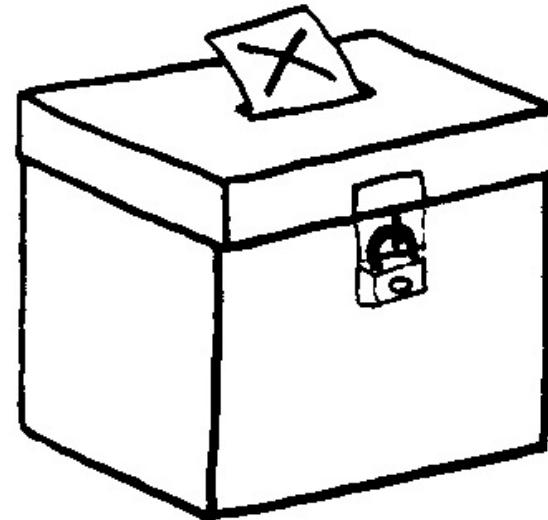




Center for Research on  
Computation and Society



# Verifying Elections with Cryptography

**Ben Adida**  
Harvard

Google – December 19th, 2007





# **Does e-voting need paper trails?**

By [Anne Broache](#)

Staff Writer, CNET News.com

Published: October 31, 2006, 4:00 AM PST

# Does e-voting need paper trails?

By Anne Broache

Staff Writer,

Published: Oct.

## State sued over lack of paper trail for ballots

By AMAN BATHEJA  
STAR-TELEGRAM STAFF WRITER

# Does e-voting need paper trails?

By Anne Broache

Staff Writer,

Published: Oct

**State sued over lack of paper trail for  
ballots**

## HBO documentary irks voting technology firm

Wed Nov 1, 2006 6:37am ET

# Does e-voting need paper trails?

By Anne Broache

Staff Writer,

Published: Oct

**State sued over lack of paper trail for  
ballots**

**HBO documentary irks voting technology firm**

Wed Nov 1, 2006 6:37am ET

© Nov 1, 2006 10:54 pm US/Pacific

**California E-Voting Machine  
Allows Multiple Votes**



**Allen Martin**  
Reporting

# Does e-voting need paper trails?

By Anne Broache

Staff Writer,

Published: Oct

**State sued over lack of paper trail for  
ballots**

**HBO documentary irks voting technology firm**

Wed Nov 1, 2006 6:37am ET

© Nov 1, 2006 10:54 pm US/Pacific

**California E-Voting Machine  
Allows Multiple Votes**



**Allen Martin**

Reporters

OCTOBER 31, 2006

**Hugo Chavez in the Voting Machine**

# Does e-voting need paper trails?

By Anne Broache

Staff Writer,

Published: Oct

## State sued over lack of paper trail for ballots

# HBO documentary irks voting technology firm

Wed Nov 1, 2006 6:37am ET

© Nov 1, 2006 10:54 pm US/Pacific

## California E-Voting Machine Allows Multiple Votes



Allen Martin

Associated Press

OCTOBER 31, 2006

## Hugo Chavez in the Voting Machine

Originally published October 26, 2006

## Your vote will count

## Hype over hacking shouldn't shatter confidence

By Paul DeGregorio  
McCLATCHY-TRIBUNE



Rogers precinct, with more than 100 percent voter turnout, alarmed both of them.

Rogers precinct, with more than 100 percent voter turnout, alarmed both of them.

## **Thief grabs voting machine from election official's car**

By ROGER H. AYLWORTH - Staff Writer

Article Launched:11/07/2006 12:00:00 AM PST

Rogers precinct, with more than 100 percent voter turnout, alarmed both of them.

## **Thief grabs voting machine from election official's car**

By ROGER H. AYLWORTH - Staff Writer

Article Launched:11/07/2006 12:00:00 AM PST

Last Updated: November 7, 2006 - 2:19 PM EST

## **Voter smashes touch-screen machine in Allentown**

# Rogers precinct with more than 100 percent vote

The  
off  
By R  
Artic



Last Updated: November 7, 2006 - 2:19 PM EST

## Voter smashes touch-screen machine in Allentown

## **State disallows some voting machines**

La Plata County technology OK; a few large counties must change

December 18, 2007

By Joe Hanel | Herald Denver Bureau

## **Ohio e-voting system security bashed in new state report**

Problems threaten the integrity of future elections, officials say

## **Report: Magnet and PDA Sufficient to Change Votes on Voting Machine**

By Kim Zetter 

December 17, 2007 | 11:36:19 PM

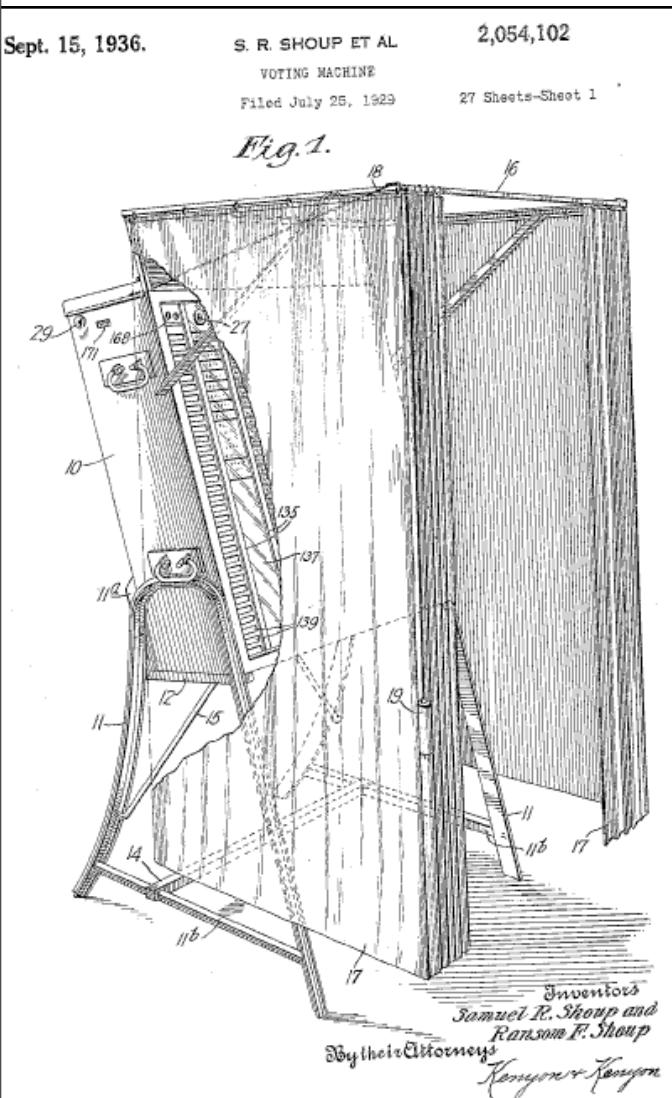
Categories: E-Voting, Election '08, Hacks And Cracks

Wooten got the news from his wife, Roxanne, who went to City Hall on Wednesday to see the election results.

"She saw my name with zero votes by it. She came home and asked me if I had voted for myself or not. I told her I did," said Wooten, owner of a local bar.

# How We Got Here

# How We Got Here



# How We Got Here

Sept. 15, 1936.

S. R. SHOUP ET AL

2,054,102

VOTING MACHINE

Filed July 25, 1923

27 Sheets-Sheet 1

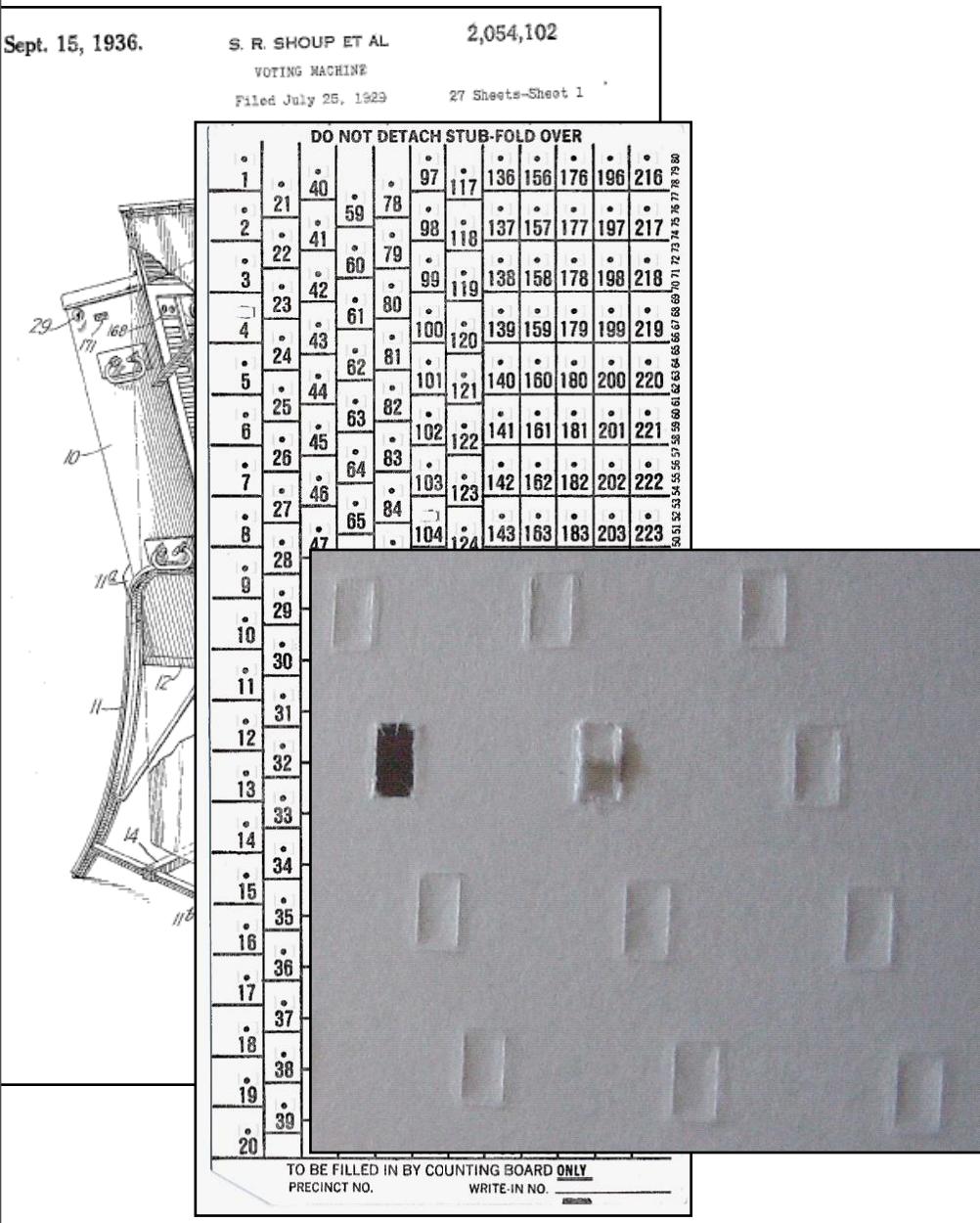
**DO NOT DETACH STUB-FOLD OVER**

|    |    |    |    |    |     |     |     |     |     |     |     |
|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|
| 1  | 21 | 40 | 59 | 78 | 97  | 117 | 136 | 156 | 176 | 196 | 216 |
| 2  | 22 | 41 | 60 | 79 | 98  | 118 | 137 | 157 | 177 | 197 | 217 |
| 3  | 23 | 42 | 61 | 80 | 99  | 119 | 138 | 158 | 178 | 198 | 218 |
| 4  | 24 | 43 | 62 | 81 | 100 | 120 | 139 | 159 | 179 | 199 | 219 |
| 5  | 25 | 44 | 63 | 82 | 101 | 121 | 140 | 160 | 180 | 200 | 220 |
| 6  | 26 | 45 | 64 | 83 | 102 | 122 | 141 | 161 | 181 | 201 | 221 |
| 7  | 27 | 46 | 65 | 84 | 103 | 123 | 142 | 162 | 182 | 202 | 222 |
| 8  | 28 | 47 | 66 | 85 | 104 | 124 | 143 | 163 | 183 | 203 | 223 |
| 9  | 29 | 48 | 67 | 86 | 105 | 125 | 144 | 164 | 184 | 204 | 224 |
| 10 | 30 | 49 | 68 | 87 | 106 | 126 | 145 | 165 | 185 | 205 | 225 |
| 11 | 31 | 50 | 69 | 88 | 107 | 127 | 146 | 166 | 186 | 206 | 226 |
| 12 | 32 | 51 | 70 | 89 | 108 | 128 | 147 | 167 | 187 | 207 | 227 |
| 13 | 33 | 52 | 71 | 90 | 109 | 129 | 148 | 168 | 188 | 208 | 228 |
| 14 | 34 | 53 | 72 | 91 | 110 | 130 | 149 | 169 | 189 | 209 | 229 |
| 15 | 35 | 54 | 73 | 92 | 111 | 131 | 150 | 170 | 190 | 210 | 230 |
| 16 | 36 | 55 | 74 | 93 | 112 | 132 | 151 | 171 | 191 | 211 | 231 |
| 17 | 37 | 56 | 75 | 94 | 113 | 133 | 152 | 172 | 192 | 212 | 232 |
| 18 | 38 | 57 | 76 | 95 | 114 | 134 | 153 | 173 | 193 | 213 | 233 |
| 19 | 39 | 58 | 77 | 96 | 115 | 135 | 154 | 174 | 194 | 214 | 234 |
| 20 |    |    |    |    | 116 | 135 | 155 | 175 | 195 | 215 | 235 |

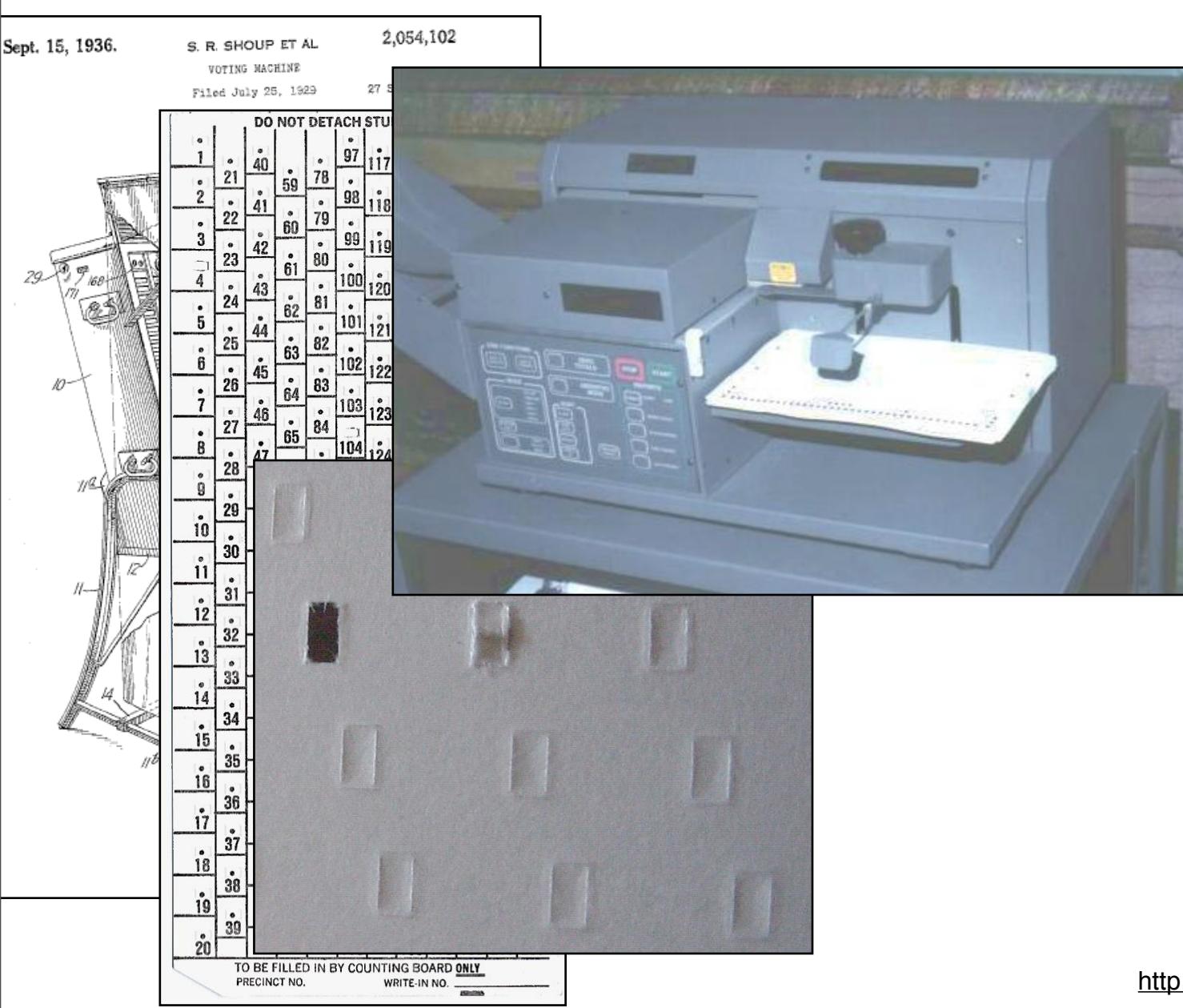
TO BE FILLED IN BY COUNTING BOARD ONLY  
PRECINCT NO. WRITE-IN NO. \_\_\_\_\_

4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

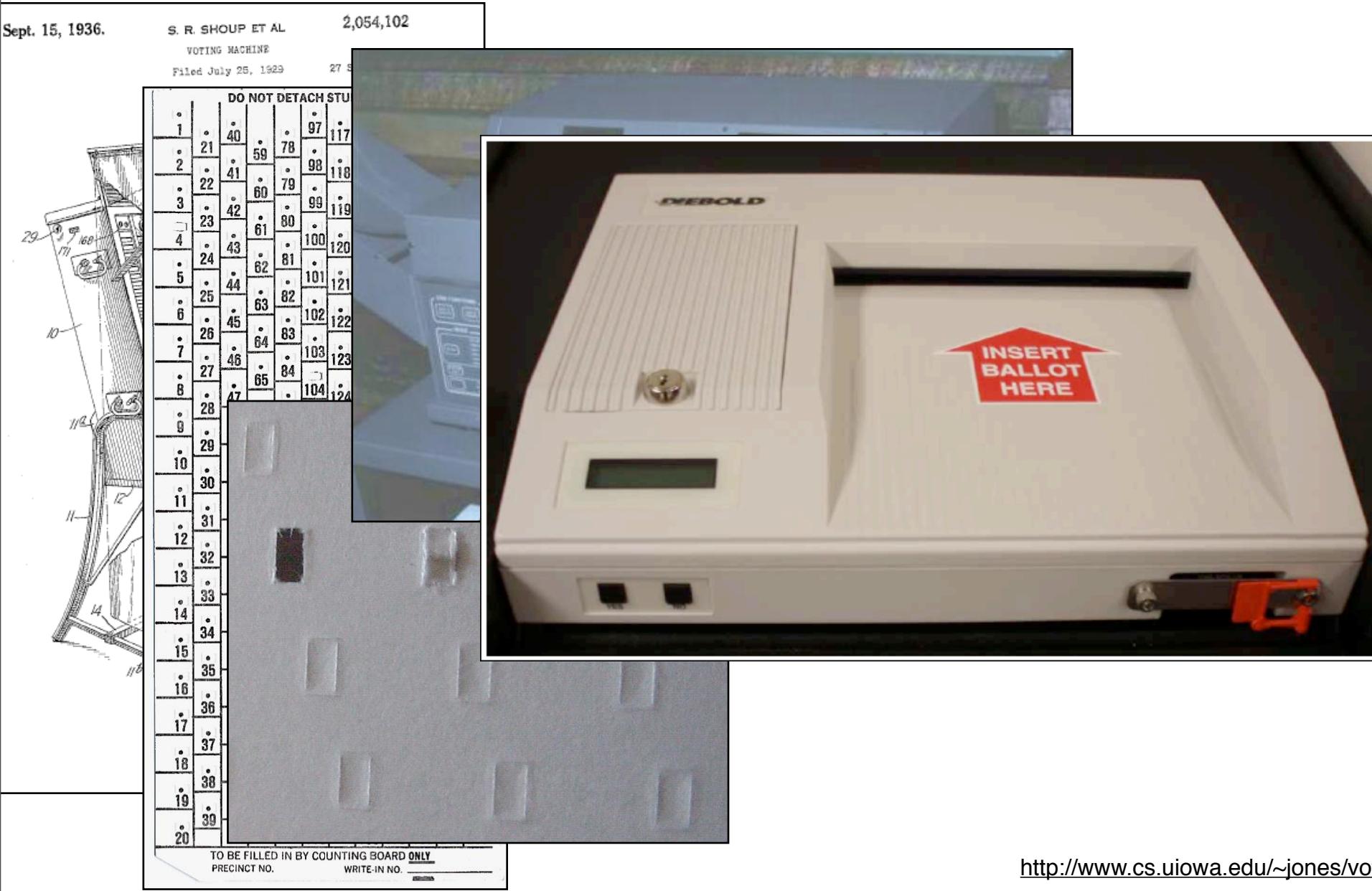
# How We Got Here



# How We Got Here



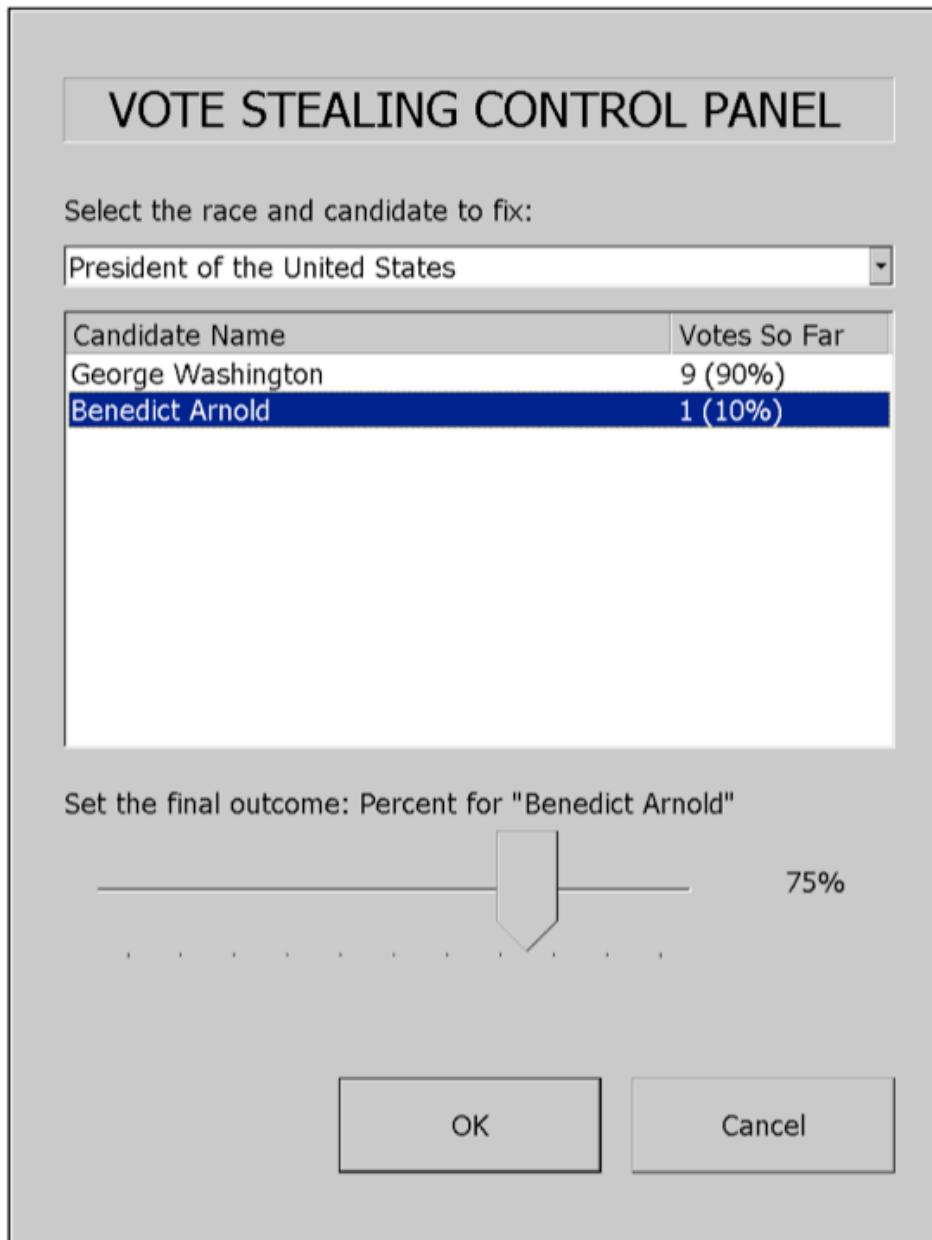
# How We Got Here



# How We Got Here



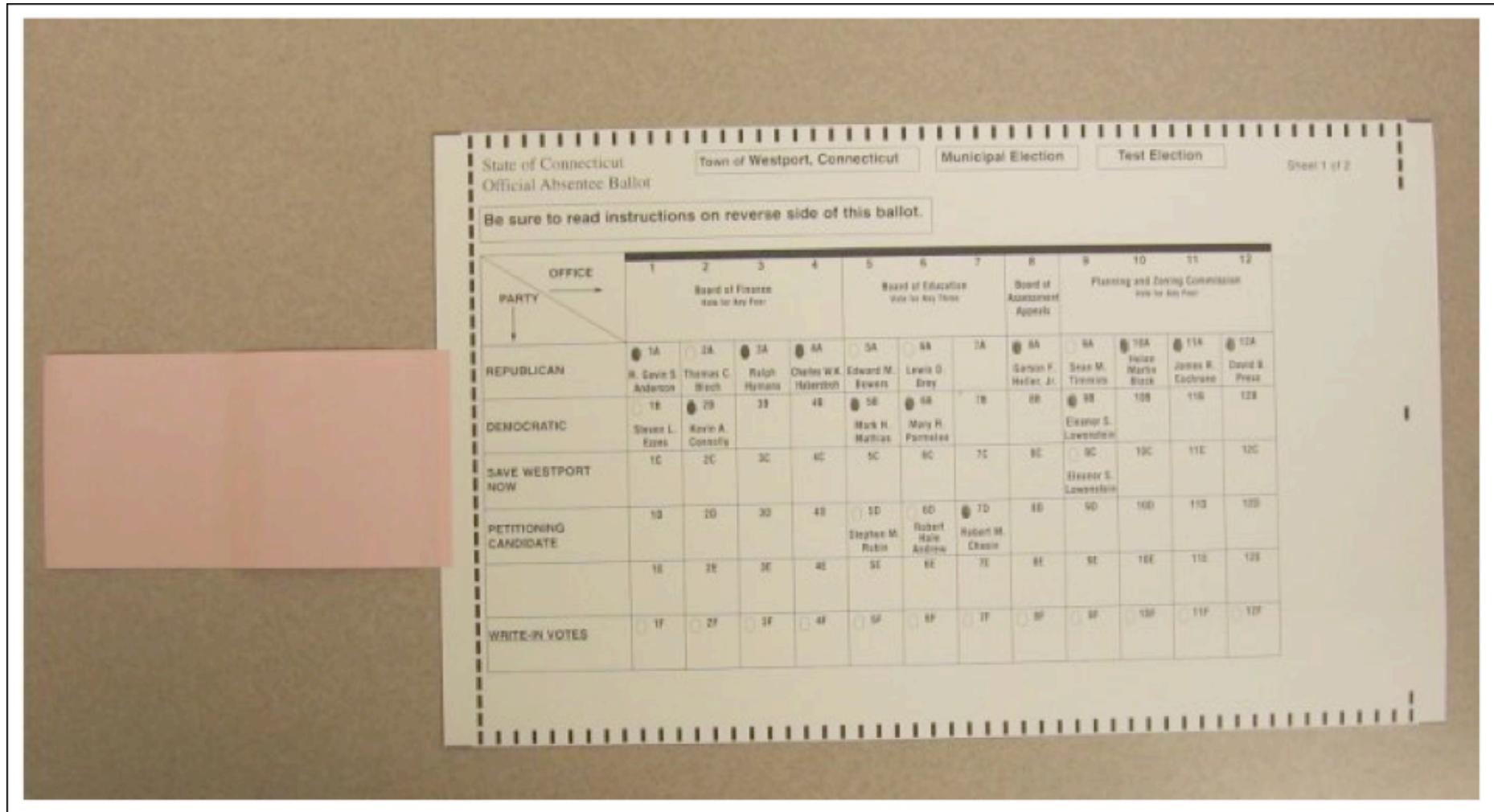
# Last Year: Princeton Report



- Diebold touch-screen runs executable code loaded from memory card
- All audit logs modified to be consistent
- Can spread virally by memory card.

[FHF2006]

# But not just DREs...



[KMRS2006]

# How can Cryptography help?

Cryptography provides more than confidentiality.

Cryptography can provide verifiability  
while maintaining ballot secrecy.

# The Point of An Election

“The People have spoken....  
the bastards!”

Dick Tuck  
1966 Concession Speech

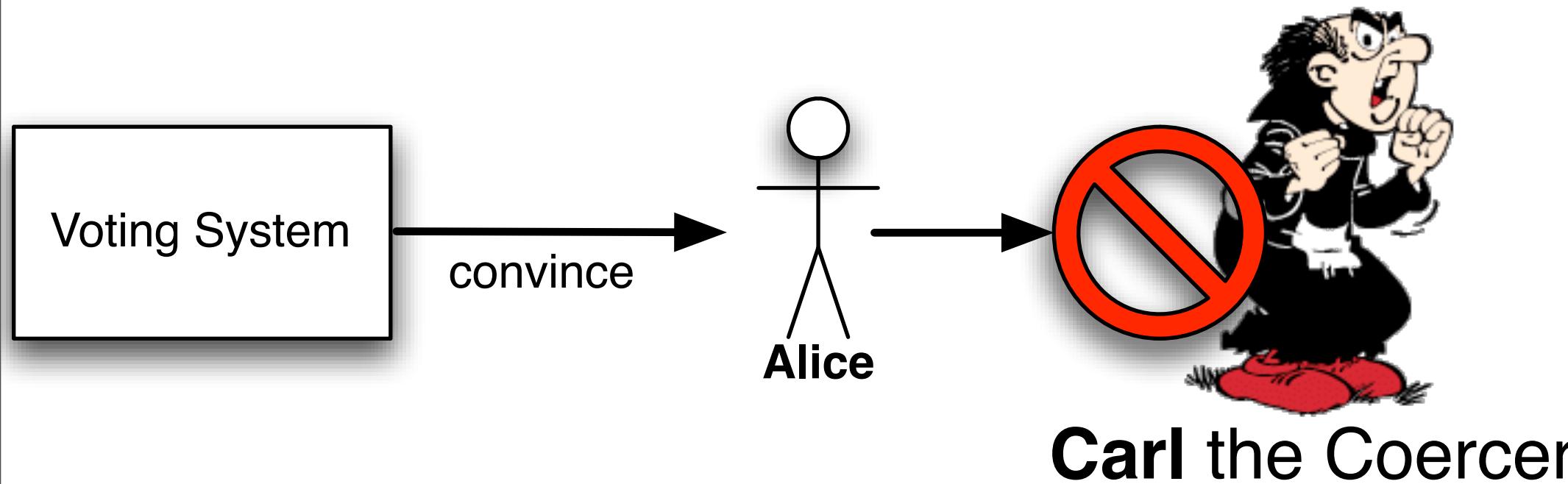
# The Point of An Election

“The People have spoken....  
the bastards!”

Dick Tuck  
1966 Concession Speech

Provide enough evidence  
to convince the loser.

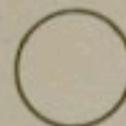
# **Secret Ballot vs. Verifiability**





# 1892 - Australian Ballot

1893



## DEMOCRATIC.

FOR MAYOR,  
AUGUST LEUZ, JR.  
CORNER BURLINGTON AND JOHNSON STREETS.

FOR TREASURER,  
GEORGE W. KOONTZ *848*

NO. 620 EAST BURLINGTON STREET.

FOR CITY SOLICITOR,  
FRANK J. HORAK

NO. 120 DODGE STREET.

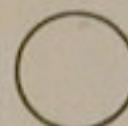
FOR ASSESSOR,  
F. A. HEINSIUS

NO. 948 EAST MARKET STREET.

## FOURTH WARD.

FOR TRUSTEE,  
JOHN U. MILLER *24*

EAST MARKET STREET.



## REPUBLICAN.

FOR MAYOR,  
CHAS. LEWIS *221*

NO. 227 NORTH CLINTON STREET.

FOR TREASURER,

FOR SOLICITOR,  
L. H. FULLER *101*

NO. 422 SOUTH DUBUQUE STREET.

FOR ASSESSOR,  
H. W. LATHROP *198*

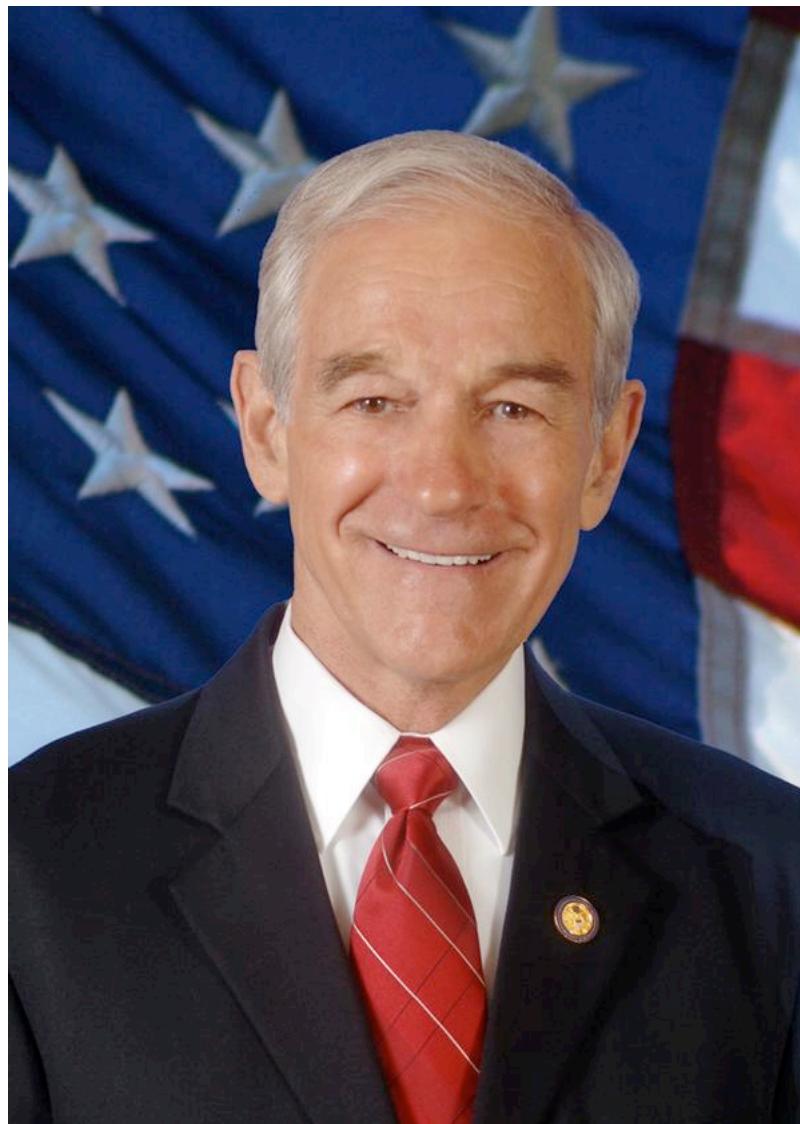
NO. 518 IOWA AVENUE.

## FOURTH WARD.

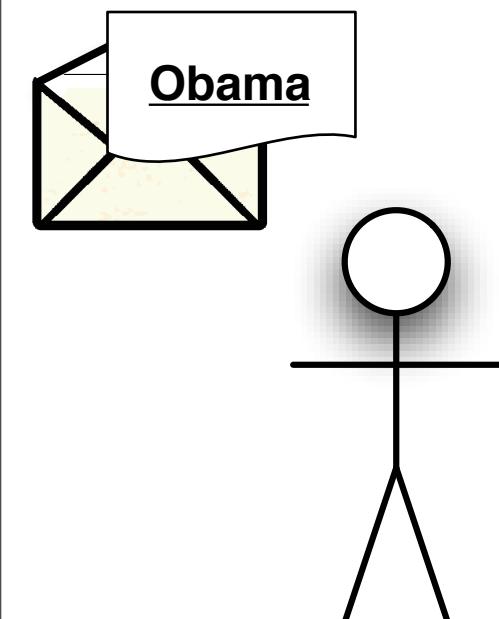
FOR TRUSTEE,  
J. C. LEASURE

COR. VAN BUREN ST. AND IOWA AVENUE.

# Election 2008

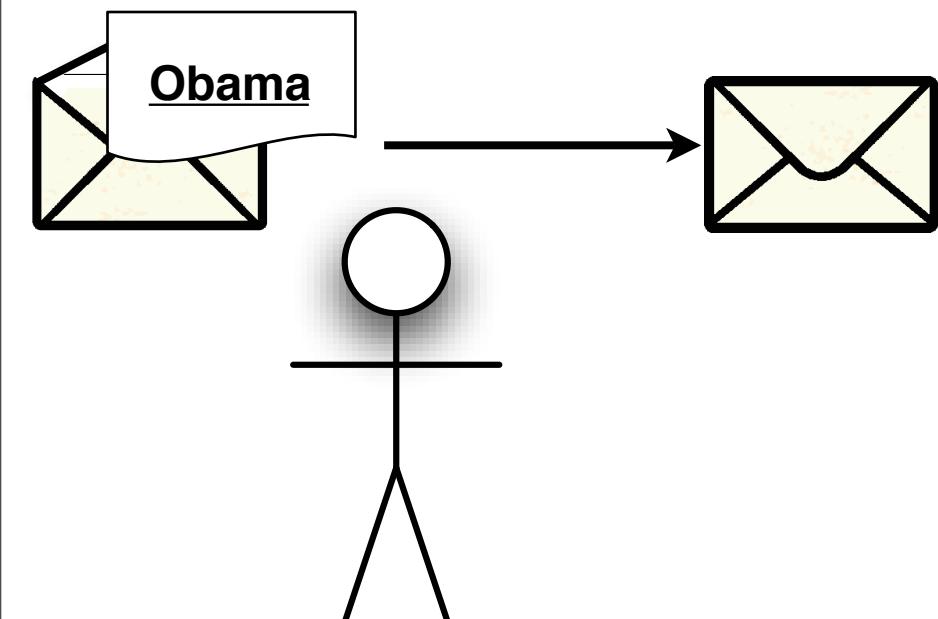


# The Ballot Handoff



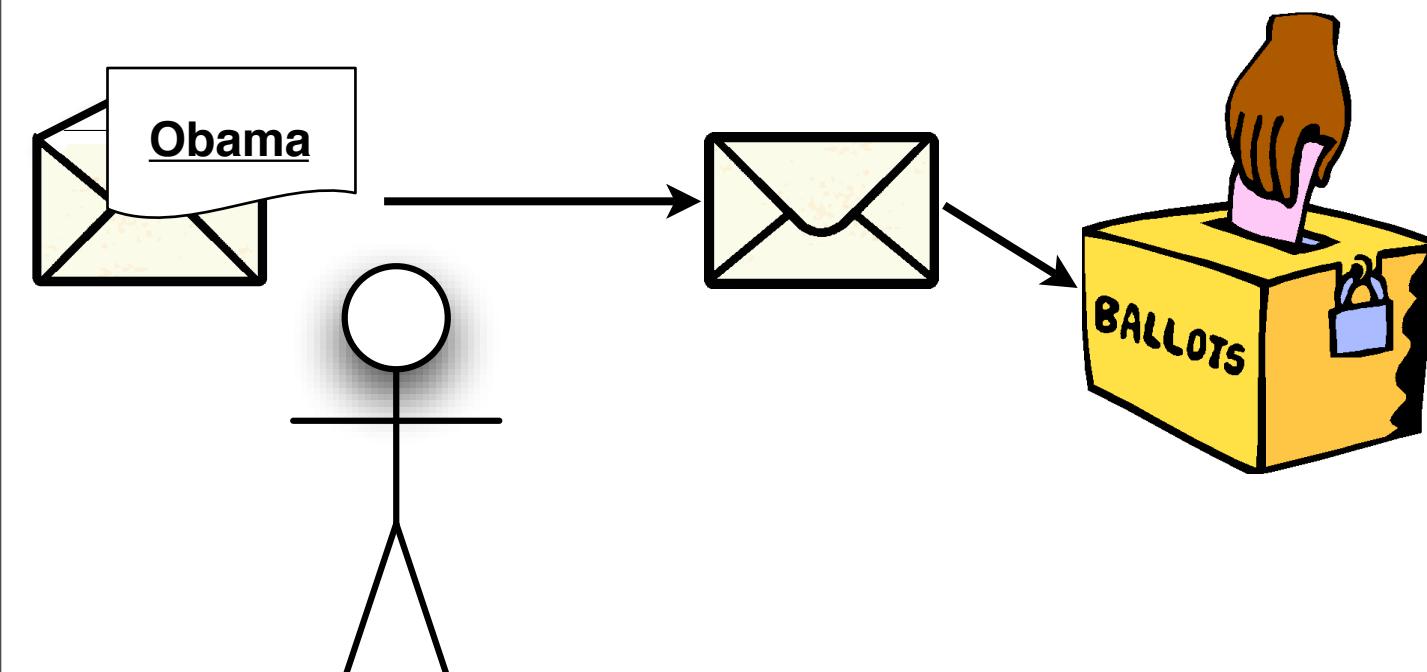
**Alice the Voter**

# The Ballot Handoff

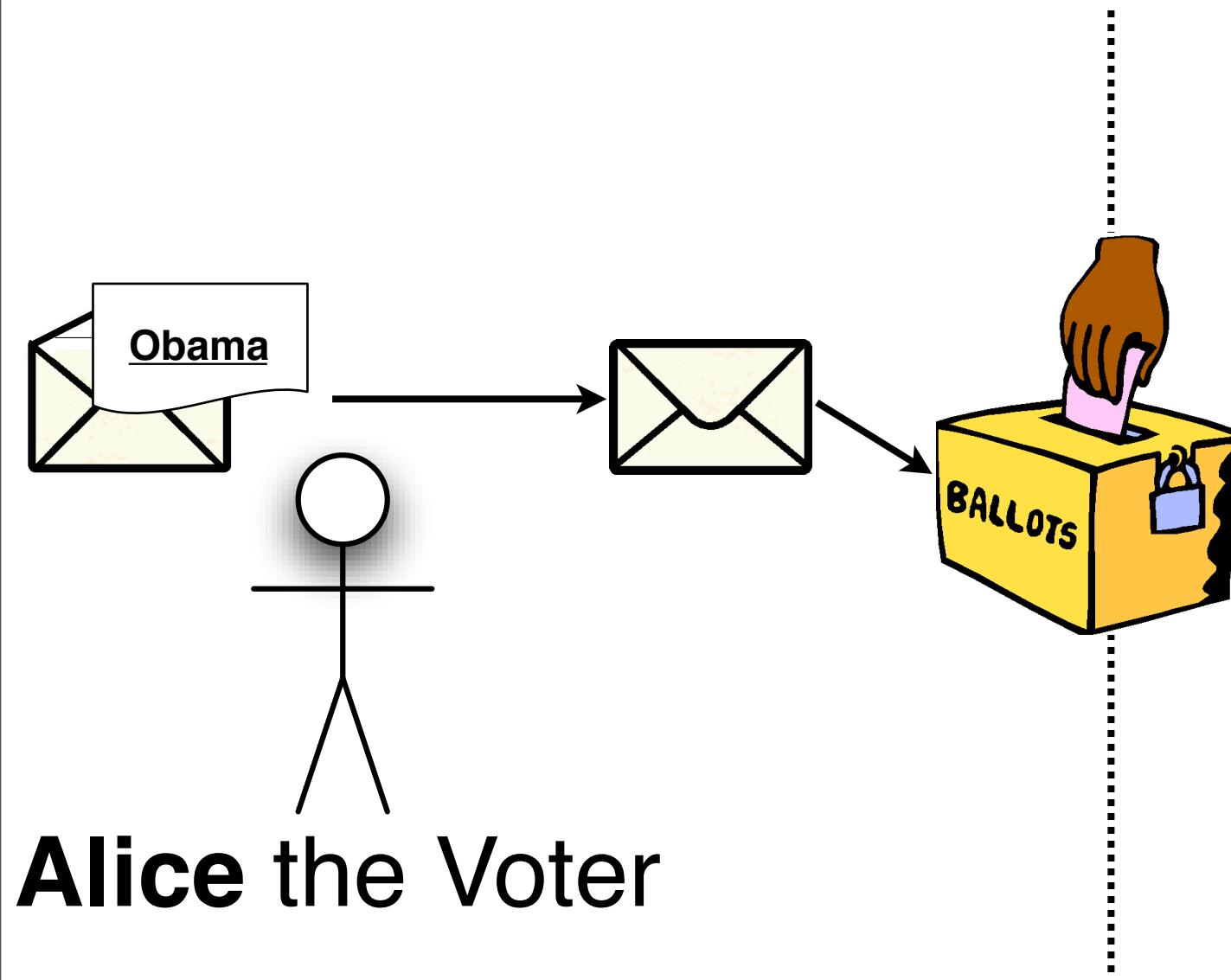


**Alice the Voter**

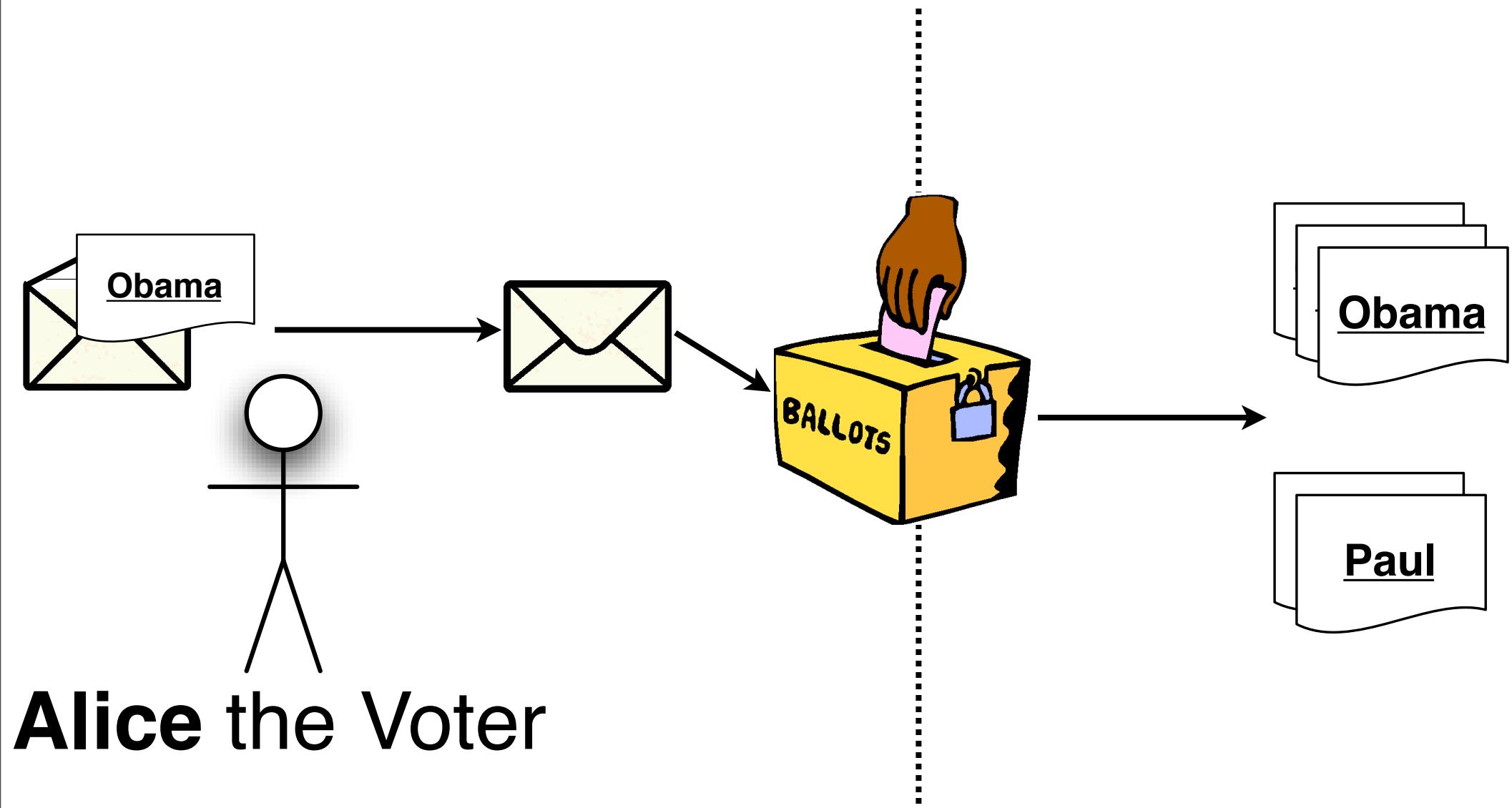
# The Ballot Handoff



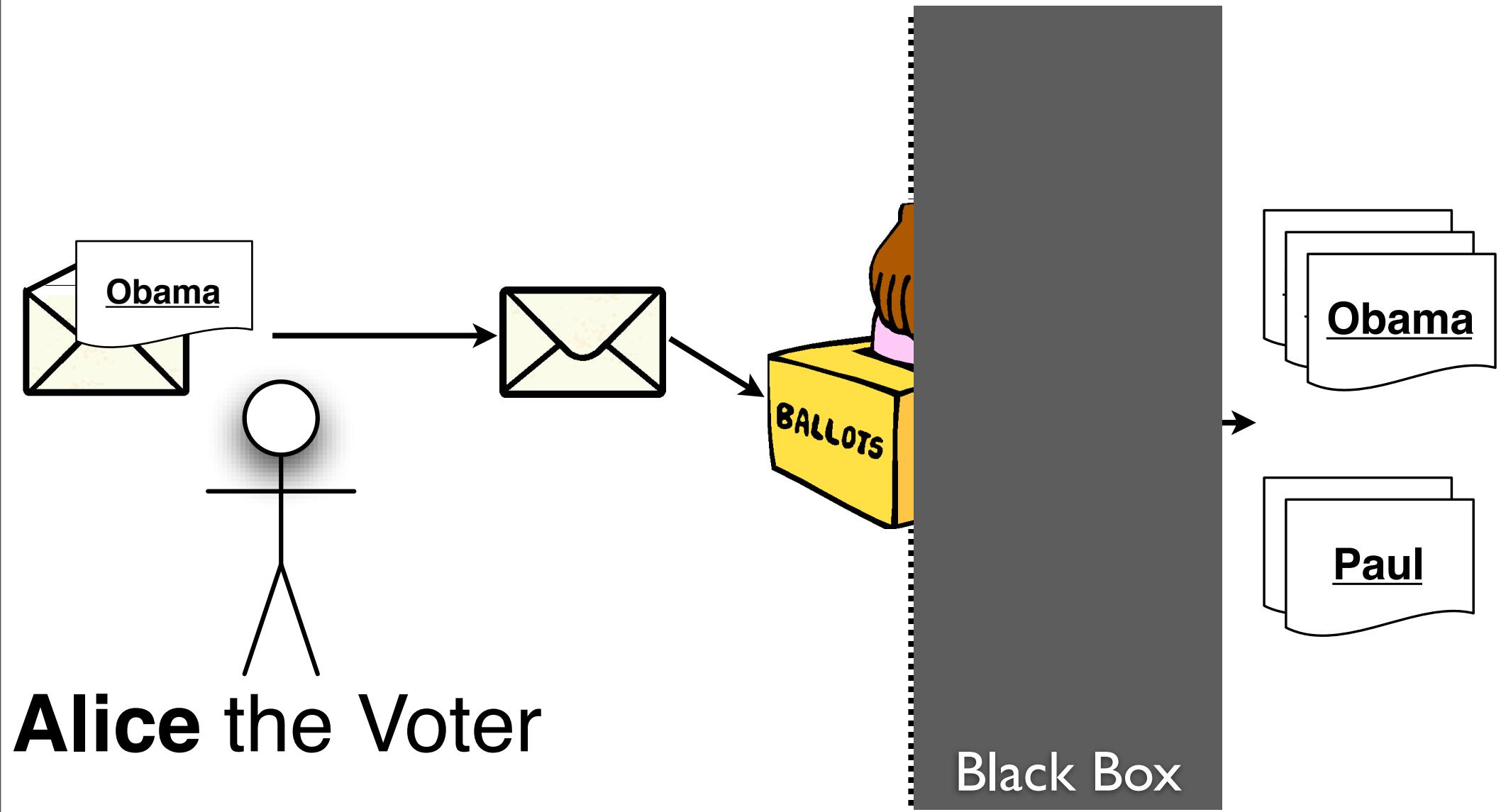
# The Ballot Handoff



# The Ballot Handoff

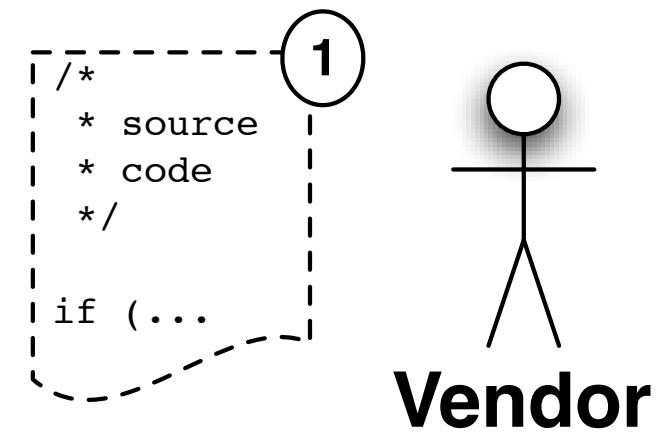


# The Ballot Handoff

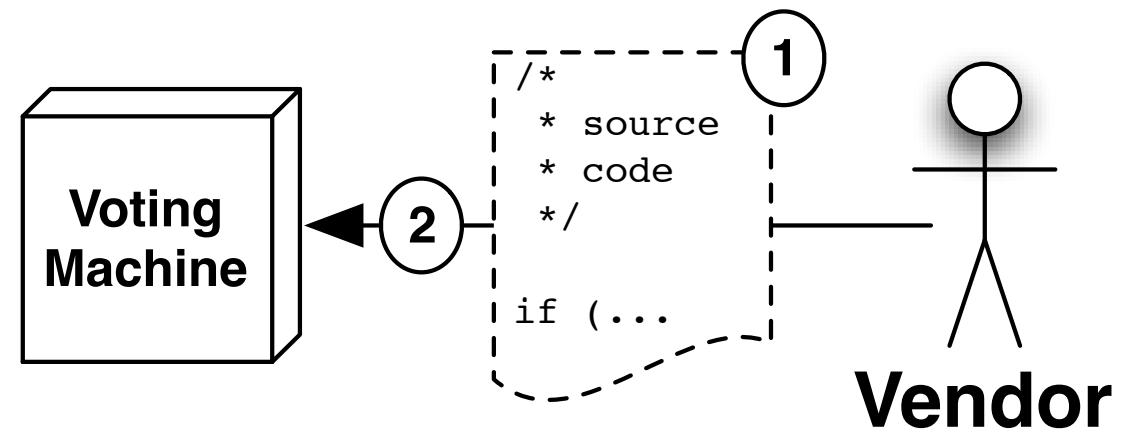


# Chain of Custody

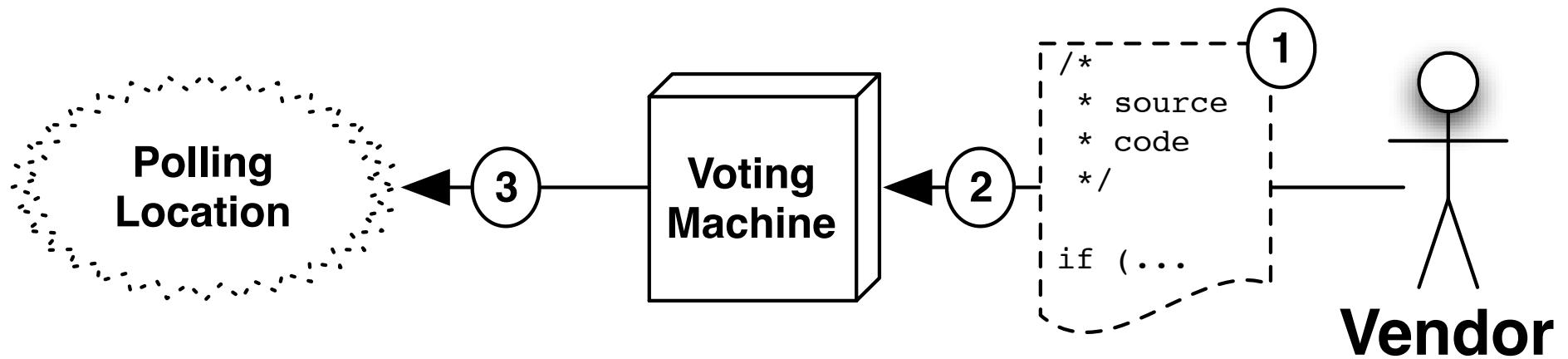
# Chain of Custody



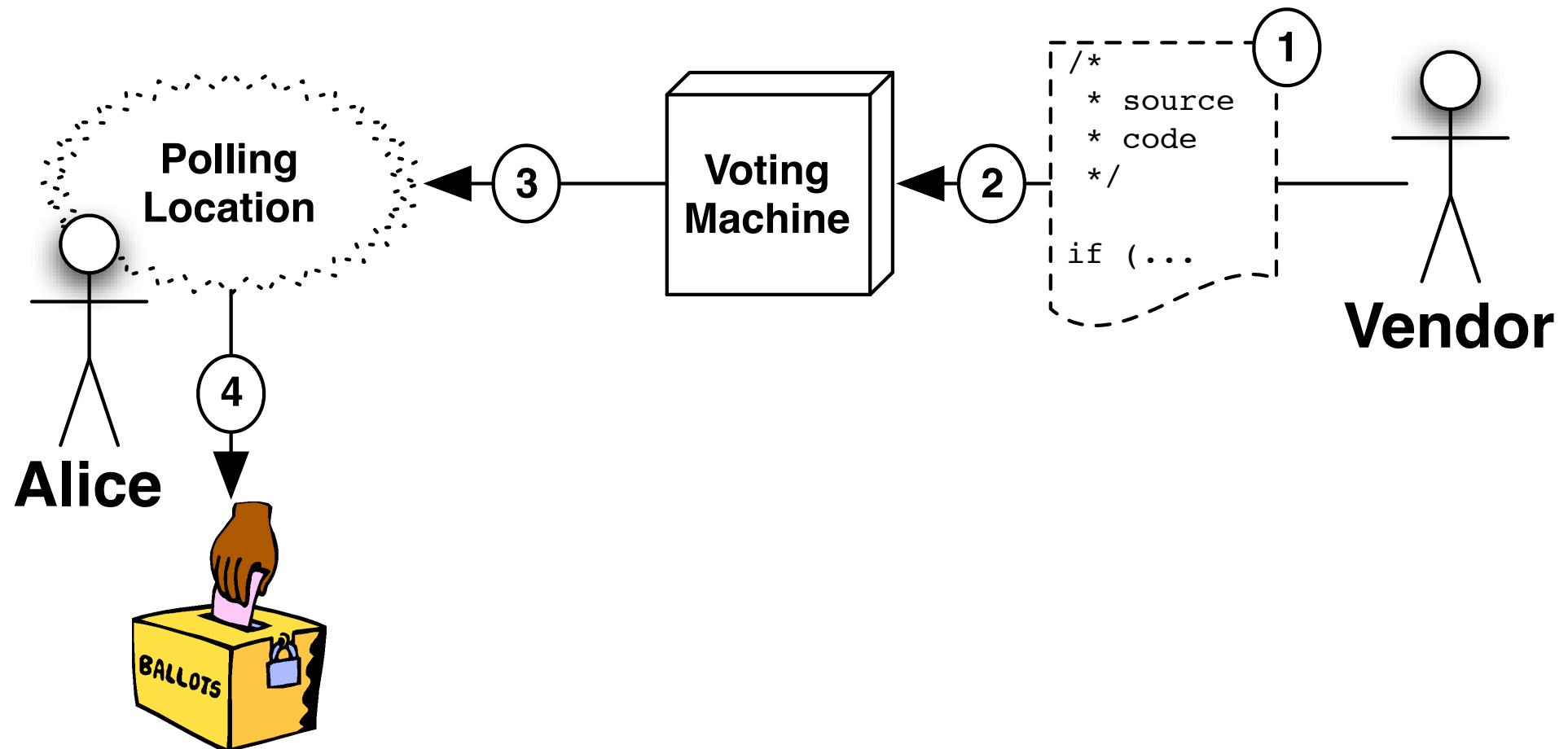
# Chain of Custody



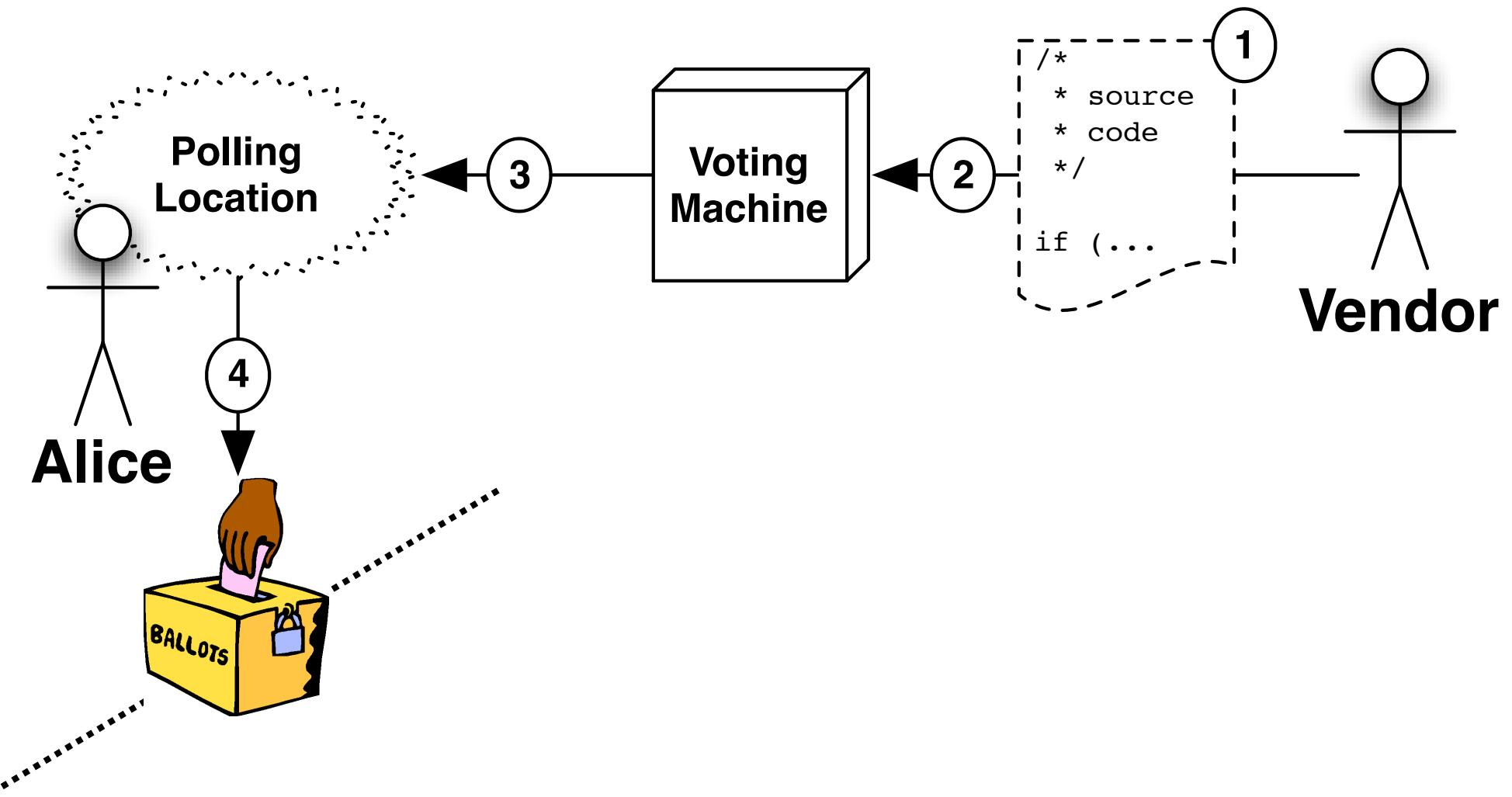
# Chain of Custody



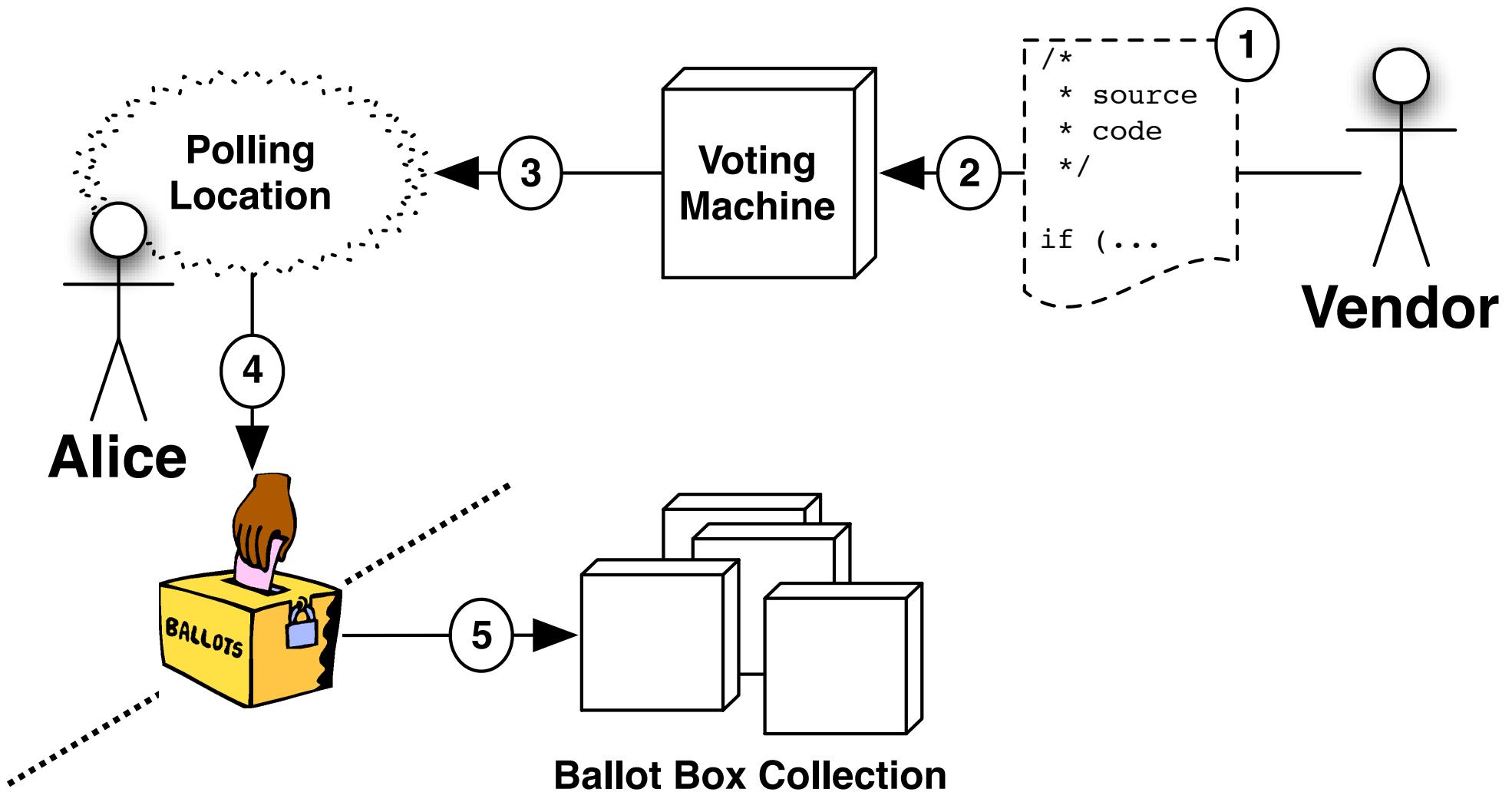
# Chain of Custody



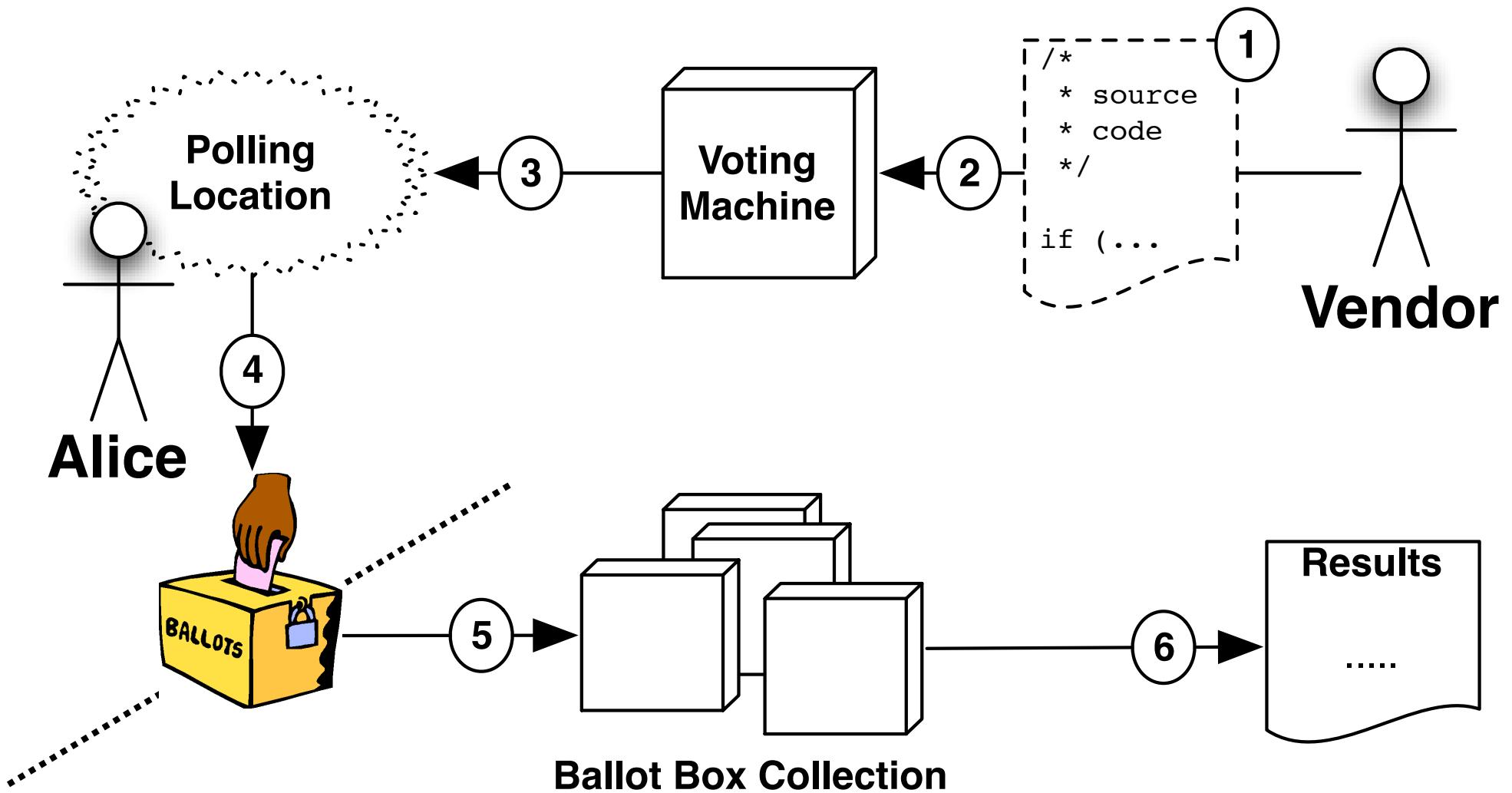
# Chain of Custody



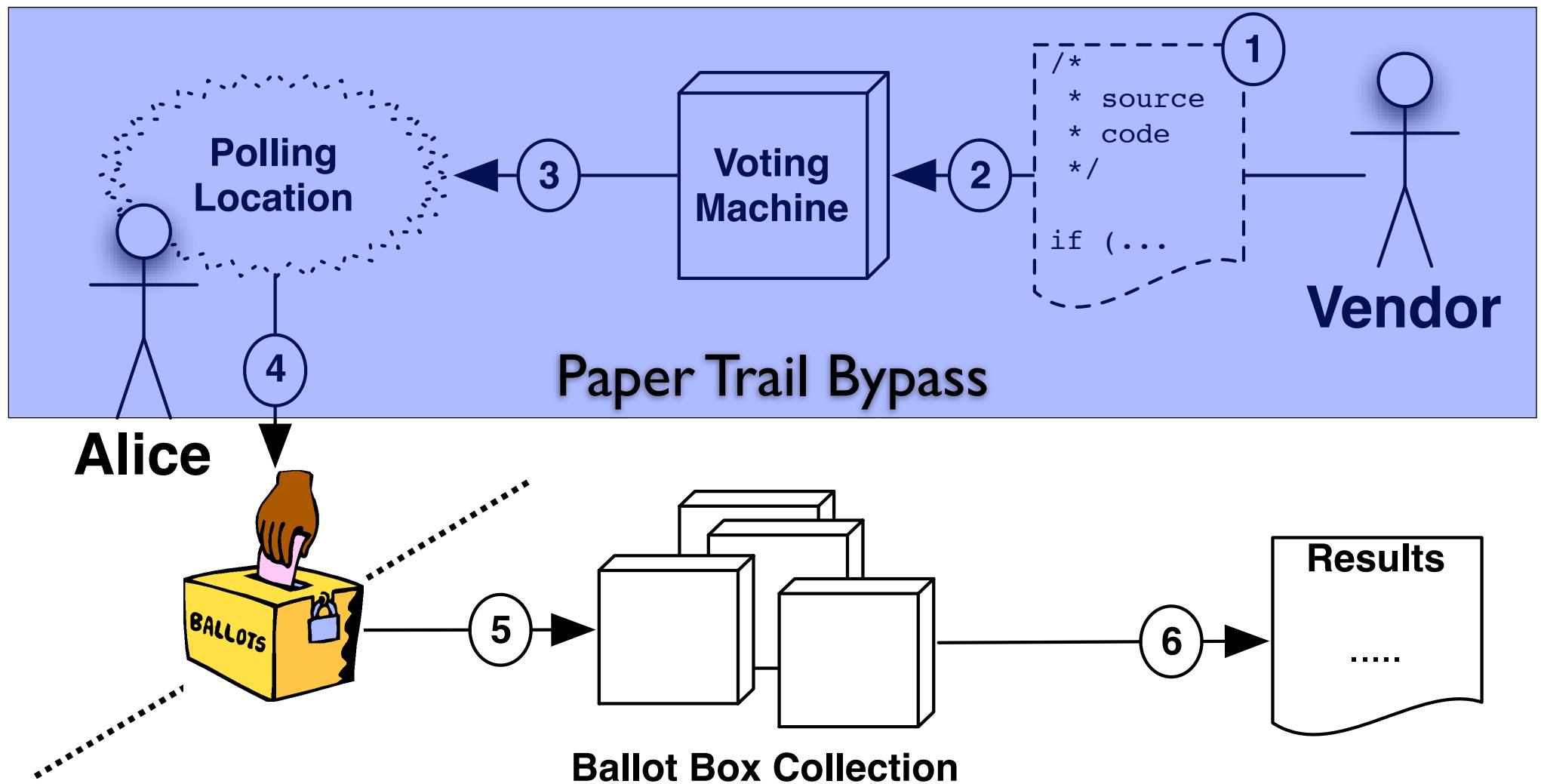
# Chain of Custody



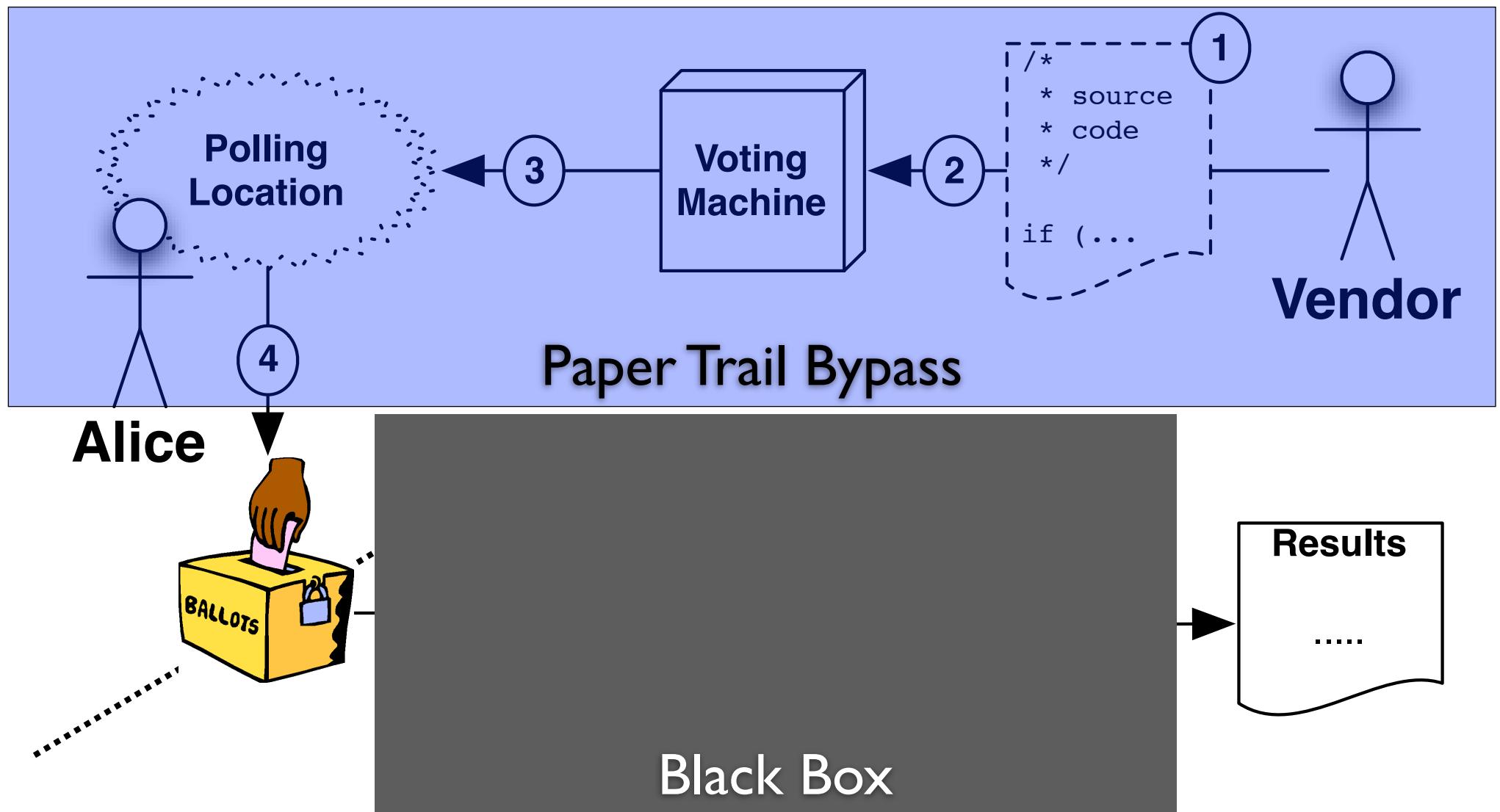
# Chain of Custody



# Chain of Custody



# Chain of Custody



# The Cost of Secrecy

# The Cost of Secrecy

**Scavenged ballot box lids haunt S.F. elections**

[Erin McCormick, Chronicle Staff Writer](#)

Monday, January 7, 2002

# The Cost of Secrecy

Scavenged **ballot box** lids haunt S.F. elections

[Erin McC](#)

Helicopter Crash Delays Afghan  
Vote Count

Monday,

Helicopter Sent to Pick Up Afghan Ballots in Remote  
Province Crash-Lands, Delaying Vote Count

# The Cost of Secrecy

Scavenged **ballot box** lids haunt S.F. elections

[Erin McClellan](#)

Helicopter Crash Delays Afghan  
Vote Count

Monday, October 25, 2004

Helicopter  
Province Cr

October 28, 2004 09:28 IST

Nearly 58,000 absentee ballots for the US presidential election may never have reached Florida's Broward County voters, who had requested them more than two weeks ago, election officials said.

# The Cost of Secrecy

**Scavenged ballot box lids haunt S.F. elections**

[Erin McC](#)

**Helicopter Crash Delays Afghan Vote Count**

Monday,

[Helicopter Crash Delays Afghan Vote Count](#)

[Province Cr](#)

October 28, 2004 09:28 IST

Nearly 58,000 absentee ballots for the U.S. presidential election may never have reached Florida's Broward County election officials said.

**Mexico Presidential Election Ballots Found in Dump**

**RAW STORY**

Published: Thursday July 6, 2006

# The Cost of Secrecy

## Scavenged ballot box lids haunt S.F. elections

[Erin McClellan](#)

### Helicopter Crash Delays Afghan Vote Count

Absentee ballots 'lost' in Florida

Helicopter Crash

Province Crash

October 28, 2004 09:28 IST

Nearly 58,000 absentee ballots for the U.S. presidential election may never have reached Florida's Broward County election officials said.

## Mexico Presidential Election Ballot Dump in Dump

SARASOTA

### 18,000 votes in U.S. House race may be lost

Thousands of votes were either not counted or not cast in Sarasota's nationally watched congressional race.

# Is Secrecy Important? Actually, it is.

Secret Ballot implemented in Chile in 1958.

“the secrecy of the ballot [...] has  
first-order implications for resource  
allocation, political outcomes, and social efficiency.”

[BalandRobinson 2004]

# Verifying with Cryptography

[Chaum81], [Benaloh85], [PIK93], [BenalohTuinstra92], [SK94], [Neff2001],  
[FS2001], [Chaum2004], [Neff2004], [Ryan2004], [Chaum2005]

# Desired Properties

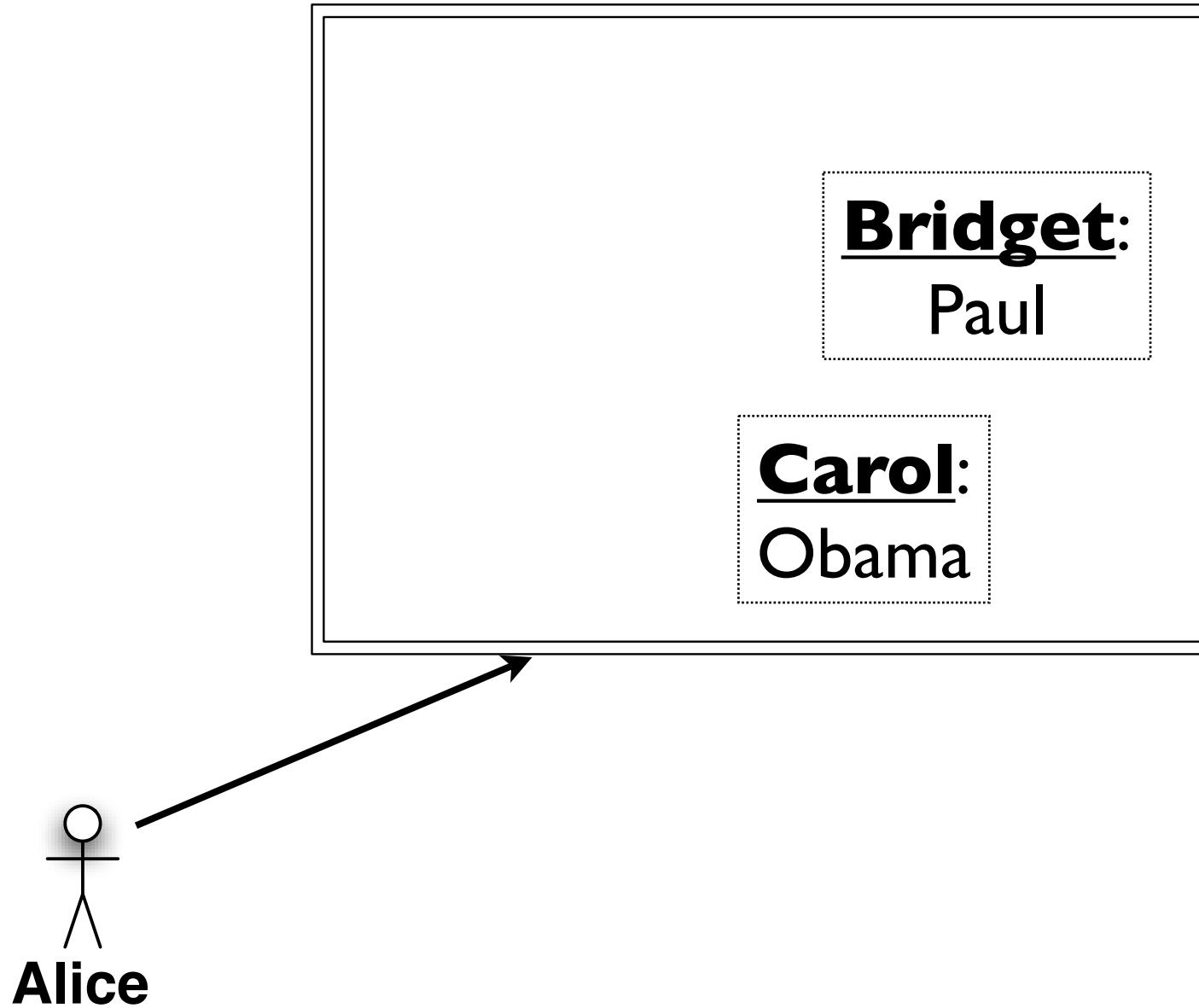
- (1) **Alice** verifies her vote.
- (2) **Everyone** verifies tallying.
- (3) Alice cannot be coerced by Eve.

# Public Ballots

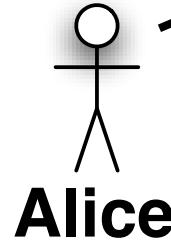
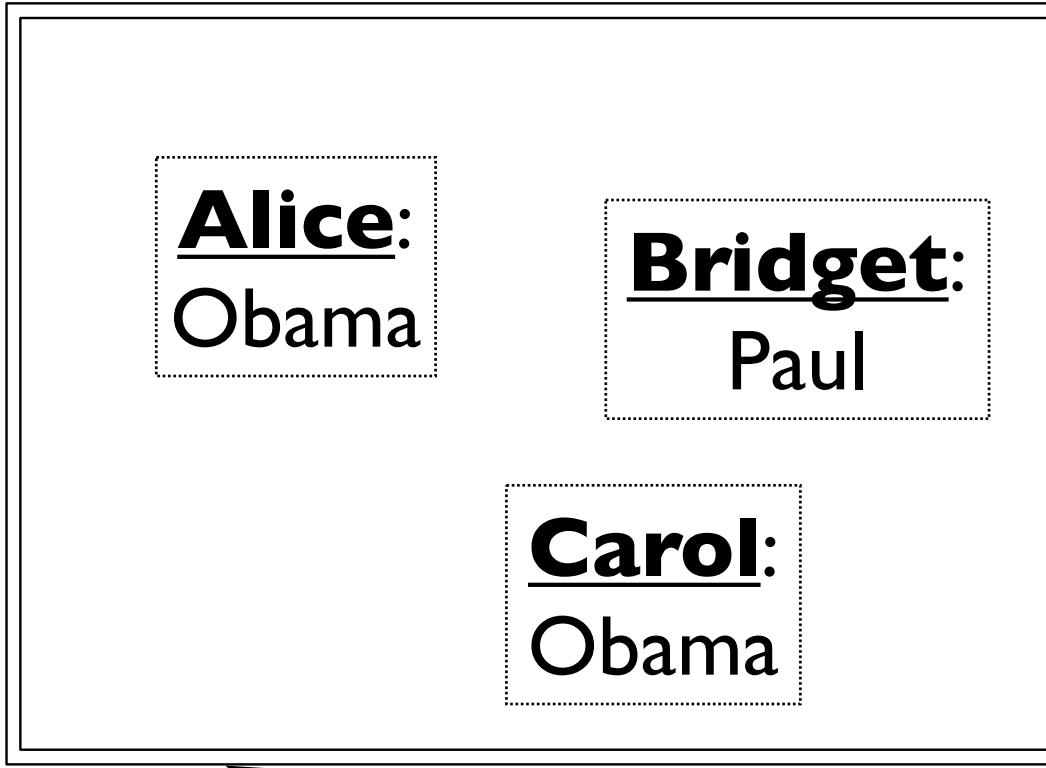
**Bridget:**  
Paul

**Carol:**  
Obama

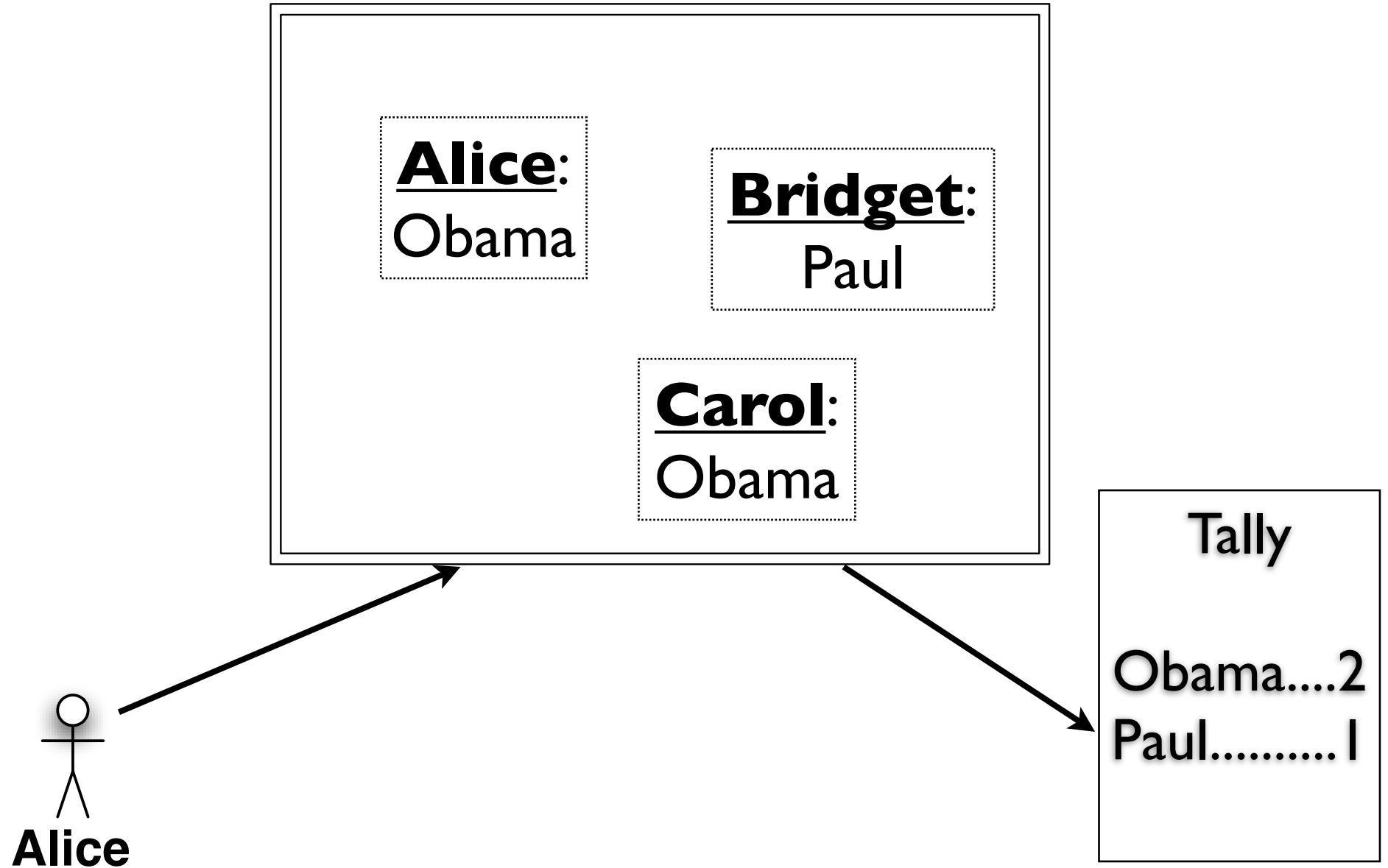
# Public Ballots



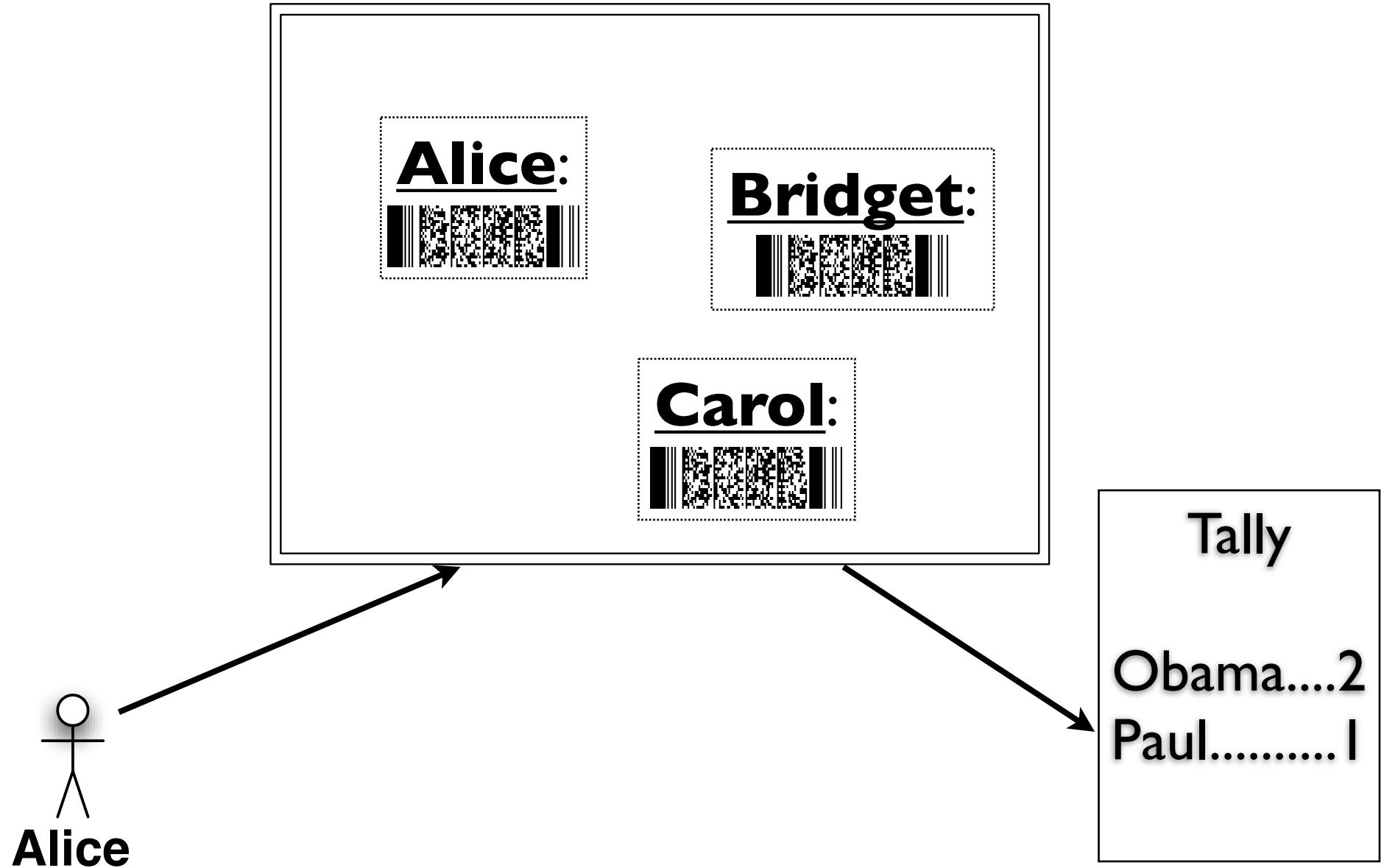
# Public Ballots



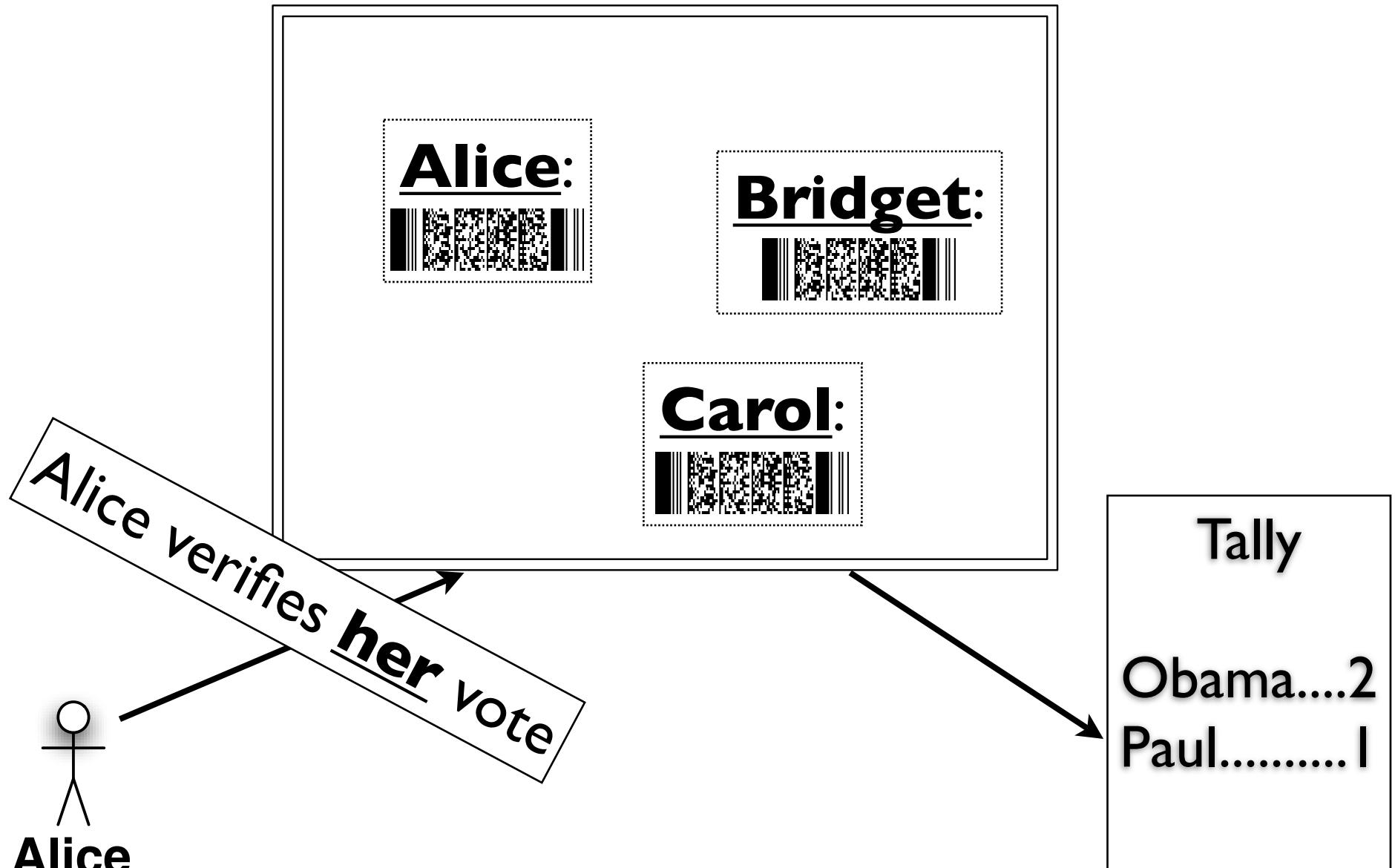
# Public Ballots



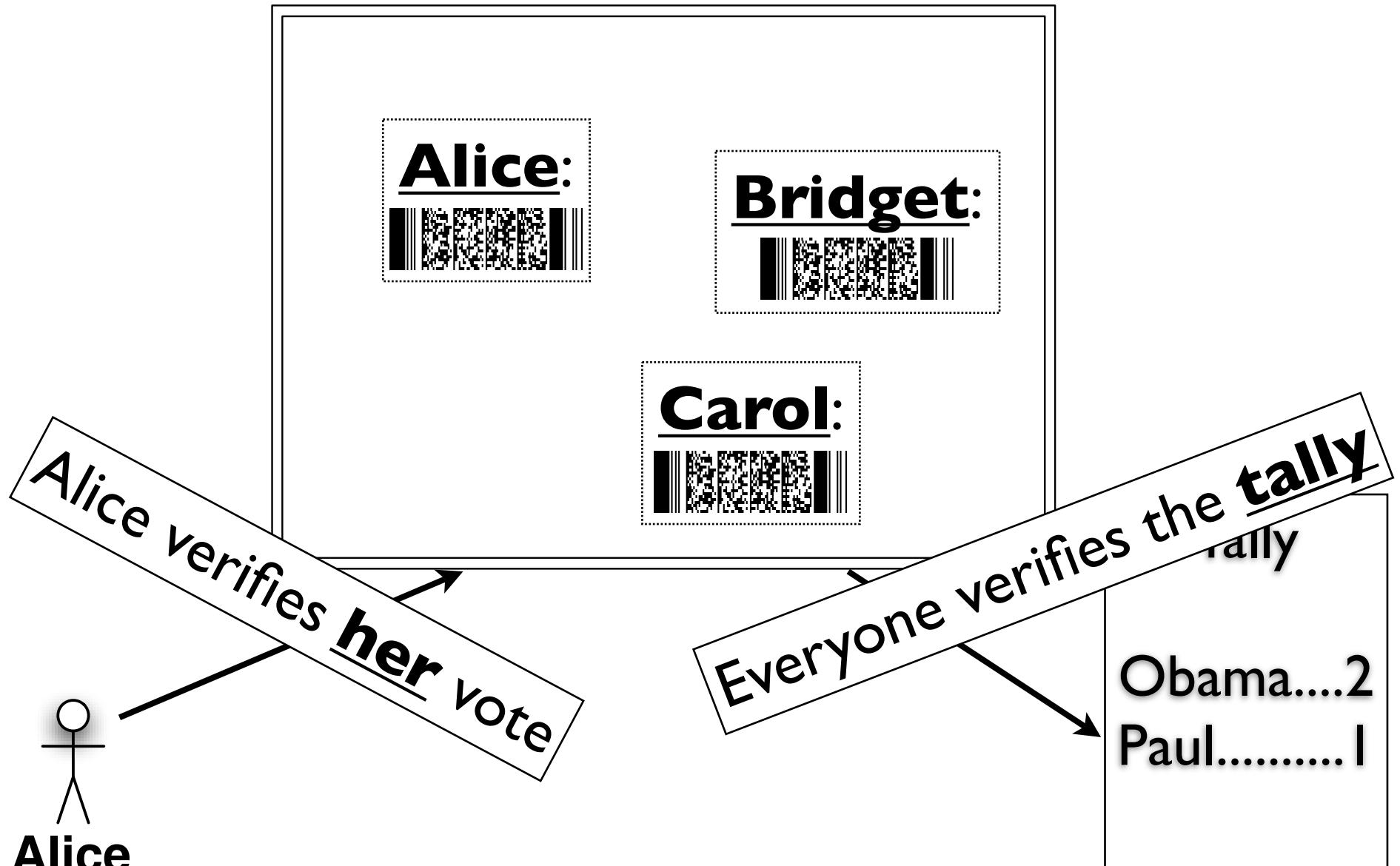
# Encrypted Public Ballots



# Encrypted Public Ballots

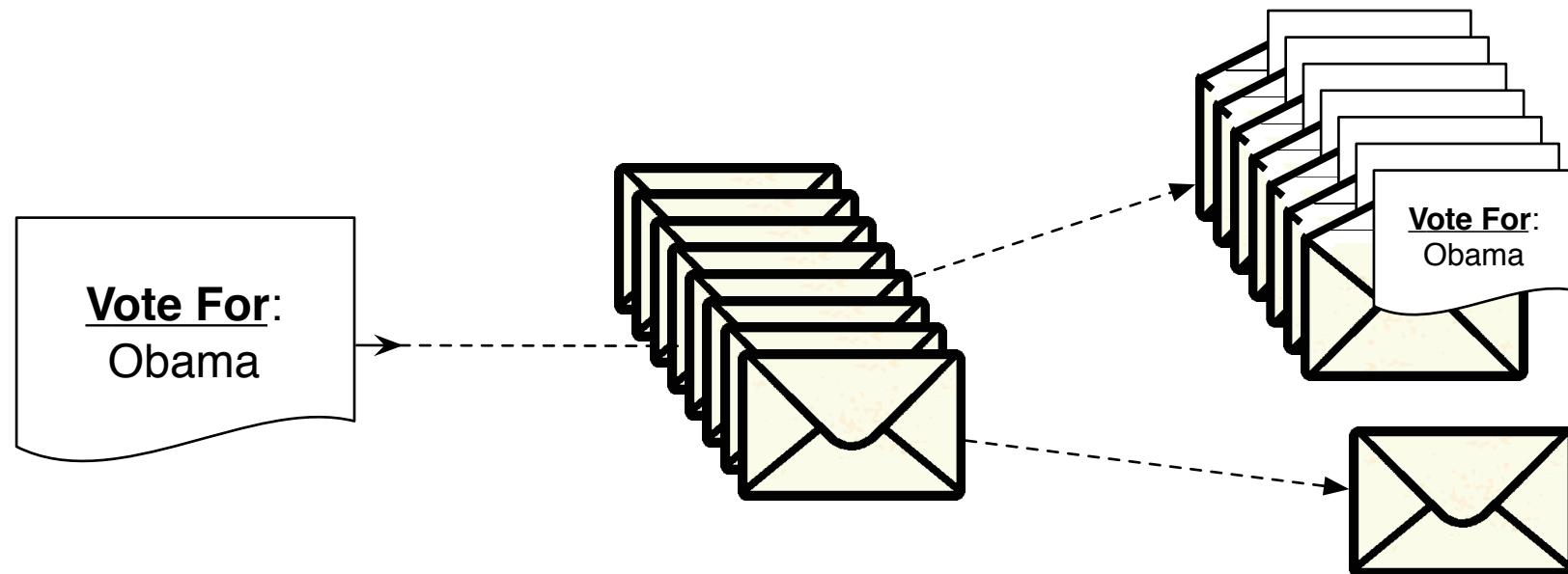


# Encrypted Public Ballots

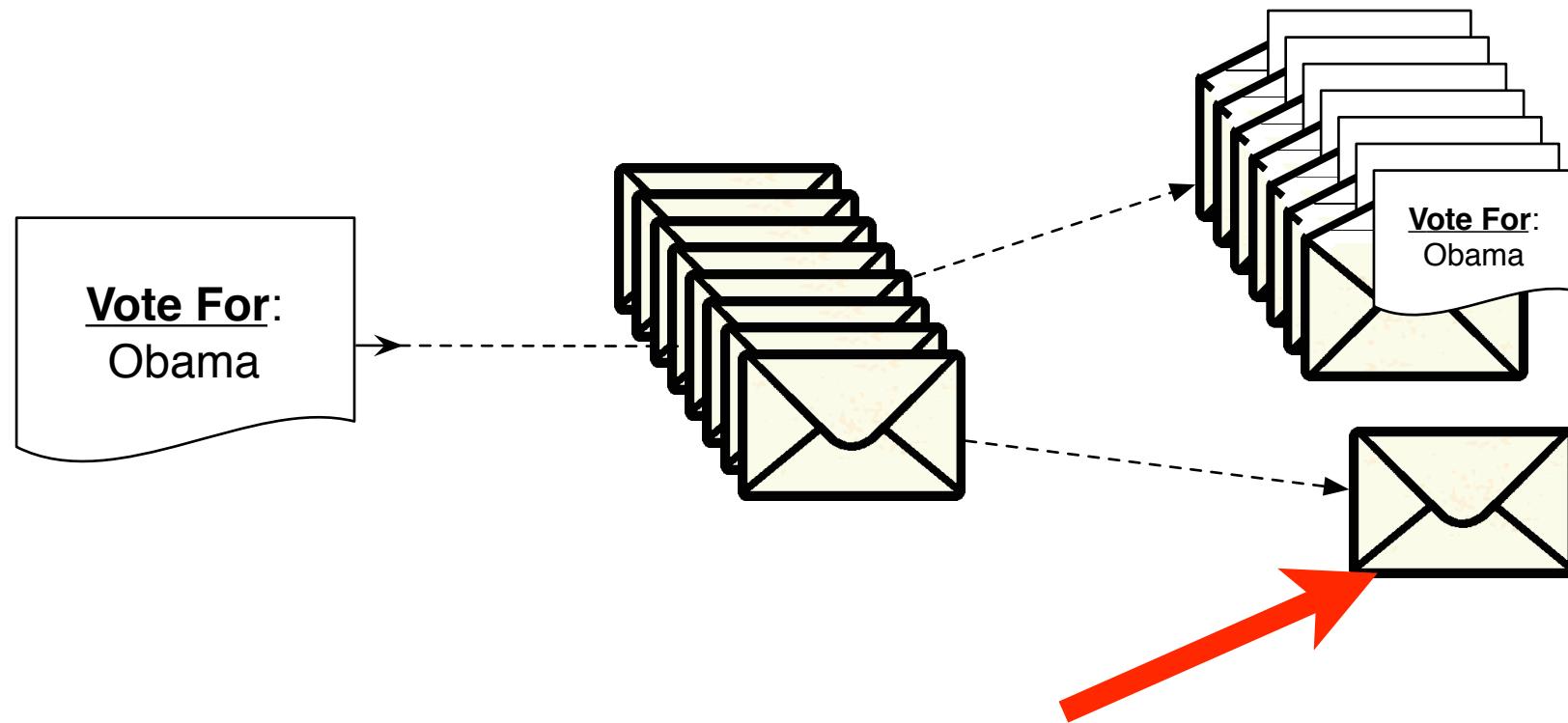


How can we **verify**  
operations on  
**encrypted** data?

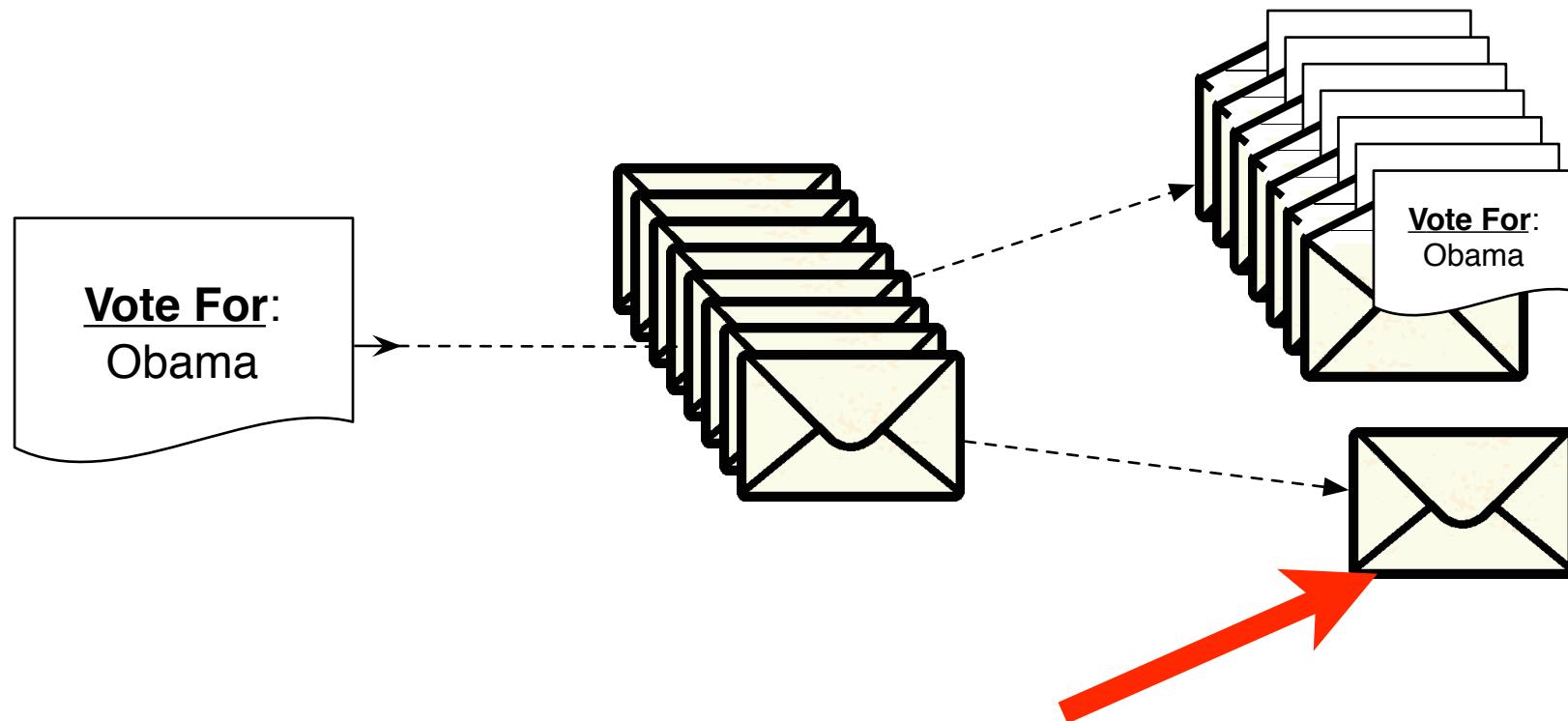
# Zero-Knowledge Proof



# Zero-Knowledge Proof

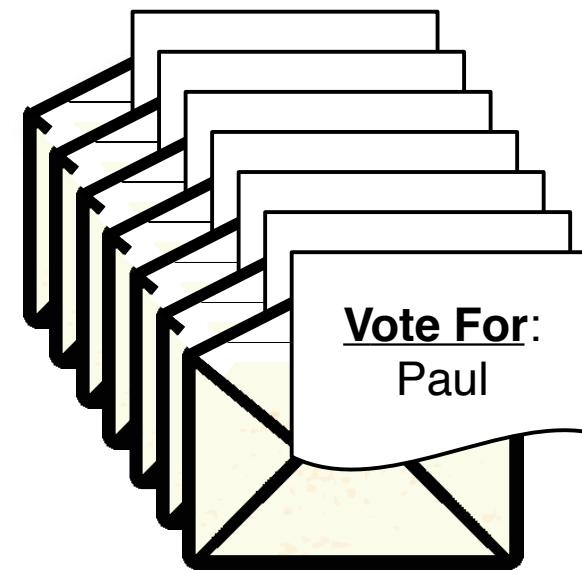
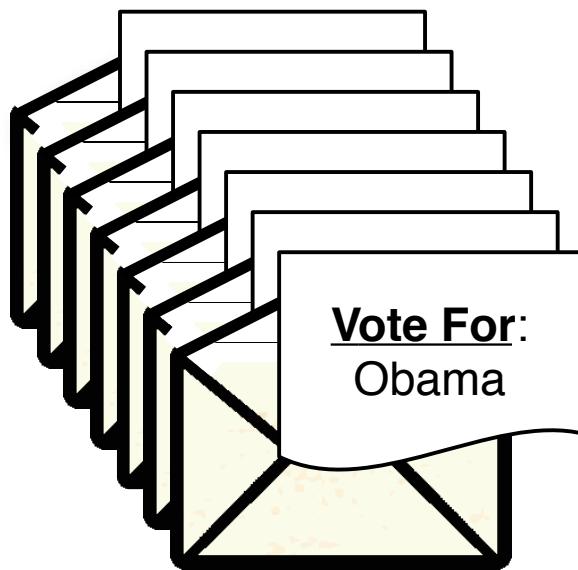


# Zero-Knowledge Proof



This last envelope  
likely contains “Obama”

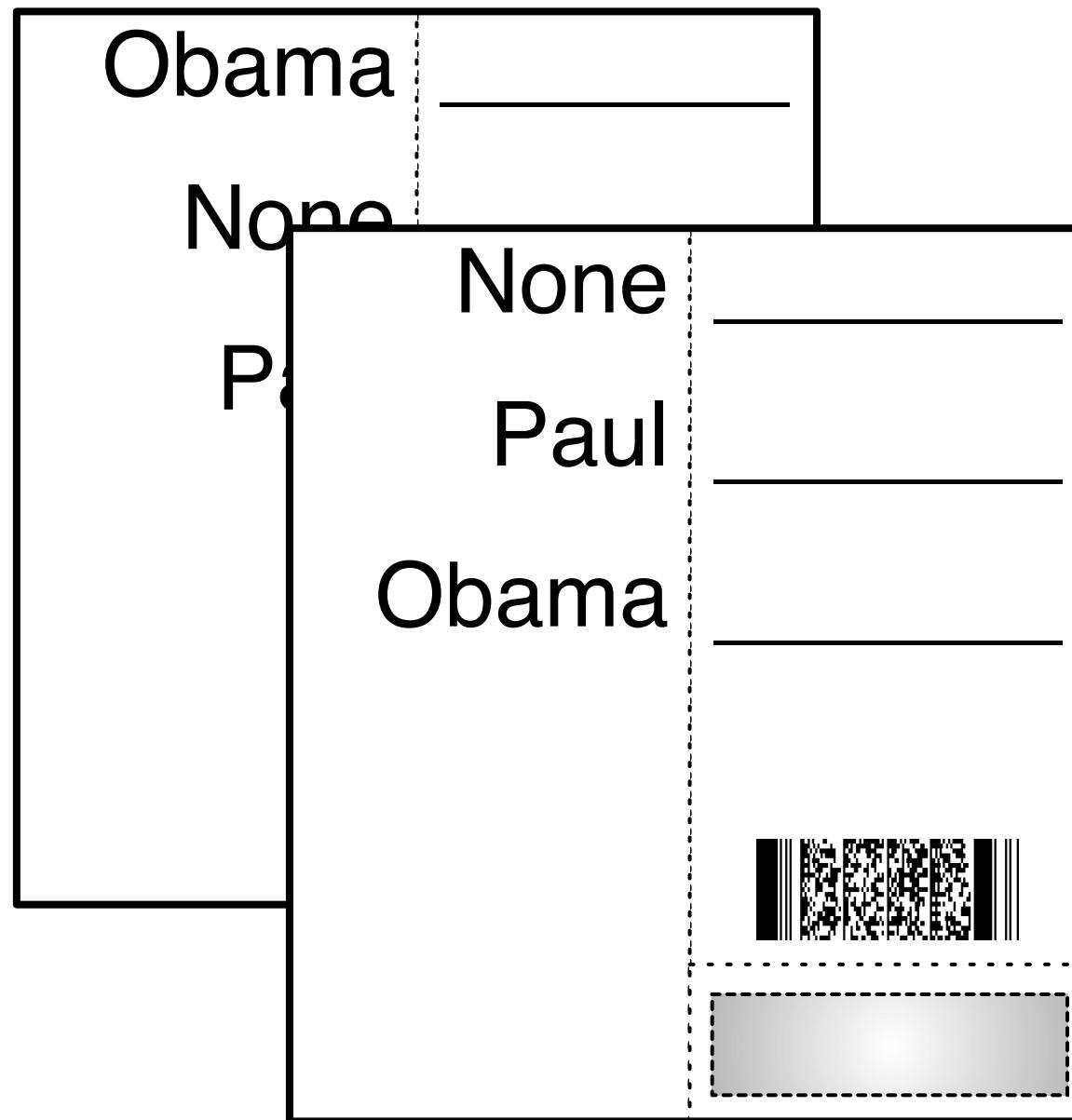
# Zero-Knowledge Proof



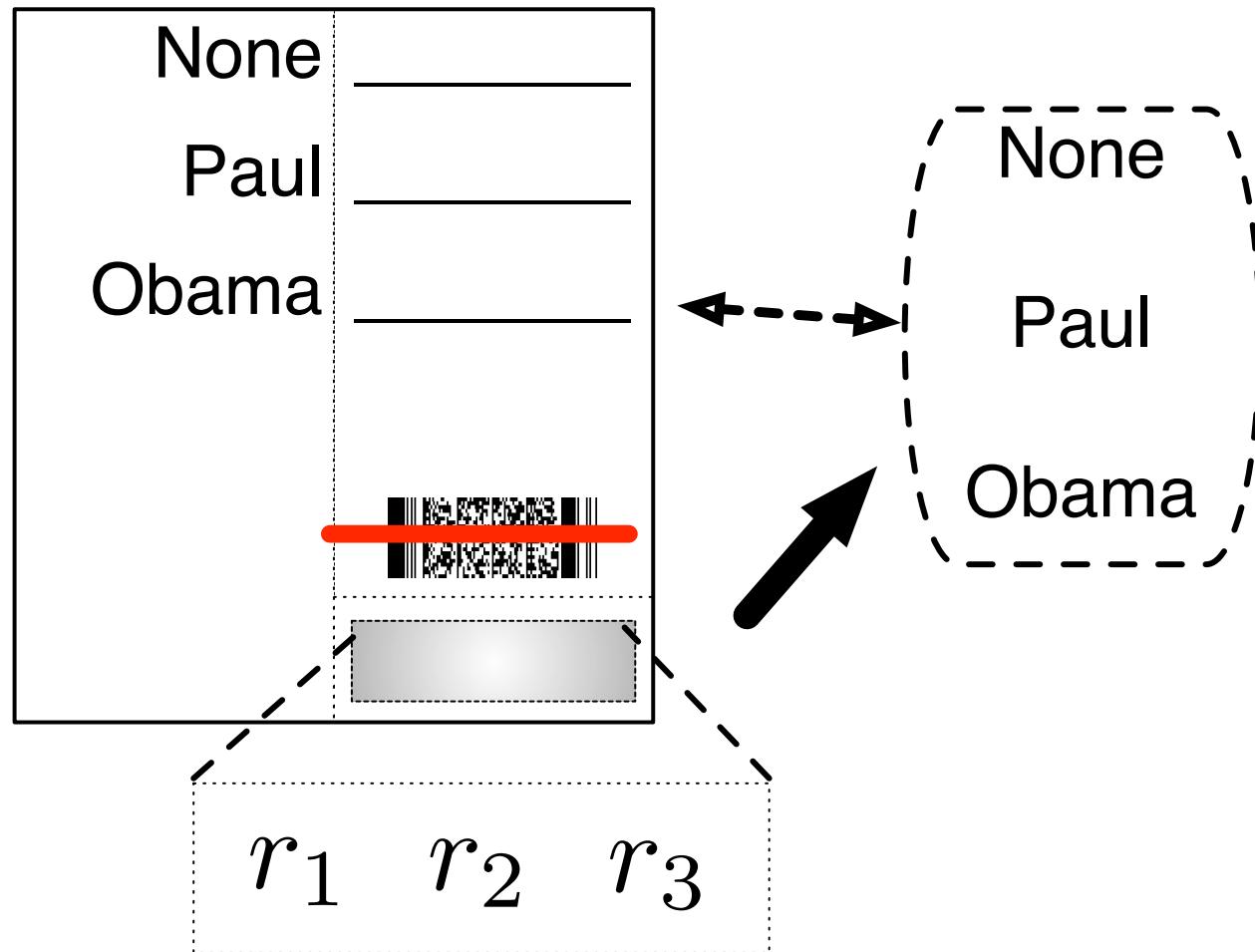
Open envelopes don't prove anything after the fact.

# **Scratch & Vote**

[AR2006]

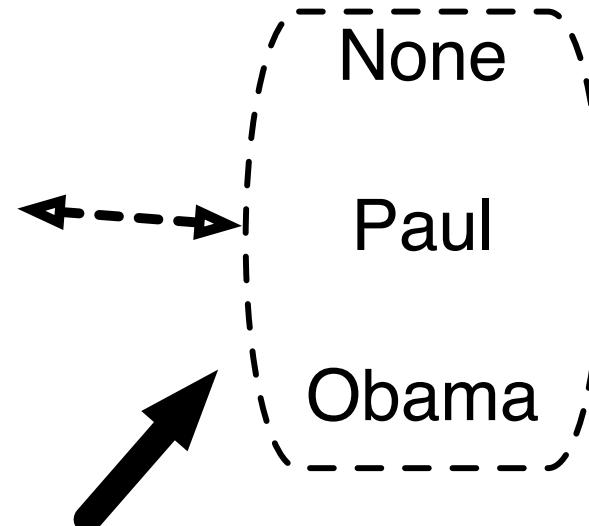


**1. Receive two ballots.**



**2.** Choose one randomly  
for auditing by scratch-off.

**Can be done by  
political organization  
of voter's choice**



$r_1 \quad r_2 \quad r_3$

2. Choose one randomly for auditing by scratch-off.

**In Private**

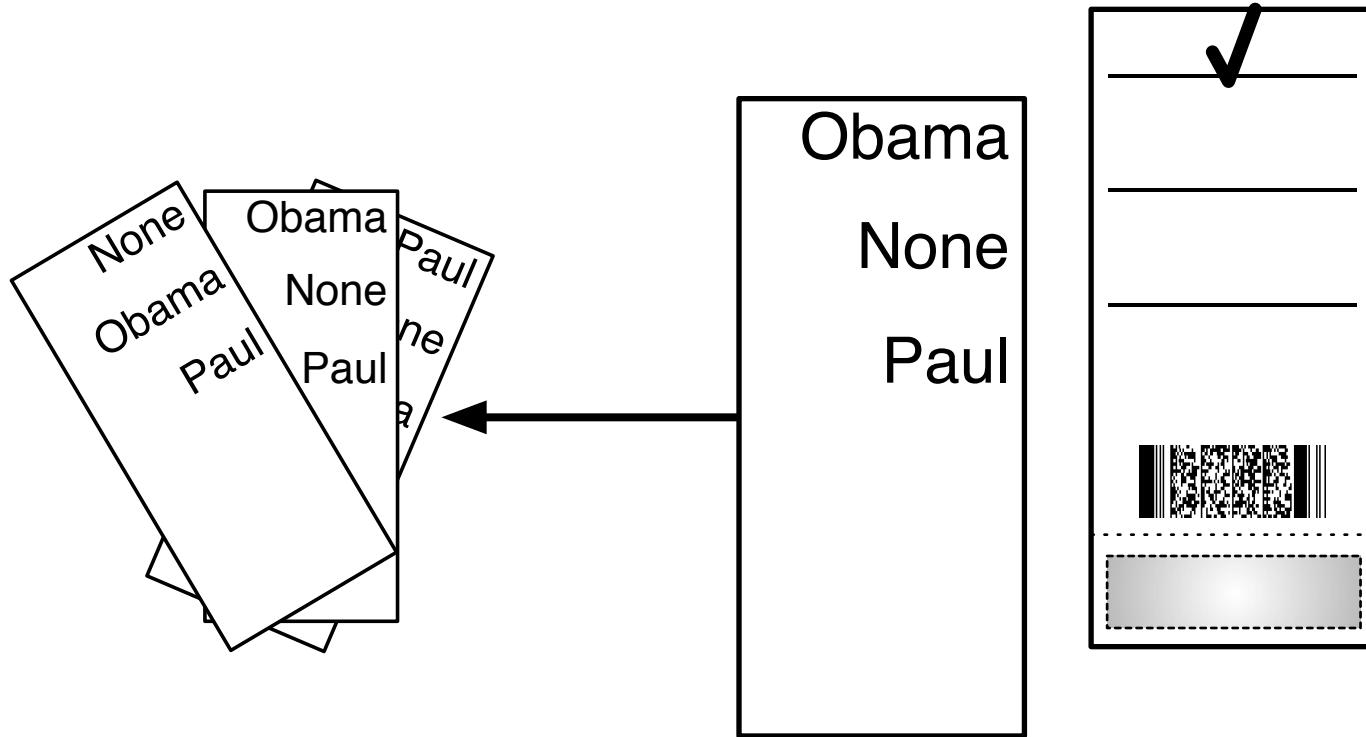
Obama

None

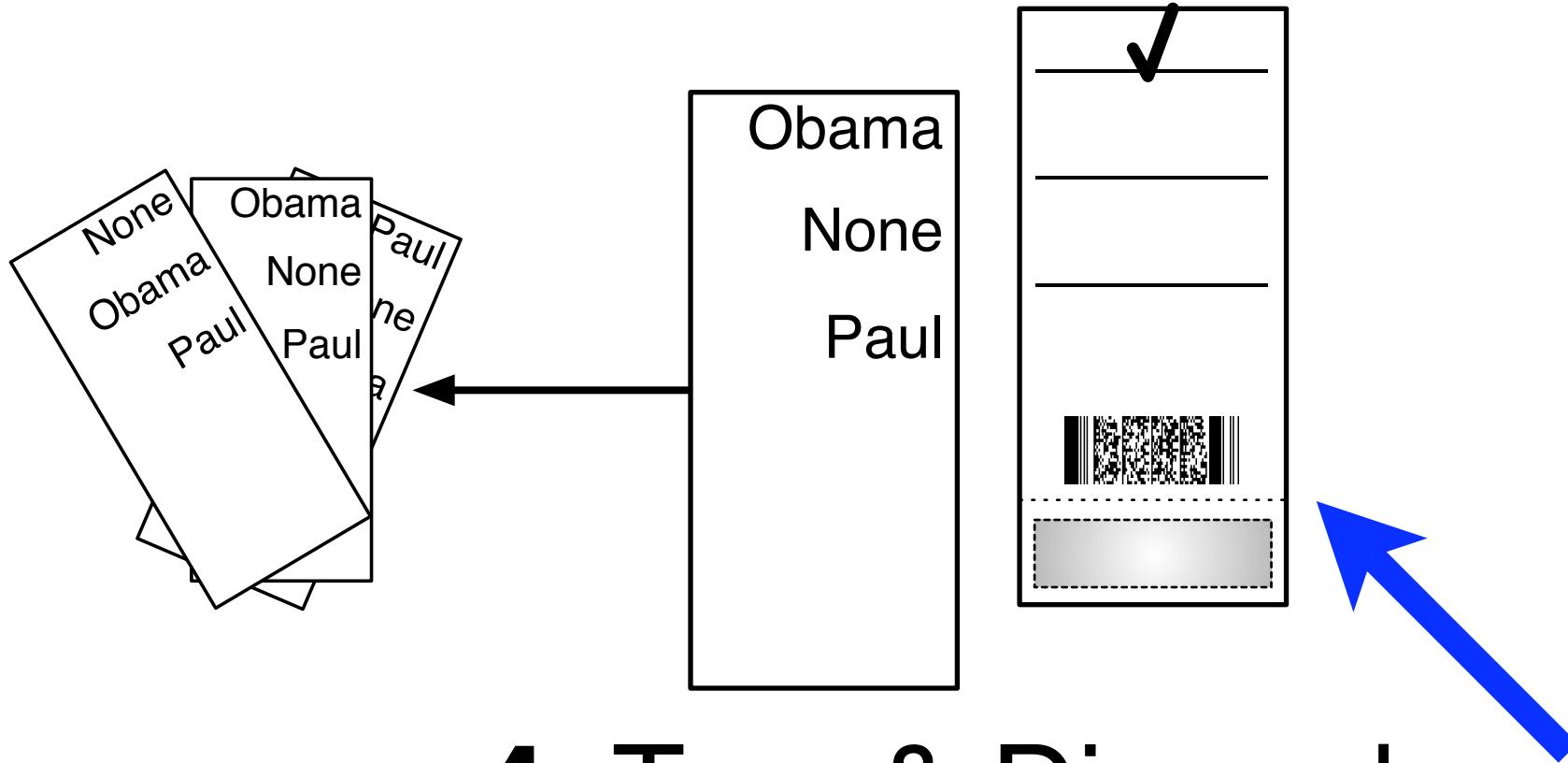
Paul



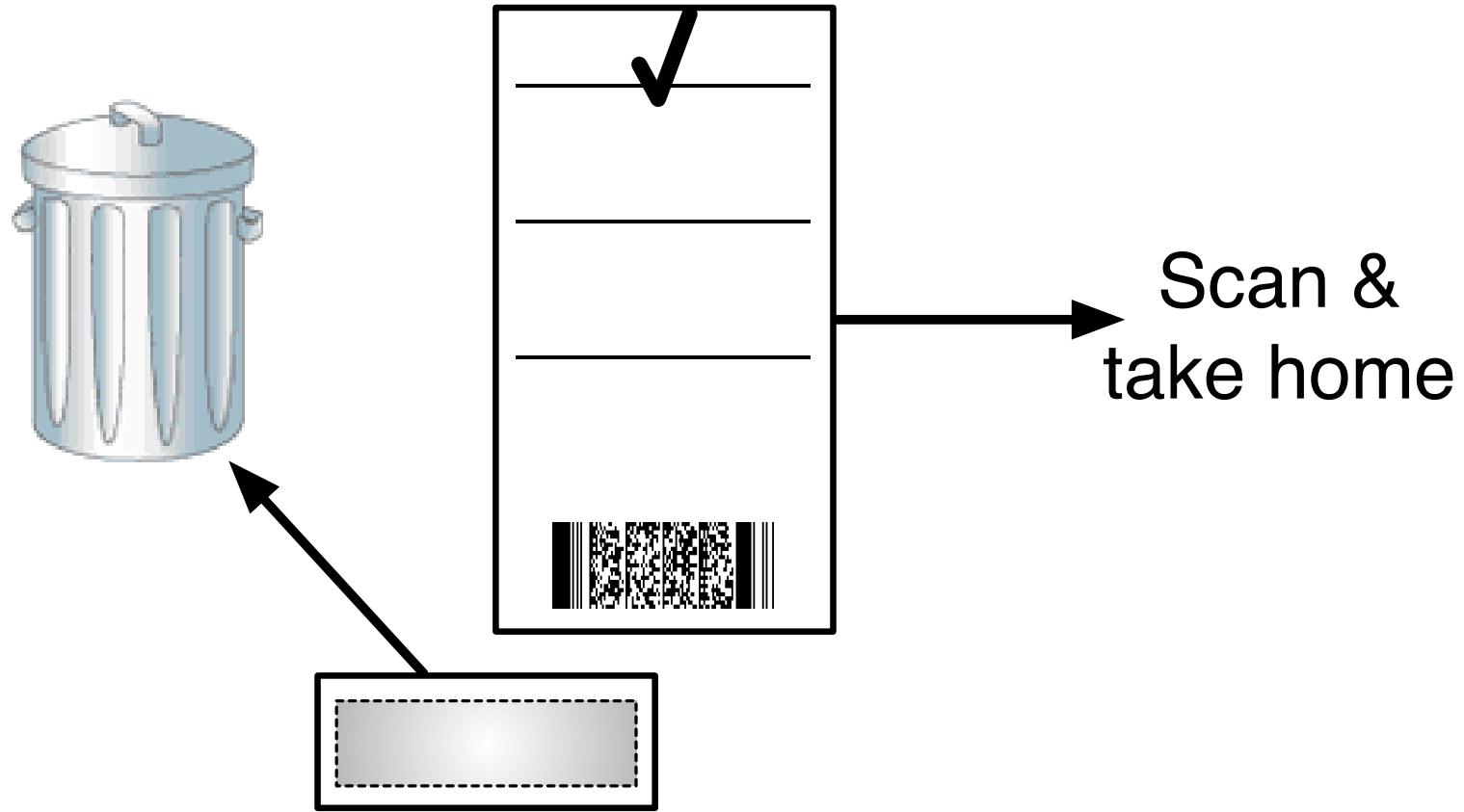
**3. Vote.**



**4. Tear & Discard  
left half of ballot.**



**4. Tear & Discard  
left half of ballot.**



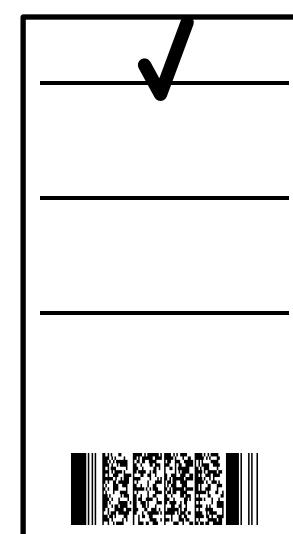
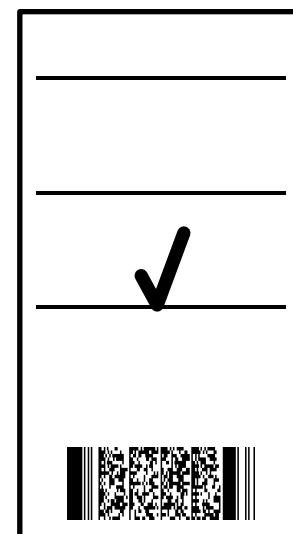
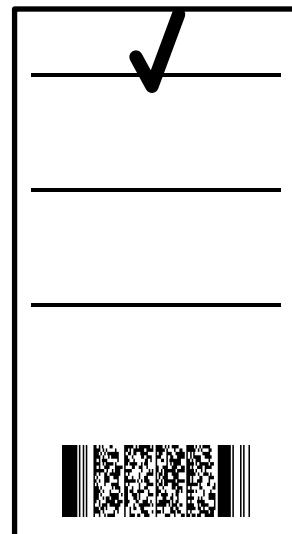
## 5. Tear & Discard scratch-off.

# Bulletin Board

Alice

Bridget

Carol



# El Gamal

**setting:**  $p$  prime,  $q$  prime,  $q|(p - 1)$

**private key:**  $x \in \mathbb{Z}_q^*$

**public key:**  $y = g^x \pmod p$

# El Gamal

**setting:**  $p$  prime,  $q$  prime,  $q|(p - 1)$

**private key:**  $x \in \mathbb{Z}_q^*$

**public key:**  $y = g^x \pmod{p}$

$$r \xleftarrow{R} \mathbb{Z}_q^*$$

$$\text{Enc}_{pk}(m; r) = (\alpha, \beta) = (g^r, m \cdot y^r)$$

# El Gamal

**setting:**  $p$  prime,  $q$  prime,  $q|(p - 1)$

**private key:**  $x \in \mathbb{Z}_q^*$

**public key:**  $y = g^x \pmod{p}$

$$r \xleftarrow{R} \mathbb{Z}_q^*$$

$$\text{Enc}_{pk}(m; r) = (\alpha, \beta) = (g^r, m \cdot y^r)$$

$$\text{Dec}_{sk}(c) = \frac{\beta}{\alpha^x}$$

# Homomorphic Property

$$\text{Enc}(m_1) \times \text{Enc}(m_2) = \text{Enc}(m_1 \times m_2)$$

# Homomorphic Property

$$\text{Enc}(m_1) \times \text{Enc}(m_2) = \text{Enc}(m_1 \times m_2)$$

$$c_1 = (g^{r_1}, m_1 y^{r_1})$$

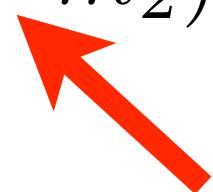
$$c_2 = (g^{r_2}, m_2 y^{r_2})$$

$$c_1 \cdot c_2 = (g^{r_1+r_2}, (m_1 \cdot m_2) y^{r_1+r_2})$$

# Wouldn't it be nice if....

# Wouldn't it be nice if....

$$\text{Enc}_{pk}(m_1) \cdot \text{Enc}_{pk}(m_2) = \text{Enc}_{pk}(m_1 + m_2)$$



# Wouldn't it be nice if....

$$\text{Enc}_{pk}(m_1) \cdot \text{Enc}_{pk}(m_2) = \text{Enc}_{pk}(m_1 + m_2)$$



then we could simply  
sum up votes homomorphically!

# Exponential El Gamal

$$\text{Enc}_{pk}(m, r) = (g^r, g^m y^r)$$

First: r'th residuosity [Benaloh85]  
Also: Paillier Cryptosystem [P99]

# Exponential El Gamal

$$\text{Enc}_{pk}(m, r) = (g^r, g^m y^r)$$



First: r'th residuosity [Benaloh85]  
Also: Paillier Cryptosystem [P99]

# Exponential El Gamal

$$\text{Enc}_{pk}(m, r) = (g^r, g^m y^r)$$



$$\text{Dec}_{sk}(c) = g^m$$

First: r'th residuosity [Benaloh85]  
Also: Paillier Cryptosystem [P99]

# Exponential El Gamal

$$\text{Enc}_{pk}(m, r) = (g^r, g^m y^r)$$



$$\text{Dec}_{sk}(c) = g^m$$

Take the discrete log base g.

First: r'th residuosity [Benaloh85]  
Also: Paillier Cryptosystem [P99]

# Homomorphic Tallying

|      |      |      |
|------|------|------|
| 0001 | 0000 | 0000 |
|------|------|------|

Vote for None

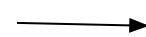
|      |      |      |
|------|------|------|
| 0000 | 0001 | 0000 |
|------|------|------|

Vote for Obama

|      |      |      |
|------|------|------|
| 0000 | 0000 | 0001 |
|------|------|------|

Vote for Paul

|      |      |      |
|------|------|------|
| 0003 | 0006 | 0005 |
|------|------|------|



Sample Tally

[B+2001, PI999]

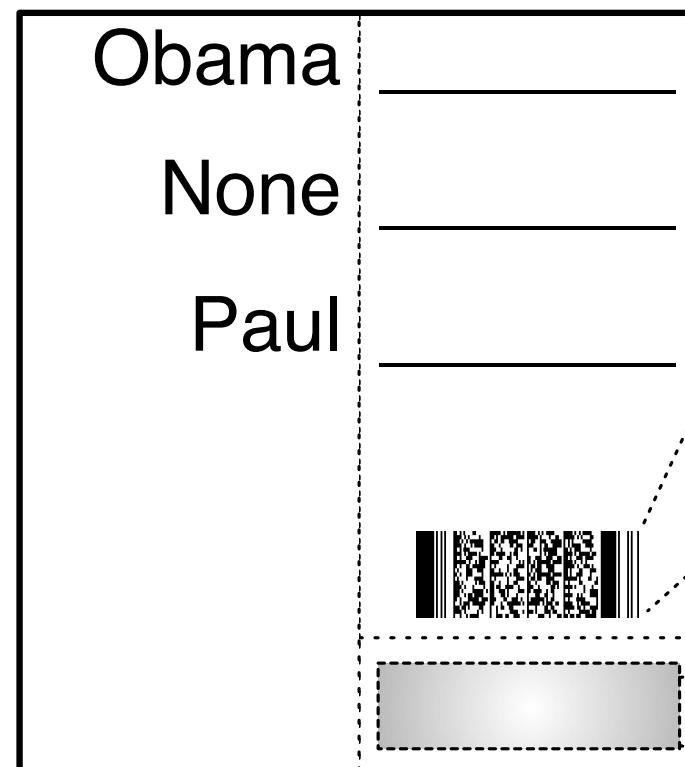
## PARAMETERS

#1 - Paul

#2 - Obama

#3 - None

M=10, Public Key =  $pk$



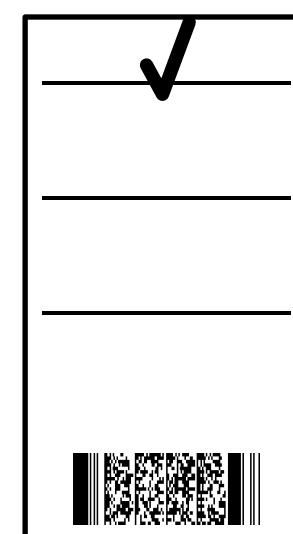
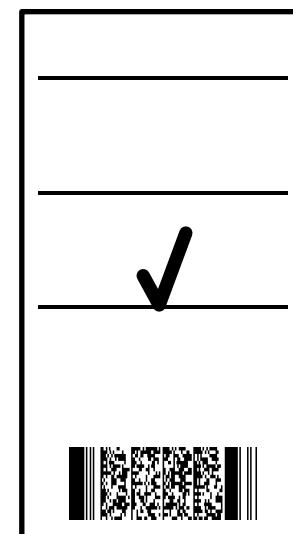
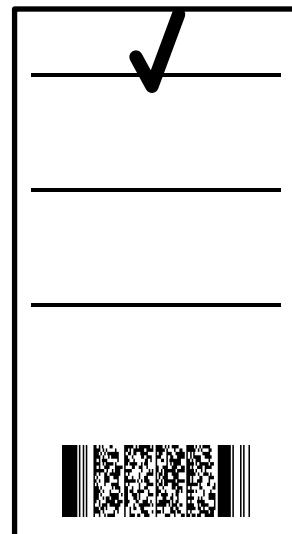
$$\begin{aligned}\mathcal{E}_{pk}(2^{10}; r_1) \\ \mathcal{E}_{pk}(2^{20}; r_2) \\ \mathcal{E}_{pk}(2^0; r_3)\end{aligned}$$

# Bulletin Board

Alice

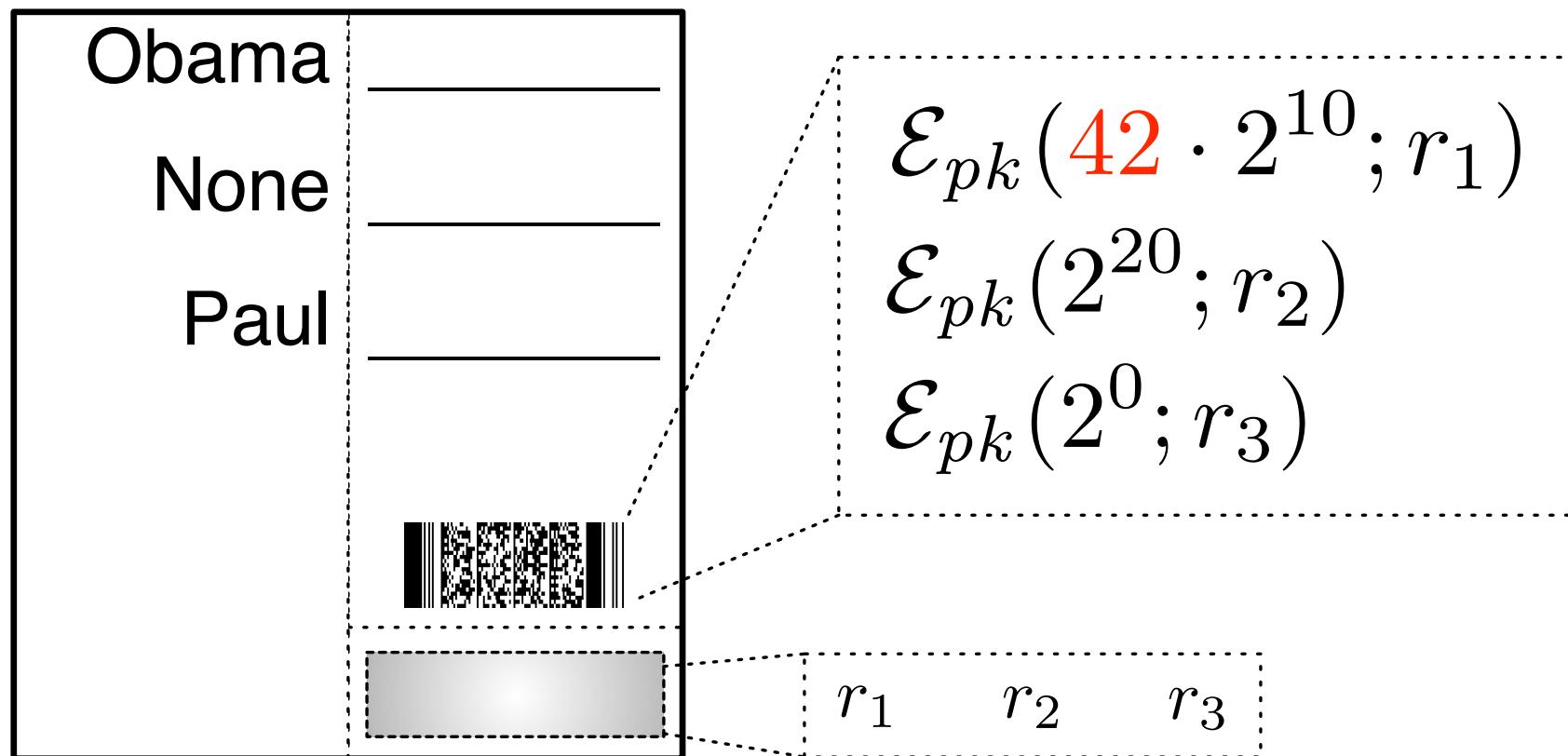
Bridget

Carol



# **Be Careful...**

# Be Careful...



# Summary of S & V

|       |                                     |
|-------|-------------------------------------|
| Obama | <input checked="" type="checkbox"/> |
| None  | <input type="checkbox"/>            |
| Paul  | <input type="checkbox"/>            |

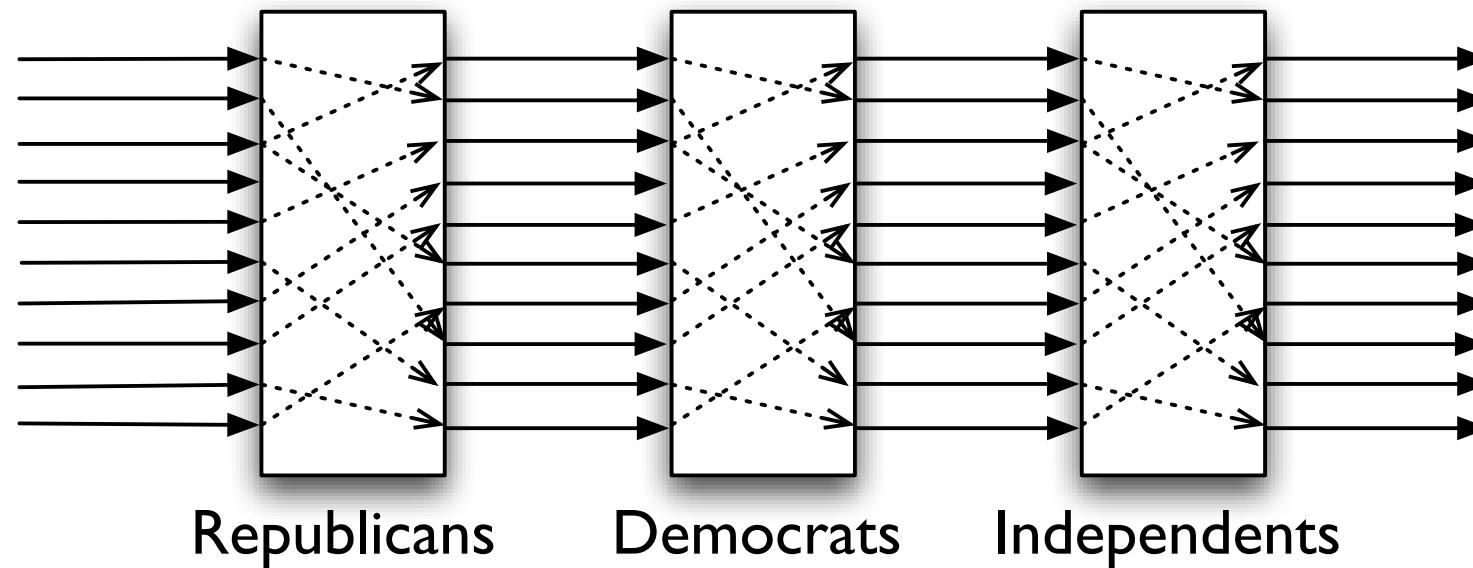


Scratch & Vote is **one** system.  
There are many others.

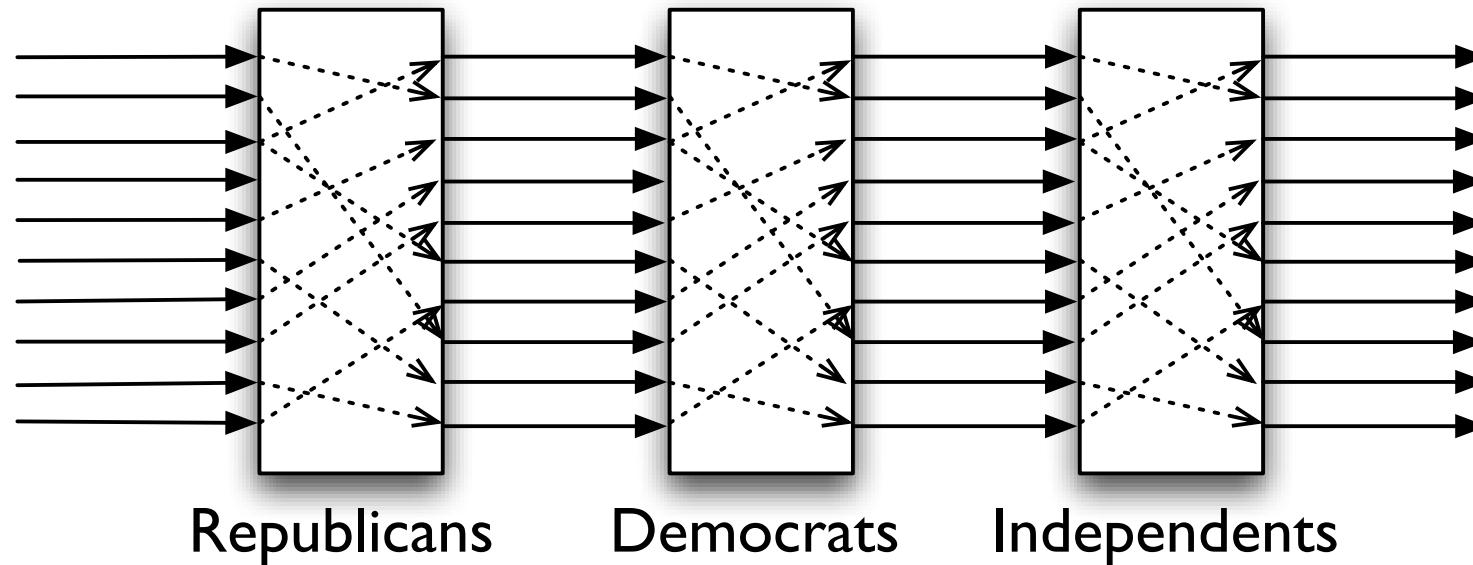
# What about write-ins?

Must preserve  
individual ballots.

# Mixnets

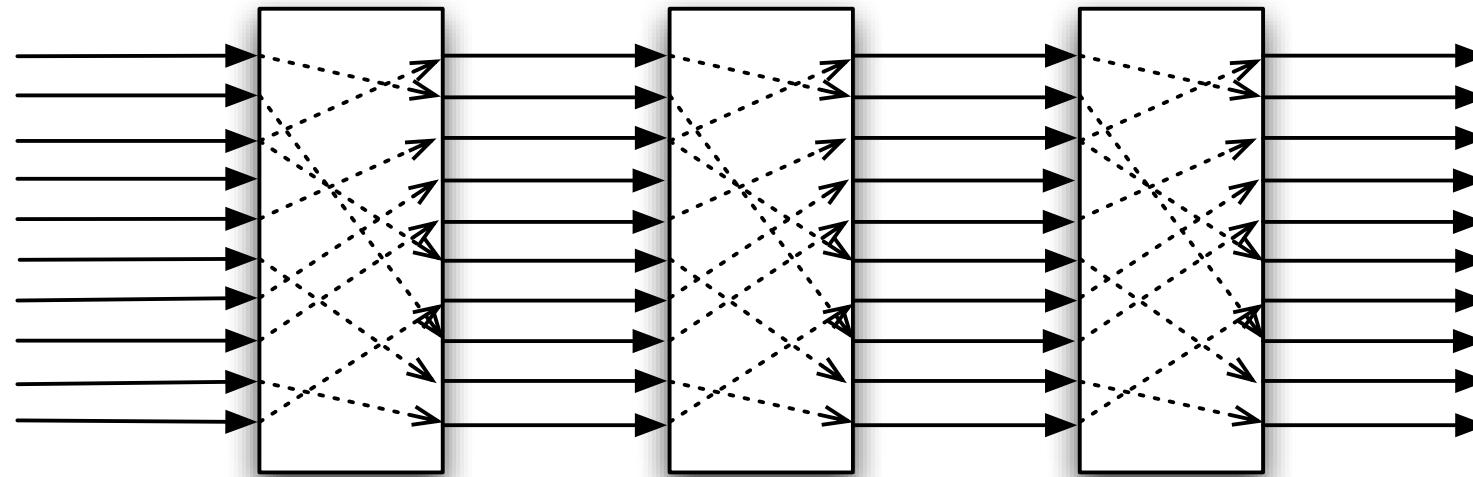


# Mixnets



Mix servers operated by  
mutually suspicious organizations.

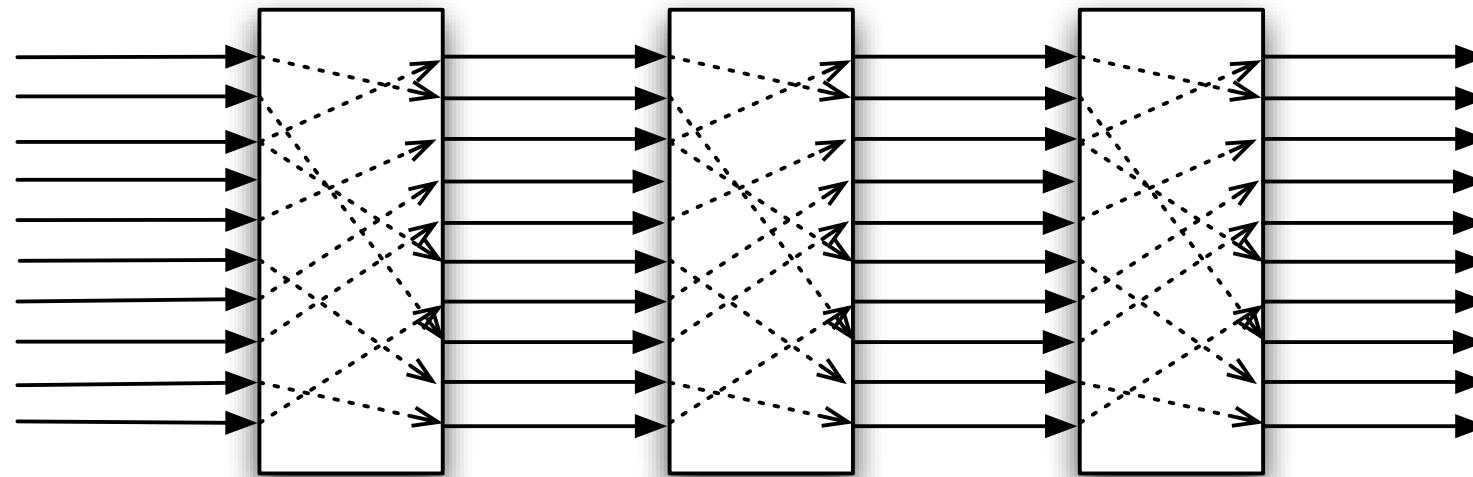
# Chaumian Mixnet (Onions)



$$c_j = \text{Enc}_{pk_1}(\text{Enc}_{pk_2}(\text{Enc}_{pk_3}(m_j)))$$

[Chaum81]

# Chaumian Mixnet (Onions)

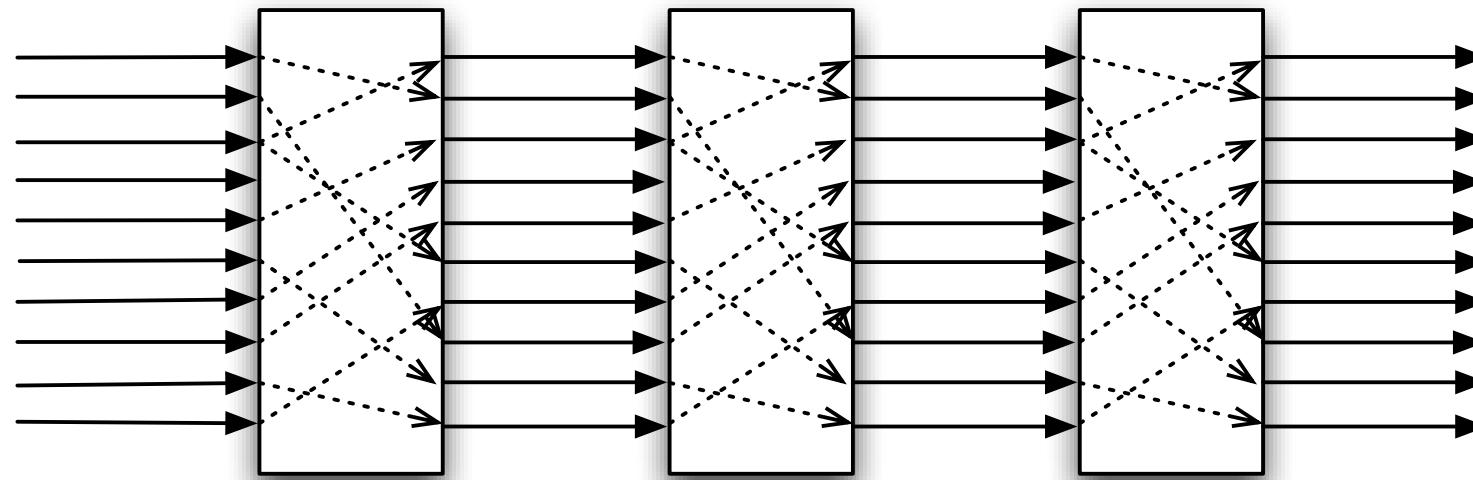


$$c_j = \text{Enc}_{pk_1}(\text{Enc}_{pk_2}(\text{Enc}_{pk_3}(m_j)))$$

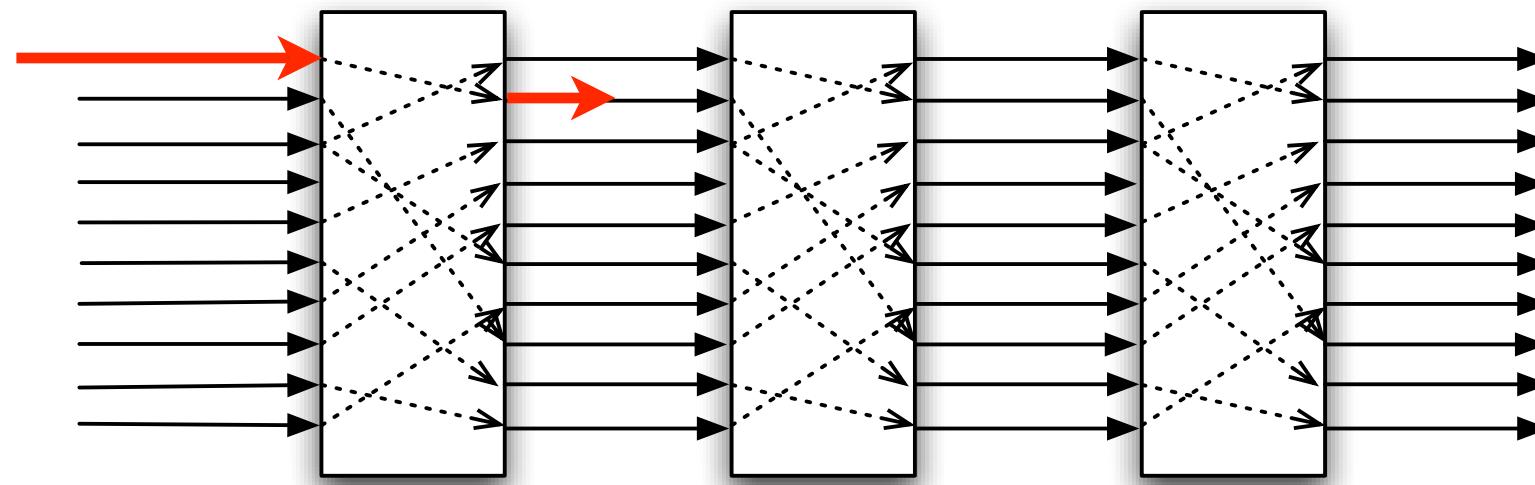
Each mix server “unwraps”  
a layer of this encryption onion.

[Chaum81]

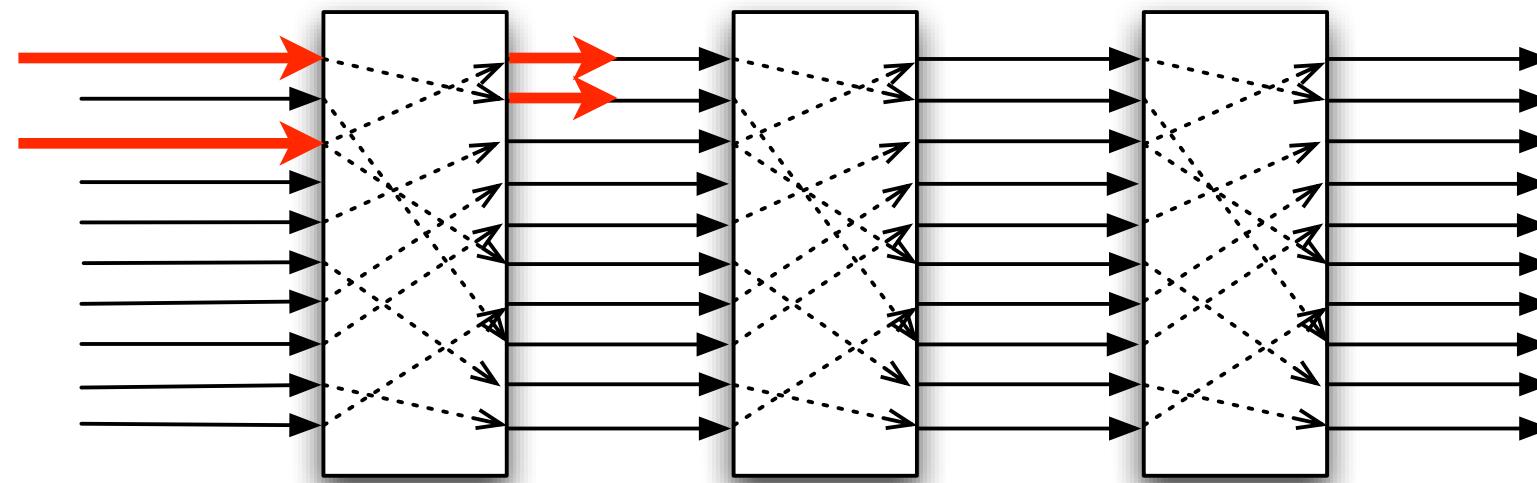
# Verifying a Mixnet



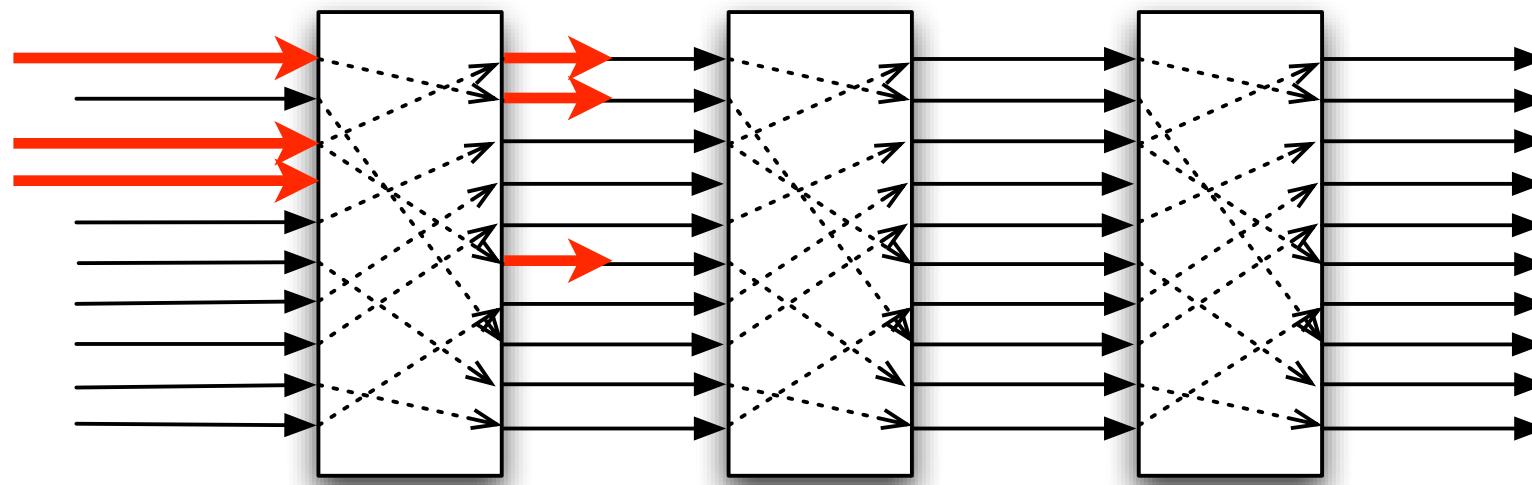
# Verifying a Mixnet



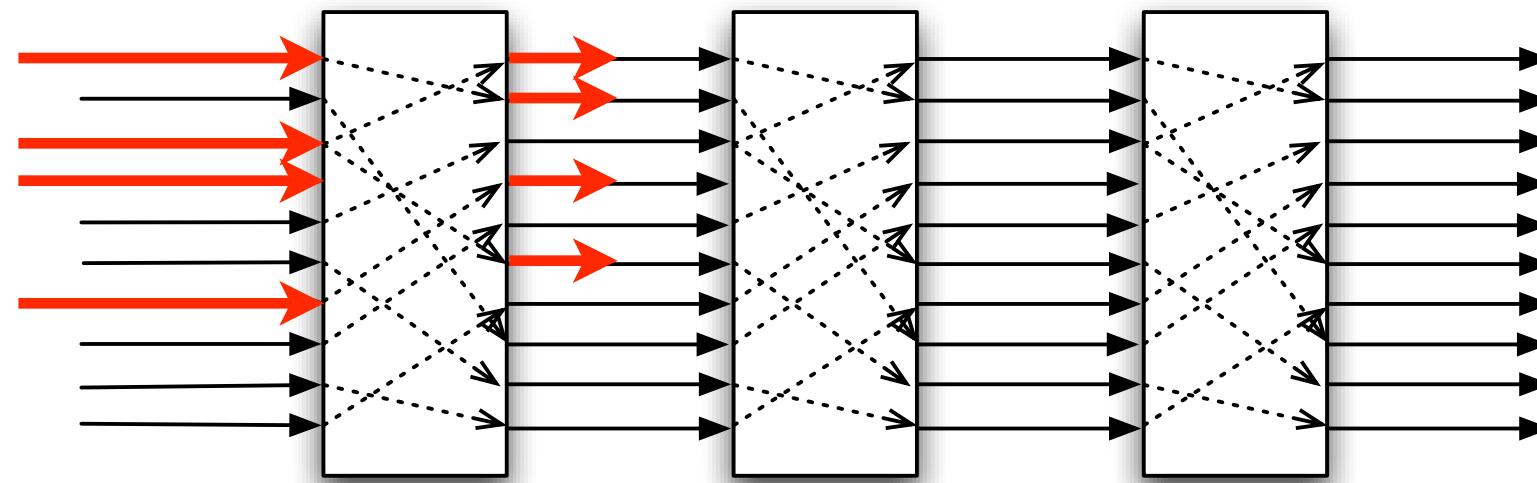
# Verifying a Mixnet



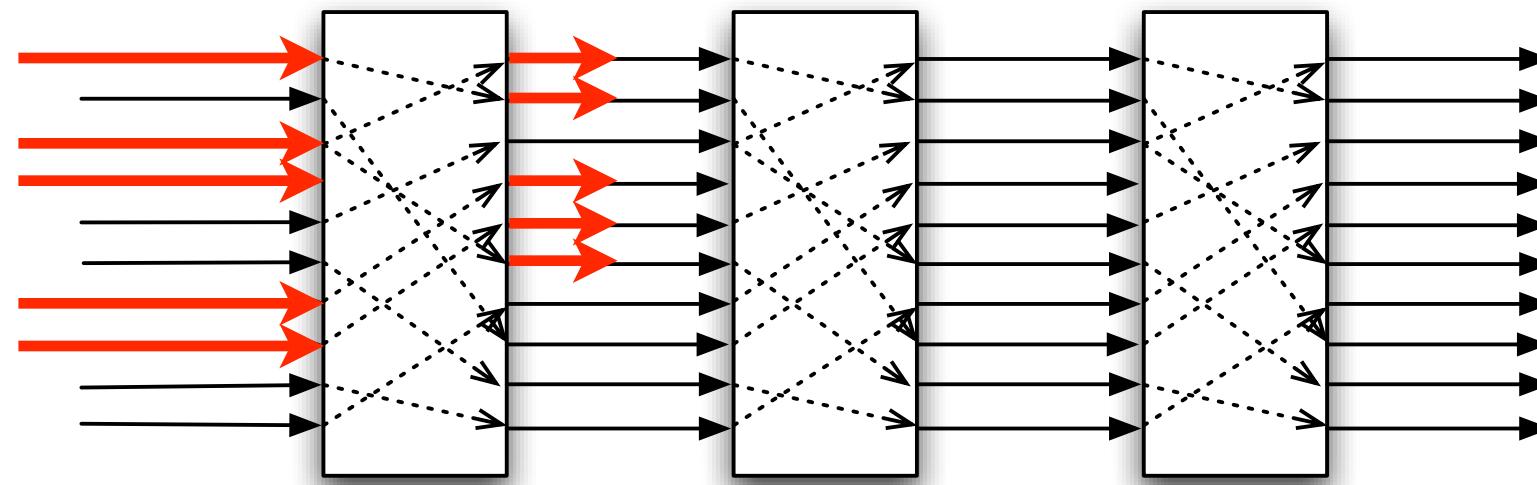
# Verifying a Mixnet



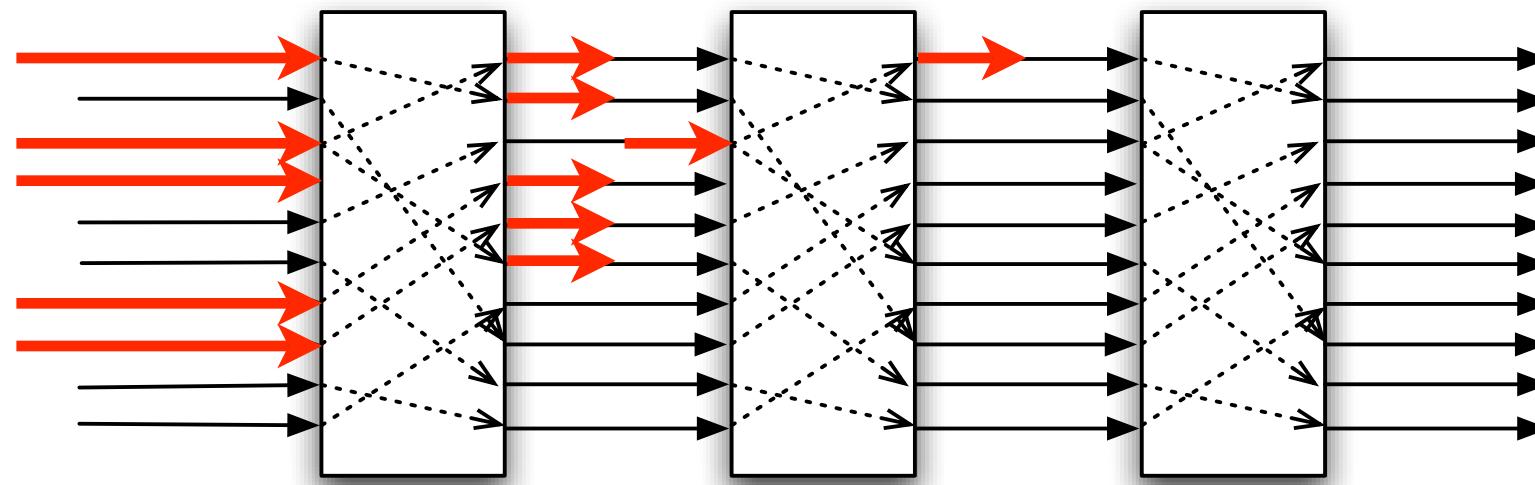
# Verifying a Mixnet



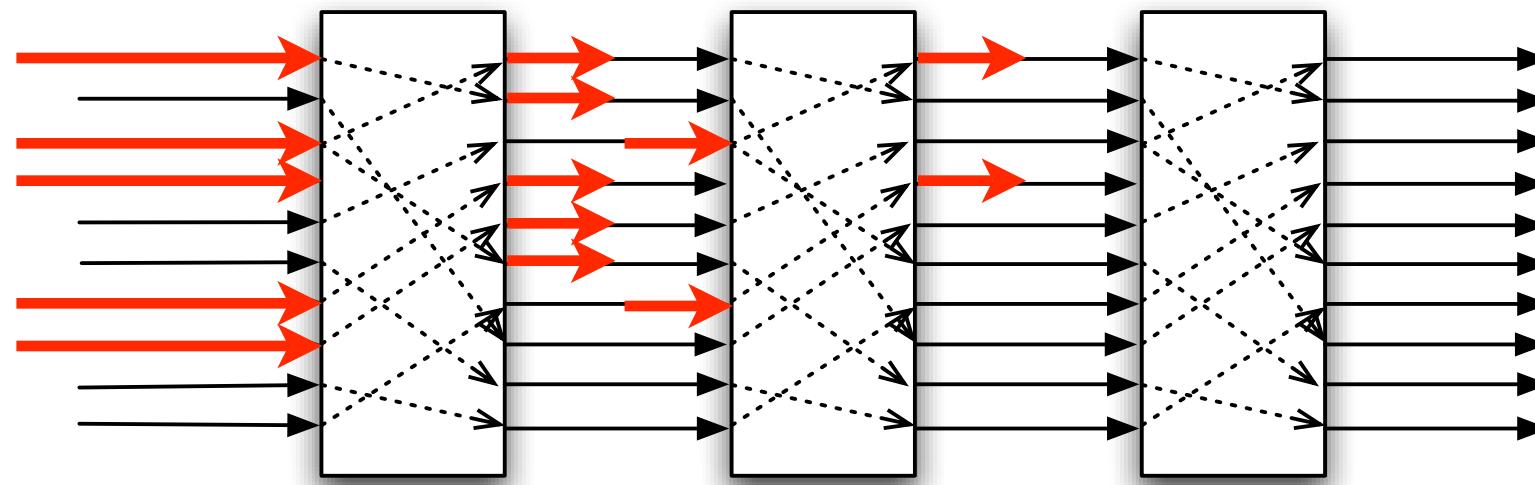
# Verifying a Mixnet



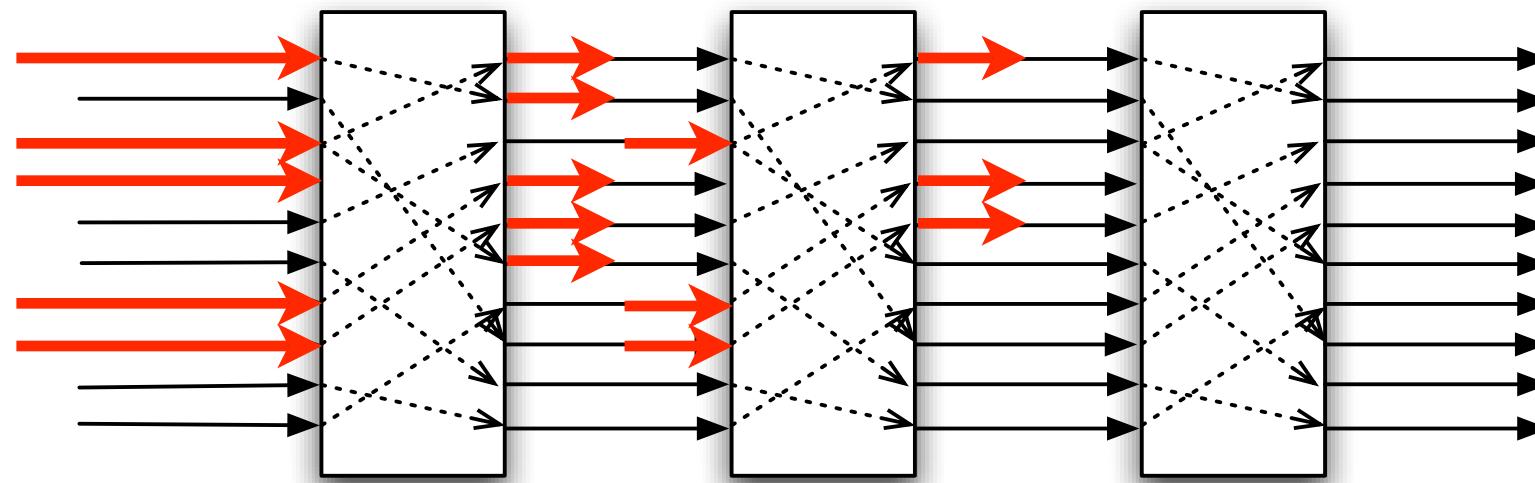
# Verifying a Mixnet



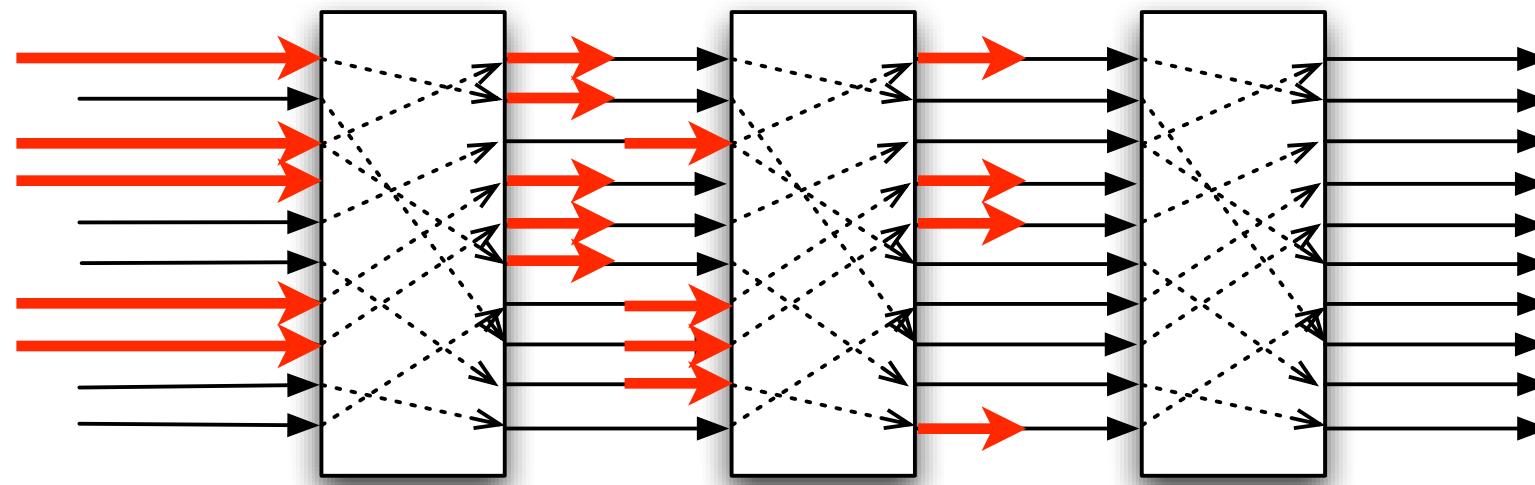
# Verifying a Mixnet



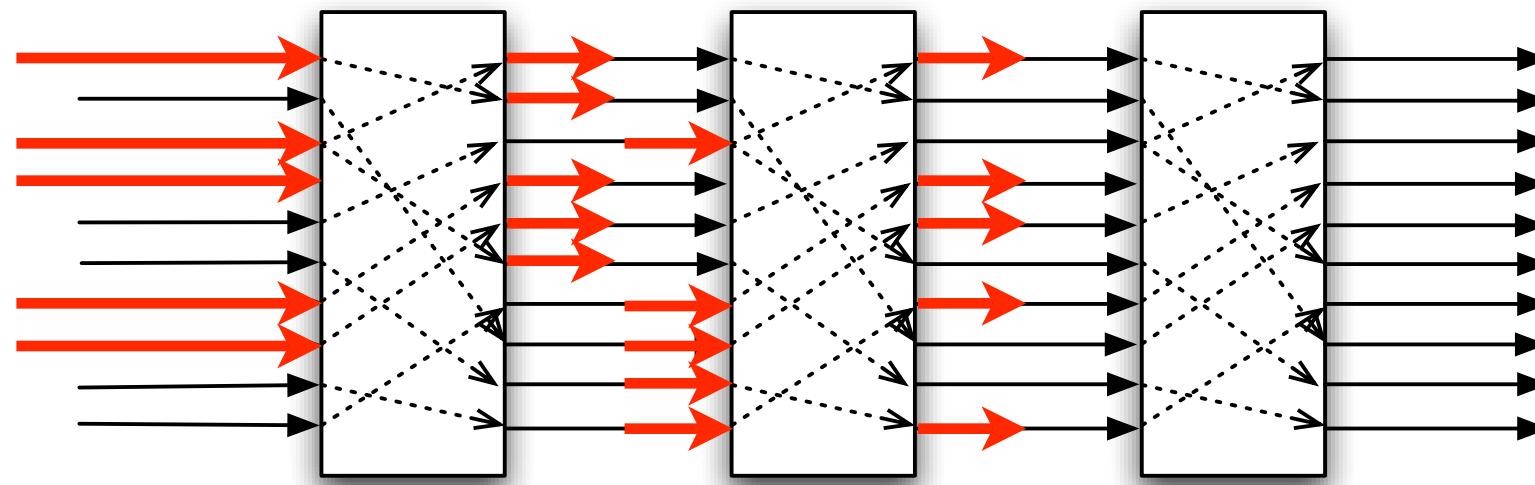
# Verifying a Mixnet



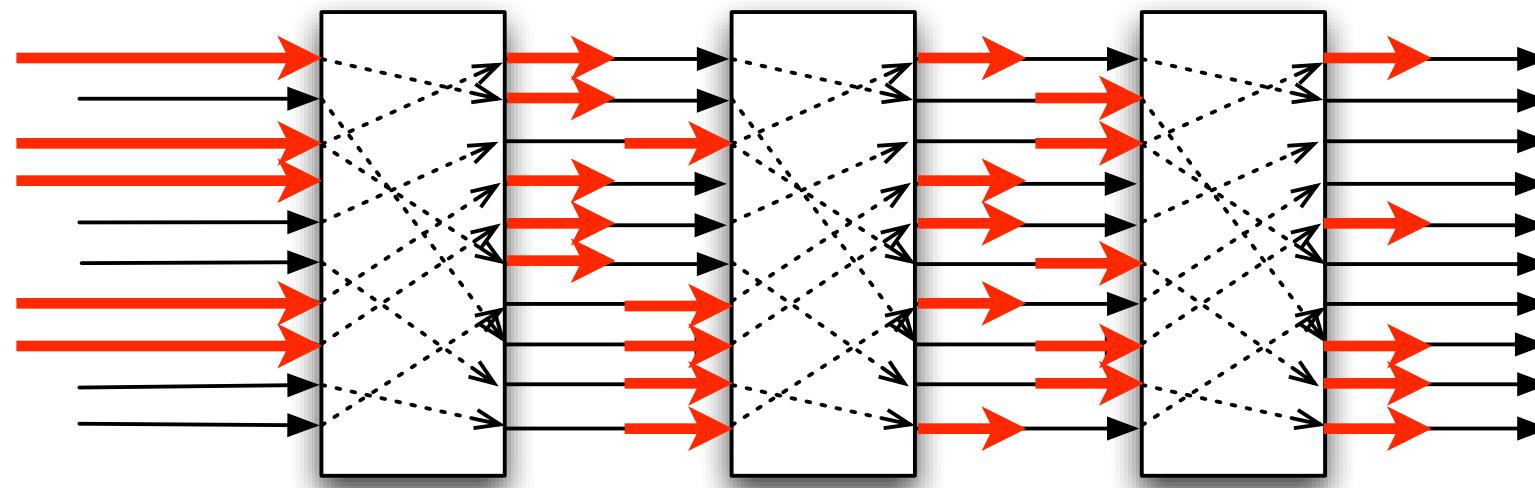
# Verifying a Mixnet



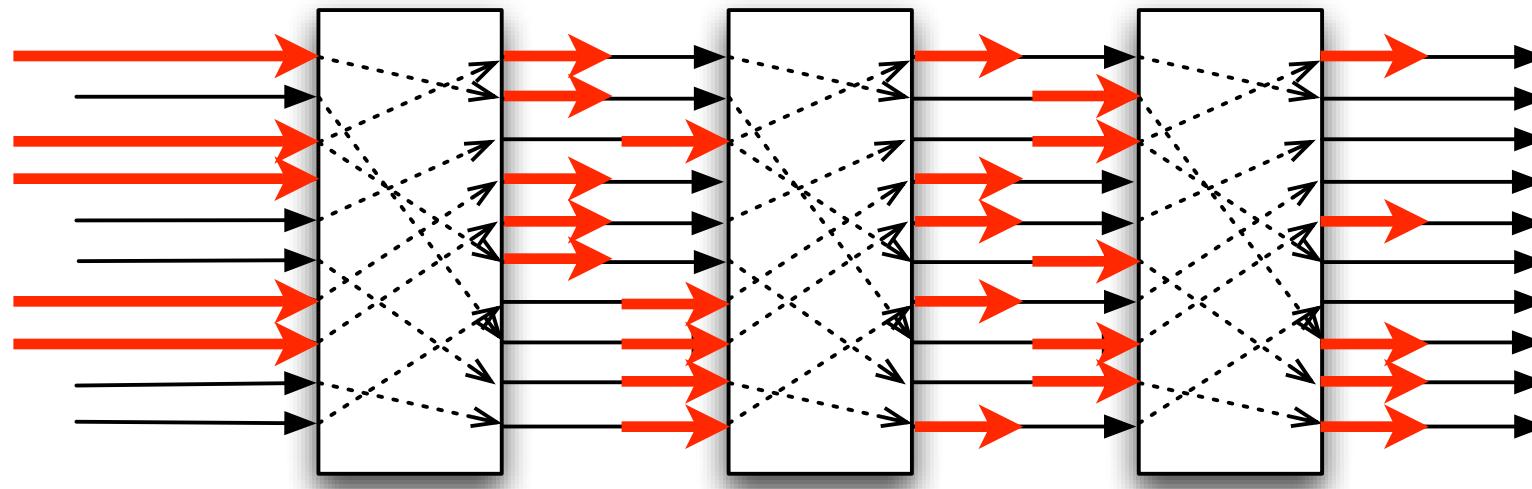
# Verifying a Mixnet



# Verifying a Mixnet



# Verifying a Mixnet



Tricks to ensure  
no complete path is revealed.

# El Gamal Reencryption

# El Gamal Reencryption

$$sk = x \bmod q \quad pk = y = g^x \bmod p$$

# El Gamal Reencryption

$$sk = x \bmod q \quad pk = y = g^x \bmod p$$

$$\text{Enc}_{pk}(m; r) = (\alpha, \beta) = (g^r, m \cdot y^r)$$

$$\text{Dec}_{sk}(c) = \frac{\beta}{\alpha^x}$$

# El Gamal Reencryption

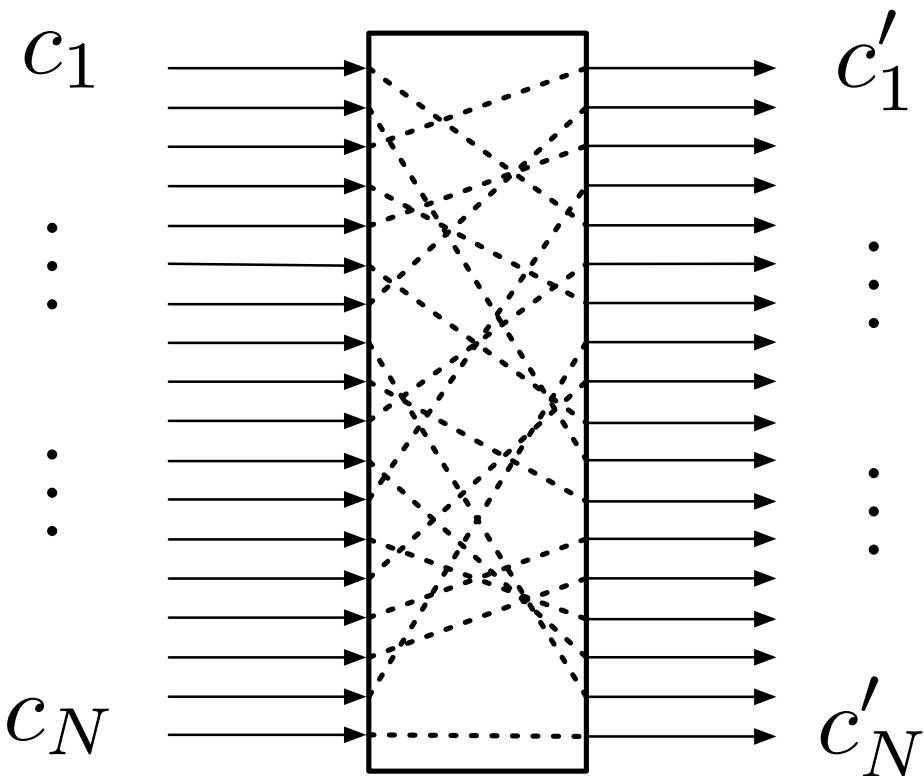
$$sk = x \bmod q \quad pk = y = g^x \bmod p$$

$$\text{Enc}_{pk}(m; r) = (\alpha, \beta) = (g^r, m \cdot y^r)$$

$$\text{Dec}_{sk}(c) = \frac{\beta}{\alpha^x}$$

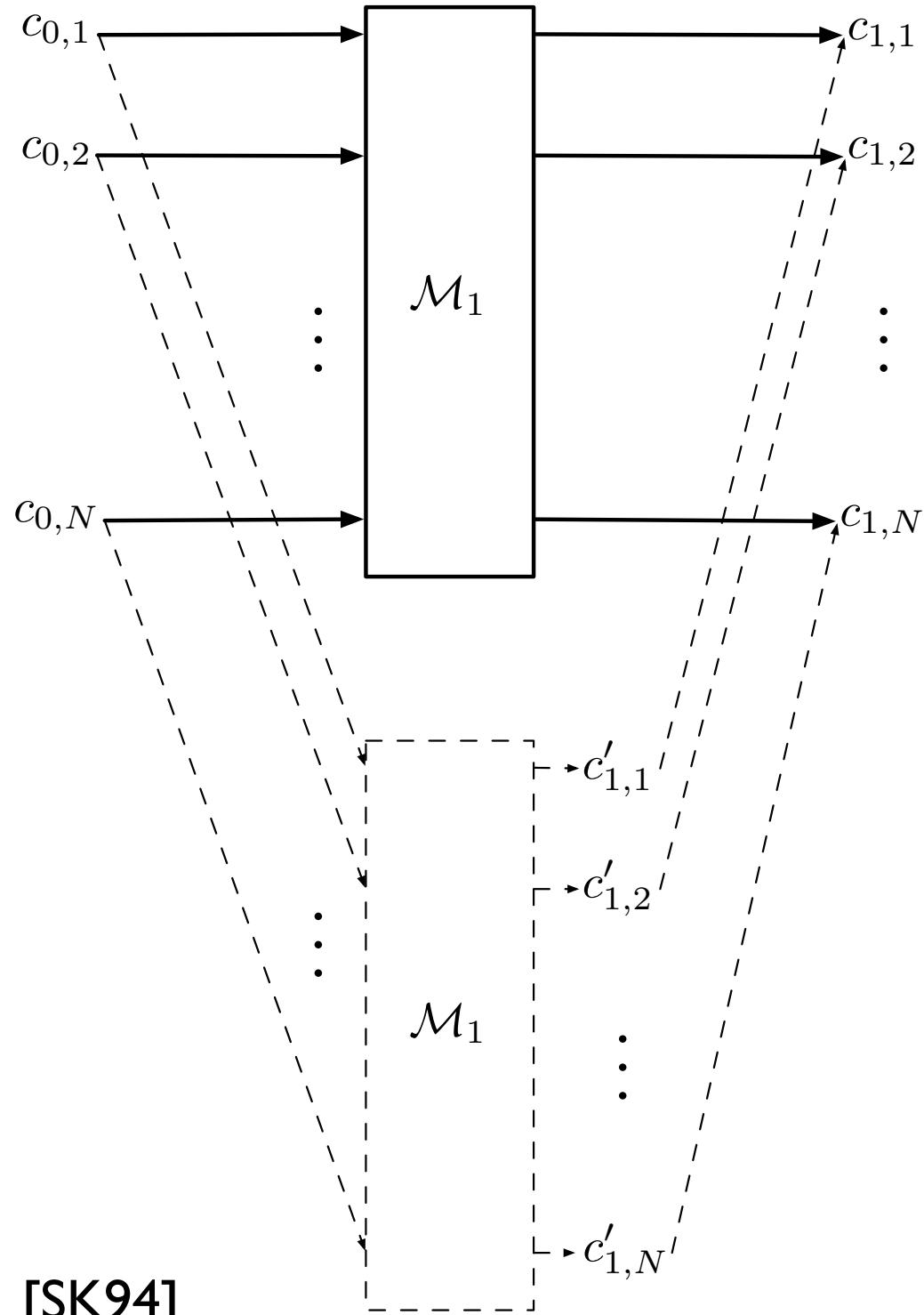
$$\begin{aligned}\text{Reenc}_{pk}(c; r') &= c \cdot \text{Enc}_{pk}(1, r') \\ &= (g^{r+r'}, m \cdot y^{r+r'})\end{aligned}$$

# Re-encryption Mixnet

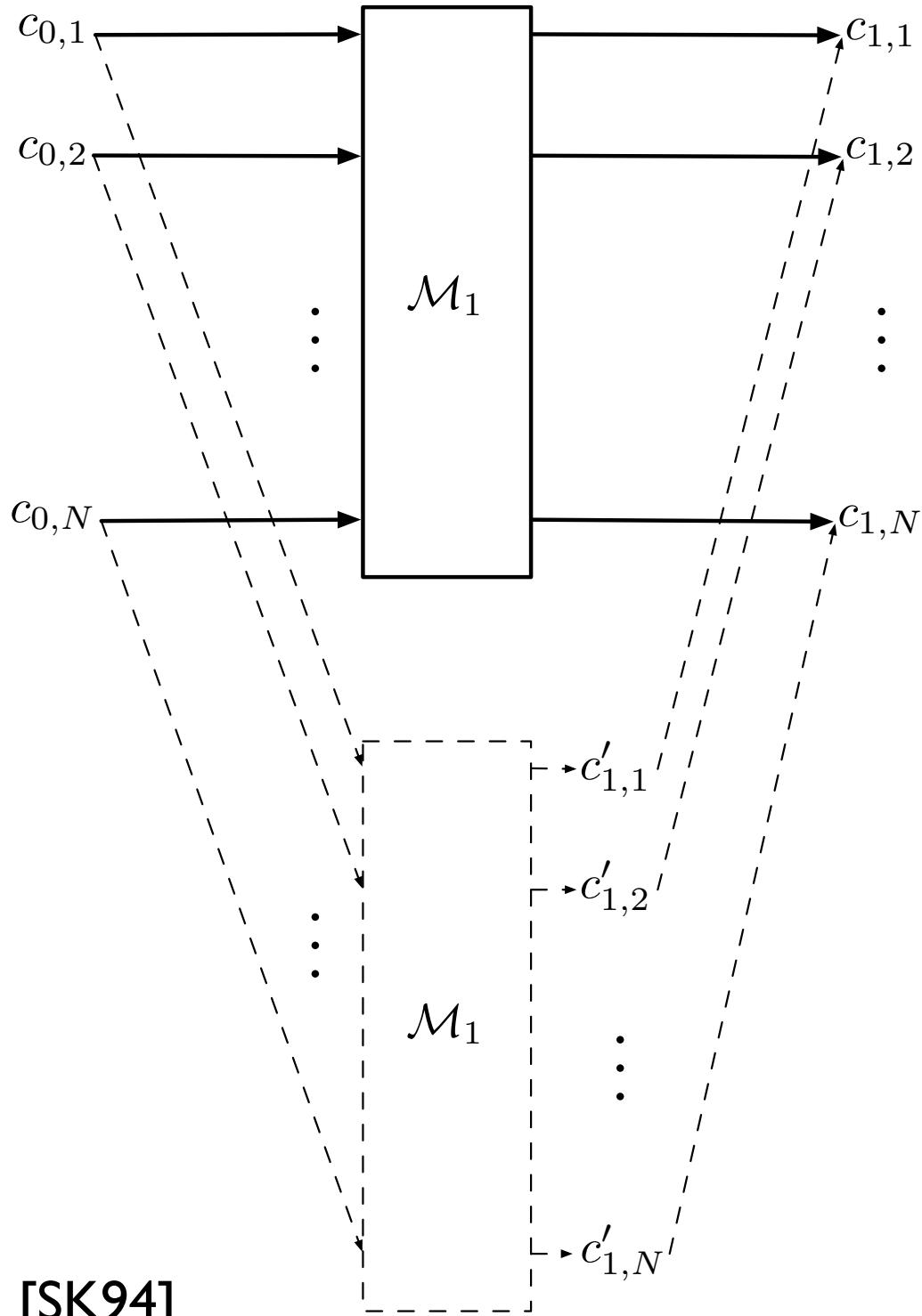


$$c'_{\pi(j)} = \text{Reenc}(c_j; r_j)$$

# Proof of Mixnet



# Proof of Mixnet

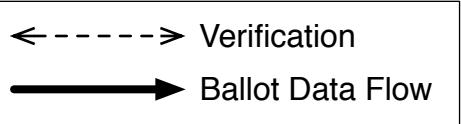


Intermediate mix.  
Coin flip determines:  
reveal first or second.

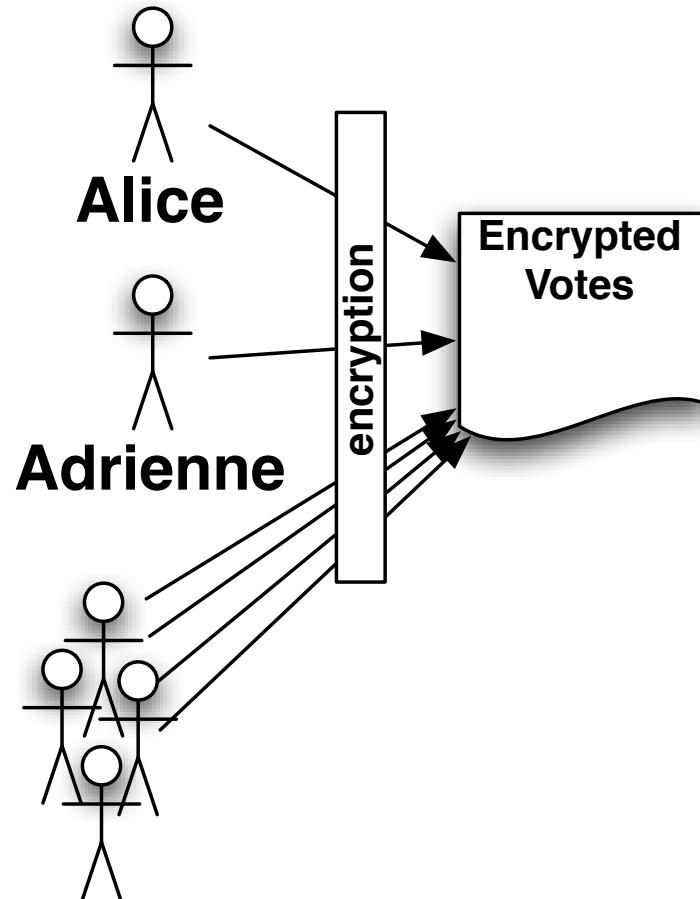
# Decryption

- **Threshold**  
multiple parties needed to decrypt
- **Provable**  
public proof of correct decryption

# Crypto Voting Schemes

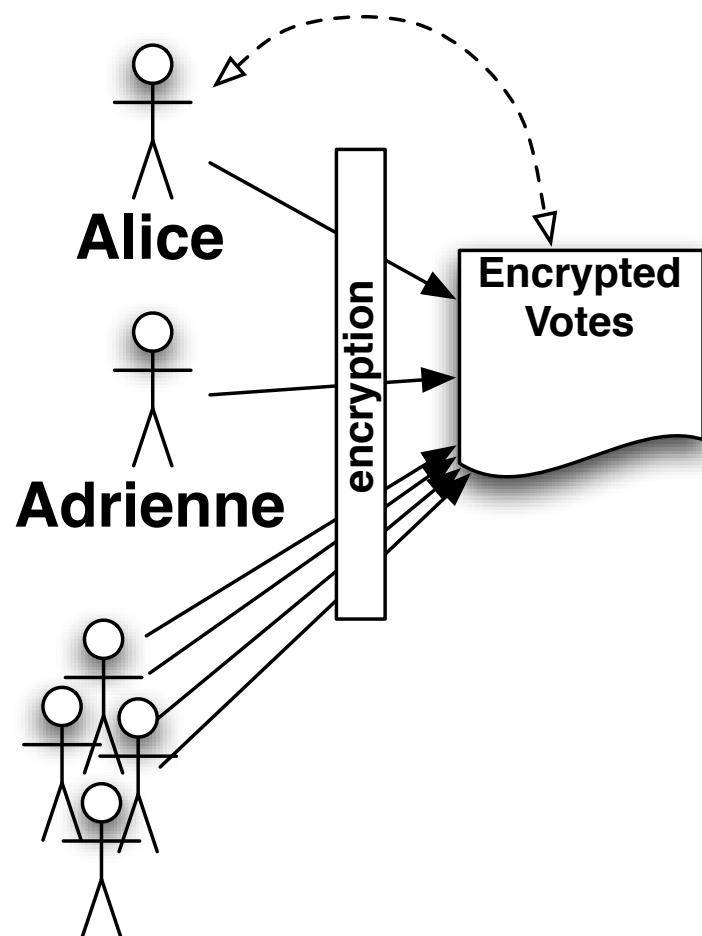


# Crypto Voting Schemes



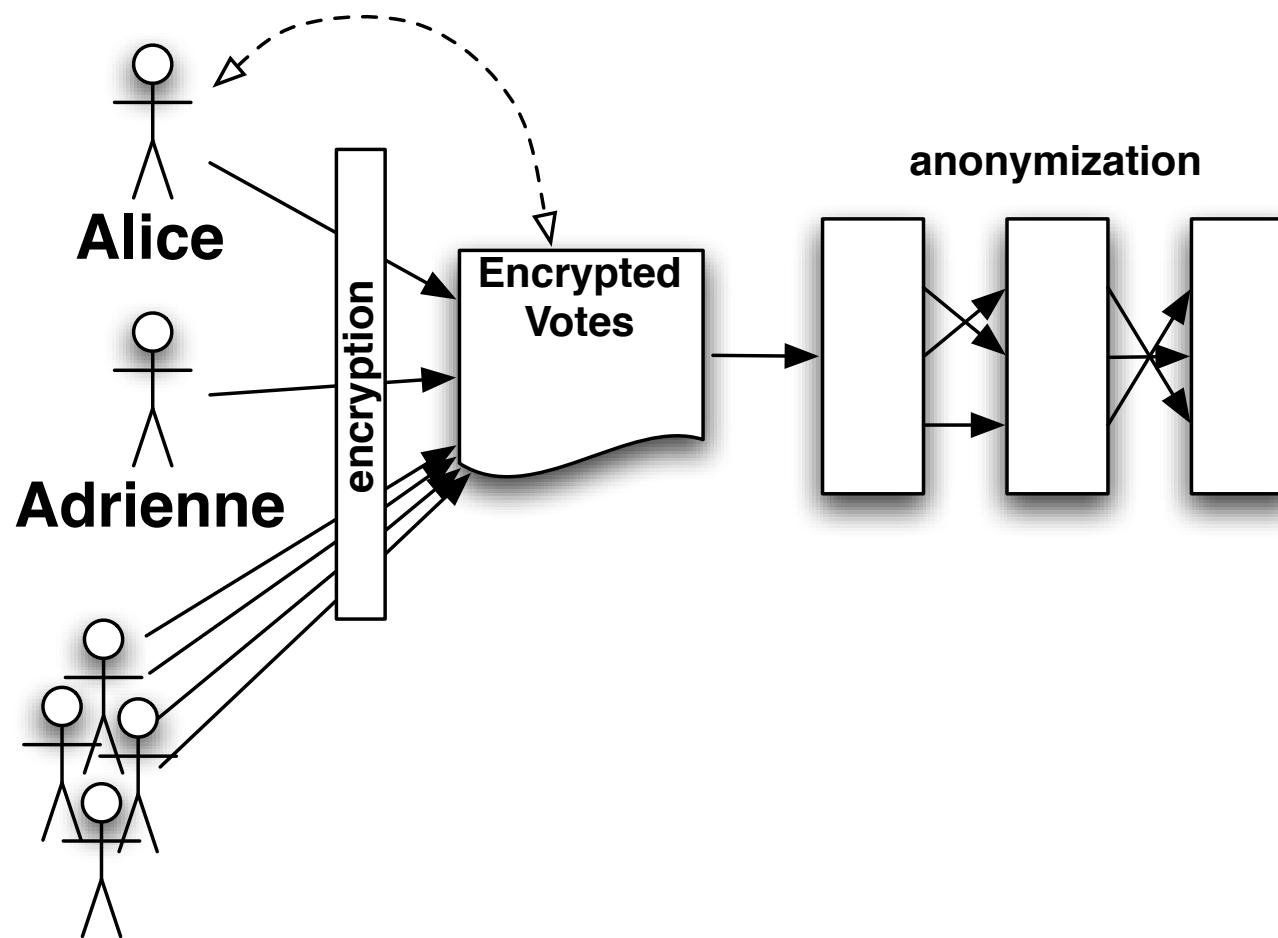
←-----→ Verification  
→ Ballot Data Flow

# Crypto Voting Schemes



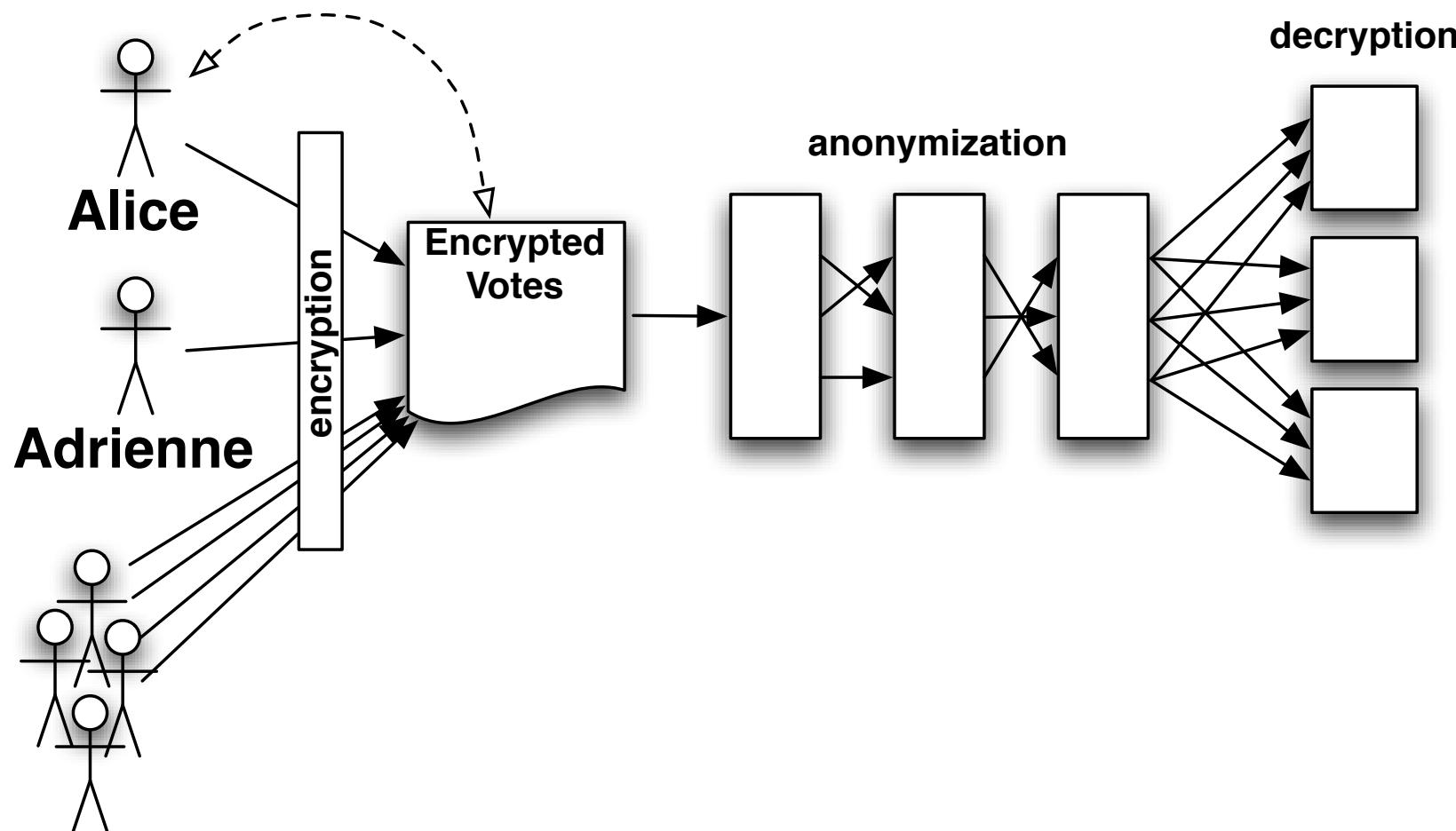
←-----→ Verification  
→ Ballot Data Flow

# Crypto Voting Schemes



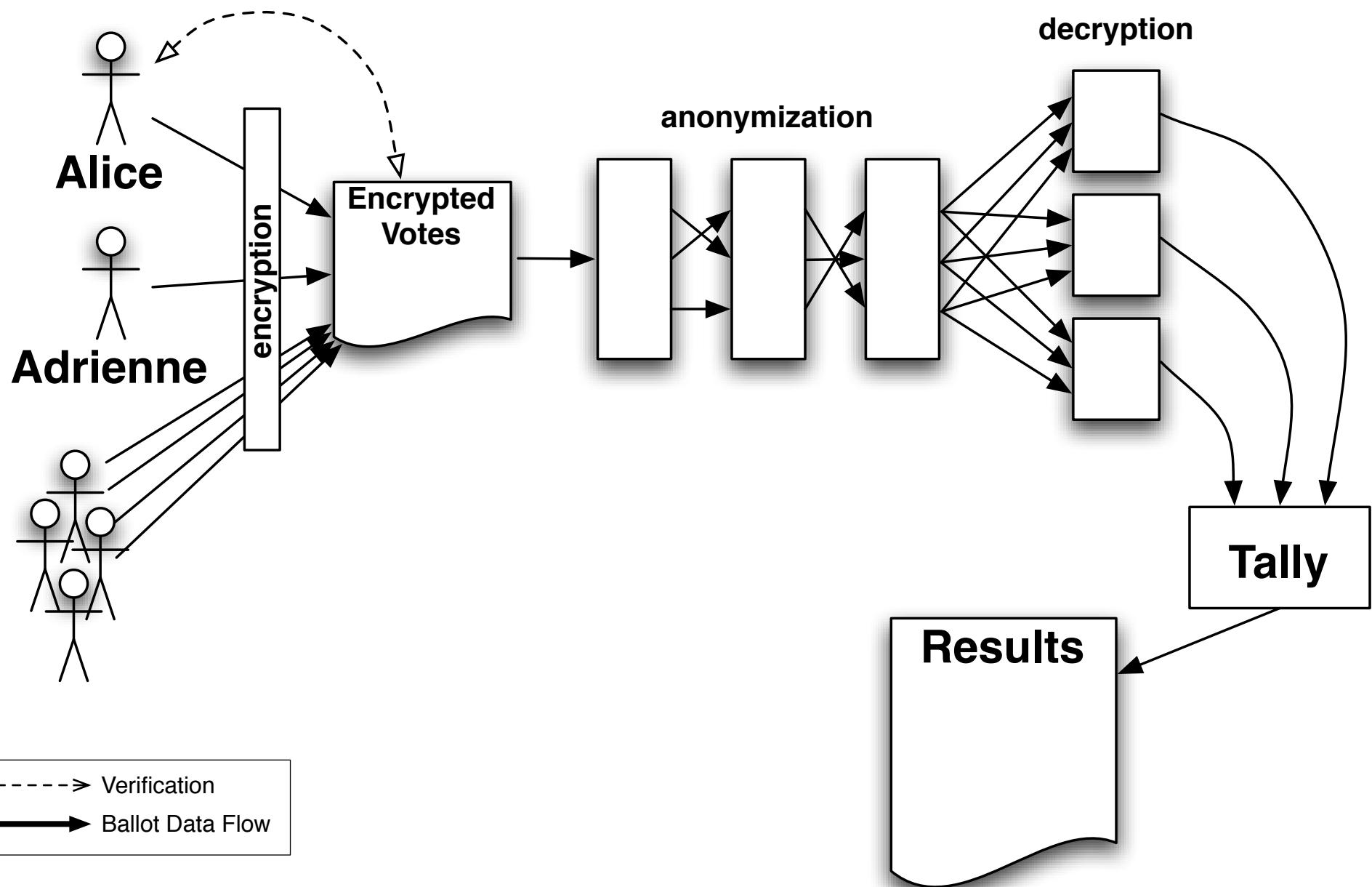
←-----→ Verification  
→ Ballot Data Flow

# Crypto Voting Schemes

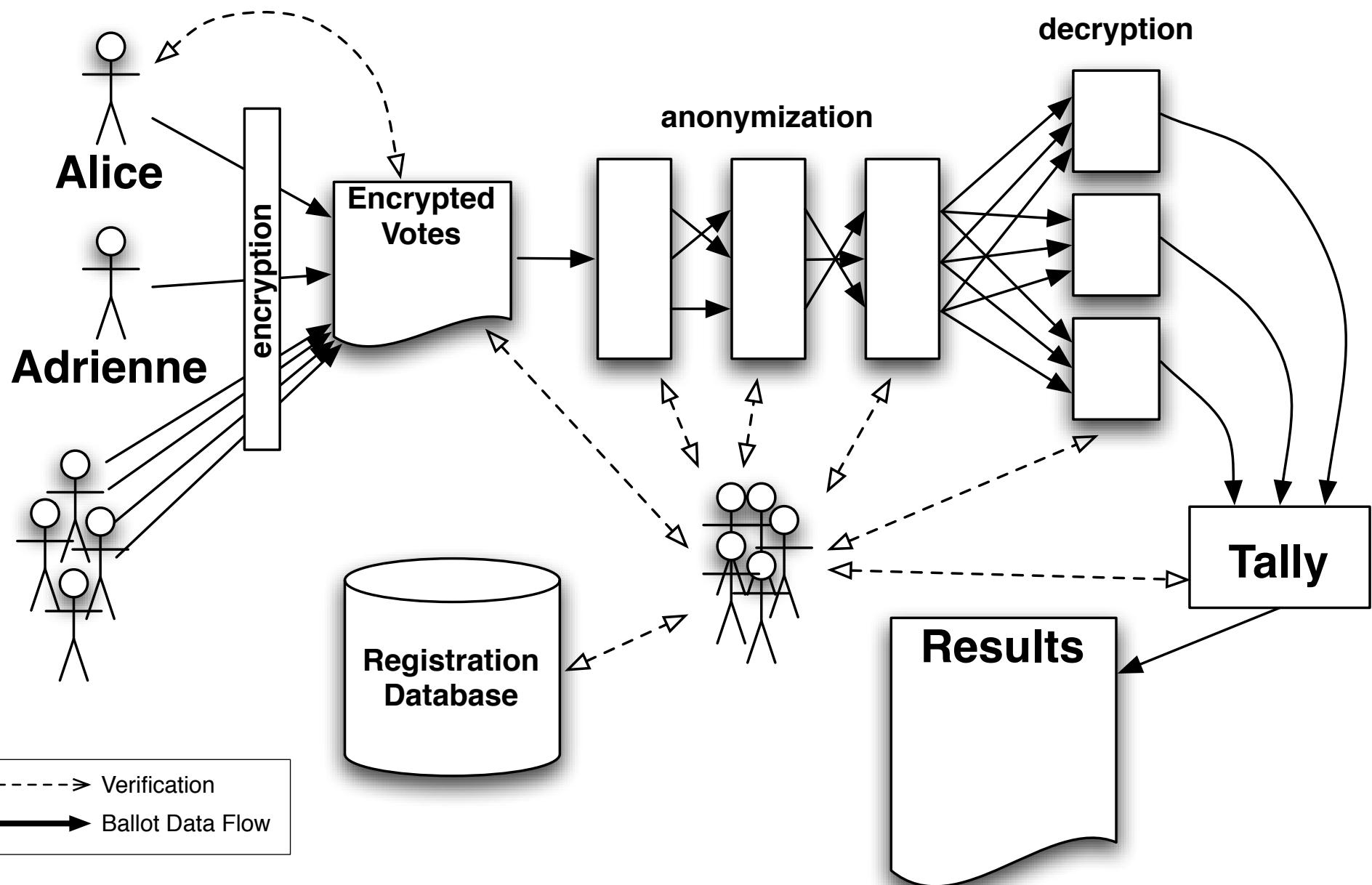


←-----> Verification  
→ Ballot Data Flow

# Crypto Voting Schemes



# Crypto Voting Schemes



# In Summary

# In Summary

- End-to-End verification

# In Summary

- End-to-End verification
- Secrecy and Verifiability are reconcilable

# In Summary

- End-to-End verification
- Secrecy and Verifiability are reconcilable
- Voting with Cryptography:  
let anyone verify.

# Questions?

