Equilibria in the cr3bp

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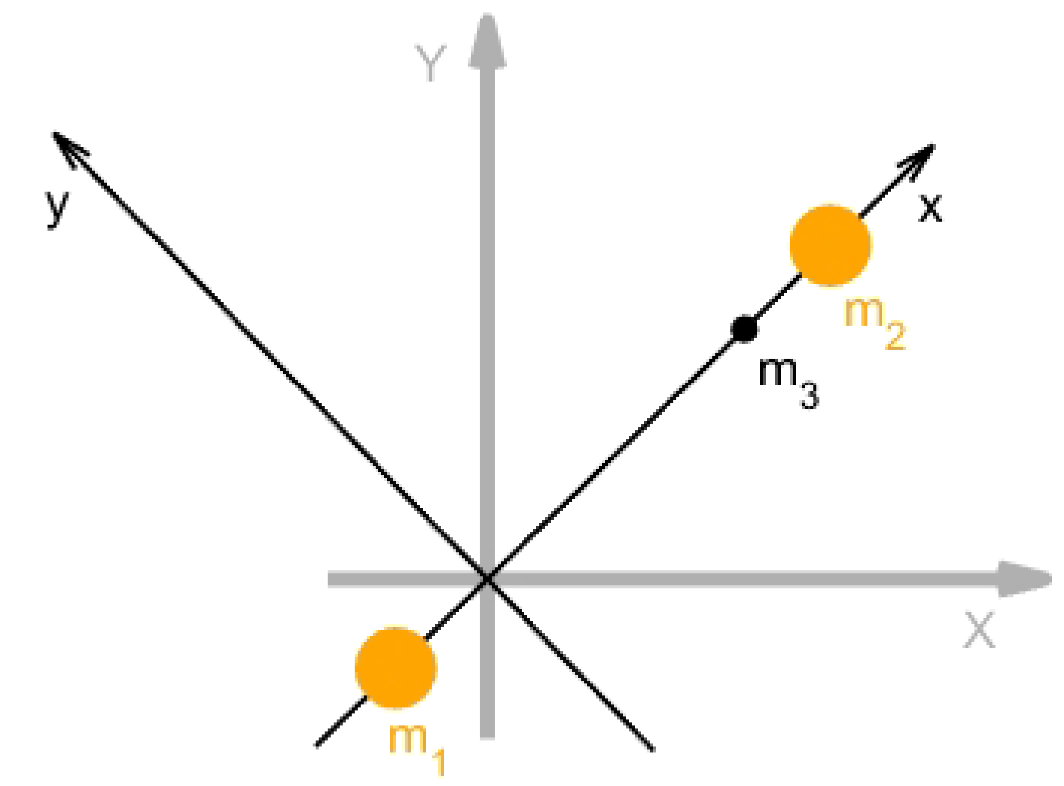
  - No net acceleration acts on m_3

  It will stay stationary *w.r.t.* the rotating frame R(x,y,z)

  Trace out an orbit *w.r.t.* the inertial frame I(X,Y,Z)



Equilibria – computation

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- Lagrange points are located where

$$\nabla U = \mathbf{0}$$

$$U = -\left(\frac{1-\mu}{r_1} + \frac{\mu}{r_2}\right) - \frac{1}{2}\left(x^2 + y^2\right)$$

Equilibria – computation

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$$\Downarrow$$

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$$\frac{\partial U}{\partial y} = 0 \qquad y\left(1 - \frac{1-\mu}{r_1^3} - \frac{\mu}{r_2^3}\right) = 0$$

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Equilibria – computation

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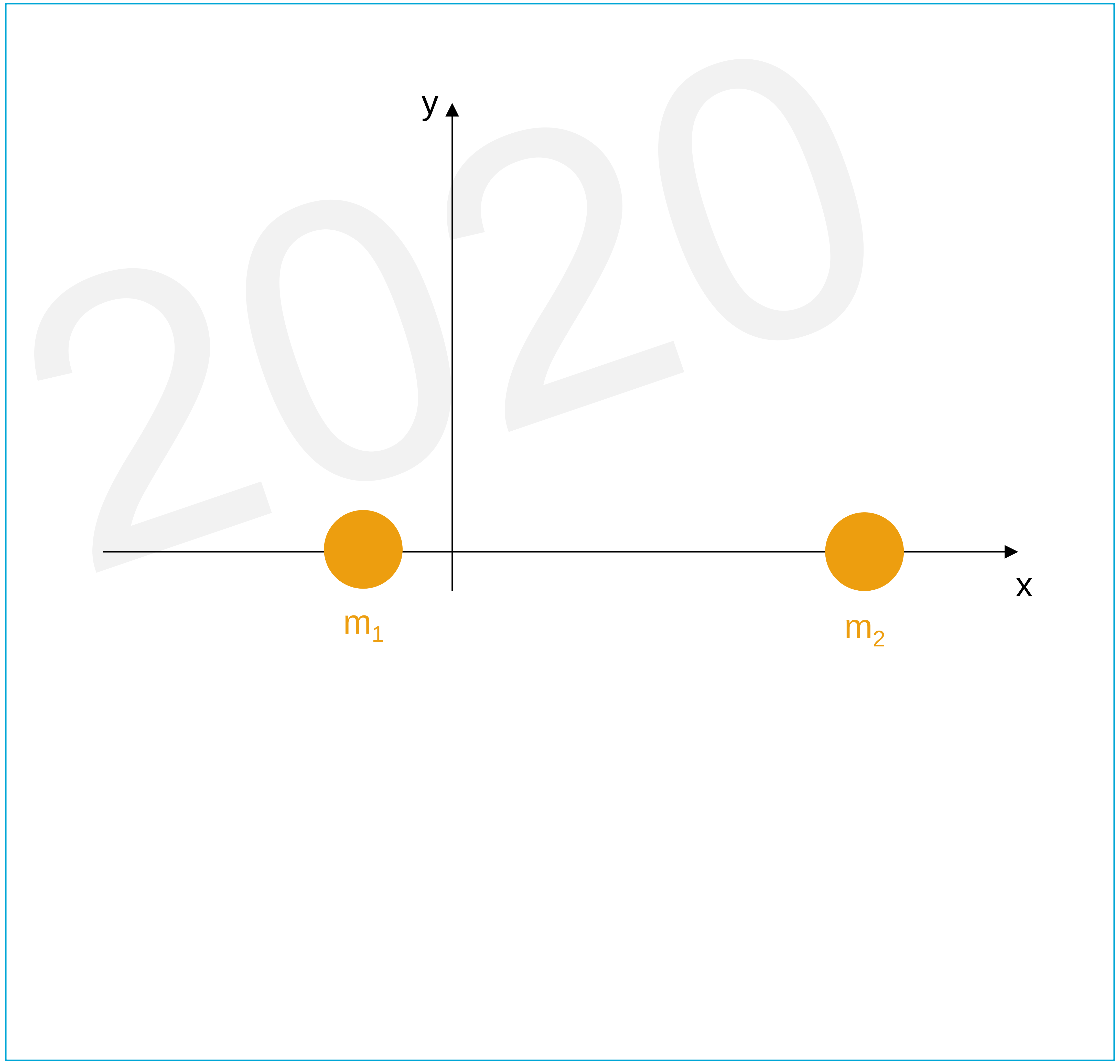
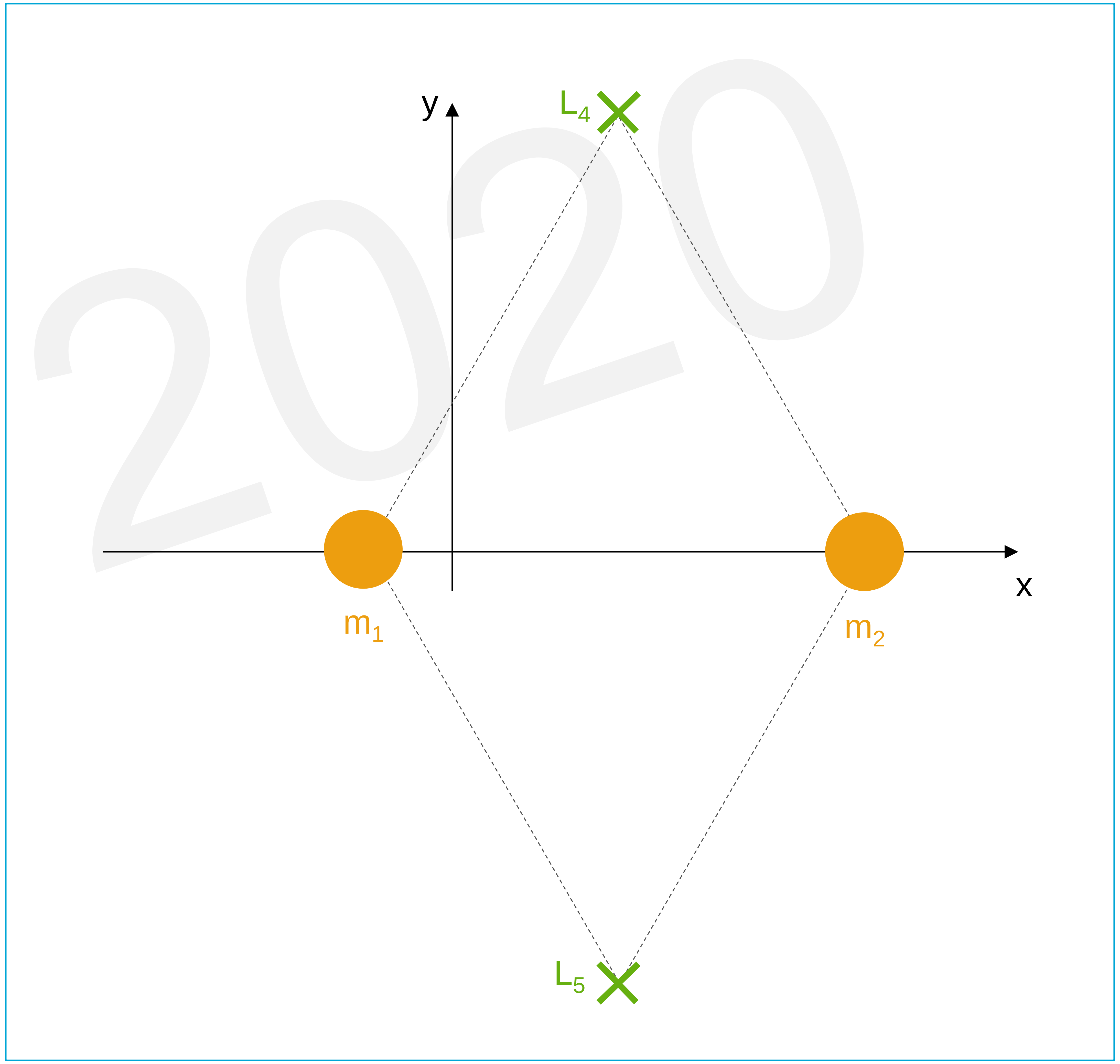


Equilibria – computation

- Five solutions
- Known from "Fundamentals of Astrodynamics"
  - **Triangular** (L_4 , L_5) - on equilateral triangles



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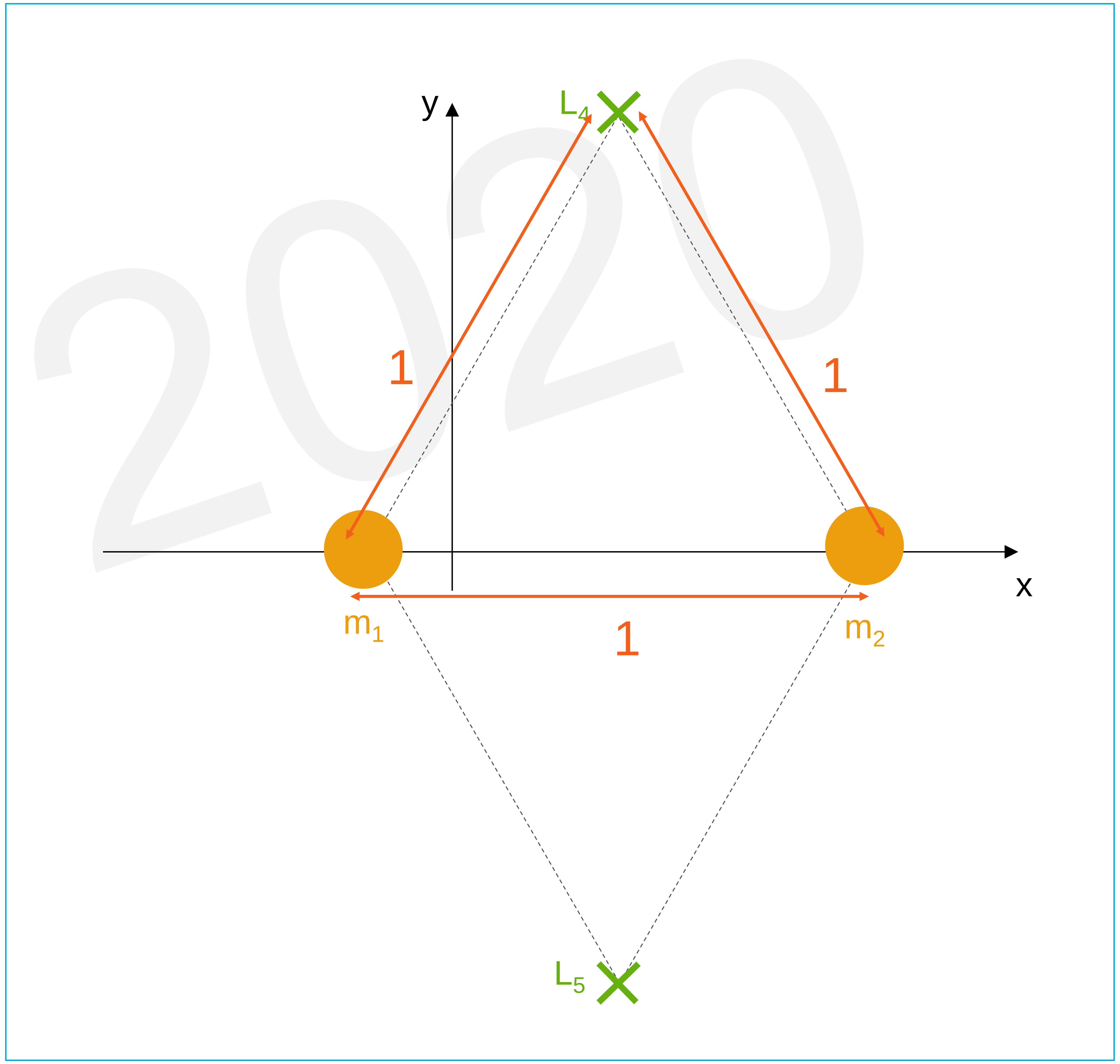

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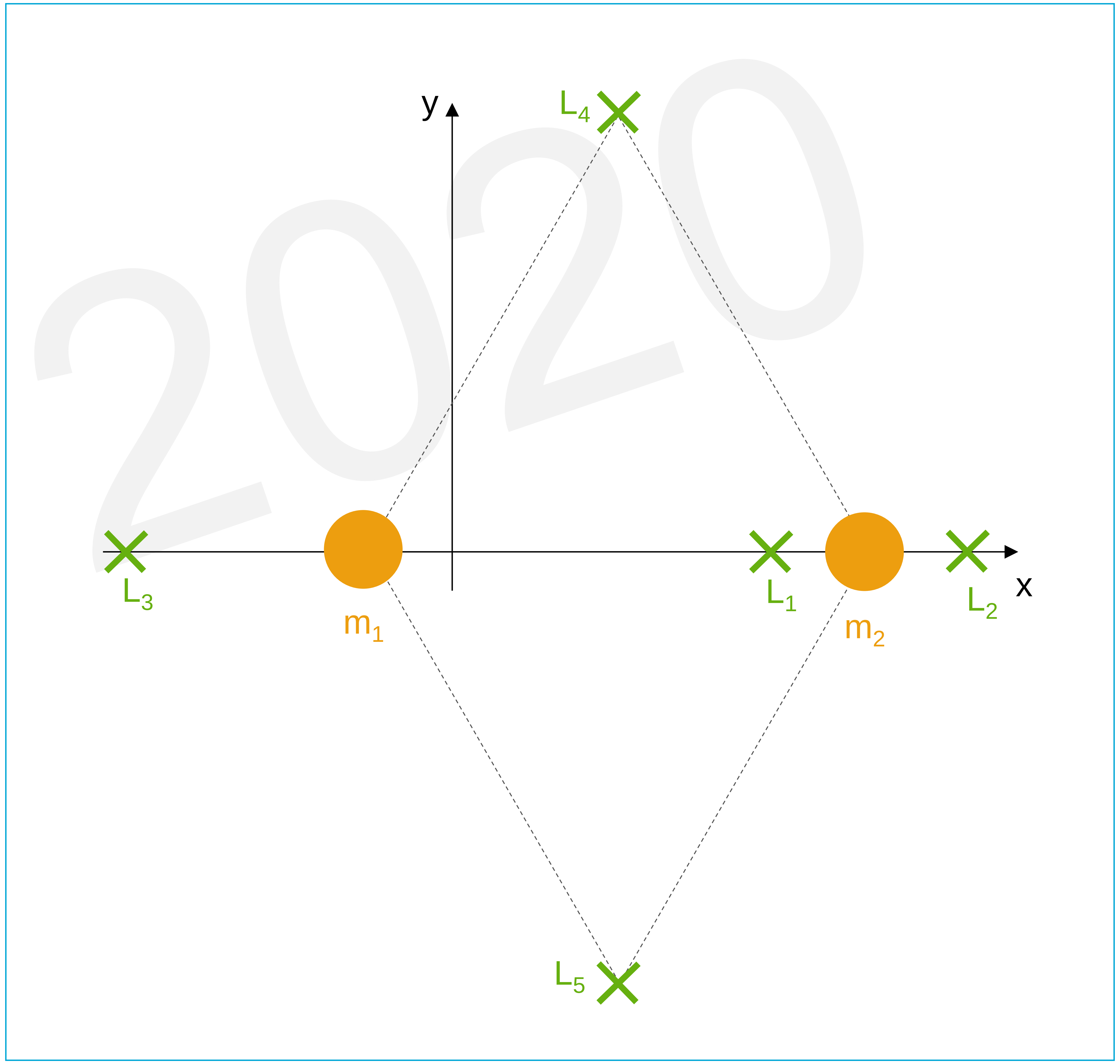

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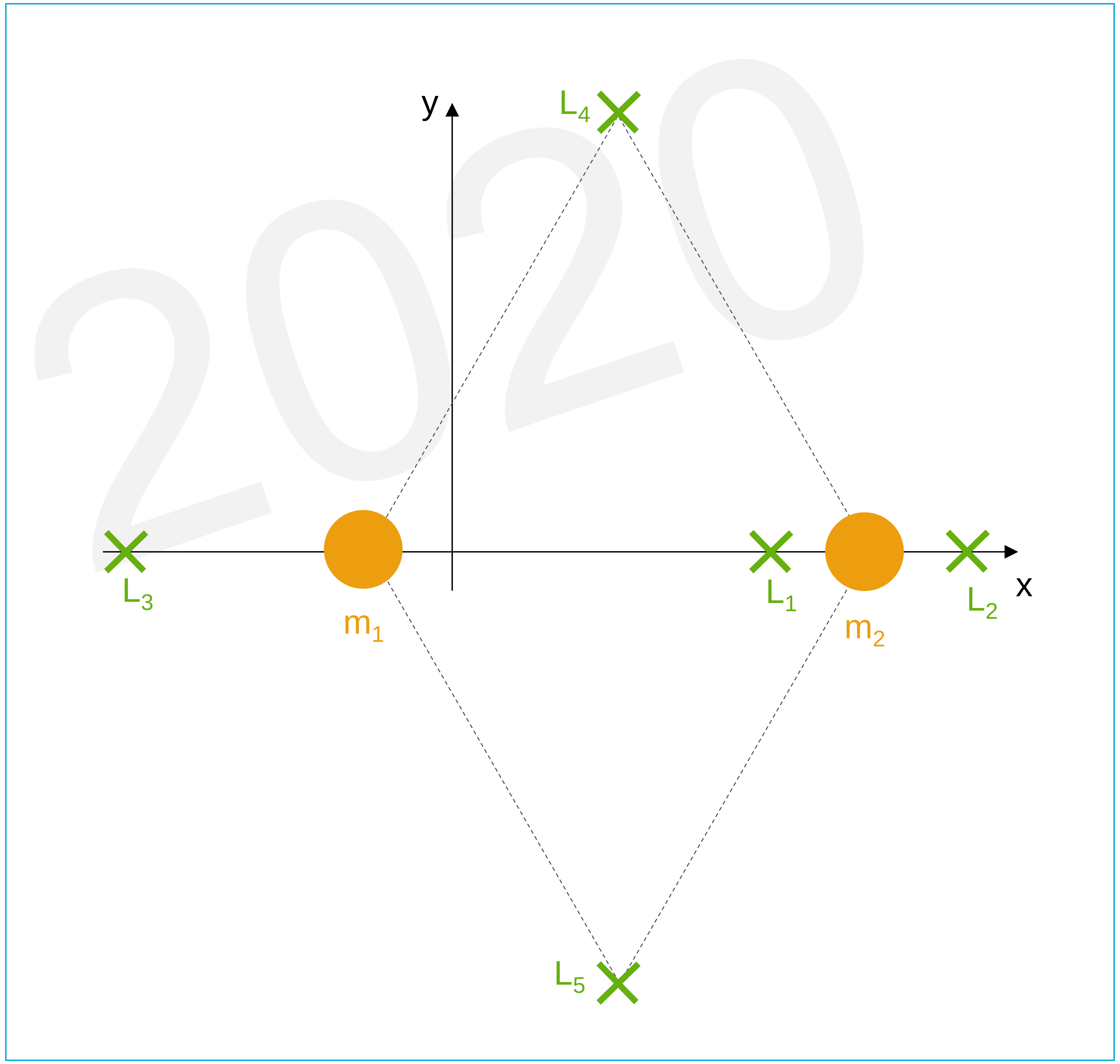

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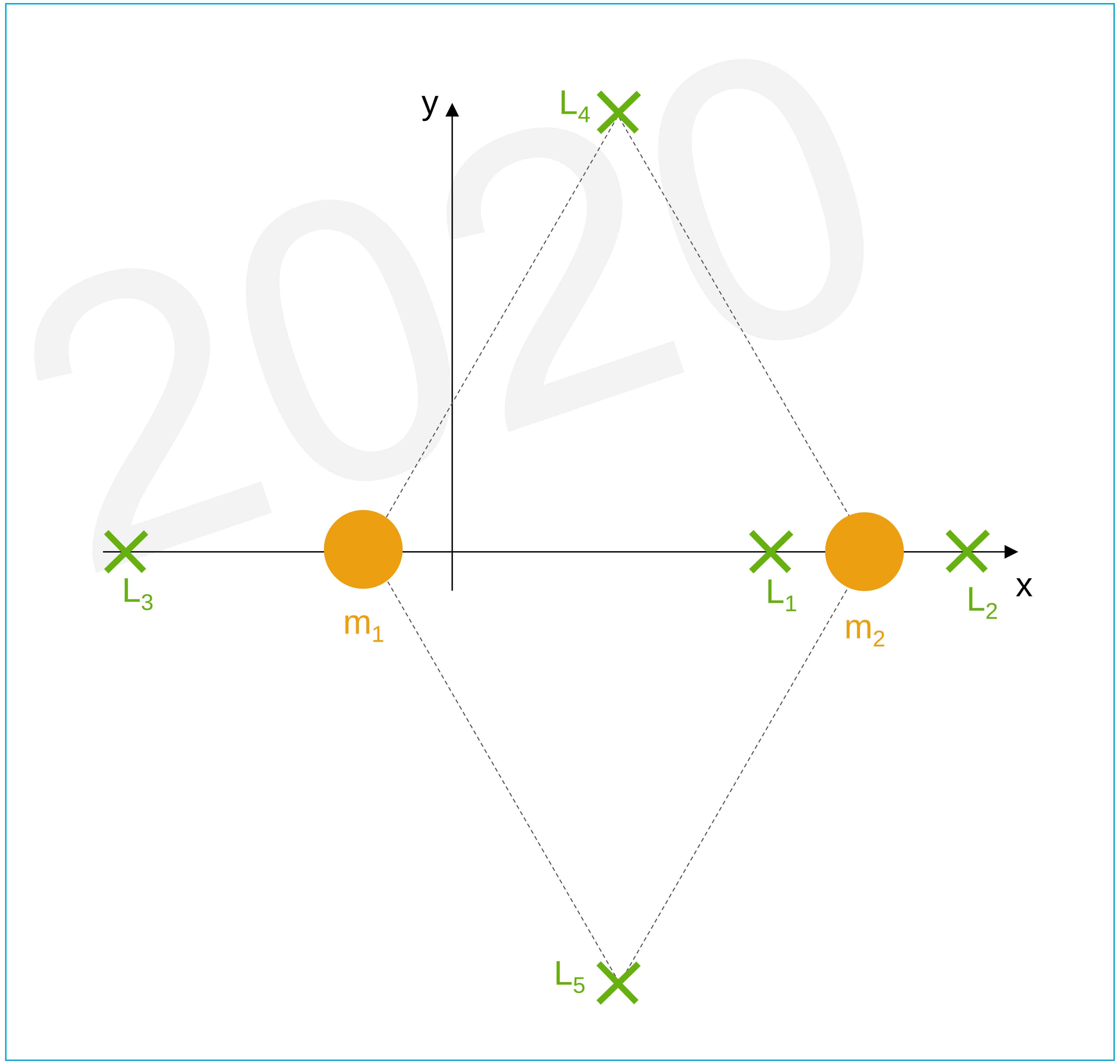

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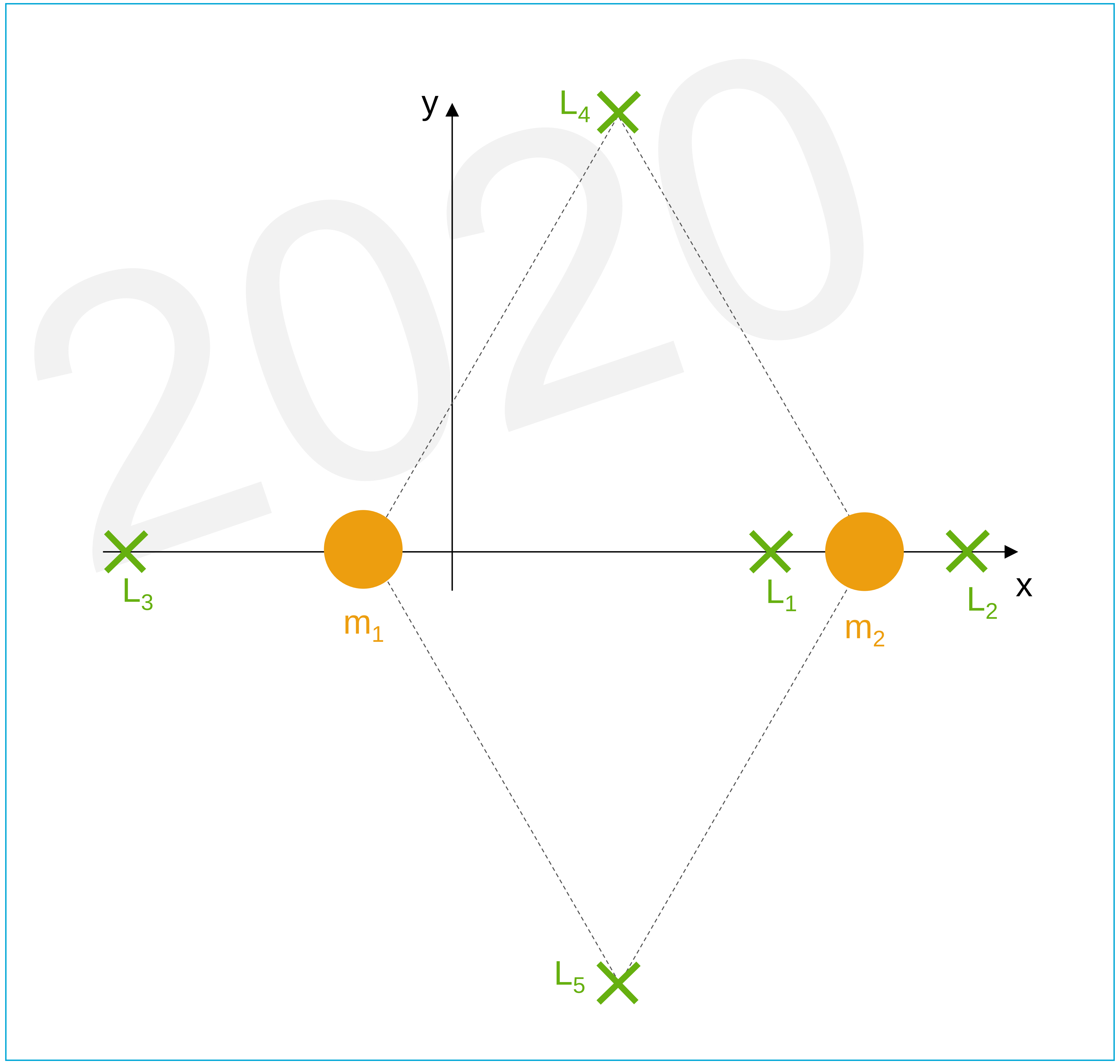

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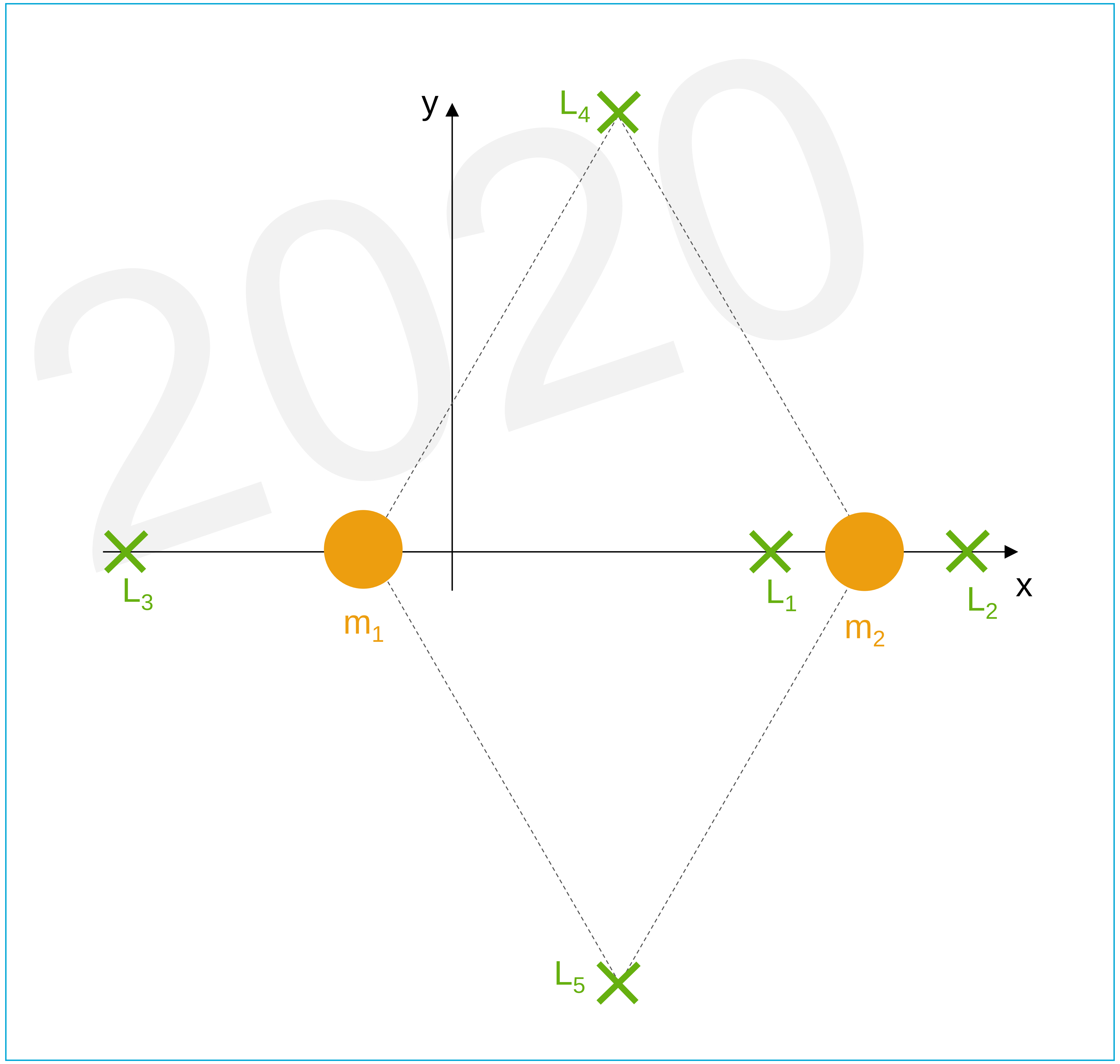

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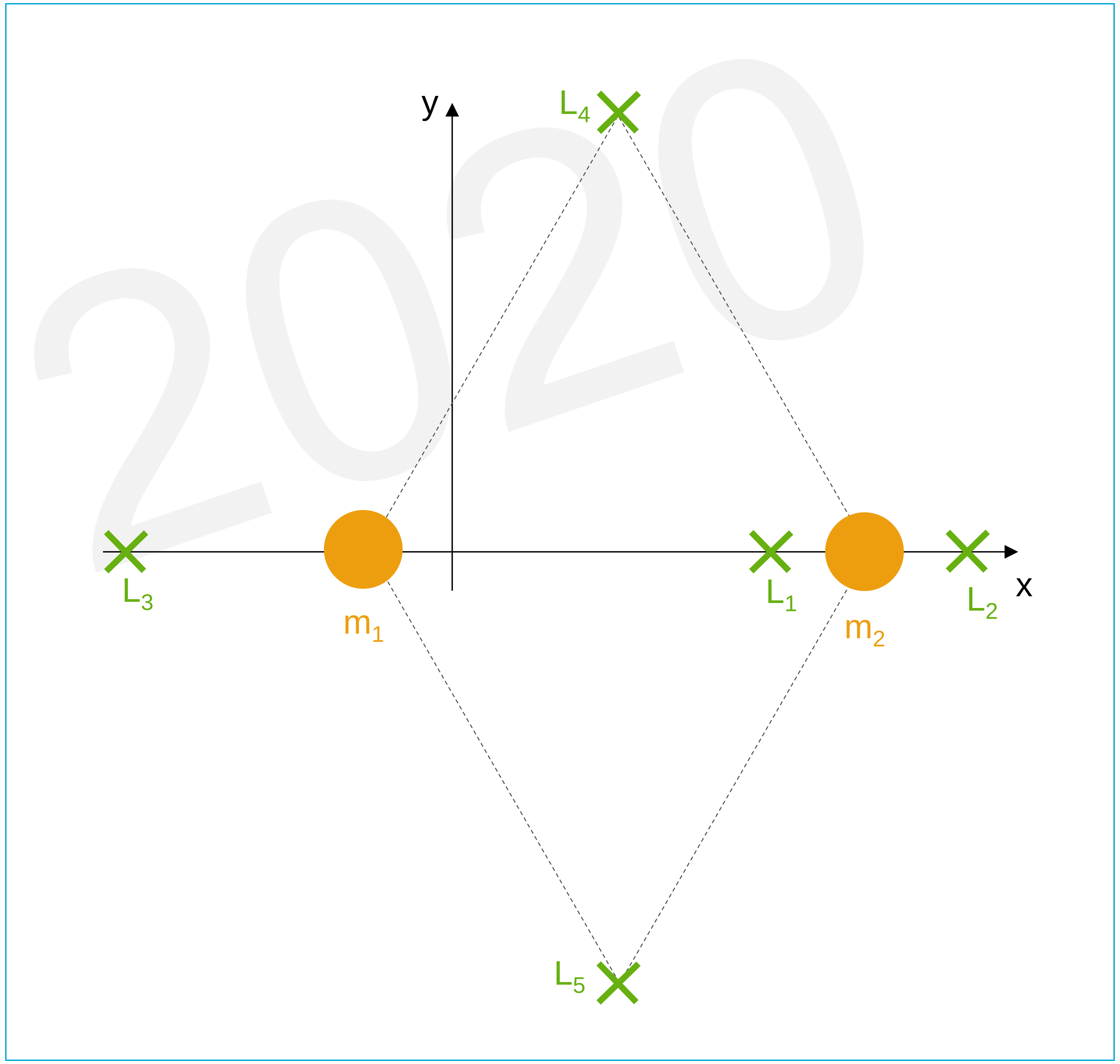
- **Assignment**
  - Compute colinear Lagrange points



Equilibria - utility

- From introductory workshop
  - Missions at **Sun-Earth L_1** for solar and Earth observation
    - SOHO (ESA/NASA)
    - WIND (NASA)
    - ACE (NASA)
    - DSCOVR (NASA)
  - Missions at **Sun-Earth L_2** for astronomy
    - GAIA (ESA)
- Missions to **Sun-Earth L_5** for heliophysics
  - Lagrange (ESA)
- Understanding natural Solar-System dynamics
  - Trojan asteroids in Sun-Jupiter / Sun-Earth systems

End of video