$tmvar$, x, y, f, m, n variables
$covar$, c coercion variables
$datacon$, K
$const$, T, F
$index$, i indices

relflag, ρ	::=			relevance flag
	|	$+$		
	|	$-$		
	|	$app\_rho\nu$	S	
	|	(ρ)	S	

appflag, ν	::=			applicative flag
	|	R		
	|	ρ		

role, R	::=			Role
	|	**Nom**		
	|	**Rep**		
	|	$R_1 \cap R_2$	S	
	|	**param** $R_1\, R_2$	S	
	|	$app\_role\nu$	S	
	|	(R)	S	

constraint, ϕ	::=			props
	|	$a \sim_{A/R} b$		
	|	(ϕ)	S	
	|	$\phi\{b/x\}$	S	
	|	$|\phi|$	S	
	|	$a \sim_R b$	S	

tm, a, b, p, v, w, A, B, C	::=			types and kinds
	|	$\star$		
	|	x		
	|	$\lambda^\rho x : A.b$	bind x in b	
	|	$\lambda^\rho x.b$	bind x in b	
	|	$a\, b^\nu$		
	|	$\Pi^\rho x : A \to B$	bind x in B	
	|	$\Lambda c : \phi.b$	bind c in b	
	|	$\Lambda c.b$	bind c in b	
	|	$a[\gamma]$		
	|	$\forall c : \phi.B$	bind c in B	
	|	$a \triangleright_R \gamma$		
	|	F		
	|	$\square$		
	|	$\mathsf{case}_R\, a\ \mathsf{of}\ F \to b_1 \| _- \to b_2$		
	|	K		
	|	**match** a **with** brs		
	|	**sub** $R\, a$		
	|	$a\{b/x\}$	S	
	|	$a\{\gamma/c\}$	S	
	|	$a\{b/x\}$	S	
	|	$a\{\gamma/c\}$	S	

|	a	S	
|	a	S	
|	(a)	S	
|	a	S	parsing precedence is hard
|	$|a|_R$	S	
|	**Int**	S	
|	**Bool**	S	
|	Nat	S	
|	**Vec**	S	
|	0	S	
|	S	S	
|	**True**	S	
|	**Fix**	S	
|	**Age**	S	
|	$a \to b$	S	
|	$\phi \Rightarrow A$	S	
|	$a\ b$	S	
|	$\lambda x.a$	S	
|	$\lambda x : A.a$	S	
|	$\forall\, x : A \to B$	S	
|	**if** ϕ **then** a **else** b	S	

brs ::=   case branches

|	**none**	
|	$K \Rightarrow a; brs$	
|	$brs\{a/x\}$	S
|	$brs\{\gamma/c\}$	S
|	(brs)	S

$co,\ \gamma$ ::=   explicit coercions

|	$\bullet$	
|	c	
|	**red** $a\ b$	
|	**refl** a	
|	$(a \mathrel{|=|_\gamma} b)$	
|	**sym** γ	
|	$\gamma_1; \gamma_2$	
|	**sub** γ	
|	$\Pi^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2
|	$\lambda^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2
|	$\gamma_1\ \gamma_2^{R,\rho}$	
|	**piFst** γ	
|	**cpiFst** γ	
|	**isoSnd** γ	
|	$\gamma_1 @ \gamma_2$	
|	$\forall c : \gamma_1.\gamma_3$	bind c in γ_3

| $\lambda c\!:\!\gamma_1.\gamma_3@\gamma_4$ bind c in γ_3
| $\gamma(\gamma_1, \gamma_2)$
| $\gamma@(\gamma_1 \sim \gamma_2)$
| $\gamma_1 \rhd_R \gamma_2$
| $\gamma_1 \sim_A \gamma_2$
| $\mathbf{conv}\ \phi_1 \sim_\gamma \phi_2$
| $\mathbf{eta}\ a$
| $\mathbf{left}\ \gamma\,\gamma'$
| $\mathbf{right}\ \gamma\,\gamma'$
| (γ) S
| γ S
| $\gamma\{a/x\}$ S

role_context, Ω	::=		role$_c$*ontexts*
	$\varnothing$		
	$x : R$		
	$\Omega, x : R$		
	Ω, Ω'	M	
	var_patp	M	
	(Ω)	M	
	Ω	M	

roles, Rs	::=		
	$\mathbf{nilR}$		
	R, Rs		
	$\mathbf{range}\,\Omega$	S	

sig_sort	::=		signature classifier
	$A@Rs$		
	$p \sim a : A/R@Rs$		

sort	::=		binding classifier
	$\mathbf{Tm}\,A$		
	$\mathbf{Co}\,\phi$		

context, Γ	::=		contexts
	$\varnothing$		
	$\Gamma, x : A$		
	$\Gamma, c : \phi$		
	$\Gamma\{b/x\}$	M	
	$\Gamma\{\gamma/c\}$	M	
	Γ, Γ'	M	
	$|\Gamma|$	M	
	(Γ)	M	
	Γ	M	

sig, Σ	::=		signatures

| $\varnothing$
| $\Sigma \cup \{F : sig\_sort\}$
| Σ_0 M
| Σ_1 M
| $|\Sigma|$ M

$available\_props,\ \Delta$ ::=

| $\varnothing$
| Δ, x
| Δ, c
| $\mathsf{fv}\,a$ M
| Δ, Δ' M
| $\widetilde{\Gamma}$ M
| $\widetilde{\Omega}$ M
| (Δ) M

$Nat,\ \mathbb{N}$ ::=
| $|a|$ S

$terminals$ ::=
| $\leftrightarrow$
| $\Leftrightarrow$
| $\longrightarrow$
| min
| $\equiv$
| $\forall$
| $\in$
| $\notin$
| $\Leftarrow$
| $\Rightarrow$
| $\Rightarrow^*$
| $\rightarrow$
| Λ
|
| $\square$
| $\vdash$
| $\dashv$
| $\vDash$
| $\vDash$
| $\neq$
| $\triangleright$
| ok
| -
| $\rightsquigarrow$
| $\rightsquigarrow^*$
| $\rightsquigarrow$
| $\varnothing$

| ∘
| fv
| dom
| ∼
| ≍
| |
| ●
| **fst**
| **snd**
| **as**
| $| \Rightarrow |$
| $\vdash_=$
| **refl₂**
| $++$
| {
| }

$formula,\ \psi \qquad ::=$

| $judgement$
| $x : A \ \in \ \Gamma$
| $x : R \ \in \ \Omega$
| $c : \phi \ \in \ \Gamma$
| $F : sig\_sort \ \in \ \Sigma$
| $x \ \in \ \Delta$
| $c \ \in \ \Delta$
| $c \ \textbf{not relevant} \ \in \ \gamma$
| $x \ \notin \ \Delta$
| $uniq\ \Gamma$
| $uniq(\Omega)$
| $c \ \notin \ \Delta$
| $T \ \notin \ \textsf{dom}\ \Sigma$
| $F \ \notin \ \textsf{dom}\ \Sigma$
| $\mathbb{N}_1 < \mathbb{N}_2$
| $\mathbb{N}_1 \le \mathbb{N}_2$
| $R_1 = R_2$
| $a = b$
| $\phi_1 = \phi_2$
| $\Gamma_1 = \Gamma_2$
| $\gamma_1 = \gamma_2$
| $\neg \psi$
| $\psi_1 \wedge \psi_2$
| $\psi_1 \vee \psi_2$
| $\psi_1 \Rightarrow \psi_2$
| (ψ)
| ψ
| $c : (a : A \sim b : B) \ \in \ \Gamma$

		suppress lc hypothesis generated
JSubRole	::=	
	$R_1 \leq R_2$	Subroling judgement
JPath	::=	
	Path $a = F@Rs$	Type headed by constant (partial
JPatCtx	::=	
	$\Omega; \Gamma \vDash p :_F B \Rightarrow A$	Contexts generated by a pattern
JRename	::=	
	rename $p \to a$ to $p' \to a'$ excluding Δ	rename with fresh variables
JMatchSubst	::=	
	match a_1 with $p \to b_1 = b_2$	match and substitute
JTmPatternAgree	::=	
	$a \leftrightarrow p$	term and pattern agree
JTmSubPatternAgree	::=	
	$a^+ = p$	sub-pattern agrees with term
JSubTmPatternAgree	::=	
	$a = p^+$	sub-term agrees with pattern
JValuePath	::=	
	ValuePath $a = F$	Type headed by constant (role-se
JCasePath	::=	
	CasePath$_R$ $a = F$	Type headed by constant (role-se
JApplyArgs	::=	
	apply args a to $b \mapsto b'$	apply arguments of a (headed by
JValue	::=	
	Value$_R$ A	values
JValueType	::=	
	ValueType$_R$ A	Types with head forms (erased la
Jconsistent	::=	
	consistent$_R$ a b	(erased) types do not differ in the
Jroleing	::=	
	$\Omega \vDash a : R$	Roleing judgment

| *JChk* | ::= | | |
| | \| | $(\rho = +) \vee (x \notin \mathsf{fv}\ A)$ | irrelevant argument check |

Jpar	::=		
	|	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	|	$\Omega \vDash a \Rightarrow_R^* b$	multistep parallel reduction
	|	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common term

Jbeta	::=		
	|	$\vDash a > b/R$	primitive reductions on erased terms
	|	$\vDash a \rightsquigarrow b/R$	single-step head reduction for implicit langu
	|	$\vDash a \rightsquigarrow^* b/R$	multistep reduction

| *JBranchTyping* | ::= | | |
| | \| | $\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'$ | Branch Typing (aligning the types of case) |

Jett	::=		
	|	$\Gamma \vDash \phi\ \mathsf{ok}$	Prop wellformedness
	|	$\Gamma \vDash a : A$	typing
	|	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	|	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	|	$\vDash \Gamma$	context wellformedness

| *Jsig* | ::= | | |
| | \| | $\vDash \Sigma$ | signature wellformedness |

Jann	::=		
	|	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	|	$\Gamma \vdash a : A/R$	typing
	|	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	|	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	|	$\vdash \Gamma$	context wellformedness

| *Jred* | ::= | | |
| | \| | $\Gamma \vdash a \rightsquigarrow b/R$ | single-step, weak head reduction to values fo |

judgement	::=		
	|	*JSubRole*	
	|	*JPath*	
	|	*JPatCtx*	
	|	*JRename*	
	|	*JMatchSubst*	
	|	*JTmPatternAgree*	
	|	*JTmSubPatternAgree*	
	|	*JSubTmPatternAgree*	
	|	*JValuePath*	
	|	*JCasePath*	

| | *JApplyArgs*
| | *JValue*
| | *JValueType*
| | *Jconsistent*
| | *Jroleing*
| | *JChk*
| | *Jpar*
| | *Jbeta*
| | *JBranchTyping*
| | *Jett*
| | *Jsig*
| | *Jann*
| | *Jred*

user_syntax   ::=
| | *tmvar*
| | *covar*
| | *datacon*
| | *const*
| | *index*
| | *relflag*
| | *appflag*
| | *role*
| | *constraint*
| | *tm*
| | *brs*
| | *co*
| | *role_context*
| | *roles*
| | *sig_sort*
| | *sort*
| | *context*
| | *sig*
| | *available_props*
| | *Nat*
| | *terminals*
| | *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \text{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \text{RepTop}$$

$$\frac{}{R \leq R} \quad \text{Refl}$$

$$\frac{R_1 \leq R_2 \quad R_2 \leq R_3}{R_1 \leq R_3} \quad \text{Trans}$$

$\boxed{\text{Path } a = F@Rs}$ Type headed by constant (partial function)

$$\frac{F : A@Rs \ \in \ \Sigma_0}{\mathsf{Path} \ F = F@Rs} \quad \text{PATH\_ABSCONST}$$

$$\frac{F : \ p \sim a : A/R_1@Rs \ \in \ \Sigma_0}{\mathsf{Path} \ F = F@Rs} \quad \text{PATH\_CONST}$$

$$\frac{\mathsf{Path} \ a = F@R_1, Rs}{\mathsf{Path} \ (a \ b'^{R_1}) = F@Rs} \quad \text{PATH\_APP}$$

$$\frac{\mathsf{Path} \ a = F@Rs}{\mathsf{Path} \ (a \ \square^-) = F@Rs} \quad \text{PATH\_IAPP}$$

$$\frac{\mathsf{Path} \ a = F@Rs}{\mathsf{Path} \ (a[\bullet]) = F@Rs} \quad \text{PATH\_CAPP}$$

$\boxed{\Omega; \Gamma \vDash p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\varnothing; \varnothing \vDash F :_F A \Rightarrow A} \quad \text{PATCTX\_CONST}$$

$$\frac{\Omega; \Gamma \vDash p :_F \Pi^+ x : A' \to A \Rightarrow B}{\Omega, x : R; \Gamma, x : A' \vDash p \ x^R :_F A \Rightarrow B} \quad \text{PATCTX\_PIREL}$$

$$\frac{\Omega; \Gamma \vDash p :_F \Pi^- x : A' \to A \Rightarrow B}{\Omega; \Gamma, x : A' \vDash p \ \square^- :_F A \Rightarrow B} \quad \text{PATCTX\_PIIRR}$$

$$\frac{\Omega; \Gamma \vDash p :_F \forall c : \phi. A \Rightarrow B}{\Omega; \Gamma, c : \phi \vDash p[\bullet] :_F A \Rightarrow B} \quad \text{PATCTX\_CPI}$$

$\boxed{\text{rename } p \to a \text{ to } p' \to a' \text{ excluding } \Delta}$ rename with fresh variables

$$\frac{}{\text{rename } F \to a \text{ to } F \to a \text{ excluding } \Delta} \quad \text{RENAME\_BASE}$$

$$\frac{\begin{array}{c}\text{rename } p_1 \to a_1 \text{ to } p_2 \to a_2 \text{ excluding } \Delta \\ y \notin \Delta\end{array}}{\text{rename } (p_1 \ x^R) \to a_1 \text{ to } (p_2 \ y^R) \to (a_2\{y/x\}) \text{ excluding } (\Delta, y)} \quad \text{RENAME\_APPREL}$$

$$\frac{\text{rename } p_1 \to a_1 \text{ to } p_2 \to a_2 \text{ excluding } \Delta}{\text{rename } (p_1 \ \square^-) \to a_1 \text{ to } (p_2 \ \square^-) \to a_2 \text{ excluding } \Delta} \quad \text{RENAME\_APPIRREL}$$

$$\frac{\text{rename } p_1 \to a_1 \text{ to } p_2 \to a_2 \text{ excluding } \Delta}{\text{rename } (p_1[\bullet]) \to a_1 \text{ to } (p_2[\bullet]) \to a_2 \text{ excluding } \Delta} \quad \text{RENAME\_CAPP}$$

$\boxed{\text{match } a_1 \text{ with } p \to b_1 = b_2}$ match and substitute

$$\frac{}{\text{match } F \text{ with } F \to b = b} \quad \text{MATCHSUBST\_CONST}$$

$$\frac{\text{match } a_1 \text{ with } a_2 \to b_1 = b_2}{\text{match } (a_1 \ a^R) \text{ with } (a_2 \ x^R) \to b_1 = (b_2\{a/x\})} \quad \text{MATCHSUBST\_APPRELR}$$

$$\frac{\text{match } a_1 \text{ with } a_2 \to b_1 = b_2}{\text{match } (a_1 \ \square^-) \text{ with } (a_2 \ \square^-) \to b_1 = b_2} \quad \text{MATCHSUBST\_APPIRREL}$$

$$\frac{\text{match } a_1 \text{ with } a_2 \to b_1 = b_2}{\text{match } (a_1[\bullet]) \text{ with } (a_2[\bullet]) \to b_1 = b_2} \quad \text{MATCHSUBST\_CAPP}$$

$\boxed{a \leftrightarrow p}$ term and pattern agree

$$\frac{}{F \leftrightarrow F} \quad \text{TM\_PATTERN\_AGREE\_CONST}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1 \; a_2{}^R) \leftrightarrow (p_1 \; x^R)} \quad \text{TM\_PATTERN\_AGREE\_APPRELR}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1 \; \square^-) \leftrightarrow (p_1 \; \square^-)} \quad \text{TM\_PATTERN\_AGREE\_APPIRREL}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1[\bullet]) \leftrightarrow (p_1[\bullet])} \quad \text{TM\_PATTERN\_AGREE\_CAPP}$$

$\boxed{a^+ = p}$ sub-pattern agrees with term

$$\frac{a \leftrightarrow p}{a^+ = p} \quad \text{TM\_SUBPATTERN\_AGREE\_BASE}$$

$$\frac{a^+ = p}{a^+ = (p \; x^R)} \quad \text{TM\_SUBPATTERN\_AGREE\_APPRELR}$$

$$\frac{a^+ = p}{a^+ = (p \; \square^-)} \quad \text{TM\_SUBPATTERN\_AGREE\_APPIRREL}$$

$$\frac{a^+ = p}{a^+ = (p[\bullet])} \quad \text{TM\_SUBPATTERN\_AGREE\_CAPPP}$$

$\boxed{a = p^+}$ sub-term agrees with pattern

$$\frac{a \leftrightarrow p}{a = p^+} \quad \text{SUBTM\_PATTERN\_AGREE\_BASE}$$

$$\frac{a = p^+}{a \; a_2{}^\nu = p^+} \quad \text{SUBTM\_PATTERN\_AGREE\_APP}$$

$$\frac{a = p^+}{a[\bullet] = p^+} \quad \text{SUBTM\_PATTERN\_AGREE\_CAPPP}$$

$\boxed{\text{ValuePath } a = F}$ Type headed by constant (role-sensitive partial function used in value)

$$\frac{F : A@Rs \in \Sigma_0}{\text{ValuePath } F = F} \quad \text{VALUEPATH\_ABSCONST}$$

$$\frac{F : p \sim a : A/R_1@Rs \in \Sigma_0}{\text{ValuePath } F = F} \quad \text{VALUEPATH\_CONST}$$

$$\frac{\text{ValuePath } a = F}{\text{ValuePath } (a \; b'^\nu) = F} \quad \text{VALUEPATH\_APP}$$

$$\frac{\text{ValuePath } a = F}{\text{ValuePath } (a[\bullet]) = F} \quad \text{VALUEPATH\_CAPP}$$

$\boxed{\text{CasePath}_R \; a = F}$ Type headed by constant (role-sensitive partial function used in case)

$$\frac{\text{ValuePath } a = F \quad F : A@Rs \in \Sigma_0}{\text{CasePath}_R \; a = F} \quad \text{CASEPATH\_ABSCONST}$$

$$\frac{\begin{array}{c} \mathsf{ValuePath}\ a = F \\ F : p \sim b : A/R_1 @Rs\ \in\ \Sigma_0 \\ \neg(R_1 \leq R) \end{array}}{\mathsf{CasePath}_R\ a = F}\ \text{CasePath\_Const}$$

$$\frac{\begin{array}{c} \mathsf{ValuePath}\ a = F \\ F : p \sim b : A/R_1 @Rs\ \in\ \Sigma_0 \\ \neg(a = p^+) \end{array}}{\mathsf{CasePath}_R\ a = F}\ \text{CasePath\_UnMatch}$$

$\boxed{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\mathsf{apply\ args}\ F\ \mathsf{to}\ b \mapsto b}\ \text{ApplyArgs\_Const}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}{\mathsf{apply\ args}\ a\ a'^\nu\ \mathsf{to}\ b \mapsto b'\ a'^\nu}\ \text{ApplyArgs\_App}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \mapsto b'}{\mathsf{apply\ args}\ a[\bullet]\ \mathsf{to}\ b \mapsto b'[\bullet]}\ \text{ApplyArgs\_CApp}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star}\ \text{Value\_Star}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^\rho x : A \to B}\ \text{Value\_Pi}$$

$$\frac{}{\mathsf{Value}_R\ \forall c : \phi.B}\ \text{Value\_CPi}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x : A.a}\ \text{Value\_AbsRel}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^+ x.a}\ \text{Value\_UAbsRel}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^- x.a}\ \text{Value\_UAbsIrrel}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c : \phi.a}\ \text{Value\_CAbs}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a}\ \text{Value\_UCAbs}$$

$$\frac{\mathsf{CasePath}_R\ a = F}{\mathsf{Value}_R\ a}\ \text{Value\_Path}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\ \text{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x : A \to B}\ \text{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c : \phi.B}\ \text{value\_type\_CPi}$$

$$\frac{\mathsf{CasePath}_R\ a = F}{\mathsf{ValueType}_R\ a}\ \text{value\_type\_ValuePath}$$

$\boxed{\mathsf{consistent}_R\ a\ b}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \star\ \star}\ \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ (\Pi^\rho x_1 : A_1 \to B_1)\ (\Pi^\rho x_2 : A_2 \to B_2)}\ \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ (\forall c_1 : \phi_1.A_1)\ (\forall c_2 : \phi_2.A_2)}\ \textsc{consistent\_a\_CPi}$$

$$\frac{\mathsf{CasePath}_R\ a_1 = F \quad \mathsf{CasePath}_R\ a_2 = F}{\mathsf{consistent}_R\ a_1\ a_2}\ \textsc{consistent\_a\_CasePath}$$

$$\frac{\neg\mathsf{ValueType}_R\ b}{\mathsf{consistent}_R\ a\ b}\ \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ a}{\mathsf{consistent}_R\ a\ b}\ \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{uniq(\Omega)}{\Omega \vDash \square : R}\ \textsc{role\_a\_Bullet}$$

$$\frac{uniq(\Omega)}{\Omega \vDash \star : R}\ \textsc{role\_a\_Star}$$

$$\frac{uniq(\Omega) \quad x : R \in \Omega \quad R \le R_1}{\Omega \vDash x : R_1}\ \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R}\ \textsc{role\_a\_Abs}$$

$$\frac{\Omega \vDash a : R \quad \Omega \vDash b : \mathbf{Nom}}{\Omega \vDash (a\ b^+) : R}\ \textsc{role\_a\_App}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a\ \square^-) : R}\ \textsc{role\_a\_IApp}$$

$$\frac{\Omega \vDash a : R \quad \mathsf{Path}\ a = F@R_1, Rs \quad \Omega \vDash b : R_1}{\Omega \vDash a\ b^{R_1} : R}\ \textsc{role\_a\_TApp}$$

$$\frac{\Omega \vDash A : R \quad \Omega, x : \mathbf{Nom} \vDash B : R}{\Omega \vDash (\Pi^\rho x : A \to B) : R}\ \textsc{role\_a\_Pi}$$

$$\frac{\Omega \vDash a : R_1 \quad \Omega \vDash b : R_1 \quad \Omega \vDash A : R_0 \quad \Omega \vDash B : R}{\Omega \vDash (\forall c : a \sim_{A/R_1} b.B) : R}\ \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R}\ \textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \text{ROLE\_A\_CApp}$$

$$\frac{\begin{array}{c} uniq(\Omega) \\ F : A@Rs \ \in \ \Sigma_0 \end{array}}{\Omega \vDash F : R} \quad \text{ROLE\_A\_CONST}$$

$$\frac{\begin{array}{c} uniq(\Omega) \\ F : \ p \sim a : A/R@Rs \ \in \ \Sigma_0 \end{array}}{\Omega \vDash F : R_1} \quad \text{ROLE\_A\_FAM}$$

$$\frac{\begin{array}{c} \Omega \vDash a : R \\ \Omega \vDash b_1 : R_1 \\ \Omega \vDash b_2 : R_1 \end{array}}{\Omega \vDash \mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \|_{\_} \to b_2 : R_1} \quad \text{ROLE\_A\_PATTERN}$$

$\boxed{(\rho = +) \vee (x \notin \mathsf{fv} \ A)}$ \quad irrelevant argument check

$$\frac{}{(+ = +) \vee (x \notin \mathsf{fv} \ A)} \quad \text{RHO\_REL}$$

$$\frac{x \ \notin \ \mathsf{fv}A}{(- = +) \vee (x \notin \mathsf{fv} \ A)} \quad \text{RHO\_IRRREL}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ \quad parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \text{PAR\_REFL}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \\ \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b' \end{array}}{\Omega \vDash a \ b^\rho \Rightarrow_R a'\{b'/x\}} \quad \text{PAR\_BETA}$$

$$\frac{\begin{array}{c} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b' \end{array}}{\Omega \vDash a \ b^\rho \Rightarrow_R a' \ b'^\rho} \quad \text{PAR\_APP}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \text{PAR\_CBETA}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \text{PAR\_CAPP}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \text{PAR\_ABS}$$

$$\frac{\begin{array}{c} \Omega \vDash A \Rightarrow_R A' \\ \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \Pi^\rho x : A \to B \Rightarrow_R \Pi^\rho x : A' \to B'} \quad \text{PAR\_PI}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \text{PAR\_CABS}$$

$$\frac{\begin{array}{c} \Omega \vDash A \Rightarrow_{R_0} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B' \end{array}}{\Omega \vDash \forall c : a \sim_{A/R_1} b.B \Rightarrow_R \forall c : a' \sim_{A'/R_1} b'.B'} \quad \text{PAR\_CPI}$$

$$\frac{\begin{array}{l} F : p \sim b : A/R_1@Rs \ \in \ \Sigma_0 \\ \Omega \vDash a : R \\ \text{rename } p \to b \text{ to } p' \to b' \text{ excluding } (\widetilde{\Omega}, \mathsf{fv}\,p) \\ \text{match } a \text{ with } p' \to b' = a' \\ R_1 \leq R \end{array}}{\Omega \vDash a \Rightarrow_R a'} \quad \text{Par\_Axiom}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \end{array}}{\Omega \vDash (\mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \| _{\_} \to b_2) \Rightarrow_{R_0} (\mathsf{case}_R \ a' \ \mathsf{of} \ F \to b_1' \| _{\_} \to b_2')} \quad \text{Par\_Pattern}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{CasePath}_R \ a' = F \\ \text{apply args } a' \text{ to } b_1' \mapsto b \end{array}}{\Omega \vDash (\mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \| _{\_} \to b_2) \Rightarrow_{R_0} b[\bullet]} \quad \text{Par\_PatternTrue}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R \ a' \\ \neg(\mathsf{CasePath}_R \ a' = F) \end{array}}{\Omega \vDash (\mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \| _{\_} \to b_2) \Rightarrow_{R_0} b_2'} \quad \text{Par\_PatternFalse}$$

$\boxed{\Omega \vDash a \Rightarrow_R^* b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow_R^* a} \quad \text{MP\_Refl}$$

$$\frac{\begin{array}{l} \Omega \vDash a \Rightarrow_R b \\ \Omega \vDash b \Rightarrow_R^* a' \end{array}}{\Omega \vDash a \Rightarrow_R^* a'} \quad \text{MP\_Step}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\begin{array}{l} \Omega \vDash a_1 \Rightarrow_R^* b \\ \Omega \vDash a_2 \Rightarrow_R^* b \end{array}}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \text{join}$$

$\boxed{\vDash a > b/R}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1} \ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v) \ b^\rho > v\{b/x\}/R_1} \quad \text{Beta\_AppAbs}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] > a'\{\bullet/c\}/R} \quad \text{Beta\_CAppCAbs}$$

$$\frac{\begin{array}{l} F : p \sim b : A/R_1@Rs \ \in \ \Sigma_0 \\ \text{rename } p \to b \text{ to } p_1 \to b_1 \text{ excluding } (\mathsf{fv}\,a, \mathsf{fv}\,p) \\ \text{match } a \text{ with } p_1 \to b_1 = b' \\ R_1 \leq R \end{array}}{\vDash a > b'/R} \quad \text{Beta\_Axiom}$$

$$\frac{\begin{array}{l} \mathsf{CasePath}_R \ a = F \\ \text{apply args } a \text{ to } b_1 \mapsto b_1' \end{array}}{\vDash \mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \| _{\_} \to b_2 > b_1'[\bullet]/R_0} \quad \text{Beta\_PatternTrue}$$

15

$$\frac{\begin{array}{c} \mathsf{Value}_R \ a \\ \neg(\mathsf{CasePath}_R \ a = F) \end{array}}{\vDash \mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \| _- \to b_2 > b_2/R_0} \quad \text{Beta\_PatternFalse}$$

$\boxed{\vDash a \rightsquigarrow b/R}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash \lambda^- x.a \rightsquigarrow \lambda^- x.a'/R_1} \quad \text{E\_AbsTerm}$$

$$\frac{\vDash a \rightsquigarrow a'/R_1}{\vDash a \ b^\rho \rightsquigarrow a' \ b^\rho/R_1} \quad \text{E\_AppLeft}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash a[\bullet] \rightsquigarrow a'[\bullet]/R} \quad \text{E\_CAppLeft}$$

$$\frac{\vDash a \rightsquigarrow a'/R}{\vDash \mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \| _- \to b_2 \rightsquigarrow \mathsf{case}_R \ a' \ \mathsf{of} \ F \to b_1 \| _- \to b_2/R_0} \quad \text{E\_Pattern}$$

$$\frac{\vDash a > b/R}{\vDash a \rightsquigarrow b/R} \quad \text{E\_Prim}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R} \quad \text{Equal}$$

$$\frac{\begin{array}{c} \vDash a \rightsquigarrow b/R \\ \vDash b \rightsquigarrow^* a'/R \end{array}}{\vDash a \rightsquigarrow^* a'/R} \quad \text{Step}$$

$\boxed{\Gamma \vDash \mathsf{case}_R \ a : A \ \mathsf{of} \ b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{c} \mathit{uniq} \ \Gamma \\ \mathtt{lc\_tm} \ C \end{array}}{\Gamma \vDash \mathsf{case}_R \ a : A \ \mathsf{of} \ b : A \Rightarrow \forall c:(a \sim_{A/R} b).C \mid C} \quad \text{BranchTyping\_Base}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R \ a : A_1 \ \mathsf{of} \ b \ x^+ : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R \ a : A_1 \ \mathsf{of} \ b : \Pi^+ x{:}A \to B \Rightarrow \Pi^+ x{:}A \to C \mid C'} \quad \text{BranchTyping\_PiRel}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R \ a : A_1 \ \mathsf{of} \ b \ \square^- : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R \ a : A_1 \ \mathsf{of} \ b : \Pi^- x{:}A \to B \Rightarrow \Pi^- x{:}A \to C \mid C'} \quad \text{BranchTyping\_PiIrrel}$$

$$\frac{\Gamma, c : \phi \vDash \mathsf{case}_R \ a : A \ \mathsf{of} \ b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R \ a : A \ \mathsf{of} \ b : \forall c{:}\phi.B \Rightarrow \forall c{:}\phi.C \mid C'} \quad \text{BranchTyping\_CPi}$$

$\boxed{\Gamma \vDash \phi \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{c} \Gamma \vDash a : A \\ \Gamma \vDash b : A \\ \Gamma \vDash A : \star \end{array}}{\Gamma \vDash a \sim_{A/R} b \ \mathsf{ok}} \quad \text{E\_Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star} \quad \text{E\_Star}$$

$$\frac{\begin{array}{c} \vDash \Gamma \\ x : A \in \Gamma \end{array}}{\Gamma \vDash x : A} \quad \text{E\_Var}$$

$$\frac{\begin{array}{c} \Gamma, x : A \vDash B : \star \\ \Gamma \vDash A : \star \end{array}}{\Gamma \vDash \Pi^\rho x : A \to B : \star} \quad \text{E\_Pi}$$

$$\frac{\begin{array}{c} \Gamma, x : A \vDash a : B \\ \Gamma \vDash A : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a) \end{array}}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x : A \to B)} \quad \text{E\_Abs}$$

$$\frac{\begin{array}{c} \Gamma \vDash b : \Pi^+ x : A \to B \\ \Gamma \vDash a : A \end{array}}{\Gamma \vDash b\ a^+ : B\{a/x\}} \quad \text{E\_App}$$

$$\frac{\begin{array}{c} \Gamma \vDash b : \Pi^+ x : A \to B \\ \Gamma \vDash a : A \\ \mathsf{Path}\ b = F@R, Rs \end{array}}{\Gamma \vDash b\ a^R : B\{a/x\}} \quad \text{E\_TApp}$$

$$\frac{\begin{array}{c} \Gamma \vDash b : \Pi^- x : A \to B \\ \Gamma \vDash a : A \end{array}}{\Gamma \vDash b\ \square^- : B\{a/x\}} \quad \text{E\_IApp}$$

$$\frac{\begin{array}{c} \Gamma \vDash a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star \end{array}}{\Gamma \vDash a : B} \quad \text{E\_Conv}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \vDash B : \star \\ \Gamma \vDash \phi\ \mathsf{ok} \end{array}}{\Gamma \vDash \forall c : \phi.B : \star} \quad \text{E\_CPi}$$

$$\frac{\begin{array}{c} \Gamma, c : \phi \vDash a : B \\ \Gamma \vDash \phi\ \mathsf{ok} \end{array}}{\Gamma \vDash \Lambda c.a : \forall c : \phi.B} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{c} \Gamma \vDash a_1 : \forall c : (a \sim_{A/R} b).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R \end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E\_CApp}$$

$$\frac{\begin{array}{c} \vDash \Gamma \\ F : A@Rs \in \Sigma_0 \\ \varnothing \vDash A : \star \end{array}}{\Gamma \vDash F : A} \quad \text{E\_Const}$$

$$\frac{\begin{array}{c} \vDash \Gamma \\ F : p \sim a : A/R_1@Rs \in \Sigma_0 \end{array}}{\Gamma \vDash F : A} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{c} \Gamma \vDash a : A \\ \Gamma \vDash F : A_1 \\ \Gamma \vDash b_1 : B \\ \Gamma \vDash b_2 : C \\ \Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ F : A_1 \Rightarrow B \mid C \end{array}}{\Gamma \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_- \to b_2 : C} \quad \text{E\_Case}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R \end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} B_1 \equiv A_2 \sim_{A/R} B_2} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ \Gamma \vDash A_1 \sim_{A/R} A_2 \ \text{ok} \\ \Gamma \vDash A_1 \sim_{B/R} A_2 \ \text{ok} \end{array}}{\Gamma; \Delta \vDash A_1 \sim_{A/R} A_2 \equiv A_1 \sim_{B/R} A_2} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c : (a_1 \sim_{A/R_1} a_2).B_1 \equiv \forall c : (b_1 \sim_{B/R_2} b_2).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{A/R_1} a_2 \equiv b_1 \sim_{B/R_2} b_2} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{c} \vDash \Gamma \\ c : (a \sim_{A/R} b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash a \equiv a_1 : A/R \\ \Gamma; \Delta \vDash a_1 \equiv b : A/R \end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Trans}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2 \end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R_2} \quad \text{E\_Sub}$$

$$\frac{\begin{array}{c} \Gamma \vDash a_1 : B \\ \Gamma \vDash a_2 : B \\ \vDash a_1 > a_2/R \end{array}}{\Gamma; \Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_Beta}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1; \Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x : A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x : A_2 \to B_2 : \star \end{array}}{\Gamma; \Delta \vDash (\Pi^\rho x : A_1 \to B_1) \equiv (\Pi^\rho x : A_2 \to B_2) : \star/R'} \quad \text{E\_PiCong}$$

$$\frac{\begin{array}{c} \Gamma, x : A_1; \Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \text{fv } b_1) \\ (\rho = +) \vee (x \notin \text{fv } b_2) \end{array}}{\Gamma; \Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x : A_1 \to B)/R'} \quad \text{E\_AbsCong}$$

$$\frac{\begin{array}{c} \Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x : A \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{Nom} \end{array}}{\Gamma; \Delta \vDash a_1 \ a_2^+ \equiv b_1 \ b_2^+ : (B\{a_2/x\})/R'} \quad \text{E\_AppCong}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^+ x : A \to B)/R' \\ \Gamma; \Delta \vDash a_2 \equiv b_2 : A/\mathbf{param}\ R\ R' \\ \mathsf{Path}\ a_1 = F@R, Rs \\ \mathsf{Path}\ b_1 = F'@R, Rs' \end{array}}{\Gamma; \Delta \vDash a_1\ a_2{}^R \equiv b_1\ b_2{}^R : (B\{a_2/x\})/R'} \quad \text{E\_TAppCong}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash a_1 \equiv b_1 : (\Pi^- x : A \to B)/R' \\ \Gamma \vDash a : A \end{array}}{\Gamma; \Delta \vDash a_1\ \square^- \equiv b_1\ \square^- : (B\{a/x\})/R'} \quad \text{E\_IAppCong}$$

$$\frac{\Gamma; \Delta \vDash \Pi^\rho x : A_1 \to B_1 \equiv \Pi^\rho x : A_2 \to B_2 : \star/R'}{\Gamma; \Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \text{E\_PiFst}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash \Pi^\rho x : A_1 \to B_1 \equiv \Pi^\rho x : A_2 \to B_2 : \star/R' \\ \Gamma; \Delta \vDash a_1 \equiv a_2 : A_1/\mathbf{Nom} \end{array}}{\Gamma; \Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PiSnd}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash a_1 \sim_{A_1/R} b_1 \equiv a_2 \sim_{A_2/R} b_2 \\ \Gamma, c : a_1 \sim_{A_1/R} b_1; \Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_{A_1/R} b_1\ \mathsf{ok} \\ \Gamma \vDash \forall c : a_1 \sim_{A_1/R} b_1.A : \star \\ \Gamma \vDash \forall c : a_2 \sim_{A_2/R} b_2.B : \star \end{array}}{\Gamma; \Delta \vDash \forall c : a_1 \sim_{A_1/R} b_1.A \equiv \forall c : a_2 \sim_{A_2/R} b_2.B : \star/R'} \quad \text{E\_CPiCong}$$

$$\frac{\begin{array}{l} \Gamma, c : \phi_1; \Delta \vDash a \equiv b : B/R \\ \Gamma \vDash \phi_1\ \mathsf{ok} \end{array}}{\Gamma; \Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c : \phi_1.B/R} \quad \text{E\_CAbsCong}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash a_1 \equiv b_1 : (\forall c : (a \sim_{A/R} b).B)/R' \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\ R\ R' \end{array}}{\Gamma; \Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \text{E\_CAppCong}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash \forall c : (a_1 \sim_{A/R} a_2).B_1 \equiv \forall c : (a_1' \sim_{A'/R'} a_2').B_2 : \star/R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\ R\ R_0 \\ \Gamma; \widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\ R'\ R_0 \end{array}}{\Gamma; \Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \text{E\_CPiSnd}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \Delta \vDash a \sim_{A/R} b \equiv a' \sim_{A'/R'} b' \end{array}}{\Gamma; \Delta \vDash a' \equiv b' : A'/R'} \quad \text{E\_Cast}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash a \equiv b : A/R \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star \end{array}}{\Gamma; \Delta \vDash a \equiv b : B/R} \quad \text{E\_EqConv}$$

$$\frac{\Gamma; \Delta \vDash a \sim_{A/R_1} b \equiv a' \sim_{A'/R_1} b'}{\Gamma; \Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \text{E\_IsoSnd}$$

$$\frac{\begin{array}{l} \Gamma; \Delta \vDash a \equiv a' : A/R \\ \Gamma; \Delta \vDash b_1 \equiv b_1' : B/R_0 \\ \Gamma; \Delta \vDash b_2 \equiv b_2' : B/R_0 \end{array}}{\Gamma; \Delta \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|_- \to b_2 \equiv \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|_- \to b_2' : B/R_0} \quad \text{E\_PatCong}$$

$$\begin{array}{l}
\text{ValuePath } a = F \\
\text{ValuePath } a' = F \\
\Gamma \vDash a : \Pi^+ x\!:\! A \to B \\
\Gamma \vDash b : A \\
\Gamma \vDash a' : \Pi^+ x\!:\! A \to B \\
\Gamma \vDash b' : A \\
\Gamma; \Delta \vDash a \ b^{R_1} \equiv a' \ b'^{R_1} : B\{b/x\}/R' \\
\underline{\Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R'} \\
\quad \Gamma; \Delta \vDash a \equiv a' : \Pi^+ x\!:\! A \to B/R'
\end{array} \quad \text{E\_LeftRel}$$

$$\begin{array}{l}
\text{ValuePath } a = F \\
\text{ValuePath } a' = F \\
\Gamma \vDash a : \Pi^- x\!:\! A \to B \\
\Gamma \vDash b : A \\
\Gamma \vDash a' : \Pi^- x\!:\! A \to B \\
\Gamma \vDash b' : A \\
\Gamma; \Delta \vDash a \ \square^- \equiv a' \ \square^- : B\{b/x\}/R' \\
\underline{\Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0} \\
\quad \Gamma; \Delta \vDash a \equiv a' : \Pi^- x\!:\! A \to B/R'
\end{array} \quad \text{E\_LeftIrrel}$$

$$\begin{array}{l}
\text{ValuePath } a = F \\
\text{ValuePath } a' = F \\
\Gamma \vDash a : \Pi^+ x\!:\! A \to B \\
\Gamma \vDash b : A \\
\Gamma \vDash a' : \Pi^+ x\!:\! A \to B \\
\Gamma \vDash b' : A \\
\Gamma; \Delta \vDash a \ b^+ \equiv a' \ b'^+ : B\{b/x\}/R' \\
\underline{\Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/R_0} \\
\quad \Gamma; \Delta \vDash b \equiv b' : A/\mathbf{param} \ R_1 \ R'
\end{array} \quad \text{E\_Right}$$

$$\begin{array}{l}
\text{ValuePath } a = F \\
\text{ValuePath } a' = F \\
\Gamma \vDash a : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\
\Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B \\
\Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/R' \\
\underline{\Gamma; \Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'} \\
\Gamma; \Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{A/R_1} a_2).B/R'
\end{array} \quad \text{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\overline{\vDash \varnothing} \quad \text{E\_Empty}$$

$$\begin{array}{l}
\vDash \Gamma \\
\Gamma \vDash A : \star \\
\underline{x \notin \widetilde{\Gamma}} \\
\vDash \Gamma, x : A
\end{array} \quad \text{E\_ConsTm}$$

$$\begin{array}{l}
\vDash \Gamma \\
\Gamma \vDash \phi \ \mathsf{ok} \\
\underline{c \notin \widetilde{\Gamma}} \\
\vDash \Gamma, c : \phi
\end{array} \quad \text{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \text{Sig\_Empty}$$

$$\frac{\begin{array}{l} \vDash \Sigma \\ \varnothing \vDash A : \star \\ F \notin \mathsf{dom}\,\Sigma \end{array}}{\vDash \Sigma \cup \{F : A@Rs\}} \quad \text{Sig\_ConsConst}$$

$$\frac{\begin{array}{l} \vDash \Sigma \\ F \notin \mathsf{dom}\,\Sigma \\ \varnothing \vDash A : \star \\ \Omega; \Gamma \vDash p :_F B \Rightarrow A \\ \Gamma \vDash a : B \\ \Omega \vDash a : R \end{array}}{\vDash \Sigma \cup \{F : p \sim a : A/R@\mathbf{range}\,\Omega\}} \quad \text{Sig\_ConsAx}$$

$\boxed{\Gamma \vdash \phi \;\mathsf{ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A/R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \rightsquigarrow b/R}$ single-step, weak head reduction to values for annotated language

```
Definition rules:        158 good    0 bad
Definition rule clauses: 434 good    0 bad
```