San Francisco Bay University

MATH208 - Probability and Statistics
2023 Fall Homework #3

Due day: 11/27/2023

Instruction:

1. Homework answer sheet should contain the original questions and corresponding answers.
2. Answer sheet must be in PDF file format with Github links for the programming questions, but MS Word file can't be accepted. As follows is the answer sheet name format.
<course_id>_week<week_number>_StudentID_FirstName_LastName.pdf
3. The program name in Github must follow the format like *<course_id>_week<week_number>_q<question_number>_StudentID_FirstName_LastName*
4. If the calculation in Excel is needed, the original file must be provided.
5. Show screenshot of all running results, including the system date/time.
6. The calculation process must be printed if needed, handwriting can't be accepted.
7. Only accept homework submission uploaded via Canvas.
8. Overdue homework submission can't be accepted.
3. Takes academic honesty and integrity seriously (Zero Tolerance of Cheating & Plagiarism)

1. Based on the experience of customs clearance inspections at the airport, the probability of luggage containing prohibited items is 10^{-4}. Given that the X-ray inspection machine has a probability of $\frac{1}{10}$ of falsely identifying a regular luggage as containing prohibited items, and the probability of falsely identifying prohibited items as regular luggage is 10^{-6}, find the probability that it actually does contain prohibited items for a luggage belonging to someone determined by the X-ray inspection machine to contain prohibited items.

Ans : Probability of prohibited items (PP) = 10^{-4}
Probability of false positive identification of prohibited items (FP) = 10^{-5}
Probability of false negative identification of prohibited items (FP) = 10^{-6}
Probability of prohibited items given that it is identified P(P|I) = ?

$P(I) = P(I|P) * P(P)+P(I|R)*P(R)$
$P(I) = 10^{-5} * 10^{-4} +(1-10^{-6})* (1-10^{-4})$
$P(I) = 1* 10^{-9} +0.99*0.99$

$P(I) \sim 0.99$

Now,

$P(P|I) = \frac{P(I|P)*P(P)}{P(I)}$

$P(P|I) = \frac{10^{-5} \times 10^{-4}}{0.99}$

$P(P|I) \sim 10^{-9}$

The probability of prohibited items given that it is identified $P(P|I) = 10^{-9}$.

2. There are two basketball teams *A* and *B* in the final for a 7-game series, where each team needs to win *4* out of *7* games. The probability of *A* winning in each game is *0.6* and *B* winning is *0.4* respectively. If defining the total game as *N*, $N \in \{4,5,6,7\}$, find the probability distribution of *N* and the expected value of *N*.

Ans: Probability of A winning a game $P(A) = 0.6$
Probability of B winning a game $P(B) = 0.4$
Probability of N games played = $P(N) = P(A)^{4-N} * P(B)^{N-4}$
$E(N) = ?$
Using the values of N{4,5,6,7} , we get:
$P(4) = P(A)^0 * P(B)^0 = 1$
$P(5) = P(A)^1 * P(B)^1 = 0.24$
$P(6) = P(A)^2 * P(B)^2 = 0.144$
$P(7) = P(A)^3 * P(B)^3 = 0.086$

To calculate the value of N, substitute the values :
$E(N) = 4*P(4) + 5*P(5) + 6*P(6) + 7*P(7)$
$E(N) = 4*1 + 5*(0.6)^1 * (0.4)^1 + 6*(0.6)^2 * (0.4)^2 + 7*(0.6)^3 * (0.4)^3$
$E(N) = 4 + 1.2 + 0.345 + 0.096$
$E(N) = 5.642$

The expected value $E(N) = 5.642$

3. The values of the discrete random variable *X* are *0, 1, 2, 3* and the probability P(X) is as follows. Find the expected value of *X*.

X	0	1	2	3
P(X)	0.2	0.1*(k+1)	0.3*(k+1)	0.2

Ans:

expected value $E(X) = ?$

$E(X) = \sum_{i=0}^{3} x_i \cdot P(X = xi)$

Substituting the values, we get:
E(X) = 0*P(X=0) +1 *P(X=1) +2*P(X=2) +3*P(X=3)
E(X) = 0*0.2 +1*(0.1*(k+1)) +2*(0.3*(k+1)) +3*0.2
E(X) = 0.1*(k+1) + 0.6*(k+1) +0.6
E(X) = 0.1k +0.1 +0.6k +0.6 +0.6
E(X) = 0.7k +1.3

The expected value of E(X) is 0.7k +1.3

4. Assuming that there are 30 male students and 20 female students in a class, the five students will be randomly selected to attend the speech contests organized by the student association. If the random variable *X* is the number of female students in the selected group, find the probability distribution of *X*.

   Ans: Binomial coefficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
   Calculating the values for 5 students ,0,1,2,3,4,5 , we get :
   $\binom{5}{0} = \frac{5!}{0!\times(5-0)!} = 1$
   $\binom{5}{1} = \frac{5!}{1!\times(5-1)!} = 5$

   $\binom{5}{2} = \frac{5!}{2!\times(5-2)!} = 10$

   $\binom{5}{3} = \frac{5!}{3!\times(5-3)!} = 10$

   $\binom{5}{4} = \frac{5!}{4!\times(5-4)!} = 5$

   $\binom{5}{5} = \frac{5!}{5!\times(5-5)!} = 1$

Substituting the values in binomial probability formula , we get:

$P(X = 0) = 1*1*\left(\frac{30}{50}\right)^5 = \frac{243}{3125}$
$P(X = 1) = 5*\frac{20}{50}*\left(\frac{30}{50}\right)^4 = \frac{4050}{3125}$
$P(X = 2) = 10*\left(\frac{20}{50}\right)^2*\left(\frac{30}{50}\right)^3 = \frac{5400}{3125}$
$P(X = 3) = 10*\left(\frac{20}{50}\right)^3*\left(\frac{30}{50}\right)^2 = \frac{3240}{3125}$
$P(X = 4) = 5*\left(\frac{20}{50}\right)^4*\left(\frac{30}{50}\right)^1 = \frac{432}{3125}$
$P(X = 5) = 1*\left(\frac{20}{50}\right)^5*\left(\frac{30}{50}\right)^0 = \frac{32}{3125}$

The probabilities of selecting female students are P(X = 0) till P(X=5).

5. In a batch of 12 TV sets, three of them are defective. Three TV sets will be randomly selected for inspection and the random variable *X* represents the number of good quality units in the inspection. If all three are good, the entire batch is accepted, otherwise, it is returned. Please answer the following questions

   a. If the sampling is done without replacement, write the probability distribution of *X*, the mean, and the variance. Also, fine the probability that the entire batch of TV sets can be accepted.

      Ans: P(X=k) = $\frac{\binom{K}{k}\cdot\binom{N-k}{n-k}}{\binom{N}{n}}$

      K is the good quality units in batch = 9

   Probability distribution od X:

P(X=0) = $\frac{\binom{9}{0}\cdot\binom{3}{3}}{\binom{12}{3}} = \frac{1}{220}$

P(X= 1) = $\frac{\binom{9}{1}\cdot\binom{3}{2}}{\binom{12}{3}} = \frac{27}{220}$

P(X=2) = $\frac{\binom{9}{2}\cdot\binom{3}{1}}{\binom{12}{3}} = \frac{108}{220}$

P(X=3) = $\frac{\binom{9}{3}\cdot\binom{3}{0}}{\binom{12}{3}} = \frac{84}{220}$

Mean = $\frac{N\cdot K}{n} = \frac{3.9}{12} = \frac{27}{12}$

Variance = $\frac{n\cdot K(N-K)\cdot(N-n)}{N^2\cdot(N-1)} = \frac{3\cdot 9\cdot 3\cdot 9}{12^2\cdot 11} = \frac{729}{440}$

Here, the probability of accepting entire batch without replacement is $\frac{84}{220}$

   b. If the sampling is done with replacement, write the probability distribution of *X*, the mean, and the variance. fine the probability that the entire batch of TV sets can be accepted.

      Ans: Probability distribution of X:

P(X=0) = $\binom{3}{0}\left(\frac{9}{12}\right)^0\left(\frac{3}{12}\right)^3 = \frac{27}{64}$

P(X= 1) = $\binom{3}{1}\left(\frac{9}{12}\right)^1\left(\frac{3}{12}\right)^2 = \frac{27}{64}$

P(X=2) =$\binom{3}{2}\left(\frac{9}{12}\right)^2\left(\frac{3}{12}\right)^1 = \frac{9}{64}$

P(X=3) = $\binom{3}{3}\left(\frac{9}{12}\right)^3\left(\frac{3}{12}\right)^0 = \frac{1}{64}$

Mean = $3 \, . \frac{9}{12} = \frac{27}{4}$

Variance =$3. \frac{9}{12} . \frac{3}{12} = \frac{27}{16}$

Here, the probability of accepting entire batch with replacement is $\frac{1}{64}$.

c. If the sampling is done without replacement, calculate the probability that the third one is one defective.

Ans: Probability of defective in third draw = $\frac{\binom{9}{2}\cdot\binom{3}{1}}{\binom{12}{3}}$

$$= \frac{\mathbf{36\cdot3}}{\mathbf{220}}$$

$$= \frac{108}{220}$$

The probability of third TV set being defective without replacement method is $\frac{108}{220}$.