tmvar, x, y, f, m, n variables

covar, c coercion variables

 $\begin{array}{c} datacon, \ K \\ const, \ T, \ F \end{array}$

index, i indices

```
relflag, \rho
                                                                                                                                                 relevance flag
                                                             ::=
                                                                       +
                                                                                                                         S
                                                                       app_rho \nu
                                                                                                                         S
                                                                       (\rho)
                                                                                                                                                 applicative flag
appflag, \ \nu
                                                             ::=
                                                                       R
                                                                       \rho
role, R
                                                                                                                                                 Role
                                                             ::=
                                                                       \mathbf{Nom}
                                                                       Rep
                                                                                                                         S
                                                                       R_1 \cap R_2
                                                                                                                         S
                                                                       \mathbf{param}\,R_1\,R_2
                                                                                                                         S
                                                                       \mathbf{app\_role}\,\nu
                                                                                                                         S
                                                                       (R)
constraint, \phi
                                                             ::=
                                                                                                                                                 props
                                                                       a \sim_{A/R} b
                                                                                                                         S
                                                                       (\phi)
                                                                                                                         S
                                                                       \phi\{b/x\}
                                                                                                                         S
                                                                       |\phi|
                                                                                                                         S
                                                                       a \sim_R b
                                                                                                                                                 types and kinds
tm, a, b, p, v, w, A, B, C
                                                                       \boldsymbol{x}
                                                                       \lambda^{\rho}x:A.b
                                                                                                                         \mathsf{bind}\ x\ \mathsf{in}\ b
                                                                       \lambda^{\rho}x.b
                                                                                                                         \mathsf{bind}\ x\ \mathsf{in}\ b
                                                                       a b^{\nu}
                                                                       \Pi^{\rho}x:A\to B
                                                                                                                         \mathsf{bind}\ x\ \mathsf{in}\ B
                                                                       \Lambda c : \phi . b
                                                                                                                         bind c in b
                                                                                                                         \mathsf{bind}\ c\ \mathsf{in}\ b
                                                                       \Lambda c.b
                                                                       a[\gamma]
                                                                                                                         \mathsf{bind}\ c\ \mathsf{in}\ B
                                                                       \forall c : \phi.B
                                                                       a \triangleright_R \gamma
                                                                       F
                                                                       \mathsf{case}_R \ a \ \mathsf{of} \ F 	o b_1 \|_{\scriptscriptstyle{-}} 	o b_2
                                                                       \mathbf{match}\ a\ \mathbf{with}\ brs
                                                                       \operatorname{\mathbf{sub}} R a
                                                                       a\{b/x\}
                                                                                                                         S
                                                                                                                         S
                                                                       a\{\gamma/c\}
                                                                                                                         S
                                                                       a\{b/x\}
                                                                                                                         S
                                                                       a\{\gamma/c\}
```

```
S
                           a
                                                            S
                           a
                                                            S
                           (a)
                                                             S
                                                                                         parsing precedence is hard
                                                             S
                           |a|_R
                                                             S
                           \mathbf{Int}
                                                            S
                           Bool
                                                            S
                           Nat
                                                            S
                           Vec
                                                             S
                           0
                                                             S
                           S
                           {\bf True}
                                                             S
                                                            S
                           Fix
                                                            S
                           Age
                                                             S
                           a \rightarrow b
                                                             S
                           \phi \Rightarrow A
                           a b
                                                             S
                                                            S
                           \lambda x.a
                                                             S
                           \lambda x : A.a
                           \forall\,x:A\to B
                                                             S
                           if \phi then a else b
                                                            S
                                                                                     case branches
brs
                  ::=
                           none
                           K \Rightarrow a; brs
                           brs\{a/x\}
                                                             S
                                                            S
                           brs\{\gamma/c\}
                                                             S
                           (brs)
co, \gamma
                                                                                    explicit coercions
                           \mathbf{red} \ a \ b
                           \mathbf{refl}\;a
                           (a \models \mid_{\gamma} b)
                           \mathbf{sym}\,\gamma
                           \gamma_1; \gamma_2
                           \mathbf{sub}\,\gamma
                           \Pi^{R,\rho}x\!:\!\gamma_1.\gamma_2
                                                             bind x in \gamma_2
                           \lambda^{R,\rho}x:\gamma_1.\gamma_2
                                                             bind x in \gamma_2
                           \gamma_1 \ \gamma_2^{R,\rho}
                           \mathbf{piFst}\,\gamma
                           \mathbf{cpiFst}\,\gamma
                           \mathbf{isoSnd}\,\gamma
                           \gamma_1@\gamma_2
                           \forall c: \gamma_1.\gamma_3
                                                            bind c in \gamma_3
```

```
\lambda c: \gamma_1.\gamma_3@\gamma_4
                                                                                  bind c in \gamma_3
                                              \gamma(\gamma_1,\gamma_2)
                                              \gamma@(\gamma_1 \sim \gamma_2)
                                              \gamma_1 \triangleright_R \gamma_2
                                              \gamma_1 \sim_A \gamma_2
                                              conv \phi_1 \sim_{\gamma} \phi_2
                                              \mathbf{eta}\,a
                                              left \gamma \gamma'
                                              right \gamma \gamma'
                                                                                  S
                                              (\gamma)
                                                                                  S
                                              \gamma
                                              \gamma\{a/x\}
                                                                                  S
role\_context, \ \Omega
                                                                                                           {\rm role}_contexts
                                               Ø
                                              x:R
                                              \Omega, x: R
                                              \Omega, \Omega'
                                                                                  Μ
                                              \mathbf{var}_{-}\mathbf{pat}\;p
                                                                                  Μ
                                              (\Omega)
                                                                                  Μ
                                              \Omega
                                                                                  Μ
roles,\ Rs
                                    ::=
                                              \mathbf{nil}\mathbf{R}
                                               R, Rs
                                                                                  S
                                              \mathbf{range}\,\Omega
                                                                                                           signature classifier
sig\_sort
                                    ::=
                                               A@Rs
                                               p \sim a : A/R@Rs
sort
                                    ::=
                                                                                                           binding classifier
                                              \operatorname{\mathbf{Tm}} A
                                               \mathbf{Co}\,\phi
context, \Gamma
                                    ::=
                                                                                                           contexts
                                              Ø
                                              \Gamma, x : A
                                              \Gamma, c: \phi
                                              \Gamma\{b/x\}
                                                                                  Μ
                                              \Gamma\{\gamma/c\}
                                                                                  Μ
                                              \Gamma, \Gamma'
                                                                                  Μ
                                              |\Gamma|
                                                                                  Μ
                                              (\Gamma)
                                                                                  Μ
                                              Γ
                                                                                  Μ
sig, \Sigma
                                                                                                           signatures
                                    ::=
```

```
\Sigma \cup \{F: sig\_sort\}
                                                              \Sigma_0
\Sigma_1
|\Sigma|
                                                                                                          Μ
available\_props, \ \Delta
                                                              Ø
                                                              \Delta, x
                                                              \Delta, c
                                                              \mathsf{fv}\,a
                                                              \Delta, \Delta'
                                                              \widetilde{\Gamma}
\widetilde{\Omega}
                                                              (\Delta)
Nat, \mathbb{N}
                                                  ::=
                                                              |a|
                                                                                                          S
terminals
                                                               \leftrightarrow
                                                              {\sf min}
                                                              \in
                                                              \Lambda
```

Μ

Μ

Μ

Μ

Μ Μ

Μ

```
0
                                           fv
                                           dom
                                           \mathbf{fst}
                                           \operatorname{snd}
                                           \mathbf{a}\mathbf{s}
                                           |\Rightarrow|
                                           \vdash_=
                                           refl_2
                                            ++
                                            {
formula, \psi
                                           judgement
                                           x:A\,\in\,\Gamma
                                           x:R\,\in\,\Omega
                                           c:\phi\in\Gamma
                                           F: sig\_sort \, \in \, \Sigma
                                           x \in \Delta
                                            c\,\in\,\Delta
                                            c \, \mathbf{not} \, \mathbf{relevant} \, \in \, \gamma
                                           x \not\in \Delta
                                           uniq \; \Gamma
                                            uniq(\Omega)
                                            c \not\in \Delta
                                            T \not\in \mathsf{dom}\, \Sigma
                                            F \not\in \operatorname{dom} \Sigma
                                           \mathbb{N}_1 < \mathbb{N}_2
                                           \mathbb{N}_1 \leq \mathbb{N}_2
                                           R_1 = R_2
                                            a = b
                                           \phi_1 = \phi_2
                                           \Gamma_1 = \Gamma_2
                                           \gamma_1 = \gamma_2
                                           \neg \psi
                                           \psi_1 \wedge \psi_2
                                           \psi_1 \vee \psi_2
                                           \psi_1 \Rightarrow \psi_2
                                           (\psi)
                                           c:(a:A\sim b:B)\in\Gamma
```

			suppress lc hypothesis generated
JSubRole	::=	$R_1 \le R_2$	Subroling judgement
JPath	::=	Path $a = F@Rs$	Type headed by constant (partial
JPatCtx	::=	$\Omega; \Gamma \vDash p :_F B \Rightarrow A$	Contexts generated by a pattern
JRename	::=	rename $p o a$ to $p' o a'$ excluding Δ	rename with fresh variables
JMatchSubst	::=	match a_1 with $p o b_1 = b_2$	match and substitute
JTmPatternAgree	::=	$a \leftrightarrow p$	term and pattern agree
JTmSubPatternAgree	::=	$a^+ = p$	sub-pattern agrees with term
JSubTmPatternAgree	::=	$a = p^+$	sub-term agrees with pattern
JValuePath	::=	$ValuePath\ a = F$	Type headed by constant (role-ser
JCasePath	::=	$CasePath_R\ a = F$	Type headed by constant (role-ser
JApplyArgs	::=	apply args a to $b\mapsto b'$	apply arguments of a (headed by
JValue	::=	$Value_R\ A$	values
JValueType	::=	$ValueType_R\ A$	Types with head forms (erased la
J consistent	::=	$consistent_R\ a\ b$	(erased) types do not differ in the
Jroleing	::=	$O \vdash a \cdot D$	Dalaing independ

Roleing judgment

 $| \quad \Omega \vDash a : R$

JChk::= $(\rho = +) \lor (x \not\in \mathsf{fv}\ A)$ irrelevant argument check Jpar::= $\Omega \vDash a \Rightarrow_R b$ parallel reduction (implicit language) $\Omega \vDash a \Rightarrow_R^* b$ multistep parallel reduction $\Omega \vDash a \Leftrightarrow_R b$ parallel reduction to a common term Jbeta $\models a > b/R$ primitive reductions on erased terms $\models a \leadsto b/R$ single-step head reduction for implicit langu $\models a \leadsto^* b/R$ multistep reduction JBranch Typing $\Gamma \vDash \mathsf{case}_R \ a : A \ \mathsf{of} \ b : B \Rightarrow C \mid C'$ Branch Typing (aligning the types of case) Jett::= $\Gamma \vDash \phi \ \, \mathsf{ok}$ Prop wellformedness $\Gamma \vDash a : A$ typing $\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$ prop equality $\Gamma; \Delta \vDash a \equiv b : A/R$ definitional equality context wellformedness Jsig::= $\models \Sigma$ signature wellformedness Jann::= $\Gamma \vdash \phi \ \, \mathsf{ok}$ prop wellformedness $\Gamma \vdash a : A/R$ typing $\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$ coercion between props $\Gamma; \Delta \vdash \gamma : A \sim_R B$ coercion between types context wellformedness Jred $\Gamma \vdash a \leadsto b/R$ single-step, weak head reduction to values for judgement::=JSubRoleJPathJPatCtxJRename

JMatchSubst JTmPatternAgreeJTmSubPatternAgree JApplyArgsJValue $JValue\,Type$ J consistentJroleingJChkJparJbeta $JBranch\,Typing$ JettJsigJannJred

 $user_syntax$

tmvarcovardata conconstindexrelflagappflagroleconstrainttmbrsco $role_context$ roles sig_sort sortcontextsig $available_props$ Natterminals

formula

 $R_1 \leq R_2$ Subroling judgement

> **NomBot** $\overline{\mathbf{Nom} \leq R}$ $\overline{R \leq \mathbf{Rep}}$ RepTop $\overline{R \leq R}$ Refl $R_1 \le R_2$ $R_2 \le R_3$ $R_1 \le R_3$ Trans

Path $a = \overline{F@Rs}$ Type headed by constant (partial function)

$$\frac{F:A @Rs \in \Sigma_0}{\mathsf{Path} \ F = F @Rs} \quad \mathsf{PATH_ABSCONST}$$

$$\frac{F:p \sim a:A/R_1@Rs \in \Sigma_0}{\mathsf{Path} \ F = F @Rs} \quad \mathsf{PATH_CONST}$$

$$\frac{F:p \sim a:A/R_1@Rs \in \Sigma_0}{\mathsf{Path} \ A = F @Rs} \quad \mathsf{PATH_ABP}$$

$$\frac{\mathsf{Path} \ a = F @Rs}{\mathsf{Path} \ (a \ b^{\circ} | h) = F @Rs} \quad \mathsf{PATH_APP}$$

$$\frac{\mathsf{Path} \ a = F @Rs}{\mathsf{Path} \ (a \ b^{\circ}) = F @Rs} \quad \mathsf{PATH_APP}$$

$$\frac{\mathsf{Path} \ a = F @Rs}{\mathsf{Path} \ (a \ b^{\circ}) = F @Rs} \quad \mathsf{PATH_APP}$$

$$\frac{\mathsf{Path} \ a = F @Rs}{\mathsf{Path} \ (a \ b^{\circ}) = F @Rs} \quad \mathsf{PATH_CAPP}$$

$$\frac{\mathsf{Path} \ a = F @Rs}{\mathsf{Path} \ (a \ b^{\circ}) = F @Rs} \quad \mathsf{PATCTX_CONST}$$

$$\frac{\mathsf{Q}; F \models p :_F \ B \Rightarrow A}{\mathsf{Q}; F \models p :_F \ H^{-}x : A' \to A \Rightarrow B} \quad \mathsf{PATCTX_CONST}$$

$$\frac{\mathsf{Q}; F \models p :_F \ \mathsf{II}^{-}x : A' \models p \ x^{R} :_F A \Rightarrow B}{\mathsf{Q}; F \vdash p :_F \ \mathsf{II}^{-}x : A' \Rightarrow A \Rightarrow B} \quad \mathsf{PATCTX_PIREL}$$

$$\frac{\mathsf{Q}; F \models p :_F \ \mathsf{II}^{-}x : A' \Rightarrow B}{\mathsf{Q}; F, c : \phi \models p \ | e \ | :_F A \Rightarrow B} \quad \mathsf{PATCTX_PIRE}$$

$$\frac{\mathsf{Q}; F \models p :_F \ \forall c : \phi A \Rightarrow B}{\mathsf{Q}; F, c : \phi \models p \ | e \ | :_F A \Rightarrow B} \quad \mathsf{PATCTX_CPI}$$

$$\frac{\mathsf{Path} \ \mathsf{Path} \ \mathsf{Pat$$

$$\overline{F} \leftrightarrow \overline{F} \qquad \text{TM_PATTERN_AGREE_CONST}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1 \ a_2^R) \leftrightarrow (p_1 \ x^R)} \qquad \text{TM_PATTERN_AGREE_APPRELR}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1 \ a^-) \leftrightarrow (p_1 \ \Box^-)} \qquad \text{TM_PATTERN_AGREE_APPIRREL}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1[\bullet]) \leftrightarrow (p_1[\bullet])} \qquad \text{TM_PATTERN_AGREE_CAPP}$$

$$\overline{a^+ = p} \qquad \text{sub-pattern agrees with term}$$

$$\underline{a \leftrightarrow p} \qquad \text{TM_SUPPATTERN_AGREE_RASE}$$

 $a^+ = p$

$$\frac{a \leftrightarrow p}{a^+ = p} \qquad \text{TM_SUBPATTERN_AGREE_BASE}$$

$$\frac{a^+ = p}{a^+ = (p \ x^R)} \qquad \text{TM_SUBPATTERN_AGREE_APPRELR}$$

$$\frac{a^+ = p}{a^+ = (p \ \Box^-)} \qquad \text{TM_SUBPATTERN_AGREE_APPIRREL}$$

$$\frac{a^+ = p}{a^+ = (p \ \Box^-)} \qquad \text{TM_SUBPATTERN_AGREE_CAPPP}$$

 $a = p^+$ sub-term agrees with pattern

$$\frac{a \leftrightarrow p}{a = p^+} \quad \text{SUBTM_PATTERN_AGREE_BASE}$$

$$\frac{a = p^+}{a \; a_2{}^{\nu} = p^+} \quad \text{SUBTM_PATTERN_AGREE_APP}$$

$$\frac{a = p^+}{a[\bullet] = p^+} \quad \text{SUBTM_PATTERN_AGREE_CAPPP}$$

ValuePath a = FType headed by constant (role-sensitive partial function used in value)

$$\frac{F:A@Rs \in \Sigma_0}{\text{ValuePath } F = F} \quad \text{ValuePath_AbsConst}$$

$$F:p \sim a:A/R_1@Rs \in \Sigma_0 \\ \text{ValuePath } F = F \\ \text{ValuePath } a = F \\ \text{ValuePath } (a\ b'^{\nu}) = F \\ \text{ValuePath_CApp} \\ \text{ValuePath } (a\ b'^{\nu}) = F \\ \text{ValuePath } (a\ b'^{\nu}) = F \\ \text{ValuePath_CApp} \\ \text{ValuePath_CApp} \\ \text{ValuePath } (a\ b'^{\nu}) = F \\ \text{ValuePath_CApp} \\ \text{ValueP$$

CasePath_R a = FType headed by constant (role-sensitive partial function used in case)

$$\begin{array}{c} \text{ValuePath } a = F \\ \underline{F: A@Rs \in \Sigma_0} \\ \hline \text{CasePath}_R \ a = F \end{array} \quad \text{CasePath_AbsConst}$$

$$\begin{array}{c} \mathsf{ValuePath} \ a = F \\ F: p \sim b: A/R_1@Rs \in \Sigma_0 \\ \neg (R_1 \leq R) \end{array} \qquad \mathsf{CASEPATH_CONST} \\ \hline \mathbf{CasePath}_R \ a = F \\ F: p \sim b: A/R_1@Rs \in \Sigma_0 \\ \neg (a = p^{\pm}) \end{array} \qquad \mathsf{CASEPATH_UNMATCH} \\ \hline \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ a'' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ a'' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ a'' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ a'' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ a'' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ a'' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ a'' \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{apply args} \ a \ \mathsf{to} \ b \mapsto b' \ \mathsf{to} \\ \mathsf{appl$$

 $\underline{\text{consistent}_R \ a \ b}$ (erased) types do not differ in their heads

```
CONSISTENT_A_PI
                         \overline{\mathsf{consistent}_{R'} \; (\Pi^{\rho} x_1 \colon A_1 \to B_1) \; (\Pi^{\rho} x_2 \colon A_2 \to B_2)}
                                                                                                         CONSISTENT_A_CPI
                                 \overline{\mathsf{consistent}_R \; (\forall c_1 : \phi_1.A_1) \; (\forall c_2 : \phi_2.A_2)}
                                            \mathsf{CasePath}_R\ a_1 = F
                                           \mathsf{CasePath}_R\ a_2 = \underline{F}
                                                                                  CONSISTENT_A_CASEPATH
                                             consistent_R a_1 a_2
                                                \neg ValueType_R b CONSISTENT_A_STEP_R
                                                \mathsf{consistent}_R \ a \ b
                                                \neg ValueType_R \ a consistent_A b CONSISTENT_A_STEP_L
|\Omega \vDash a : R|
                        Roleing judgment
                                                            \frac{uniq(\Omega)}{\Omega \vDash \square : R} \quad \text{ROLE\_A\_BULLET}
                                                               \frac{uniq(\Omega)}{\Omega \vDash \star : R} \quad \text{ROLE\_A\_STAR}
                                                               uniq(\Omega)
                                                               x:R\in\Omega
                                                               \frac{R \le R_1}{\Omega \vDash x : R_1} \quad \text{ROLE\_A\_VAR}
                                                       \frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^{\rho} x.a) : R} \quad \text{ROLE\_A\_ABS}
                                                            \Omega \vDash a : R
                                                           \frac{1}{\Omega \vDash (a \ b^{\rho}) : R} \quad \text{ROLE\_A\_APP}
                                                     \Omega \vDash a : R
                                                     Path a = F@R_1, Rs
                                                          \frac{\models b : R_1}{\Omega \models a \ b^{R_1} : R} ROLE_A_TAPP
                                                     \Omega \vDash b : R_1
                                                        \Omega \vDash A : R
                                                      \frac{\Omega, x: \mathbf{Nom} \vDash B: R}{\Omega \vDash (\Pi^{\rho}x: A \to B): R} \quad \text{ROLE\_A\_PI}
                                                               \Omega \vDash a : R_1
                                                               \Omega \vDash b : R_1
                                                               \Omega \vDash A : R_0
                                                               \Omega \vDash B:R
                                                                                                    role_a_CPi
                                                 \overline{\Omega \vDash (\forall c : a \sim_{A/R_1} b.B) : R}
                                                          \frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \text{ROLE\_A\_CABS}
                                                          \frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \text{ROLE\_A\_CAPP}
```

$$\frac{uniq(\Omega)}{\Omega \vDash F: R} = R \text{ ROLE_A_CONST}$$

$$\frac{uniq(\Omega)}{F: p \sim a: A/R@Rs} \in \Sigma_0}{\Omega \vDash F: R_1} \text{ ROLE_A_FAM}$$

$$\frac{F: p \sim a: A/R@Rs}{\Omega \vDash b_1 \coloneqq R_1} = R \text{ ROLE_A_FAM}$$

$$\frac{\Omega \vDash a: R}{\Omega \vDash b_1 \coloneqq R_1}$$

$$\frac{\Omega \vDash a \bowtie R}{\Omega \vDash b_2 \coloneqq R_1} = R \text{ ROLE_A_PATTERN}$$

$$(\rho = +) \lor (x \not \in \text{ fv } A) \text{ irrelevant argument check}$$

$$\frac{x \not \in \text{ fv } A}{(-=+) \lor (x \not \in \text{ fv } A)} = R \text{ RHO_IRREL}$$

$$\frac{x \not \in \text{ fv } A}{(-=+) \lor (x \not \in \text{ fv } A)} = R \text{ RHO_IRREL}$$

$$\frac{\Omega \vDash a \Rightarrow_R b}{\Omega \vDash a \Rightarrow_R a} = P \text{ AR_REFL}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\lambda^p x. a')}{\Omega \vDash a b^p \Rightarrow_R a' \{b'/x\}} = P \text{ AR_BETA}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a \Rightarrow_R a'} = P \text{ AR_APP}$$

$$\frac{\Omega \vDash a \Rightarrow_R (Ac.a')}{\Omega \vDash a [\bullet] \Rightarrow_R a' b'^p} = P \text{ AR_APP}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a \Rightarrow_R a'} = P \text{ AR_CAPP}$$

$$\frac{\Omega}{\Omega} \vDash a \Rightarrow_R a' \Rightarrow_R a'$$

 $\frac{1}{\Omega \vDash \forall c : a \sim_{A/R_1} b.B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'.B'} \quad \text{PAR_CPI}$

 $\Omega \vDash B \Rightarrow_R B'$

$$F: F \sim b: A/R_1@Rs \in \Sigma_0$$

$$R_1 \leq R$$

$$umiq(\Omega)$$

$$D \models F \Rightarrow_R b$$

$$F: p \sim b: A/R_1@Rs \in \Sigma_0$$

$$a^+ = p \land \neg (a \leftrightarrow p)$$

$$\Omega \models a \Rightarrow_R a'$$

$$\Omega \models a \Rightarrow_R a'$$

$$\Omega \models a \Rightarrow_R a \Rightarrow_$$

 $\models a > b/R$ primitive reductions on erased terms $\frac{\mathsf{Value}_{R_1} (\lambda^{\rho} x. v)}{\vDash (\lambda^{\rho} x. v) \ b^{\rho} > v\{b/x\}/R_1} \quad \mathsf{BETA_APPABS}$ $= (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R$ BETA_CAPPCABS $F: p \sim b: A/R_1@Rs \in \Sigma_0$ rename $p \to b$ to $p_1 \to b_1$ excluding (fva, fvp) match a with $p_1 \rightarrow b_1 = b'$ $R_1 \leq R$ Beta_Axiom $\models a > b'/R$ $\mathsf{CasePath}_R\ a = F$ $\frac{\text{apply args } a \text{ to } b_1 \mapsto b_1'}{\models \mathsf{case}_R \ a \text{ of } F \to b_1 \|_{-} \to b_2 > b_1' [\bullet] / R_0} \quad \text{Beta_PatternTrue}$ $\frac{\neg(\mathsf{CasePath}_R\ a = F)}{\models \mathsf{case}_R\ a \text{ of } F \to b_1 \|_- \to b_2 > b_2/R_0} \quad \text{Beta_PatternFalse}$ single-step head reduction for implicit language $\frac{\models a \leadsto a'/R_1}{\models \lambda^- x. a \leadsto \lambda^- x. a'/R_1} \quad \text{E_AbsTerm}$ $\frac{\models a \leadsto a'/R_1}{\models a \ b^{\nu} \leadsto a' \ b^{\nu}/R_1} \quad \text{E-Appleft}$ $\frac{\vDash a \leadsto a'/R}{\vDash a[\bullet] \leadsto a'[\bullet]/R} \quad \text{E_CAPPLEFT}$ $\frac{\models a \leadsto a'/R}{\models \mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \|_{\text{-}} \to b_2 \leadsto \mathsf{case}_R \ a' \ \mathsf{of} \ F \to b_1 \|_{\text{-}} \to b_2/R_0}$ $\frac{\models a > b/R}{\models a \leadsto b/R}$ E_PRIM $\vDash a \leadsto^* b/R$ multistep reduction $= a \leadsto^* a/R$ Equal $\Gamma \vDash \mathsf{case}_R \ a : A \ \mathsf{of} \ b : B \Rightarrow C \mid C'$ Branch Typing (aligning the types of case)

 $\frac{1c_tm \ C}{\Gamma \vDash \mathsf{case}_R \ a : A \text{ of } b : A \Rightarrow \forall c : (a \sim_{A/R} b) . C \mid C} \quad \mathsf{BRANCHTYPING_BASE}$

 $\frac{\Gamma, x: A \vDash \mathsf{case}_R \ a: A_1 \ \mathsf{of} \ b \ x^+: B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R \ a: A_1 \ \mathsf{of} \ b: \Pi^+ x: A \to B \Rightarrow \Pi^+ x: A \to C \mid C'} \quad \text{BranchTyping_PiRel}$

 $\frac{\Gamma, x: A \vDash \mathsf{case}_R \ a: A_1 \ \mathsf{of} \ b \ \Box^-: B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R \ a: A_1 \ \mathsf{of} \ b: \Pi^- x: A \to B \Rightarrow \Pi^- x: A \to C \mid C'} \quad \text{BranchTyping_PiIrrel}$

$$\frac{\Gamma,\,c:\phi\vDash\mathsf{case}_R\;a:A\;\mathsf{of}\;b[\bullet]:B\Rightarrow C\;|\;C'}{\Gamma\vDash\mathsf{case}_R\;a:A\;\mathsf{of}\;b:\forall c\!:\!\phi.B\Rightarrow\forall c\!:\!\phi.C\;|\;C'}\quad\mathsf{BranchTyping_CPi}$$

 $\Gamma \vDash \phi$ ok Prop wellformedness

$$\begin{split} & \Gamma \vDash a : A \\ & \Gamma \vDash b : A \\ & \frac{\Gamma \vDash A : \star}{\Gamma \vDash a \sim_{A/R} b \text{ ok}} \quad \text{E-Wff} \end{split}$$

 $\Gamma \vDash a : A$ typing

$$\begin{array}{c} \models \Gamma \\ F:A@Rs \in \Sigma_0 \\ \underline{\varnothing \vDash A: \star} \\ \hline \Gamma \vDash F:A \end{array} \quad \text{E-Const} \\ \\ \frac{\vdash \Gamma}{F:F:A} \qquad \qquad E_{FAM} \\ \\ \frac{F:p \sim a:A/R_1@Rs \in \Sigma_0}{\Gamma \vDash F:A} \qquad E_{FAM} \\ \\ \Gamma \vDash a:A \\ \Gamma \vDash F:A_1 \\ \Gamma \vDash b_1:B \\ \Gamma \vDash b_2:C \\ \underline{\Gamma \vDash \mathsf{case}_R \; a:A \; \mathsf{of} \; F:A_1 \Rightarrow B \; | \; C} \\ \underline{\Gamma \vDash \mathsf{case}_R \; a:A \; \mathsf{of} \; F:A_1 \Rightarrow B \; | \; C} \\ \overline{\Gamma \vDash \mathsf{case}_R \; a \; \mathsf{of} \; F \to b_1 \|_{-} \to b_2:C} \end{array} \quad \text{E-Case} \\ \underline{\Gamma;\Delta \vDash A_1 \equiv A_2:A/R} \\ \underline{\Gamma;\Delta \vDash A_1 \equiv A_2:A/R} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_1 \equiv B_2:A/R} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_2 \approx A_1 \equiv A_2 \approx A_2 \approx A_3 \equiv B_2} \qquad \text{E-Prop-Const} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_2 \approx A_3 \approx B_2} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_2 \approx A_3 \approx B_2} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_2 \approx A_3 \approx B_2} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_2 \approx A_3 \approx B_2} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_2 \approx A_3 \approx B_2} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \vDash A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \succeq A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \succeq A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \succeq A_1 \approx A_3 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{\Gamma;\Delta \succeq A_1 \approx A_3 \approx B_3} \qquad \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{E} = P_{\mathsf{Prop-Const}} \\ \underline{E} = P_{\mathsf{Prop-Const}}$$

$$\begin{split} &\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ &\Gamma; \Delta \vDash B_1 \equiv B_2 : A/R \\ \hline &\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2 \end{split} \quad \text{E_PropCong} \\ &\Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ &\Gamma \vDash A_1 \sim_{A/R} A_2 \text{ ok} \\ &\Gamma \vDash A_1 \sim_{B/R} A_2 \text{ ok} \\ \hline &\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2 \end{split} \quad \text{E_IsoConv} \end{split}$$

$$\frac{\Gamma; \Delta \vDash \forall c : (a_1 \sim_{A/R_1} a_2) . B_1 \equiv \forall c : (b_1 \sim_{B/R_2} b_2) . B_2 : \star / R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E_CPIFST}$$

 $\Gamma; \Delta \vDash a \equiv b : A/R$ definitional equality

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\Gamma; \Delta \vDash A_1 \equiv A_2 : \star / R'
                            \Gamma, x: A_1; \Delta \vDash B_1 \equiv B_2: \star/R'
                            \Gamma \vDash A_1 : \star
                            \Gamma \vDash \Pi^{\rho} x : A_1 \to B_1 : \star
                            \Gamma \vDash \Pi^{\rho} x : A_2 \to B_2 : \star
                                                                                                             E_PICONG
         \Gamma; \Delta \models (\Pi^{\rho}x : A_1 \to B_1) \equiv (\Pi^{\rho}x : A_2 \to B_2) : \star / R'
                           \Gamma, x: A_1; \Delta \vDash b_1 \equiv b_2: B/R'
                           \Gamma \vDash A_1 : \star
                           (\rho = +) \lor (x \not\in \mathsf{fv}\ b_1)
                           (\rho = +) \lor (x \not\in \mathsf{fv}\ b_2)
                                                                                                        E_AbsCong
        \overline{\Gamma;\Delta\vDash(\lambda^{\rho}x.b_{1})\equiv(\lambda^{\rho}x.b_{2}):(\Pi^{\rho}x:A_{1}\to B)/R'}
                    \Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x : A \to B)/R'
                    \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{Nom}
                                                                                                  E_AppCong
                \overline{\Gamma; \Delta} \models a_1 \ a_2^+ \equiv b_1 \ b_2^+ : (B\{a_2/x\})/R'
                   \Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x : A \to B)/R'
                   \Gamma; \Delta \vDash a_2 \equiv b_2 : A/R
                   Path a_1 = F@R, Rs
                   Path b_1 = F'@R, Rs'
              \overline{\Gamma; \Delta \vDash a_1 \ a_2{}^R \equiv b_1 \ b_2{}^R : (B\{a_2/x\})/R'}
                                                                                                E_TAPPCONG
                    \Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x : A \to B)/R'
                    \Gamma \vDash a : A
                                                                                             E_IAppCong
                \Gamma : \Delta \vDash a_1 \square^- \equiv b_1 \square^- : (B\{a/x\})/R'
              \frac{\Gamma; \Delta \vDash \Pi^{\rho} x : A_1 \to B_1 \equiv \Pi^{\rho} x : A_2 \to B_2 : \star / R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star / R'}
              \Gamma; \Delta \vDash \Pi^{\rho} x : A_1 \to B_1 \equiv \Pi^{\rho} x : A_2 \to B_2 : \star / R'
              \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/\mathbf{Nom}
                                                                                                             E_PiSnd
                      \Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'
                   \Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2
                   \Gamma, c: a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B: \star/R'
                   \Gamma \vDash a_1 \sim_{A_1/R} b_1 ok
                   \Gamma \vDash \forall c : a_1 \sim_{A_1/R} b_1.A : \star
                   \Gamma \vDash \forall c : a_2 \sim_{A_2/R} b_2.B : \star
                                                                                                               E_CPICONG
   \overline{\Gamma; \Delta \vDash \forall c : a_1 \sim_{A_1/R} b_1.A \equiv \forall c : a_2 \sim_{A_2/R} b_2.B : \star/R'}
                           \Gamma, c: \phi_1; \Delta \vDash a \equiv b: B/R
                           \Gamma \vDash \phi_1 ok
                                                                                           E_CABSCONG
                 \overline{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c : \phi_1.B/R}
              \Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b).B)/R'
              \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param} R R'
                   \Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R' E_CAPPCONG
\Gamma; \Delta \vDash \forall c : (a_1 \sim_{A/R} a_2).B_1 \equiv \forall c : (a'_1 \sim_{A'/R'} a'_2).B_2 : \star/R_0
\Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param} \ R \ R_0
\Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param} R' R_0
                                                                                                                        E_CPiSnd
                       \Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0
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\Gamma; \Delta \vDash a \equiv b : A/R
                                               \frac{\Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b'}{\Gamma; \Delta \vDash a' \equiv b' : A'/R'} \quad \text{E-CAST}
                                                      \Gamma; \Delta \vDash a \equiv b : A/R
                                                      \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star / \mathbf{Rep}
                                                      \Gamma \vDash B : \star
                                                      \frac{\Gamma \vDash B : \star}{\Gamma; \Delta \vDash a \equiv b : B/R} \quad \text{E\_EQCONV}
                                          \frac{\Gamma; \Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_ISOSND}
                                                     \Gamma; \Delta \vDash a \equiv a' : A/R
                                                     \Gamma; \Delta \vDash b_1 \equiv b'_1 : B/R_0
                                                    \Gamma; \Delta \vDash b_2 \equiv b_2' : B/R_0
\frac{1}{\Gamma; \Delta \vDash \mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \parallel_{-} \to b_2 \equiv \mathsf{case}_R \ a' \ \mathsf{of} \ F \to b_1' \parallel_{-} \to b_2' : B/R_0} \quad \text{E\_PATCONG}
                                      ValuePath a = F
                                      ValuePath a' = F
                                      \Gamma \vDash a : \Pi^+ x : A \to B
                                      \Gamma \vDash b : A
                                      \Gamma \vDash a' : \Pi^+ x : A \to B
                                      \Gamma \vDash b' : A
                                      \Gamma; \Delta \vDash a \ b^{R_1} \equiv a' \ b'^{R_1} : B\{b/x\}/R'
                                          \frac{\mathcal{L}_{\{0\}} x_{f} = \mathcal{B}\{0'/x\} : \star/R'}{\Gamma; \Delta \vDash a \equiv a' : \Pi^{+}x : A \to B/R'} \quad \text{E-LeftRel}
                                      \Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R'
                                     ValuePath a = F
                                     ValuePath a' = F
                                     \Gamma \vDash a : \Pi^- x : A \to B
                                     \Gamma \vDash b : A
                                     \Gamma \vDash a' : \Pi^- x : A \to B
                                     \Gamma \vDash b' : A
                                     \Gamma; \Delta \vDash a \square^- \equiv a' \square^- : B\{b/x\}/R'
                                    \frac{\Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0}{\Gamma; \Delta \vDash a \equiv a' : \Pi^- x : A \to B/R'} \quad \text{E_LEFTIRREL}
                                           ValuePath a = F
                                           ValuePath a' = F
                                           \Gamma \vDash a : \Pi^+ x : A \to B
                                           \Gamma \vDash b : A
                                           \Gamma \vDash a' : \Pi^+ x \colon\! A \to B
                                           \Gamma \vDash b' : A
                                           \Gamma; \Delta \vDash a \ b^+ \equiv a' \ b'^+ : B\{b/x\}/R'
                                           \frac{\Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0}{\Gamma; \Delta \vDash b \equiv b' : A/\mathbf{param} R_1 R'}
                                                                                                                           E_Right
                                              ValuePath a = F
                                              ValuePath a' = F
                                              \Gamma \vDash a : \forall c : (a_1 \sim_{A/R_1} a_2).B
                                              \Gamma \vDash a' : \forall c : (a_1 \sim_{A/R_1} a_2).B
                                              \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R'
                                      \frac{\Gamma;\Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'}{\Gamma;\Delta \vDash a \equiv a' : \forall c : (a_1 \sim_{A/R_1} a_2).B/R'}
                                                                                                                              E_{-}CLeft
```

$\models \Gamma$ context wellformedness

$\models \Sigma$ signature wellformedness

 $\Gamma \vdash \phi$ ok prop wellformedness

 $\Gamma \vdash a : A/R$ typing

 $\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$ coercion between props

 $\Gamma; \Delta \vdash \gamma : A \sim_R B$ coercion between types

 $\vdash \Gamma$ context wellformedness

 $\Gamma \vdash a \leadsto b/R$ single-step, weak head reduction to values for annotated language

Definition rules: 159 good 0 bad Definition rule clauses: 445 good 0 bad