b : \Pi^+ x\!:\!A \to B\\ \Gamma \vDash \rho\, a : A\end{array}}{\Gamma \vDash \rho\, b\ a^+ : B\{a/x\}}$$

$$\textsc{E-IApp}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, b : \Pi^{-} x\!:\!A \to B \\ \Gamma \vDash -\, a : A\end{array}}{\Gamma \vDash \rho\, b\ \square^{-} : B\{a/x\}}$$

$$\textsc{E-Conv}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, a : A \\ \Gamma; \widetilde{\Gamma} \vDash -\, A \equiv B : \star \\ [\Gamma \vDash -\, B : \star]\end{array}}{\Gamma \vDash \rho\, a : B}$$

$$\textsc{E-CPi}\quad \frac{\begin{array}{c}\Gamma, c : \phi \vDash \rho\, B : \star \\ \Gamma \vDash \rho\, \phi\ \ \mathsf{ok}\end{array}}{\Gamma \vDash \rho \forall c\!:\!\phi.B : \star}$$

$$\textsc{E-CAbs}\quad \frac{\begin{array}{c}\Gamma, c : \phi \vDash \rho\, a : B \\ [\Gamma \vDash -\, \phi\ \ \mathsf{ok}]\end{array}}{\Gamma \vDash \rho\, \Lambda c.a : \forall c\!:\!\phi.B}$$

$$\textsc{E-CApp}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, a_1 : \forall c\!:\!(a \sim_A b).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash -\, a \equiv b : A\end{array}}{\Gamma \vDash \rho\, a_1[\bullet] : B_1\{\bullet/c\}}$$

$$\textsc{E-Fam}\quad \frac{\begin{array}{c}\vDash \Gamma \\ F \sim a : A \in \Sigma_0 \\ [\varnothing \vDash -\, A : \star]\end{array}}{\Gamma \vDash \rho\, F : A}$$

$$\textsc{E-Sigma}\quad \frac{\begin{array}{c}\Gamma, x : \rho\, A \vDash \rho\, B : \star \\ \Gamma \vDash \rho\, A : \star\end{array}}{\Gamma \vDash \rho\, \Sigma^{\rho}_{1} x\!:\!A.B : \star}$$

$$\textsc{E-Pair}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, a : A \\ \Gamma \vDash \rho\, b : B\{a/x\}\end{array}}{\Gamma \vDash \rho\, ({}^{+}a, b) : \Sigma^{+} x\!:\!A.B}$$

$$\textsc{E-PairIrrel}\quad \frac{\begin{array}{c}\Gamma \vDash -\, a : A \\ \Gamma \vDash \rho\, b : B\{a/x\}\end{array}}{\Gamma \vDash \rho\, ({}^{-}\square, b) : \Sigma^{-} x\!:\!A.B}$$

$$\textsc{E-Fst}\quad \frac{\Gamma \vDash \rho\, a : \Sigma^{+} x\!:\!A.B}{\Gamma \vDash \rho\, \mathbf{fst}\ a : A}$$

$$\textsc{E-FstIrrel}\quad \frac{\Gamma \vDash -\, a : \Sigma^{-} x\!:\!A.B}{\Gamma \vDash -\, \mathbf{fst}\ a : A}$$

$$\textsc{E-Snd}\quad \frac{\Gamma \vDash \rho_1\, a : \Sigma^{\rho} x\!:\!A.B}{\Gamma \vDash \rho_1\, \mathbf{snd}\ a : B\{\mathbf{fst}\ a/x\}}$$

$\boxed{\Gamma; \Delta \vDash \rho\, \phi_1 \equiv \phi_2}$ *(prop equality)*

$$\textsc{E-PropCong}\quad \frac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, A_1 \equiv A_2 : A \\ \Gamma; \Delta \vDash \rho\, B_1 \equiv B_2 : A\end{array}}{\Gamma; \Delta \vDash \rho\, A_1 \sim_A B_1 \equiv A_2 \sim_A B_2}$$

$$\textsc{E-IsoConv}\quad \frac{\begin{array}{c}\Gamma; \Delta \vDash -\, A \equiv B : \star \\ \Gamma \vDash \rho\, A_1 \sim_A A_2\ \ \mathsf{ok} \\ \Gamma \vDash \rho\, A_1 \sim_B A_2\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash \rho\, A_1 \sim_A A_2 \equiv A_1 \sim_B A_2}$$

$$\textsc{E-CPiFst}\quad \frac{\Gamma; \Delta \vDash \rho \forall c\!:\!\phi_1.B_1 \equiv \forall c\!:\!\phi_2.B_2 : \star}{\Gamma; \Delta \vDash \rho\, \phi_1 \equiv \phi_2}$$

$\boxed{\Gamma; \Delta \vDash \rho\, a \equiv b : A}$ *(definitional equality)*

$$\textsc{E-Assn}\quad \frac{\begin{array}{c}\vDash \Gamma \\ c : (a \sim_A b) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash \rho\, a \equiv b : A}$$

$$\textsc{E-Refl}\quad \frac{\Gamma \vDash \rho\, a : A}{\Gamma; \Delta \vDash \rho\, a \equiv a : A}$$

$$\textsc{E-Sym}\quad \frac{\Gamma; \Delta \vDash \rho\, b \equiv a : A}{\Gamma; \Delta \vDash \rho\, a \equiv b : A}$$

$$\textsc{E-Trans}\quad \frac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, a \equiv a_1 : A \\ \Gamma; \Delta \vDash \rho\, a_1 \equiv b : A\end{array}}{\Gamma; \Delta \vDash \rho\, a \equiv b : A}$$

$$\textsc{E-Beta}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, a_1 : B \\ \Gamma \vDash \rho\, a_2 : B \\ \vDash a_1 > a_2\end{array}}{\Gamma; \Delta \vDash \rho\, a_1 \equiv a_2 : B}$$

E-PiCong

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash \rho_1\, A_1 \equiv A_2 : \star \\ \Gamma, x : \rho_1\, A_1; \Delta \vDash \rho_1\, B_1 \equiv B_2 : \star \\ [\Gamma \vDash \rho_1\, A_1 : \star] \\ [\Gamma \vDash \rho_1\, \Pi^\rho x\!:\!A_1 \to B_1 : \star] \\ [\Gamma \vDash \rho_1\, \Pi^\rho x\!:\!A_2 \to B_2 : \star]\end{array}}{\Gamma; \Delta \vDash \rho_1\, (\Pi^\rho x\!:\!A_1 \to B_1) \equiv (\Pi^\rho x\!:\!A_2 \to B_2) : \star}$$

E-AbsCong

$$\frac{\begin{array}{c}\Gamma, x : \rho\, A_1; \Delta \vDash \rho_1\, b_1 \equiv b_2 : B \\ [\Gamma \vDash -\, A_1 : \star] \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash \rho_1\, (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : \Pi^\rho x\!:\!A_1 \to B}$$

E-AppCong

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, a_1 \equiv b_1 : \Pi^+ x\!:\!A \to B \\ \Gamma; \Delta \vDash \rho\, a_2 \equiv b_2 : A\end{array}}{\Gamma; \Delta \vDash \rho\, a_1\ \ {a_2}^+ \equiv b_1\ \ {b_2}^+ : B\{a_2/x\}}$$

E-IAppCong

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, a_1 \equiv b_1 : \Pi^- x\!:\!A \to B \\ \Gamma \vDash -\, a : A\end{array}}{\Gamma; \Delta \vDash \rho\, a_1\ \Box^- \equiv b_1\ \Box^- : B\{a/x\}}$$

E-PiFst

$$\frac{\Gamma; \Delta \vDash \rho_1\, \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star}{\Gamma; \Delta \vDash \rho_1\, A_1 \equiv A_2 : \star}$$

E-PiSnd

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash \rho_1\, \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star \\ \Gamma; \Delta \vDash \rho_1\, a_1 \equiv a_2 : A_1\end{array}}{\Gamma; \Delta \vDash \rho_1\, B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star}$$

E-CPiCong

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, \phi_1 \equiv \phi_2 \\ \Gamma, c : \phi_1; \Delta \vDash \rho\, A \equiv B : \star \\ [\Gamma \vDash \rho\, \phi_1\ \ \mathsf{ok}] \\ [\Gamma \vDash \rho\, \forall c\!:\!\phi_1.A : \star] \\ [\Gamma \vDash \rho\, \forall c\!:\!\phi_2.B : \star]\end{array}}{\Gamma; \Delta \vDash \rho\, \forall c\!:\!\phi_1.A \equiv \forall c\!:\!\phi_2.B : \star}$$

E-CAbsCong

$$\frac{\begin{array}{c}\Gamma, c : \phi_1; \Delta \vDash \rho\, a \equiv b : B \\ [\Gamma \vDash -\, \phi_1\ \ \mathsf{ok}]\end{array}}{\Gamma; \Delta \vDash \rho\, (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B}$$

E-CAppCong

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, a_1 \equiv b_1 : \forall c\!:\!(a \sim_A b).B \\ \Gamma; \widetilde{\Gamma} \vDash -\, a \equiv b : A\end{array}}{\Gamma; \Delta \vDash \rho\, a_1[\bullet] \equiv b_1[\bullet] : B\{\bullet/c\}}$$

$$\textsc{E-CPiSnd}\quad \frac{\begin{array}{c}\Gamma;\Delta \vDash \rho\,\forall c\!:\!(a_1 \sim_A a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'} a_2').B_2 : \star \\ \Gamma;\widetilde{\Gamma} \vDash -\, a_1 \equiv a_2 : A \\ \Gamma;\widetilde{\Gamma} \vDash -\, a_1' \equiv a_2' : A'\end{array}}{\Gamma;\Delta \vDash \rho\, B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star}$$

$$\textsc{E-Cast}\quad \frac{\begin{array}{c}\Gamma;\Delta \vDash \rho\, a \equiv b : A \\ \Gamma;\Delta \vDash \rho\, a \sim_A b \equiv a' \sim_{A'} b'\end{array}}{\Gamma;\Delta \vDash \rho\, a' \equiv b' : A'}$$

$$\textsc{E-EqConv}\quad \frac{\begin{array}{c}\Gamma;\Delta \vDash \rho\, a \equiv b : A \\ \Gamma;\widetilde{\Gamma} \vDash -\, A \equiv B : \star\end{array}}{\Gamma;\Delta \vDash \rho\, a \equiv b : B}$$

$$\textsc{E-IsoSnd}\quad \frac{\Gamma;\Delta \vDash -\, a \sim_A b \equiv a' \sim_{A'} b'}{\Gamma;\Delta \vDash -\, A \equiv A' : \star}$$

$$\textsc{E-EtaRel}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, b : \Pi^+ x\!:\!A \to B \\ a = b\ x^+\end{array}}{\Gamma;\Delta \vDash \rho\, \lambda^+ x.a \equiv b : \Pi^+ x\!:\!A \to B}$$

$$\textsc{E-EtaIrrel}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, b : \Pi^- x\!:\!A \to B \\ a = b\ \Box^-\end{array}}{\Gamma;\Delta \vDash \rho\, \lambda^- x.a \equiv b : \Pi^- x\!:\!A \to B}$$

$$\textsc{E-EtaC}\quad \frac{\begin{array}{c}\Gamma \vDash \rho\, b : \forall c\!:\!\phi.B \\ a = b[\bullet]\end{array}}{\Gamma;\Delta \vDash \rho\, \Lambda c.a \equiv b : \forall c\!:\!\phi.B}$$

$$\textsc{E-SigmaCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vDash \rho\, A_1 \equiv A_2 : \star \\ \Gamma, x : \rho\, A_1;\Delta \vDash \rho\, B_1 \equiv B_2 : \star \\ [\Gamma \vDash \rho\, A_1 : \star] \\ [\Gamma \vDash \rho\, \Sigma_1^\rho x\!:\!A_1.B_1 : \star] \\ [\Gamma \vDash \rho\, \Sigma_1^\rho x\!:\!A_2.B_2 : \star]\end{array}}{\Gamma;\Delta \vDash \rho\, (\Sigma_1^\rho x\!:\!A_1.B_1) \equiv (\Sigma_1^\rho x\!:\!A_2.B_2) : \star}$$

$$\textsc{E-PairCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vDash \rho\, a_1 \equiv b_1 : A \\ \Gamma;\Delta \vDash \rho\, a_2 \equiv b_2 : B\{a_1/x\}\end{array}}{\Gamma;\Delta \vDash \rho\, ({}^+a_1, a_2) \equiv ({}^+b_1, b_2) : \Sigma^+ x\!:\!A.B}$$

$$\textsc{E-PairCongIrrel}\quad \frac{\begin{array}{c}\Gamma \vDash -\, a : A \\ \Gamma;\Delta \vDash \rho\, a_2 \equiv b_2 : B\{a/x\}\end{array}}{\Gamma;\Delta \vDash \rho\, ({}^-\Box, a_2) \equiv ({}^-\Box, b_2) : \Sigma^- x\!:\!A.B}$$

$$\textsc{E-FstCong}\quad \frac{\Gamma;\Delta \vDash \rho\, a_1 \equiv b_1 : \Sigma^+ x\!:\!A.B}{\Gamma;\Delta \vDash \rho\, \mathbf{fst}\ a_1 \equiv \mathbf{fst}\ b_1 : A}$$

$$\textsc{E-FstCongIrrel}\quad \frac{\Gamma;\Delta \vDash -\, a_1 \equiv b_1 : \Sigma^- x\!:\!A.B}{\Gamma;\Delta \vDash -\, \mathbf{fst}\ a_1 \equiv \mathbf{fst}\ b_1 : A}$$

$$\textsc{E-SndCong}\quad \frac{\Gamma;\Delta \vDash \rho\, a_1 \equiv b_1 : \Sigma^\rho x\!:\!A.B}{\Gamma;\Delta \vDash \rho\, \mathbf{snd}\ a_1 \equiv \mathbf{snd}\ b_1 : B\{\mathbf{fst}\ a_1/x\}}$$

$\boxed{\vDash \Gamma}$ *(context wellformedness)*

$$\textsc{E-Empty}\quad \frac{}{\vDash \varnothing}$$

$$\textsc{E-ConsTm}\quad \frac{\vDash \Gamma \qquad \Gamma \vDash -\, A : \star \qquad x \notin \mathsf{dom}\,\Gamma}{\vDash \Gamma, x : \rho\, A}$$

$$\textsc{E-ConsCo}\quad \frac{\vDash \Gamma \qquad \Gamma \vDash -\, \phi\ \mathsf{ok} \qquad c \notin \mathsf{dom}\,\Gamma}{\vDash \Gamma, c : \phi}$$

$\boxed{\vDash \Sigma}$ *(signature wellformedness)*

$$\textsc{Sig-Empty}\quad \frac{}{\vDash \varnothing}$$

$$\textsc{Sig-ConsAx}\quad \frac{\vDash \Sigma \qquad \varnothing \vDash -\, A : \star \qquad \varnothing \vDash +\, a : A \qquad F \notin \mathsf{dom}\,\Sigma}{\vDash \Sigma \cup \{F \sim a : A\}}$$

$\boxed{\Gamma \vdash \phi\ \mathsf{ok}}$ *(prop wellformedness)*

$$\textsc{An-Wff}\quad \frac{\Gamma \vdash a : A \qquad \Gamma \vdash b : B \qquad |A| = |B|}{\Gamma \vdash a \sim_A b\ \mathsf{ok}}$$

$\boxed{\Gamma \vdash a : A}$ *(typing)*

$$\textsc{An-Star}\quad \frac{\vdash \Gamma}{\Gamma \vdash \star : \star}$$

$$\textsc{An-Var}\quad \frac{\vdash \Gamma \qquad x : \rho\, A \in \Gamma}{\Gamma \vdash x : A}$$

$$\textsc{An-Pi}\quad \frac{\Gamma, x : \rho\, A \vdash B : \star \qquad [\Gamma \vdash A : \star]}{\Gamma \vdash \Pi^\rho x\!:\!A \to B : \star}$$

$$\textsc{An-Abs}\quad \frac{[\Gamma \vdash A : \star] \qquad \Gamma, x : \rho\, A \vdash a : B \qquad (\rho = +) \lor (x \notin \mathsf{fv}\ |a|)}{\Gamma \vdash \lambda^\rho x\!:\!A.a : \Pi^\rho x\!:\!A \to B}$$

$$\textsc{An-App}\quad \frac{\Gamma \vdash b : \Pi^\rho x\!:\!A \to B \qquad \Gamma \vdash a : A}{\Gamma \vdash b\ a^\rho : B\{a/x\}}$$

$$\textsc{An-Conv}\quad \frac{\Gamma \vdash a : A \qquad \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim B \qquad \Gamma \vdash B : \star}{\Gamma \vdash a \rhd \gamma : B}$$

$$\textsc{An-CPi}\quad \frac{[\Gamma \vdash \phi\ \mathsf{ok}] \qquad \Gamma, c : \phi \vdash B : \star}{\Gamma \vdash \forall c\!:\!\phi.B : \star}$$

$$\textsc{An-CAbs}\quad \frac{[\Gamma \vdash \phi\ \mathsf{ok}] \qquad \Gamma, c : \phi \vdash a : B}{\Gamma \vdash \Lambda c\!:\!\phi.a : \forall c\!:\!\phi.B}$$

$$\textsc{An-CApp}\quad \frac{\Gamma \vdash a_1 : \forall c\!:\!a \sim_{A_1} b.B \qquad \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim b}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}}$$

$$\textsc{An-Fam}\quad \frac{\vdash \Gamma \qquad F \sim a : A \in \Sigma_1 \qquad [\varnothing \vdash A : \star]}{\Gamma \vdash F : A}$$

$$\textsc{An-Sigma}\quad \frac{\Gamma, x : \rho\, A \vdash B : \star \qquad [\Gamma \vdash A : \star]}{\Gamma \vdash \Sigma^\rho x\!:\!A.B : \star}$$

$$\textsc{An-Pair}\quad \frac{\Gamma \vdash a : A \qquad \Gamma \vdash b : B\{a/x\}}{\Gamma \vdash ({}^\rho a, b)\ \mathsf{as}\ \Sigma^\rho x\!:\!A.B : \Sigma^\rho x\!:\!A.B}$$

$$\textsc{An-Fst}\quad \frac{\Gamma \vdash a : \Sigma^\rho x\!:\!A.B}{\Gamma \vdash \mathbf{fst}\ a : A}$$

$$\textsc{An-Snd}\quad \frac{\Gamma \vdash a : \Sigma^\rho x\!:\!A.B}{\Gamma \vdash \mathbf{snd}\ a : B\{\mathbf{fst}\ a/x\}}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ *(coercion between props)*

$$\textsc{An-PropCong}\quad \frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim B_2 \\ \Gamma \vdash A_1 \sim_A B_1\ \ \mathsf{ok} \\ \Gamma \vdash A_2 \sim_A B_2\ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_A B_1) \sim (A_2 \sim_A B_2)}$$

$$\textsc{An-CPiFst}\quad \frac{\Gamma; \Delta \vdash \gamma : \forall c\!:\!\phi_1.A_2 \sim \forall c\!:\!\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\,\gamma : \phi_1 \sim \phi_2}$$

$$\textsc{An-IsoSym}\quad \frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\,\gamma : \phi_2 \sim \phi_1}$$

$$\textsc{An-IsoConv}\quad \frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma : A \sim B \\ \Gamma \vdash a_1 \sim_A a_2\ \ \mathsf{ok} \\ \Gamma \vdash a_1' \sim_B a_2'\ \ \mathsf{ok} \\ |a_1| = |a_1'| \\ |a_2| = |a_2'|\end{array}}{\Gamma; \Delta \vdash \mathbf{conv}\ (a_1 \sim_A a_2) \sim_\gamma (a_1' \sim_B a_2') : (a_1 \sim_A a_2) \sim (a_1' \sim_B a_2')}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim B}$ *(coercion between types)*

$$\textsc{An-Assn}\quad \frac{\begin{array}{c}\vdash \Gamma \\ c : a \sim_A b\ \in\ \Gamma \\ c\ \in\ \Delta\end{array}}{\Gamma; \Delta \vdash c : a \sim b}$$

$$\textsc{An-Refl}\quad \frac{\Gamma \vdash a : A}{\Gamma; \Delta \vdash \mathbf{refl}\,a : a \sim a}$$

$$\textsc{An-EraseEq}\quad \frac{\begin{array}{c}\Gamma \vdash a : A \\ \Gamma \vdash b : B \qquad |a| = |b| \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim B\end{array}}{\Gamma; \Delta \vdash (a \mathrel{|{=}|_\gamma} b) : a \sim b}$$

$$\textsc{An-Sym}\quad \frac{\begin{array}{c}\Gamma \vdash b : B \\ \Gamma \vdash a : A \\ [\Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim A] \\ \Gamma; \Delta \vdash \gamma : b \sim a\end{array}}{\Gamma; \Delta \vdash \mathbf{sym}\,\gamma : a \sim b}$$

$$\textsc{An-Trans}\quad \frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma_1 : a \sim a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim b \\ [\Gamma \vdash a : A] \\ [\Gamma \vdash a_1 : A_1] \\ [\Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim A_1]\end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim b}$$

$$\textsc{An-Beta}\quad \frac{\begin{array}{c}\Gamma \vdash a_1 : B_0 \\ \Gamma \vdash a_2 : B_1 \\ |B_0| = |B_1| \\ \models |a_1| > |a_2|\end{array}}{\Gamma; \Delta \vdash \mathbf{red}\ a_1\ a_2 : a_1 \sim a_2}$$

$$\textsc{An-PiCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma, x : \rho\, A_1;\Delta \vdash \gamma_2 : B_1 \sim B_2 \\ B_3 = B_2\{x \triangleright \mathbf{sym}\,\gamma_1/x\} \\ \Gamma \vdash \Pi^\rho x\!:\!A_1 \to B_1 : \star \\ \Gamma \vdash \Pi^\rho x\!:\!A_2 \to B_3 : \star \\ \Gamma \vdash (\Pi^\rho x\!:\!A_1 \to B_2) : \star\end{array}}{\Gamma;\Delta \vdash \Pi^\rho x\!:\!\gamma_1.\gamma_2 : (\Pi^\rho x\!:\!A_1 \to B_1) \sim (\Pi^\rho x\!:\!A_2 \to B_3)}$$

$$\textsc{An-AbsCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma, x : \rho\, A_1;\Delta \vdash \gamma_2 : b_1 \sim b_2 \\ b_3 = b_2\{x \triangleright \mathbf{sym}\,\gamma_1/x\} \\ [\Gamma \vdash A_1 : \star] \\ \Gamma \vdash A_2 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\,|b_1|) \\ (\rho = +) \vee (x \notin \mathsf{fv}\,|b_3|) \\ [\Gamma \vdash (\lambda^\rho x\!:\!A_1.b_2) : B]\end{array}}{\Gamma;\Delta \vdash (\lambda^\rho x\!:\!\gamma_1.\gamma_2) : (\lambda^\rho x\!:\!A_1.b_1) \sim (\lambda^\rho x\!:\!A_2.b_3)}$$

$$\textsc{An-AppCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim b_1 \\ \Gamma;\Delta \vdash \gamma_2 : a_2 \sim b_2 \\ \Gamma \vdash a_1\ {a_2}^\rho : A \\ \Gamma \vdash b_1\ {b_2}^\rho : B \\ [\Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim B]\end{array}}{\Gamma;\Delta \vdash \gamma_1\ \gamma_2^\rho : a_1\ {a_2}^\rho \sim b_1\ {b_2}^\rho}$$

$$\textsc{An-PiFst}\quad \frac{\Gamma;\Delta \vdash \gamma : \Pi^\rho x\!:\!A_1 \to B_1 \sim \Pi^\rho x\!:\!A_2 \to B_2}{\Gamma;\Delta \vdash \mathbf{piFst}\,\gamma : A_1 \sim A_2}$$

$$\textsc{An-PiSnd}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : \Pi^\rho x\!:\!A_1 \to B_1 \sim \Pi^\rho x\!:\!A_2 \to B_2 \\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim a_2 \\ \Gamma \vdash a_1 : A_1 \\ \Gamma \vdash a_2 : A_2\end{array}}{\Gamma;\Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim B_2\{a_2/x\}}$$

$$\textsc{An-CPiCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : \phi_1 \sim \phi_2 \\ \Gamma, c : \phi_1;\Delta \vdash \gamma_3 : B_1 \sim B_2 \\ B_3 = B_2\{c \triangleright \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash \forall c\!:\!\phi_1.B_1 : \star \\ [\Gamma \vdash \forall c\!:\!\phi_2.B_3 : \star] \\ \Gamma \vdash \forall c\!:\!\phi_1.B_2 : \star\end{array}}{\Gamma;\Delta \vdash (\forall c\!:\!\gamma_1.\gamma_3) : (\forall c\!:\!\phi_1.B_1) \sim (\forall c\!:\!\phi_2.B_3)}$$

$$\textsc{An-CAbsCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : \phi_1 \sim \phi_2 \\ \Gamma, c:\phi_1;\Delta \vdash \gamma_3 : a_1 \sim a_2 \\ a_3 = a_2\{c \triangleright \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash (\Lambda c\!:\!\phi_1.a_1) : \forall c\!:\!\phi_1.B_1 \\ \Gamma \vdash (\Lambda c\!:\!\phi_2.a_3) : \forall c\!:\!\phi_2.B_2 \\ \Gamma \vdash (\Lambda c\!:\!\phi_1.a_2) : B \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : \forall c\!:\!\phi_1.B_1 \sim \forall c\!:\!\phi_2.B_2\end{array}}{\Gamma;\Delta \vdash (\lambda c\!:\!\gamma_1.\gamma_3@\gamma_4) : (\Lambda c\!:\!\phi_1.a_1) \sim (\Lambda c\!:\!\phi_2.a_3)}$$

$$\textsc{An-CAppCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim b_1 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a_2 \sim b_2 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : a_3 \sim b_3 \\ \Gamma \vdash a_1[\gamma_2] : A \\ \Gamma \vdash b_1[\gamma_3] : B \\ [\Gamma;\widetilde{\Gamma} \vdash \gamma_4 : A \sim B]\end{array}}{\Gamma;\Delta \vdash \gamma_1(\gamma_2,\gamma_3) : a_1[\gamma_2] \sim b_1[\gamma_3]}$$

$$\textsc{An-CPiSnd}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : (\forall c_1\!:\!a \sim_A a'.B_1) \sim (\forall c_2\!:\!b \sim_B b'.B_2) \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a \sim a' \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : b \sim b'\end{array}}{\Gamma;\Delta \vdash \gamma_1@(\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim B_2\{\gamma_3/c_2\}}$$

$$\textsc{An-Cast}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : a \sim a' \\ \Gamma;\Delta \vdash \gamma_2 : a \sim_A a' \sim b \sim_B b'\end{array}}{\Gamma;\Delta \vdash \gamma_1 \triangleright \gamma_2 : b \sim b'}$$

$$\textsc{An-IsoSnd}\quad \frac{\Gamma;\Delta \vdash \gamma : (a \sim_A a') \sim (b \sim_B b')}{\Gamma;\Delta \vdash \mathbf{isoSnd}\,\gamma : A \sim B}$$

$$\textsc{An-Eta}\quad \frac{\begin{array}{c}\Gamma \vdash b : \Pi^\rho x\!:\!A \to B \\ a = b\ x^\rho\end{array}}{\Gamma;\Delta \vdash \mathbf{eta}\,b : (\lambda^\rho x\!:\!A.a) \sim b}$$

$$\textsc{An-EtaC}\quad \frac{\begin{array}{c}\Gamma \vdash b : \forall c\!:\!\phi.B \\ a = b[c]\end{array}}{\Gamma;\Delta \vdash \mathbf{eta}\,b : (\Lambda c\!:\!\phi.a) \sim b}$$

$$\textsc{An-SigmaCong}\quad \frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma, x:\rho\,A_1;\Delta \vdash \gamma_2 : B_1 \sim B_2 \\ B_3 = B_2\{x \triangleright \mathbf{sym}\,\gamma_1/x\} \\ \Gamma \vdash \Sigma^\rho x\!:\!A_1.B_1 : \star \\ \Gamma \vdash \Sigma^\rho x\!:\!A_2.B_3 : \star \\ \Gamma \vdash (\Sigma^\rho x\!:\!A_1.B_2) : \star\end{array}}{\Gamma;\Delta \vdash \Sigma x\!:^\rho\gamma_1.\gamma_2 : (\Sigma^\rho x\!:\!A_1.B_1) \sim (\Sigma^\rho x\!:\!A_2.B_3)}$$

$$\textsc{An-PairCong}\quad \frac{\Gamma;\Delta \vdash \gamma_1 : a_1 \sim b_1 \qquad \Gamma;\Delta \vdash \gamma_2 : a_2 \sim b_2 \qquad \Gamma;\Delta \vdash \gamma_3 : A_1 \sim A_2}{\Gamma;\Delta \vdash (\rho\,\gamma_1, \gamma_2)\,\mathbf{as}\,\gamma_3 : ({}^{\rho}\,a_1, a_2)\ \mathsf{as}\ A_1 \sim ({}^{\rho}\,b_1, b_2)\ \mathsf{as}\ A_2}$$

$$\textsc{An-FstCong}\quad \frac{\Gamma;\Delta \vdash \gamma : a_1 \sim b_1}{\Gamma;\Delta \vdash \mathbf{fst}\,\gamma : \mathbf{fst}\,a_1 \sim \mathbf{fst}\,b_1} \qquad \textsc{An-SndCong}\quad \frac{\Gamma;\Delta \vdash \gamma : a_1 \sim b_1}{\Gamma;\Delta \vdash \mathbf{snd}\,\gamma : \mathbf{snd}\,a_1 \sim \mathbf{snd}\,b_1}$$

$$\textsc{An-SigmaFst}\quad \frac{\Gamma;\Delta \vdash \gamma : \Sigma^{\rho} x\!:\!A_1.B_1 \sim \Sigma^{\rho} x\!:\!A_2.B_2}{\Gamma;\Delta \vdash \mathbf{sigmaFst}\,\gamma : A_1 \sim A_2}$$

$$\textsc{An-SigmaSnd}\quad \frac{\Gamma;\Delta \vdash \gamma_1 : \Sigma^{\rho} x\!:\!A_1.B_1 \sim \Sigma^{\rho} x\!:\!A_2.B_2 \qquad \Gamma;\Delta \vdash \gamma_2 : a_1 \sim a_2 \qquad \Gamma \vdash a_1 : A_1 \qquad \Gamma \vdash a_2 : A_2}{\Gamma;\Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim B_2\{a_2/x\}}$$

$\boxed{\vdash \Gamma}$ *(context wellformedness)*

$$\textsc{An-Empty}\quad \frac{}{\vdash \varnothing} \qquad \textsc{An-ConsTm}\quad \frac{\vdash \Gamma \qquad \Gamma \vdash A : \star \qquad x \notin \mathsf{dom}\,\Gamma}{\vdash \Gamma, x : \rho\,A} \qquad \textsc{An-ConsCo}\quad \frac{\vdash \Gamma \qquad \Gamma \vdash \phi\ \mathsf{ok} \qquad c \notin \mathsf{dom}\,\Gamma}{\vdash \Gamma, c : \phi}$$

$\boxed{\vdash \Sigma}$ *(signature wellformedness)*

$$\textsc{An-Sig-Empty}\quad \frac{}{\vdash \varnothing} \qquad \textsc{An-Sig-ConsAx}\quad \frac{\vdash \Sigma \qquad \varnothing \vdash A : \star \qquad \varnothing \vdash a : A \qquad F \notin \mathsf{dom}\,\Sigma}{\vdash \Sigma \cup \{F \sim a : A\}}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b}$ *(single-step, weak head reduction to values for annotated language)*

$$\textsc{An-AppLeft}\quad \frac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a\ b^{\rho} \rightsquigarrow a'\ b^{\rho}} \qquad \textsc{An-AppAbs}\quad \frac{[\mathsf{Value}\ (\lambda^{\rho} x\!:\!A.w)]}{\Gamma \vdash (\lambda^{\rho} x\!:\!A.w)\ a^{\rho} \rightsquigarrow w\{a/x\}} \qquad \textsc{An-CAppLeft}\quad \frac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma]}$$

$$\textsc{An-CAppCAbs}\quad \frac{}{\Gamma \vdash (\Lambda c\!:\!\phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\}} \qquad \textsc{An-AbsTerm}\quad \frac{\Gamma \vdash A : \star \qquad \Gamma, x : -\,A \vdash b \rightsquigarrow b'}{\Gamma \vdash (\lambda^{-} x\!:\!A.b) \rightsquigarrow (\lambda^{-} x\!:\!A.b')}$$

$$\frac{\textsc{An-Axiom}\quad F \sim a : A \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a}$$

$$\textsc{An-FstRed}\quad \frac{}{\Gamma \vdash \mathbf{fst}\,({}^{\rho}a, b) \rightsquigarrow a}$$

$$\textsc{An-SndRed}\quad \frac{}{\Gamma \vdash \mathbf{snd}\,({}^{\rho}a, b) \rightsquigarrow b}$$

$$\textsc{An-ConvTerm}\quad \frac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a \rhd \gamma \rightsquigarrow a' \rhd \gamma}$$

$$\textsc{An-Combine}\quad \frac{[\mathsf{Value}\ v]}{\Gamma \vdash (v \rhd \gamma_1) \rhd \gamma_2 \rightsquigarrow v \rhd (\gamma_1; \gamma_2)}$$

$$\textsc{An-Push}\quad \frac{\begin{array}{c}[\mathsf{Value}\ v] \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Pi^{\rho} x_1 \!:\! A_1 \to B_1 \sim \Pi^{\rho} x_2 \!:\! A_2 \to B_2 \\ b' = b \rhd \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \\ \gamma' = \gamma @ (b' \mid=\mid_{(\mathbf{piFst}\,\gamma)}\ b)\end{array}}{\Gamma \vdash (v \rhd \gamma)\ b^{\rho} \rightsquigarrow (v\ b'^{\rho}) \rhd \gamma'}$$

$$\textsc{An-CPush}\quad \frac{\begin{array}{c}[\mathsf{Value}\ v] \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \forall c_1 \!:\! \phi_1 . A_1 \sim \forall c_2 \!:\! \phi_2 . A_2 \\ \gamma_1' = \gamma_1 \rhd \mathbf{sym}\,(\mathbf{cpiFst}\,\gamma) \\ \gamma' = \gamma @ (\gamma_1' \sim \gamma_1)\end{array}}{\Gamma \vdash (v \rhd \gamma)[\gamma_1] \rightsquigarrow (v[\gamma_1']) \rhd \gamma'}$$

$$\textsc{An-FstPush}\quad \frac{\begin{array}{c}[\mathsf{Value}\ v] \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Sigma_1^{\rho} x_1 \!:\! A_1 . B_1 \sim \Sigma_2^{\rho} x_2 \!:\! A_2 . B_2\end{array}}{\Gamma \vdash \mathbf{fst}\,(v \rhd \gamma) \rightsquigarrow (\mathbf{fst}\ v) \rhd \mathbf{sigmaFst}\,\gamma}$$

$$\textsc{An-SndPush}\quad \frac{\begin{array}{c}[\mathsf{Value}\ v] \\ \Gamma; \widetilde{\Gamma} \vdash \gamma : \Sigma_1^{\rho} x_1 \!:\! A_1 . B_1 \sim \Sigma_2^{\rho} x_2 \!:\! A_2 . B_2 \\ b' = b \rhd \mathbf{sym}\,(\mathbf{sigmaFst}\,\gamma) \\ \gamma' = \gamma @ (b' \mid=\mid_{(\mathbf{sigmaFst}\,\gamma)}\ b)\end{array}}{\Gamma \vdash \mathbf{snd}\,(v \rhd \gamma) \rightsquigarrow (\mathbf{snd}\ v) \rhd \gamma'}$$