Solar-sail perturbed cr3bp

Previous videos:

- Recap of the cr3bp
- Equilibria in the cr3bp
- Solar-sail acceleration model

cr3bp – recap from video “recap of the cr3bp”

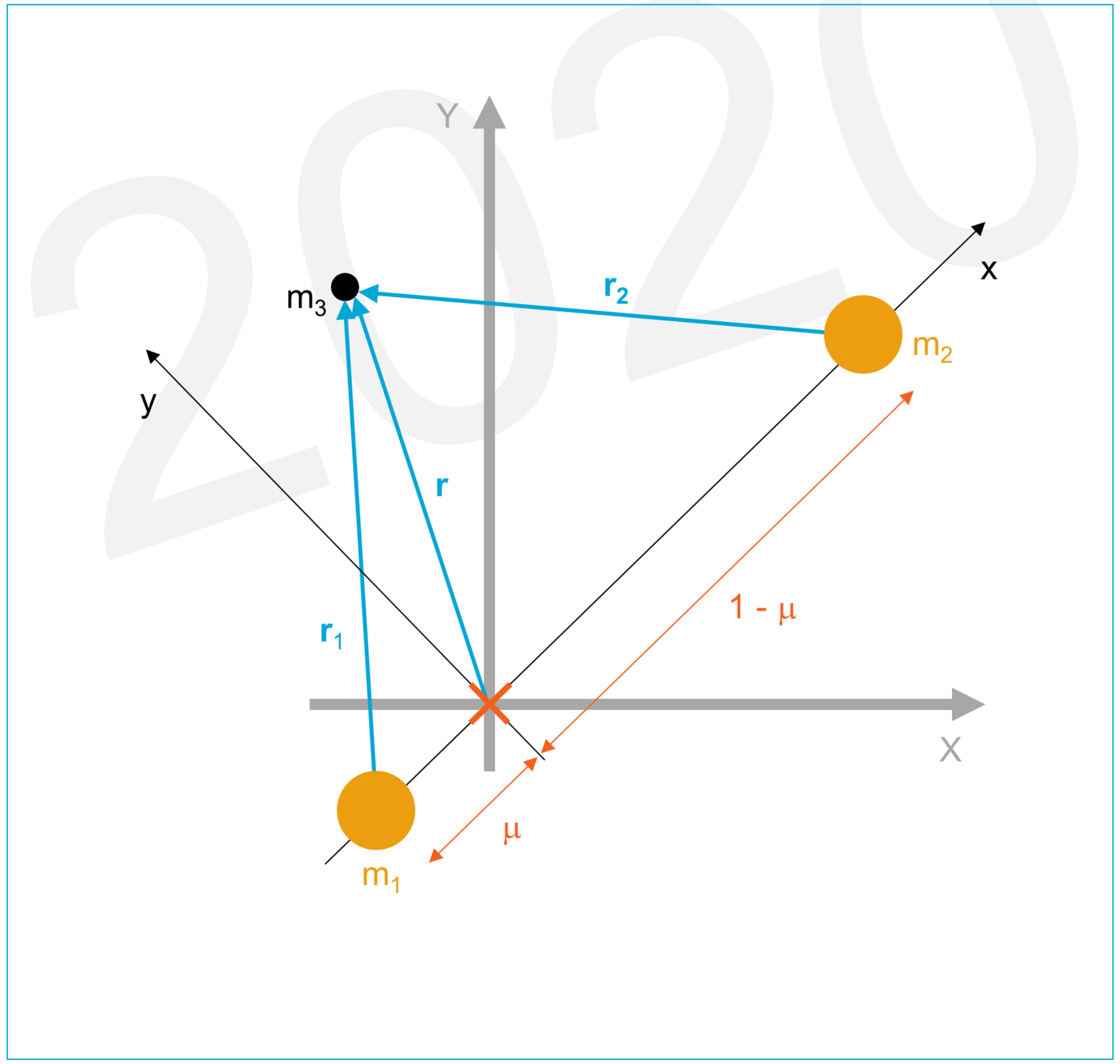
cr3bp – the equations of motion **in dimensionless form**

- Equations of motion in rotating frame – **dimensionless**

$$\ddot{\mathbf{r}} = -\left(\frac{1-\mu}{r_1^3}\mathbf{r}_1 + \frac{\mu}{r_2^3}\mathbf{r}_2\right) - 2\boldsymbol{\omega}\times\dot{\mathbf{r}} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})$$

  - $\mathbf{r} = [x \quad y \quad z]^T, \dot{\mathbf{r}} = [\dot{x} \quad \dot{y} \quad \dot{z}]^T, \ddot{\mathbf{r}} = [\ddot{x} \quad \ddot{y} \quad \ddot{z}]^T$
  - $\mu = \dfrac{m_1}{m_1 + m_2}$ Erratum: this should be $\mu = \frac{m_2}{m_1 + m_2}$
  - $\mathbf{r}_1 = [x+\mu \quad y \quad z]^T$
  - $\mathbf{r}_2 = [x-(1-\mu) \quad y \quad z]^T$
  - $\boldsymbol{\omega} = [0 \quad 0 \quad \omega]^T = [0 \quad 0 \quad 1]^T$

- Equations of motion are fully parameterized by μ



cr3bp – recap from video “recap of the cr3bp”

cr3bp – the equations of motion in dimensionless form **+ potential formulation**

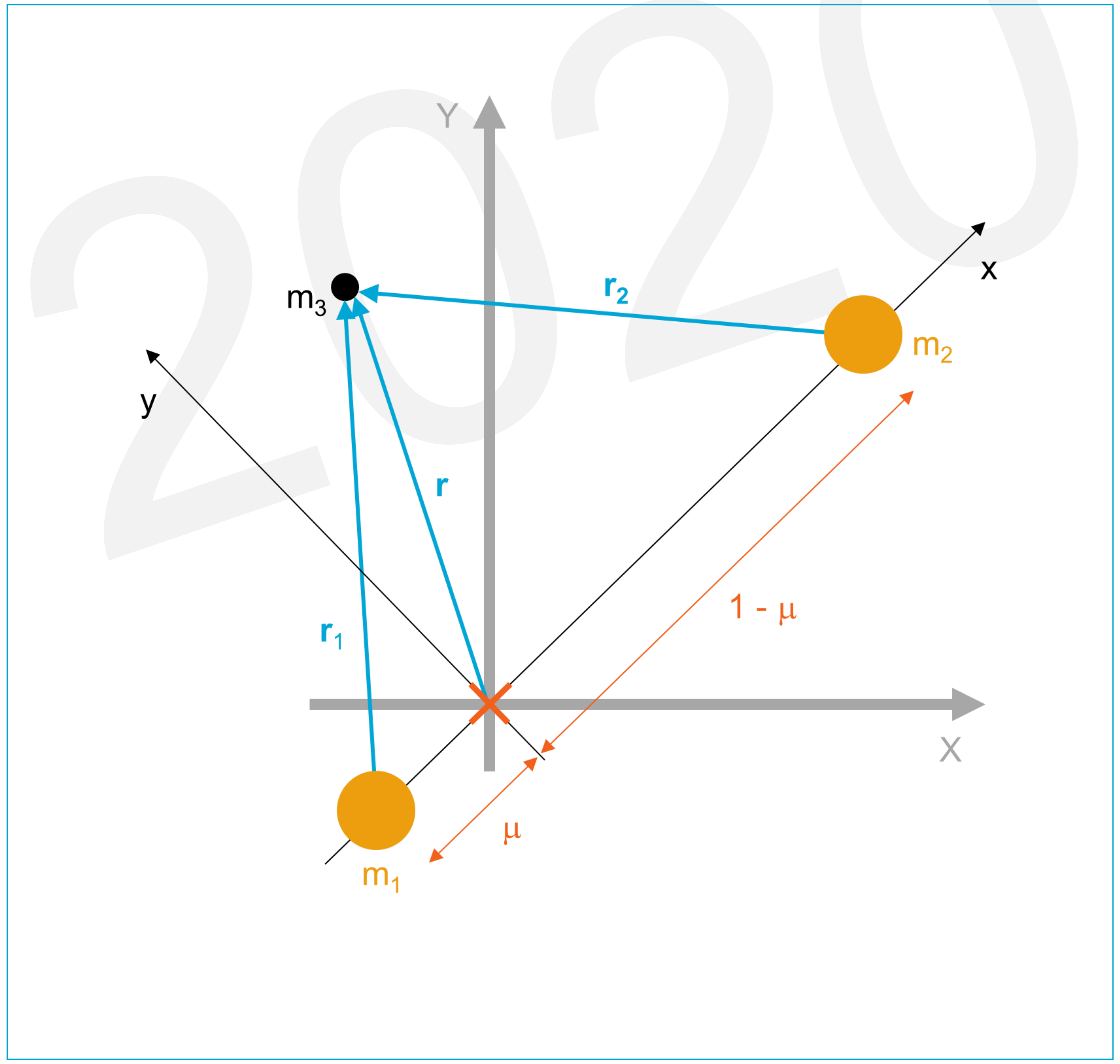
- Equations of motion in rotating frame – **dimensionless**

$$\ddot{\mathbf{r}} = \underbrace{-\left(\frac{1-\mu}{r_1^3}\mathbf{r}_1 + \frac{\mu}{r_2^3}\mathbf{r}_2\right)} - 2\boldsymbol{\omega}\times\dot{\mathbf{r}} \underbrace{- \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})}$$

These terms can be written in potential form

$$U = -\left(\frac{1-\mu}{r_1} + \frac{\mu}{r_2}\right) - \frac{1}{2}\left(x^2+y^2\right)$$ Effective potential

$$\ddot{\mathbf{r}} = -2\boldsymbol{\omega}\times\dot{\mathbf{r}} - \nabla U$$



cr3bp – recap from video “equilibria in the cr3bp”

Equilibria – computation

- Also called “Lagrange points”
- Locations where, if
  - The body with neglible mass, m_3, has zero velocity
  - No net acceleration acts on m_3

  It will stay stationary *w.r.t.* the rotating frame R(x,y,z)

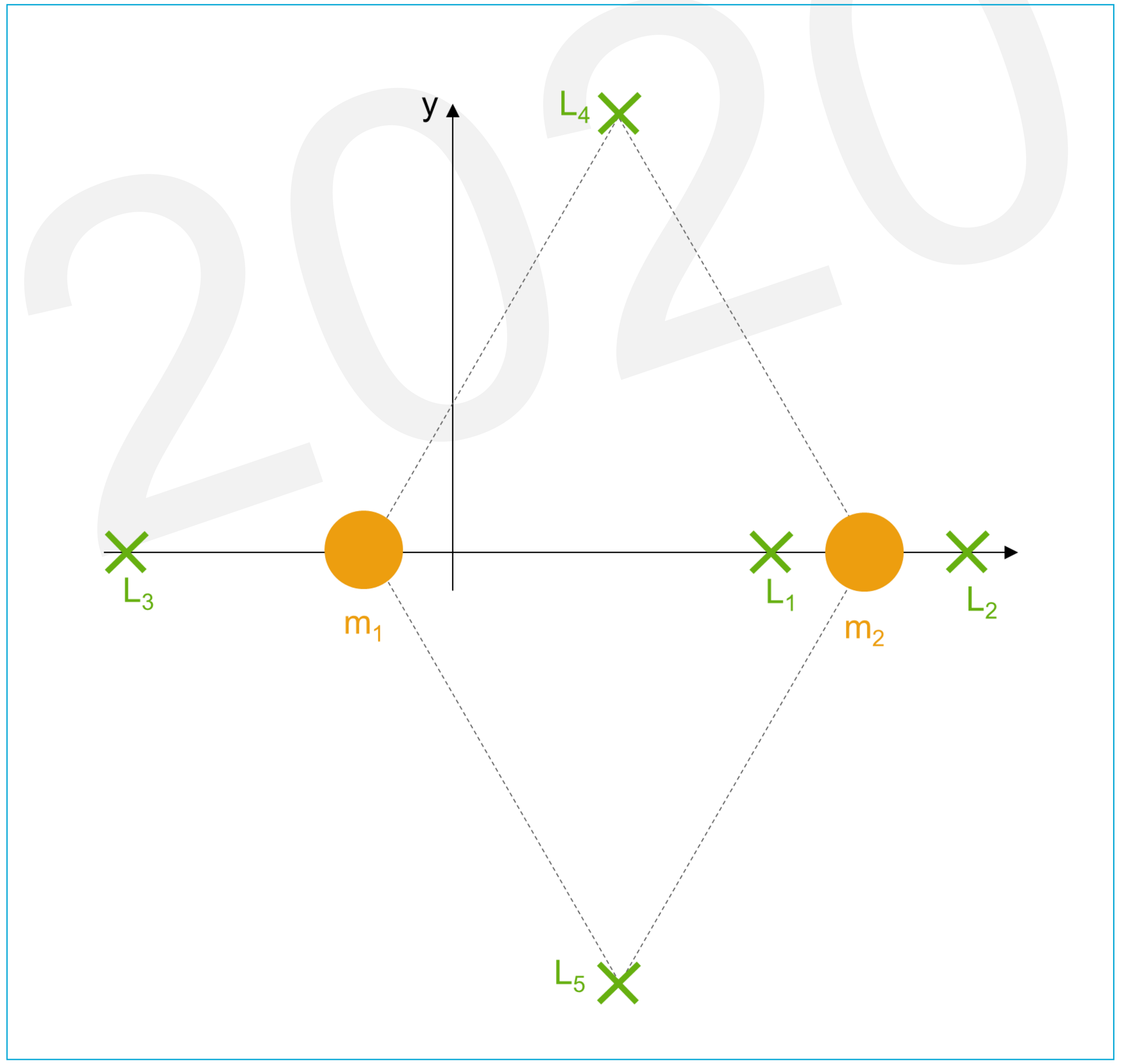
  Trace out an orbit *w.r.t.* the inertial frame I(X,Y,Z)

$$\underbrace{\cancel{\ddot{\mathbf{r}}}}_{=0} = -2\boldsymbol{\omega} \times \underbrace{\cancel{\dot{\mathbf{r}}}}_{=0} - \nabla U$$

- Lagrange points are located where

$$\nabla U = \mathbf{0}$$

$$U = -\left(\frac{1-\mu}{r_1} + \frac{\mu}{r_2}\right) - \frac{1}{2}\left(x^2 + y^2\right)$$

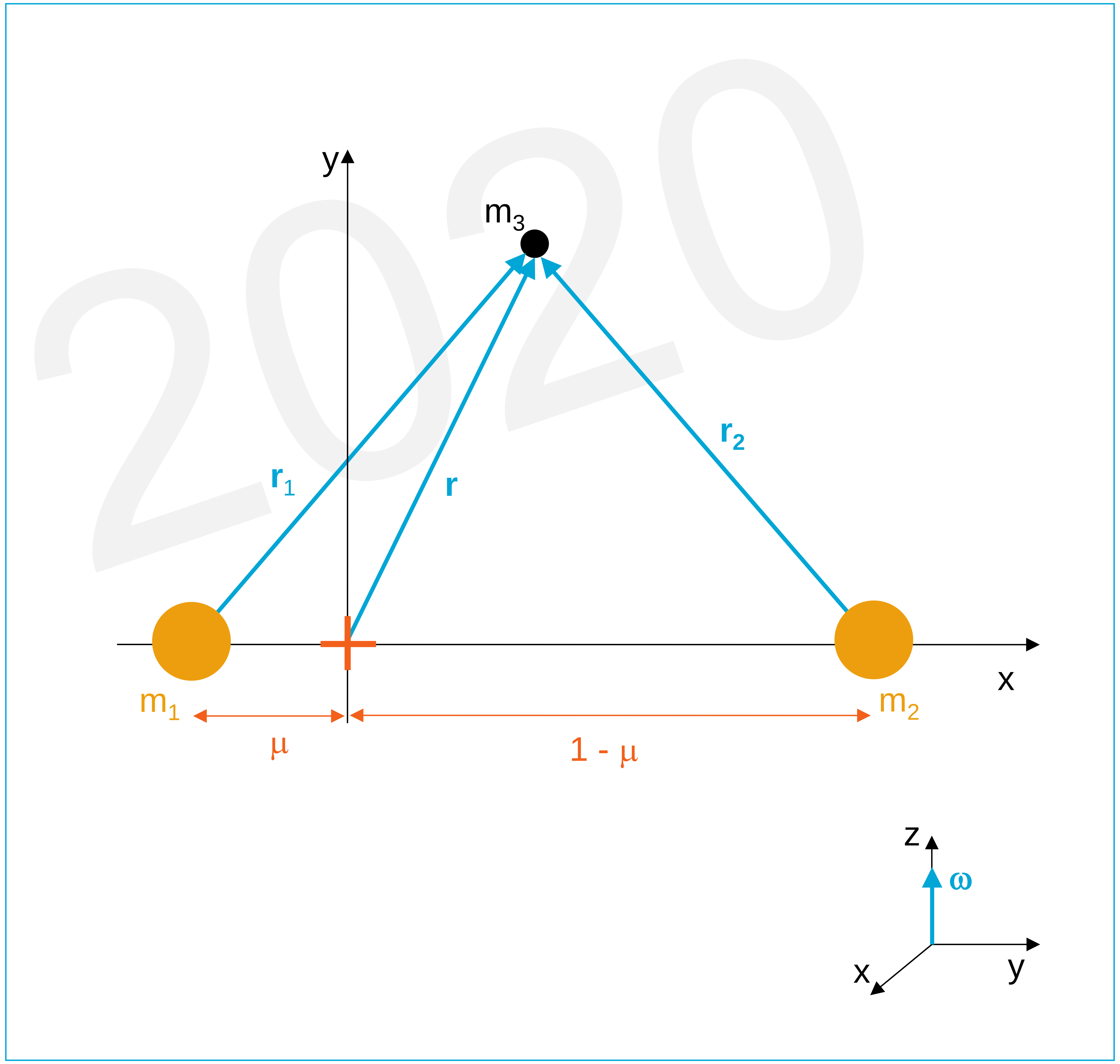


Solar-sail perturbed cr3bp

- Equations of motion

$$\ddot{\mathbf{r}} = -2\boldsymbol{\omega}\times\dot{\mathbf{r}} - \nabla U$$

  - $\mathbf{r} = \begin{bmatrix} x & y & z \end{bmatrix}^T, \dot{\mathbf{r}} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T, \ddot{\mathbf{r}} = \begin{bmatrix} \ddot{x} & \ddot{y} & \ddot{z} \end{bmatrix}^T$
  - $U = -\left(\frac{1-\mu}{r_1} + \frac{\mu}{r_2}\right) - \frac{1}{2}\left(x^2 + y^2\right)$
  - $\mu = \frac{m_2}{m_1 + m_2}$
  - $\mathbf{r}_1 = \begin{bmatrix} x+\mu & y & z \end{bmatrix}^T$
  - $\mathbf{r}_2 = \begin{bmatrix} x-(1-\mu) & y & z \end{bmatrix}^T$
  - $\boldsymbol{\omega} = \begin{bmatrix} 0 & 0 & \omega \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

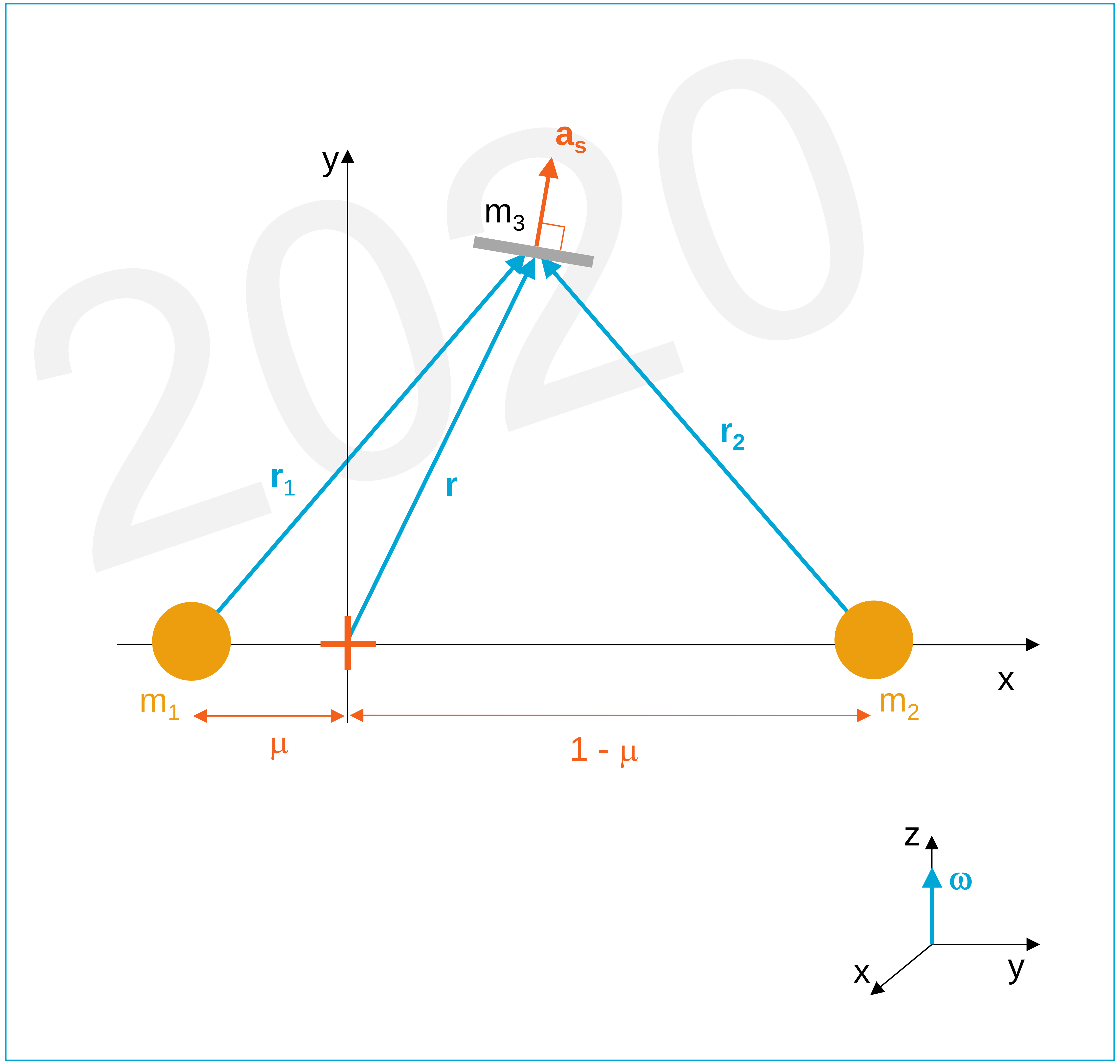


Solar-sail perturbed cr3bp

- Equations of motion

$$\ddot{\mathbf{r}} = -2\boldsymbol{\omega}\times\dot{\mathbf{r}} - \nabla U + \mathbf{a}_s$$

  - $\mathbf{r} = \begin{bmatrix} x & y & z \end{bmatrix}^T, \dot{\mathbf{r}} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T, \ddot{\mathbf{r}} = \begin{bmatrix} \ddot{x} & \ddot{y} & \ddot{z} \end{bmatrix}^T$
  - $U = -\left(\frac{1-\mu}{r_1} + \frac{\mu}{r_2}\right) - \frac{1}{2}\left(x^2 + y^2\right)$
  - $\mu = \frac{m_2}{m_1 + m_2}$
  - $\mathbf{r}_1 = \begin{bmatrix} x+\mu & y & z \end{bmatrix}^T$
  - $\mathbf{r}_2 = \begin{bmatrix} x-(1-\mu) & y & z \end{bmatrix}^T$
  - $\boldsymbol{\omega} = \begin{bmatrix} 0 & 0 & \omega \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

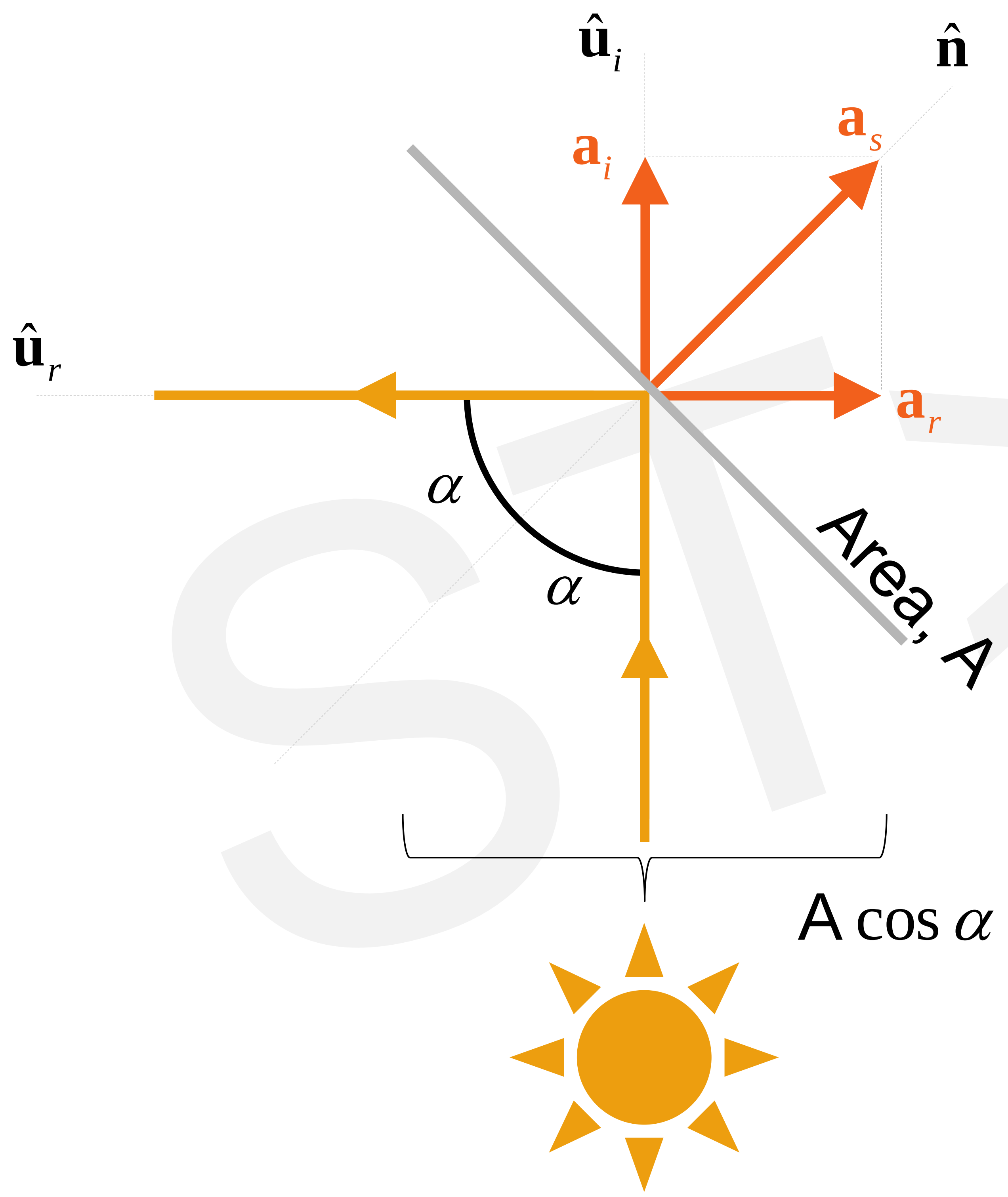


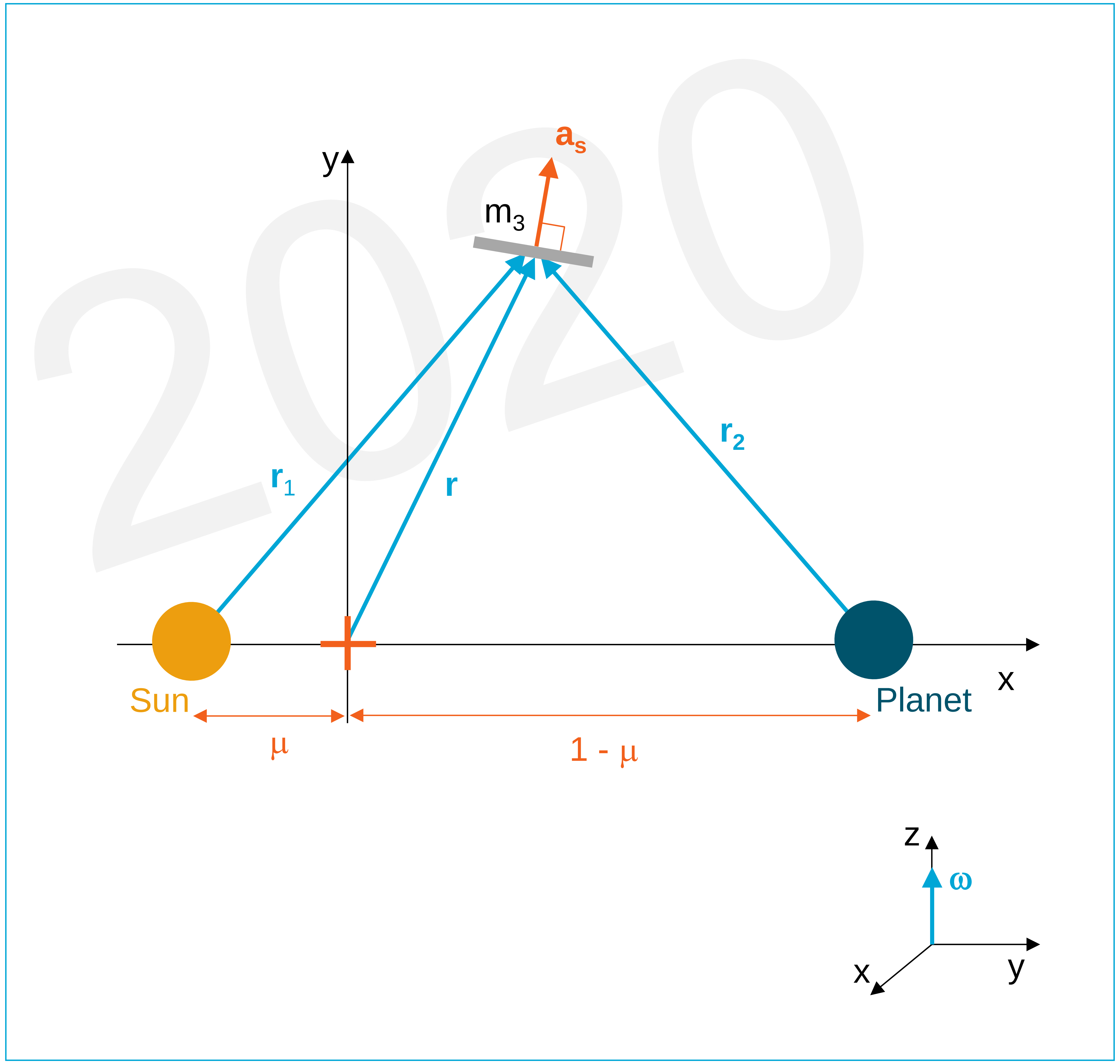
Solar-sail acceleration – recap from video “solar-sail acceleration model”

- Dimensional solar-sail acceleration

$$\mathbf{a}_s = \boxed{\beta} \frac{\mu_\odot}{r^2} (\cos\alpha)^2 \, \hat{\mathbf{n}}$$

Solar-sail lightness number



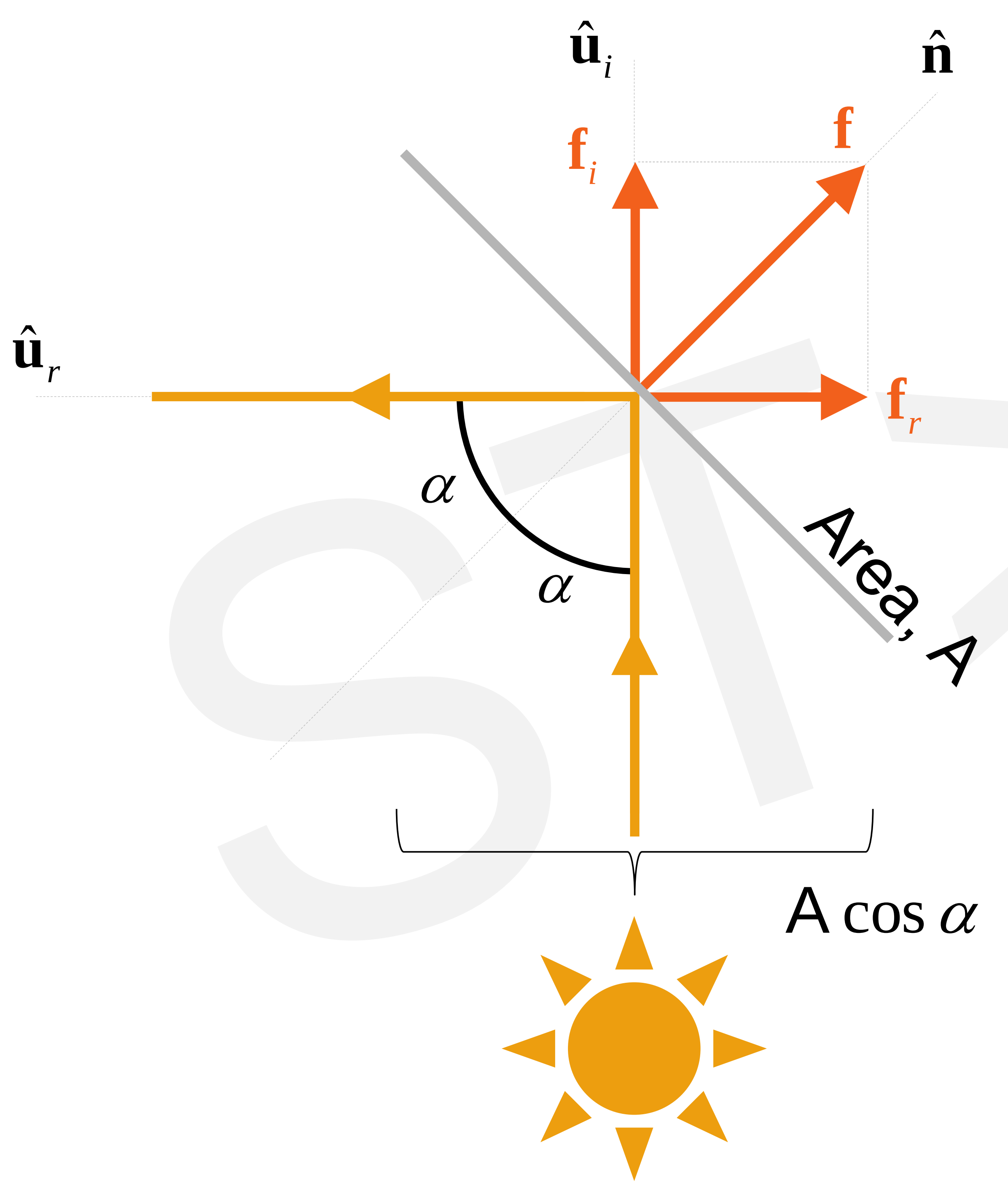


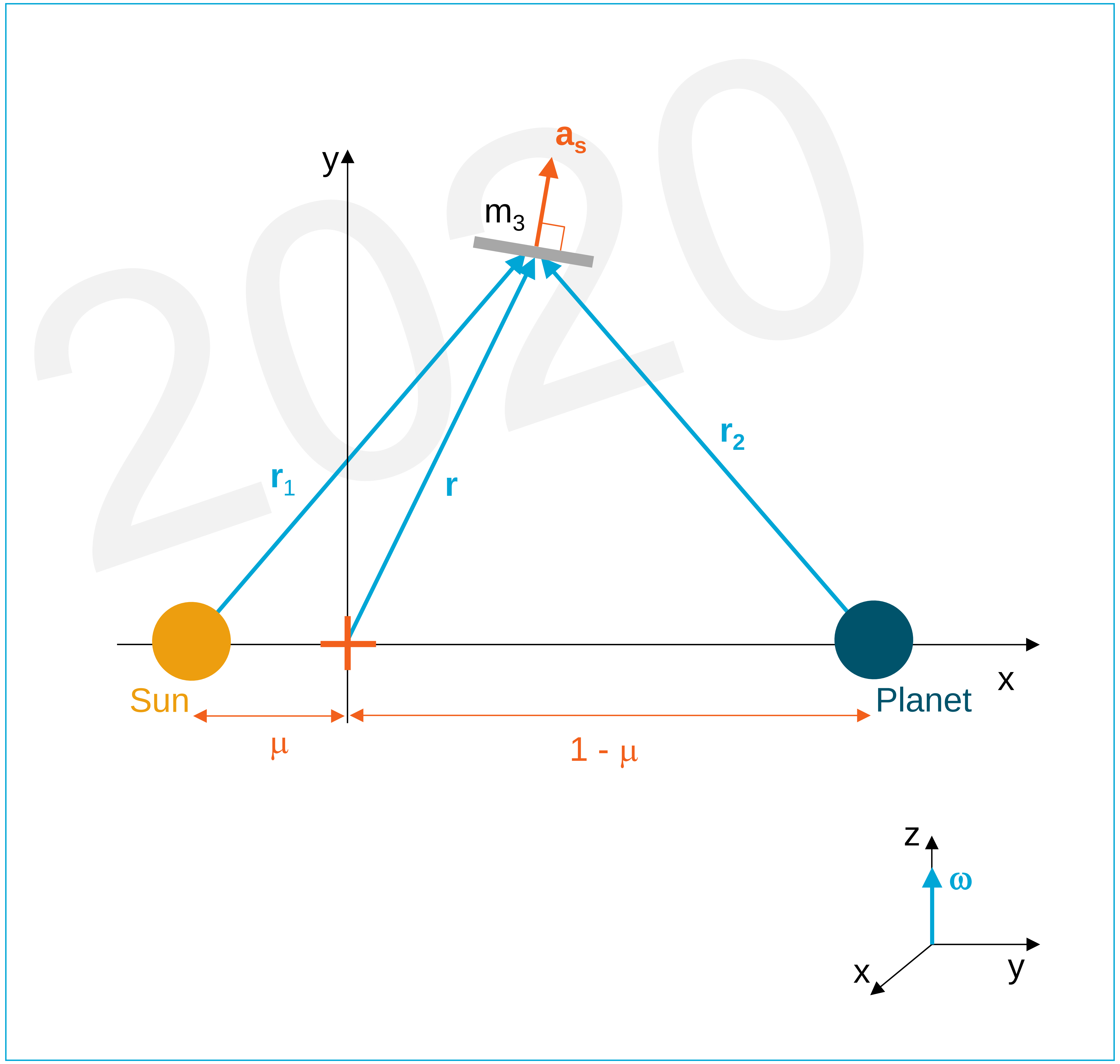
Solar-sail acceleration – recap from video “solar-sail acceleration model”

- Dimensional solar-sail acceleration

$$\mathbf{a}_s = \beta \boxed{\frac{\mu_\odot}{r^2}} (\cos\alpha)^2 \,\hat{\mathbf{n}}$$

Solar gravitational acceleration



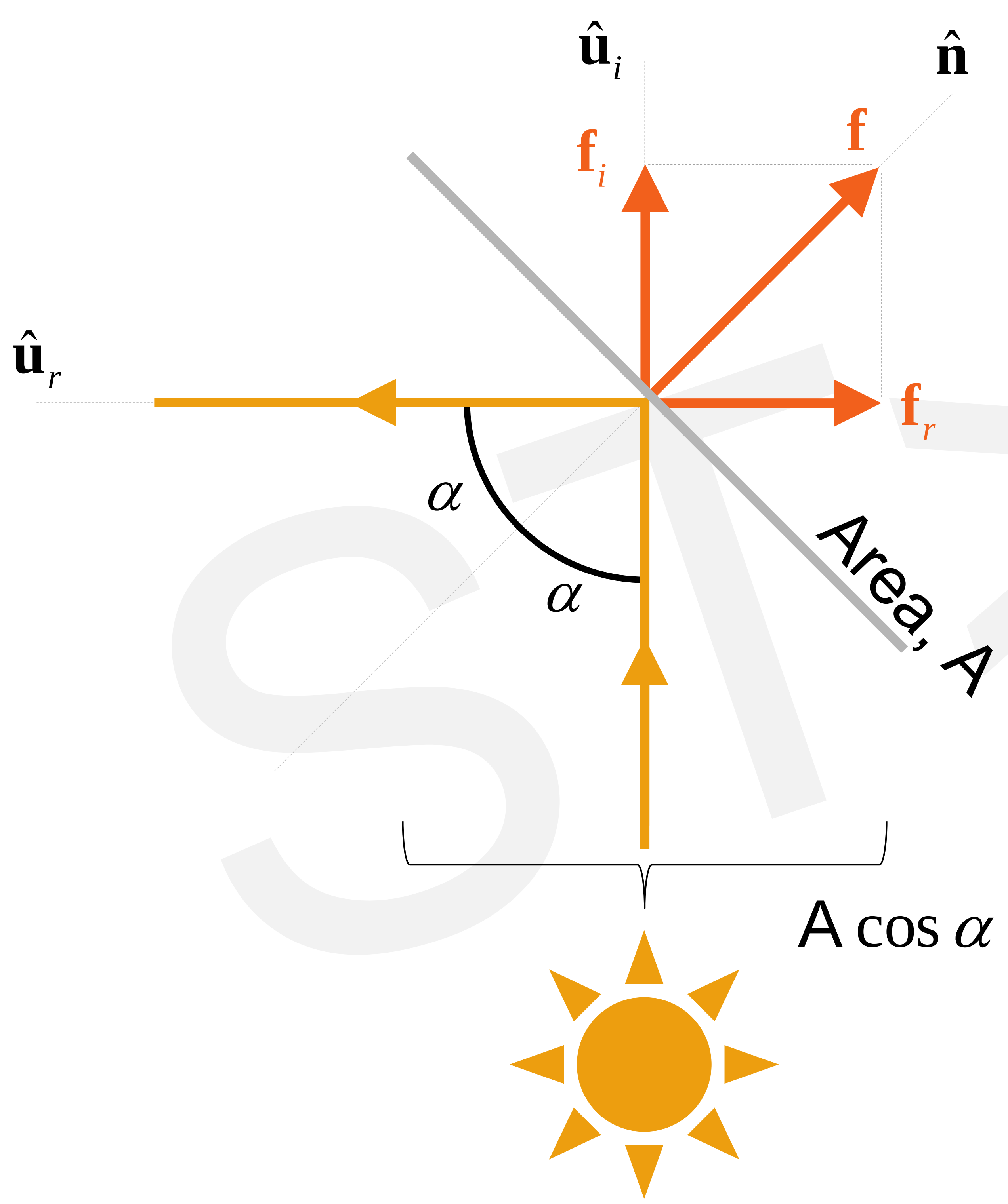


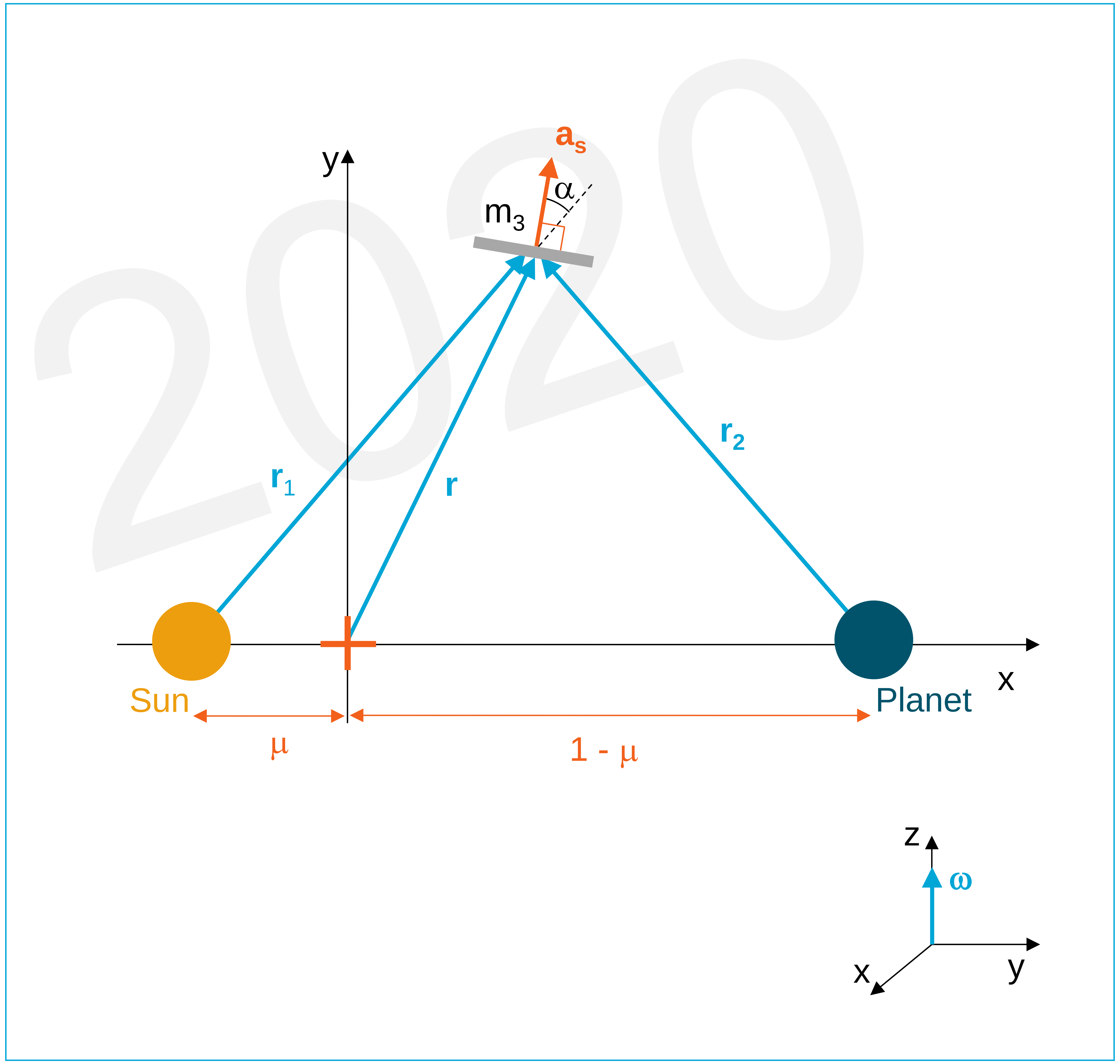
Solar-sail acceleration – recap from video “solar-sail acceleration model”

- Dimensional solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{\mu_\odot}{r^2} \boxed{(\cos\alpha)^2} \hat{\mathbf{n}}$$

Accounting for sail attitude



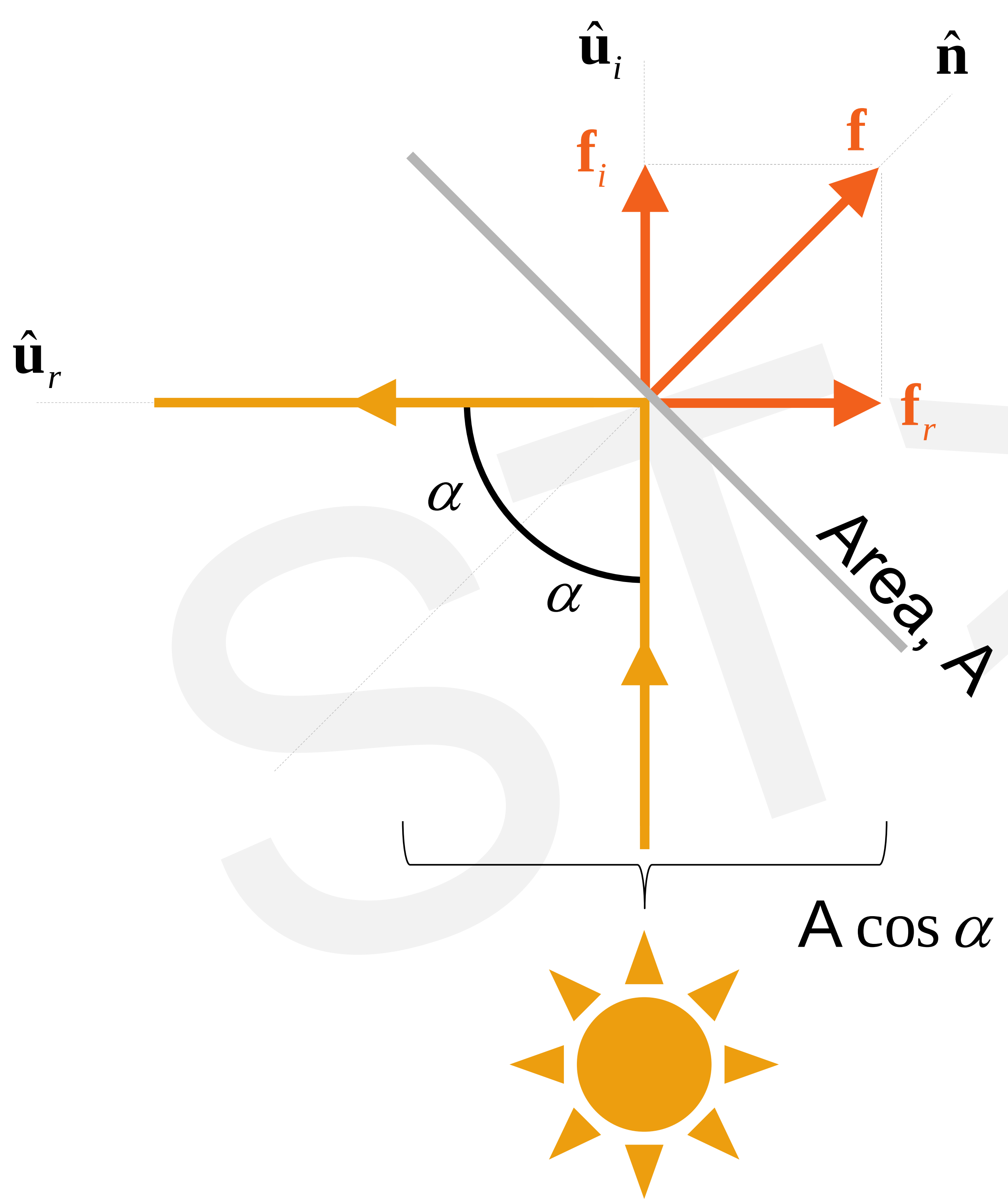


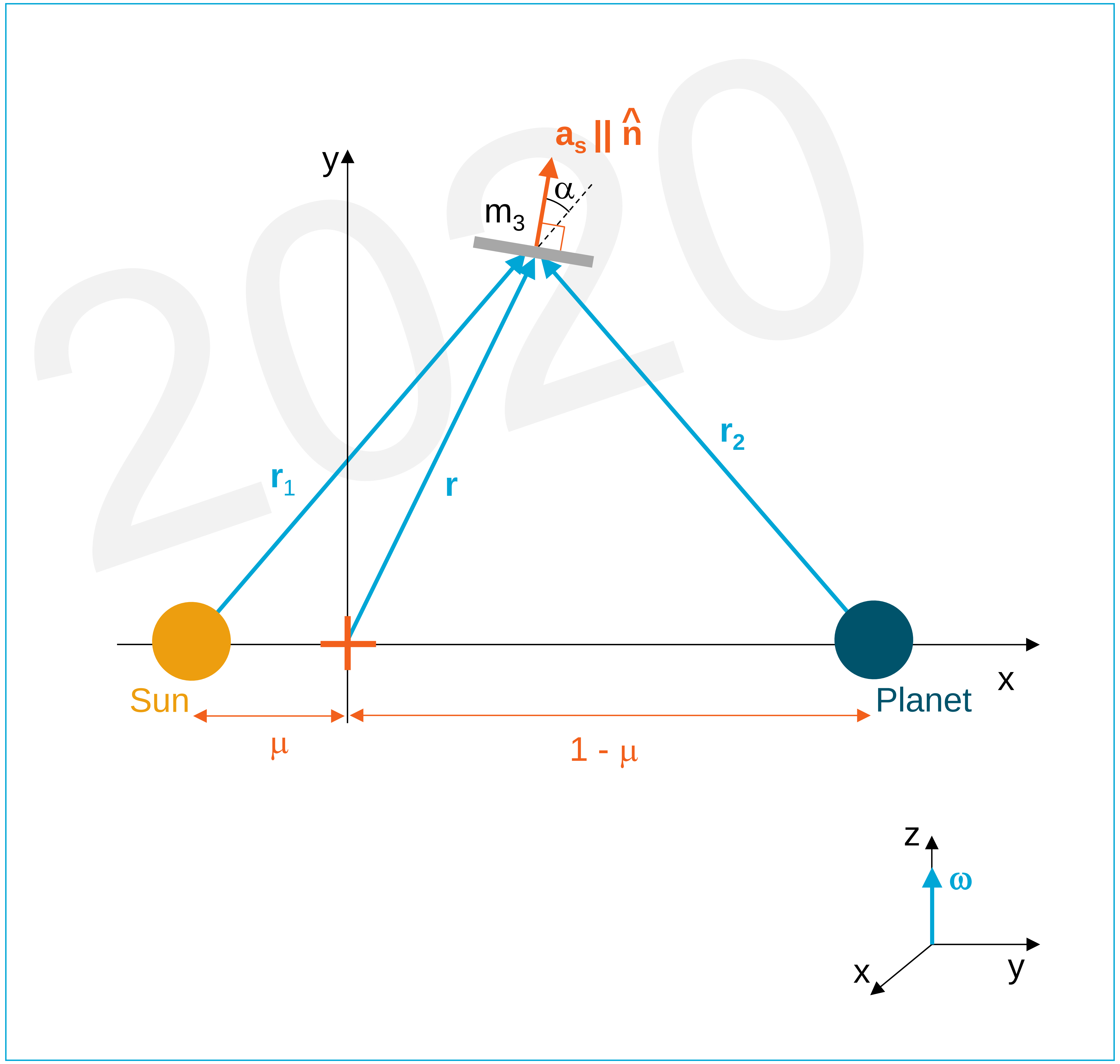
Solar-sail acceleration – recap from video “solar-sail acceleration model”

- Dimensional solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{\mu_\odot}{r^2} (\cos\alpha)^2 \boxed{\hat{\mathbf{n}}}$$

Unit vector normal to the sail





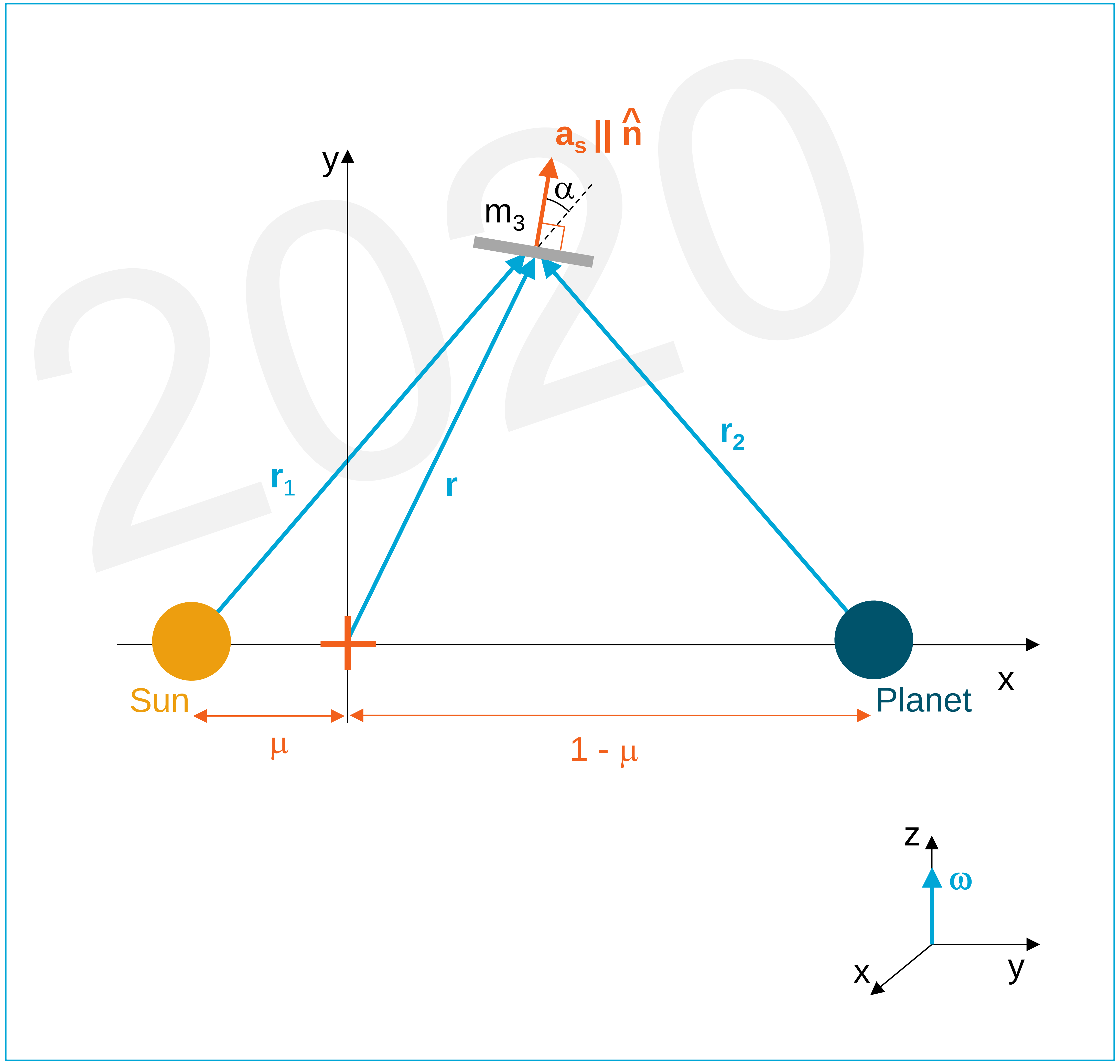
Solar-sail acceleration in the dimensionless cr3bp

- Dimensional solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{\mu_\odot}{r^2} (\cos\alpha)^2 \, \hat{\mathbf{n}}$$

- **Dimensionless** solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2} (\cos\alpha)^2 \, \hat{\mathbf{n}}$$



Solar-sail acceleration in the dimensionless cr3bp

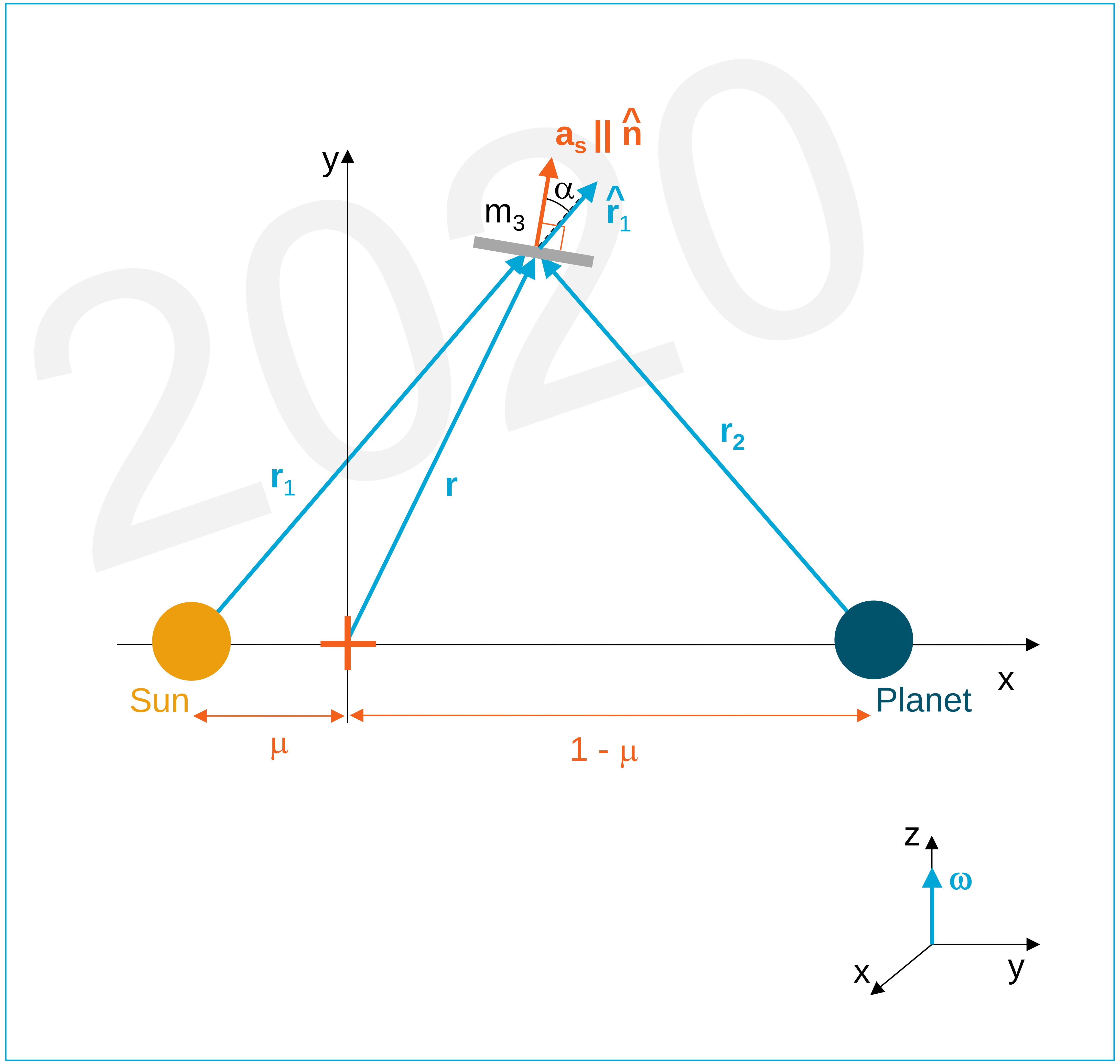
- Dimensional solar-sail acceleration in

$$\mathbf{a}_s = \beta \frac{\mu_\odot}{r^2} (\cos\alpha)^2 \,\hat{\mathbf{n}}$$

- Dimensionless solar-sail acceleration in

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2} (\cos\alpha)^2 \,\hat{\mathbf{n}}$$

- More **general form** of solar-sail acceleration



Solar-sail acceleration in the dimensionless cr3bp

- Dimensional solar-sail acceleration

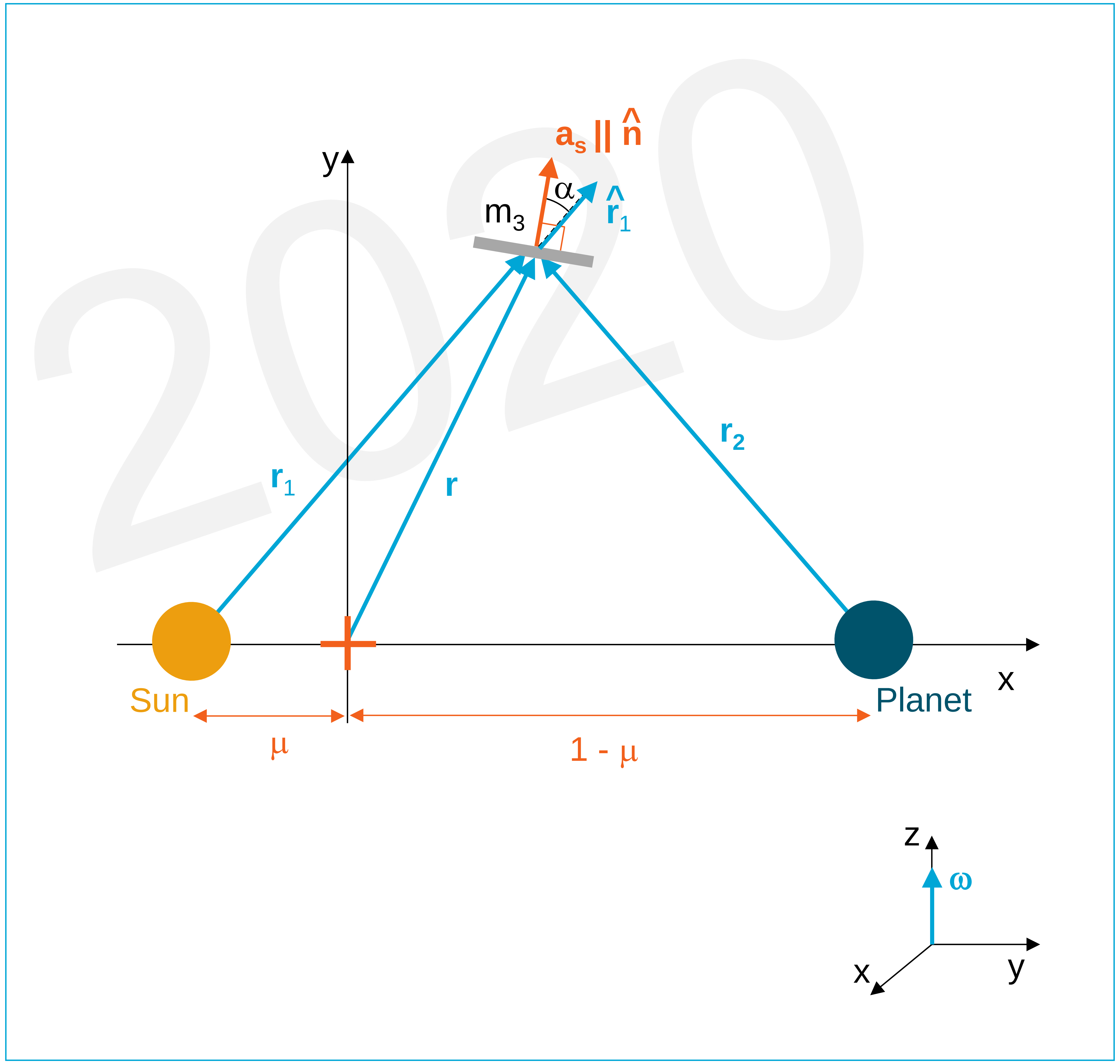
$$\mathbf{a}_s = \beta \frac{\mu_\odot}{r^2} (\cos\alpha)^2 \,\hat{\mathbf{n}}$$

- Dimensionless solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2} (\cos\alpha)^2 \,\hat{\mathbf{n}}$$

- More **general form** of solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2} (\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}})^2 \,\hat{\mathbf{n}}$$



Solar-sail acceleration in the dimensionless cr3bp

- Dimensional solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{\mu_\odot}{r^2} (\cos\alpha)^2 \hat{\mathbf{n}}$$

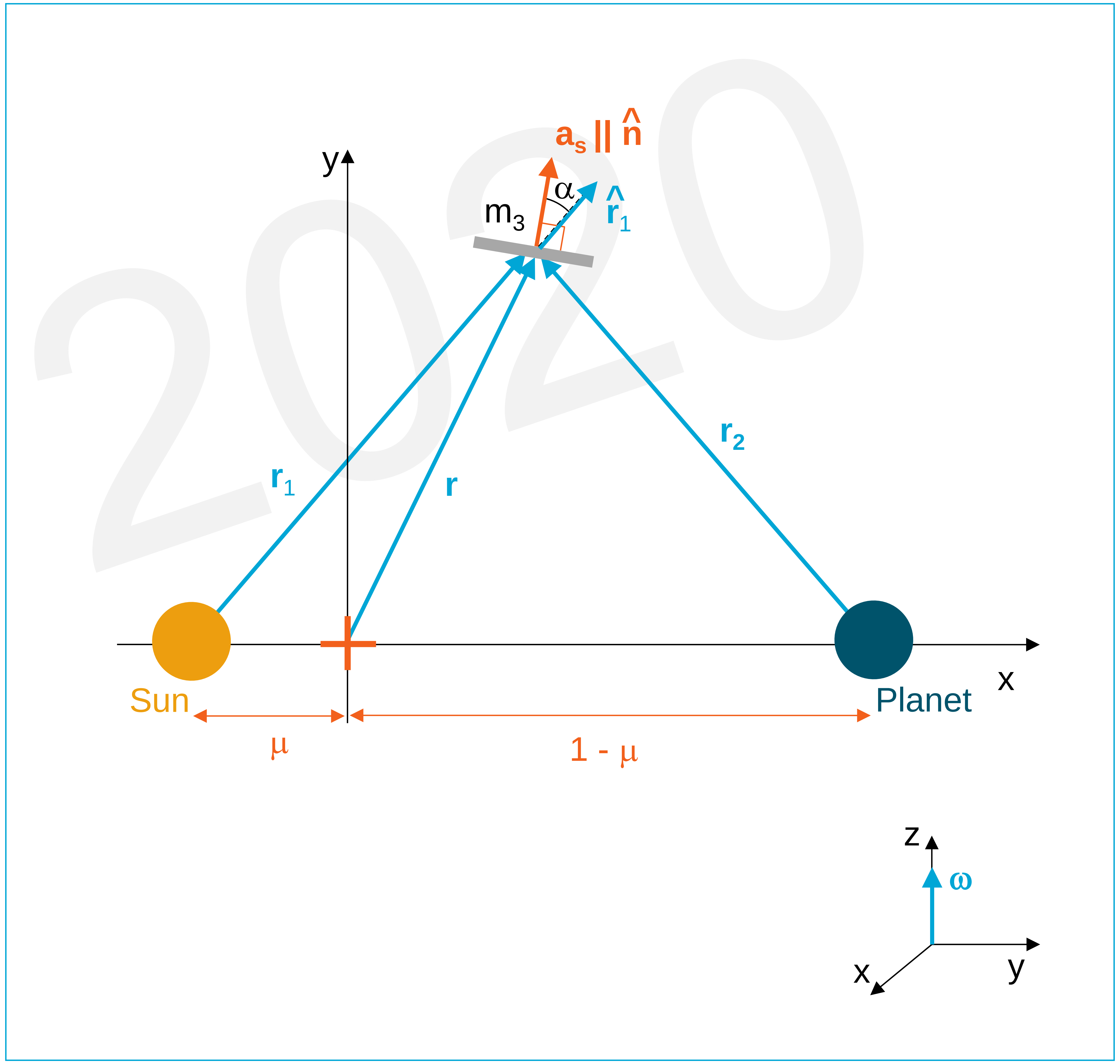
- Dimensionless solar-sail acceleration in

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2} \left(\boxed{\cos\alpha}\right)^2 \hat{\mathbf{n}}$$

- More **general form** of solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2} \left(\boxed{\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}}\right)^2 \hat{\mathbf{n}}$$

$$\cos\alpha = \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}$$

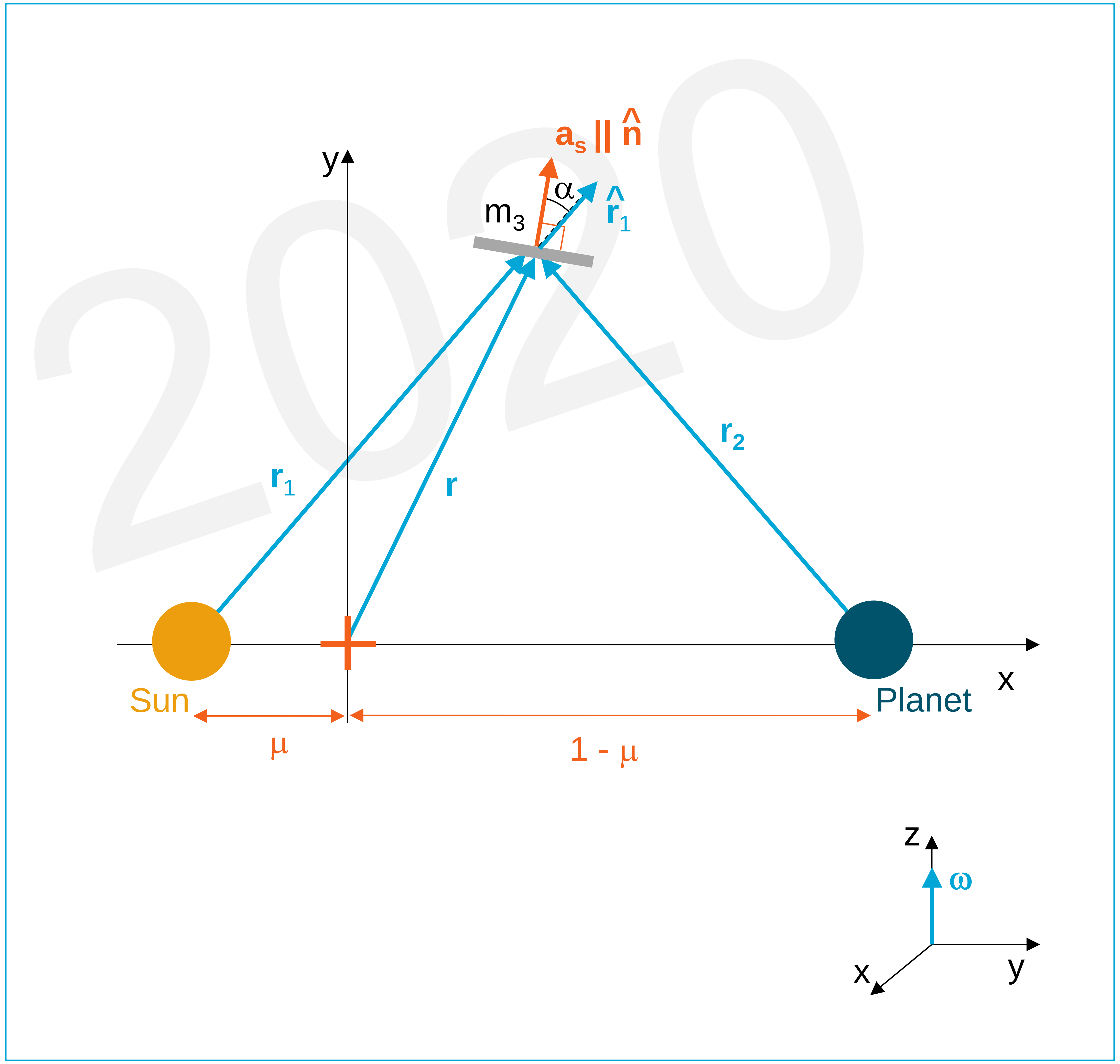


Solar-sail acceleration in the dimensionless cr3bp

- Dimensionless solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2} \left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}} \right)^2 \hat{\mathbf{n}}$$

- Define the sail orientation in **3D**



Solar-sail acceleration in the dimensionless cr3bp

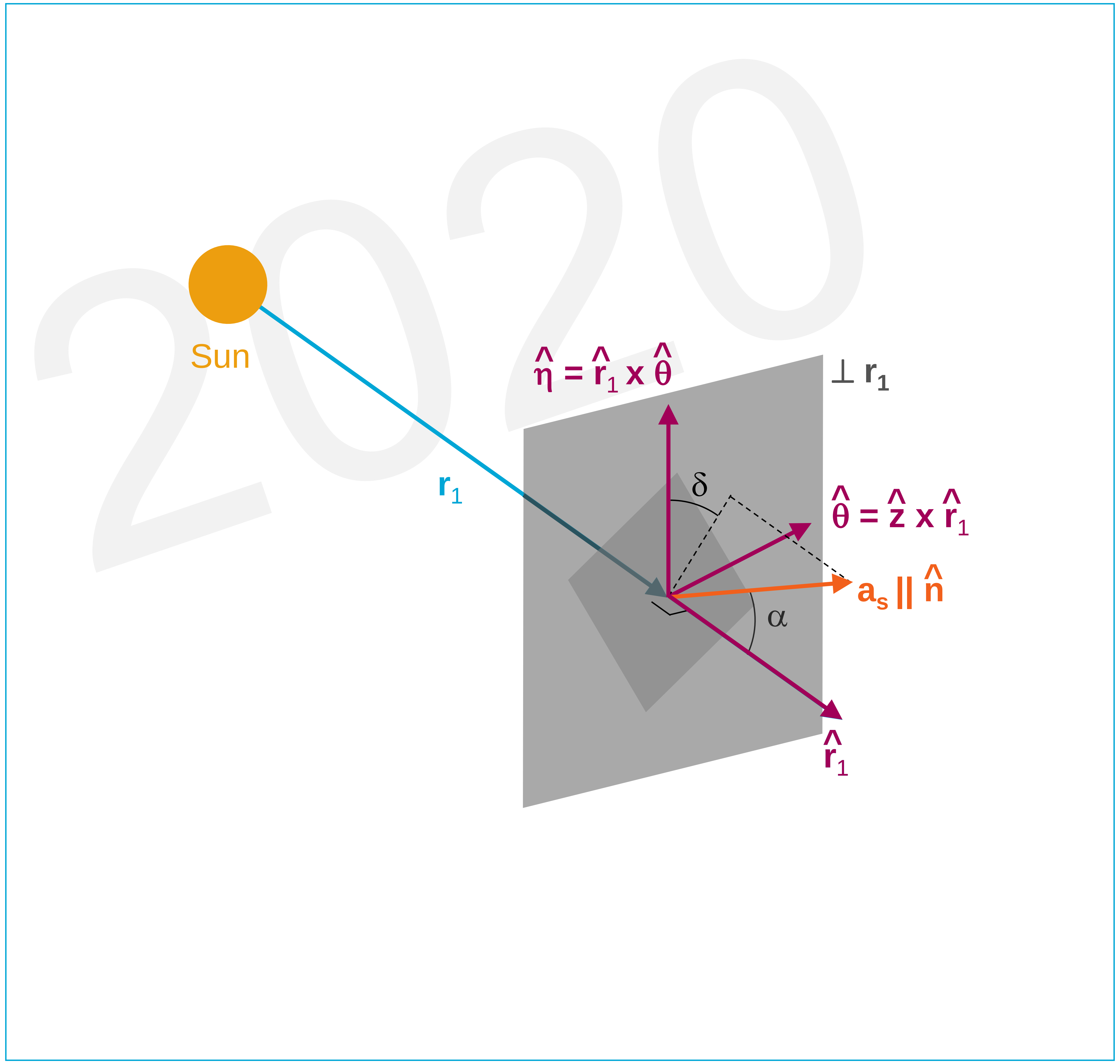
- Dimensionless solar-sail acceleration

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2} \left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}} \right)^2 \hat{\mathbf{n}}$$

- Define the sail orientation in **3D**

$$\hat{\mathbf{n}}\Big|_{\left(\hat{\mathbf{r}}_1, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\eta}}\right)} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \sin\delta \\ \sin\alpha \cos\delta \end{bmatrix}$$

α = cone angle
δ = clock angle



Solar-sail acceleration in the dimensionless cr3bp

- Dimensionless solar-sail acceleration

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- Define the sail orientation in **3D**

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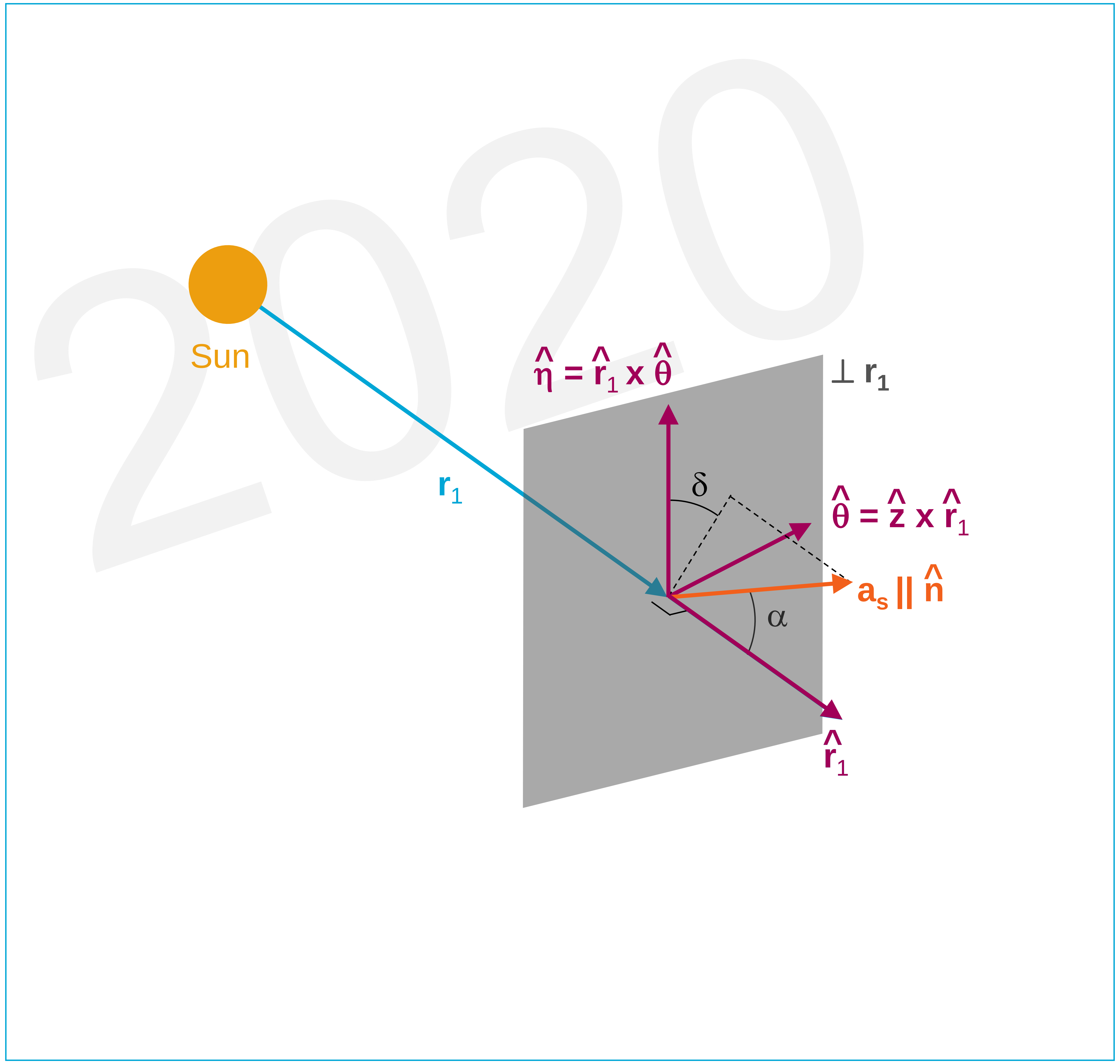
Reference frame transformation!



Solar-sail acceleration in the dimensionless cr3bp

- Dimensionless solar-sail acceleration

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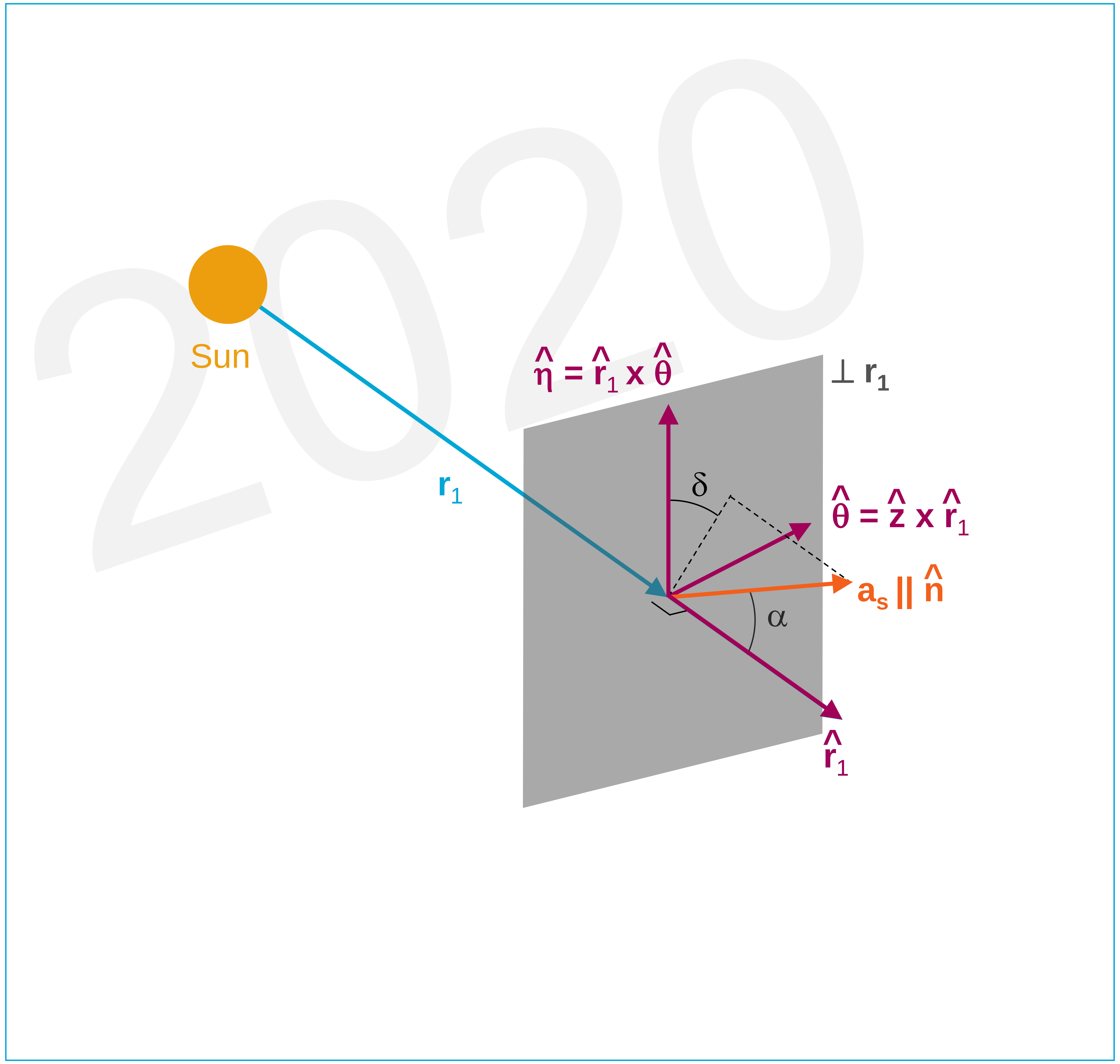
Reference frame transformation!

$$\hat{\mathbf{n}}\Big|_{(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})} = \hat{\mathbf{n}} = \begin{bmatrix} \hat{\mathbf{r}}_1 & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\eta}} \end{bmatrix} \hat{\mathbf{n}}\Big|_{(\hat{\mathbf{r}}_1, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\eta}})}$$



Solar-sail perturbed cr3bp

- Equations of motion

$$\ddot{\mathbf{r}} = -2\boldsymbol{\omega}\times\dot{\mathbf{r}} - \nabla U + \mathbf{a}_s$$

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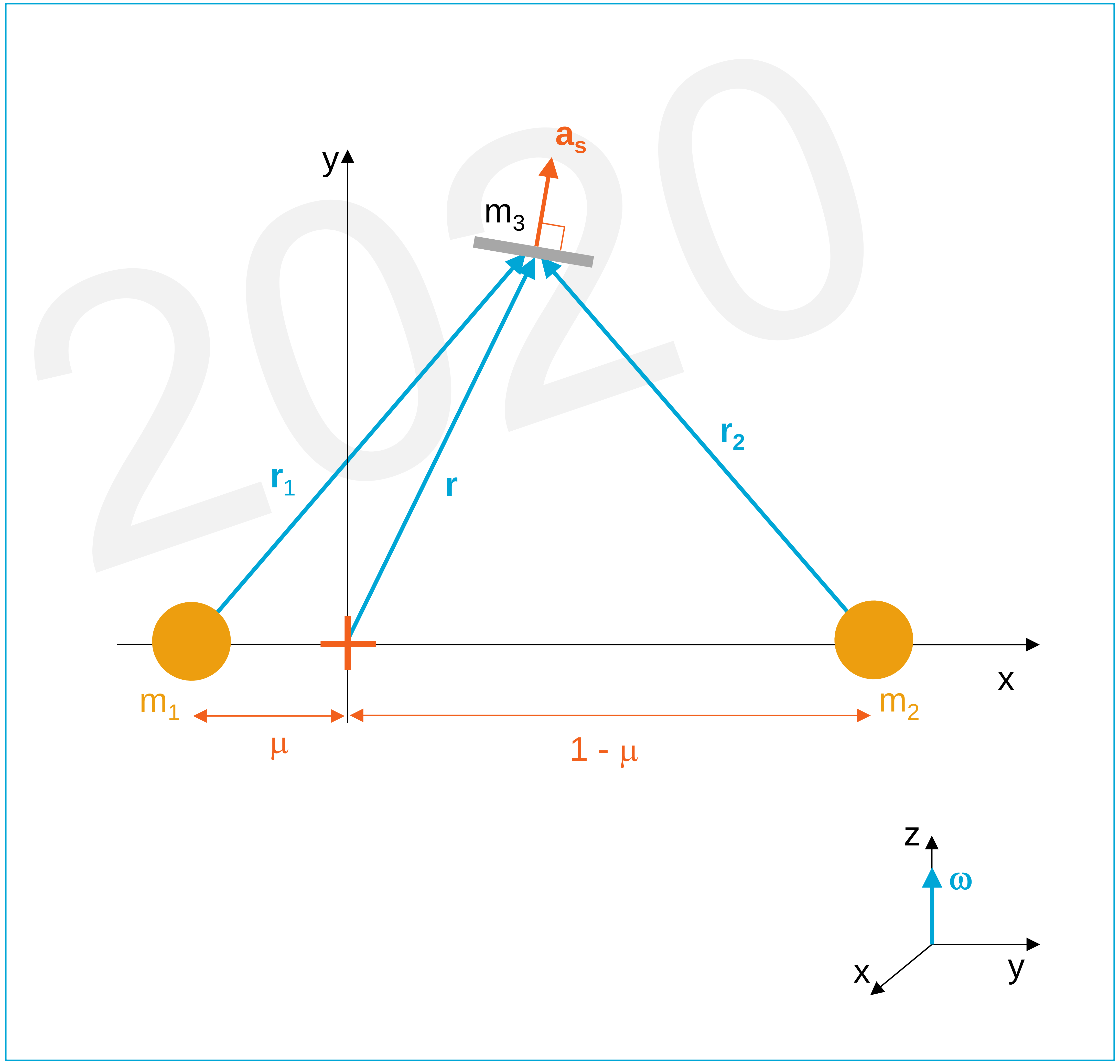
  - $\boldsymbol{\omega} = \begin{bmatrix} 0 & 0 & \omega \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$
  - $\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}}$



Solar-sail perturbed cr3bp

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  - $\mathbf{r} = \begin{bmatrix} x & y & z \end{bmatrix}^T, \dot{\mathbf{r}} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T, \ddot{\mathbf{r}} = \begin{bmatrix} \ddot{x} & \ddot{y} & \ddot{z} \end{bmatrix}^T$
  - $U = -\left(\frac{1-\mu}{r_1} + \frac{\mu}{r_2}\right) - \frac{1}{2}\left(x^2 + y^2\right)$
  - $\mu = \frac{m_2}{m_1 + m_2}$
  - $\mathbf{r}_1 = \begin{bmatrix} x+\mu & y & z \end{bmatrix}^T$
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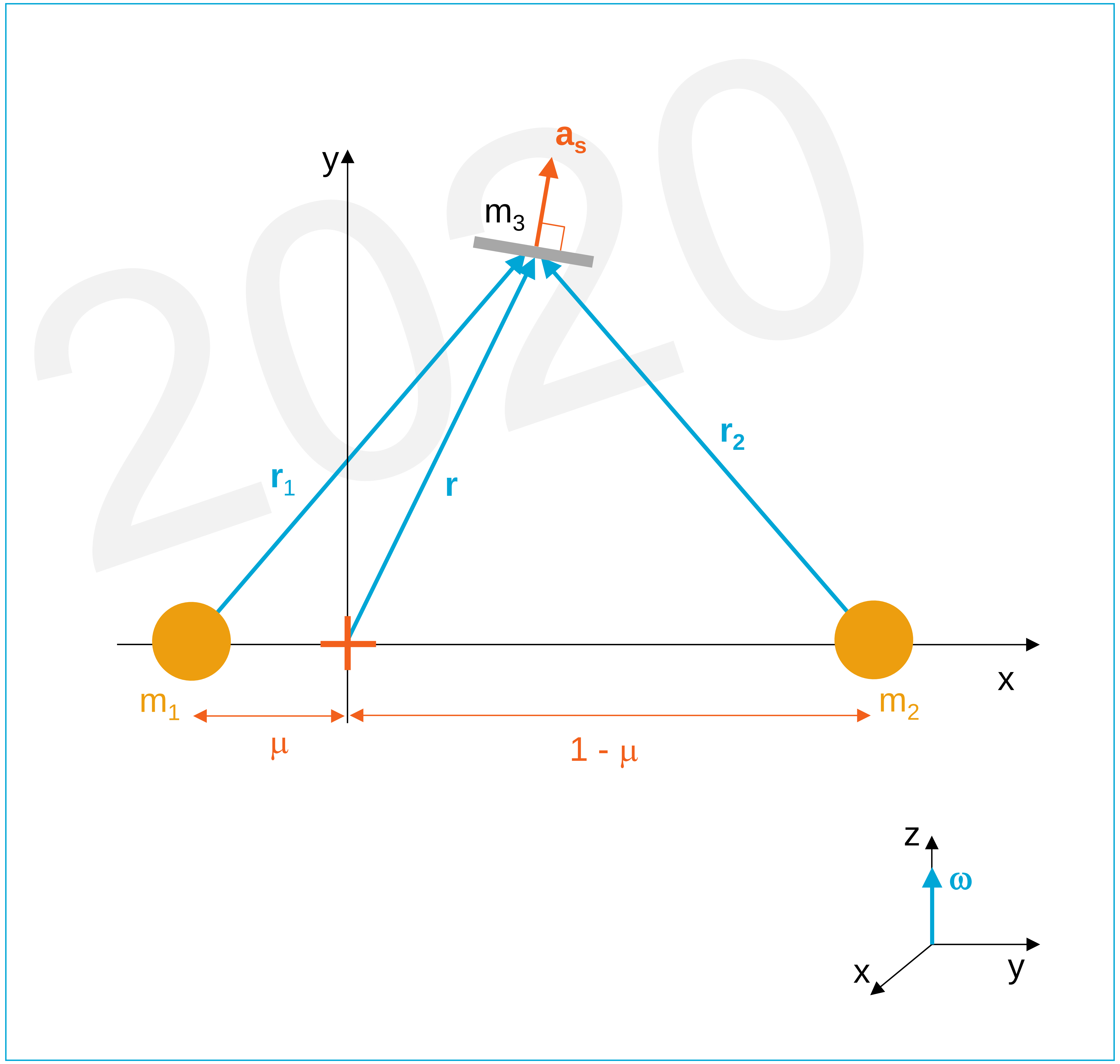
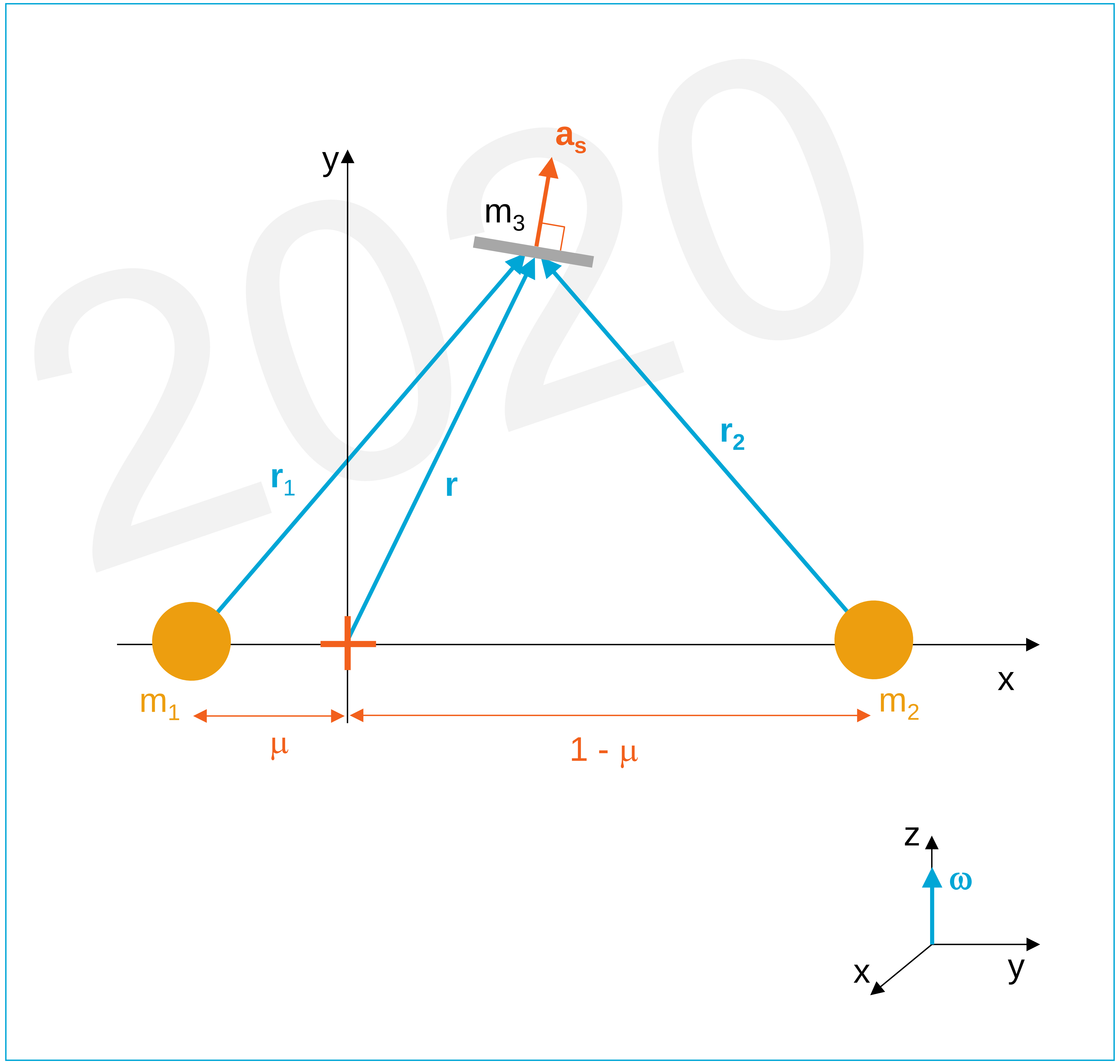
  - $\boldsymbol{\omega} = \begin{bmatrix} 0 & 0 & \omega \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$
  - $\mathbf{a}_s = \beta\frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}}$
  - $\hat{\mathbf{n}} = \begin{bmatrix} \hat{\mathbf{r}}_1 & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\eta}} \end{bmatrix} \hat{\mathbf{n}}\Big|_{\left(\hat{\mathbf{r}}_1, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\eta}}\right)} \qquad \hat{\mathbf{n}}\Big|_{\left(\hat{\mathbf{r}}_1, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\eta}}\right)} = \begin{bmatrix} \cos\alpha \\ \sin\alpha\sin\delta \\ \sin\alpha\cos\delta \end{bmatrix}$



Solar-sail Lagrange points

- Deriving equilibria in the solar-sail perturbed cr3bp

$$\underbrace{\cancel{\ddot{\mathbf{r}}}}_{=0} = -2\boldsymbol{\omega}\times\underbrace{\cancel{\dot{\mathbf{r}}}}_{=0} - \nabla U + \mathbf{a}_s$$



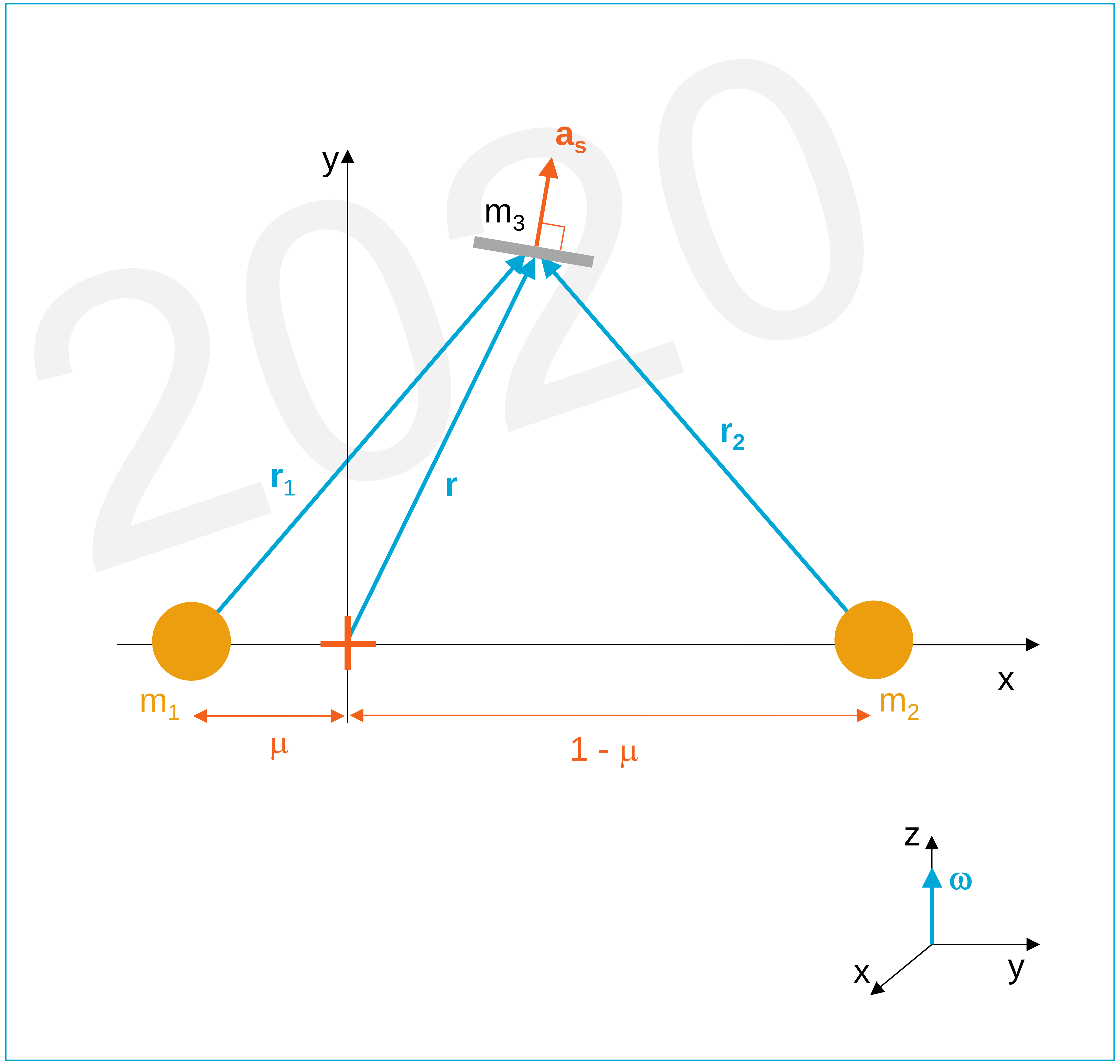
Solar-sail Lagrange points

- Deriving equilibria in the solar-sail perturbed cr3bp

$$\underbrace{\cancel{\ddot{\mathbf{r}}}}_{=0} = -2\boldsymbol{\omega}\times\underbrace{\cancel{\dot{\mathbf{r}}}}_{=0} - \nabla U + \mathbf{a}_s$$

- Solar-sail Lagrange points (aka **artificial Lagrange points**)

$$\nabla U = \mathbf{a}_s$$



Solar-sail Lagrange points – required sail orientation + performance

- Deriving equilibria in the solar-sail perturbed cr3bp

Solar-sail Lagrange points – required sail orientation + performance

- Deriving equilibria in the solar-sail perturbed cr3bp
- Required sail acceleration

$$\mathbf{a}_s = \nabla U$$

Solar-sail Lagrange points – required sail orientation + performance

- Deriving equilibria in the solar-sail perturbed cr3bp
- Required sail acceleration
- Substitute the sail acceleration

$$\mathbf{a}_s = \nabla U$$

$$\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}} = \nabla U$$

Solar-sail Lagrange points – required sail orientation + performance

- Deriving equilibria in the solar-sail perturbed cr3bp
- Required sail acceleration
- Substitute the sail acceleration
- Take the cross product of both sides

$$\mathbf{a}_s = \nabla U$$

$$\beta \frac{1-\mu}{r_1^2} \left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}} = \nabla U$$

$$\left(\beta \frac{1-\mu}{r_1^2} \left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}}\right) \times \hat{\mathbf{n}} = \nabla U \times \hat{\mathbf{n}}$$

Solar-sail Lagrange points – required sail orientation + performance

- Deriving equilibria in the solar-sail perturbed cr3bp
- Required sail acceleration
- Substitute the sail acceleration
- Take the cross product of both sides
- Rewrite

$$\mathbf{a}_s = \nabla U$$

$$\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}} = \nabla U$$

$$\left(\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}}\right) \times \hat{\mathbf{n}} = \nabla U \times \hat{\mathbf{n}}$$

$$\underbrace{\left(\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2\right)}_{\text{Scalar term}} \underbrace{\left(\hat{\mathbf{n}} \times \hat{\mathbf{n}}\right)}_{=0} = \nabla U \times \hat{\mathbf{n}}$$

Solar-sail Lagrange points – required sail orientation + performance

- Deriving equilibria in the solar-sail perturbed cr3bp
- Required sail acceleration
- Substitute the sail acceleration
- Take the cross product of both sides
- Rewrite
- To obtain the required sail orientation

$$\mathbf{a}_s = \nabla U$$

$$\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}} = \nabla U$$

$$\left(\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}}\right) \times \hat{\mathbf{n}} = \nabla U \times \hat{\mathbf{n}}$$

$$\left(\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2\right)\left(\hat{\mathbf{n}} \times \hat{\mathbf{n}}\right) = \nabla U \times \hat{\mathbf{n}}$$

$$\mathbf{0} = \nabla U \times \hat{\mathbf{n}} \Rightarrow \hat{\mathbf{n}} = \frac{\nabla U}{\left|\nabla U\right|}$$

Solar-sail Lagrange points – required sail orientation + performance

- Deriving equilibria in the solar-sail perturbed cr3bp
- Required sail acceleration
- Substitute the sail acceleration
- Take the cross product of both sides
- Rewrite
- To obtain the required sail orientation
- Take the dot product of both sides

$$\mathbf{a}_s = \nabla U$$

$$\beta \frac{1-\mu}{r_1^2} \left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}} \right)^2 \hat{\mathbf{n}} = \nabla U$$

$$\left(\beta \frac{1-\mu}{r_1^2} \left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}} \right)^2 \hat{\mathbf{n}} \right) \times \hat{\mathbf{n}} = \nabla U \times \hat{\mathbf{n}}$$

$$\left(\beta \frac{1-\mu}{r_1^2} \left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}} \right)^2 \right) \left(\hat{\mathbf{n}} \times \hat{\mathbf{n}} \right) = \nabla U \times \hat{\mathbf{n}}$$

$$\mathbf{0} = \nabla U \times \hat{\mathbf{n}} \Rightarrow \hat{\mathbf{n}} = \frac{\nabla U}{\left| \nabla U \right|}$$

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Solar-sail Lagrange points – required sail orientation + performance

- Deriving equilibria in the solar-sail perturbed cr3bp
- Required sail acceleration
- Substitute the sail acceleration
- Take the cross product of both sides
- Rewrite
- To obtain the required sail orientation
- Take the dot product of both sides
- Rewrite

$$\mathbf{a}_s = \nabla U$$

$$\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}} = \nabla U$$

$$\left(\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}}\right) \times \hat{\mathbf{n}} = \nabla U \times \hat{\mathbf{n}}$$

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$$\underbrace{\left(\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2\right)}_{\text{Scalar term}}\underbrace{\left(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}\right)}_{=1} = \nabla U \cdot \hat{\mathbf{n}}$$

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- To obtain the required sail orientation
- Take the dot product of both sides
- Rewrite
- To obtain the required sail performance (i.e., the solar-sail lightness number)

$$\mathbf{a}_s = \nabla U$$

$$\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}} = \nabla U$$

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Only a function of the position coordinates (x,y,z)

$$\left(\beta \frac{1-\mu}{r_1^2}\left(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}}\right)^2 \hat{\mathbf{n}}\right) \cdot \hat{\mathbf{n}} = \nabla U \cdot \hat{\mathbf{n}}$$

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Example

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  - in Sun-Earth cr3bp
  - above the ecliptic
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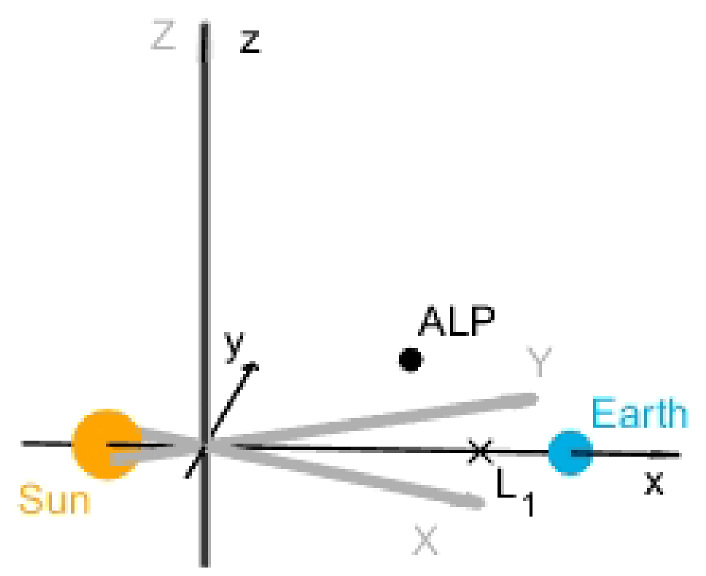

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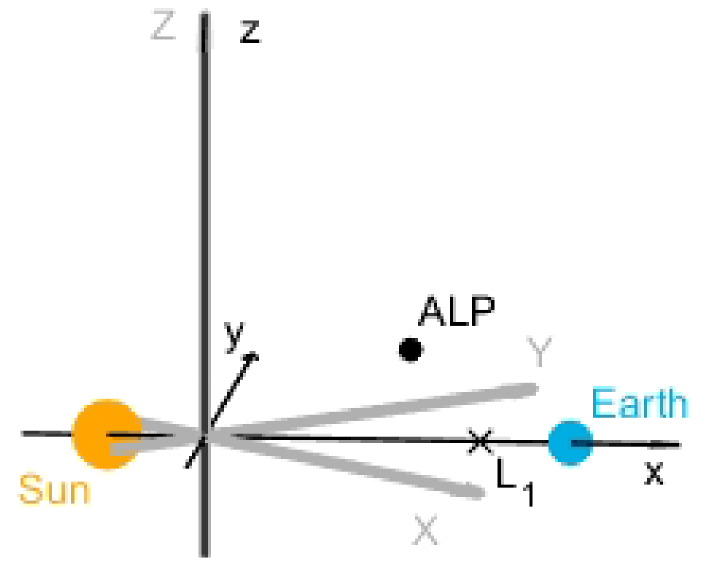
- What is the required sail orientation and performance?



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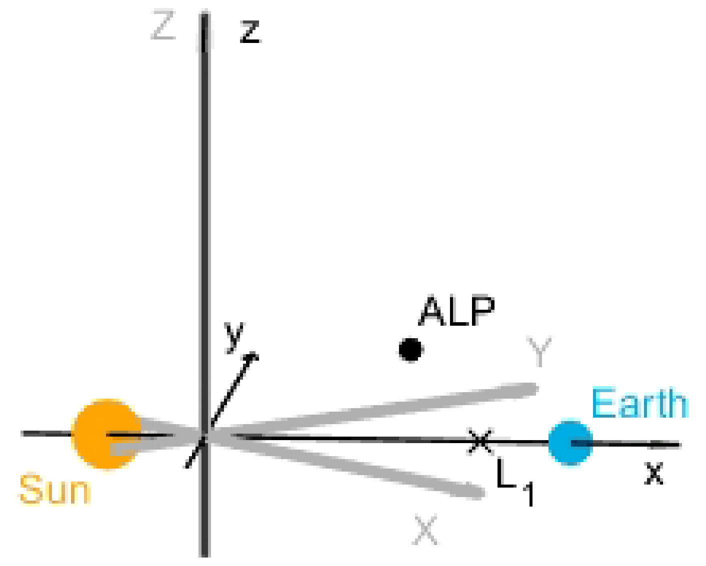

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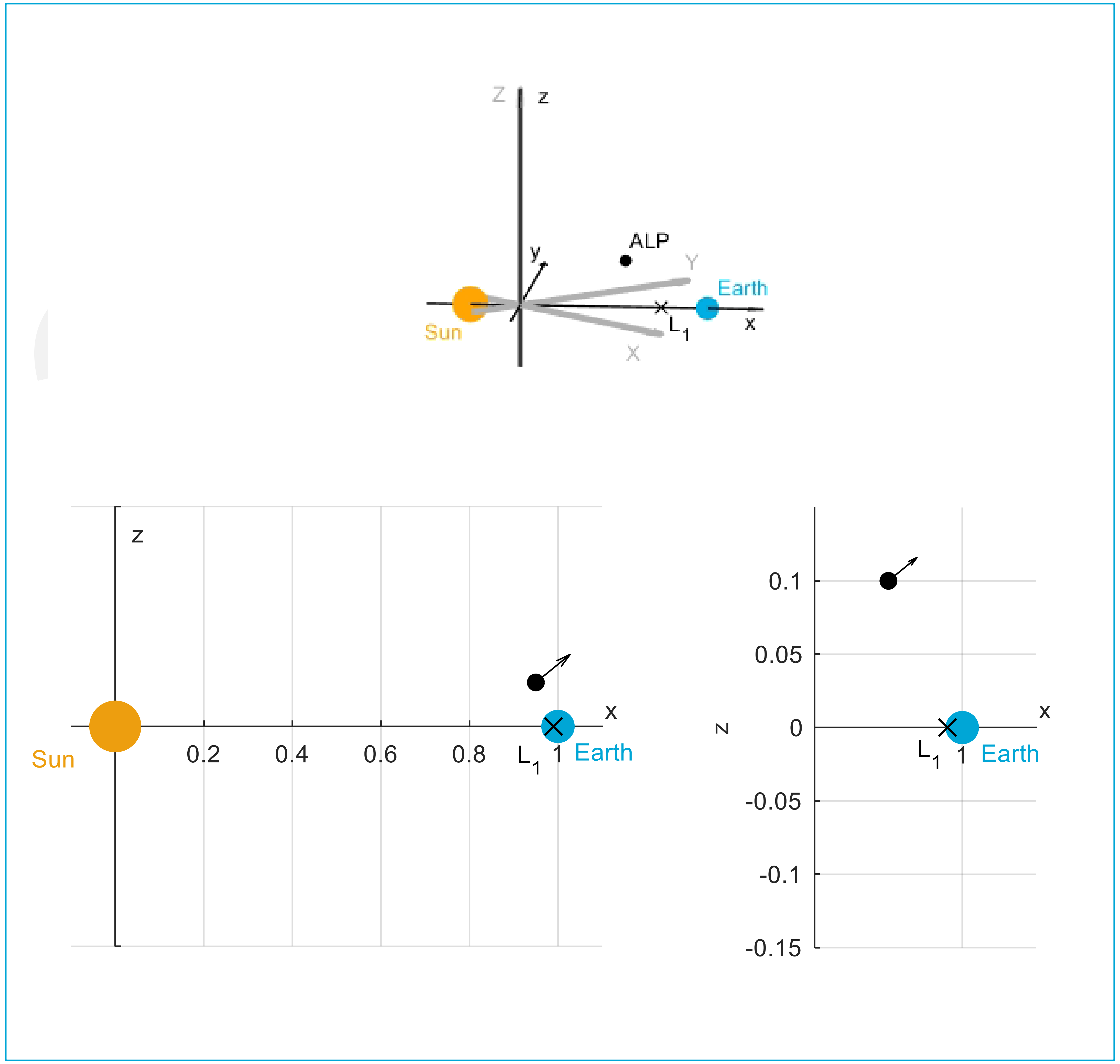

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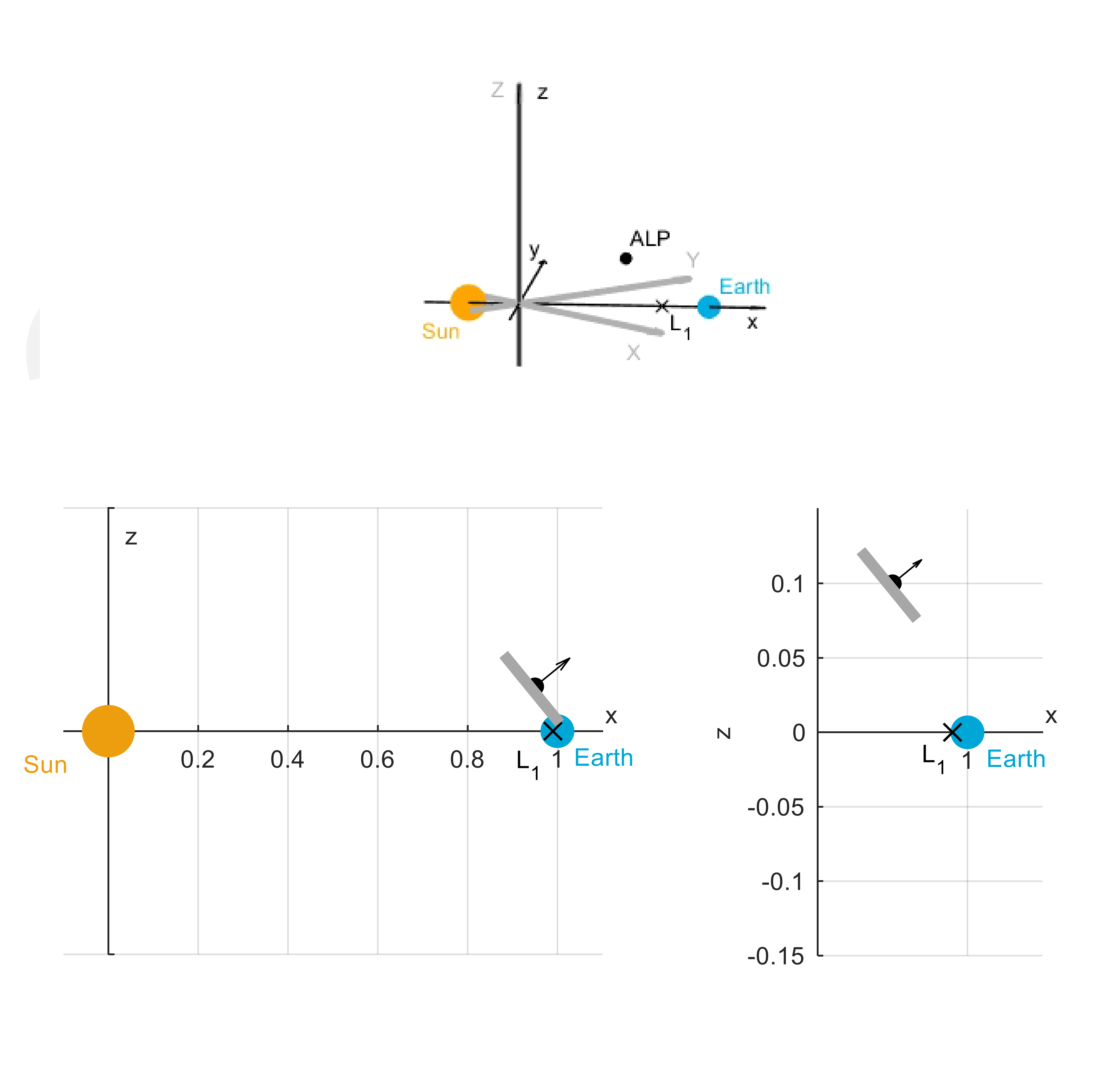

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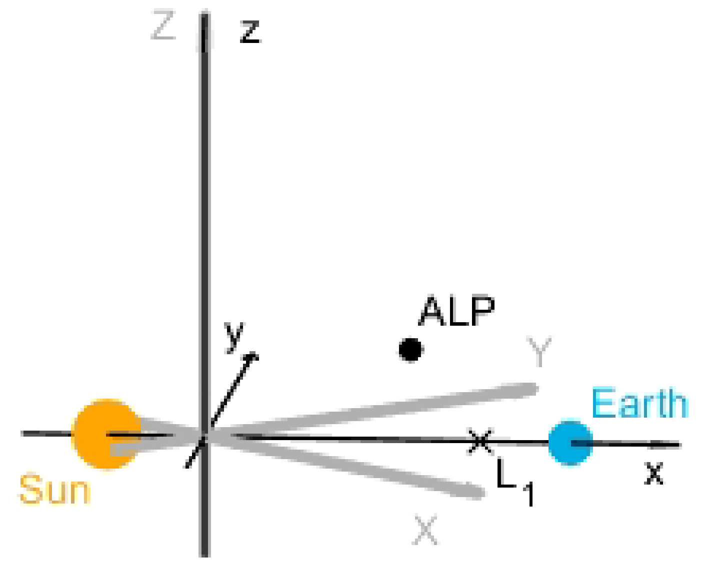

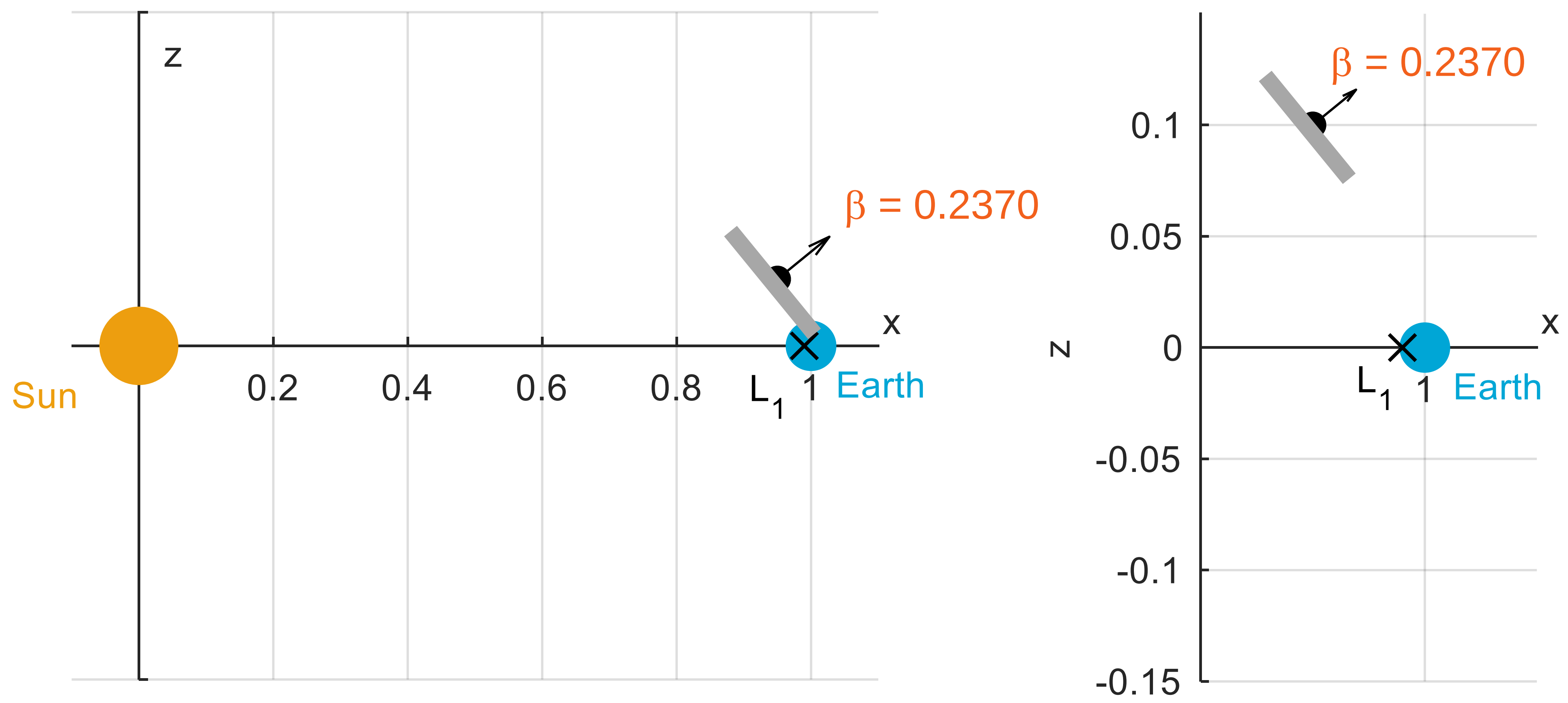

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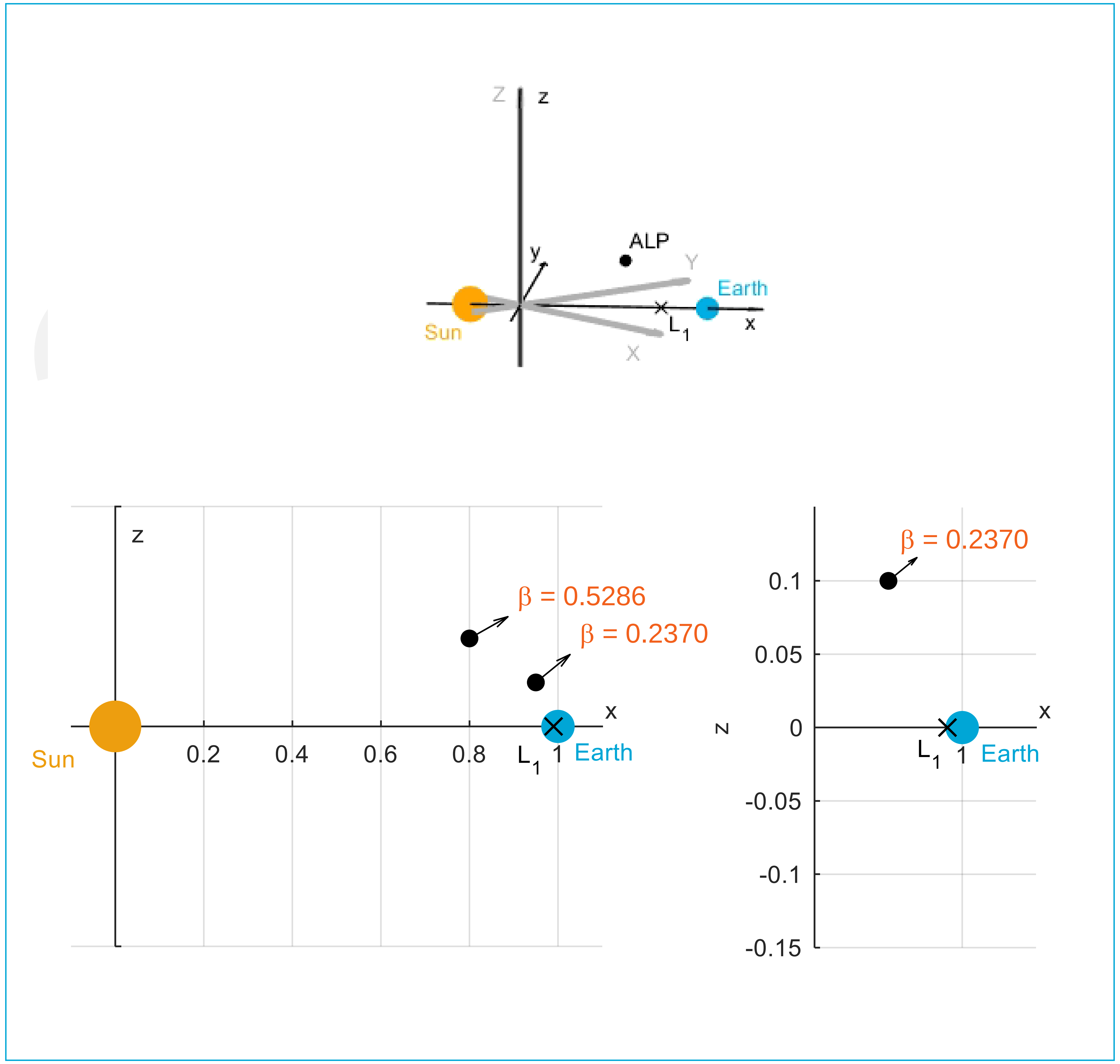

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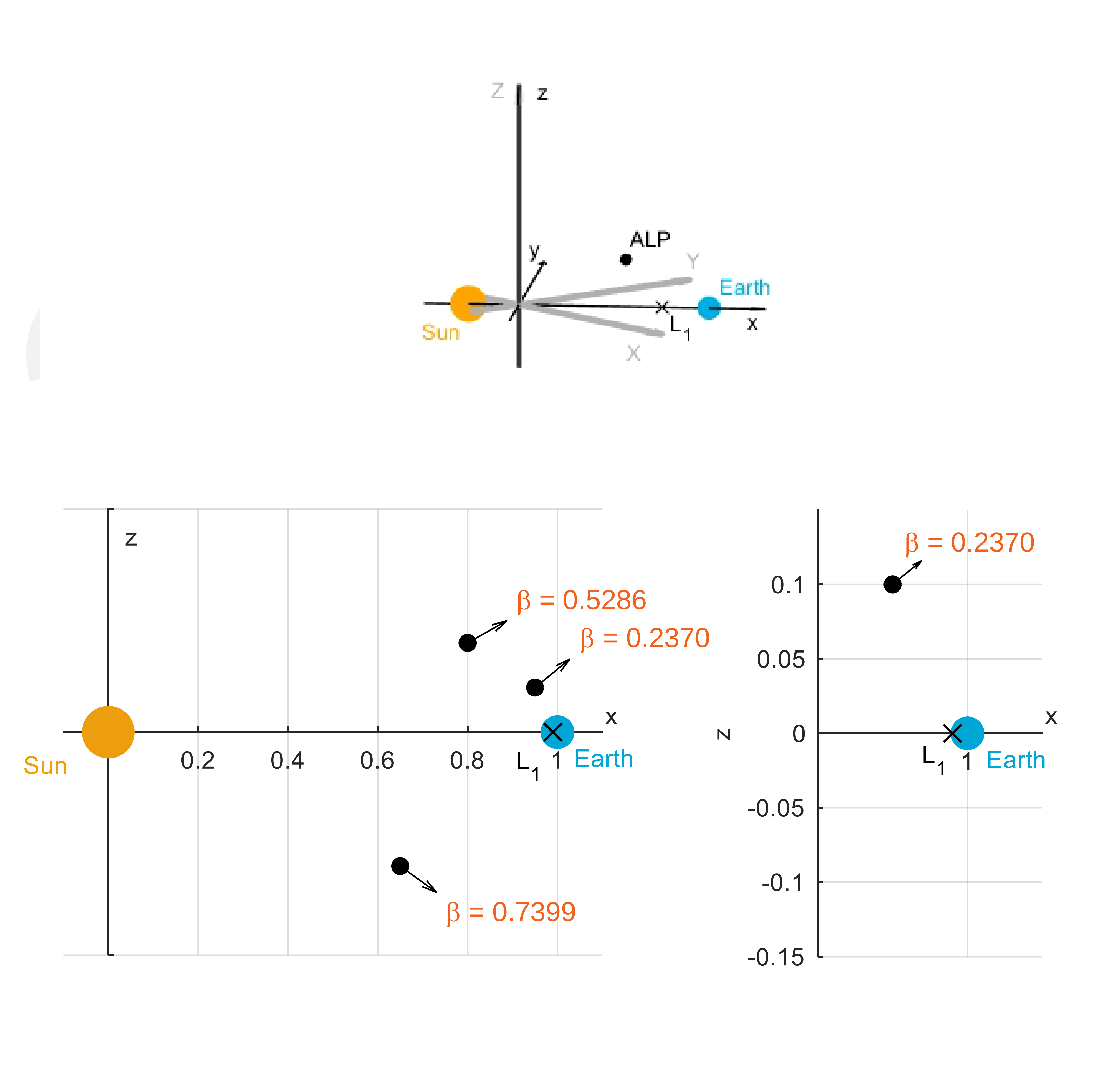

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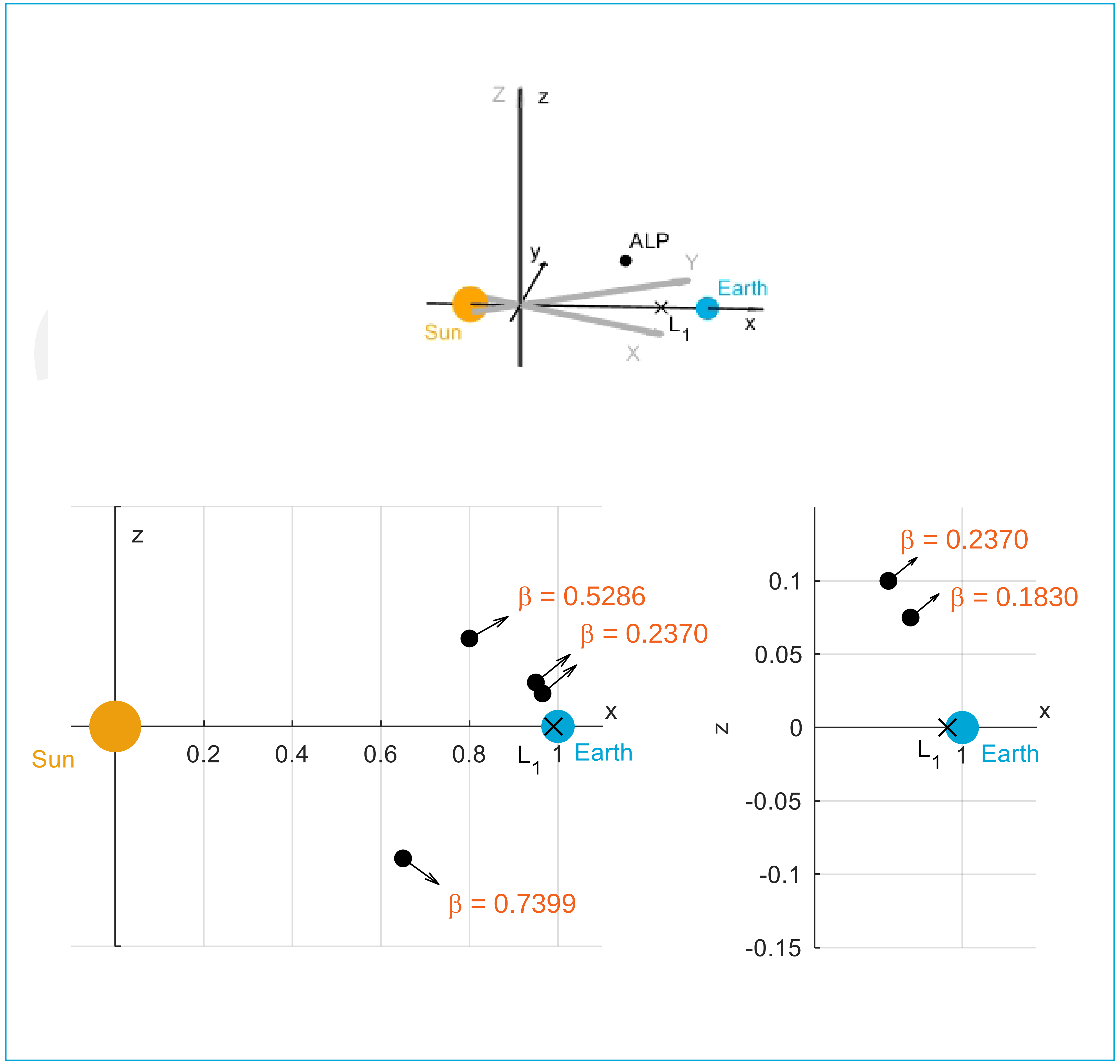

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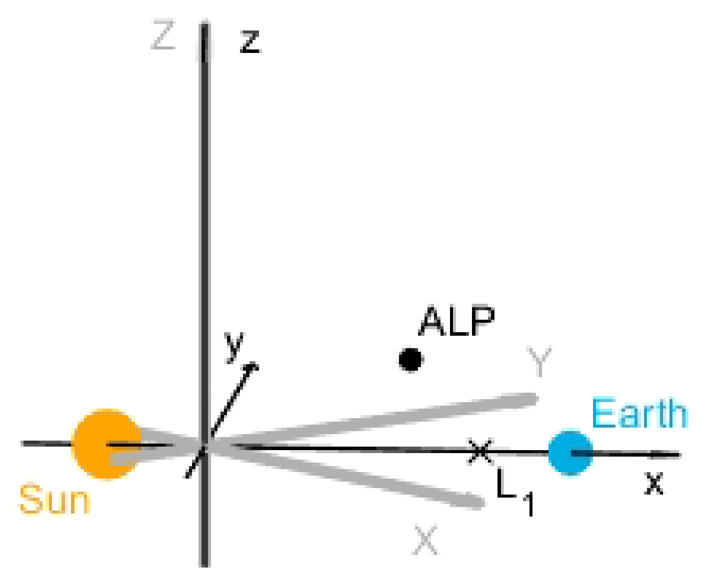

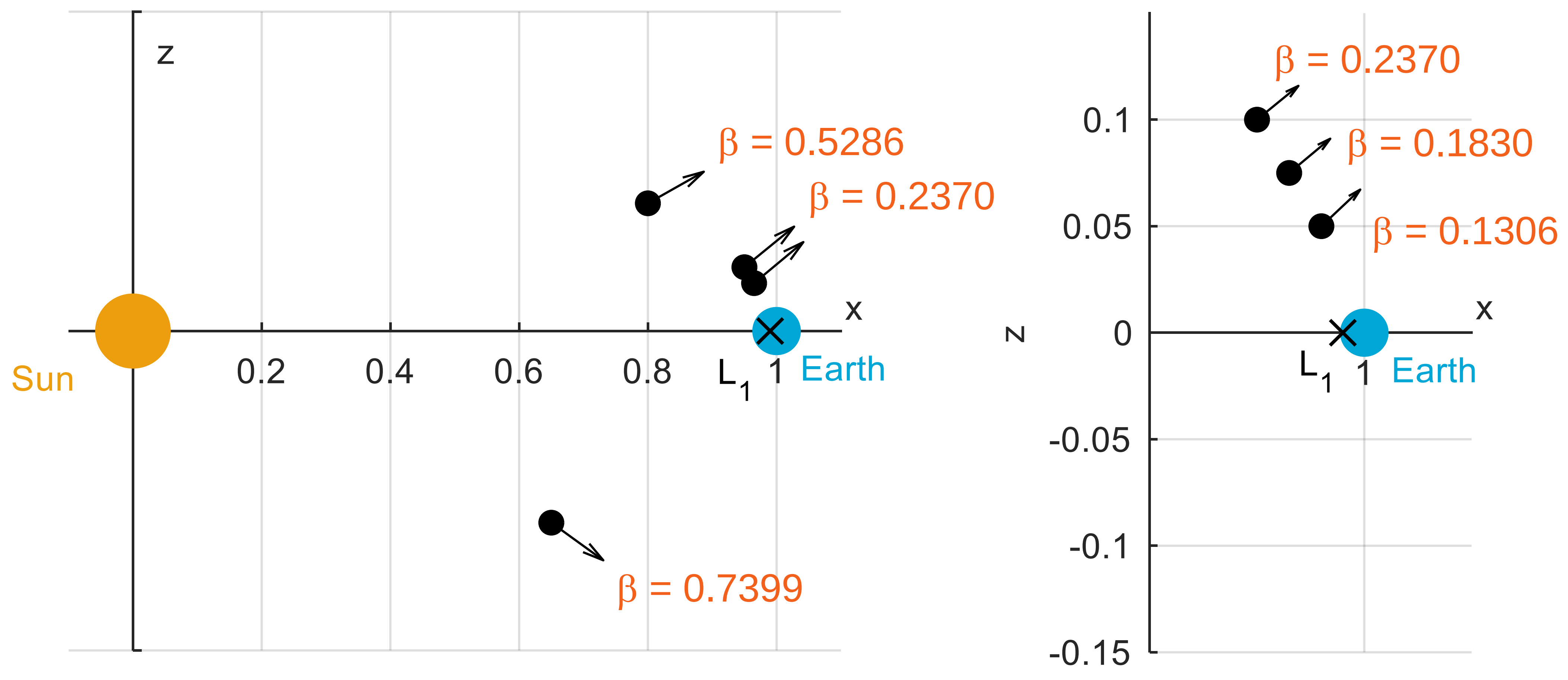

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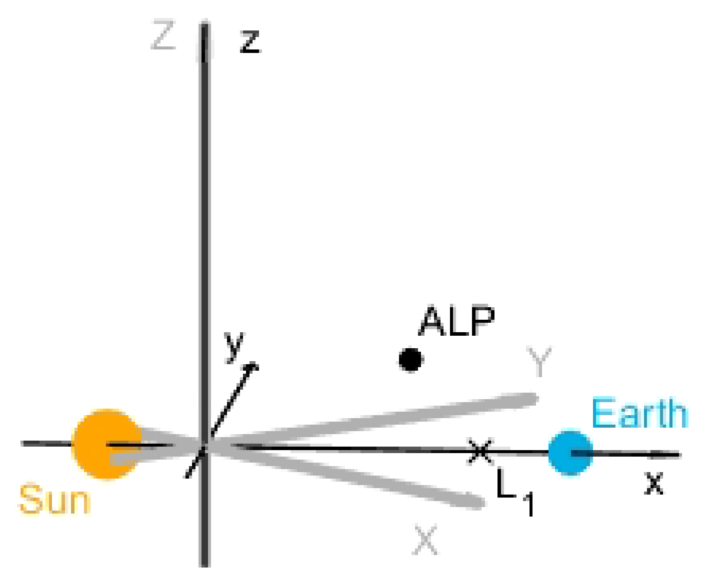

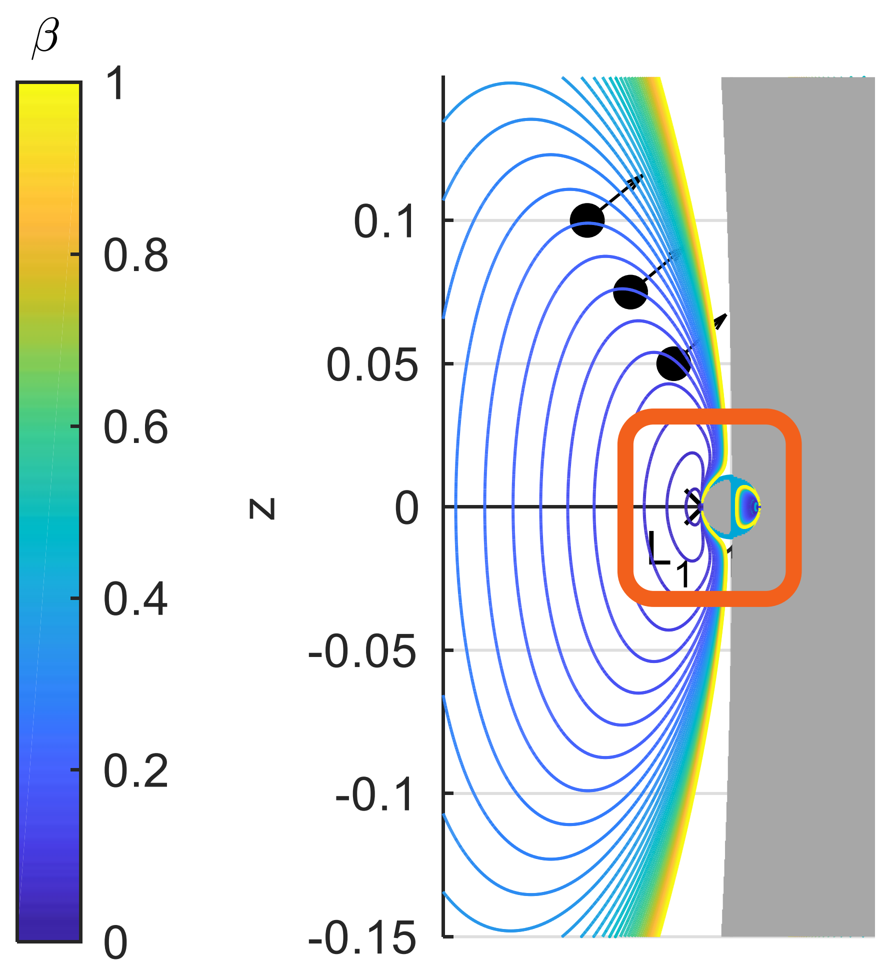

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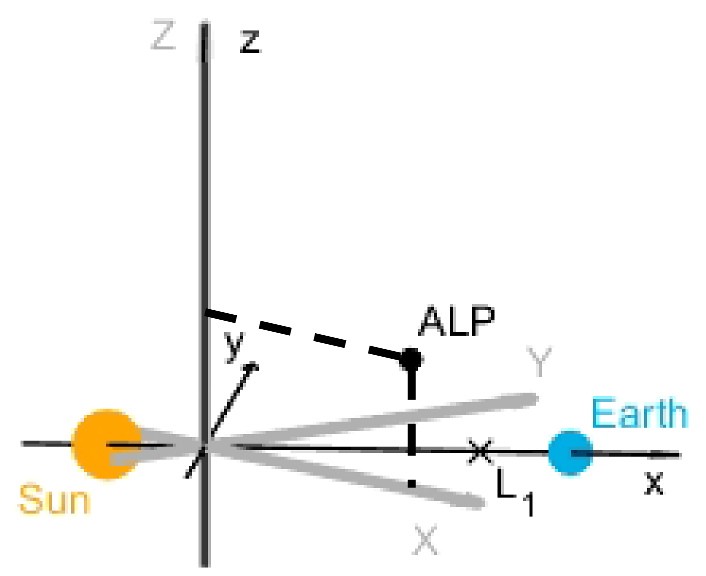

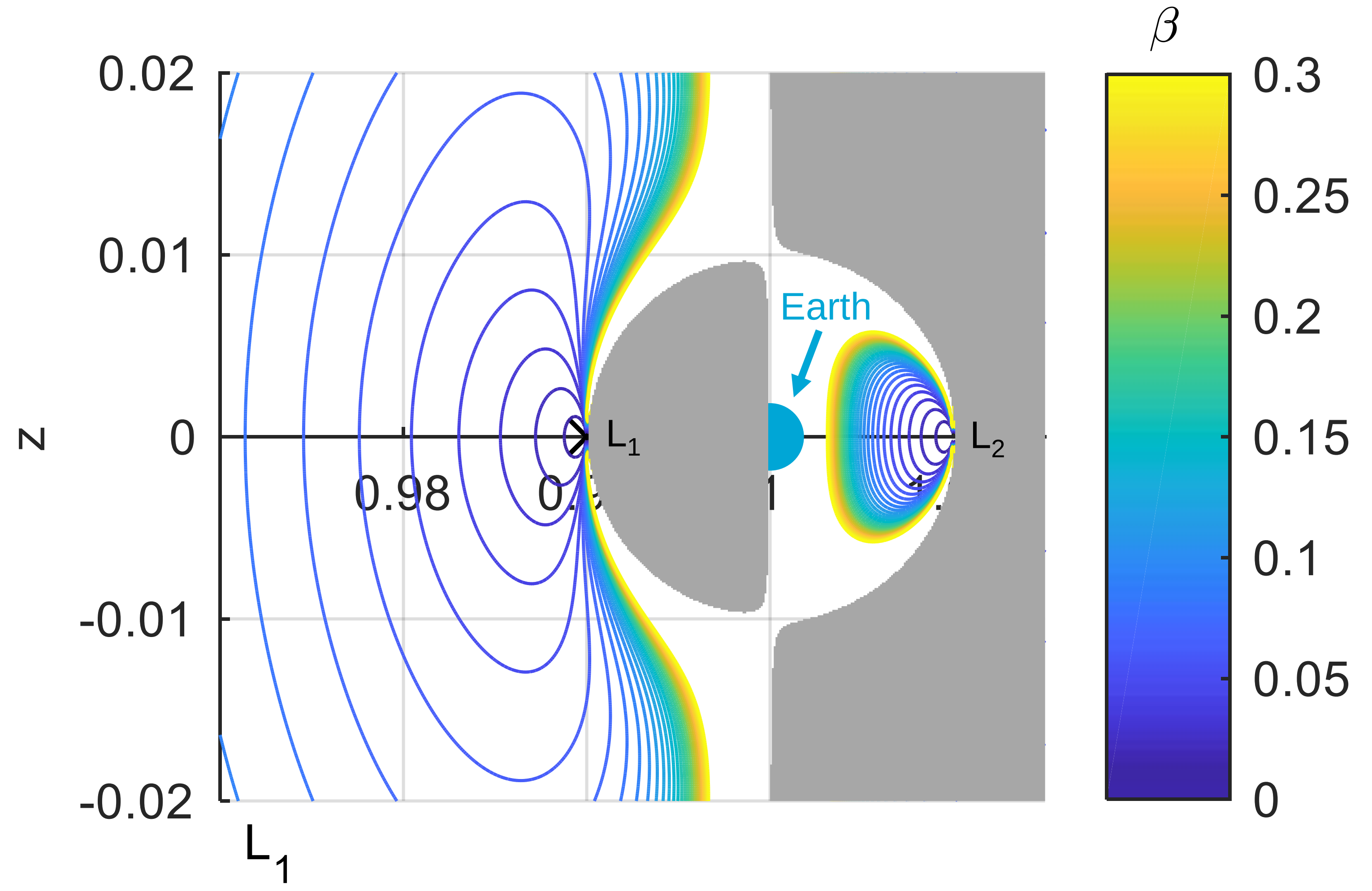

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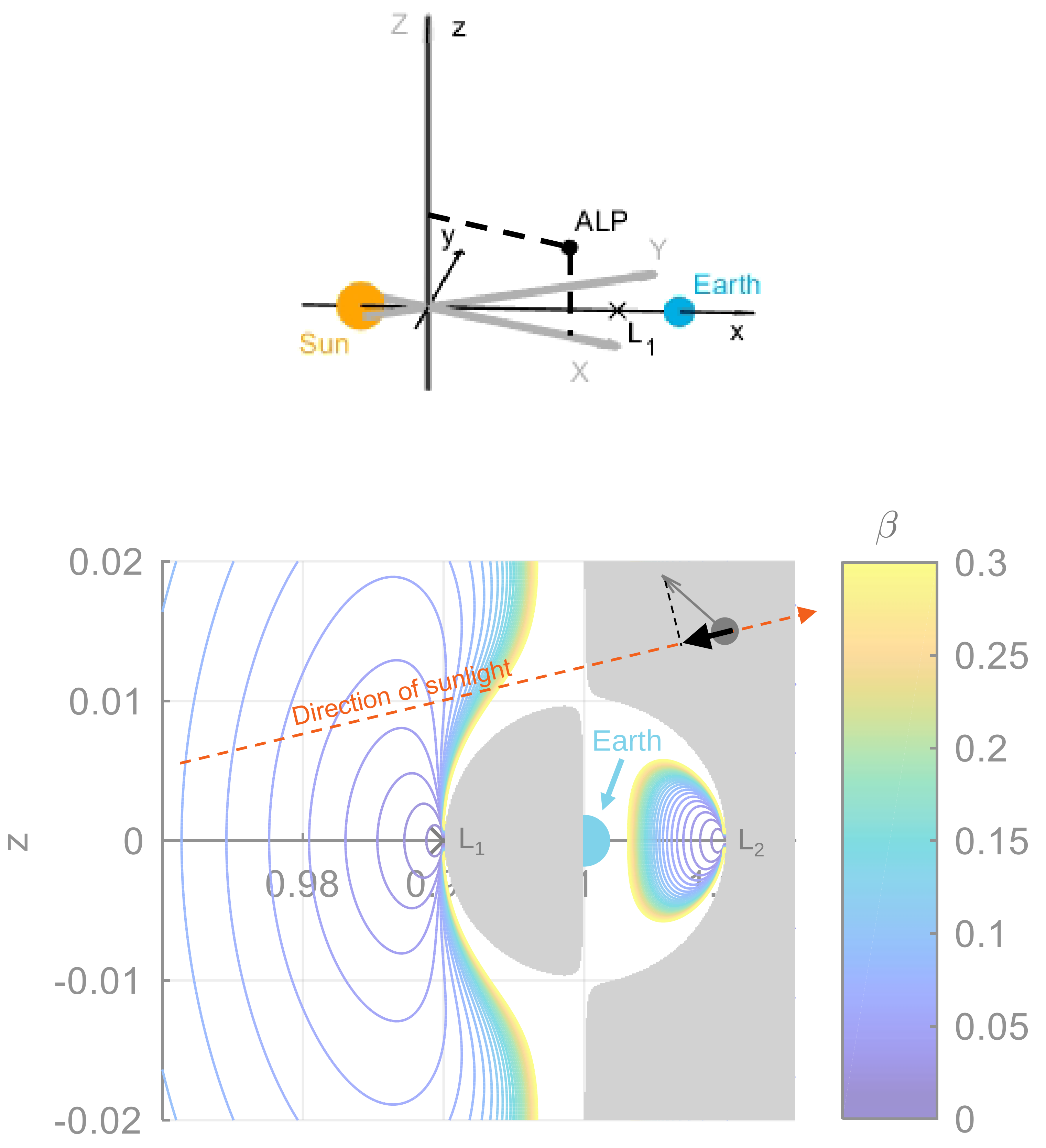

Artificial Lagrange points - utility

- Previously proposed Sunjammer mission
  - Mass: 45 kg
  - Sail size: 38 x 38 m^2
  - Lightness number ~0.04

Credit: NASA

Artificial Lagrange points - utility

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  - Mass: 45 kg
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- **Mission objective** – increase the warning time for solar storms

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  - Sail size: 38 x 38 m^2
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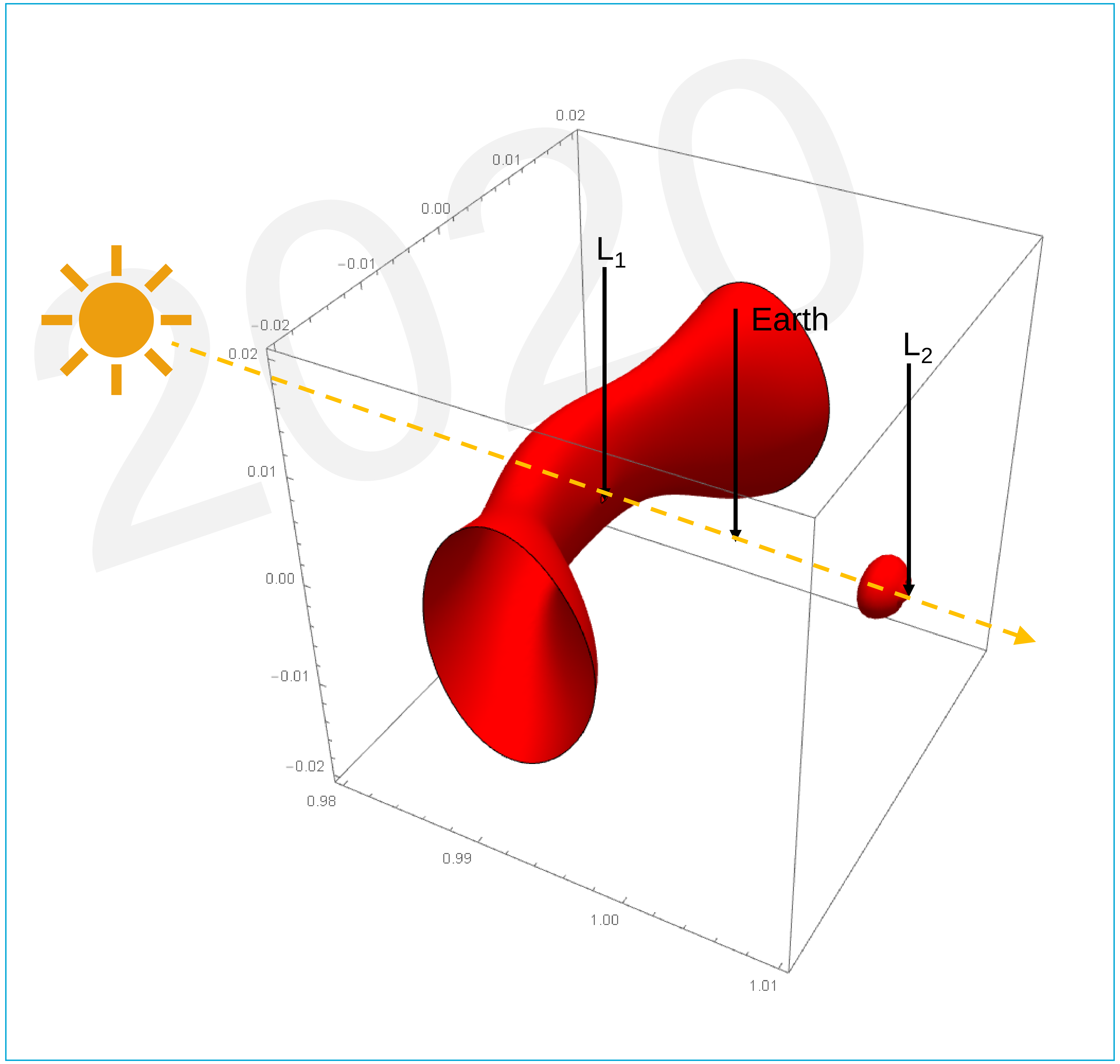
- Mission objective – increase the warning time for solar storms
- **Mission destination** – sub-L_1 point



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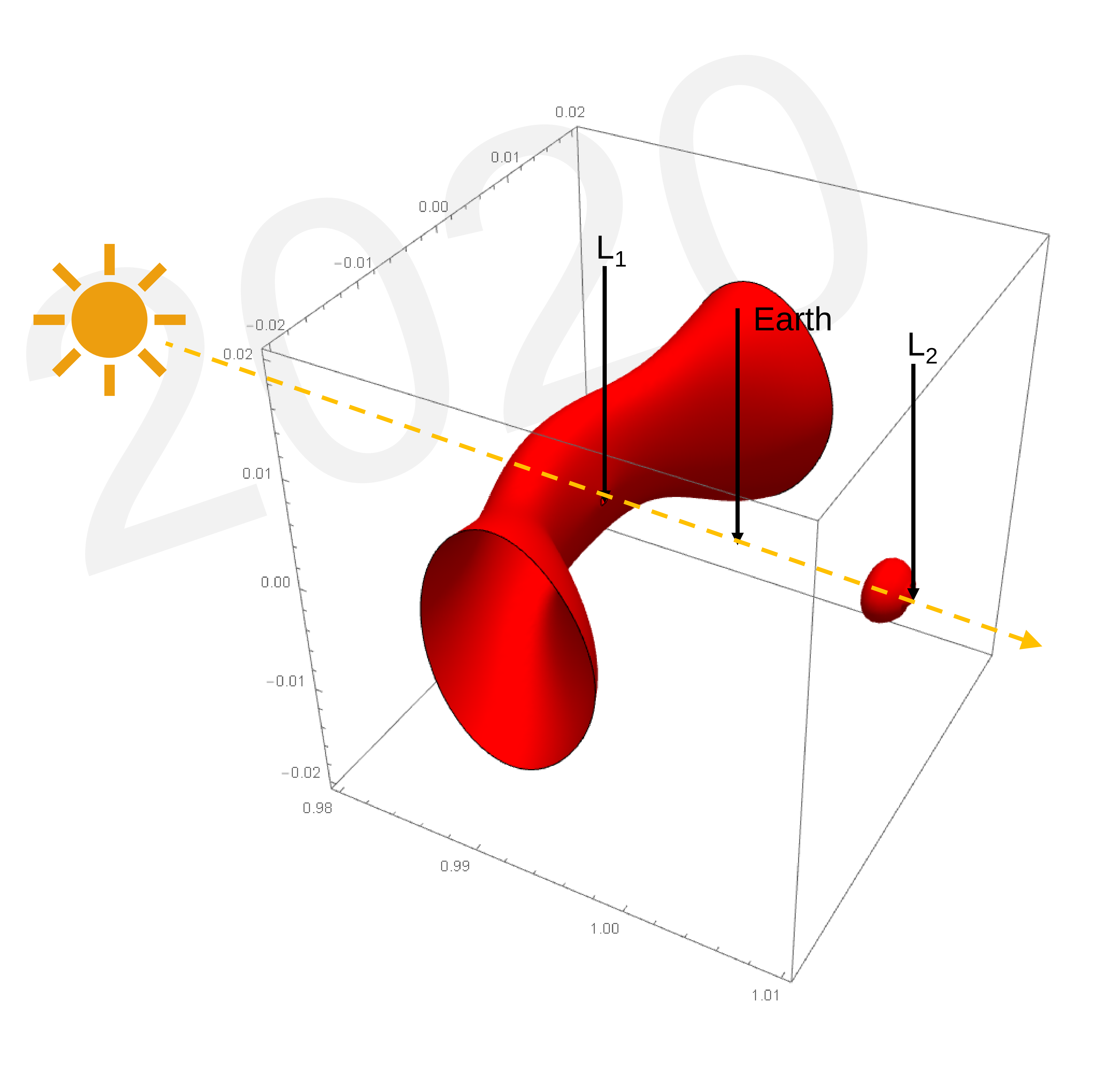
- Mission objective – increase the warning time for solar storms
- Mission destination – sub-L_1 point
- Satellites with similar objective are currently stationed at the L_1 point



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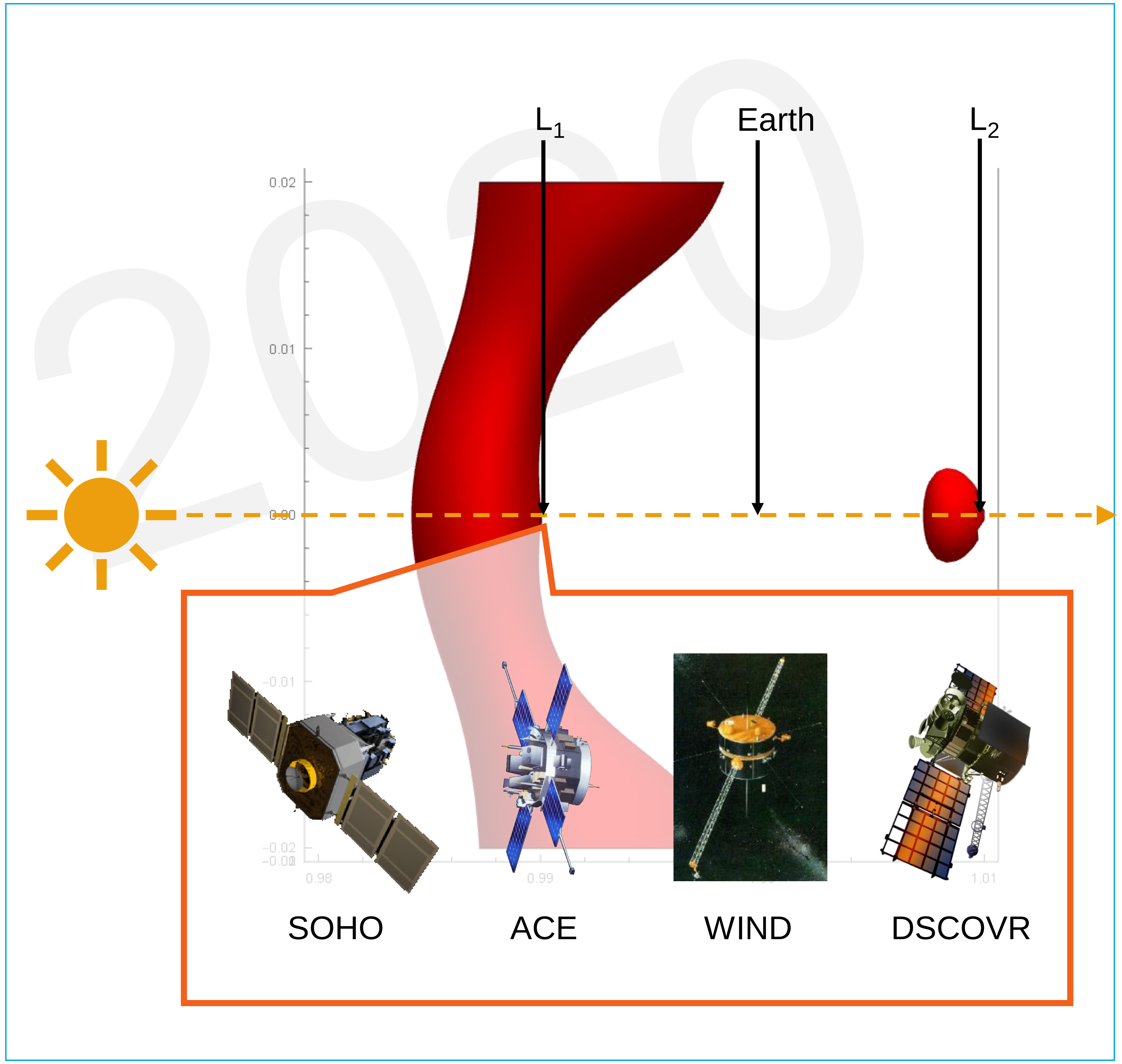

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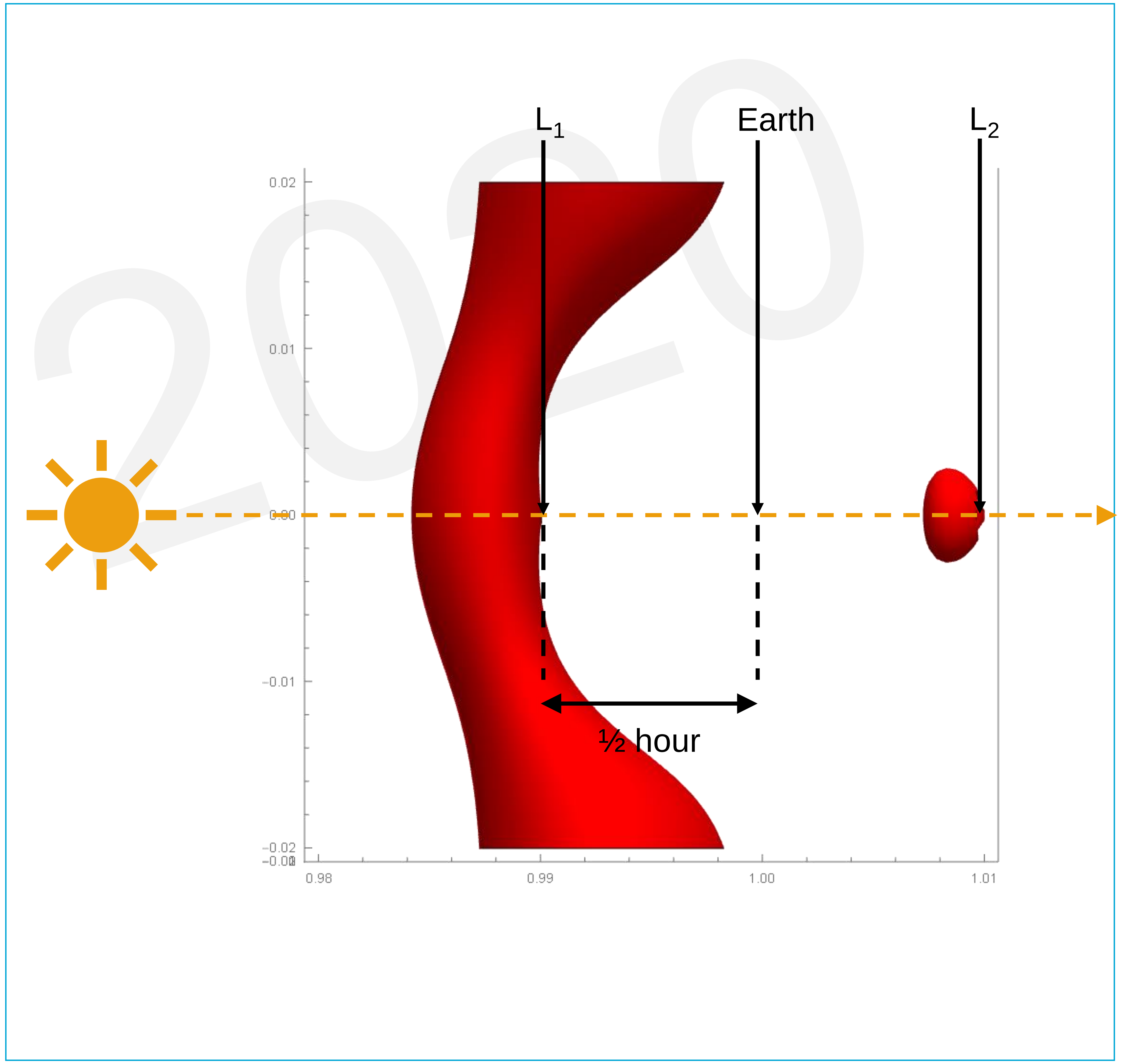

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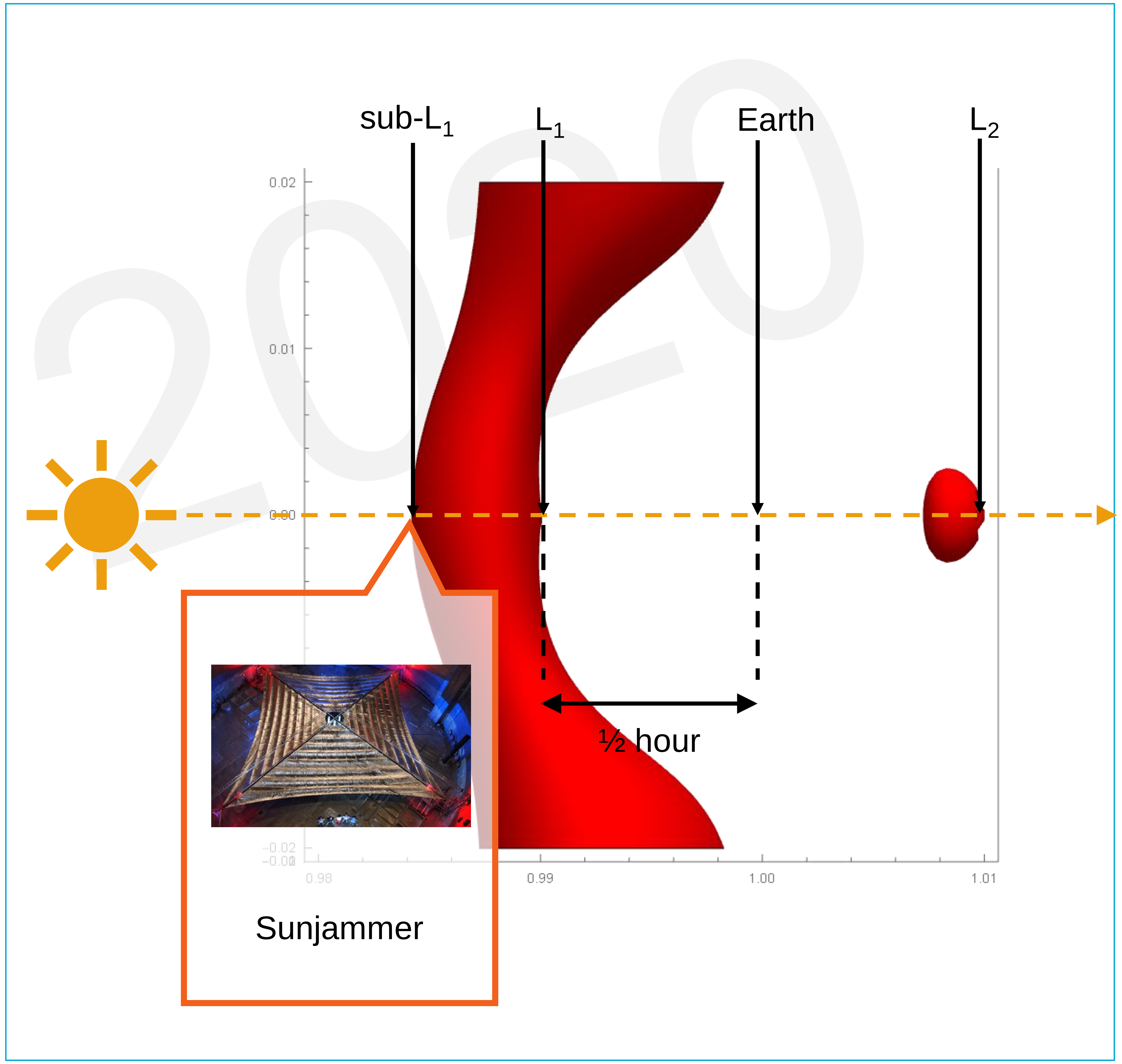

Artificial Lagrange points - utility

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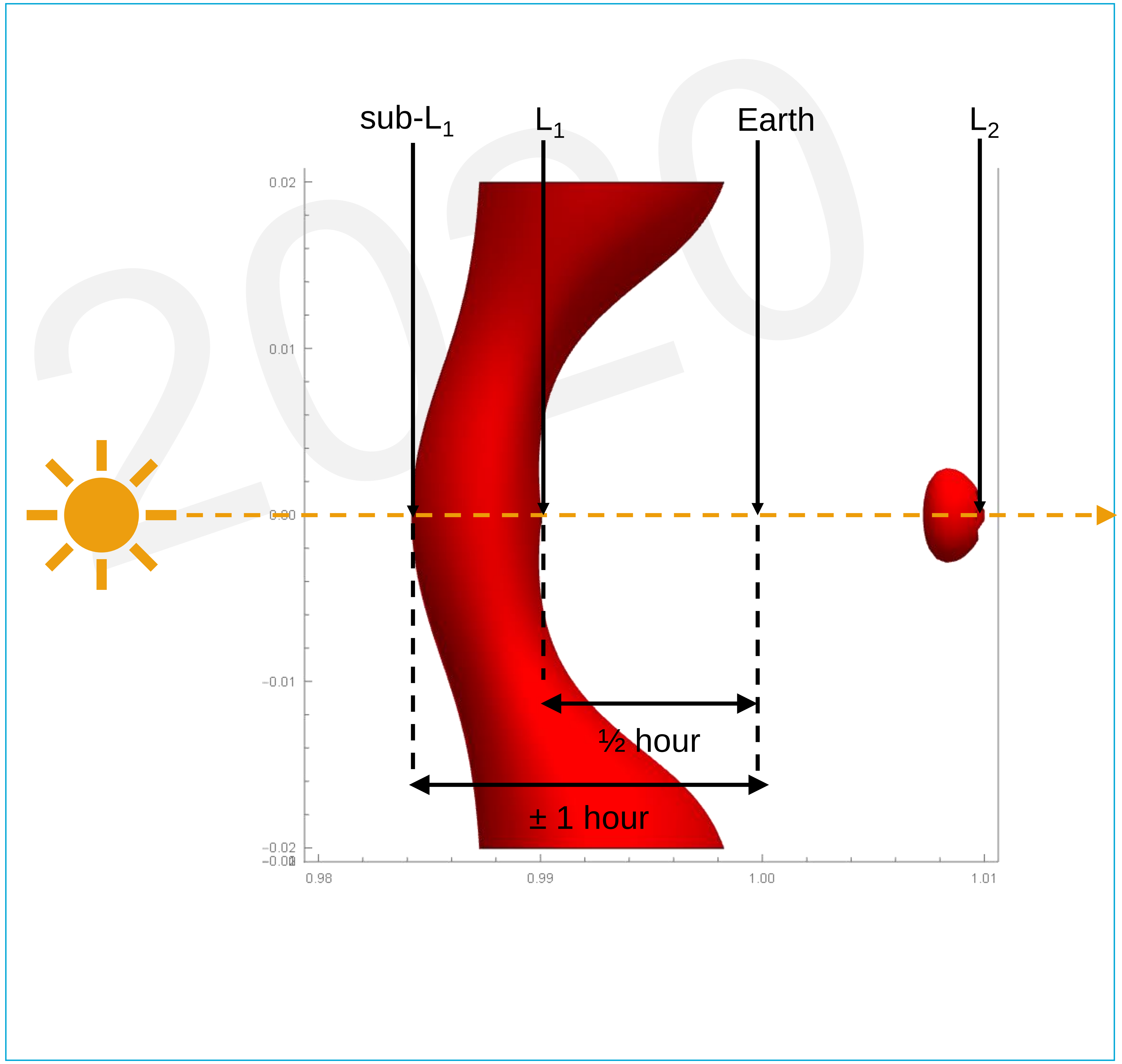
- Sunjammer could have **doubled the warning time** for solar storms



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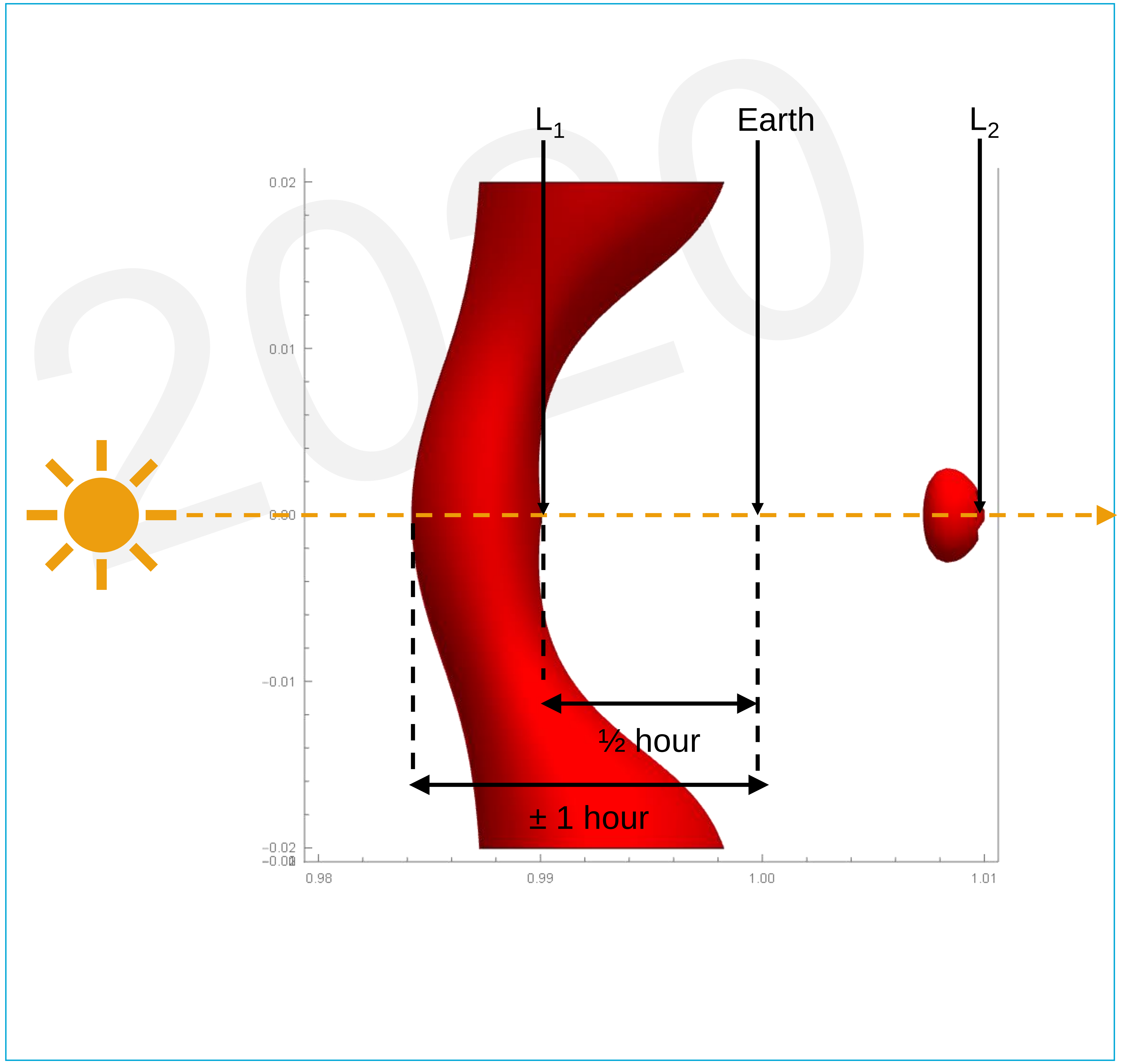
- Sunjammer could have doubled the warning time for solar storms
- Due to budgetary reasons, Sunjammer was cancelled in 2015



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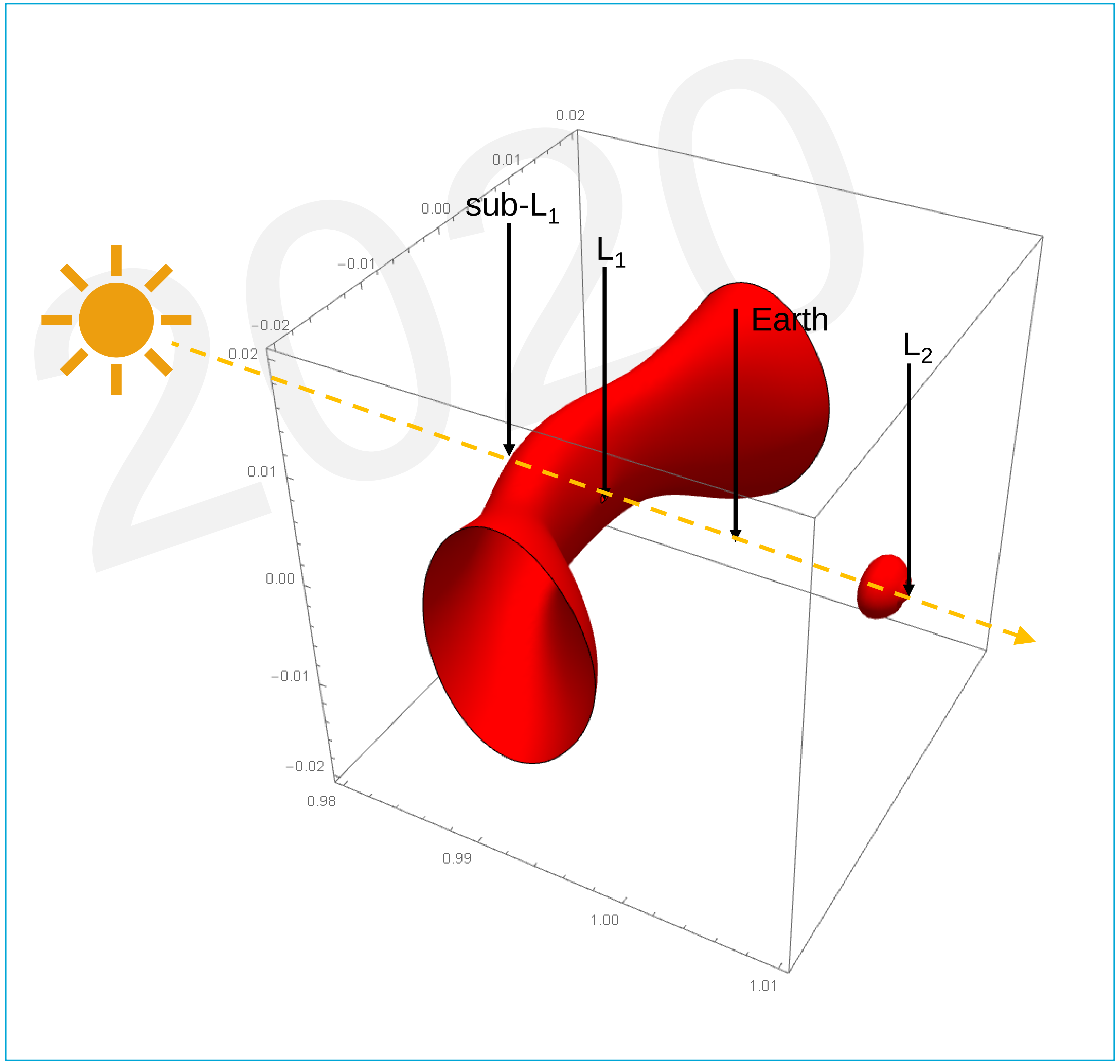
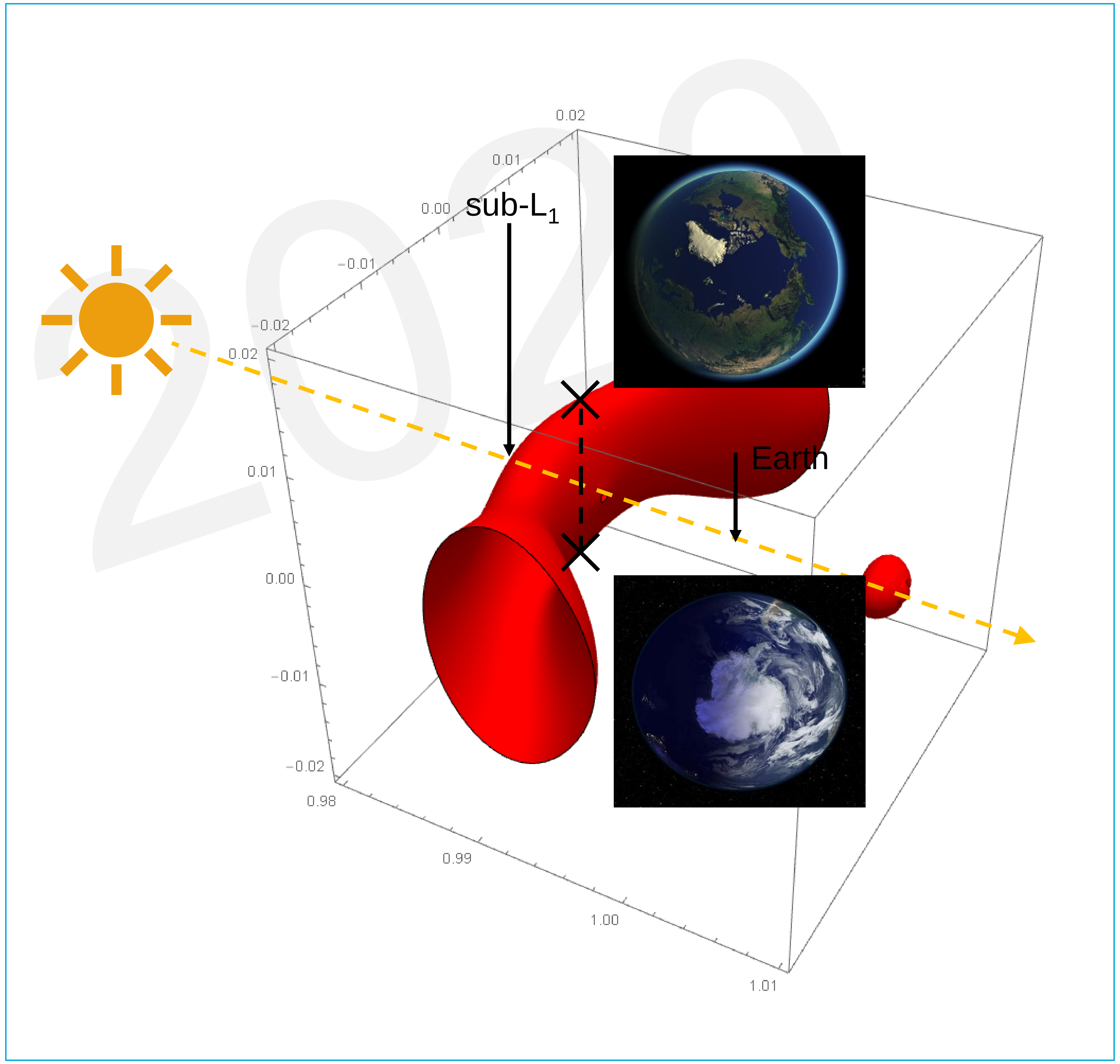


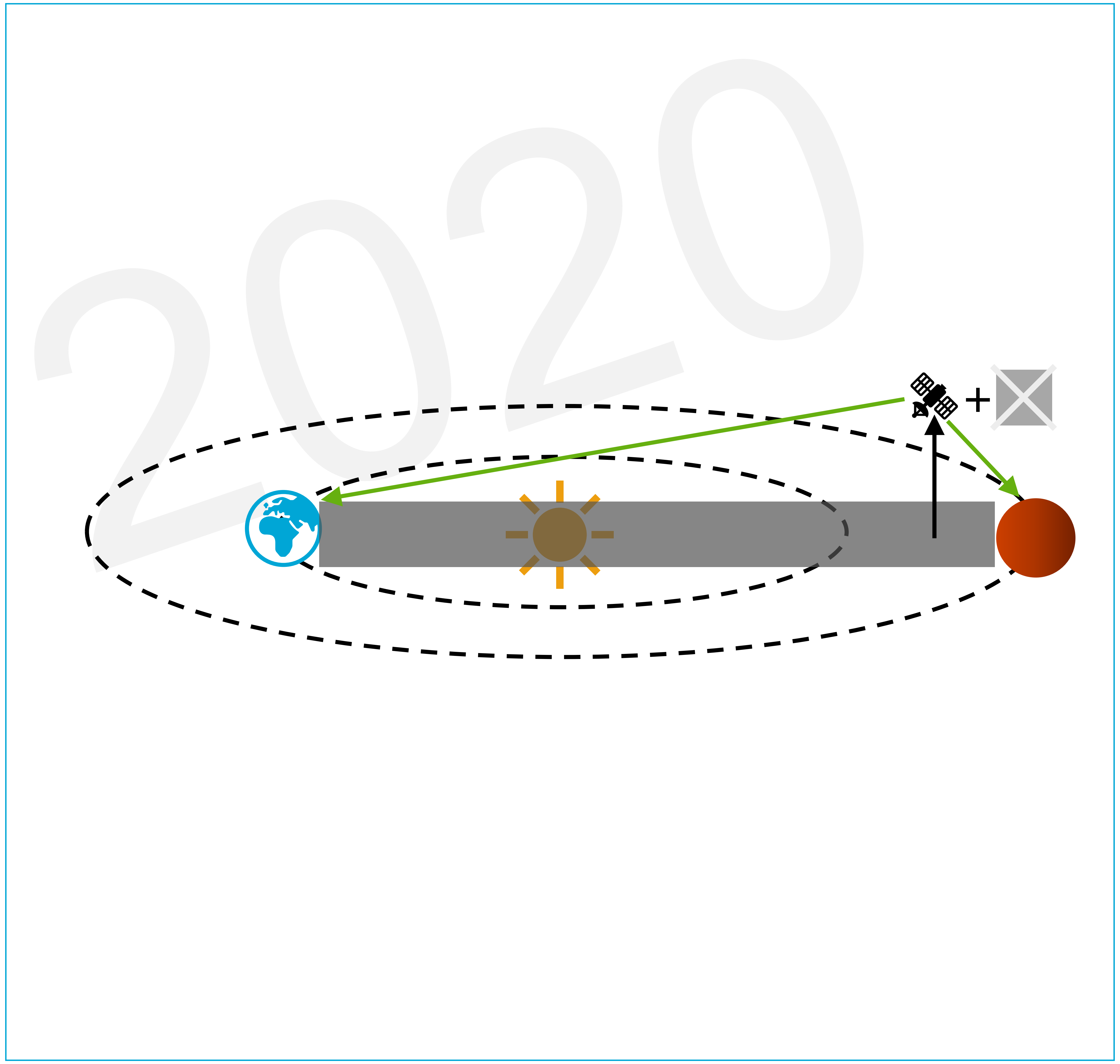
Artificial Lagrange points - utility

- High-latitude observation of the Earth



Artificial Lagrange points - utility

- High-latitude observation of the Earth
- Earth-Mars communication during conjunction



ST>A – Learning objectives today – revisited

• Summarize a short history on solar sailing	✓
• Explain the basic workings, advantages and disadvantages of solar-sail propulsion	✓
• Derive a solar-sail acceleration model	✓
• Compute artificial Lagrange points in the solar-sail perturbed cr3bp	✓
• **Identify mission applications of solar-sail propulsion in the cr3bp**	✓

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End of video