

$tmvar, x, y, f, m, n$	variables
$covar, c$	coercion variables
$datacon, K$	
$const, T, F$	
$index, i$	indices

<i>relflag</i> , ρ	::=	relevance flag
		+
		-

$$\begin{array}{ccc} \text{constraint, } \phi & ::= & \text{props} \\ & | & a \sim_A b \end{array}$$

tm, a, b, v, w, A, B	$::=$	types and kinds
	\star	
	x	
	$\lambda^\rho x : A. b$	annotated lambda
	$\lambda^\rho x. b$	erased lambda
	$a \ b^\rho$	
	T	constant
	$\Pi^\rho x : A \rightarrow B$	
	$a \triangleright \gamma$	coercion
	$\forall c : \phi. B$	
	$\Lambda c : \phi. b$	annotated coercion abstraction
	$\Lambda c. b$	erased coercion abstraction
	$a[\gamma]$	
	\square	erased term
	$\Sigma^\rho x : A. B$	
	$(^\rho a, b)$	erased pair
	$(^\rho a, b) \text{ as } A$	annotated pair
	fst a	
	snd a	

co, γ	$::=$	<ul style="list-style-type: none"> • c red $a\ b$ refl a $(a \models_{\gamma} b)$ sym γ $\gamma_1; \gamma_2$ $\Pi^{\rho} x : \gamma_1. \gamma_2$ $\lambda^{\rho} x : \gamma_1. \gamma_2$ $\gamma_1 \ \gamma_2^{\rho}$ piFst γ cpiFst γ isoSnd γ $\gamma_1 @ \gamma_2$ $\forall c : \gamma_1. \gamma_3$ $\lambda c : \gamma_1. \gamma_3 @ \gamma_4$ $\gamma(\gamma_1, \gamma_2)$ $\gamma @ (\gamma_1 \sim \gamma_2)$ $\gamma_1 \triangleright \gamma_2$ $\gamma_1 \sim_A \gamma_2$ conv $\phi_1 \sim_{\gamma} \phi_2$ eta a left $\gamma \ \gamma'$ right $\gamma \ \gamma'$ $\Sigma x :^{\rho} \gamma_1. \gamma_2$ $(\rho \ \gamma_1, \gamma_2) \text{ as } \gamma_3$ fst γ_1 snd γ_1 sigmaFst γ 	<p>explicit coercions</p> <p>erased coercion</p>
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sig_sort	$::=$	signature classifier
	$\begin{array}{ l} : A \\ \sim a : A \end{array}$	
$sort$	$::=$	binding classifier
	$\begin{array}{ l} \mathbf{Tm} \rho A \\ \mathbf{Co} \phi \end{array}$	
$context, \Gamma$	$::=$	contexts
	$\begin{array}{ l} \emptyset \\ \Gamma, x : \rho A \\ \Gamma, c : \phi \end{array}$	
$available_props, \Delta$	$::=$	
	$\begin{array}{ l} \emptyset \\ \Delta, c \end{array}$	
sig, Σ	$::=$	signatures
	$\begin{array}{ l} \emptyset \\ \Sigma \cup \{T sig_sort\} \end{array}$	

$$\boxed{\rho_1 \Leftarrow \rho_2} \quad ()$$

$$\begin{array}{ccc} \text{BOT} & \text{TOP} & \text{REFL} \\ \hline - \Leftarrow \rho & \rho \Leftarrow + & \rho \Leftarrow \rho \end{array}$$

$$\boxed{\Gamma_1 \Leftarrow \Gamma_2} \quad ()$$

$$\begin{array}{ccc} \text{SUB-EMPTY} & \text{SUB-CONSTM} & \text{SUB-CONSCo} \\ \hline \emptyset \Leftarrow \emptyset & \frac{\rho_1 \Leftarrow \rho_2 \quad \Gamma_1 \Leftarrow \Gamma_2}{\Gamma_1, x : \rho_1 A \Leftarrow \Gamma_2, x : \rho_2 A} & \frac{\Gamma_1 \Leftarrow \Gamma_2}{\Gamma_1, c : \phi \Leftarrow \Gamma_2, c : \phi} \end{array}$$

$$\boxed{\mathbf{CoercedValue} A} \quad (Values \text{ with at most one coercion at the top})$$

$$\begin{array}{cc} \text{CV} & \text{CC} \\ \frac{[\mathbf{Value} a]}{\mathbf{CoercedValue} a} & \frac{[\mathbf{Value} a]}{\mathbf{CoercedValue} (a \triangleright \gamma)} \end{array}$$

$$\boxed{[\mathbf{Value} A]} \quad (values)$$

$$\begin{array}{ccc} \text{VALUE-STAR} & \text{VALUE-PI} & \text{VALUE-CPi} \\ \hline [\mathbf{Value} \star] & [\mathbf{Value} \Pi^\rho x : A \rightarrow B] & [\mathbf{Value} \forall c : \phi. B] \end{array}$$

VALUE-ABSREL $\frac{}{[\text{Value } \lambda^+ x : A.a]}$	VALUE-UABSREL $\frac{}{[\text{Value } \lambda^+ x.a]}$	VALUE-UABSIRREL $\frac{[\text{Value } a]}{[\text{Value } \lambda^- x.a]}$
VALUE-ABSIRREL CoercedValue a $\frac{}{[\text{Value } \lambda^- x : A.a]}$	VALUE-CABS $\frac{}{[\text{Value } \Lambda c : \phi.a]}$	VALUE-UCABS $\frac{}{[\text{Value } \Lambda c.a]}$
VALUE-SIGMA $\frac{}{[\text{Value } \Sigma^\rho x : A.B]}$	VALUE-UPAIR $\frac{}{[\text{Value } (^\rho a, b)]}$	VALUE-PAIR $\frac{}{[\text{Value } (^\rho a, b) \text{ as } A]}$

ValueType A *(Types with head forms (erased language))*

VALUE-TYPE-STAR $\frac{}{\mathbf{ValueType} \star}$	VALUE-TYPE-PI $\frac{}{\mathbf{ValueType} \Pi^\rho x : A \rightarrow B}$	VALUE-TYPE-CPI $\frac{}{\mathbf{ValueType} \forall c : \phi.B}$
VALUE-TYPE-SIGMA $\frac{}{\mathbf{ValueType} \Sigma^\rho x : A.B}$		

consistent $a \ b$ *((erased) types do not differ in their heads)*

CONSISTENT-A-STAR $\frac{}{\mathbf{consistent} \star \star}$	CONSISTENT-A-PI $\frac{}{\mathbf{consistent} (\Pi^\rho x_1 : A_1 \rightarrow B_1) (\Pi^\rho x_2 : A_2 \rightarrow B_2)}$
CONSISTENT-A-CPI $\frac{}{\mathbf{consistent} (\forall c_1 : \phi_1.A_1) (\forall c_2 : \phi_2.A_2)}$	
CONSISTENT-A-SIGMA $\frac{}{\mathbf{consistent} (\Sigma^\rho x : A_1.B_1) (\Sigma^\rho x : A_2.B_2)}$	CONSISTENT-A-STEP-R $\frac{\mathbf{not \ ValueType} \ b}{\mathbf{consistent} \ a \ b}$
CONSISTENT-A-STEP-L $\frac{\mathbf{not \ ValueType} \ a}{\mathbf{consistent} \ a \ b}$	

$(\rho = +) \vee (x \notin \text{fv } A)$ *(irrelevant argument check)*

RHO-REL $\frac{}{(+ = +) \vee (x \notin \text{fv } A)}$	RHO-IRRREL $\frac{x \notin \text{fv } A}{(- = +) \vee (x \notin \text{fv } A)}$
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$\boxed{\text{erased_tm } a}$ ()

ERASED-A-BULLET	ERASED-A-STAR	ERASED-A-VAR
$\frac{}{\text{erased_tm } \square}$	$\frac{}{\text{erased_tm } \star}$	$\frac{}{\text{erased_tm } x}$
ERASED-A-ABS	ERASED-A-APP	ERASED-A-APP _{IRREL}
$\frac{\text{erased_tm } a}{(\rho = +) \vee (x \notin \text{fv } a)}$	$\frac{\text{erased_tm } a \quad \text{erased_tm } b}{\text{erased_tm } (a \ b^+)}$	$\frac{\text{erased_tm } a}{\text{erased_tm } (a \ \square^-)}$
ERASED-A-PI	ERASED-A-CPI	
$\frac{\text{erased_tm } A \quad \text{erased_tm } B}{\text{erased_tm } (\Pi^\rho x : A \rightarrow B)}$	$\frac{\text{erased_tm } a \quad \text{erased_tm } b \quad \text{erased_tm } A \quad \text{erased_tm } B}{\text{erased_tm } (\forall c : a \sim_A b. B)}$	
ERASED-A-CABS	ERASED-A-CAPP	ERASED-A-FAM
$\frac{\text{erased_tm } b}{\text{erased_tm } (\Lambda c. b)}$	$\frac{\text{erased_tm } a}{\text{erased_tm } (a[\bullet])}$	$\frac{}{\text{erased_tm } F}$
ERASED-A-SIGMA	ERASED-A-UPAIRREL	ERASED-A-UPAIR _{IRREL}
$\frac{\text{erased_tm } A \quad \text{erased_tm } B}{\text{erased_tm } (\Sigma^\rho x : A. B)}$	$\frac{\text{erased_tm } a \quad \text{erased_tm } b}{\text{erased_tm } (^+ a, b)}$	$\frac{\text{erased_tm } b}{\text{erased_tm } (^- \square, b)}$
ERASED-A-FST	ERASED-A-SND	
$\frac{\text{erased_tm } a}{\text{erased_tm } (\text{fst } a)}$	$\frac{\text{erased_tm } a}{\text{erased_tm } (\text{snd } a)}$	

$\boxed{\models a \Rightarrow b}$ (parallel reduction (implicit language))

PAR-BETA	PAR-BETA _{IRREL}	
$\frac{\models a \Rightarrow (\lambda^+ x. a') \quad \models b \Rightarrow b'}{\models a \ b^+ \Rightarrow a' \{b'/x\}}$	$\frac{\models a \Rightarrow (\lambda^- x. a')}{\models a \ \square^- \Rightarrow a' \{\square/x\}}$	
PAR-REFL	PAR-APP	PAR-APP _{IRREL}
$\frac{}{\models a \Rightarrow a}$	$\frac{\models a \Rightarrow a' \quad \models b \Rightarrow b'}{\models a \ b^+ \Rightarrow a' \ b'^+}$	$\frac{\models a \Rightarrow a'}{\models a \ \square^- \Rightarrow a' \ \square^-}$
PAR-CBETA	PAR-CAPP	PAR-ABS
$\frac{\models a \Rightarrow (\Lambda c. a')}{\models a[\bullet] \Rightarrow a' \{\bullet/c\}}$	$\frac{\models a \Rightarrow a'}{\models a[\bullet] \Rightarrow a'[\bullet]}$	$\frac{\models \lambda^\rho x. a \Rightarrow \lambda^\rho x. a'}{\models \lambda^\rho x. a \Rightarrow \lambda^\rho x. a'}$
PAR-PI		
$\frac{\models A \Rightarrow A' \quad \models B \Rightarrow B'}{\models \Pi^\rho x : A \rightarrow B \Rightarrow \Pi^\rho x : A' \rightarrow B'}$		

$$\begin{array}{c}
\text{PAR-CP1} \\
\frac{\vdash A \Rightarrow A' \quad \vdash B \Rightarrow B' \quad \vdash a \Rightarrow a' \quad \vdash A_1 \Rightarrow A'_1}{\vdash \forall c : A \sim_{A_1} B.a \Rightarrow \forall c : A' \sim_{A'_1} B'.a'} \\
\\
\text{PAR-CABS} \quad \frac{\vdash a \Rightarrow a'}{\vdash \Lambda c.a \Rightarrow \Lambda c.a'} \quad \text{PAR-AXIOM} \quad \frac{F \sim a : A \in \Sigma_0}{\vdash F \Rightarrow a} \quad \text{PAR-ETA} \quad \frac{\vdash b \Rightarrow b' \quad a = b \ x^+}{\vdash \lambda^+ x.a \Rightarrow b'} \quad \text{PAR-ETAIrREL} \quad \frac{\vdash b \Rightarrow b' \quad a = b \ \Box^-}{\vdash \lambda^- x.a \Rightarrow b'} \\
\\
\text{PAR-ETAC} \quad \frac{\vdash b \Rightarrow b' \quad a = b[\bullet]}{\vdash \Lambda c.a \Rightarrow b'} \quad \text{PAR-SIGMA} \quad \frac{\vdash A \Rightarrow A' \quad \vdash B \Rightarrow B'}{\vdash \Sigma^\rho x : A.B \Rightarrow \Sigma^\rho x : A'.B'} \\
\\
\text{PAR-PAIR} \quad \frac{\vdash a \Rightarrow a' \quad \vdash b \Rightarrow b'}{\vdash (+a, b) \Rightarrow (+a', b')} \quad \text{PAR-PAIRIrREL} \quad \frac{\vdash b \Rightarrow b'}{\vdash (-\Box, b) \Rightarrow (-\Box, b')} \quad \text{PAR-FST} \quad \frac{\vdash a \Rightarrow (\rho a_1, a_2)}{\vdash \mathbf{fst} \ a \Rightarrow a_1} \\
\\
\text{PAR-SND} \quad \frac{\vdash a \Rightarrow (\rho a_1, a_2)}{\vdash \mathbf{snd} \ a \Rightarrow a_2} \quad \text{PAR-FSTCONG} \quad \frac{\vdash a \Rightarrow a_1}{\vdash \mathbf{fst} \ a \Rightarrow \mathbf{fst} \ a_1} \quad \text{PAR-SNDCONG} \quad \frac{\vdash a \Rightarrow a_2}{\vdash \mathbf{snd} \ a \Rightarrow \mathbf{snd} \ a_2}
\end{array}$$

$$\boxed{\vdash a \Rightarrow^* b} \quad (multistep \ parallel \ reduction)$$

$$\text{MP-REFL} \quad \frac{}{\vdash a \Rightarrow^* a} \quad \text{MP-STEP} \quad \frac{\vdash a \Rightarrow b \quad \vdash b \Rightarrow^* a'}{\vdash a \Rightarrow^* a'}$$

$$\boxed{\vdash a \Leftrightarrow b} \quad (parallel \ reduction \ to \ a \ common \ term)$$

$$\text{JOIN} \quad \frac{\vdash a_1 \Rightarrow^* b \quad \vdash a_2 \Rightarrow^* b}{\vdash a_1 \Leftrightarrow a_2}$$

$$\boxed{\vdash a > b} \quad (primitive \ reductions \ on \ erased \ terms)$$

$$\text{BETA-APPABS} \quad \frac{}{\vdash (\lambda^+ x.v) \ b^+ > v\{b/x\}} \quad \text{BETA-APPABSIrREL} \quad \frac{\text{[Value } (\lambda^- x.v)]}{\vdash (\lambda^- x.v) \ \Box^- > v\{\Box/x\}}$$

BETA-CAPPCABS

$$\frac{}{\models (\Lambda c.a')[\bullet] > a'\{\bullet/c\}}$$

BETA-AXIOM

$$\frac{F \sim a : A \in \Sigma_0}{\models F > a}$$

BETA-FST

$$\frac{}{\models \mathbf{fst}({}^\rho a, b) > a}$$

BETA-SND

$$\frac{}{\models \mathbf{snd}({}^\rho a, b) > b}$$

$$\boxed{\models a \rightsquigarrow b}$$

(single-step head reduction for implicit language)

E-ABSTERM

$$\frac{\models a \rightsquigarrow a'}{\models \lambda^- x.a \rightsquigarrow \lambda^- x.a'}$$

E-APPLEFT

$$\frac{\models a \rightsquigarrow a'}{\models a b^+ \rightsquigarrow a' b^+}$$

E-APPLEFTIRREL

$$\frac{\models a \rightsquigarrow a'}{\models a \square^- \rightsquigarrow a' \square^-}$$

E-CAPPLEFT

$$\frac{\models a \rightsquigarrow a'}{\models a[\bullet] \rightsquigarrow a'[\bullet]}$$

E-APPABS

$$\frac{}{\models (\lambda^+ x.v) a^+ \rightsquigarrow v\{a/x\}}$$

E-APPABSIRREL

$$\frac{[\mathbf{Value}(\lambda^- x.v)]}{\models (\lambda^- x.v) \square^- \rightsquigarrow v\{\square^-/x\}}$$

E-CAPPCABS

$$\frac{}{\models (\Lambda c.b)[\bullet] \rightsquigarrow b\{\bullet/c\}}$$

E-AXIOM

$$\frac{F \sim a : A \in \Sigma_0}{\models F \rightsquigarrow a}$$

E-FSTRED

$$\frac{}{\models \mathbf{fst}({}^\rho a, b) \rightsquigarrow a}$$

E-SNDRED

$$\frac{}{\models \mathbf{snd}({}^\rho a, b) \rightsquigarrow b}$$

E-FSTCONGRED

$$\frac{\models a \rightsquigarrow a'}{\models \mathbf{fst} a \rightsquigarrow \mathbf{fst} a'}$$

E-SNDCONGRED

$$\frac{\models a \rightsquigarrow a'}{\models \mathbf{snd} a \rightsquigarrow \mathbf{snd} a'}$$

$$\boxed{\models a \rightsquigarrow^* b}$$

(multistep reduction)

EQUAL

$$\frac{}{\models a \rightsquigarrow^* a}$$

STEP

$$\frac{\models a \rightsquigarrow b \quad \models b \rightsquigarrow^* a'}{\models a \rightsquigarrow^* a'}$$

$$\boxed{\Gamma \models^\rho \phi \text{ ok}}$$

(Prop wellformedness)

E-WFF

$$\frac{\begin{array}{l} \Gamma \models^\rho a : A \\ \Gamma \models^\rho b : A \\ [\Gamma \models^- A : \star] \end{array}}{\Gamma \models^\rho a \sim_A b \text{ ok}}$$

$\boxed{\Gamma \models^\rho a : A}$ (typing)

$\frac{\text{E-STAR} \quad \vdash \Gamma}{\Gamma \models^\rho \star : \star}$	$\frac{\text{E-VAR} \quad \vdash \Gamma \quad x : \rho_1 A \in \Gamma \quad \rho_2 \Leftarrow \rho_1}{\Gamma \models^{\rho_2} x : A}$	$\frac{\text{E-PI} \quad \Gamma, x : -A \models^{\rho_1} B : \star \quad \Gamma \models^{\rho_1} A : \star}{\Gamma \models^{\rho_1} \Pi^\rho x : A \rightarrow B : \star}$
$\frac{\text{E-ABS} \quad \Gamma, x : \rho A \models^{\rho_1} a : B \quad [\Gamma \vdash^- A : \star] \quad \Gamma \vdash^- \Pi^\rho x : A \rightarrow B : \star \quad (\rho = +) \vee (x \notin \text{fv } a)}{\Gamma \models^{\rho_1} \lambda^\rho x. a : \Pi^\rho x : A \rightarrow B}$	$\frac{\text{E-APP} \quad \Gamma \models^\rho b : \Pi^+ x : A \rightarrow B \quad \Gamma \models^\rho a : A}{\Gamma \models^\rho b \ a^+ : B\{a/x\}}$	
$\frac{\text{E-IAPP} \quad \Gamma \models^\rho b : \Pi^- x : A \rightarrow B \quad \Gamma \vdash^- a : A}{\Gamma \models^\rho b \ \Box^- : B\{a/x\}}$	$\frac{\text{E-CONV} \quad \Gamma \models^\rho a : A \quad \Gamma; \tilde{\Gamma} \vdash -A \equiv B : \star \quad [\Gamma \vdash^- B : \star]}{\Gamma \models^\rho a : B}$	$\frac{\text{E-CP1} \quad \Gamma, c : \phi \models^\rho B : \star \quad \Gamma \models^\rho \phi \text{ ok}}{\Gamma \models^\rho \forall c : \phi. B : \star}$
$\frac{\text{E-CABS} \quad \Gamma, c : \phi \models^\rho a : B \quad [\Gamma \vdash^- \phi \text{ ok}]}{\Gamma \models^\rho \Lambda c. a : \forall c : \phi. B}$	$\frac{\text{E-CAPP} \quad \Gamma \models^\rho a_1 : \forall c : (a \sim_A b). B_1 \quad \Gamma; \tilde{\Gamma} \vdash -a \equiv b : A}{\Gamma \models^\rho a_1[\bullet] : B_1\{\bullet/c\}}$	$\frac{\text{E-FAM} \quad \vdash \Gamma \quad F \sim a : A \in \Sigma_0 \quad [\emptyset \vdash^- A : \star]}{\Gamma \models^\rho F : A}$
$\frac{\text{E-SIGMA} \quad \Gamma, x : -A \models^{\rho_1} B : \star \quad \Gamma \models^{\rho_1} A : \star}{\Gamma \models^{\rho_1} \Sigma^\rho x : A. B : \star}$	$\frac{\text{E-PAIR} \quad \Gamma \models^+ a : A \quad \Gamma \models^\rho b : B\{a/x\} \quad \Gamma \vdash^- \Sigma^+ x : A. B : \star}{\Gamma \models^\rho (+a, b) : \Sigma^+ x : A. B}$	$\frac{\text{E-PAIRIRREL} \quad \Gamma \vdash^- a : A \quad \Gamma \models^\rho b : B\{a/x\} \quad \Gamma \vdash^- \Sigma^- x : A. B : \star}{\Gamma \models^\rho (-\Box, b) : \Sigma^- x : A. B}$
$\frac{\text{E-FST} \quad \Gamma \models^\rho a : \Sigma^+ x : A. B}{\Gamma \models^\rho \mathbf{fst} a : A}$	$\frac{\text{E-FSTIRREL} \quad \Gamma \vdash^- a : \Sigma^- x : A. B}{\Gamma \vdash^- \mathbf{fst} a : A}$	$\frac{\text{E-SND} \quad \Gamma \models^{\rho_1} a : \Sigma^\rho x : A. B}{\Gamma \models^{\rho_1} \mathbf{snd} a : B\{\mathbf{fst} a/x\}}$

$\boxed{\Gamma; \Delta \models \rho \phi_1 \equiv \phi_2}$ (prop equality)

$\frac{\text{E-PROP CONG} \quad \Gamma; \Delta \models \rho A_1 \equiv A_2 : A \quad \Gamma; \Delta \models \rho B_1 \equiv B_2 : A}{\Gamma; \Delta \models \rho A_1 \sim_A B_1 \equiv A_2 \sim_A B_2}$	$\frac{\text{E-ISOCONG} \quad \Gamma; \Delta \vdash -A \equiv B : \star \quad \Gamma \models^\rho A_1 \sim_A A_2 \text{ ok} \quad \Gamma \models^\rho A_1 \sim_B A_2 \text{ ok}}{\Gamma; \Delta \models \rho A_1 \sim_A A_2 \equiv A_1 \sim_B A_2}$
$\frac{\text{E-CP1FST} \quad \Gamma; \Delta \models \rho \forall c : \phi_1. B_1 \equiv \forall c : \phi_2. B_2 : \star}{\Gamma; \Delta \models \rho \phi_1 \equiv \phi_2}$	

$$\boxed{\Gamma; \Delta \models \rho a \equiv b : A}$$

(definitional equality)

E-ASSN

$$\frac{\begin{array}{c} \vdash \Gamma \\ c : (a \sim_A b) \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \models \rho a \equiv b : A}$$

E-REFL

$$\frac{\Gamma \models^\rho a : A}{\Gamma; \Delta \models \rho a \equiv a : A}$$

E-SYM

$$\frac{\Gamma; \Delta \models \rho b \equiv a : A}{\Gamma; \Delta \models \rho a \equiv b : A}$$

E-TRANS

$$\frac{\begin{array}{c} \Gamma; \Delta \models \rho a \equiv a_1 : A \\ \Gamma; \Delta \models \rho a_1 \equiv b : A \end{array}}{\Gamma; \Delta \models \rho a \equiv b : A}$$

E-BETA

$$\frac{\begin{array}{c} \Gamma \models^\rho a_1 : B \\ \Gamma \models^\rho a_2 : B \\ \vdash a_1 > a_2 \end{array}}{\Gamma; \Delta \models \rho a_1 \equiv a_2 : B}$$

E-PICONG

$$\frac{\begin{array}{c} \Gamma; \Delta \models \rho_1 A_1 \equiv A_2 : \star \\ \Gamma, x : -A_1; \Delta \models \rho_1 B_1 \equiv B_2 : \star \\ \Gamma \models^{\rho_1} A_1 : \star \\ [\Gamma \models^{\rho_1} \Pi^\rho x : A_1 \rightarrow B_1 : \star] \\ [\Gamma \models^{\rho_1} \Pi^\rho x : A_2 \rightarrow B_2 : \star] \end{array}}{\Gamma; \Delta \models \rho_1 (\Pi^\rho x : A_1 \rightarrow B_1) \equiv (\Pi^\rho x : A_2 \rightarrow B_2) : \star}$$

E-ABSCONG

$$\frac{\begin{array}{c} \Gamma, x : \rho A_1; \Delta \models \rho_1 b_1 \equiv b_2 : B \\ [\Gamma \models^- A_1 : \star] \\ (\rho = +) \vee (x \notin \text{fv } b_1) \\ (\rho = +) \vee (x \notin \text{fv } b_2) \end{array}}{\Gamma; \Delta \models \rho_1 (\lambda^\rho x. b_1) \equiv (\lambda^\rho x. b_2) : \Pi^\rho x : A_1 \rightarrow B}$$

E-APPCONG

$$\frac{\begin{array}{c} \Gamma; \Delta \models \rho a_1 \equiv b_1 : \Pi^+ x : A \rightarrow B \\ \Gamma; \Delta \models + a_2 \equiv b_2 : A \end{array}}{\Gamma; \Delta \models \rho a_1 a_2^+ \equiv b_1 b_2^+ : B\{a_2/x\}}$$

E-IAPPCONG

$$\frac{\begin{array}{c} \Gamma; \Delta \models \rho a_1 \equiv b_1 : \Pi^- x : A \rightarrow B \\ \Gamma \models^- a : A \end{array}}{\Gamma; \Delta \models \rho a_1 \square^- \equiv b_1 \square^- : B\{a/x\}}$$

E-PIFST

$$\frac{\Gamma; \Delta \models \rho_1 \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star}{\Gamma; \Delta \models \rho_1 A_1 \equiv A_2 : \star}$$

E-PISND

$$\frac{\begin{array}{c} \Gamma; \Delta \models \rho_1 \Pi^\rho x : A_1 \rightarrow B_1 \equiv \Pi^\rho x : A_2 \rightarrow B_2 : \star \\ \Gamma; \Delta \models \rho a_1 \equiv a_2 : A_1 \end{array}}{\Gamma; \Delta \models \rho_1 B_1\{a_1/x\} \equiv B_2\{a_2/x\} : \star}$$

$$\begin{array}{c}
\text{E-CPiCONG} \\
\frac{\Gamma; \Delta \models \rho \phi_1 \equiv \phi_2 \quad \Gamma, c : \phi_1; \Delta \models \rho A \equiv B : \star \quad \begin{array}{c} [\Gamma \models^\rho \phi_1 \text{ ok}] \\ [\Gamma \models^\rho \forall c : \phi_1. A : \star] \\ [\Gamma \models^\rho \forall c : \phi_2. B : \star] \end{array}}{\Gamma; \Delta \models \rho \forall c : \phi_1. A \equiv \forall c : \phi_2. B : \star}
\end{array}
\quad
\begin{array}{c}
\text{E-CABSCONG} \\
\frac{\Gamma, c : \phi_1; \Delta \models \rho a \equiv b : B \quad [\Gamma \models^- \phi_1 \text{ ok}]}{\Gamma; \Delta \models \rho (\Lambda c. a) \equiv (\Lambda c. b) : \forall c : \phi_1. B}
\end{array}$$

$$\begin{array}{c}
\text{E-CAPPCONG} \\
\frac{\Gamma; \Delta \models \rho a_1 \equiv b_1 : \forall c : (a \sim_A b). B \quad \Gamma; \tilde{\Gamma} \models - a \equiv b : A}{\Gamma; \Delta \models \rho a_1[\bullet] \equiv b_1[\bullet] : B\{\bullet/c\}}
\end{array}$$

$$\begin{array}{c}
\text{E-CPiSND} \\
\frac{\Gamma; \Delta \models \rho \forall c : (a_1 \sim_A a_2). B_1 \equiv \forall c : (a'_1 \sim_{A'} a'_2). B_2 : \star \quad \begin{array}{c} \Gamma; \tilde{\Gamma} \models - a_1 \equiv a_2 : A \\ \Gamma; \tilde{\Gamma} \models - a'_1 \equiv a'_2 : A' \end{array}}{\Gamma; \Delta \models \rho B_1\{\bullet/c\} \equiv B_2\{\bullet/c\} : \star}
\end{array}$$

$$\begin{array}{c}
\text{E-CAST} \\
\frac{\Gamma; \Delta \models \rho a \equiv b : A \quad \Gamma; \Delta \models \rho a \sim_A b \equiv a' \sim_{A'} b'}{\Gamma; \Delta \models \rho a' \equiv b' : A'}
\end{array}
\quad
\begin{array}{c}
\text{E-EQCONV} \\
\frac{\Gamma; \Delta \models \rho a \equiv b : A \quad \Gamma; \tilde{\Gamma} \models - A \equiv B : \star}{\Gamma; \Delta \models \rho a \equiv b : B}
\end{array}$$

$$\begin{array}{c}
\text{E-ISOSND} \\
\frac{\Gamma; \Delta \models - a \sim_A b \equiv a' \sim_{A'} b'}{\Gamma; \Delta \models - A \equiv A' : \star}
\end{array}
\quad
\begin{array}{c}
\text{E-ETAREL} \\
\frac{\Gamma \models^\rho b : \Pi^+ x : A \rightarrow B \quad a = b x^+}{\Gamma; \Delta \models \rho \lambda^+ x. a \equiv b : \Pi^+ x : A \rightarrow B}
\end{array}$$

$$\begin{array}{c}
\text{E-ETAIrREL} \\
\frac{\Gamma \models^\rho b : \Pi^- x : A \rightarrow B \quad a = b \Box^-}{\Gamma; \Delta \models \rho \lambda^- x. a \equiv b : \Pi^- x : A \rightarrow B}
\end{array}
\quad
\begin{array}{c}
\text{E-ETAC} \\
\frac{\Gamma \models^\rho b : \forall c : \phi. B \quad a = b[\bullet]}{\Gamma; \Delta \models \rho \Lambda c. a \equiv b : \forall c : \phi. B}
\end{array}$$

$$\begin{array}{c}
\text{E-SIGMACONG} \\
\frac{\Gamma; \Delta \models \rho_1 A_1 \equiv A_2 : \star \quad \Gamma, x : - A_1; \Delta \models \rho_1 B_1 \equiv B_2 : \star \quad \begin{array}{c} [\Gamma \models^{\rho_1} A_1 : \star] \\ [\Gamma \models^{\rho_1} \Sigma^\rho x : A_1. B_1 : \star] \\ [\Gamma \models^{\rho_1} \Sigma^\rho x : A_2. B_2 : \star] \end{array}}{\Gamma; \Delta \models \rho_1 (\Sigma^\rho x : A_1. B_1) \equiv (\Sigma^\rho x : A_2. B_2) : \star}
\end{array}$$

$$\begin{array}{c}
\text{E-PAIRCONG} \\
\frac{\Gamma; \Delta \vdash + a_1 \equiv b_1 : A \quad \Gamma; \Delta \vdash \rho a_2 \equiv b_2 : B\{a_1/x\} \quad \Gamma \vdash^- \Sigma^+ x : A.B : \star}{\Gamma; \Delta \vdash \rho(^+ a_1, a_2) \equiv (^+ b_1, b_2) : \Sigma^+ x : A.B}
\end{array}$$

$$\begin{array}{c}
\text{E-PAIRCONGIRREL} \\
\frac{\Gamma \vdash^- a : A \quad \Gamma; \Delta \vdash \rho a_2 \equiv b_2 : B\{a/x\} \quad \Gamma \vdash^- \Sigma^- x : A.B : \star}{\Gamma; \Delta \vdash \rho(-\square, a_2) \equiv (-\square, b_2) : \Sigma^- x : A.B}
\end{array}
\quad
\begin{array}{c}
\text{E-FSTCONG} \\
\frac{\Gamma; \Delta \vdash \rho a_1 \equiv b_1 : \Sigma^+ x : A.B}{\Gamma; \Delta \vdash \rho \mathbf{fst} a_1 \equiv \mathbf{fst} b_1 : A}
\end{array}$$

$$\begin{array}{c}
\text{E-FSTCONGIRREL} \\
\frac{\Gamma; \Delta \vdash - a_1 \equiv b_1 : \Sigma^- x : A.B}{\Gamma; \Delta \vdash - \mathbf{fst} a_1 \equiv \mathbf{fst} b_1 : A}
\end{array}
\quad
\begin{array}{c}
\text{E-SNDCONG} \\
\frac{\Gamma; \Delta \vdash \rho a_1 \equiv b_1 : \Sigma_1^\rho x : A.B}{\Gamma; \Delta \vdash \rho \mathbf{snd} a_1 \equiv \mathbf{snd} b_1 : B\{\mathbf{fst} a_1/x\}}
\end{array}$$

$$\boxed{\vdash \Gamma} \quad (\text{context wellformedness})$$

$$\begin{array}{c}
\text{E-EMPTY} \\
\frac{}{\vdash \emptyset}
\end{array}
\quad
\begin{array}{c}
\text{E-CONSTM} \\
\frac{\vdash \Gamma \quad \Gamma \vdash^- A : \star \quad x \notin \mathbf{dom} \Gamma}{\vdash \Gamma, x : \rho A}
\end{array}
\quad
\begin{array}{c}
\text{E-CONSCo} \\
\frac{\vdash \Gamma \quad \Gamma \vdash^- \phi \text{ ok} \quad c \notin \mathbf{dom} \Gamma}{\vdash \Gamma, c : \phi}
\end{array}$$

$$\boxed{\vdash \Sigma} \quad (\text{signature wellformedness})$$

$$\begin{array}{c}
\text{SIG-EMPTY} \\
\frac{}{\vdash \emptyset}
\end{array}
\quad
\begin{array}{c}
\text{SIG-CONSAx} \\
\frac{\vdash \Sigma \quad \emptyset \vdash^- A : \star \quad \emptyset \vdash^+ a : A \quad F \notin \mathbf{dom} \Sigma}{\vdash \Sigma \cup \{F \sim a : A\}}
\end{array}$$

$$\boxed{\Gamma \vdash \phi \text{ ok}} \quad (\text{prop wellformedness})$$

$$\begin{array}{c}
\text{AN-WFF} \\
\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B \quad |A| = |B|}{\Gamma \vdash a \sim_A b \text{ ok}}
\end{array}$$

$$\boxed{\Gamma \vdash a : A} \quad (\text{typing})$$

$$\begin{array}{c}
\text{AN-STAR} \\
\frac{\vdash \Gamma}{\Gamma \vdash \star : \star}
\end{array}
\quad
\begin{array}{c}
\text{AN-VAR} \\
\frac{\vdash \Gamma \quad x : \rho A \in \Gamma}{\Gamma \vdash x : A}
\end{array}
\quad
\begin{array}{c}
\text{AN-PI} \\
\frac{\Gamma, x : \rho A \vdash B : \star \quad [\Gamma \vdash A : \star]}{\Gamma \vdash \Pi^\rho x : A \rightarrow B : \star}
\end{array}$$

$$\begin{array}{c}
\text{AN-ABS} \\
\frac{\begin{array}{c} [\Gamma \vdash A : \star] \\ \Gamma, x : \rho A \vdash a : B \\ (\rho = +) \vee (x \notin \text{fv } |a|) \end{array}}{\Gamma \vdash \lambda^\rho x : A. a : \Pi^\rho x : A \rightarrow B} \\
\\
\text{AN-APP} \\
\frac{\begin{array}{c} \Gamma \vdash b : \Pi^\rho x : A \rightarrow B \\ \Gamma \vdash a : A \end{array}}{\Gamma \vdash b \ a^\rho : B\{a/x\}} \\
\\
\text{AN-CONV} \\
\frac{\begin{array}{c} \Gamma \vdash a : A \\ \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim B \\ \Gamma \vdash B : \star \end{array}}{\Gamma \vdash a \triangleright \gamma : B} \\
\\
\text{AN-CPI} \\
\frac{\begin{array}{c} [\Gamma \vdash \phi \text{ ok}] \\ \Gamma, c : \phi \vdash B : \star \end{array}}{\Gamma \vdash \forall c : \phi. B : \star} \\
\\
\text{AN-CABS} \\
\frac{\begin{array}{c} [\Gamma \vdash \phi \text{ ok}] \\ \Gamma, c : \phi \vdash a : B \end{array}}{\Gamma \vdash \Lambda c : \phi. a : \forall c : \phi. B} \\
\\
\text{AN-CAPP} \\
\frac{\begin{array}{c} \Gamma \vdash a_1 : \forall c : a \sim_{A_1} b. B \\ \Gamma; \tilde{\Gamma} \vdash \gamma : a \sim b \end{array}}{\Gamma \vdash a_1[\gamma] : B\{\gamma/c\}} \\
\\
\text{AN-FAM} \\
\frac{\begin{array}{c} \vdash \Gamma \\ F \sim a : A \in \Sigma_1 \\ [\emptyset \vdash A : \star] \end{array}}{\Gamma \vdash F : A} \\
\\
\text{AN-SIGMA} \\
\frac{\begin{array}{c} \Gamma, x : \rho A \vdash B : \star \\ [\Gamma \vdash A : \star] \end{array}}{\Gamma \vdash \Sigma^\rho x : A. B : \star} \\
\\
\text{AN-PAIR} \\
\frac{\begin{array}{c} \Gamma \vdash a : A \\ \Gamma \vdash b : B\{a/x\} \end{array}}{\Gamma \vdash (\rho a, b) \text{ as } \Sigma^\rho x : A. B : \Sigma^\rho x : A. B} \\
\\
\text{AN-FST} \\
\frac{\Gamma \vdash a : \Sigma^\rho x : A. B}{\Gamma \vdash \mathbf{fst} \ a : A} \\
\\
\text{AN-SND} \\
\frac{\Gamma \vdash a : \Sigma^\rho x : A. B}{\Gamma \vdash \mathbf{snd} \ a : B\{\mathbf{fst} \ a/x\}}
\end{array}$$

$$\boxed{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2} \quad (\text{coercion between props})$$

$$\begin{array}{c}
\text{AN-PROP CONG} \\
\frac{\begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma; \Delta \vdash \gamma_2 : B_1 \sim B_2 \\ \Gamma \vdash A_1 \sim_A B_1 \text{ ok} \\ \Gamma \vdash A_2 \sim_A B_2 \text{ ok} \end{array}}{\Gamma; \Delta \vdash (\gamma_1 \sim_A \gamma_2) : (A_1 \sim_A B_1) \sim (A_2 \sim_A B_2)} \\
\\
\text{AN-CPIFST} \\
\frac{\Gamma; \Delta \vdash \gamma : \forall c : \phi_1. A_2 \sim \forall c : \phi_2. B_2}{\Gamma; \Delta \vdash \mathbf{cpiFst} \ \gamma : \phi_1 \sim \phi_2} \\
\\
\text{AN-ISOSYM} \\
\frac{\Gamma; \Delta \vdash \gamma : \phi_1 \sim \phi_2}{\Gamma; \Delta \vdash \mathbf{sym} \ \gamma : \phi_2 \sim \phi_1} \\
\\
\text{AN-ISOCONV} \\
\frac{\begin{array}{c} \Gamma; \Delta \vdash \gamma : A \sim B \\ \Gamma \vdash a_1 \sim_A a_2 \text{ ok} \\ \Gamma \vdash a'_1 \sim_B a'_2 \text{ ok} \\ |a_1| = |a'_1| \\ |a_2| = |a'_2| \end{array}}{\Gamma; \Delta \vdash \mathbf{conv} \ (a_1 \sim_A a_2) \sim_\gamma (a'_1 \sim_B a'_2) : (a_1 \sim_A a_2) \sim (a'_1 \sim_B a'_2)}
\end{array}$$

$$\boxed{\Gamma; \Delta \vdash \gamma : A \sim B}$$

(coercion between types)

$$\frac{\text{AN-ASSN} \quad \begin{array}{c} \vdash \Gamma \\ c : a \sim_A b \in \Gamma \\ c \in \Delta \end{array}}{\Gamma; \Delta \vdash c : a \sim b}$$

$$\frac{\text{AN-REFL} \quad \Gamma \vdash a : A}{\Gamma; \Delta \vdash \mathbf{refl} \, a : a \sim a}$$

$$\frac{\text{AN-ERASEEQ} \quad \begin{array}{c} \Gamma \vdash a : A \\ \Gamma \vdash b : B \quad |a| = |b| \\ \Gamma; \tilde{\Gamma} \vdash \gamma : A \sim B \end{array}}{\Gamma; \Delta \vdash (a \mid_{\gamma} b) : a \sim b}$$

$$\frac{\text{AN-SYM} \quad \begin{array}{c} \Gamma \vdash b : B \\ \Gamma \vdash a : A \\ [\Gamma; \tilde{\Gamma} \vdash \gamma_1 : B \sim A] \\ \Gamma; \Delta \vdash \gamma : b \sim a \end{array}}{\Gamma; \Delta \vdash \mathbf{sym} \, \gamma : a \sim b}$$

$$\frac{\text{AN-TRANS} \quad \begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : a \sim a_1 \\ \Gamma; \Delta \vdash \gamma_2 : a_1 \sim b \\ [\Gamma \vdash a : A] \\ [\Gamma \vdash a_1 : A_1] \\ [\Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim A_1] \end{array}}{\Gamma; \Delta \vdash (\gamma_1; \gamma_2) : a \sim b}$$

$$\frac{\text{AN-BETA} \quad \begin{array}{c} \Gamma \vdash a_1 : B_0 \\ \Gamma \vdash a_2 : B_1 \\ |B_0| = |B_1| \\ \models |a_1| > |a_2| \end{array}}{\Gamma; \Delta \vdash \mathbf{red} \, a_1 \, a_2 : a_1 \sim a_2}$$

$$\frac{\text{AN-PICONG} \quad \begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma, x : \rho \, A_1; \Delta \vdash \gamma_2 : B_1 \sim B_2 \\ B_3 = B_2 \{x \triangleright \mathbf{sym} \, \gamma_1 / x\} \\ \Gamma \vdash \Pi^\rho x : A_1 \rightarrow B_1 : \star \\ \Gamma \vdash \Pi^\rho x : A_2 \rightarrow B_3 : \star \\ \Gamma \vdash (\Pi^\rho x : A_1 \rightarrow B_2) : \star \end{array}}{\Gamma; \Delta \vdash \Pi^\rho x : \gamma_1. \gamma_2 : (\Pi^\rho x : A_1 \rightarrow B_1) \sim (\Pi^\rho x : A_2 \rightarrow B_3)}$$

$$\frac{\text{AN-ABSCONG} \quad \begin{array}{c} \Gamma; \Delta \vdash \gamma_1 : A_1 \sim A_2 \\ \Gamma, x : \rho \, A_1; \Delta \vdash \gamma_2 : b_1 \sim b_2 \\ b_3 = b_2 \{x \triangleright \mathbf{sym} \, \gamma_1 / x\} \\ [\Gamma \vdash A_1 : \star] \\ \Gamma \vdash A_2 : \star \\ (\rho = +) \vee (x \notin \mathbf{fv} \, |b_1|) \\ (\rho = +) \vee (x \notin \mathbf{fv} \, |b_3|) \\ [\Gamma \vdash (\lambda^\rho x : A_1. b_2) : B] \end{array}}{\Gamma; \Delta \vdash (\lambda^\rho x : \gamma_1. \gamma_2) : (\lambda^\rho x : A_1. b_1) \sim (\lambda^\rho x : A_2. b_3)}$$

$$\begin{array}{c}
\text{AN-APPCONG} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : a_1 \sim b_1 \quad \Gamma; \Delta \vdash \gamma_2 : a_2 \sim b_2 \quad \Gamma \vdash a_1 \ a_2^\rho : A \quad \Gamma \vdash b_1 \ b_2^\rho : B \quad [\Gamma; \tilde{\Gamma} \vdash \gamma_3 : A \sim B]}{\Gamma; \Delta \vdash \gamma_1 \ \gamma_2^\rho : a_1 \ a_2^\rho \sim b_1 \ b_2^\rho}
\end{array}
\quad
\begin{array}{c}
\text{AN-PIFST} \\
\frac{\Gamma; \Delta \vdash \gamma : \Pi^\rho x : A_1 \rightarrow B_1 \sim \Pi^\rho x : A_2 \rightarrow B_2}{\Gamma; \Delta \vdash \mathbf{piFst} \ \gamma : A_1 \sim A_2}
\end{array}$$

$$\begin{array}{c}
\text{AN-PIsND} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : \Pi^\rho x : A_1 \rightarrow B_1 \sim \Pi^\rho x : A_2 \rightarrow B_2 \quad \Gamma; \Delta \vdash \gamma_2 : a_1 \sim a_2 \quad \Gamma \vdash a_1 : A_1 \quad \Gamma \vdash a_2 : A_2}{\Gamma; \Delta \vdash \gamma_1 @ \gamma_2 : B_1 \{a_1/x\} \sim B_2 \{a_2/x\}}
\end{array}$$

$$\begin{array}{c}
\text{AN-CPiCONG} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : \phi_1 \sim \phi_2 \quad \Gamma, c : \phi_1; \Delta \vdash \gamma_3 : B_1 \sim B_2 \quad B_3 = B_2 \{c \triangleright \mathbf{sym} \ \gamma_1 / c\} \quad \Gamma \vdash \forall c : \phi_1. B_1 : \star \quad [\Gamma \vdash \forall c : \phi_2. B_3 : \star] \quad \Gamma \vdash \forall c : \phi_1. B_2 : \star}{\Gamma; \Delta \vdash (\forall c : \gamma_1. \gamma_3) : (\forall c : \phi_1. B_1) \sim (\forall c : \phi_2. B_3)}
\end{array}$$

$$\begin{array}{c}
\text{AN-CABSCONG} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : \phi_1 \sim \phi_2 \quad \Gamma, c : \phi_1; \Delta \vdash \gamma_3 : a_1 \sim a_2 \quad a_3 = a_2 \{c \triangleright \mathbf{sym} \ \gamma_1 / c\} \quad \Gamma \vdash (\Lambda c : \phi_1. a_1) : \forall c : \phi_1. B_1 \quad \Gamma \vdash (\Lambda c : \phi_2. a_3) : \forall c : \phi_2. B_2 \quad \Gamma \vdash (\Lambda c : \phi_1. a_2) : B \quad \Gamma; \tilde{\Gamma} \vdash \gamma_4 : \forall c : \phi_1. B_1 \sim \forall c : \phi_2. B_2}{\Gamma; \Delta \vdash (\lambda c : \gamma_1. \gamma_3 @ \gamma_4) : (\Lambda c : \phi_1. a_1) \sim (\Lambda c : \phi_2. a_3)}
\end{array}$$

$$\begin{array}{c}
\text{AN-CAPPCONG} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : a_1 \sim b_1 \quad \Gamma; \tilde{\Gamma} \vdash \gamma_2 : a_2 \sim b_2 \quad \Gamma; \tilde{\Gamma} \vdash \gamma_3 : a_3 \sim b_3 \quad \Gamma \vdash a_1[\gamma_2] : A \quad \Gamma \vdash b_1[\gamma_3] : B \quad [\Gamma; \tilde{\Gamma} \vdash \gamma_4 : A \sim B]}{\Gamma; \Delta \vdash \gamma_1(\gamma_2, \gamma_3) : a_1[\gamma_2] \sim b_1[\gamma_3]}
\end{array}$$

$$\begin{array}{c}
\text{AN-CPIsND} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : (\forall c_1 : a \sim_A a'. B_1) \sim (\forall c_2 : b \sim_B b'. B_2) \quad \Gamma; \tilde{\Gamma} \vdash \gamma_2 : a \sim a' \quad \Gamma; \tilde{\Gamma} \vdash \gamma_3 : b \sim b'}{\Gamma; \Delta \vdash \gamma_1 @ (\gamma_2 \sim \gamma_3) : B_1\{\gamma_2/c_1\} \sim B_2\{\gamma_3/c_2\}}
\end{array}$$

$$\begin{array}{c}
\text{AN-CAST} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : a \sim a' \quad \Gamma; \Delta \vdash \gamma_2 : a \sim_A a' \sim b \sim_B b'}{\Gamma; \Delta \vdash \gamma_1 \triangleright \gamma_2 : b \sim b'}
\end{array}
\quad
\begin{array}{c}
\text{AN-ISOSND} \\
\frac{\Gamma; \Delta \vdash \gamma : (a \sim_A a') \sim (b \sim_B b')}{\Gamma; \Delta \vdash \mathbf{isoSnd} \gamma : A \sim B}
\end{array}$$

$$\begin{array}{c}
\text{AN-ETA} \\
\frac{\Gamma \vdash b : \Pi^\rho x : A \rightarrow B \quad a = b \ x^\rho}{\Gamma; \Delta \vdash \mathbf{eta} b : (\lambda^\rho x : A. a) \sim b}
\end{array}
\quad
\begin{array}{c}
\text{AN-ETAC} \\
\frac{\Gamma \vdash b : \forall c : \phi. B \quad a = b[c]}{\Gamma; \Delta \vdash \mathbf{eta} b : (\Lambda c : \phi. a) \sim b}
\end{array}$$

$$\begin{array}{c}
\text{AN-SIGMACONG} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : A_1 \sim A_2 \quad \Gamma, x : \rho A_1; \Delta \vdash \gamma_2 : B_1 \sim B_2 \quad B_3 = B_2\{x \triangleright \mathbf{sym} \gamma_1/x\} \quad \Gamma \vdash \Sigma^\rho x : A_1. B_1 : \star \quad \Gamma \vdash \Sigma^\rho x : A_2. B_3 : \star \quad \Gamma \vdash (\Sigma^\rho x : A_1. B_2) : \star}{\Gamma; \Delta \vdash \Sigma x : ^\rho \gamma_1. \gamma_2 : (\Sigma^\rho x : A_1. B_1) \sim (\Sigma^\rho x : A_2. B_3)}
\end{array}$$

$$\begin{array}{c}
\text{AN-PAIRCONG} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : a_1 \sim b_1 \quad \Gamma; \Delta \vdash \gamma_2 : a_2 \sim b_2 \quad \Gamma; \Delta \vdash \gamma_3 : A_1 \sim A_2}{\Gamma; \Delta \vdash (\rho \gamma_1, \gamma_2) \mathbf{as} \gamma_3 : (^\rho a_1, a_2) \mathbf{as} A_1 \sim (^\rho b_1, b_2) \mathbf{as} A_2}
\end{array}$$

$$\begin{array}{c}
\text{AN-FSTCONG} \\
\frac{\Gamma; \Delta \vdash \gamma : a_1 \sim b_1}{\Gamma; \Delta \vdash \mathbf{fst} \gamma : \mathbf{fst} a_1 \sim \mathbf{fst} b_1}
\end{array}
\quad
\begin{array}{c}
\text{AN-SNDCONG} \\
\frac{\Gamma; \Delta \vdash \gamma : a_1 \sim b_1}{\Gamma; \Delta \vdash \mathbf{snd} \gamma : \mathbf{snd} a_1 \sim \mathbf{snd} b_1}
\end{array}$$

$$\begin{array}{c}
\text{AN-SIGMAFST} \\
\frac{\Gamma; \Delta \vdash \gamma : \Sigma^\rho x : A_1. B_1 \sim \Sigma^\rho x : A_2. B_2}{\Gamma; \Delta \vdash \mathbf{sigmaFst} \gamma : A_1 \sim A_2}
\end{array}$$

$$\begin{array}{c}
\text{AN-SIGMASND} \\
\frac{\Gamma; \Delta \vdash \gamma_1 : \Sigma^\rho x : A_1. B_1 \sim \Sigma^\rho x : A_2. B_2 \quad \Gamma; \Delta \vdash \gamma_2 : a_1 \sim a_2 \quad \Gamma \vdash a_1 : A_1 \quad \Gamma \vdash a_2 : A_2}{\Gamma; \Delta \vdash \gamma_1 @ \gamma_2 : B_1\{a_1/x\} \sim B_2\{a_2/x\}}
\end{array}$$

$\boxed{\vdash \Gamma}$ (context wellformedness)

$$\begin{array}{c}
\text{AN-EMPTY} \\
\hline
\vdash \emptyset
\end{array}
\quad
\begin{array}{c}
\text{AN-CONSTM} \\
\vdash \Gamma \quad \Gamma \vdash A : \star \\
x \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, x : \rho A
\end{array}
\quad
\begin{array}{c}
\text{AN-CONSCo} \\
\vdash \Gamma \quad \Gamma \vdash \phi \text{ ok} \\
c \notin \text{dom } \Gamma \\
\hline
\vdash \Gamma, c : \phi
\end{array}$$

$\boxed{\vdash \Sigma}$ (signature wellformedness)

$$\begin{array}{c}
\text{AN-SIG-EMPTY} \\
\hline
\vdash \emptyset
\end{array}
\quad
\begin{array}{c}
\text{AN-SIG-CONSAX} \\
\vdash \Sigma \quad \emptyset \vdash A : \star \\
\emptyset \vdash a : A \\
F \notin \text{dom } \Sigma \\
\hline
\vdash \Sigma \cup \{F \sim a : A\}
\end{array}$$

$\boxed{\Gamma \vdash a \rightsquigarrow b}$ (single-step, weak head reduction to values for annotated language)

$$\begin{array}{c}
\text{AN-APPLEFT} \\
\Gamma \vdash a \rightsquigarrow a' \\
\hline
\Gamma \vdash a b^\rho \rightsquigarrow a' b^\rho
\end{array}
\quad
\begin{array}{c}
\text{AN-APPABS} \\
[\text{Value } (\lambda^\rho x : A. w)] \\
\hline
\Gamma \vdash (\lambda^\rho x : A. w) a^\rho \rightsquigarrow w\{a/x\}
\end{array}
\quad
\begin{array}{c}
\text{AN-CAPPLEFT} \\
\Gamma \vdash a \rightsquigarrow a' \\
\hline
\Gamma \vdash a[\gamma] \rightsquigarrow a'[\gamma]
\end{array}$$

$$\begin{array}{c}
\text{AN-CAPPCABS} \\
\hline
\Gamma \vdash (\Lambda c : \phi. b)[\gamma] \rightsquigarrow b\{\gamma/c\}
\end{array}
\quad
\begin{array}{c}
\text{AN-ABSTERM} \\
\Gamma \vdash A : \star \\
\Gamma, x : - A \vdash b \rightsquigarrow b' \\
\hline
\Gamma \vdash (\lambda^- x : A. b) \rightsquigarrow (\lambda^- x : A. b')
\end{array}$$

$$\begin{array}{c}
\text{AN-AXIOM} \\
F \sim a : A \in \Sigma_1 \\
\hline
\Gamma \vdash F \rightsquigarrow a
\end{array}
\quad
\begin{array}{c}
\text{AN-FSTRED} \\
\hline
\Gamma \vdash \mathbf{fst}(\rho a, b) \rightsquigarrow a
\end{array}
\quad
\begin{array}{c}
\text{AN-SNDRRED} \\
\hline
\Gamma \vdash \mathbf{snd}(\rho a, b) \rightsquigarrow b
\end{array}$$

$$\begin{array}{c}
\text{AN-CONVTERM} \\
\Gamma \vdash a \rightsquigarrow a' \\
\hline
\Gamma \vdash a \triangleright \gamma \rightsquigarrow a' \triangleright \gamma
\end{array}
\quad
\begin{array}{c}
\text{AN-COMBINE} \\
[\text{Value } v] \\
\hline
\Gamma \vdash (v \triangleright \gamma_1) \triangleright \gamma_2 \rightsquigarrow v \triangleright (\gamma_1; \gamma_2)
\end{array}$$

$$\begin{array}{c}
\text{AN-PUSH} \\
[\text{Value } v] \\
\Gamma; \tilde{\Gamma} \vdash \gamma : \Pi^\rho x_1 : A_1 \rightarrow B_1 \sim \Pi^\rho x_2 : A_2 \rightarrow B_2 \\
b' = b \triangleright \mathbf{sym}(\mathbf{piFst} \gamma) \\
\gamma' = \gamma @ (b' \models_{(\mathbf{piFst} \gamma)} b) \\
\hline
\Gamma \vdash (v \triangleright \gamma) b^\rho \rightsquigarrow (v b'^\rho) \triangleright \gamma'
\end{array}$$

$$\begin{array}{c}
\text{AN-CPUSH} \\
[\text{Value } v] \\
\Gamma; \tilde{\Gamma} \vdash \gamma : \forall c_1 : \phi_1. A_1 \sim \forall c_2 : \phi_2. A_2 \\
\gamma'_1 = \gamma_1 \triangleright \mathbf{sym}(\mathbf{cpiFst} \gamma) \\
\gamma' = \gamma @ (\gamma'_1 \sim \gamma_1) \\
\hline
\Gamma \vdash (v \triangleright \gamma)[\gamma_1] \rightsquigarrow (v[\gamma'_1]) \triangleright \gamma'
\end{array}
\quad
\begin{array}{c}
\text{AN-FSTPUSH} \\
[\text{Value } v] \\
\Gamma; \tilde{\Gamma} \vdash \gamma : \Sigma_1^\rho x_1 : A_1. B_1 \sim \Sigma_2^\rho x_2 : A_2. B_2 \\
\hline
\Gamma \vdash \mathbf{fst}(v \triangleright \gamma) \rightsquigarrow (\mathbf{fst} v) \triangleright \mathbf{sigmaFst} \gamma
\end{array}$$

$$\begin{array}{c}
\text{AN-SNDPUSH} \\
\text{[Value } v\text{]} \\
\Gamma; \tilde{\Gamma} \vdash \gamma : \Sigma_1^\rho x_1 : A_1.B_1 \sim \Sigma_2^\rho x_2 : A_2.B_2 \\
b' = b \triangleright \mathbf{sym}(\mathbf{sigmaFst} \gamma) \\
\gamma' = \gamma @ (b' \mid_{(\mathbf{sigmaFst} \gamma)} b) \\
\hline
\Gamma \vdash \mathbf{snd}(v \triangleright \gamma) \rightsquigarrow (\mathbf{snd} v) \triangleright \gamma'
\end{array}$$