Specification of System D and System DC

$tmvar,\ x,\ y,\ f,\ m,\ n$	variables
$covar,\ c$	coercion variables
$datacon,\ K$	
$const,\ T,\ F$	
$index,\ i$	indices

$relflag,\ \rho$	$::=$		relevance flag
	$\mid$	$+$	
	$\mid$	$-$	
$constraint,\ \phi$	$::=$		props
	$\mid$	$a \sim_A b$	
$tm,\ a,\ b,\ v,\ w,\ A,\ B$	$::=$		types and kinds
	$\mid$	$\star$	
	$\mid$	x	
	$\mid$	$\lambda^\rho x\!:\!A.b$	annotated lambda
	$\mid$	$\lambda^\rho x.b$	erased lambda
	$\mid$	$a\ b^\rho$	
	$\mid$	T	constant
	$\mid$	$\Pi^\rho x\!:\!A \to B$	
	$\mid$	$a \triangleright \gamma$	coercion
	$\mid$	$\forall c\!:\!\phi.B$	
	$\mid$	$\Lambda c\!:\!\phi.b$	annotated coercion abstraction
	$\mid$	$\Lambda c.b$	erased coercion abstraction
	$\mid$	$a[\gamma]$	
	$\mid$	$\square$	erased term
	$\mid$	$\Sigma^\rho x\!:\!A.B$	
	$\mid$	$({}^\rho a, b)$	erased pair
	$\mid$	$({}^\rho a, b)\ \mathsf{as}\ A$	annotated pair
	$\mid$	$\mathbf{fst}\ a$	
	$\mid$	$\mathbf{snd}\ a$	

$co,\ \gamma$	$::=$		explicit coercions
	$\mid$	$\bullet$	erased coercion
	$\mid$	c	
	$\mid$	$\mathbf{red}\,a\,b$	
	$\mid$	$\mathbf{refl}\,a$	
	$\mid$	$(a\ \vert{=}\vert_{\gamma}\ b)$	
	$\mid$	$\mathbf{sym}\,\gamma$	
	$\mid$	$\gamma_1;\gamma_2$	
	$\mid$	$\Pi^{\rho}x\!:\!\gamma_1.\gamma_2$	
	$\mid$	$\lambda^{\rho}x\!:\!\gamma_1.\gamma_2$	
	$\mid$	$\gamma_1\ \gamma_2^{\rho}$	
	$\mid$	$\mathbf{piFst}\,\gamma$	
	$\mid$	$\mathbf{cpiFst}\,\gamma$	
	$\mid$	$\mathbf{isoSnd}\,\gamma$	
	$\mid$	$\gamma_1 @ \gamma_2$	
	$\mid$	$\forall c\!:\!\gamma_1.\gamma_3$	
	$\mid$	$\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4$	
	$\mid$	$\gamma(\gamma_1, \gamma_2)$	
	$\mid$	$\gamma@(\gamma_1 \sim \gamma_2)$	
	$\mid$	$\gamma_1 \triangleright \gamma_2$	
	$\mid$	$\gamma_1 \sim_A \gamma_2$	
	$\mid$	$\mathbf{conv}\ \phi_1 \sim_{\gamma} \phi_2$	
	$\mid$	$\mathbf{eta}\,a$	
	$\mid$	$\mathbf{left}\,\gamma\,\gamma'$	
	$\mid$	$\mathbf{right}\,\gamma\,\gamma'$	
	$\mid$	$\Sigma x\!:^{\rho}\gamma_1.\gamma_2$	
	$\mid$	$(\rho\,\gamma_1, \gamma_2)\,\mathbf{as}\,\gamma_3$	
	$\mid$	$\mathbf{fst}\,\gamma_1$	
	$\mid$	$\mathbf{snd}\,\gamma_1$	
	$\mid$	$\mathbf{sigmaFst}\,\gamma$	

sig_sort	::=		signature classifier
	|	$: A$	
	|	$\sim a : A$	
sort	::=		binding classifier
	|	$\mathbf{Tm}\,\rho\,A$	
	|	$\mathbf{Co}\,\phi$	
context, Γ	::=		contexts
	|	$\varnothing$	
	|	$\Gamma, x : \rho\,A$	
	|	$\Gamma, c : \phi$	
available_props, Δ	::=		
	|	$\varnothing$	
	|	Δ, c	
sig, Σ	::=		signatures
	|	$\varnothing$	
	|	$\Sigma \cup \{\, T\, sig\_sort \}$	

$\boxed{\rho_1 \Leftarrow \rho_2}$ *()*

$$\textsc{Bot}\ \frac{}{- \Leftarrow \rho} \qquad \textsc{Top}\ \frac{}{\rho \Leftarrow +} \qquad \textsc{Refl}\ \frac{}{\rho \Leftarrow \rho}$$

$\boxed{\Gamma_1 \Leftarrow \Gamma_2}$ *()*

$$\textsc{Sub-Empty}\ \frac{}{\varnothing \Leftarrow \varnothing} \qquad \textsc{Sub-ConsTm}\ \frac{\rho_1 \Leftarrow \rho_2 \qquad \Gamma_1 \Leftarrow \Gamma_2}{\Gamma_1, x : \rho_1\, A \Leftarrow \Gamma_2, x : \rho_2\, A} \qquad \textsc{Sub-ConsCo}\ \frac{\Gamma_1 \Leftarrow \Gamma_2}{\Gamma_1, c : \phi \Leftarrow \Gamma_2, c : \phi}$$

$\boxed{\mathbf{CoercedValue}\, A}$ *(Values with at most one coercion at the top)*

$$\text{CV}\ \frac{[\mathsf{Value}\ a]}{\mathbf{CoercedValue}\, a} \qquad \text{CC}\ \frac{[\mathsf{Value}\ a]}{\mathbf{CoercedValue}\,(a \triangleright \gamma)}$$

$\boxed{[\mathsf{Value}\ A]}$ *(values)*

$$\textsc{Value-Star}\ \frac{}{[\mathsf{Value}\ \star]} \qquad \textsc{Value-Pi}\ \frac{}{[\mathsf{Value}\ \Pi^{\rho} x{:}A \to B]} \qquad \textsc{Value-CPi}\ \frac{}{[\mathsf{Value}\ \forall c{:}\phi.B]}$$

$$\frac{}{[\mathsf{Value}\ \lambda^{+} x : A.a]}\ \textsc{Value-AbsRel} \qquad \frac{}{[\mathsf{Value}\ \lambda^{+} x.a]}\ \textsc{Value-UAbsRel} \qquad \frac{[\mathsf{Value}\ a]}{[\mathsf{Value}\ \lambda^{-} x.a]}\ \textsc{Value-UAbsIrrel}$$

$$\frac{\mathbf{CoercedValue}\ a}{[\mathsf{Value}\ \lambda^{-} x : A.a]}\ \textsc{Value-AbsIrrel} \qquad \frac{}{[\mathsf{Value}\ \Lambda c : \phi.a]}\ \textsc{Value-CAbs} \qquad \frac{}{[\mathsf{Value}\ \Lambda c.a]}\ \textsc{Value-UCAbs}$$

$$\frac{}{[\mathsf{Value}\ \Sigma^{\rho} x : A.B]}\ \textsc{Value-Sigma} \qquad \frac{}{[\mathsf{Value}\ ({}^{\rho}\, a, b)]}\ \textsc{Value-UPair} \qquad \frac{}{[\mathsf{Value}\ ({}^{\rho}\, a, b)\ \mathsf{as}\ A]}\ \textsc{Value-Pair}$$

$\boxed{\mathbf{ValueType}\ A}$ *(Types with head forms (erased language))*

$$\frac{}{\mathbf{ValueType}\ \star}\ \textsc{value-type-Star} \qquad \frac{}{\mathbf{ValueType}\ \Pi^{\rho} x : A \to B}\ \textsc{value-type-Pi} \qquad \frac{}{\mathbf{ValueType}\ \forall c : \phi.B}\ \textsc{value-type-CPi}$$

$$\frac{}{\mathbf{ValueType}\ \Sigma^{\rho} x : A.B}\ \textsc{value-type-Sigma}$$

$\boxed{\mathbf{consistent}\ a\ b}$ *((erased) types do not differ in their heads)*

$$\frac{}{\mathbf{consistent}\ \star\ \star}\ \textsc{consistent-a-Star} \qquad \frac{}{\mathbf{consistent}\ (\Pi^{\rho} x_1 : A_1 \to B_1)\ (\Pi^{\rho} x_2 : A_2 \to B_2)}\ \textsc{consistent-a-Pi}$$

$$\frac{}{\mathbf{consistent}\ (\forall c_1 : \phi_1.A_1)\ (\forall c_2 : \phi_2.A_2)}\ \textsc{consistent-a-CPi}$$

$$\frac{}{\mathbf{consistent}\ (\Sigma^{\rho} x : A_1.B_1)\ (\Sigma^{\rho} x : A_2.B_2)}\ \textsc{consistent-a-Sigma} \qquad \frac{\mathbf{not\ ValueType}\ b}{\mathbf{consistent}\ a\ b}\ \textsc{consistent-a-Step-R}$$

$$\frac{\mathbf{not\ ValueType}\ a}{\mathbf{consistent}\ a\ b}\ \textsc{consistent-a-Step-L}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv}\ A)}$ *(irrelevant argument check)*

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv}\ A)}\ \textsc{Rho-Rel} \qquad \frac{x \notin \mathsf{fv} A}{(- = +) \vee (x \notin \mathsf{fv}\ A)}\ \textsc{Rho-IrrRel}$$

$\boxed{\textbf{erased\_tm}\ a}$ *()*

$$\frac{}{\textbf{erased\_tm}\ \Box}\ \textsc{erased-a-Bullet} \qquad \frac{}{\textbf{erased\_tm}\ \star}\ \textsc{erased-a-Star} \qquad \frac{}{\textbf{erased\_tm}\ x}\ \textsc{erased-a-Var}$$

$$\frac{\textbf{erased\_tm}\ a \quad (\rho = +) \vee (x \notin \mathsf{fv}\ a)}{\textbf{erased\_tm}\ (\lambda^{\rho} x.a)}\ \textsc{erased-a-Abs} \qquad \frac{\textbf{erased\_tm}\ a \quad \textbf{erased\_tm}\ b}{\textbf{erased\_tm}\ (a\ \ b^{+})}\ \textsc{erased-a-App} \qquad \frac{\textbf{erased\_tm}\ a}{\textbf{erased\_tm}\ (a\ \Box^{-})}\ \textsc{erased-a-AppIrrel}$$

$$\frac{\textbf{erased\_tm}\ A \quad \textbf{erased\_tm}\ B}{\textbf{erased\_tm}\ (\Pi^{\rho} x\!:\!A \to B)}\ \textsc{erased-a-Pi} \qquad \frac{\textbf{erased\_tm}\ a \quad \textbf{erased\_tm}\ b \quad \textbf{erased\_tm}\ A \quad \textbf{erased\_tm}\ B}{\textbf{erased\_tm}\ (\forall c\!:\!a \sim_A b.B)}\ \textsc{erased-a-CPi}$$

$$\frac{\textbf{erased\_tm}\ b}{\textbf{erased\_tm}\ (\Lambda c.b)}\ \textsc{erased-a-CAbs} \qquad \frac{\textbf{erased\_tm}\ a}{\textbf{erased\_tm}\ (a[\bullet])}\ \textsc{erased-a-CApp} \qquad \frac{}{\textbf{erased\_tm}\ F}\ \textsc{erased-a-Fam}$$

$$\frac{\textbf{erased\_tm}\ A \quad \textbf{erased\_tm}\ B}{\textbf{erased\_tm}\ (\Sigma^{\rho} x\!:\!A.B)}\ \textsc{erased-a-Sigma} \qquad \frac{\textbf{erased\_tm}\ a \quad \textbf{erased\_tm}\ b}{\textbf{erased\_tm}\ ({}^{+}a, b)}\ \textsc{erased-a-UPairRel} \qquad \frac{\textbf{erased\_tm}\ b}{\textbf{erased\_tm}\ ({}^{-}\Box, b)}\ \textsc{erased-a-UPairIrrel}$$

$$\frac{\textbf{erased\_tm}\ a}{\textbf{erased\_tm}\ (\textbf{fst}\ a)}\ \textsc{erased-a-Fst} \qquad \frac{\textbf{erased\_tm}\ a}{\textbf{erased\_tm}\ (\textbf{snd}\ a)}\ \textsc{erased-a-Snd}$$

$\boxed{\vDash a \Rightarrow b}$ *(parallel reduction (implicit language))*

$$\frac{}{\vDash a \Rightarrow a}\ \textsc{Par-Refl} \qquad \frac{\vDash a \Rightarrow (\lambda^{+} x.a') \quad \vDash b \Rightarrow b'}{\vDash a\ \ b^{+} \Rightarrow a'\{b'/x\}}\ \textsc{Par-Beta} \qquad \frac{\vDash a \Rightarrow (\lambda^{-} x.a')}{\vDash a\ \Box^{-} \Rightarrow a'\{\Box/x\}}\ \textsc{Par-BetaIrrel}$$

$$\frac{\vDash a \Rightarrow a' \qquad \vDash b \Rightarrow b'}{\vDash a\ \ b^{+} \Rightarrow a'\ \ b'^{+}}\ \textsc{Par-App} \qquad \frac{\vDash a \Rightarrow a'}{\vDash a\ \Box^{-} \Rightarrow a'\ \Box^{-}}\ \textsc{Par-AppIrrel} \qquad \frac{\vDash a \Rightarrow (\Lambda c.a')}{\vDash a[\bullet] \Rightarrow a'\{\bullet/c\}}\ \textsc{Par-CBeta}$$

$$\frac{\vDash a \Rightarrow a'}{\vDash a[\bullet] \Rightarrow a'[\bullet]}\ \textsc{Par-CApp} \qquad \frac{\vDash a \Rightarrow a'}{\vDash \lambda^{\rho} x.a \Rightarrow \lambda^{\rho} x.a'}\ \textsc{Par-Abs} \qquad \frac{\vDash A \Rightarrow A' \quad \vDash B \Rightarrow B'}{\vDash \Pi^{\rho} x\!:\!A \to B \Rightarrow \Pi^{\rho} x\!:\!A' \to B'}\ \textsc{Par-Pi}$$

$$\textsc{Par-CAbs}\quad \frac{\vDash a \Rightarrow a'}{\vDash \Lambda c.a \Rightarrow \Lambda c.a'}$$

$$\textsc{Par-CPi}\quad \frac{\begin{array}{c}\vDash A \Rightarrow A' \\ \vDash B \Rightarrow B' \\ \vDash a \Rightarrow a' \\ \vDash A_1 \Rightarrow A_1'\end{array}}{\vDash \forall c\!:\!A \sim_{A_1} B.a \Rightarrow \forall c\!:\!A' \sim_{A_1'} B'.a'}$$

$$\textsc{Par-Axiom}\quad \frac{F \sim a : A \in \Sigma_0}{\vDash F \Rightarrow a}$$

$$\textsc{Par-Eta}\quad \frac{\vDash b \Rightarrow b' \qquad a = b\ x^+}{\vDash \lambda^+ x.a \Rightarrow b'}$$

$$\textsc{Par-EtaIrrel}\quad \frac{\begin{array}{c}\vDash b \Rightarrow b' \\ a = b\ \Box^-\end{array}}{\vDash \lambda^- x.a \Rightarrow b'}$$

$$\textsc{Par-EtaC}\quad \frac{\vDash b \Rightarrow b' \qquad a = b[\bullet]}{\vDash \Lambda c.a \Rightarrow b'}$$

$$\textsc{Par-Sigma}\quad \frac{\begin{array}{c}\vDash A \Rightarrow A' \\ \vDash B \Rightarrow B'\end{array}}{\vDash \Sigma^\rho x\!:\!A.B \Rightarrow \Sigma^\rho x\!:\!A'.B'}$$

$$\textsc{Par-Pair}\quad \frac{\vDash a \Rightarrow a' \qquad \vDash b \Rightarrow b'}{\vDash ({}^+a, b) \Rightarrow ({}^+a', b')}$$

$$\textsc{Par-PairIrrel}\quad \frac{\vDash b \Rightarrow b'}{\vDash ({}^-\Box, b) \Rightarrow ({}^-\Box, b')}$$

$$\textsc{Par-Fst}\quad \frac{\vDash a \Rightarrow ({}^\rho a_1, a_2)}{\vDash \mathbf{fst}\ a \Rightarrow a_1}$$

$$\textsc{Par-Snd}\quad \frac{\vDash a \Rightarrow ({}^\rho a_1, a_2)}{\vDash \mathbf{snd}\ a \Rightarrow a_2}$$

$$\textsc{Par-FstCong}\quad \frac{\vDash a \Rightarrow a_1}{\vDash \mathbf{fst}\ a \Rightarrow \mathbf{fst}\ a_1}$$

$$\textsc{Par-SndCong}\quad \frac{\vDash a \Rightarrow a_2}{\vDash \mathbf{snd}\ a \Rightarrow \mathbf{snd}\ a_2}$$

$\boxed{\vdash a \Rightarrow^* b}$ *(multistep parallel reduction)*

$$\textsc{MP-Refl}\quad \frac{}{\vdash a \Rightarrow^* a}$$

$$\textsc{MP-Step}\quad \frac{\begin{array}{c}\vDash a \Rightarrow b \\ \vdash b \Rightarrow^* a'\end{array}}{\vdash a \Rightarrow^* a'}$$

$\boxed{\vdash a \Leftrightarrow b}$ *(parallel reduction to a common term)*

$$\textsc{join}\quad \frac{\begin{array}{c}\vdash a_1 \Rightarrow^* b \\ \vdash a_2 \Rightarrow^* b\end{array}}{\vdash a_1 \Leftrightarrow a_2}$$

$\boxed{\vDash a > b}$ *(primitive reductions on erased terms)*

$$\textsc{Beta-AppAbs}\quad \frac{}{\vDash (\lambda^+ x.v)\ b^+ > v\{b/x\}}$$

$$\textsc{Beta-AppAbsIrrel}\quad \frac{[\mathsf{Value}\ (\lambda^- x.v)]}{\vDash (\lambda^- x.v)\ \Box^- > v\{\Box/x\}}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}}\ \textsc{Beta-CAppCAbs} \qquad \frac{F \sim a : A \in \Sigma_0}{\vDash F > a}\ \textsc{Beta-Axiom} \qquad \frac{}{\vDash \mathbf{fst}\,({}^{\rho}a, b) > a}\ \textsc{Beta-Fst}$$

$$\frac{}{\vDash \mathbf{snd}\,({}^{\rho}a, b) > b}\ \textsc{Beta-Snd}$$

$\boxed{\vDash a \rightsquigarrow b}$ *(single-step head reduction for implicit language)*

$$\frac{\vDash a \rightsquigarrow a'}{\vDash \lambda^{-}x.a \rightsquigarrow \lambda^{-}x.a'}\ \textsc{E-AbsTerm} \qquad \frac{\vDash a \rightsquigarrow a'}{\vDash a\ b^{+} \rightsquigarrow a'\ b^{+}}\ \textsc{E-AppLeft} \qquad \frac{\vDash a \rightsquigarrow a'}{\vDash a\ \Box^{-} \rightsquigarrow a'\ \Box^{-}}\ \textsc{E-AppLeftIrrel}$$

$$\frac{\vDash a \rightsquigarrow a'}{\vDash a[\bullet] \rightsquigarrow a'[\bullet]}\ \textsc{E-CAppLeft} \qquad \frac{}{\vDash (\lambda^{+}x.v)\ a^{+} \rightsquigarrow v\{a/x\}}\ \textsc{E-AppAbs} \qquad \frac{[\mathsf{Value}\ (\lambda^{-}x.v)]}{\vDash (\lambda^{-}x.v)\ \Box^{-} \rightsquigarrow v\{\Box/x\}}\ \textsc{E-AppAbsIrrel}$$

$$\frac{}{\vDash (\Lambda c.b)[\bullet] \rightsquigarrow b\{\bullet/c\}}\ \textsc{E-CAppCAbs} \qquad \frac{F \sim a : A \in \Sigma_0}{\vDash F \rightsquigarrow a}\ \textsc{E-Axiom} \qquad \frac{}{\vDash \mathbf{fst}\,({}^{\rho}a, b) \rightsquigarrow a}\ \textsc{E-FstRed}$$

$$\frac{}{\vDash \mathbf{snd}\,({}^{\rho}a, b) \rightsquigarrow b}\ \textsc{E-SndRed} \qquad \frac{\vDash a \rightsquigarrow a'}{\vDash \mathbf{fst}\ a \rightsquigarrow \mathbf{fst}\ a'}\ \textsc{E-FstCongRed} \qquad \frac{\vDash a \rightsquigarrow a'}{\vDash \mathbf{snd}\ a \rightsquigarrow \mathbf{snd}\ a'}\ \textsc{E-SndCongRed}$$

$\boxed{\vDash a \rightsquigarrow^{*} b}$ *(multistep reduction)*

$$\frac{}{\vDash a \rightsquigarrow^{*} a}\ \textsc{Equal} \qquad \frac{\begin{array}{c}\vDash a \rightsquigarrow b \\ \vDash b \rightsquigarrow^{*} a'\end{array}}{\vDash a \rightsquigarrow^{*} a'}\ \textsc{Step}$$

$\boxed{\Gamma \vDash^{\rho} \phi\ \mathsf{ok}}$ *(Prop wellformedness)*

$$\frac{\begin{array}{c}\Gamma \vDash^{\rho} a : A \\ \Gamma \vDash^{\rho} b : A \\ [\Gamma \vDash^{-} A : \star]\end{array}}{\Gamma \vDash^{\rho} a \sim_{A} b\ \mathsf{ok}}\ \textsc{E-Wff}$$

$\boxed{\Gamma \vDash^{\rho} a : A}$ *(typing)*

$$\text{E-Star}\quad \frac{\vDash \Gamma}{\Gamma \vDash^{\rho} \star : \star}$$

$$\text{E-Var}\quad \frac{\vDash \Gamma \qquad x : \rho_1\, A \in \Gamma \qquad \rho_2 \Leftarrow \rho_1}{\Gamma \vDash^{\rho_2} x : A}$$

$$\text{E-Pi}\quad \frac{\Gamma, x : -\, A \vDash^{\rho_1} B : \star \qquad \Gamma \vDash^{\rho_1} A : \star}{\Gamma \vDash^{\rho_1} \Pi^{\rho} x\!:\!A \to B : \star}$$

$$\text{E-Abs}\quad \frac{\Gamma, x : \rho\, A \vDash^{\rho_1} a : B \qquad [\Gamma \vDash^{-} A : \star] \qquad \Gamma \vDash^{-} \Pi^{\rho} x\!:\!A \to B : \star \qquad (\rho = +) \vee (x \notin \mathsf{fv}\ a)}{\Gamma \vDash^{\rho_1} \lambda^{\rho} x.a : \Pi^{\rho} x\!:\!A \to B}$$

$$\text{E-App}\quad \frac{\Gamma \vDash^{\rho} b : \Pi^{+} x\!:\!A \to B \qquad \Gamma \vDash^{\rho} a : A}{\Gamma \vDash^{\rho} b\ a^{+} : B\{a/x\}}$$

$$\text{E-IApp}\quad \frac{\Gamma \vDash^{\rho} b : \Pi^{-} x\!:\!A \to B \qquad \Gamma \vDash^{-} a : A}{\Gamma \vDash^{\rho} b\ \Box^{-} : B\{a/x\}}$$

$$\text{E-Conv}\quad \frac{\Gamma \vDash^{\rho} a : A \qquad \Gamma; \widetilde{\Gamma} \vDash -\, A \equiv B : \star \qquad [\Gamma \vDash^{-} B : \star]}{\Gamma \vDash^{\rho} a : B}$$

$$\text{E-CPi}\quad \frac{\Gamma, c : \phi \vDash^{\rho} B : \star \qquad \Gamma \vDash^{\rho} \phi\ \mathsf{ok}}{\Gamma \vDash^{\rho} \forall c\!:\!\phi.B : \star}$$

$$\text{E-CAbs}\quad \frac{\Gamma, c : \phi \vDash^{\rho} a : B \qquad [\Gamma \vDash^{-} \phi\ \mathsf{ok}]}{\Gamma \vDash^{\rho} \Lambda c.a : \forall c\!:\!\phi.B}$$

$$\text{E-CApp}\quad \frac{\Gamma \vDash^{\rho} a_1 : \forall c\!:\!(a \sim_A b).B_1 \qquad \Gamma; \widetilde{\Gamma} \vDash -\, a \equiv b : A}{\Gamma \vDash^{\rho} a_1[\bullet] : B_1\{\bullet/c\}}$$

$$\text{E-Fam}\quad \frac{\vDash \Gamma \qquad F \sim a : A \in \Sigma_0 \qquad [\varnothing \vDash^{-} A : \star]}{\Gamma \vDash^{\rho} F : A}$$

$$\text{E-Sigma}\quad \frac{\Gamma, x : -\, A \vDash^{\rho_1} B : \star \qquad \Gamma \vDash^{\rho_1} A : \star}{\Gamma \vDash^{\rho_1} \Sigma^{\rho} x\!:\!A.B : \star}$$

$$\text{E-Pair}\quad \frac{\Gamma \vDash^{+} a : A \qquad \Gamma \vDash^{\rho} b : B\{a/x\} \qquad \Gamma \vDash^{-} \Sigma^{+} x\!:\!A.B : \star}{\Gamma \vDash^{\rho} ({}^{+}a, b) : \Sigma^{+} x\!:\!A.B}$$

$$\text{E-PairIrrel}\quad \frac{\Gamma \vDash^{-} a : A \qquad \Gamma \vDash^{\rho} b : B\{a/x\} \qquad \Gamma \vDash^{-} \Sigma^{-} x\!:\!A.B : \star}{\Gamma \vDash^{\rho} ({}^{-}\Box, b) : \Sigma^{-} x\!:\!A.B}$$

$$\text{E-Fst}\quad \frac{\Gamma \vDash^{\rho} a : \Sigma^{+} x\!:\!A.B}{\Gamma \vDash^{\rho} \mathbf{fst}\, a : A}$$

$$\text{E-FstIrrel}\quad \frac{\Gamma \vDash^{-} a : \Sigma^{-} x\!:\!A.B}{\Gamma \vDash^{-} \mathbf{fst}\, a : A}$$

$$\text{E-Snd}\quad \frac{\Gamma \vDash^{\rho_1} a : \Sigma^{\rho} x\!:\!A.B}{\Gamma \vDash^{\rho_1} \mathbf{snd}\, a : B\{\mathbf{fst}\, a/x\}}$$

$\boxed{\Gamma; \Delta \vDash \rho\, \phi_1 \equiv \phi_2}$ *(prop equality)*

$$\text{E-PropCong}\quad \frac{\Gamma; \Delta \vDash \rho\, A_1 \equiv A_2 : A \qquad \Gamma; \Delta \vDash \rho\, B_1 \equiv B_2 : A}{\Gamma; \Delta \vDash \rho\, A_1 \sim_A B_1 \equiv A_2 \sim_A B_2}$$

$$\text{E-IsoConv}\quad \frac{\Gamma; \Delta \vDash -\, A \equiv B : \star \qquad \Gamma \vDash^{\rho} A_1 \sim_A A_2\ \mathsf{ok} \qquad \Gamma \vDash^{\rho} A_1 \sim_B A_2\ \mathsf{ok}}{\Gamma; \Delta \vDash \rho\, A_1 \sim_A A_2 \equiv A_1 \sim_B A_2}$$

$$\text{E-CPiFst}\quad \frac{\Gamma; \Delta \vDash \rho\, \forall c\!:\!\phi_1.B_1 \equiv \forall c\!:\!\phi_2.B_2 : \star}{\Gamma; \Delta \vDash \rho\, \phi_1 \equiv \phi_2}$$

$\boxed{\Gamma; \Delta \vDash \rho\, a \equiv b : A}$ *(definitional equality)*

E-Assn
$$\frac{\vDash \Gamma \quad c : (a \sim_A b) \in \Gamma \quad c \in \Delta}{\Gamma; \Delta \vDash \rho\, a \equiv b : A}$$

E-Refl
$$\frac{\Gamma \vDash^{\rho} a : A}{\Gamma; \Delta \vDash \rho\, a \equiv a : A}$$

E-Sym
$$\frac{\Gamma; \Delta \vDash \rho\, b \equiv a : A}{\Gamma; \Delta \vDash \rho\, a \equiv b : A}$$

E-Trans
$$\frac{\Gamma; \Delta \vDash \rho\, a \equiv a_1 : A \quad \Gamma; \Delta \vDash \rho\, a_1 \equiv b : A}{\Gamma; \Delta \vDash \rho\, a \equiv b : A}$$

E-Beta
$$\frac{\Gamma \vDash^{\rho} a_1 : B \quad \Gamma \vDash^{\rho} a_2 : B \quad \vDash a_1 > a_2}{\Gamma; \Delta \vDash \rho\, a_1 \equiv a_2 : B}$$

E-PiCong
$$\frac{\Gamma; \Delta \vDash \rho_1\, A_1 \equiv A_2 : \star \quad \Gamma, x : -\, A_1; \Delta \vDash \rho_1\, B_1 \equiv B_2 : \star \quad \Gamma \vDash^{\rho_1} A_1 : \star \quad [\Gamma \vDash^{\rho_1} \Pi^{\rho} x\!:\!A_1 \to B_1 : \star] \quad [\Gamma \vDash^{\rho_1} \Pi^{\rho} x\!:\!A_2 \to B_2 : \star]}{\Gamma; \Delta \vDash \rho_1\, (\Pi^{\rho} x\!:\!A_1 \to B_1) \equiv (\Pi^{\rho} x\!:\!A_2 \to B_2) : \star}$$

E-AbsCong
$$\frac{\Gamma, x : \rho\, A_1; \Delta \vDash \rho_1\, b_1 \equiv b_2 : B \quad [\Gamma \vDash^{-} A_1 : \star] \quad (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \quad (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)}{\Gamma; \Delta \vDash \rho_1\, (\lambda^{\rho} x.b_1) \equiv (\lambda^{\rho} x.b_2) : \Pi^{\rho} x\!:\!A_1 \to B}$$

E-AppCong
$$\frac{\Gamma; \Delta \vDash \rho\, a_1 \equiv b_1 : \Pi^{+} x\!:\!A \to B \quad \Gamma; \Delta \vDash +\, a_2 \equiv b_2 : A}{\Gamma; \Delta \vDash \rho\, a_1\ {a_2}^{+} \equiv b_1\ {b_2}^{+} : B\{a_2/x\}}$$

E-IAppCong
$$\frac{\Gamma; \Delta \vDash \rho\, a_1 \equiv b_1 : \Pi^{-} x\!:\!A \to B \quad \Gamma \vDash^{-} a : A}{\Gamma; \Delta \vDash \rho\, a_1\ \square^{-} \equiv b_1\ \square^{-} : B\{a/x\}}$$

E-PiFst
$$\frac{\Gamma; \Delta \vDash \rho_1\, \Pi^{\rho} x\!:\!A_1 \to B_1 \equiv \Pi^{\rho} x\!:\!A_2 \to B_2 : \star}{\Gamma; \Delta \vDash \rho_1\, A_1 \equiv A_2 : \star}$$

E-PiSnd
$$\frac{\Gamma; \Delta \vDash \rho_1\, \Pi^{\rho} x\!:\!A_1 \to B_1 \equiv \Pi^{\rho} x\!:\!A_2 \to B_2 : \star \quad \Gamma; \Delta \vDash \rho\, a_1 \equiv a_2 : A_1}{\Gamma; \Delta \vDash \rho_1\, B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star}$$

$$\dfrac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, \phi_1 \equiv \phi_2 \\ \Gamma, c : \phi_1; \Delta \vDash \rho\, A \equiv B : \star \\ [\Gamma \vDash^{\rho} \phi_1\ \mathsf{ok}] \\ [\Gamma \vDash^{\rho} \forall c\!:\!\phi_1.A : \star] \\ [\Gamma \vDash^{\rho} \forall c\!:\!\phi_2.B : \star]\end{array}}{\Gamma; \Delta \vDash \rho \forall c\!:\!\phi_1.A \equiv \forall c\!:\!\phi_2.B : \star}\ \textsc{E-CPiCong}$$

$$\dfrac{\begin{array}{c}\Gamma, c : \phi_1; \Delta \vDash \rho\, a \equiv b : B \\ [\Gamma \vDash^{-} \phi_1\ \mathsf{ok}]\end{array}}{\Gamma; \Delta \vDash \rho\, (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B}\ \textsc{E-CAbsCong}$$

$$\dfrac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, a_1 \equiv b_1 : \forall c\!:\!(a \sim_A b).B \\ \Gamma; \widetilde{\Gamma} \vDash -\, a \equiv b : A\end{array}}{\Gamma; \Delta \vDash \rho\, a_1[\bullet] \equiv b_1[\bullet] : B\{\bullet/c\}}\ \textsc{E-CAppCong}$$

$$\dfrac{\begin{array}{c}\Gamma; \Delta \vDash \rho \forall c\!:\!(a_1 \sim_A a_2).B_1 \equiv \forall c\!:\!(a_1' \sim_{A'} a_2').B_2 : \star \\ \Gamma; \widetilde{\Gamma} \vDash -\, a_1 \equiv a_2 : A \\ \Gamma; \widetilde{\Gamma} \vDash -\, a_1' \equiv a_2' : A'\end{array}}{\Gamma; \Delta \vDash \rho\, B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star}\ \textsc{E-CPiSnd}$$

$$\dfrac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, a \equiv b : A \\ \Gamma; \Delta \vDash \rho\, a \sim_A b \equiv a' \sim_{A'} b'\end{array}}{\Gamma; \Delta \vDash \rho\, a' \equiv b' : A'}\ \textsc{E-Cast}$$

$$\dfrac{\begin{array}{c}\Gamma; \Delta \vDash \rho\, a \equiv b : A \\ \Gamma; \widetilde{\Gamma} \vDash -\, A \equiv B : \star\end{array}}{\Gamma; \Delta \vDash \rho\, a \equiv b : B}\ \textsc{E-EqConv}$$

$$\dfrac{\Gamma; \Delta \vDash -\, a \sim_A b \equiv a' \sim_{A'} b'}{\Gamma; \Delta \vDash -\, A \equiv A' : \star}\ \textsc{E-IsoSnd}$$

$$\dfrac{\begin{array}{c}\Gamma \vDash^{\rho} b : \Pi^{+} x\!:\!A \to B \\ a = b\ x^{+}\end{array}}{\Gamma; \Delta \vDash \rho\, \lambda^{+} x.a \equiv b : \Pi^{+} x\!:\!A \to B}\ \textsc{E-EtaRel}$$

$$\dfrac{\begin{array}{c}\Gamma \vDash^{\rho} b : \Pi^{-} x\!:\!A \to B \\ a = b\ \Box^{-}\end{array}}{\Gamma; \Delta \vDash \rho\, \lambda^{-} x.a \equiv b : \Pi^{-} x\!:\!A \to B}\ \textsc{E-EtaIrrel}$$

$$\dfrac{\begin{array}{c}\Gamma \vDash^{\rho} b : \forall c\!:\!\phi.B \\ a = b[\bullet]\end{array}}{\Gamma; \Delta \vDash \rho\, \Lambda c.a \equiv b : \forall c\!:\!\phi.B}\ \textsc{E-EtaC}$$

$$\dfrac{\begin{array}{c}\Gamma; \Delta \vDash \rho_1\, A_1 \equiv A_2 : \star \\ \Gamma, x : -\, A_1; \Delta \vDash \rho_1\, B_1 \equiv B_2 : \star \\ [\Gamma \vDash^{\rho_1} A_1 : \star] \\ [\Gamma \vDash^{\rho_1} \Sigma^{\rho} x\!:\!A_1.B_1 : \star] \\ [\Gamma \vDash^{\rho_1} \Sigma^{\rho} x\!:\!A_2.B_2 : \star]\end{array}}{\Gamma; \Delta \vDash \rho_1\, (\Sigma^{\rho} x\!:\!A_1.B_1) \equiv (\Sigma^{\rho} x\!:\!A_2.B_2) : \star}\ \textsc{E-SigmaCong}$$

$$\textsc{E-PairCong}\quad \frac{\Gamma;\Delta \vDash +\, a_1 \equiv b_1 : A \qquad \Gamma;\Delta \vDash \rho\, a_2 \equiv b_2 : B\{a_1/x\} \qquad \Gamma \vDash^{-} \Sigma^{+}x\!:\!A.B : \star}{\Gamma;\Delta \vDash \rho\, ({}^{+}a_1, a_2) \equiv ({}^{+}b_1, b_2) : \Sigma^{+}x\!:\!A.B}$$

$$\textsc{E-PairCongIrrel}\quad \frac{\Gamma \vDash^{-} a : A \qquad \Gamma;\Delta \vDash \rho\, a_2 \equiv b_2 : B\{a/x\} \qquad \Gamma \vDash^{-} \Sigma^{-}x\!:\!A.B : \star}{\Gamma;\Delta \vDash \rho\, ({}^{-}\Box, a_2) \equiv ({}^{-}\Box, b_2) : \Sigma^{-}x\!:\!A.B}$$

$$\textsc{E-FstCong}\quad \frac{\Gamma;\Delta \vDash \rho\, a_1 \equiv b_1 : \Sigma^{+}x\!:\!A.B}{\Gamma;\Delta \vDash \rho\, \mathbf{fst}\, a_1 \equiv \mathbf{fst}\, b_1 : A}$$

$$\textsc{E-FstCongIrrel}\quad \frac{\Gamma;\Delta \vDash -\, a_1 \equiv b_1 : \Sigma^{-}x\!:\!A.B}{\Gamma;\Delta \vDash -\, \mathbf{fst}\, a_1 \equiv \mathbf{fst}\, b_1 : A}$$

$$\textsc{E-SndCong}\quad \frac{\Gamma;\Delta \vDash \rho\, a_1 \equiv b_1 : \Sigma^{\rho}_{1}x\!:\!A.B}{\Gamma;\Delta \vDash \rho\, \mathbf{snd}\, a_1 \equiv \mathbf{snd}\, b_1 : B\{\mathbf{fst}\, a_1/x\}}$$

$\boxed{\vDash \Gamma}$ *(context wellformedness)*

$$\textsc{E-Empty}\quad \frac{}{\vDash \varnothing}$$

$$\textsc{E-ConsTm}\quad \frac{\vDash \Gamma \qquad \Gamma \vDash^{-} A : \star \qquad x \notin \mathsf{dom}\,\Gamma}{\vDash \Gamma, x : \rho\, A}$$

$$\textsc{E-ConsCo}\quad \frac{\vDash \Gamma \qquad \Gamma \vDash^{-} \phi\ \mathsf{ok} \qquad c \notin \mathsf{dom}\,\Gamma}{\vDash \Gamma, c : \phi}$$

$\boxed{\vDash \Sigma}$ *(signature wellformedness)*

$$\textsc{Sig-Empty}\quad \frac{}{\vDash \varnothing}$$

$$\textsc{Sig-ConsAx}\quad \frac{\vDash \Sigma \qquad \varnothing \vDash^{-} A : \star \qquad \varnothing \vDash^{+} a : A \qquad F \notin \mathsf{dom}\,\Sigma}{\vDash \Sigma \cup \{F \sim a : A\}}$$

$\boxed{\Gamma \vdash \phi\ \mathsf{ok}}$ *(prop wellformedness)*

$$\textsc{An-Wff}\quad \frac{\Gamma \vdash a : A \qquad \Gamma \vdash b : B \qquad |A| = |B|}{\Gamma \vdash a \sim_A b\ \ \mathsf{ok}}$$

$\boxed{\Gamma \vdash a : A}$ *(typing)*

$$\textsc{An-Star}\quad \frac{\vdash \Gamma}{\Gamma \vdash \star : \star}$$

$$\textsc{An-Var}\quad \frac{\vdash \Gamma \qquad x : \rho\, A \in \Gamma}{\Gamma \vdash x : A}$$

$$\textsc{An-Pi}\quad \frac{\Gamma, x : \rho\, A \vdash B : \star \qquad [\Gamma \vdash A : \star]}{\Gamma \vdash \Pi^{\rho}x\!:\!A \to B : \star}$$

$$\textsc{An-Abs}\quad \frac{[\Gamma \vdash A : \star] \quad \Gamma, x : \rho\, A \vdash a : B \quad (\rho = +) \vee (x \notin \mathsf{fv}\ |a|)}{\Gamma \vdash \lambda^\rho x\!:\!A.a : \Pi^\rho x\!:\!A \to B}$$

$$\textsc{An-App}\quad \frac{\Gamma \vdash b : \Pi^\rho x\!:\!A \to B \quad \Gamma \vdash a : A}{\Gamma \vdash b\ a^\rho : B\{a/x\}}$$

$$\textsc{An-Conv}\quad \frac{\Gamma \vdash a : A \quad \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim B \quad \Gamma \vdash B : \star}{\Gamma \vdash a \triangleright \gamma : B}$$

$$\textsc{An-CPi}\quad \frac{[\Gamma \vdash \phi\ \ \mathsf{ok}] \quad \Gamma, c : \phi \vdash B : \star}{\Gamma \vdash \forall c\!:\!\phi.B : \star}$$

$$\textsc{An-CAbs}\quad \frac{[\Gamma \vdash \phi\ \ \mathsf{ok}] \quad \Gamma, c : \phi \vdash a : B}{\Gamma \vdash \Lambda c\!:\!\phi.a : \forall c\!:\!\phi.B}$$

$$\textsc{An-CApp}\quad \frac{\Gamma \vdash a_1 : \forall c\!:\!a \sim_{A_1} b.B \quad \Gamma; \widetilde{\Gamma} \vdash \gamma : a \sim b}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}}$$

$$\textsc{An-Fam}\quad \frac{\vdash \Gamma \quad F \sim a : A \in \Sigma_1 \quad [\varnothing \vdash A : \star]}{\Gamma \vdash F : A}$$

$$\textsc{An-Sigma}\quad \frac{\Gamma, x : \rho\, A \vdash B : \star \quad [\Gamma \vdash A : \star]}{\Gamma \vdash \Sigma^\rho x\!:\!A.B : \star}$$

$$\textsc{An-Pair}\quad \frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B\{a/x\}}{\Gamma \vdash ({}^\rho a, b)\ \mathsf{as}\ \Sigma^\rho x\!:\!A.B : \Sigma^\rho x\!:\!A.B}$$

$$\textsc{An-Fst}\quad \frac{\Gamma \vdash a : \Sigma^\rho x\!:\!A.B}{\Gamma \vdash \mathbf{fst}\ a : A}$$

$$\textsc{An-Snd}\quad \frac{\Gamma \vdash a : \Sigma^\rho x\!:\!A.B}{\Gamma \vdash \mathbf{snd}\ a : B\{\mathbf{fst}\ a/x\}}$$

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ *(coercion between props)*

$$\textsc{An-PropCong}\quad \frac{\Gamma; \Delta \vdash \gamma_1 : A_1 \sim A_2 \quad \Gamma; \Delta \vdash \gamma_2 : B_1 \sim B_2 \quad \Gamma \vdash A_1 \sim_A B_1\ \ \mathsf{ok} \quad \Gamma \vdash A_2 \sim_A B_2\ \ \mathsf{ok}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_A B_1) \sim (A_2 \sim_A B_2)}$$

$$\textsc{An-CPiFst}\quad \frac{\Gamma; \Delta \vdash \gamma : \forall c\!:\!\phi_1.A_2 \sim \forall c\!:\!\phi_2.B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst}\,\gamma : \phi_1 \sim \phi_2}$$

$$\textsc{An-IsoSym}\quad \frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym}\,\gamma : \phi_2 \sim \phi_1}$$

$$\textsc{An-IsoConv}\quad \frac{\Gamma; \Delta \vdash \gamma : A \sim B \quad \Gamma \vdash a_1 \sim_A a_2\ \ \mathsf{ok} \quad \Gamma \vdash a_1' \sim_B a_2'\ \ \mathsf{ok} \quad |a_1| = |a_1'| \quad |a_2| = |a_2'|}{\Gamma; \Delta \vdash \mathbf{conv}\ (a_1 \sim_A a_2) \sim_\gamma (a_1' \sim_B a_2') : (a_1 \sim_A a_2) \sim (a_1' \sim_B a_2')}$$

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim B}$ *(coercion between types)*

$$\textsc{An-Assn}\quad \frac{\vdash \Gamma \qquad c : a \sim_A b \in \Gamma \qquad c \in \Delta}{\Gamma; \Delta \vdash c : a \sim b}$$

$$\textsc{An-Refl}\quad \frac{\Gamma \vdash a : A}{\Gamma; \Delta \vdash \mathbf{refl}\, a : a \sim a}$$

$$\textsc{An-EraseEq}\quad \frac{\Gamma \vdash a : A \qquad \Gamma \vdash b : B \qquad |a| = |b| \qquad \Gamma; \widetilde{\Gamma} \vdash \gamma : A \sim B}{\Gamma; \Delta \vdash (a \mid=\mid_\gamma b) : a \sim b}$$

$$\textsc{An-Sym}\quad \frac{\Gamma \vdash b : B \qquad \Gamma \vdash a : A \qquad [\Gamma; \widetilde{\Gamma} \vdash \gamma_1 : B \sim A] \qquad \Gamma; \Delta \vdash \gamma : b \sim a}{\Gamma; \Delta \vdash \mathbf{sym}\, \gamma : a \sim b}$$

$$\textsc{An-Trans}\quad \frac{\Gamma; \Delta \vdash \gamma_1 : a \sim a_1 \qquad \Gamma; \Delta \vdash \gamma_2 : a_1 \sim b \qquad [\Gamma \vdash a : A] \qquad [\Gamma \vdash a_1 : A_1] \qquad [\Gamma; \widetilde{\Gamma} \vdash \gamma_3 : A \sim A_1]}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim b}$$

$$\textsc{An-Beta}\quad \frac{\Gamma \vdash a_1 : B_0 \qquad \Gamma \vdash a_2 : B_1 \qquad |B_0| = |B_1| \qquad \models |a_1| > |a_2|}{\Gamma; \Delta \vdash \mathbf{red}\, a_1\, a_2 : a_1 \sim a_2}$$

$$\textsc{An-PiCong}\quad \frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma, x : \rho\, A_1; \Delta \vdash \gamma_2 : B_1 \sim B_2 \\ B_3 = B_2\{x \triangleright \mathbf{sym}\, \gamma_1 / x\} \\ \Gamma \vdash \Pi^\rho x\!:\!A_1 \to B_1 : \star \\ \Gamma \vdash \Pi^\rho x\!:\!A_2 \to B_3 : \star \\ \Gamma \vdash (\Pi^\rho x\!:\!A_1 \to B_2) : \star\end{array}}{\Gamma; \Delta \vdash \Pi^\rho x\!:\!\gamma_1.\gamma_2 : (\Pi^\rho x\!:\!A_1 \to B_1) \sim (\Pi^\rho x\!:\!A_2 \to B_3)}$$

$$\textsc{An-AbsCong}\quad \frac{\begin{array}{c}\Gamma; \Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma, x : \rho\, A_1; \Delta \vdash \gamma_2 : b_1 \sim b_2 \\ b_3 = b_2\{x \triangleright \mathbf{sym}\, \gamma_1 / x\} \\ [\Gamma \vdash A_1 : \star] \\ \Gamma \vdash A_2 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_1|) \\ (\rho = +) \vee (x \notin \mathsf{fv}\, |b_3|) \\ [\Gamma \vdash (\lambda^\rho x\!:\!A_1.b_2) : B]\end{array}}{\Gamma; \Delta \vdash (\lambda^\rho x\!:\!\gamma_1.\gamma_2) : (\lambda^\rho x\!:\!A_1.b_1) \sim (\lambda^\rho x\!:\!A_2.b_3)}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim b_1 \\ \Gamma;\Delta \vdash \gamma_2 : a_2 \sim b_2 \\ \Gamma \vdash a_1\ a_2{}^{\rho} : A \\ \Gamma \vdash b_1\ b_2{}^{\rho} : B \\ [\Gamma;\widetilde{\Gamma} \vdash \gamma_3 : A \sim B]\end{array}}{\Gamma;\Delta \vdash \gamma_1\ \gamma_2^{\rho} : a_1\ a_2{}^{\rho} \sim b_1\ b_2{}^{\rho}}\ \textsc{An-AppCong}$$

$$\frac{\Gamma;\Delta \vdash \gamma : \Pi^{\rho} x\!:\!A_1 \to B_1 \sim \Pi^{\rho} x\!:\!A_2 \to B_2}{\Gamma;\Delta \vdash \mathbf{piFst}\,\gamma : A_1 \sim A_2}\ \textsc{An-PiFst}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : \Pi^{\rho} x\!:\!A_1 \to B_1 \sim \Pi^{\rho} x\!:\!A_2 \to B_2 \\ \Gamma;\Delta \vdash \gamma_2 : a_1 \sim a_2 \\ \Gamma \vdash a_1 : A_1 \\ \Gamma \vdash a_2 : A_2\end{array}}{\Gamma;\Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim B_2\{a_2/x\}}\ \textsc{An-PiSnd}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : \phi_1 \sim \phi_2 \\ \Gamma, c : \phi_1;\Delta \vdash \gamma_3 : B_1 \sim B_2 \\ B_3 = B_2\{c \triangleright \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash \forall c\!:\!\phi_1.B_1 : \star \\ [\Gamma \vdash \forall c\!:\!\phi_2.B_3 : \star] \\ \Gamma \vdash \forall c\!:\!\phi_1.B_2 : \star\end{array}}{\Gamma;\Delta \vdash (\forall c\!:\!\gamma_1.\gamma_3) : (\forall c\!:\!\phi_1.B_1) \sim (\forall c\!:\!\phi_2.B_3)}\ \textsc{An-CPiCong}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : \phi_1 \sim \phi_2 \\ \Gamma, c : \phi_1;\Delta \vdash \gamma_3 : a_1 \sim a_2 \\ a_3 = a_2\{c \triangleright \mathbf{sym}\,\gamma_1/c\} \\ \Gamma \vdash (\Lambda c\!:\!\phi_1.a_1) : \forall c\!:\!\phi_1.B_1 \\ \Gamma \vdash (\Lambda c\!:\!\phi_2.a_3) : \forall c\!:\!\phi_2.B_2 \\ \Gamma \vdash (\Lambda c\!:\!\phi_1.a_2) : B \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_4 : \forall c\!:\!\phi_1.B_1 \sim \forall c\!:\!\phi_2.B_2\end{array}}{\Gamma;\Delta \vdash (\lambda c\!:\!\gamma_1.\gamma_3 @ \gamma_4) : (\Lambda c\!:\!\phi_1.a_1) \sim (\Lambda c\!:\!\phi_2.a_3)}\ \textsc{An-CAbsCong}$$

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : a_1 \sim b_1 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a_2 \sim b_2 \\ \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : a_3 \sim b_3 \\ \Gamma \vdash a_1[\gamma_2] : A \\ \Gamma \vdash b_1[\gamma_3] : B \\ [\Gamma;\widetilde{\Gamma} \vdash \gamma_4 : A \sim B]\end{array}}{\Gamma;\Delta \vdash \gamma_1(\gamma_2, \gamma_3) : a_1[\gamma_2] \sim b_1[\gamma_3]}\ \textsc{An-CAppCong}$$

An-CPiSnd

$$\frac{\Gamma;\Delta \vdash \gamma_1 : (\forall c_1 \!:\! a \sim_A a'.B_1) \sim (\forall c_2 \!:\! b \sim_B b'.B_2) \qquad \Gamma;\widetilde{\Gamma} \vdash \gamma_2 : a \sim a' \qquad \Gamma;\widetilde{\Gamma} \vdash \gamma_3 : b \sim b'}{\Gamma;\Delta \vdash \gamma_1@(\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim B_2\{\gamma_3/c_2\}}$$

An-Cast

$$\frac{\Gamma;\Delta \vdash \gamma_1 : a \sim a' \qquad \Gamma;\Delta \vdash \gamma_2 : a \sim_A a' \sim b \sim_B b'}{\Gamma;\Delta \vdash \gamma_1 \triangleright \gamma_2 : b \sim b'}$$

An-IsoSnd

$$\frac{\Gamma;\Delta \vdash \gamma : (a \sim_A a') \sim (b \sim_B b')}{\Gamma;\Delta \vdash \mathbf{isoSnd}\,\gamma : A \sim B}$$

An-Eta

$$\frac{\Gamma \vdash b : \Pi^\rho x \!:\! A \to B \qquad a = b\ x^\rho}{\Gamma;\Delta \vdash \mathbf{eta}\,b : (\lambda^\rho x \!:\! A.a) \sim b}$$

An-EtaC

$$\frac{\Gamma \vdash b : \forall c \!:\! \phi.B \qquad a = b[c]}{\Gamma;\Delta \vdash \mathbf{eta}\,b : (\Lambda c \!:\! \phi.a) \sim b}$$

An-SigmaCong

$$\frac{\begin{array}{c}\Gamma;\Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma, x : \rho\, A_1;\Delta \vdash \gamma_2 : B_1 \sim B_2 \\ B_3 = B_2\{x \triangleright \mathbf{sym}\,\gamma_1/x\} \\ \Gamma \vdash \Sigma^\rho x \!:\! A_1.B_1 : \star \\ \Gamma \vdash \Sigma^\rho x \!:\! A_2.B_3 : \star \\ \Gamma \vdash (\Sigma^\rho x \!:\! A_1.B_2) : \star\end{array}}{\Gamma;\Delta \vdash \Sigma x \!:^\rho \gamma_1.\gamma_2 : (\Sigma^\rho x \!:\! A_1.B_1) \sim (\Sigma^\rho x \!:\! A_2.B_3)}$$

An-PairCong

$$\frac{\Gamma;\Delta \vdash \gamma_1 : a_1 \sim b_1 \qquad \Gamma;\Delta \vdash \gamma_2 : a_2 \sim b_2 \qquad \Gamma;\Delta \vdash \gamma_3 : A_1 \sim A_2}{\Gamma;\Delta \vdash (\rho\,\gamma_1, \gamma_2)\,\mathbf{as}\,\gamma_3 : ({}^\rho a_1, a_2)\ \mathsf{as}\ A_1 \sim ({}^\rho b_1, b_2)\ \mathsf{as}\ A_2}$$

An-FstCong

$$\frac{\Gamma;\Delta \vdash \gamma : a_1 \sim b_1}{\Gamma;\Delta \vdash \mathbf{fst}\,\gamma : \mathbf{fst}\,a_1 \sim \mathbf{fst}\,b_1}$$

An-SndCong

$$\frac{\Gamma;\Delta \vdash \gamma : a_1 \sim b_1}{\Gamma;\Delta \vdash \mathbf{snd}\,\gamma : \mathbf{snd}\,a_1 \sim \mathbf{snd}\,b_1}$$

An-SigmaFst

$$\frac{\Gamma;\Delta \vdash \gamma : \Sigma^\rho x \!:\! A_1.B_1 \sim \Sigma^\rho x \!:\! A_2.B_2}{\Gamma;\Delta \vdash \mathbf{sigmaFst}\,\gamma : A_1 \sim A_2}$$

An-SigmaSnd

$$\frac{\Gamma;\Delta \vdash \gamma_1 : \Sigma^\rho x \!:\! A_1.B_1 \sim \Sigma^\rho x \!:\! A_2.B_2 \qquad \Gamma;\Delta \vdash \gamma_2 : a_1 \sim a_2 \qquad \Gamma \vdash a_1 : A_1 \qquad \Gamma \vdash a_2 : A_2}{\Gamma;\Delta \vdash \gamma_1@\gamma_2 : B_1\{a_1/x\} \sim B_2\{a_2/x\}}$$

$\boxed{\vdash \Gamma}$ *(context wellformedness)*

$$\dfrac{}{\vdash \varnothing}\ \textsc{An-Empty}$$

$$\dfrac{\vdash \Gamma \qquad \Gamma \vdash A : \star \qquad x \notin \mathsf{dom}\,\Gamma}{\vdash \Gamma, x : \rho\, A}\ \textsc{An-ConsTm}$$

$$\dfrac{\vdash \Gamma \qquad \Gamma \vdash \phi\ \mathsf{ok} \qquad c \notin \mathsf{dom}\,\Gamma}{\vdash \Gamma, c : \phi}\ \textsc{An-ConsCo}$$

$\boxed{\vdash \Sigma}$ *(signature wellformedness)*

$$\dfrac{}{\vdash \varnothing}\ \textsc{An-Sig-Empty}$$

$$\dfrac{\vdash \Sigma \qquad \varnothing \vdash A : \star \qquad \varnothing \vdash a : A \qquad F \notin \mathsf{dom}\,\Sigma}{\vdash \Sigma \cup \{F \sim a : A\}}\ \textsc{An-Sig-ConsAx}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b}$ *(single-step, weak head reduction to values for annotated language)*

$$\dfrac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a\ b^{\rho} \rightsquigarrow a'\ b^{\rho}}\ \textsc{An-AppLeft}$$

$$\dfrac{[\mathsf{Value}\ (\lambda^{\rho} x\!:\!A.w)]}{\Gamma \vdash (\lambda^{\rho} x\!:\!A.w)\ a^{\rho} \rightsquigarrow w\{a/x\}}\ \textsc{An-AppAbs}$$

$$\dfrac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma]}\ \textsc{An-CAppLeft}$$

$$\dfrac{}{\Gamma \vdash (\Lambda c\!:\!\phi.b)[\gamma] \rightsquigarrow b\{\gamma/c\}}\ \textsc{An-CAppCAbs}$$

$$\dfrac{\Gamma \vdash A : \star \qquad \Gamma, x : -\, A \vdash b \rightsquigarrow b'}{\Gamma \vdash (\lambda^{-} x\!:\!A.b) \rightsquigarrow (\lambda^{-} x\!:\!A.b')}\ \textsc{An-AbsTerm}$$

$$\dfrac{F \sim a : A \in \Sigma_1}{\Gamma \vdash F \rightsquigarrow a}\ \textsc{An-Axiom}$$

$$\dfrac{}{\Gamma \vdash \mathbf{fst}\,({}^{\rho} a, b) \rightsquigarrow a}\ \textsc{An-FstRed}$$

$$\dfrac{}{\Gamma \vdash \mathbf{snd}\,({}^{\rho} a, b) \rightsquigarrow b}\ \textsc{An-SndRed}$$

$$\dfrac{\Gamma \vdash a \rightsquigarrow a'}{\Gamma \vdash a \triangleright \gamma \rightsquigarrow a' \triangleright \gamma}\ \textsc{An-ConvTerm}$$

$$\dfrac{[\mathsf{Value}\ v]}{\Gamma \vdash (v \triangleright \gamma_1) \triangleright \gamma_2 \rightsquigarrow v \triangleright (\gamma_1 ; \gamma_2)}\ \textsc{An-Combine}$$

$$\dfrac{[\mathsf{Value}\ v] \qquad \Gamma; \widetilde{\Gamma} \vdash \gamma : \Pi^{\rho} x_1\!:\!A_1 \to B_1 \sim \Pi^{\rho} x_2\!:\!A_2 \to B_2 \qquad b' = b \triangleright \mathbf{sym}\,(\mathbf{piFst}\,\gamma) \qquad \gamma' = \gamma @ (b' \mid=\mid_{(\mathbf{piFst}\,\gamma)}\ b)}{\Gamma \vdash (v \triangleright \gamma)\ b^{\rho} \rightsquigarrow (v\ b'^{\rho}) \triangleright \gamma'}\ \textsc{An-Push}$$

$$\dfrac{[\mathsf{Value}\ v] \qquad \Gamma; \widetilde{\Gamma} \vdash \gamma : \forall c_1\!:\!\phi_1.A_1 \sim \forall c_2\!:\!\phi_2.A_2 \qquad \gamma_1' = \gamma_1 \triangleright \mathbf{sym}\,(\mathbf{cpiFst}\,\gamma) \qquad \gamma' = \gamma @ (\gamma_1' \sim \gamma_1)}{\Gamma \vdash (v \triangleright \gamma)[\gamma_1] \rightsquigarrow (v[\gamma_1']) \triangleright \gamma'}\ \textsc{An-CPush}$$

$$\dfrac{[\mathsf{Value}\ v] \qquad \Gamma; \widetilde{\Gamma} \vdash \gamma : \Sigma^{\rho}_1 x_1\!:\!A_1.B_1 \sim \Sigma^{\rho}_2 x_2\!:\!A_2.B_2}{\Gamma \vdash \mathbf{fst}\,(v \triangleright \gamma) \rightsquigarrow (\mathbf{fst}\ v) \triangleright \mathbf{sigmaFst}\,\gamma}\ \textsc{An-FstPush}$$

An-SndPush

$$\frac{\begin{array}{c}[\mathsf{Value}\ v] \\ \Gamma;\widetilde{\Gamma}\vdash\gamma : \Sigma_1^{\rho} x_1 : A_1.B_1 \sim \Sigma_2^{\rho} x_2 : A_2.B_2 \\ b' = b \triangleright \mathbf{sym}\,(\mathbf{sigmaFst}\,\gamma) \\ \gamma' = \gamma @ (b' \mid = \mid_{(\mathbf{sigmaFst}\,\gamma)}\ b)\end{array}}{\Gamma\vdash\mathbf{snd}\,(v\triangleright\gamma)\leadsto(\mathbf{snd}\,v)\triangleright\gamma'}$$