$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F	
$index$, i	indices

relflag, ρ	$::=$			relevance flag
	$\mid$	$+$		
	$\mid$	$-$		
	$\mid$	$\textbf{app\_rho}\, \nu$	S	
	$\mid$	(ρ)	S	
appflag, ν	$::=$			applicative flag
	$\mid$	R		
	$\mid$	ρ		
role, R	$::=$			Role
	$\mid$	$\mathbf{Nom}$		
	$\mid$	$\mathbf{Rep}$		
	$\mid$	$R_1 \cap R_2$	S	
	$\mid$	$R_1 \sqcap R_2$	S	
	$\mid$	$\textbf{app\_role}\, \nu\, R$	S	
	$\mid$	(R)	S	
constraint, ϕ	$::=$			props
	$\mid$	$a \sim_R b : A$		
	$\mid$	(ϕ)	S	
	$\mid$	$\phi\{b/x\}$	S	
	$\mid$	$\lvert\phi\rvert$	S	
	$\mid$	$a \sim_R b$	S	
	$\mid$	$a \sim b$	S	
tm, a, b, p, v, w, A, B, C	$::=$			types and kinds
	$\mid$	$\star$		
	$\mid$	x		
	$\mid$	$\lambda^\rho x\!:\!A.b$	bind x in b	
	$\mid$	$\lambda^\rho x.b$	bind x in b	
	$\mid$	$a\ b^\nu$		
	$\mid$	$\Pi^\rho x\!:\!A \to B$	bind x in B	
	$\mid$	$\Lambda c\!:\!\phi.b$	bind c in b	
	$\mid$	$\Lambda c.b$	bind c in b	
	$\mid$	$a[\gamma]$		
	$\mid$	$\forall c\!:\!\phi.B$	bind c in B	
	$\mid$	$a \triangleright_R \gamma$		
	$\mid$	F		
	$\mid$	$\square$		
	$\mid$	$\mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \Vert_{\_} \to b_2$		
	$\mid$	$a\overline{\mu}$	M	
	$\mid$	K		
	$\mid$	$\mathbf{match}\, a\, \mathbf{with}\, brs$		
	$\mid$	$\mathbf{sub}\, R\, a$		
	$\mid$	$\mathbf{coerce}\, a$		
	$\mid$	$a\ b$		

	$\vert$	$a\{b/x\}$	S	
	$\vert$	$a\{\gamma/c\}$	S	
	$\vert$	$a\{b/x\}$	S	
	$\vert$	$a\{\gamma/c\}$	S	
	$\vert$	a	S	
	$\vert$	a	S	
	$\vert$	(a)	S	
	$\vert$	a	S	parsing precedence is hard
	$\vert$	$\vert a\vert_R$	S	
	$\vert$	**Int**	S	
	$\vert$	**Age**	S	
	$\vert$	**Bool**	S	
	$\vert$	Nat	S	
	$\vert$	**Vec**	S	
	$\vert$	0	S	
	$\vert$	S	S	
	$\vert$	**True**	S	
	$\vert$	**Fix**	S	
	$\vert$	**Maybe**	S	
	$\vert$	**Just**	S	
	$\vert$	**Nothing**	S	
	$\vert$	$a \rightarrow b$	S	
	$\vert$	$\phi \Rightarrow A$	S	
	$\vert$	$\lambda x.a$	S	
	$\vert$	$\lambda x\!:\!A.a$	S	
	$\vert$	$\forall\, x : A \rightarrow B$	S	
	$\vert$	**if** ϕ **then** a **else** b	S	
brs	$::=$			case branches
	$\vert$	**none**		
	$\vert$	$K \Rightarrow a; brs$		
	$\vert$	$brs\{a/x\}$	S	
	$\vert$	$brs\{\gamma/c\}$	S	
	$\vert$	(brs)	S	
$co,\ \gamma$	$::=$			explicit coercions
	$\vert$	$\bullet$		
	$\vert$	c		
	$\vert$	**red** $a\ b$		
	$\vert$	**refl** a		
	$\vert$	$(a \vert\!=\!\vert_{\gamma}\ b)$		
	$\vert$	**sym** γ		
	$\vert$	$\gamma_1;\gamma_2$		
	$\vert$	**sub** γ		
	$\vert$	$\Pi^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	
	$\vert$	$\lambda^{R,\rho} x\!:\!\gamma_1.\gamma_2$	bind x in γ_2	

	|	$\gamma_1\ \gamma_2^{R,\rho}$		
	|	$\mathbf{piFst}\,\gamma$		
	|	$\mathbf{cpiFst}\,\gamma$		
	|	$\mathbf{isoSnd}\,\gamma$		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c\!:\!\gamma_1.\gamma_3$	bind c in γ_3	
	|	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	
	|	$\gamma(\gamma_1, \gamma_2)$		
	|	$\gamma @ (\gamma_1 \sim \gamma_2)$		
	|	$\gamma_1 \triangleright_R \gamma_2$		
	|	$\gamma_1 \sim_A \gamma_2$		
	|	$\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	|	$\mathbf{eta}\,a$		
	|	$\mathbf{left}\,\gamma\,\gamma'$		
	|	$\mathbf{right}\,\gamma\,\gamma'$		
	|	(γ)	S	
	|	γ	S	
	|	$\gamma\{a/x\}$	S	
υ	::=			
	|	ν		
	|	$\sim$		
$\overline{\upsilon}$	::=			
	|			
	|	$\overline{\upsilon}\upsilon$	M	
	|	$\upsilon\overline{\upsilon}$		
	|	$(\overline{\upsilon})$	M	
role_context, Ω	::=			$\mathrm{role}_c ontexts$
	|	$\varnothing$		
	|	$x : R$		
	|	$\Omega, x : R$		
	|	Ω, Ω'	M	
	|	$\mathbf{var\_pat}\,p$	M	
	|	(Ω)	M	
	|	Ω	M	
roles, $\overline{R}$	::=			
	|	$\cdot$		
	|	$R, \overline{R}$		
	|	$\mathbf{range}\,\Omega$	S	
	|	$(\overline{R})$	S	
	|	$\overline{R} +\!\!+ \overline{R}'$	S	
	|		S	
	|	R_1	S	
	|	R_1, R_2	S	

	$\|$	R_1, R_2, R_3	S	
sig_sort	$::=$			signature classifier
	$\|$	$A \mathbin{@} \overline{R}$		
	$\|$	$p \sim_R a : A \mathbin{@} \overline{R}$		
$sort$	$::=$			binding classifier
	$\|$	$\mathbf{Tm}\, A$		
	$\|$	$\mathbf{Co}\, \phi$		
$context,\ \Gamma$	$::=$			contexts
	$\|$	$\varnothing$		
	$\|$	$\Gamma, x : A$		
	$\|$	$\Gamma, c : \phi$		
	$\|$	$\Gamma\{b/x\}$	M	
	$\|$	$\Gamma\{\gamma/c\}$	M	
	$\|$	Γ, Γ'	M	
	$\|$	$\|\Gamma\|$	M	
	$\|$	(Γ)	M	
	$\|$	Γ	M	
$sig,\ \Sigma$	$::=$			signatures
	$\|$	$\varnothing$		
	$\|$	$\Sigma \cup \{F : sig\_sort\}$		
	$\|$	Σ_0	M	
	$\|$	Σ_1	M	
	$\|$	$\|\Sigma\|$	M	
$available\_props,\ \Delta$	$::=$			
	$\|$	$\varnothing$		
	$\|$	Δ, x		
	$\|$	Δ, c		
	$\|$	$\mathsf{fv}\, a$	M	
	$\|$	Δ, Δ'	M	
	$\|$	$\widetilde{\Gamma}$	M	
	$\|$	$\widetilde{\Omega}$	M	
	$\|$	(Δ)	M	
$Nat,\ \mathbb{N}$	$::=$			
	$\|$	0	M	
	$\|$	$\Sigma\, \mathbb{N}$	M	
	$\|$	$(\mathbb{N})$	M	
	$\|$	$\|a\|$	S	
μ	$::=$			Pattern arguments
	$\|$	a^ν		
	$\|$	$[\gamma]$		

$\overline{\mu},\ PA$ ::=
|
| μPA
| $PA\mu$

terminals ::=
| $\leftrightarrow$
| $\Leftrightarrow$
| $\longrightarrow$
| min
| $\equiv$
| $\forall$
| $\in$
| $\not\in$
| $\Leftarrow$
| $\Rightarrow$
| $\Rightarrow^*$
| $\rightarrow$
| Λ
|
| $\Box$
| $\vdash$
| $\dashv$
| $\models$
| $\models$
| $\models_{\mathsf{src}}$
| $\neq$
| $\triangleright$
| ok
| $\_$
| $\leadsto$
| $\leadsto^*$
| $\leadsto$
| $\varnothing$
| $\circ$
| fv
| dom
| $\sim$
| $\asymp$
| $|$
| $\bullet$
| $\mathbf{fst}$
| $\mathbf{snd}$
| $\mathbf{as}$
| $|\Rightarrow|$
| $\vdash_{=}$

	$\mid$	$\mathbf{refl_2}$	
	$\mid$	$++$	
	$\mid$	$\{$	
	$\mid$	$\}$	
	$\mid$	$\not\mapsto$	
formula, ψ	$::=$		
	$\mid$	*judgement*	
	$\mid$	$x : A \in \Gamma$	
	$\mid$	$x : R \in \Omega$	
	$\mid$	$c : \phi \in \Gamma$	
	$\mid$	$F : sig\_sort \in \Sigma$	
	$\mid$	$x \in \Delta$	
	$\mid$	$c \in \Delta$	
	$\mid$	$c\, \textbf{not relevant} \in \gamma$	
	$\mid$	$x \notin \Delta$	
	$\mid$	$uniq\, \Gamma$	
	$\mid$	$uniq(\Omega)$	
	$\mid$	$c \notin \Delta$	
	$\mid$	$T \notin \mathsf{dom}\, \Sigma$	
	$\mid$	$F \notin \mathsf{dom}\, \Sigma$	
	$\mid$	$\mathbb{N}_1 < \mathbb{N}_2$	
	$\mid$	$\mathbb{N}_1 \leq \mathbb{N}_2$	
	$\mid$	$\nu = \rho$	
	$\mid$	$R_1 = R_2$	
	$\mid$	$a = b$	
	$\mid$	$\phi_1 = \phi_2$	
	$\mid$	$\Gamma_1 = \Gamma_2$	
	$\mid$	$\gamma_1 = \gamma_2$	
	$\mid$	$\neg\psi$	
	$\mid$	$\psi_1 \wedge \psi_2$	
	$\mid$	$\psi_1 \vee \psi_2$	
	$\mid$	$\psi_1 \Rightarrow \psi_2$	
	$\mid$	(ψ)	
	$\mid$	ψ	
	$\mid$	$c : (a : A \sim b : B) \in \Gamma$	
	$\mid$		suppress lc hypothesis generated by Ott
	$\mid$	$\Delta \# \Delta_2$	
JSubRole	$::=$		
	$\mid$	$R_1 \leq R_2$	Subroling judgement
JRolePath	$::=$		
	$\mid$	$\mathsf{RolePath}\ a = F@\overline{R}$	Type headed by constant (partial function)
JAppsPath	$::=$		
	$\mid$	$\mathsf{AppsPath}\, a \not\mapsto F\overline{v}$	Type headed by constant (partial function)

$JSat$	$::=$		
	$\|$	$\overline{v} \sim \overline{R}$	
	$\|$	$\mathbf{Sat}\, F\, \overline{v}$	
$JPatCtx$	$::=$		
	$\|$	$\Omega; \Gamma; \Delta \vDash p :_F B \Rightarrow A$	Contexts generated by a p
$JRename$	$::=$		
	$\|$	$\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'$	rename with fresh variable
$JMatchSubst$	$::=$		
	$\|$	$\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 \nleftrightarrow b_2$	match and substitute
$JIsPattern$	$::=$		
	$\|$	$\mathbf{pattern}\, p$	
$JSubPat$	$::=$		
	$\|$	$\mathbf{subpattern}\, p'\, p$	Subpattern
$JTmPatternAgree$	$::=$		
	$\|$	$a \leftrightarrow p$	term and pattern agree
$JTmSubPatternAgree$	$::=$		
	$\|$	$a \sqsubseteq p$	sub-pattern agrees with te
$JSubTmPatternAgree$	$::=$		
	$\|$	$a \sqsupseteq p$	sub-term agrees with patt
$JValuePath$	$::=$		
	$\|$	$\mathsf{ValuePath}\ a \nleftrightarrow F$	Path headed by valid cons
$JCasePath$	$::=$		
	$\|$	$\mathsf{CasePath}_R\ a \nleftrightarrow F$	Path that is a value
$JApplyArgs$	$::=$		
	$\|$	$\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nleftrightarrow b'$	apply arguments of a (hea
$JValue$	$::=$		
	$\|$	$\mathsf{Value}_R\ A$	values
$JValueType$	$::=$		
	$\|$	$\mathsf{ValueType}_R\ A$	Types with head forms (e
$Jconsistent$	$::=$		
	$\|$	$\mathsf{consistent}_R\ a\ b$	Types do not differ in thei
$Jroleing$	$::=$		

	$\mid$	$\Omega \vDash a : R$	Roleing judgment
JChk	$::=$		
	$\mid$	$(\rho = +) \lor (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	$::=$		
	$\mid$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction
	$\mid$	$\Omega \vDash a \Rightarrow^*_R b$	multistep parallel reduction
	$\mid$	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	$::=$		
	$\mid$	$\vDash a \to^{\beta}_R b$	primitive reductions
	$\mid$	$\vDash a \rightsquigarrow_R b$	single-step head reduction for implicit lang
	$\mid$	$\vDash a \rightsquigarrow^* b/R$	multistep reduction
JBranchTyping	$::=$		
	$\mid$	$\Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b\overline{\mu} :^{\overline{v}} B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
Jett	$::=$		
	$\mid$	$\Gamma \vDash \phi\ \ \mathsf{ok}$	Prop wellformedness
	$\mid$	$\Gamma \vDash a : A$	typing
	$\mid$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	$\mid$	$\Gamma; \Delta \vDash a \equiv_R b : A$	definitional equality
	$\mid$	$\vDash \Gamma$	context wellformedness
Jsig	$::=$		
	$\mid$	$\vDash \Sigma$	signature wellformedness
Jhiding	$::=$		
	$\mid$	$\overline{R}_1 \Leftarrow \overline{R}_2$	
	$\mid$	$\Sigma_1 \Leftarrow \Sigma_2$	
JSrc	$::=$		
	$\mid$	$\Gamma \vDash_{\mathsf{src}} a : A$	source typing
	$\mid$	$\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A$	source translation
	$\mid$	$\Gamma \vDash_{\mathsf{src}} \phi \rightsquigarrow \phi'\ \ \mathsf{ok}$	Prop wellformedness
Jann	$::=$		
	$\mid$	$\Gamma \vdash \phi\ \ \mathsf{ok}$	prop wellformedness
	$\mid$	$\Gamma \vdash a : A/R$	typing
	$\mid$	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	$\mid$	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	$\mid$	$\vdash \Gamma$	context wellformedness
Jred	$::=$		
	$\mid$	$\Gamma \vdash a \rightsquigarrow b/R$	single-step, weak head reduction to values f

$$
\begin{array}{lcl}
\mathit{judgement} & ::= & \\
 & | & \mathit{JSubRole} \\
 & | & \mathit{JRolePath} \\
 & | & \mathit{JAppsPath} \\
 & | & \mathit{JSat} \\
 & | & \mathit{JPatCtx} \\
 & | & \mathit{JRename} \\
 & | & \mathit{JMatchSubst} \\
 & | & \mathit{JIsPattern} \\
 & | & \mathit{JSubPat} \\
 & | & \mathit{JTmPatternAgree} \\
 & | & \mathit{JTmSubPatternAgree} \\
 & | & \mathit{JSubTmPatternAgree} \\
 & | & \mathit{JValuePath} \\
 & | & \mathit{JCasePath} \\
 & | & \mathit{JApplyArgs} \\
 & | & \mathit{JValue} \\
 & | & \mathit{JValueType} \\
 & | & \mathit{Jconsistent} \\
 & | & \mathit{Jroleing} \\
 & | & \mathit{JChk} \\
 & | & \mathit{Jpar} \\
 & | & \mathit{Jbeta} \\
 & | & \mathit{JBranchTyping} \\
 & | & \mathit{Jett} \\
 & | & \mathit{Jsig} \\
 & | & \mathit{Jhiding} \\
 & | & \mathit{JSrc} \\
 & | & \mathit{Jann} \\
 & | & \mathit{Jred} \\
 & & \\
\mathit{user\_syntax} & ::= & \\
 & | & \mathit{tmvar} \\
 & | & \mathit{covar} \\
 & | & \mathit{datacon} \\
 & | & \mathit{const} \\
 & | & \mathit{index} \\
 & | & \mathit{relflag} \\
 & | & \mathit{appflag} \\
 & | & \mathit{role} \\
 & | & \mathit{constraint} \\
 & | & \mathit{tm} \\
 & | & \mathit{brs} \\
 & | & \mathit{co} \\
 & | & \upsilon \\
 & | & \overline{\upsilon}
\end{array}
$$

| *role_context*
| *roles*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *Nat*
| μ
| $\overline{\mu}$
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{RolePath}\ a = F@\overline{R}}$ Type headed by constant (partial function)

$$\frac{F : A \mathbin{@} \overline{R} \in \Sigma_0}{\mathsf{RolePath}\ F = F@\overline{R}} \quad \textsc{RolePath\_AbsConst}$$

$$\frac{F : p \sim_{R_1} a : A \mathbin{@} \overline{R} \in \Sigma_0}{\mathsf{RolePath}\ F = F@\overline{R}} \quad \textsc{RolePath\_Const}$$

$$\frac{\mathsf{RolePath}\ a = F@R_1, \overline{R}}{\mathsf{RolePath}\ (a\ b'^{R_1}) = F@\overline{R}} \quad \textsc{RolePath\_TApp}$$

$$\frac{\mathsf{RolePath}\ a = F@\overline{R}}{\mathsf{RolePath}\ (a\ b'^{+}) = F@\overline{R}} \quad \textsc{RolePath\_App}$$

$$\frac{\mathsf{RolePath}\ a = F@\overline{R}}{\mathsf{RolePath}\ (a\ \Box^{-}) = F@\overline{R}} \quad \textsc{RolePath\_IApp}$$

$$\frac{\mathsf{RolePath}\ a = F@\overline{R}}{\mathsf{RolePath}\ (a[\bullet]) = F@\overline{R}} \quad \textsc{RolePath\_CApp}$$

$\boxed{\mathsf{AppsPath} a \nrightarrow F\overline{v}}$ Type headed by constant (partial function)

$$\frac{F : A \mathbin{@} \overline{R} \in \Sigma_0}{\mathsf{AppsPath} F \nrightarrow F} \quad \textsc{AppsPath\_AbsConst}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} a : A \mathbin{@} \overline{R} \in \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{AppsPath} F \nrightarrow F} \quad \textsc{AppsPath\_Const}$$

$$\frac{\mathsf{AppsPath} a \nrightarrow F\overline{v}}{\mathsf{AppsPath}(a\ b'^{R_1}) \nrightarrow F(\overline{v}R_1)} \quad \textsc{AppsPath\_App}$$

$$\frac{\mathsf{AppsPath}\, a \nrightarrow F\overline{v}}{\mathsf{AppsPath}(a\ b^-) \nrightarrow F(\overline{v}-)} \quad \text{AppsPath\_IApp}$$

$$\frac{\mathsf{AppsPath}\, a \nrightarrow F\overline{v}}{\mathsf{AppsPath}(a[\bullet]) \nrightarrow F(\overline{v}\ \sim)} \quad \text{AppsPath\_CApp}$$

$\boxed{\overline{v} \sim \overline{R}}$

$$\frac{}{\sim \cdot} \quad \text{AR\_nil}$$

$$\frac{\overline{v} \sim \overline{R}}{R_1\overline{v} \sim R_1, \overline{R}} \quad \text{AR\_consTApp}$$

$$\frac{\overline{v} \sim \overline{R}}{+\overline{v} \sim \overline{R}} \quad \text{AR\_consApp}$$

$$\frac{\overline{v} \sim \overline{R}}{-\overline{v} \sim \overline{R}} \quad \text{AR\_consIApp}$$

$$\frac{\overline{v} \sim \overline{R}}{\sim \overline{v} \sim \overline{R}} \quad \text{AR\_consCApp}$$

$\boxed{\mathbf{Sat}\, F\, \overline{v}}$

$$\frac{\begin{array}{l} F : A\ @\ \overline{R} \in \Sigma_0 \\ \overline{v} \sim \overline{R} \end{array}}{\mathbf{Sat}\, F\, \overline{v}} \quad \text{Sat\_Const}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} a_0 : A_1\ @\ \overline{R} \in \Sigma_0 \\ \neg(R_1 \leq \mathbf{Nom}) \\ \overline{v} \sim \overline{R} \end{array}}{\mathbf{Sat}\, F\, \overline{v}} \quad \text{Sat\_Axiom}$$

$\boxed{\Omega; \Gamma; \Delta \vDash p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\varnothing; \varnothing; \varnothing \vDash F :_F A \Rightarrow A} \quad \text{PatCtx\_Const}$$

$$\frac{\Omega; \Gamma; \Delta \vDash p :_F \Pi^+ x\!:\!A' \to A \Rightarrow B}{\Omega, x : R; \Gamma, x : A'; \Delta \vDash p\ x^R :_F A \Rightarrow B} \quad \text{PatCtx\_PiRel}$$

$$\frac{\Omega; \Gamma; \Delta \vDash p :_F \Pi^- x\!:\!A' \to A \Rightarrow B}{\Omega; \Gamma, x : A'; \Delta, x \vDash p\ \Box^- :_F A \Rightarrow B} \quad \text{PatCtx\_PiIrr}$$

$$\frac{\Omega; \Gamma; \Delta \vDash p :_F \forall c\!:\!\phi.A \Rightarrow B}{\Omega; \Gamma, c : \phi; \Delta \vDash p[\bullet] :_F A \Rightarrow B} \quad \text{PatCtx\_CPi}$$

$\boxed{\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}$ rename with fresh variables

$$\frac{}{\mathsf{rename}\ F \to a\ \mathsf{to}\ F \to a\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \varnothing} \quad \text{Rename\_Base}$$

$$\frac{\begin{array}{l} \mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta' \\ y \notin (\Delta, \Delta') \end{array}}{\mathsf{rename}\ (p_1\ x^R) \to a_1\ \mathsf{to}\ (p_2\ y^R) \to (a_2\{y/x\})\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ (\Delta', y)} \quad \text{Rename\_AppRel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}{\mathsf{rename}\ (p_1\ \Box^-) \to a_1\ \mathsf{to}\ (p_2\ \Box^-) \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'} \quad \text{Rename\_AppIrrel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}{\mathsf{rename}\ (p_1[\bullet]) \to a_1\ \mathsf{to}\ (p_2[\bullet]) \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'} \quad \textsc{Rename\_CApp}$$

$\boxed{\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 \nleftrightarrow b_2}$ match and substitute

$$\frac{}{\mathsf{match}\ F\ \mathsf{with}\ F \to b \nleftrightarrow b} \quad \textsc{MatchSubst\_Const}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ p_1 \to b_1 \nleftrightarrow b_2}{\mathsf{match}\ (a_1\ a^R)\ \mathsf{with}\ (p_1\ x^R) \to b_1 \nleftrightarrow (b_2\{a/x\})} \quad \textsc{MatchSubst\_AppRelR}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ p_1 \to b_1 \nleftrightarrow b_2}{\mathsf{match}\ (a_1\ a^-)\ \mathsf{with}\ (p_1\ \Box^-) \to b_1 \nleftrightarrow b_2} \quad \textsc{MatchSubst\_AppIrrel}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 \nleftrightarrow b_2}{\mathsf{match}\ (a_1[\bullet])\ \mathsf{with}\ (a_2[\bullet]) \to b_1 \nleftrightarrow b_2} \quad \textsc{MatchSubst\_CApp}$$

$\boxed{\mathbf{pattern}\, p}$

$$\frac{}{\mathbf{pattern}\, F} \quad \textsc{Pattern\_Head}$$

$$\frac{\mathbf{pattern}\, p}{\mathbf{pattern}\, (p\ a^R)} \quad \textsc{Pattern\_Rel}$$

$$\frac{\mathbf{pattern}\, p}{\mathbf{pattern}\, (p\ a^-)} \quad \textsc{Pattern\_Irrel}$$

$$\frac{\mathbf{pattern}\, p}{\mathbf{pattern}\, (p[\gamma])} \quad \textsc{Pattern\_Co}$$

$\boxed{\mathbf{subpattern}\, p'\ p}$ Subpattern

$$\frac{\mathbf{pattern}\, p}{\mathbf{subpattern}\, p\ p} \quad \textsc{SubPat\_Refl}$$

$$\frac{\mathbf{subpattern}\, p'\ p}{\mathbf{subpattern}\, p'\ (p\ x^R)} \quad \textsc{SubPat\_Rel}$$

$$\frac{\mathbf{subpattern}\, p'\ p}{\mathbf{subpattern}\, p'\ (p\ \Box^-)} \quad \textsc{SubPat\_Irrel}$$

$$\frac{\mathbf{subpattern}\, p'\ p}{\mathbf{subpattern}\, p'\ (p[\bullet])} \quad \textsc{SubPat\_Co}$$

$\boxed{a \leftrightarrow p}$ term and pattern agree

$$\frac{}{F \leftrightarrow F} \quad \textsc{tm\_pattern\_agree\_Const}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ {a_2}^R) \leftrightarrow (p_1\ x^R)} \quad \textsc{tm\_pattern\_agree\_AppRelR}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ a^-) \leftrightarrow (p_1\ \Box^-)} \quad \textsc{tm\_pattern\_agree\_AppIrrel}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1[\bullet]) \leftrightarrow (p_1[\bullet])} \quad \textsc{tm\_pattern\_agree\_CApp}$$

$\boxed{a \sqsubseteq p}$ sub-pattern agrees with term

$$\frac{a \leftrightarrow p}{a \sqsubseteq p} \quad \textsc{tm\_subpattern\_agree\_Base}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ x^R)} \quad \textsc{tm\_subpattern\_agree\_AppRelR}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ \Box^-)} \quad \textsc{tm\_subpattern\_agree\_AppIrrel}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p[\bullet])} \quad \textsc{tm\_subpattern\_agree\_CAppp}$$

$\boxed{a \sqsupseteq p}$ sub-term agrees with pattern

$$\frac{a \leftrightarrow p}{a \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_Base}$$

$$\frac{a \sqsupseteq p}{a\ a_2{}^\nu \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_App}$$

$$\frac{a \sqsupseteq p}{a[\bullet] \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_CAppp}$$

$\boxed{\mathsf{ValuePath}\ a \nleftrightarrow F}$ Path headed by valid constructor

$$\frac{F : A\ @\ \overline{R} \in \Sigma_0}{\mathsf{ValuePath}\ F \nleftrightarrow F} \quad \textsc{ValuePath\_AbsConst}$$

$$\frac{F : p \sim_{R_1} a : A\ @\ \overline{R} \in \Sigma_0}{\mathsf{ValuePath}\ F \nleftrightarrow F} \quad \textsc{ValuePath\_Const}$$

$$\frac{\mathsf{ValuePath}\ a \nleftrightarrow F}{\mathsf{ValuePath}\ (a\ b'^\nu) \nleftrightarrow F} \quad \textsc{ValuePath\_App}$$

$$\frac{\mathsf{ValuePath}\ a \nleftrightarrow F}{\mathsf{ValuePath}\ (a[\bullet]) \nleftrightarrow F} \quad \textsc{ValuePath\_CApp}$$

$\boxed{\mathsf{CasePath}_R\ a \nleftrightarrow F}$ Path that is a value

$$\frac{\begin{array}{c}\mathsf{ValuePath}\ a \nleftrightarrow F \\ F : A\ @\ \overline{R} \in \Sigma_0\end{array}}{\mathsf{CasePath}_R\ a \nleftrightarrow F} \quad \textsc{CasePath\_AbsConst}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \nleftrightarrow F \\ F : p \sim_{R_1} b : A\ @\ \overline{R} \in \Sigma_0 \\ \neg(R_1 \leq R)\end{array}}{\mathsf{CasePath}_R\ a \nleftrightarrow F} \quad \textsc{CasePath\_Const}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \nleftrightarrow F \\ F : p \sim_{R_1} b : A\ @\ \overline{R} \in \Sigma_0 \\ \neg(a \sqsupseteq p)\end{array}}{\mathsf{CasePath}_R\ a \nleftrightarrow F} \quad \textsc{CasePath\_UnMatch}$$

$\boxed{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nleftrightarrow b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\mathsf{apply\ args}\ F\ \mathsf{to}\ b \nleftrightarrow b} \quad \textsc{ApplyArgs\_Const}$$

$$\frac{\textsf{apply args } a \textsf{ to } b \not\mapsto b'}{\textsf{apply args } (a\ a'^{R}) \textsf{ to } b \not\mapsto (b'\ a'^{+})} \quad \textsc{ApplyArgs\_AppRole}$$

$$\frac{\textsf{apply args } a \textsf{ to } b \not\mapsto b'}{\textsf{apply args } (a\ a'^{\rho}) \textsf{ to } b \not\mapsto (b'\ a'^{\rho})} \quad \textsc{ApplyArgs\_AppRho}$$

$$\frac{\textsf{apply args } a \textsf{ to } b \not\mapsto b'}{\textsf{apply args } a[\bullet] \textsf{ to } b \not\mapsto b'[\bullet]} \quad \textsc{ApplyArgs\_CApp}$$

$\boxed{\textsf{Value}_R\ A}$ values

$$\frac{}{\textsf{Value}_R\ \star} \quad \textsc{Value\_Star}$$

$$\frac{}{\textsf{Value}_R\ \Pi^{\rho} x\!:\!A \to B} \quad \textsc{Value\_Pi}$$

$$\frac{}{\textsf{Value}_R\ \forall c\!:\!\phi.B} \quad \textsc{Value\_CPi}$$

$$\frac{}{\textsf{Value}_R\ \lambda^{+} x\!:\!A.a} \quad \textsc{Value\_AbsRel}$$

$$\frac{}{\textsf{Value}_R\ \lambda^{+} x.a} \quad \textsc{Value\_UAbsRel}$$

$$\frac{\textsf{Value}_R\ a}{\textsf{Value}_R\ \lambda^{-} x.a} \quad \textsc{Value\_UAbsIrrel}$$

$$\frac{}{\textsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \textsc{Value\_CAbs}$$

$$\frac{}{\textsf{Value}_R\ \Lambda c.a} \quad \textsc{Value\_UCAbs}$$

$$\frac{\textsf{CasePath}_R\ a \not\mapsto F}{\textsf{Value}_R\ a} \quad \textsc{Value\_Path}$$

$\boxed{\textsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\textsf{ValueType}_R\ \star} \quad \textsc{value\_type\_Star}$$

$$\frac{}{\textsf{ValueType}_R\ \Pi^{\rho} x\!:\!A \to B} \quad \textsc{value\_type\_Pi}$$

$$\frac{}{\textsf{ValueType}_R\ \forall c\!:\!\phi.B} \quad \textsc{value\_type\_CPi}$$

$$\frac{\textsf{CasePath}_R\ a \not\mapsto F}{\textsf{ValueType}_R\ a} \quad \textsc{value\_type\_ValuePath}$$

$\boxed{\textsf{consistent}_R\ a\ b}$ Types do not differ in their heads

$$\frac{}{\textsf{consistent}_R\ \star\ \star} \quad \textsc{consistent\_a\_Star}$$

$$\frac{}{\textsf{consistent}_{R'}\ (\Pi^{\rho} x_1\!:\!A_1 \to B_1)\ (\Pi^{\rho} x_2\!:\!A_2 \to B_2)} \quad \textsc{consistent\_a\_Pi}$$

$$\frac{}{\textsf{consistent}_R\ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)} \quad \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{c}\textsf{CasePath}_R\ a_1 \not\mapsto F\\ \textsf{CasePath}_R\ a_2 \not\mapsto F\end{array}}{\textsf{consistent}_R\ a_1\ a_2} \quad \textsc{consistent\_a\_CasePath}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ a\ b}\quad\textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ a\ b}\quad\textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \Box : R}\quad\textsc{role\_a\_Bullet}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R}\quad\textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ x : R \in \Omega\\ R \leq R_1\end{array}}{\Omega \vDash x : R_1}\quad\textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R}\quad\textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : \mathbf{Nom}\end{array}}{\Omega \vDash (a\ b^\rho) : R}\quad\textsc{role\_a\_App}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : R_1 \sqcap R\end{array}}{\Omega \vDash a\ b^{R_1} : R}\quad\textsc{role\_a\_TApp}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R\\ \Omega, x : \mathbf{Nom} \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A \to B) : R}\quad\textsc{role\_a\_Pi}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R_1\\ \Omega \vDash b : R_1\\ \Omega \vDash A : \mathbf{Rep}\\ \Omega \vDash B : R\end{array}}{\Omega \vDash (\forall c\!:\!a \sim_{R_1} b : A.B) : R}\quad\textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R}\quad\textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R}\quad\textsc{role\_a\_CApp}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ F : A\ @\ \overline{R} \in \Sigma_0\end{array}}{\Omega \vDash F : R}\quad\textsc{role\_a\_Const}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ F : p \sim_R a : A\ @\ \overline{R} \in \Sigma_0\end{array}}{\Omega \vDash F : R_1}\quad\textsc{role\_a\_Fam}$$

$$\frac{\begin{array}{l}\Omega \vDash a : \mathbf{Nom}\\ \Omega \vDash b_1 : R_1\\ \Omega \vDash b_2 : R_1\end{array}}{\Omega \vDash \mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : R_1}\quad\textsc{role\_a\_Pattern}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b'\end{array}}{\Omega \vDash a\ b^\rho \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{(\mathbf{app\_role}\,\nu\,R)} b'\end{array}}{\Omega \vDash a\ b^\nu \Rightarrow_R a'\ b'^\nu} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \Pi^\rho x\!:\!A \to B \Rightarrow_R \Pi^\rho x\!:\!A' \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \textsc{Par\_CAbs}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_{\mathbf{Rep}} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \forall c\!:\!a \sim_{R_1} b : A.B \Rightarrow_R \forall c\!:\!a' \sim_{R_1} b' : A'.B'} \quad \textsc{Par\_CPi}$$

$$\frac{\begin{array}{l}F : F \sim_{R_1} b : A \ @\ \overline{R} \in \Sigma_0 \\ R_1 \leq R \\ \mathit{uniq}(\Omega)\end{array}}{\Omega \vDash F \Rightarrow_R b} \quad \textsc{Par\_AxiomBase}$$

$$\frac{\begin{array}{l}F : p \sim_{R_1} b : A \ @\ \overline{R} \in \Sigma_0 \\ a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash a_1 \Rightarrow_{(\mathbf{app\_role}\,\nu\,R)} a_1' \\ \text{rename } p \to b \text{ to } p' \to b' \text{ excluding } (\widetilde{\Omega}, \mathsf{fv} p) \text{ and } \Delta' \\ \text{match } (a'\ a_1'^{\nu}) \text{ with } p' \to b' \nrightarrow a_2 \\ R_1 \leq R\end{array}}{\Omega \vDash a\ a_1{}^\nu \Rightarrow_R a_2} \quad \textsc{Par\_AxiomApp}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} b : A \mathbin{@} \overline{R} \in \Sigma_0 \\ a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\ \Omega \vDash a \Rightarrow_R a' \\ \textsf{rename } p \rightarrow b \textsf{ to } p' \rightarrow b' \textsf{ excluding } (\widetilde{\Omega}, \textsf{fv}p) \textsf{ and } \Delta' \\ \textsf{match } (a'[\bullet]) \textsf{ with } p' \rightarrow b' \nrightarrow a_2 \\ R_1 \leq R \end{array}}{\Omega \vDash a[\bullet] \Rightarrow_R a_2} \quad \textsc{Par\_AxiomCApp}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_{\mathbf{Nom}} a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \end{array}}{\Omega \vDash (\textsf{case } a \textsf{ of } F \rightarrow b_1 \| \_ \rightarrow b_2) \Rightarrow_{R_0} (\textsf{case } a' \textsf{ of } F \rightarrow b_1' \| \_ \rightarrow b_2')} \quad \textsc{Par\_Pattern}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_{\mathbf{Nom}} a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \textsf{AppsPath} a' \nrightarrow F \overline{v} \\ \textsf{apply args } a' \textsf{ to } b_1' \nrightarrow b \\ \mathbf{Sat}\, F\, \overline{v}' \end{array}}{\Omega \vDash (\textsf{case } a \textsf{ of } F \rightarrow b_1 \| \_ \rightarrow b_2) \Rightarrow_{R_0} b[\bullet]} \quad \textsc{Par\_PatternTrue}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_{\mathbf{Nom}} a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \textsf{Value}_{\mathbf{Nom}}\ a' \\ \neg(\textsf{AppsPath} a' \nrightarrow F \overline{v}) \end{array}}{\Omega \vDash (\textsf{case } a \textsf{ of } F \rightarrow b_1 \| \_ \rightarrow b_2) \Rightarrow_{R_0} b_2'} \quad \textsc{Par\_PatternFalse}$$

$\boxed{\Omega \vDash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow_R^* a} \quad \textsc{MP\_Refl}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_R b \\ \Omega \vDash b \Rightarrow_R^* a' \end{array}}{\Omega \vDash a \Rightarrow_R^* a'} \quad \textsc{MP\_Step}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{c} \Omega \vDash a_1 \Rightarrow_R^* b \\ \Omega \vDash a_2 \Rightarrow_R^* b \end{array}}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \textsc{join}$$

$\boxed{\vDash a \rightarrow_R^\beta b}$ primitive reductions

$$\frac{\textsf{Value}_{R_1}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ b^\rho \rightarrow_{R_1}^\beta v\{b/x\}} \quad \textsc{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] \rightarrow_R^\beta a'\{\bullet/c\}} \quad \textsc{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l} F : p \sim_{R_1} b : A \mathbin{@} \overline{R} \in \Sigma_0 \\ \textsf{rename } p \rightarrow b \textsf{ to } p_1 \rightarrow b_1 \textsf{ excluding } (\textsf{fv}a, \textsf{fv}p) \textsf{ and } \Delta' \\ \textsf{match } a \textsf{ with } p_1 \rightarrow b_1 \nrightarrow b' \\ R_1 \leq R \end{array}}{\vDash a \rightarrow_R^\beta b'} \quad \textsc{Beta\_Axiom}$$

$$\frac{\begin{array}{l}\mathsf{AppsPath}\, a \nrightarrow F\overline{\upsilon}\\ \mathsf{apply\ args}\ a\ \mathsf{to}\ b_1 \nrightarrow b_1'\\ \mathbf{Sat}\, F\,\overline{\upsilon}'\end{array}}{\vDash \mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 \to^{\beta}_{R_0} b_1'[\bullet]}\quad \text{BETA\_PATTERNTRUE}$$

$$\frac{\begin{array}{l}\mathsf{Value}_{\mathbf{Nom}}\ a\\ \neg(\mathsf{AppsPath}\, a \nrightarrow F\overline{\upsilon})\end{array}}{\vDash \mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 \to^{\beta}_{R_0} b_2}\quad \text{BETA\_PATTERNFALSE}$$

$\boxed{\vDash a \rightsquigarrow_R b}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow_{R_1} a'}{\vDash \lambda^- x.a \rightsquigarrow_{R_1} \lambda^- x.a'}\quad \text{E\_ABSTERM}$$

$$\frac{\vDash a \rightsquigarrow_{R_1} a'}{\vDash a\ b^{\nu} \rightsquigarrow_{R_1} a'\ b^{\nu}}\quad \text{E\_APPLEFT}$$

$$\frac{\vDash a \rightsquigarrow_{R} a'}{\vDash a[\bullet] \rightsquigarrow_{R} a'[\bullet]}\quad \text{E\_CAPPLEFT}$$

$$\frac{\vDash a \rightsquigarrow_{\mathbf{Nom}} a'}{\vDash \mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \| \_ \to b_2 \rightsquigarrow_{R_0} \mathsf{case}\ a'\ \mathsf{of}\ F \to b_1 \| \_ \to b_2}\quad \text{E\_PATTERN}$$

$$\frac{\vDash a \to^{\beta}_{R} b}{\vDash a \rightsquigarrow_{R} b}\quad \text{E\_PRIM}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R}\quad \text{EQUAL}$$

$$\frac{\begin{array}{l}\vDash a \rightsquigarrow_R b\\ \vDash b \rightsquigarrow^* a'/R\end{array}}{\vDash a \rightsquigarrow^* a'/R}\quad \text{STEP}$$

$\boxed{\Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b\overline{\mu} :^{\overline{\upsilon}} B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{l}\mathit{uniq}\ \Gamma\\ C_1\{\bullet/c\} = C_2\end{array}}{\Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b\overline{\mu} : A \Rightarrow \forall c\!:\!(a \sim_{\mathbf{Nom}} b\overline{\mu} : A).C_1 \mid C_2}\quad \text{BRANCHTYPING\_BASE}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b\overline{\mu}x^R :^{\overline{\upsilon}} B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b\overline{\mu} :^{(R\overline{\upsilon})} \Pi^+ x\!:\!A \to B \Rightarrow \Pi^+ x\!:\!A \to C \mid C'}\quad \text{BRANCHTYPING\_PIROLE}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b\overline{\mu}x^+ :^{\overline{\upsilon}} B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b\overline{\mu} :^{(+\overline{\upsilon})} \Pi^+ x\!:\!A \to B \Rightarrow \Pi^+ x\!:\!A \to C \mid C'}\quad \text{BRANCHTYPING\_PIREL}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b\overline{\mu}\square^- :^{\overline{\upsilon}} B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}\ a : A_1\ \mathsf{of}\ b\overline{\mu} :^{(-\overline{\upsilon})} \Pi^- x\!:\!A \to B \Rightarrow \Pi^- x\!:\!A \to C \mid C'}\quad \text{BRANCHTYPING\_PIIRREL}$$

$$\frac{\Gamma, c : \phi \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b\overline{\mu}[\bullet] :^{\overline{\upsilon}} B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ b\overline{\mu} :^{(\sim\overline{\upsilon})} \forall c\!:\!\phi.B \Rightarrow \forall c\!:\!\phi.C \mid C'}\quad \text{BRANCHTYPING\_CPI}$$

$\boxed{\Gamma \vDash \phi\ \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\Gamma \vDash a : A \quad \Gamma \vDash b : A \quad \Gamma \vDash A : \star}{\Gamma \vDash a \sim_R b : A \ \mathsf{ok}} \quad \text{E\_Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \text{E\_Star}$$

$$\frac{\vDash \Gamma \quad x : A \in \Gamma}{\Gamma \vDash x : A} \quad \text{E\_Var}$$

$$\frac{\Gamma, x : A \vDash B : \star \quad \Gamma \vDash A : \star}{\Gamma \vDash \Pi^\rho x\!:\!A \to B : \star} \quad \text{E\_Pi}$$

$$\frac{\Gamma, x : A \vDash a : B \quad \Gamma \vDash A : \star \quad (\rho = +) \vee (x \notin \mathsf{fv}\ a)}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x\!:\!A \to B)} \quad \text{E\_Abs}$$

$$\frac{\Gamma \vDash b : \Pi^+ x\!:\!A \to B \quad \Gamma \vDash a : A}{\Gamma \vDash b\ a^+ : B\{a/x\}} \quad \text{E\_App}$$

$$\frac{\Gamma \vDash b : \Pi^+ x\!:\!A \to B \quad \Gamma \vDash a : A \quad \mathsf{RolePath}\ b = F@R, \overline{R}}{\Gamma \vDash b\ a^R : B\{a/x\}} \quad \text{E\_TApp}$$

$$\frac{\Gamma \vDash b : \Pi^- x\!:\!A \to B \quad \Gamma \vDash a : A}{\Gamma \vDash b\ \Box^- : B\{a/x\}} \quad \text{E\_IApp}$$

$$\frac{\Gamma \vDash a : A \quad \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Rep}} B : \star \quad \Gamma \vDash B : \star}{\Gamma \vDash a : B} \quad \text{E\_Conv}$$

$$\frac{\Gamma, c : \phi \vDash B : \star \quad \Gamma \vDash \phi\ \mathsf{ok}}{\Gamma \vDash \forall c\!:\!\phi.B : \star} \quad \text{E\_CPi}$$

$$\frac{\Gamma, c : \phi \vDash a : B \quad \Gamma \vDash \phi\ \mathsf{ok}}{\Gamma \vDash \Lambda c.a : \forall c\!:\!\phi.B} \quad \text{E\_CAbs}$$

$$\frac{\Gamma \vDash a_1 : \forall c\!:\!(a \sim_R b : A).B_1 \quad \Gamma; \widetilde{\Gamma} \vDash a \equiv_R b : A}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E\_CApp}$$

$$\frac{\vDash \Gamma \quad F : A\ @\ \overline{R} \in \Sigma_0 \quad \varnothing \vDash A : \star}{\Gamma \vDash F : A} \quad \text{E\_Const}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F : p \sim_{R_1} a : A \mathbin{@} \overline{R} \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash b_1 : B \\ \Gamma \vDash b_2 : C \\ \Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ F :^{\overline{v}} A_1 \Rightarrow B \mid C \\ \Gamma \vDash F : A_1 \\ \textbf{Sat}\ F\ \overline{v}\end{array}}{\Gamma \vDash \mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : C} \quad \text{E\_Case}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv_R A_2 : A \\ \Gamma; \Delta \vDash B_1 \equiv_R B_2 : A\end{array}}{\Gamma; \Delta \vDash A_1 \sim_R B_1 : A \equiv A_2 \sim_R B_2 : A} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv_{R_0} B : \star \\ \Gamma \vDash A_1 \sim_R A_2 : A \ \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_R A_2 : B \ \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_R A_2 : A \equiv A_1 \sim_R A_2 : B} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_{R_1} a_2 : A).B_1 \equiv_{R'} \forall c\!:\!(b_1 \sim_{R_2} b_2 : B).B_2 : \star}{\Gamma; \Delta \vDash a_1 \sim_{R_1} a_2 : A \equiv b_1 \sim_{R_2} b_2 : B} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv_R b : A}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_R b : A) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv_R b : A} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv_R a : A} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv_R a : A}{\Gamma; \Delta \vDash a \equiv_R b : A} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv_R a_1 : A \\ \Gamma; \Delta \vDash a_1 \equiv_R b : A\end{array}}{\Gamma; \Delta \vDash a \equiv_R b : A} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv_{R_1} b : A \\ R_1 \leq R_2\end{array}}{\Gamma; \Delta \vDash a \equiv_{R_2} b : A} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B \\ \Gamma \vDash a_2 : B \\ \vDash a_1 \to^{\beta}_{R} a_2\end{array}}{\Gamma; \Delta \vDash a_1 \equiv_R a_2 : B} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash A_1 \equiv_{R'} A_2 : \star \\ \Gamma, x : A_1; \Delta \vDash B_1 \equiv_{R'} B_2 : \star \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^{\rho} x\!:\!A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^{\rho} x\!:\!A_2 \to B_2 : \star\end{array}}{\Gamma; \Delta \vDash (\Pi^{\rho} x\!:\!A_1 \to B_1) \equiv_{R'} (\Pi^{\rho} x\!:\!A_2 \to B_2) : \star} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1; \Delta \vDash b_1 \equiv_{R'} b_2 : B \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma; \Delta \vDash (\lambda^\rho x.b_1) \equiv_{R'} (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv_{R'} b_1 : (\Pi^+ x\!:\!A \to B) \\ \Gamma; \Delta \vDash a_2 \equiv_{\mathbf{Nom}} b_2 : A\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^+ \equiv_{R'} b_1\ {b_2}^+ : (B\{a_2/x\})} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv_{R'} b_1 : (\Pi^+ x\!:\!A \to B) \\ \Gamma; \Delta \vDash a_2 \equiv_{R \sqcap R'} b_2 : A \\ \mathsf{RolePath}\ a_1 = F@R, \overline{R} \\ \mathsf{RolePath}\ b_1 = F@R, \overline{R} \\ \Gamma \vDash b_1\ {b_2}^R : B\{a_2/x\}\end{array}}{\Gamma; \Delta \vDash a_1\ {a_2}^R \equiv_{R'} b_1\ {b_2}^R : (B\{a_2/x\})} \quad \text{E\_TAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv_{R'} b_1 : (\Pi^- x\!:\!A \to B) \\ \Gamma \vDash a : A\end{array}}{\Gamma; \Delta \vDash a_1\ \Box^- \equiv_{R'} b_1\ \Box^- : (B\{a/x\})} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv_{R'} \Pi^\rho x\!:\!A_2 \to B_2 : \star}{\Gamma; \Delta \vDash A_1 \equiv_{R'} A_2 : \star} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv_{R'} \Pi^\rho x\!:\!A_2 \to B_2 : \star \\ \Gamma; \Delta \vDash a_1 \equiv_{\mathbf{Nom}} a_2 : A_1\end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv_{R'} B_2\{a_2/x\} : \star} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \sim_R b_1 : A_1 \equiv a_2 \sim_R b_2 : A_2 \\ \Gamma, c : a_1 \sim_R b_1 : A_1; \Delta \vDash A \equiv_{R'} B : \star \\ \Gamma \vDash a_1 \sim_R b_1 : A_1\ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_R b_1 : A_1.A : \star \\ \Gamma \vDash \forall c\!:\!a_2 \sim_R b_2 : A_2.B : \star\end{array}}{\Gamma; \Delta \vDash \forall c\!:\!a_1 \sim_R b_1 : A_1.A \equiv_{R'} \forall c\!:\!a_2 \sim_R b_2 : A_2.B : \star} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi_1; \Delta \vDash a \equiv_R b : B \\ \Gamma \vDash \phi_1\ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv_R (\Lambda c.b) : \forall c\!:\!\phi_1.B} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a_1 \equiv_{R'} b_1 : (\forall c\!:\!(a \sim_R b : A).B) \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv_R b : A\end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv_{R'} b_1[\bullet] : (B\{\bullet/c\})} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash \forall c\!:\!(a_1 \sim_R a_2 : A).B_1 \equiv_{R_0} \forall c\!:\!(a_1' \sim_{R'} a_2' : A').B_2 : \star \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv_R a_2 : A \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv_{R'} a_2' : A'\end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv_{R_0} B_2\{\bullet/c\} : \star} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv_R b : A \\ \Gamma; \Delta \vDash a \sim_R b : A \equiv a' \sim_{R'} b' : A'\end{array}}{\Gamma; \Delta \vDash a' \equiv_{R'} b' : A'} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma; \Delta \vDash a \equiv_R b : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Rep}} B : \star \\ \Gamma \vDash B : \star\end{array}}{\Gamma; \Delta \vDash a \equiv_R b : B} \quad \text{E\_EqConv}$$

$$\frac{\Gamma; \Delta \vDash a \sim_{R_1} b : A \equiv a' \sim_{R_1} b' : A'}{\Gamma; \Delta \vDash A \equiv_{\mathbf{Rep}} A' : \star} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash a \equiv_{\mathbf{Nom}} a' : A \\ \Gamma; \Delta \vDash b_1 \equiv_{R_0} b_1' : B \\ \Gamma; \Delta \vDash b_2 \equiv_{R_0} b_2' : C \\ \Gamma \vDash \mathsf{case}\ a : A\ \mathsf{of}\ F :^{\overline{v}} A_1 \Rightarrow B \mid C \\ \Gamma \vDash \mathsf{case}\ a' : A\ \mathsf{of}\ F :^{\overline{v}} A_1 \Rightarrow B' \mid C \\ \Gamma; \Delta \vDash B \equiv_{\mathbf{Rep}} B' : \star \\ \mathbf{Sat}\ F\, \overline{v} \\ \Gamma \vDash F : A_1 \end{array}}{\Gamma; \Delta \vDash \mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 \equiv_{R_0} \mathsf{case}\ a'\ \mathsf{of}\ F \to b_1' \|_{\_} \to b_2' : C} \quad \text{E\_PatCong}$$

$$\frac{\begin{array}{l} \mathsf{CasePath}_{R'}\ (a\ b^{R_1}) \nrightarrow F \\ \mathsf{CasePath}_{R'}\ (a'\ b'^{R_1}) \nrightarrow F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma; \Delta \vDash a\ b^{R_1} \equiv_{R'} a'\ b'^{R_1} : B\{b/x\} \\ \Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv_{\mathbf{Rep}} B\{b'/x\} : \star \end{array}}{\Gamma; \Delta \vDash a \equiv_{R'} a' : \Pi^+ x\!:\!A \to B} \quad \text{E\_LeftRel}$$

$$\frac{\begin{array}{l} \mathsf{CasePath}_{R'}\ (a\ \Box^-) \nrightarrow F \\ \mathsf{CasePath}_{R'}\ (a'\ \Box^-) \nrightarrow F \\ \Gamma \vDash a : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^- x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma; \Delta \vDash a\ \Box^- \equiv_{R'} a'\ \Box^- : B\{b/x\} \\ \Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv_{\mathbf{Rep}} B\{b'/x\} : \star \end{array}}{\Gamma; \Delta \vDash a \equiv_{R'} a' : \Pi^- x\!:\!A \to B} \quad \text{E\_LeftIrrel}$$

$$\frac{\begin{array}{l} \mathsf{CasePath}_{R'}\ (a\ b^{R_1}) \nrightarrow F \\ \mathsf{CasePath}_{R'}\ (a'\ b'^{R_1}) \nrightarrow F \\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b : A \\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash b' : A \\ \Gamma; \Delta \vDash a\ b^{R_1} \equiv_{R'} a'\ b'^{R_1} : B\{b/x\} \\ \Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv_{\mathbf{Rep}} B\{b'/x\} : \star \end{array}}{\Gamma; \Delta \vDash b \equiv_{R_1 \sqcap R'} b' : A} \quad \text{E\_Right}$$

$$\frac{\begin{array}{l} \mathsf{CasePath}_{R'}\ (a[\bullet]) \nrightarrow F \\ \mathsf{CasePath}_{R'}\ (a'[\bullet]) \nrightarrow F \\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{R_1} a_2 : A).B \\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{R_1} a_2 : A).B \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv_{R_1 \sqcap R'} a_2 : A \\ \Gamma; \Delta \vDash a[\bullet] \equiv_{R'} a'[\bullet] : B\{\bullet/c\} \end{array}}{\Gamma; \Delta \vDash a \equiv_{R'} a' : \forall c\!:\!(a_1 \sim_{R_1} a_2 : A).B} \quad \text{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing}\quad \textsc{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash A : \star\\ x \notin \widetilde{\Gamma}\end{array}}{\vDash \Gamma, x : A}\quad \textsc{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash \phi\ \mathsf{ok}\\ c \notin \widetilde{\Gamma}\end{array}}{\vDash \Gamma, c : \phi}\quad \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing}\quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma\\ \varnothing \vDash A : \star\\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vDash \Sigma \cup \{F : A \mathbin{@} \overline{R}\}}\quad \textsc{Sig\_ConsConst}$$

$$\frac{\begin{array}{l}\vDash \Sigma\\ F \notin \mathsf{dom}\,\Sigma\\ \varnothing \vDash A : \star\\ \Omega; \Gamma; \Delta \vDash p :_F B \Rightarrow A\\ \Gamma \vDash a : B\\ \Delta \# \mathsf{fv}\, a\\ \Omega \vDash a : R\end{array}}{\vDash \Sigma \cup \{F : p \sim_R a : A \mathbin{@} \mathbf{range}\, \Omega\}}\quad \textsc{Sig\_ConsAx}$$

$\boxed{\overline{R}_1 \Leftarrow \overline{R}_2}$

$$\frac{}{\cdot \Leftarrow \cdot}\quad \textsc{R\_Nil}$$

$$\frac{\begin{array}{c}R_2 \leq R_1\\ \overline{R}_1 \Leftarrow \overline{R}_2\end{array}}{R_1, \overline{R}_1 \Leftarrow R_2, \overline{R}_2}\quad \textsc{R\_Cons}$$

$\boxed{\Sigma_1 \Leftarrow \Sigma_2}$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2}{\Sigma_1 \Leftarrow \Sigma_2 \cup \{F : \mathit{sig\_sort}\}}\quad \textsc{S\_Forget}$$

$$\frac{\begin{array}{c}\Sigma_1 \Leftarrow \Sigma_2\\ \overline{R}_1 \Leftarrow \overline{R}_2\end{array}}{\Sigma_1 \cup \{F : A \mathbin{@} \overline{R}_1\} \Leftarrow \Sigma_2 \cup \{F : p \sim_R a : A \mathbin{@} \overline{R}_2\}}\quad \textsc{S\_Hide}$$

$$\frac{\begin{array}{c}\Sigma_1 \Leftarrow \Sigma_2\\ \overline{R}_1 \Leftarrow \overline{R}_2\end{array}}{\Sigma_1 \cup \{F : A \mathbin{@} \overline{R}_1\} \Leftarrow \Sigma_2 \cup \{F : A \mathbin{@} \overline{R}_2\}}\quad \textsc{S\_WeakenConst}$$

$$\frac{\begin{array}{c}\Sigma_1 \Leftarrow \Sigma_2\\ \overline{R}_1 \Leftarrow \overline{R}_2\end{array}}{\Sigma_1 \cup \{F : p' \sim_R a : A \mathbin{@} \overline{R}_1\} \Leftarrow \Sigma_2 \cup \{F : p \sim_R a : A \mathbin{@} \overline{R}_2\}}\quad \textsc{S\_WeakenAxiom}$$

$$\frac{}{\varnothing \Leftarrow \varnothing} \quad \textsc{S\_Empty}$$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2}{\Sigma_1 \cup \{F : \mathit{sig\_sort}\} \Leftarrow \Sigma_2 \cup \{F : \mathit{sig\_sort}\}} \quad \textsc{S\_Same}$$

$\boxed{\Gamma \vDash_{\mathsf{src}} a : A}$ source typing

$$\frac{\vDash \Gamma}{\Gamma \vDash_{\mathsf{src}} \star : \star} \quad \textsc{S\_Star}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ x : A \in \Gamma \end{array}}{\Gamma \vDash_{\mathsf{src}} x : A} \quad \textsc{S\_Var}$$

$$\frac{\begin{array}{l} \Gamma \vDash_{\mathsf{src}} A : \star \\ \Gamma \vDash_{\mathsf{src}} A \rightsquigarrow A' : \star \\ \Gamma, x : A' \vDash_{\mathsf{src}} B : \star \end{array}}{\Gamma \vDash_{\mathsf{src}} (\Pi^{\rho} x\!:\!A \to B) : \star} \quad \textsc{S\_Pi}$$

$$\frac{\Gamma, x : A \vDash_{\mathsf{src}} a : B}{\Gamma \vDash_{\mathsf{src}} \lambda x.a : (\Pi^{+} x\!:\!A \to B)} \quad \textsc{S\_Abs}$$

$$\frac{\Gamma, x : A \vDash_{\mathsf{src}} a : B}{\Gamma \vDash_{\mathsf{src}} a : (\Pi^{-} x\!:\!A \to B)} \quad \textsc{S\_IAbs}$$

$$\frac{\begin{array}{l} \Gamma \vDash_{\mathsf{src}} a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Nom}} B : \star \end{array}}{\Gamma \vDash_{\mathsf{src}} a : B} \quad \textsc{S\_Conv}$$

$$\frac{\begin{array}{l} \Gamma \vDash_{\mathsf{src}} a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Rep}} B : \star \end{array}}{\Gamma \vDash_{\mathsf{src}} \mathbf{coerce}\, a : B} \quad \textsc{S\_Coerce}$$

$$\frac{\begin{array}{l} \Gamma \vDash_{\mathsf{src}} b : \Pi^{+} x\!:\!A \to B \\ \Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A \end{array}}{\Gamma \vDash_{\mathsf{src}} b\ \ a : B\{a'/x\}} \quad \textsc{S\_App}$$

$$\frac{\begin{array}{l} \Gamma \vDash_{\mathsf{src}} b : \Pi^{-} x\!:\!A \to B \\ \Gamma \vDash a' : A \end{array}}{\Gamma \vDash_{\mathsf{src}} b : B\{a'/x\}} \quad \textsc{S\_IApp}$$

$$\frac{\begin{array}{l} \Gamma \vDash_{\mathsf{src}} \phi \rightsquigarrow \phi'\ \ \mathsf{ok} \\ \Gamma, c : \phi' \vDash_{\mathsf{src}} B : \star \end{array}}{\Gamma \vDash_{\mathsf{src}} \forall c\!:\!\phi.B : \star} \quad \textsc{S\_CPi}$$

$$\frac{\Gamma, c : \phi' \vDash_{\mathsf{src}} a : B}{\Gamma \vDash_{\mathsf{src}} a : \forall c\!:\!\phi.B} \quad \textsc{S\_CAbs}$$

$$\frac{\begin{array}{l} \Gamma \vDash_{\mathsf{src}} a_1 : \forall c\!:\!(a \sim_R b : A).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv_R b : A \end{array}}{\Gamma \vDash_{\mathsf{src}} a_1 : B_1\{\bullet/c\}} \quad \textsc{S\_CApp}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ F : A \mathbin{@} \overline{R} \in \Sigma_0 \end{array}}{\Gamma \vDash_{\mathsf{src}} F : A} \quad \textsc{S\_Const}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash_{\mathsf{src}} F : A} \quad \textsc{S\_Fam}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} a : A \quad \Gamma \vDash_{\mathsf{src}} b_1' : B \quad \Gamma \vDash_{\mathsf{src}} b_2' : C}{\Gamma \vDash_{\mathsf{src}} \mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \|_{\_} \to b_2 : C} \quad \textsc{S\_Case}$$

$\boxed{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A}$ source translation

$$\frac{\vDash \Gamma}{\Gamma \vDash_{\mathsf{src}} \star \rightsquigarrow \star : \star} \quad \textsc{ST\_Star}$$

$$\frac{\vDash \Gamma \quad x : A \in \Gamma}{\Gamma \vDash_{\mathsf{src}} x \rightsquigarrow x : A} \quad \textsc{ST\_Var}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} A \rightsquigarrow A' : \star \quad \Gamma, x : A' \vDash_{\mathsf{src}} B \rightsquigarrow B' : \star}{\Gamma \vDash_{\mathsf{src}} (\Pi^{\rho} x\!:\!A \to B) \rightsquigarrow (\Pi^{\rho} x\!:\!A' \to B') : \star} \quad \textsc{ST\_Pi}$$

$$\frac{\Gamma, x : A \vDash_{\mathsf{src}} a \rightsquigarrow a' : B}{\Gamma \vDash_{\mathsf{src}} \lambda x.a \rightsquigarrow \lambda^{+} x.a' : (\Pi^{+} x\!:\!A \to B)} \quad \textsc{ST\_Abs}$$

$$\frac{\Gamma, x : A \vDash_{\mathsf{src}} a \rightsquigarrow a' : B \quad x \notin \mathsf{fv}\, a}{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow \lambda^{-} x.a : (\Pi^{-} x\!:\!A \to B)} \quad \textsc{ST\_IAbs}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} b \rightsquigarrow b' : \Pi^{+} x\!:\!A \to B \quad \Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A}{\Gamma \vDash_{\mathsf{src}} b\ a \rightsquigarrow b'\ a'^{+} : B\{a'/x\}} \quad \textsc{ST\_App}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} b \rightsquigarrow b' : \Pi^{+} x\!:\!A \to B \quad \Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A \quad \mathsf{RolePath}\ b = F@R, \overline{R}}{\Gamma \vDash_{\mathsf{src}} b\ a \rightsquigarrow b'\ a'^{R} : B\{a/x\}} \quad \textsc{ST\_TApp}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} b \rightsquigarrow b' : \Pi^{-} x\!:\!A \to B \quad \Gamma \vDash a : A}{\Gamma \vDash_{\mathsf{src}} b \rightsquigarrow b'\ \Box^{-} : B\{a/x\}} \quad \textsc{ST\_IApp}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A \quad \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Nom}} B : \star}{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : B} \quad \textsc{ST\_Conv}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow a' : A \quad \Gamma; \widetilde{\Gamma} \vDash A \equiv_{\mathbf{Rep}} B : \star}{\Gamma \vDash_{\mathsf{src}} \mathbf{coerce}\, a \rightsquigarrow a' : B} \quad \textsc{ST\_Coerce}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} \phi \rightsquigarrow \phi'\ \mathsf{ok} \quad \Gamma, c : \phi' \vDash_{\mathsf{src}} B \rightsquigarrow B' : \star}{\Gamma \vDash_{\mathsf{src}} \forall c\!:\!\phi.B \rightsquigarrow \forall c\!:\!\phi.B' : \star} \quad \textsc{ST\_CPi}$$

$$\frac{\Gamma, c : \phi \vDash_{\mathsf{src}} a \rightsquigarrow a' : B}{\Gamma \vDash_{\mathsf{src}} a \rightsquigarrow \Lambda c.a' : \forall c\!:\!\phi.B} \quad \textsc{ST\_CAbs}$$

$$\frac{\Gamma \vDash_{\mathsf{src}} a_1 \rightsquigarrow a_1' : \forall c\!:\!(a \sim_R b : A).B_1 \quad \Gamma; \widetilde{\Gamma} \vDash a \equiv_R b : A}{\Gamma \vDash_{\mathsf{src}} a_1 \rightsquigarrow a_1'[\bullet] : B_1\{\bullet/c\}} \quad \textsc{ST\_CApp}$$

$$\frac{\begin{array}{c}\vDash \Gamma \\ F : A \mathbin{@} \overline{R} \in \Sigma_0\end{array}}{\Gamma \vDash_{\mathsf{src}} F \leadsto F : A} \quad \text{ST\_CONST}$$

$$\frac{\vDash \Gamma}{\Gamma \vDash_{\mathsf{src}} F \leadsto F : A} \quad \text{ST\_FAM}$$

$$\frac{\begin{array}{c}\Gamma \vDash_{\mathsf{src}} a \leadsto a' : A \\ \Gamma \vDash_{\mathsf{src}} b_1 \leadsto b_1' : B \\ \Gamma \vDash_{\mathsf{src}} b_2 \leadsto b_2' : C\end{array}}{\Gamma \vDash_{\mathsf{src}} \mathsf{case}\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 \leadsto \mathsf{case}\ a'\ \mathsf{of}\ F \to b_1' \|\_ \to b_2' : C} \quad \text{ST\_CASE}$$

$\boxed{\Gamma \vDash_{\mathsf{src}} \phi \leadsto \phi' \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{c}\Gamma \vDash_{\mathsf{src}} a \leadsto a' : A \\ \Gamma \vDash_{\mathsf{src}} b \leadsto b' : A\end{array}}{\Gamma \vDash_{\mathsf{src}} (a \sim_{\mathbf{Nom}} b : A) \leadsto (a' \sim_{\mathbf{Nom}} b' : A)\ \mathsf{ok}} \quad \text{S\_WFF}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A/R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \leadsto b/R}$ single-step, weak head reduction to values for annotated language

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Definition rules:        222 good    0 bad
Definition rule clauses: 612 good    0 bad
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