$tmvar$, x, y, f, m, n	variables
$covar$, c	coercion variables
$datacon$, K	
$const$, T, F, Age	
$index$, i	indices

relflag, ρ	::=			relevance flag
	|	$+$		
	|	$-$		
	|	**app_rho** ν	S	
	|	(ρ)	S	
appflag, ν	::=			applicative flag
	|	R		
	|	ρ		
role, R	::=			Role
	|	**Nom**		
	|	**Rep**		
	|	$R_1 \cap R_2$	S	
	|	**param** R_1 R_2	S	
	|	**app_role** ν	S	
	|	(R)	S	
constraint, ϕ	::=			props
	|	$a \sim_R b : A$		
	|	(ϕ)	S	
	|	$\phi\{b/x\}$	S	
	|	$\lvert\phi\rvert$	S	
	|	$a \sim_R b$	S	
tm, a, b, p, v, w, A, B, C	::=			types and kinds
	|	$\star$		
	|	x		
	|	$\lambda^\rho x\!:\!A.b$	bind x in b	
	|	$\lambda^\rho x.b$	bind x in b	
	|	$a\ b^\nu$		
	|	$\Pi^\rho x\!:\!A \to B$	bind x in B	
	|	$\Lambda c\!:\!\phi.b$	bind c in b	
	|	$\Lambda c.b$	bind c in b	
	|	$a[\gamma]$		
	|	$\forall c\!:\!\phi.B$	bind c in B	
	|	$a \triangleright_R \gamma$		
	|	F		
	|	$\square$		
	|	$\mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \| _- \to b_2$		
	|	K		
	|	**match** a **with** brs		
	|	**sub** R a		
	|	$a\{b/x\}$	S	
	|	$a\{\gamma/c\}$	S	
	|	$a\{b/x\}$	S	
	|	$a\{\gamma/c\}$	S	

	|	a	S	
	|	a	S	
	|	(a)	S	
	|	a	S	parsing precedence is hard
	|	$\|a\|_R$	S	
	|	**Int**	S	
	|	**Bool**	S	
	|	Nat	S	
	|	**Vec**	S	
	|	0	S	
	|	S	S	
	|	**True**	S	
	|	**Fix**	S	
	|	$a \to b$	S	
	|	$\phi \Rightarrow A$	S	
	|	$a\ b$	S	
	|	$\lambda x.a$	S	
	|	$\lambda x : A.a$	S	
	|	$\forall\, x : A \to B$	S	
	|	**if** ϕ **then** a **else** b	S	
brs	::=			case branches
	|	**none**		
	|	$K \Rightarrow a; brs$		
	|	$brs\{a/x\}$	S	
	|	$brs\{\gamma/c\}$	S	
	|	(brs)	S	
$co,\ \gamma$	::=			explicit coercions
	|	$\bullet$		
	|	c		
	|	**red** $a\ b$		
	|	**refl** a		
	|	$(a \mid=\mid_{\gamma}\ b)$		
	|	**sym** γ		
	|	$\gamma_1; \gamma_2$		
	|	**sub** γ		
	|	$\Pi^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2	
	|	$\lambda^{R,\rho} x : \gamma_1.\gamma_2$	bind x in γ_2	
	|	$\gamma_1\ \gamma_2^{R,\rho}$		
	|	**piFst** γ		
	|	**cpiFst** γ		
	|	**isoSnd** γ		
	|	$\gamma_1 @ \gamma_2$		
	|	$\forall c : \gamma_1.\gamma_3$	bind c in γ_3	
	|	$\lambda c : \gamma_1.\gamma_3 @ \gamma_4$	bind c in γ_3	

	$\vert$	$\gamma(\gamma_1, \gamma_2)$		
	$\vert$	$\gamma@(\gamma_1 \sim \gamma_2)$		
	$\vert$	$\gamma_1 \triangleright_R \gamma_2$		
	$\vert$	$\gamma_1 \sim_A \gamma_2$		
	$\vert$	$\textbf{conv}\ \phi_1 \sim_\gamma \phi_2$		
	$\vert$	$\textbf{eta}\, a$		
	$\vert$	$\textbf{left}\, \gamma\, \gamma'$		
	$\vert$	$\textbf{right}\, \gamma\, \gamma'$		
	$\vert$	(γ)	S	
	$\vert$	γ	S	
	$\vert$	$\gamma\{a/x\}$	S	
role_context, Ω	::=			role$_c$*ontexts*
	$\vert$	$\varnothing$		
	$\vert$	$x : R$		
	$\vert$	$\Omega, x : R$		
	$\vert$	Ω, Ω'	M	
	$\vert$	$\textbf{var\_pat}\, p$	M	
	$\vert$	(Ω)	M	
	$\vert$	Ω	M	
roles, Rs	::=			
	$\vert$	$\cdot$		
	$\vert$	R, Rs		
	$\vert$	$\textbf{range}\, \Omega$	S	
sig_sort	::=			signature classifier
	$\vert$	$_{Rs}\, A$		
	$\vert$	$_{Rs}\, p \sim_R a : A$		
sort	::=			binding classifier
	$\vert$	$\textbf{Tm}\, A$		
	$\vert$	$\textbf{Co}\, \phi$		
context, Γ	::=			contexts
	$\vert$	$\varnothing$		
	$\vert$	$\Gamma, x : A$		
	$\vert$	$\Gamma, c : \phi$		
	$\vert$	$\Gamma\{b/x\}$	M	
	$\vert$	$\Gamma\{\gamma/c\}$	M	
	$\vert$	Γ, Γ'	M	
	$\vert$	$\vert\Gamma\vert$	M	
	$\vert$	(Γ)	M	
	$\vert$	Γ	M	
sig, Σ	::=			signatures
	$\vert$	$\varnothing$		

	|	$\Sigma \cup \{F : sig\_sort\}$		
	|	Σ_0	M	
	|	Σ_1	M	
	|	$\|\Sigma\|$	M	
$available\_props,\ \Delta$	::=			
	|	$\varnothing$		
	|	Δ, x		
	|	Δ, c		
	|	$\mathsf{fv}\, a$	M	
	|	Δ, Δ'	M	
	|	$\widetilde{\Gamma}$	M	
	|	$\widetilde{\Omega}$	M	
	|	(Δ)	M	
$Nat,\ \mathbb{N}$	::=			
	|	$\|a\|$	S	
$pattern\_arg$	::=			Pattern arguments
	|	$\backslash Rel\, a\, R$		
	|	$\backslash Irr\, a$		
	|	$\backslash Coe \gamma$		
$pattern\_args,\ PA$	::=			
	|	**none**		
	|	$PA, pattern\_arg$		
$terminals$	::=			
	|	$\leftrightarrow$		
	|	$\Leftrightarrow$		
	|	$\longrightarrow$		
	|	min		
	|	$\equiv$		
	|	$\forall$		
	|	$\in$		
	|	$\not\in$		
	|	$\Leftarrow$		
	|	$\Rightarrow$		
	|	$\Rightarrow^*$		
	|	$\rightarrow$		
	|	Λ		
	|			
	|	$\Box$		
	|	$\vdash$		
	|	$\dashv$		
	|	$\models$		
	|	$\models$		

$$
\begin{array}{lll}
 & | & \neq \\
 & | & \triangleright \\
 & | & \ \mathsf{ok} \\
 & | & \_ \\
 & | & \rightsquigarrow \\
 & | & \rightsquigarrow^{*} \\
 & | & \rightsquigarrow \\
 & | & \varnothing \\
 & | & \circ \\
 & | & \mathsf{fv} \\
 & | & \mathsf{dom} \\
 & | & \sim \\
 & | & \asymp \\
 & | & | \\
 & | & \bullet \\
 & | & \mathbf{fst} \\
 & | & \mathbf{snd} \\
 & | & \mathbf{as} \\
 & | & |\Rightarrow| \\
 & | & \vdash_{=} \\
 & | & \mathbf{refl_2} \\
 & | & ++ \\
 & | & \{ \\
 & | & \} \\
 & | & \not\rightarrow \\
 & & \\
\mathit{formula},\ \psi & ::= & \\
 & | & \mathit{judgement} \\
 & | & x : A\ \in\ \Gamma \\
 & | & x : R\ \in\ \Omega \\
 & | & c : \phi\ \in\ \Gamma \\
 & | & F : \mathit{sig\_sort}\ \in\ \Sigma \\
 & | & x\ \in\ \Delta \\
 & | & c\ \in\ \Delta \\
 & | & c\,\mathbf{not\,relevant}\ \in\ \gamma \\
 & | & x\ \notin\ \Delta \\
 & | & \mathit{uniq}\,\Gamma \\
 & | & \mathit{uniq}(\Omega) \\
 & | & c\ \notin\ \Delta \\
 & | & T\ \notin\ \mathsf{dom}\,\Sigma \\
 & | & F\ \notin\ \mathsf{dom}\,\Sigma \\
 & | & \mathbb{N}_1 < \mathbb{N}_2 \\
 & | & \mathbb{N}_1 \leq \mathbb{N}_2 \\
 & | & \nu = \rho \\
 & | & R_1 = R_2 \\
 & | & a = b
\end{array}
$$

	$\vert$	$\phi_1 = \phi_2$	
	$\vert$	$\Gamma_1 = \Gamma_2$	
	$\vert$	$\gamma_1 = \gamma_2$	
	$\vert$	$\neg\psi$	
	$\vert$	$\psi_1 \wedge \psi_2$	
	$\vert$	$\psi_1 \vee \psi_2$	
	$\vert$	$\psi_1 \Rightarrow \psi_2$	
	$\vert$	(ψ)	
	$\vert$	ψ	
	$\vert$	$c : (a : A \sim b : B) \in \Gamma$	
	$\vert$		suppress lc hypothesis gen
JSubRole	::=		
	$\vert$	$R_1 \leq R_2$	Subroling judgement
JRolePath	::=		
	$\vert$	$\mathsf{RolePath}\ a = F@Rs$	Type headed by constant
JPatCtx	::=		
	$\vert$	$\Omega; \Gamma \vDash p :_F B \Rightarrow A$	Contexts generated by a p
JRename	::=		
	$\vert$	$\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'$	rename with fresh variable
JMatchSubst	::=		
	$\vert$	$\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 \nleftrightarrow b_2$	match and substitute
JPatData	::=		
	$\vert$	$\mathbf{uncurry}\, p = F@PA$	Pattern data (head argun
JIsPattern	::=		
	$\vert$	$\mathbf{pattern}\, p$	
JSubPat	::=		
	$\vert$	$\mathbf{subpattern}\, p'\, p$	Subpattern
JTmPatternAgree	::=		
	$\vert$	$a \leftrightarrow p$	term and pattern agree
JTmSubPatternAgree	::=		
	$\vert$	$a \sqsubseteq p$	sub-pattern agrees with te
JSubTmPatternAgree	::=		
	$\vert$	$a \sqsupseteq p$	sub-term agrees with patt
JValuePath	::=		
	$\vert$	$\mathsf{ValuePath}\ a \nleftrightarrow F$	Type headed by constant

JCasePath	$::=$		
	$\mid$	$\mathsf{CasePath}_R\ a \not\rightarrow F$	Type headed by constant (role-sensitive part
JApplyArgs	$::=$		
	$\mid$	$\mathsf{apply\ args}\ a\ \mathsf{to}\ b \not\rightarrow b'$	apply arguments of a (headed by a constant
JValue	$::=$		
	$\mid$	$\mathsf{Value}_R\ A$	values
JValueType	$::=$		
	$\mid$	$\mathsf{ValueType}_R\ A$	Types with head forms (erased language)
Jconsistent	$::=$		
	$\mid$	$\mathsf{consistent}_R\ a\ b$	(erased) types do not differ in their heads
Jroleing	$::=$		
	$\mid$	$\Omega \vDash a : R$	Roleing judgment
JChk	$::=$		
	$\mid$	$(\rho = +) \vee (x \notin \mathsf{fv}\ A)$	irrelevant argument check
Jpar	$::=$		
	$\mid$	$\Omega \vDash a \Rightarrow_R b$	parallel reduction (implicit language)
	$\mid$	$\Omega \vDash a \Rightarrow^*_R b$	multistep parallel reduction
	$\mid$	$\Omega \vDash a \Leftrightarrow_R b$	parallel reduction to a common term
Jbeta	$::=$		
	$\mid$	$\vDash a >_R b$	primitive reductions on erased terms
	$\mid$	$\vDash a \rightsquigarrow_R b$	single-step head reduction for implicit langu
	$\mid$	$\vDash a \rightsquigarrow^* b/R$	multistep reduction
JBranchTyping	$::=$		
	$\mid$	$\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'$	Branch Typing (aligning the types of case)
Jett	$::=$		
	$\mid$	$\Gamma \vDash \phi\ \ \mathsf{ok}$	Prop wellformedness
	$\mid$	$\Gamma \vDash a : A$	typing
	$\mid$	$\Gamma; \Delta \vDash \phi_1 \equiv \phi_2$	prop equality
	$\mid$	$\Gamma; \Delta \vDash a \equiv b : A/R$	definitional equality
	$\mid$	$\vDash \Gamma$	context wellformedness
Jsig	$::=$		
	$\mid$	$\vDash \Sigma$	signature wellformedness
Jhiding	$::=$		
	$\mid$	$Rs_1 \Leftarrow Rs_2$	
	$\mid$	$\Sigma_1 \Leftarrow \Sigma_2$	

Jann	::=		
	|	$\Gamma \vdash \phi\ \mathsf{ok}$	prop wellformedness
	|	$\Gamma \vdash a : A / R$	typing
	|	$\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2$	coercion between props
	|	$\Gamma; \Delta \vdash \gamma : A \sim_R B$	coercion between types
	|	$\vdash \Gamma$	context wellformedness
Jred	::=		
	|	$\Gamma \vdash a \leadsto b / R$	single-step, weak head reduction to values for annotated la
JAlt	::=		
	|	$\Gamma \vDash a : A$	
	|	$\vDash a > b / R$	fake rules for the paper
judgement	::=		
	|	*JSubRole*	
	|	*JRolePath*	
	|	*JPatCtx*	
	|	*JRename*	
	|	*JMatchSubst*	
	|	*JPatData*	
	|	*JIsPattern*	
	|	*JSubPat*	
	|	*JTmPatternAgree*	
	|	*JTmSubPatternAgree*	
	|	*JSubTmPatternAgree*	
	|	*JValuePath*	
	|	*JCasePath*	
	|	*JApplyArgs*	
	|	*JValue*	
	|	*JValueType*	
	|	*Jconsistent*	
	|	*Jroleing*	
	|	*JChk*	
	|	*Jpar*	
	|	*Jbeta*	
	|	*JBranchTyping*	
	|	*Jett*	
	|	*Jsig*	
	|	*Jhiding*	
	|	*Jann*	
	|	*Jred*	
	|	*JAlt*	
user_syntax	::=		
	|	*tmvar*	
	|	*covar*	

| *datacon*
| *const*
| *index*
| *relflag*
| *appflag*
| *role*
| *constraint*
| *tm*
| *brs*
| *co*
| *role_context*
| *roles*
| *sig_sort*
| *sort*
| *context*
| *sig*
| *available_props*
| *Nat*
| *pattern_arg*
| *pattern_args*
| *terminals*
| *formula*

$\boxed{R_1 \leq R_2}$ Subroling judgement

$$\frac{}{\mathbf{Nom} \leq R} \quad \textsc{NomBot}$$

$$\frac{}{R \leq \mathbf{Rep}} \quad \textsc{RepTop}$$

$$\frac{}{R \leq R} \quad \textsc{Refl}$$

$$\frac{\begin{array}{c} R_1 \leq R_2 \\ R_2 \leq R_3 \end{array}}{R_1 \leq R_3} \quad \textsc{Trans}$$

$\boxed{\mathsf{RolePath}\ a = F@Rs}$ Type headed by constant (partial function)

$$\frac{F :_{Rs} A \in \Sigma_0}{\mathsf{RolePath}\ F = F@Rs} \quad \textsc{RolePath\_AbsConst}$$

$$\frac{F :_{Rs} p \sim_{R_1} a : A \in \Sigma_0}{\mathsf{RolePath}\ F = F@Rs} \quad \textsc{RolePath\_Const}$$

$$\frac{\mathsf{RolePath}\ a = F@R_1, Rs}{\mathsf{RolePath}\ (a\ b'^{R_1}) = F@Rs} \quad \textsc{RolePath\_App}$$

$$\frac{\mathsf{RolePath}\ a = F@Rs}{\mathsf{RolePath}\ (a\ \square^{-}) = F@Rs} \quad \textsc{RolePath\_IApp}$$

$$\frac{\mathsf{RolePath}\ a = F@Rs}{\mathsf{RolePath}\ (a[\bullet]) = F@Rs} \quad \textsc{RolePath\_CApp}$$

$\boxed{\Omega; \Gamma \vDash p :_F B \Rightarrow A}$ Contexts generated by a pattern (variables bound by the pattern)

$$\frac{}{\varnothing; \varnothing \vDash F :_F A \Rightarrow A} \quad \textsc{PatCtx\_Const}$$

$$\frac{\Omega;\Gamma \vDash p :_F \Pi^+ x\!:\!A' \to A \Rightarrow B}{\Omega, x : R;\Gamma, x : A' \vDash p\ x^R :_F A \Rightarrow B} \quad \text{PatCtx\_PiRel}$$

$$\frac{\Omega;\Gamma \vDash p :_F \Pi^- x\!:\!A' \to A \Rightarrow B}{\Omega;\Gamma, x : A' \vDash p\ \Box^- :_F A \Rightarrow B} \quad \text{PatCtx\_PiIrr}$$

$$\frac{\Omega;\Gamma \vDash p :_F \forall c\!:\!\phi.A \Rightarrow B}{\Omega;\Gamma, c : \phi \vDash p[\bullet] :_F A \Rightarrow B} \quad \text{PatCtx\_CPi}$$

$\boxed{\mathsf{rename}\ p \to a\ \mathsf{to}\ p' \to a'\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}$ rename with fresh variables

$$\frac{}{\mathsf{rename}\ F \to a\ \mathsf{to}\ F \to a\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \varnothing} \quad \text{Rename\_Base}$$

$$\frac{\begin{array}{l}\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta' \\ y \notin (\Delta, \Delta')\end{array}}{\mathsf{rename}\ (p_1\ x^R) \to a_1\ \mathsf{to}\ (p_2\ y^R) \to (a_2\{y/x\})\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ (\Delta', y)} \quad \text{Rename\_AppRel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}{\mathsf{rename}\ (p_1\ \Box^-) \to a_1\ \mathsf{to}\ (p_2\ \Box^-) \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'} \quad \text{Rename\_AppIrrel}$$

$$\frac{\mathsf{rename}\ p_1 \to a_1\ \mathsf{to}\ p_2 \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'}{\mathsf{rename}\ (p_1[\bullet]) \to a_1\ \mathsf{to}\ (p_2[\bullet]) \to a_2\ \mathsf{excluding}\ \Delta\ \mathsf{and}\ \Delta'} \quad \text{Rename\_CApp}$$

$\boxed{\mathsf{match}\ a_1\ \mathsf{with}\ p \to b_1 \not\leftrightarrow b_2}$ match and substitute

$$\frac{}{\mathsf{match}\ F\ \mathsf{with}\ F \to b \not\leftrightarrow b} \quad \text{MatchSubst\_Const}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ p_1 \to b_1 \not\leftrightarrow b_2}{\mathsf{match}\ (a_1\ a^R)\ \mathsf{with}\ (p_1\ x^R) \to b_1 \not\leftrightarrow (b_2\{a/x\})} \quad \text{MatchSubst\_AppRelR}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ p_1 \to b_1 \not\leftrightarrow b_2}{\mathsf{match}\ (a_1\ a^-)\ \mathsf{with}\ (p_1\ \Box^-) \to b_1 \not\leftrightarrow b_2} \quad \text{MatchSubst\_AppIrrel}$$

$$\frac{\mathsf{match}\ a_1\ \mathsf{with}\ a_2 \to b_1 \not\leftrightarrow b_2}{\mathsf{match}\ (a_1[\bullet])\ \mathsf{with}\ (a_2[\bullet]) \to b_1 \not\leftrightarrow b_2} \quad \text{MatchSubst\_CApp}$$

$\boxed{\mathbf{uncurry}\ p = F@PA}$ Pattern data (head arguments)

$$\frac{}{\mathbf{uncurry}\ F = F@\mathbf{none}} \quad \text{PatData\_Head}$$

$$\frac{\mathbf{uncurry}\ p = F@PA}{\mathbf{uncurry}\ (p\ a^R) = F@PA, \backslash Rel a\ R} \quad \text{PatData\_Rel}$$

$\boxed{\mathbf{pattern}\ p}$

$$\frac{}{\mathbf{pattern}\ F} \quad \text{Pattern\_Head}$$

$$\frac{\mathbf{pattern}\ p}{\mathbf{pattern}\ (p\ a^R)} \quad \text{Pattern\_Rel}$$

$$\frac{\mathbf{pattern}\ p}{\mathbf{pattern}\ (p\ a^-)} \quad \text{Pattern\_Irr}$$

$$\frac{\mathbf{pattern}\ p}{\mathbf{pattern}\ (p[\gamma])} \quad \text{Pattern\_Coe}$$

$\boxed{\mathbf{subpattern}\ p'\ p}$ Subpattern

$$\frac{\textbf{pattern}\, p}{\textbf{subpattern}\, p\ p} \quad \textsc{SubPat\_Refl}$$

$$\frac{\textbf{subpattern}\, p'\ p}{\textbf{subpattern}\, p'\ (p\ \ x^R)} \quad \textsc{SubPat\_Rel}$$

$$\frac{\textbf{subpattern}\, p'\ p}{\textbf{subpattern}\, p'\ (p\ \ \Box^-)} \quad \textsc{SubPat\_Irr}$$

$$\frac{\textbf{subpattern}\, p'\ p}{\textbf{subpattern}\, p'\ (p[\bullet])} \quad \textsc{SubPat\_Coe}$$

$\boxed{a \leftrightarrow p}$ term and pattern agree

$$\frac{}{F \leftrightarrow F} \quad \textsc{tm\_pattern\_agree\_Const}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \ a_2{}^R) \leftrightarrow (p_1\ \ x^R)} \quad \textsc{tm\_pattern\_agree\_AppRelR}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1\ \ a^-) \leftrightarrow (p_1\ \ \Box^-)} \quad \textsc{tm\_pattern\_agree\_AppIrrel}$$

$$\frac{a_1 \leftrightarrow p_1}{(a_1[\bullet]) \leftrightarrow (p_1[\bullet])} \quad \textsc{tm\_pattern\_agree\_CApp}$$

$\boxed{a \sqsubseteq p}$ sub-pattern agrees with term

$$\frac{a \leftrightarrow p}{a \sqsubseteq p} \quad \textsc{tm\_subpattern\_agree\_Base}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ \ x^R)} \quad \textsc{tm\_subpattern\_agree\_AppRelR}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p\ \ \Box^-)} \quad \textsc{tm\_subpattern\_agree\_AppIrrel}$$

$$\frac{a \sqsubseteq p}{a \sqsubseteq (p[\bullet])} \quad \textsc{tm\_subpattern\_agree\_CAppp}$$

$\boxed{a \sqsupseteq p}$ sub-term agrees with pattern

$$\frac{a \leftrightarrow p}{a \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_Base}$$

$$\frac{a \sqsupseteq p}{a\ \ a_2{}^\nu \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_App}$$

$$\frac{a \sqsupseteq p}{a[\bullet] \sqsupseteq p} \quad \textsc{subtm\_pattern\_agree\_CAppp}$$

$\boxed{\mathsf{ValuePath}\ \ a \nleftrightarrow F}$ Type headed by constant (role-sensitive partial function used in value)

$$\frac{F :_{Rs}\ A \in \Sigma_0}{\mathsf{ValuePath}\ \ F \nleftrightarrow F} \quad \textsc{ValuePath\_AbsConst}$$

$$\frac{F :_{Rs}\ p \sim_{R_1} a : A \in \Sigma_0}{\mathsf{ValuePath}\ \ F \nleftrightarrow F} \quad \textsc{ValuePath\_Const}$$

$$\frac{\mathsf{ValuePath}\ a \nrightarrow F}{\mathsf{ValuePath}\ (a\ b'^{\nu}) \nrightarrow F} \quad \text{VALUEPATH\_APP}$$

$$\frac{\mathsf{ValuePath}\ a \nrightarrow F}{\mathsf{ValuePath}\ (a[\bullet]) \nrightarrow F} \quad \text{VALUEPATH\_CAPP}$$

$\boxed{\mathsf{CasePath}_R\ a \nrightarrow F}$ Type headed by constant (role-sensitive partial function used in case)

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \nrightarrow F\\ F :_{Rs}\ A \in \Sigma_0\end{array}}{\mathsf{CasePath}_R\ a \nrightarrow F} \quad \text{CASEPATH\_ABSCONST}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \nrightarrow F\\ F :_{Rs}\ p \sim_{R_1} b : A \in \Sigma_0\\ \neg(R_1 \leq R)\end{array}}{\mathsf{CasePath}_R\ a \nrightarrow F} \quad \text{CASEPATH\_CONST}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \nrightarrow F\\ F :_{Rs}\ p \sim_{R_1} b : A \in \Sigma_0\\ \neg(a \sqsupseteq p)\end{array}}{\mathsf{CasePath}_R\ a \nrightarrow F} \quad \text{CASEPATH\_UNMATCH}$$

$\boxed{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nrightarrow b'}$ apply arguments of a (headed by a constant) to b

$$\frac{}{\mathsf{apply\ args}\ F\ \mathsf{to}\ b \nrightarrow b} \quad \text{APPLYARGS\_CONST}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nrightarrow b'}{\mathsf{apply\ args}\ a\ a'^{\nu}\ \mathsf{to}\ b \nrightarrow b'\ a'^{\nu}} \quad \text{APPLYARGS\_APP}$$

$$\frac{\mathsf{apply\ args}\ a\ \mathsf{to}\ b \nrightarrow b'}{\mathsf{apply\ args}\ a[\bullet]\ \mathsf{to}\ b \nrightarrow b'[\bullet]} \quad \text{APPLYARGS\_CAPP}$$

$\boxed{\mathsf{Value}_R\ A}$ values

$$\frac{}{\mathsf{Value}_R\ \star} \quad \text{VALUE\_STAR}$$

$$\frac{}{\mathsf{Value}_R\ \Pi^{\rho} x\!:\!A \to B} \quad \text{VALUE\_PI}$$

$$\frac{}{\mathsf{Value}_R\ \forall c\!:\!\phi.B} \quad \text{VALUE\_CPI}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x\!:\!A.a} \quad \text{VALUE\_ABSREL}$$

$$\frac{}{\mathsf{Value}_R\ \lambda^{+} x.a} \quad \text{VALUE\_UABSREL}$$

$$\frac{\mathsf{Value}_R\ a}{\mathsf{Value}_R\ \lambda^{-} x.a} \quad \text{VALUE\_UABSIRREL}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c\!:\!\phi.a} \quad \text{VALUE\_CABS}$$

$$\frac{}{\mathsf{Value}_R\ \Lambda c.a} \quad \text{VALUE\_UCABS}$$

$$\frac{\mathsf{CasePath}_R\ a \nrightarrow F}{\mathsf{Value}_R\ a} \quad \text{VALUE\_PATH}$$

$\boxed{\mathsf{ValueType}_R\ A}$ Types with head forms (erased language)

$$\frac{}{\mathsf{ValueType}_R\ \star}\ \textsc{value\_type\_Star}$$

$$\frac{}{\mathsf{ValueType}_R\ \Pi^\rho x\!:\!A \to B}\ \textsc{value\_type\_Pi}$$

$$\frac{}{\mathsf{ValueType}_R\ \forall c\!:\!\phi.B}\ \textsc{value\_type\_CPi}$$

$$\frac{\mathsf{CasePath}_R\ \ a \nrightarrow F}{\mathsf{ValueType}_R\ \ a}\ \textsc{value\_type\_ValuePath}$$

$\boxed{\mathsf{consistent}_R\ \ a\ \ b}$ (erased) types do not differ in their heads

$$\frac{}{\mathsf{consistent}_R\ \ \star\ \ \star}\ \textsc{consistent\_a\_Star}$$

$$\frac{}{\mathsf{consistent}_{R'}\ \ (\Pi^\rho x_1\!:\!A_1 \to B_1)\ (\Pi^\rho x_2\!:\!A_2 \to B_2)}\ \textsc{consistent\_a\_Pi}$$

$$\frac{}{\mathsf{consistent}_R\ \ (\forall c_1\!:\!\phi_1.A_1)\ (\forall c_2\!:\!\phi_2.A_2)}\ \textsc{consistent\_a\_CPi}$$

$$\frac{\begin{array}{c}\mathsf{CasePath}_R\ \ a_1 \nrightarrow F\\ \mathsf{CasePath}_R\ \ a_2 \nrightarrow F\end{array}}{\mathsf{consistent}_R\ \ a_1\ \ a_2}\ \textsc{consistent\_a\_CasePath}$$

$$\frac{\neg\mathsf{ValueType}_R\ \ b}{\mathsf{consistent}_R\ \ a\ \ b}\ \textsc{consistent\_a\_Step\_R}$$

$$\frac{\neg\mathsf{ValueType}_R\ \ a}{\mathsf{consistent}_R\ \ a\ \ b}\ \textsc{consistent\_a\_Step\_L}$$

$\boxed{\Omega \vDash a : R}$ Roleing judgment

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \square : R}\ \textsc{role\_a\_Bullet}$$

$$\frac{\mathit{uniq}(\Omega)}{\Omega \vDash \star : R}\ \textsc{role\_a\_Star}$$

$$\frac{\begin{array}{l}\mathit{uniq}(\Omega)\\ x : R \in \Omega\\ R \leq R_1\end{array}}{\Omega \vDash x : R_1}\ \textsc{role\_a\_Var}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a : R}{\Omega \vDash (\lambda^\rho x.a) : R}\ \textsc{role\_a\_Abs}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : \mathbf{Nom}\end{array}}{\Omega \vDash (a\ \ b^\rho) : R}\ \textsc{role\_a\_App}$$

$$\frac{\begin{array}{l}\Omega \vDash a : R\\ \Omega \vDash b : R_1\end{array}}{\Omega \vDash a\ \ b^{R_1} : R}\ \textsc{role\_a\_TApp}$$

$$\frac{\begin{array}{l}\Omega \vDash A : R\\ \Omega, x : \mathbf{Nom} \vDash B : R\end{array}}{\Omega \vDash (\Pi^\rho x\!:\!A \to B) : R}\ \textsc{role\_a\_Pi}$$

$$\frac{\Omega \vDash a : R_1 \quad \Omega \vDash b : R_1 \quad \Omega \vDash A : \mathbf{Rep} \quad \Omega \vDash B : R}{\Omega \vDash (\forall c{:}a \sim_{R_1} b : A.B) : R} \quad \textsc{role\_a\_CPi}$$

$$\frac{\Omega \vDash b : R}{\Omega \vDash (\Lambda c.b) : R} \quad \textsc{role\_a\_CAbs}$$

$$\frac{\Omega \vDash a : R}{\Omega \vDash (a[\bullet]) : R} \quad \textsc{role\_a\_CApp}$$

$$\frac{\mathit{uniq}(\Omega) \quad F :_{Rs} \ A \in \Sigma_0}{\Omega \vDash F : R} \quad \textsc{role\_a\_Const}$$

$$\frac{\mathit{uniq}(\Omega) \quad F :_{Rs} \ p \sim_R a : A \in \Sigma_0}{\Omega \vDash F : R_1} \quad \textsc{role\_a\_Fam}$$

$$\frac{\Omega \vDash a : R \quad \Omega \vDash b_1 : R_1 \quad \Omega \vDash b_2 : R_1}{\Omega \vDash \mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \|_{\_} \to b_2 : R_1} \quad \textsc{role\_a\_Pattern}$$

$\boxed{(\rho = +) \lor (x \notin \mathsf{fv}\ A)}$ irrelevant argument check

$$\frac{}{(+ = +) \lor (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_Rel}$$

$$\frac{x \notin \mathsf{fv} A}{(- = +) \lor (x \notin \mathsf{fv}\ A)} \quad \textsc{Rho\_IrrRel}$$

$\boxed{\Omega \vDash a \Rightarrow_R b}$ parallel reduction (implicit language)

$$\frac{\Omega \vDash a : R}{\Omega \vDash a \Rightarrow_R a} \quad \textsc{Par\_Refl}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\lambda^\rho x.a') \quad \Omega \vDash b \Rightarrow_{\mathbf{Nom}} b'}{\Omega \vDash a \ b^\rho \Rightarrow_R a'\{b'/x\}} \quad \textsc{Par\_Beta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a' \quad \Omega \vDash b \Rightarrow_{(\mathbf{app\_role}\,\nu)} b'}{\Omega \vDash a \ b^\nu \Rightarrow_R a' \ b'^\nu} \quad \textsc{Par\_App}$$

$$\frac{\Omega \vDash a \Rightarrow_R (\Lambda c.a')}{\Omega \vDash a[\bullet] \Rightarrow_R a'\{\bullet/c\}} \quad \textsc{Par\_CBeta}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash a[\bullet] \Rightarrow_R a'[\bullet]} \quad \textsc{Par\_CApp}$$

$$\frac{\Omega, x : \mathbf{Nom} \vDash a \Rightarrow_R a'}{\Omega \vDash \lambda^\rho x.a \Rightarrow_R \lambda^\rho x.a'} \quad \textsc{Par\_Abs}$$

$$\frac{\Omega \vDash A \Rightarrow_R A' \quad \Omega, x : \mathbf{Nom} \vDash B \Rightarrow_R B'}{\Omega \vDash \Pi^\rho x{:}A \to B \Rightarrow_R \Pi^\rho x{:}A' \to B'} \quad \textsc{Par\_Pi}$$

$$\frac{\Omega \vDash a \Rightarrow_R a'}{\Omega \vDash \Lambda c.a \Rightarrow_R \Lambda c.a'} \quad \text{PAR\_CABS}$$

$$\frac{\begin{array}{l}\Omega \vDash A \Rightarrow_{\mathbf{Rep}} A' \\ \Omega \vDash a \Rightarrow_{R_1} a' \\ \Omega \vDash b \Rightarrow_{R_1} b' \\ \Omega \vDash B \Rightarrow_R B'\end{array}}{\Omega \vDash \forall c{:}a \sim_{R_1} b : A.B \Rightarrow_R \forall c{:}a' \sim_{R_1} b' : A'.B'} \quad \text{PAR\_CPI}$$

$$\frac{\begin{array}{l}F :_{Rs} F \sim_{R_1} b : A \in \Sigma_0 \\ R_1 \leq R \\ uniq(\Omega)\end{array}}{\Omega \vDash F \Rightarrow_R b} \quad \text{PAR\_AXIOMBASE}$$

$$\frac{\begin{array}{l}F :_{Rs} p \sim_{R_1} b : A \in \Sigma_0 \\ a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\ \Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash a_1 \Rightarrow_{(\mathbf{app\_role}\,\nu)} a_1' \\ \text{rename } p \to b \text{ to } p' \to b' \text{ excluding } (\widetilde{\Omega}, \mathsf{fv} p) \text{ and } \Delta' \\ \text{match } (a' \ {a_1'}^{\nu}) \text{ with } p' \to b' \nrightarrow a_2 \\ R_1 \leq R\end{array}}{\Omega \vDash a \ {a_1}^{\nu} \Rightarrow_R a_2} \quad \text{PAR\_AXIOMAPP}$$

$$\frac{\begin{array}{l}F :_{Rs} p \sim_{R_1} b : A \in \Sigma_0 \\ a \sqsubseteq p \wedge \neg(a \leftrightarrow p) \\ \Omega \vDash a \Rightarrow_R a' \\ \text{rename } p \to b \text{ to } p' \to b' \text{ excluding } (\widetilde{\Omega}, \mathsf{fv} p) \text{ and } \Delta' \\ \text{match } (a'[\bullet]) \text{ with } p' \to b' \nrightarrow a_2 \\ R_1 \leq R\end{array}}{\Omega \vDash a[\bullet] \Rightarrow_R a_2} \quad \text{PAR\_AXIOMCAPP}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2'\end{array}}{\Omega \vDash (\mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \|_{\_} \to b_2) \Rightarrow_{R_0} (\mathsf{case}_R \ a' \ \mathsf{of} \ F \to b_1' \|_{\_} \to b_2')} \quad \text{PAR\_PATTERN}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{CasePath}_R \ a' \nrightarrow F \\ \text{apply args } a' \text{ to } b_1' \nrightarrow b\end{array}}{\Omega \vDash (\mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \|_{\_} \to b_2) \Rightarrow_{R_0} b[\bullet]} \quad \text{PAR\_PATTERNTRUE}$$

$$\frac{\begin{array}{l}\Omega \vDash a \Rightarrow_R a' \\ \Omega \vDash b_1 \Rightarrow_{R_0} b_1' \\ \Omega \vDash b_2 \Rightarrow_{R_0} b_2' \\ \mathsf{Value}_R \ a' \\ \neg(\mathsf{CasePath}_R \ a' \nrightarrow F)\end{array}}{\Omega \vDash (\mathsf{case}_R \ a \ \mathsf{of} \ F \to b_1 \|_{\_} \to b_2) \Rightarrow_{R_0} b_2'} \quad \text{PAR\_PATTERNFALSE}$$

$\boxed{\Omega \vDash a \Rightarrow^*_R b}$ multistep parallel reduction

$$\frac{}{\Omega \vDash a \Rightarrow^*_R a} \quad \text{MP\_REFL}$$

$$\frac{\Omega \vDash a \Rightarrow_R b \quad \Omega \vDash b \Rightarrow^*_R a'}{\Omega \vDash a \Rightarrow^*_R a'} \quad \text{MP\_STEP}$$

$\boxed{\Omega \vDash a \Leftrightarrow_R b}$ parallel reduction to a common term

$$\frac{\Omega \vDash a_1 \Rightarrow^*_R b \quad \Omega \vDash a_2 \Rightarrow^*_R b}{\Omega \vDash a_1 \Leftrightarrow_R a_2} \quad \text{JOIN}$$

$\boxed{\vDash a >_R b}$ primitive reductions on erased terms

$$\frac{\mathsf{Value}_{R_1}\ (\lambda^\rho x.v)}{\vDash (\lambda^\rho x.v)\ b^\rho >_{R_1} v\{b/x\}} \quad \text{BETA\_APPABS}$$

$$\frac{}{\vDash (\Lambda c.a')[\bullet] >_R a'\{\bullet/c\}} \quad \text{BETA\_CAPPCABS}$$

$$\frac{\begin{array}{l} F :_{Rs}\ p \sim_{R_1} b : A \in \Sigma_0 \\ \mathsf{rename}\ p \to b\ \mathsf{to}\ p_1 \to b_1\ \mathsf{excluding}\ (\mathsf{fv}a, \mathsf{fv}p)\ \mathsf{and}\ \Delta' \\ \mathsf{match}\ a\ \mathsf{with}\ p_1 \to b_1 \nrightarrow b' \\ R_1 \leq R \end{array}}{\vDash a >_R b'} \quad \text{BETA\_AXIOM}$$

$$\frac{\mathsf{CasePath}_R\ a \nrightarrow F \quad \mathsf{apply\ args}\ a\ \mathsf{to}\ b_1 \nrightarrow b_1'}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 >_{R_0} b_1'[\bullet]} \quad \text{BETA\_PATTERNTRUE}$$

$$\frac{\mathsf{Value}_R\ a \quad \neg(\mathsf{CasePath}_R\ a \nrightarrow F)}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 >_{R_0} b_2} \quad \text{BETA\_PATTERNFALSE}$$

$\boxed{\vDash a \rightsquigarrow_R b}$ single-step head reduction for implicit language

$$\frac{\vDash a \rightsquigarrow_{R_1} a'}{\vDash \lambda^- x.a \rightsquigarrow_{R_1} \lambda^- x.a'} \quad \text{E\_ABSTERM}$$

$$\frac{\vDash a \rightsquigarrow_{R_1} a'}{\vDash a\ b^\nu \rightsquigarrow_{R_1} a'\ b^\nu} \quad \text{E\_APPLEFT}$$

$$\frac{\vDash a \rightsquigarrow_R a'}{\vDash a[\bullet] \rightsquigarrow_R a'[\bullet]} \quad \text{E\_CAPPLEFT}$$

$$\frac{\vDash a \rightsquigarrow_R a'}{\vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 \rightsquigarrow_{R_0} \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1 \|\_ \to b_2} \quad \text{E\_PATTERN}$$

$$\frac{\vDash a >_R b}{\vDash a \rightsquigarrow_R b} \quad \text{E\_PRIM}$$

$\boxed{\vDash a \rightsquigarrow^* b/R}$ multistep reduction

$$\frac{}{\vDash a \rightsquigarrow^* a/R} \quad \text{EQUAL}$$

$$\frac{\vDash a \rightsquigarrow_R b \quad \vDash b \rightsquigarrow^* a'/R}{\vDash a \rightsquigarrow^* a'/R} \quad \text{STEP}$$

$\boxed{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : B \Rightarrow C \mid C'}$ Branch Typing (aligning the types of case)

$$\frac{\begin{array}{l} \mathit{uniq}\,\Gamma \\ \texttt{lc\_tm}\ \ C \end{array}}{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : A \Rightarrow \forall c\!:\!(a \sim_R b : A).C \mid C}\ \textsc{BranchTyping\_Base}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b\ x^+ : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b : \Pi^+ x\!:\!A \to B \Rightarrow \Pi^+ x\!:\!A \to C \mid C'}\ \textsc{BranchTyping\_PiRel}$$

$$\frac{\Gamma, x : A \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b\ \square^- : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A_1\ \mathsf{of}\ b : \Pi^- x\!:\!A \to B \Rightarrow \Pi^- x\!:\!A \to C \mid C'}\ \textsc{BranchTyping\_PiIrrel}$$

$$\frac{\Gamma, c : \phi \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b[\bullet] : B \Rightarrow C \mid C'}{\Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ b : \forall c\!:\!\phi.B \Rightarrow \forall c\!:\!\phi.C \mid C'}\ \textsc{BranchTyping\_CPi}$$

$\boxed{\Gamma \vDash \phi\ \ \mathsf{ok}}$ Prop wellformedness

$$\frac{\begin{array}{l} \Gamma \vDash a : A \\ \Gamma \vDash b : A \\ \Gamma \vDash A : \star \end{array}}{\Gamma \vDash a \sim_R b : A\ \ \mathsf{ok}}\ \textsc{E\_Wff}$$

$\boxed{\Gamma \vDash a : A}$ typing

$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star}\ \textsc{E\_Star}$$

$$\frac{\begin{array}{l} \vDash \Gamma \\ x : A \in \Gamma \end{array}}{\Gamma \vDash x : A}\ \textsc{E\_Var}$$

$$\frac{\begin{array}{l} \Gamma, x : A \vDash B : \star \\ \Gamma \vDash A : \star \end{array}}{\Gamma \vDash \Pi^\rho x\!:\!A \to B : \star}\ \textsc{E\_Pi}$$

$$\frac{\begin{array}{l} \Gamma, x : A \vDash a : B \\ \Gamma \vDash A : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ a) \end{array}}{\Gamma \vDash \lambda^\rho x.a : (\Pi^\rho x\!:\!A \to B)}\ \textsc{E\_Abs}$$

$$\frac{\begin{array}{l} \Gamma \vDash b : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash a : A \end{array}}{\Gamma \vDash b\ a^+ : B\{a/x\}}\ \textsc{E\_App}$$

$$\frac{\begin{array}{l} \Gamma \vDash b : \Pi^+ x\!:\!A \to B \\ \Gamma \vDash a : A \\ \mathsf{RolePath}\ \ b = F@R, Rs \end{array}}{\Gamma \vDash b\ a^R : B\{a/x\}}\ \textsc{E\_TApp}$$

$$\frac{\begin{array}{l} \Gamma \vDash b : \Pi^- x\!:\!A \to B \\ \Gamma \vDash a : A \end{array}}{\Gamma \vDash b\ \square^- : B\{a/x\}}\ \textsc{E\_IApp}$$

$$\frac{\begin{array}{l} \Gamma \vDash a : A \\ \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep} \\ \Gamma \vDash B : \star \end{array}}{\Gamma \vDash a : B}\ \textsc{E\_Conv}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash B : \star \\ \Gamma \vDash \phi \ \mathsf{ok}\end{array}}{\Gamma \vDash \forall c{:}\phi.B : \star} \quad \text{E\_CPi}$$

$$\frac{\begin{array}{l}\Gamma, c : \phi \vDash a : B \\ \Gamma \vDash \phi \ \mathsf{ok}\end{array}}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B} \quad \text{E\_CAbs}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : \forall c{:}(a \sim_R b : A).B_1 \\ \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A/R\end{array}}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}} \quad \text{E\_CApp}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F :_{Rs} A \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_Const}$$

$$\frac{\begin{array}{l}\vDash \Gamma \\ F :_{Rs} p \sim_{R_1} a : A \in \Sigma_0 \\ \varnothing \vDash A : \star\end{array}}{\Gamma \vDash F : A} \quad \text{E\_Fam}$$

$$\frac{\begin{array}{l}\Gamma \vDash a : A \\ \Gamma \vDash F : A_1 \\ \Gamma \vDash b_1 : B \\ \Gamma \vDash b_2 : C \\ \Gamma \vDash \mathsf{case}_R\ a : A \ \mathsf{of}\ F : A_1 \Rightarrow B \mid C\end{array}}{\Gamma \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 : C} \quad \text{E\_Case}$$

$\boxed{\Gamma; \Delta \vDash \phi_1 \equiv \phi_2}$ prop equality

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A_1 \equiv A_2 : A/R \\ \Gamma; \Delta \vDash B_1 \equiv B_2 : A/R\end{array}}{\Gamma; \Delta \vDash A_1 \sim_R B_1 : A \equiv A_2 \sim_R B_2 : A} \quad \text{E\_PropCong}$$

$$\frac{\begin{array}{c}\Gamma; \Delta \vDash A \equiv B : \star/R_0 \\ \Gamma \vDash A_1 \sim_R A_2 : A \ \mathsf{ok} \\ \Gamma \vDash A_1 \sim_R A_2 : B \ \mathsf{ok}\end{array}}{\Gamma; \Delta \vDash A_1 \sim_R A_2 : A \equiv A_1 \sim_R A_2 : B} \quad \text{E\_IsoConv}$$

$$\frac{\Gamma; \Delta \vDash \forall c{:}(a_1 \sim_{R_1} a_2 : A).B_1 \equiv \forall c{:}(b_1 \sim_{R_2} b_2 : B).B_2 : \star/R'}{\Gamma; \Delta \vDash a_1 \sim_{R_1} a_2 : A \equiv b_1 \sim_{R_2} b_2 : B} \quad \text{E\_CPiFst}$$

$\boxed{\Gamma; \Delta \vDash a \equiv b : A/R}$ definitional equality

$$\frac{\begin{array}{l}\vDash \Gamma \\ c : (a \sim_R b : A) \in \Gamma \\ c \in \Delta\end{array}}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Assn}$$

$$\frac{\Gamma \vDash a : A}{\Gamma; \Delta \vDash a \equiv a : A/R} \quad \text{E\_Refl}$$

$$\frac{\Gamma; \Delta \vDash b \equiv a : A/R}{\Gamma; \Delta \vDash a \equiv b : A/R} \quad \text{E\_Sym}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv a_1 : A/R \\ \Gamma;\Delta \vDash a_1 \equiv b : A/R\end{array}}{\Gamma;\Delta \vDash a \equiv b : A/R} \quad \text{E\_TRANS}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R_1 \\ R_1 \leq R_2\end{array}}{\Gamma;\Delta \vDash a \equiv b : A/R_2} \quad \text{E\_SUB}$$

$$\frac{\begin{array}{l}\Gamma \vDash a_1 : B \\ \Gamma \vDash a_2 : B \\ \vDash a_1 >_R a_2\end{array}}{\Gamma;\Delta \vDash a_1 \equiv a_2 : B/R} \quad \text{E\_BETA}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R' \\ \Gamma, x : A_1;\Delta \vDash B_1 \equiv B_2 : \star/R' \\ \Gamma \vDash A_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_1 \to B_1 : \star \\ \Gamma \vDash \Pi^\rho x\!:\!A_2 \to B_2 : \star\end{array}}{\Gamma;\Delta \vDash (\Pi^\rho x\!:\!A_1 \to B_1) \equiv (\Pi^\rho x\!:\!A_2 \to B_2) : \star/R'} \quad \text{E\_PICONG}$$

$$\frac{\begin{array}{l}\Gamma, x : A_1;\Delta \vDash b_1 \equiv b_2 : B/R' \\ \Gamma \vDash A_1 : \star \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_1) \\ (\rho = +) \vee (x \notin \mathsf{fv}\ b_2)\end{array}}{\Gamma;\Delta \vDash (\lambda^\rho x.b_1) \equiv (\lambda^\rho x.b_2) : (\Pi^\rho x\!:\!A_1 \to B)/R'} \quad \text{E\_ABSCONG}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma;\Delta \vDash a_2 \equiv b_2 : A/\mathbf{Nom}\end{array}}{\Gamma;\Delta \vDash a_1\ {a_2}^+ \equiv b_1\ {b_2}^+ : (B\{a_2/x\})/R'} \quad \text{E\_APPCONG}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^+ x\!:\!A \to B)/R' \\ \Gamma;\Delta \vDash a_2 \equiv b_2 : A/\mathbf{param}\, R\, R' \\ \mathsf{RolePath}\ a_1 = F@R, Rs \\ \mathsf{RolePath}\ b_1 = F'@R, Rs' \\ \Gamma \vDash b_1\ {b_2}^R : B\{a_2/x\}\end{array}}{\Gamma;\Delta \vDash a_1\ {a_2}^R \equiv b_1\ {b_2}^R : (B\{a_2/x\})/R'} \quad \text{E\_TAPPCONG}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\Pi^- x\!:\!A \to B)/R' \\ \Gamma \vDash a : A\end{array}}{\Gamma;\Delta \vDash a_1\ \Box^- \equiv b_1\ \Box^- : (B\{a/x\})/R'} \quad \text{E\_IAPPCONG}$$

$$\frac{\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R'}{\Gamma;\Delta \vDash A_1 \equiv A_2 : \star/R'} \quad \text{E\_PIFST}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \Pi^\rho x\!:\!A_1 \to B_1 \equiv \Pi^\rho x\!:\!A_2 \to B_2 : \star/R' \\ \Gamma;\Delta \vDash a_1 \equiv a_2 : A_1/\mathbf{Nom}\end{array}}{\Gamma;\Delta \vDash B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star/R'} \quad \text{E\_PISND}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \sim_R b_1 : A_1 \equiv a_2 \sim_R b_2 : A_2 \\ \Gamma, c : a_1 \sim_R b_1 : A_1;\Delta \vDash A \equiv B : \star/R' \\ \Gamma \vDash a_1 \sim_R b_1 : A_1\ \mathsf{ok} \\ \Gamma \vDash \forall c\!:\!a_1 \sim_R b_1 : A_1.A : \star \\ \Gamma \vDash \forall c\!:\!a_2 \sim_R b_2 : A_2.B : \star\end{array}}{\Gamma;\Delta \vDash \forall c\!:\!a_1 \sim_R b_1 : A_1.A \equiv \forall c\!:\!a_2 \sim_R b_2 : A_2.B : \star/R'} \quad \text{E\_CPICONG}$$

$$\frac{\begin{array}{l}\Gamma, c:\phi_1;\Delta \vDash a \equiv b : B/R\\ \Gamma \vDash \phi_1\ \ \mathsf{ok}\end{array}}{\Gamma;\Delta \vDash (\Lambda c.a) \equiv (\Lambda c.b) : \forall c\!:\!\phi_1.B/R} \quad \textsc{E\_CAbsCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a_1 \equiv b_1 : (\forall c\!:\!(a \sim_R b : A).B)/R'\\ \Gamma;\widetilde{\Gamma} \vDash a \equiv b : A/\mathbf{param}\, R\, R'\end{array}}{\Gamma;\Delta \vDash a_1[\bullet] \equiv b_1[\bullet] : (B\{\bullet/c\})/R'} \quad \textsc{E\_CAppCong}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash \forall c\!:\!(a_1 \sim_R a_2 : A).B_1 \equiv \forall c\!:\!(a_1' \sim_{R'} a_2' : A').B_2 : \star/R_0\\ \Gamma;\widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\, R\, R_0\\ \Gamma;\widetilde{\Gamma} \vDash a_1' \equiv a_2' : A'/\mathbf{param}\, R'\, R_0\end{array}}{\Gamma;\Delta \vDash B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star/R_0} \quad \textsc{E\_CPiSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R\\ \Gamma;\Delta \vDash a \sim_R b : A \equiv a' \sim_{R'} b' : A'\end{array}}{\Gamma;\Delta \vDash a' \equiv b' : A'/R'} \quad \textsc{E\_Cast}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv b : A/R\\ \Gamma;\widetilde{\Gamma} \vDash A \equiv B : \star/\mathbf{Rep}\\ \Gamma \vDash B : \star\end{array}}{\Gamma;\Delta \vDash a \equiv b : B/R} \quad \textsc{E\_EqConv}$$

$$\frac{\Gamma;\Delta \vDash a \sim_{R_1} b : A \equiv a' \sim_{R_1} b' : A'}{\Gamma;\Delta \vDash A \equiv A' : \star/\mathbf{Rep}} \quad \textsc{E\_IsoSnd}$$

$$\frac{\begin{array}{l}\Gamma;\Delta \vDash a \equiv a' : A/R\\ \Gamma \vDash F : A_1\\ \Gamma;\Delta \vDash b_1 \equiv b_1' : B/R_0\\ \Gamma;\Delta \vDash b_2 \equiv b_2' : C/R_0\\ \Gamma \vDash \mathsf{case}_R\ a : A\ \mathsf{of}\ F : A_1 \Rightarrow B \mid C\\ \Gamma \vDash \mathsf{case}_R\ a' : A\ \mathsf{of}\ F : A_1 \Rightarrow B \mid C\end{array}}{\Gamma;\Delta \vDash \mathsf{case}_R\ a\ \mathsf{of}\ F \to b_1 \|\_ \to b_2 \equiv \mathsf{case}_R\ a'\ \mathsf{of}\ F \to b_1' \|\_ \to b_2' : C/R_0} \quad \textsc{E\_PatCong}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \not\mapsto F\\ \mathsf{ValuePath}\ a' \not\mapsto F\\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B\\ \Gamma \vDash b : A\\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B\\ \Gamma \vDash b' : A\\ \Gamma;\Delta \vDash a\ b^{R_1} \equiv a'\ b'^{R_1} : B\{b/x\}/R'\\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/\mathbf{Rep}\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^+ x\!:\!A \to B/R'} \quad \textsc{E\_LeftRel}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \not\mapsto F\\ \mathsf{ValuePath}\ a' \not\mapsto F\\ \Gamma \vDash a : \Pi^- x\!:\!A \to B\\ \Gamma \vDash b : A\\ \Gamma \vDash a' : \Pi^- x\!:\!A \to B\\ \Gamma \vDash b' : A\\ \Gamma;\Delta \vDash a\ \Box^- \equiv a'\ \Box^- : B\{b/x\}/R'\\ \Gamma;\widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/\mathbf{Rep}\end{array}}{\Gamma;\Delta \vDash a \equiv a' : \Pi^- x\!:\!A \to B/R'} \quad \textsc{E\_LeftIrrel}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \nrightarrow F\\ \mathsf{ValuePath}\ a' \nrightarrow F\\ \Gamma \vDash a : \Pi^+ x\!:\!A \to B\\ \Gamma \vDash b : A\\ \Gamma \vDash a' : \Pi^+ x\!:\!A \to B\\ \Gamma \vDash b' : A\\ \Gamma; \Delta \vDash a\ b^{R_1} \equiv a'\ b'^{R_1} : B\{b/x\}/R'\\ \Gamma; \widetilde{\Gamma} \vDash B\{b/x\} \equiv B\{b'/x\} : \star/\mathbf{Rep}\end{array}}{\Gamma; \Delta \vDash b \equiv b' : A/\mathbf{param}\, R_1\ R'} \quad \textsc{E\_Right}$$

$$\frac{\begin{array}{l}\mathsf{ValuePath}\ a \nrightarrow F\\ \mathsf{ValuePath}\ a' \nrightarrow F\\ \Gamma \vDash a : \forall c\!:\!(a_1 \sim_{R_1} a_2 : A).B\\ \Gamma \vDash a' : \forall c\!:\!(a_1 \sim_{R_1} a_2 : A).B\\ \Gamma; \widetilde{\Gamma} \vDash a_1 \equiv a_2 : A/\mathbf{param}\, R_1\ R'\\ \Gamma; \Delta \vDash a[\bullet] \equiv a'[\bullet] : B\{\bullet/c\}/R'\end{array}}{\Gamma; \Delta \vDash a \equiv a' : \forall c\!:\!(a_1 \sim_{R_1} a_2 : A).B/R'} \quad \textsc{E\_CLeft}$$

$\boxed{\vDash \Gamma}$ context wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{E\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash A : \star\\ x \notin \widetilde{\Gamma}\end{array}}{\vDash \Gamma, x : A} \quad \textsc{E\_ConsTm}$$

$$\frac{\begin{array}{l}\vDash \Gamma\\ \Gamma \vDash \phi\ \mathsf{ok}\\ c \notin \widetilde{\Gamma}\end{array}}{\vDash \Gamma, c : \phi} \quad \textsc{E\_ConsCo}$$

$\boxed{\vDash \Sigma}$ signature wellformedness

$$\frac{}{\vDash \varnothing} \quad \textsc{Sig\_Empty}$$

$$\frac{\begin{array}{l}\vDash \Sigma\\ \varnothing \vDash A : \star\\ F \notin \mathsf{dom}\,\Sigma\end{array}}{\vDash \Sigma \cup \{F :_{Rs} A\}} \quad \textsc{Sig\_ConsConst}$$

$$\frac{\begin{array}{l}\vDash \Sigma\\ F \notin \mathsf{dom}\,\Sigma\\ \varnothing \vDash A : \star\\ \Omega; \Gamma \vDash p :_F B \Rightarrow A\\ \Gamma \vDash a : B\\ \Omega \vDash a : R\end{array}}{\vDash \Sigma \cup \{F :_{\mathbf{range}} \Omega\, p \sim_R a : A\}} \quad \textsc{Sig\_ConsAx}$$

$\boxed{Rs_1 \Leftarrow Rs_2}$

$$\frac{}{\cdot \Leftarrow \cdot} \quad \textsc{R\_Nil}$$

$$\frac{R_2 \leq R_1 \quad Rs_1 \Leftarrow Rs_2}{R_1, Rs_1 \Leftarrow R_2, Rs_2} \quad \text{R\_CONS}$$

$\boxed{\Sigma_1 \Leftarrow \Sigma_2}$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2}{\Sigma_1 \Leftarrow \Sigma_2 \cup \{F : sig\_sort\}} \quad \text{S\_FORGET}$$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2 \quad Rs_1 \Leftarrow Rs_2}{\Sigma_1 \cup \{F :_{Rs_1} \; A\} \Leftarrow \Sigma_2 \cup \{F :_{Rs_2} \; p \sim_R a : A\}} \quad \text{S\_HIDE}$$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2 \quad Rs_1 \Leftarrow Rs_2}{\Sigma_1 \cup \{F :_{Rs_1} \; A\} \Leftarrow \Sigma_2 \cup \{F :_{Rs_2} \; A\}} \quad \text{S\_WEAKENCONST}$$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2 \quad Rs_1 \Leftarrow Rs_2}{\Sigma_1 \cup \{F :_{Rs_1} \; p' \sim_R a : A\} \Leftarrow \Sigma_2 \cup \{F :_{Rs_2} \; p \sim_R a : A\}} \quad \text{S\_WEAKENAXIOM}$$

$$\frac{}{\varnothing \Leftarrow \varnothing} \quad \text{S\_EMPTY}$$

$$\frac{\Sigma_1 \Leftarrow \Sigma_2}{\Sigma_1 \cup \{F : sig\_sort\} \Leftarrow \Sigma_2 \cup \{F : sig\_sort\}} \quad \text{S\_SAME}$$

$\boxed{\Gamma \vdash \phi \ \mathsf{ok}}$ prop wellformedness

$\boxed{\Gamma \vdash a : A / R}$ typing

$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}$ coercion between props

$\boxed{\Gamma; \Delta \vdash \gamma : A \sim_R B}$ coercion between types

$\boxed{\vdash \Gamma}$ context wellformedness

$\boxed{\Gamma \vdash a \leadsto b / R}$ single-step, weak head reduction to values for annotated language

$\boxed{\Gamma \vDash a : A}$

$$\frac{\Gamma \vDash a : A \quad \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star / \mathbf{Nom} \quad \Gamma \vDash B : \star}{\Gamma \vDash a : B} \quad \text{ATYPING\_CONV}$$

$\boxed{\vDash a > b / R}$ fake rules for the paper

$$\frac{F :_{Rs} \; p \sim_{R_1} b : A \in \Sigma_0 \quad \mathsf{match}\ a\ \mathsf{with}\ p \to b \nrightarrow b' \quad R_1 \leq R}{\vDash a > b' / R} \quad \text{ABETA\_AXIOM}$$

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Definition rules:        179 good    0 bad
Definition rule clauses: 493 good    0 bad
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