Question 1

a.

i.
{0, -1}
ii.
{1, 4, 7, 10}

b.

i.
we get $|A \cup B| = |A| + |B| - |A \cap B| = 2n - 2$
ii.
we get $|P(A \cup B)| = 2^{2n-2}$

c.

i.
It's true.
Because $(A - B) \cap C = (A \cap \bar{B}) \cap C = A \cap \bar{B} \cap C = (C \cap \bar{B}) \cap A = (C - B) \cap A$
ii.
It's false.
Because $(A - B) \cup C = (A \cap \bar{B}) \cup C = (A \cup C) \cap (\bar{B} \cup C)$
and $(C - B) \cup A = (C \cap \bar{B}) \cup A = (C \cup A) \cap (\bar{B} \cup A) = (A \cup C) \cap (\bar{B} \cup A)$
Obviously, they are different.
iii.
It's true.
Because$(A - C) \cap (C - B) = (A \cap \bar{C}) \cap (C \cap \bar{B}) = A \cap (\bar{C} \cap C) \cap \bar{B} = A \cap \emptyset \cap \bar{B} = \emptyset$

d.

i.

p	q	r	$\neg q$	$p \wedge \neg q$	$r \vee q$	$(p \wedge \neg q) \rightarrow (r \vee q)$
F	F	F	T	F	F	T
F	F	T	T	F	T	T

F	T	F	F	F	T	T
F	T	T	F	F	T	T
T	F	F	T	T	F	F
T	F	T	T	T	T	T
T	T	F	F	F	T	T
T	T	T	F	F	T	T

ii.

This compound proposition is not a tautology, because when p is True, q is False and r is False ,the compound proposition will be False.

e.

i.

False. Because x>0,y>0, so x+y > 0 and x*y >0

ii.

False. If x=1, y=1, then x*y=1, x+y =2, it will be false.

f.

i.

$$\exists a \in Z \text{ and } b \in N, \forall c \in N \text{ such that } ac \leq ab$$

ii.

The original statement is false.

When a=0, it is wrong.

g.

Assume that $x \in A - C, so\ x \in B \cap (A - C)$

We can get that $B \cap (A - C) = B \cap (A \cap \bar{C}) = (A \cap B) \cap \bar{C}$

We know that $A \cap B \subseteq C$, so$(A \cap B) \cap C = \emptyset$

We can conclude that the assumption is wrong, so $x \notin A - C$

Question 3

a.

Euler path and Euler are both use each edge precisely once, but Euler path is a path while Euler cycle is a cycle, which means Euler cycle is a closed trail.

b.

set the maximum number of cimparisons are x.

$$2^x > 3000 > 2^{x-1}$$

2^{12}=4096,2^{11}=2048,so x=12

The maximum numer of comparisons are 12.

c.

i.

A path is a trail which neither vertices nor edges are repeated.

ii.

we study on each point to get the minimum distance to them. D(n) means the shortest distance from A to n.

D(B)=23

D(C)=39

D(D)=MIN((D(B)+28),(D(C)+20,56)=51

D(E)=MIN((D(D)+11),(D(B)+27))=50

D(F)=MIN((D(D)+21),(D(C)+44))=72

D(G)=MIN((D(D)+37),(D(F)+15))=87

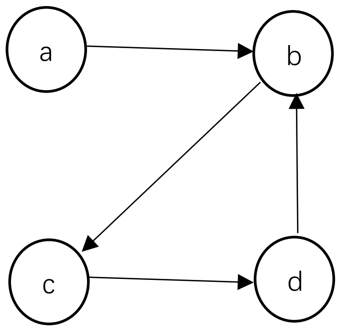
D(H)=MIN((D(F)+27),(D(C)+83))=99

D(I)=MIN((D(H)+20),(D(F)+44),(D(G)+21))=108

So the shortest path from A to I is A-B-D-F-G-I, Which length is 108.

d.

i.



ii.

R is not symmetric because aRb, but not bRa.

R is anti-symmetric because there is not a pairwise such that aRb with bRa.

R is not transitive because bRc and cRd but not bRd.

iii.

$R*=$

	a	b	c	d
a	1	1	1	1
b	0	1	1	1
c	0	0	1	1
d	0	1	0	1

iv.

$M_{R^*}=$

	a	b	c	d
a	0	1	0	0
b	0	0	1	0
c	0	0	0	1
d	0	1	0	0

e.

i.

S(2)=$\sum_{i=1}^{2} 2^{i-1}$=2^0+2^1=3

ii.

Basic step:

S(1)=1=2^1-1

Induction hypothesis:

For all integer k>=1,assume that S(k)=2^k-1

Induction step:

As for k+1,S(k+1)= $S(k) + 2^k = 2^k + 2^k - 1 = 2^{k+1} - 1.$

We can conclude that for any integer n>= 1,S(n)= 2^k-1

f.

we get:$(x.x.x.y.y + \bar{x}.y.y)\left(\overline{x.x + x.\bar{y}.\bar{y}.\bar{\bar{y}}}\right)$

$= (x.y + \bar{x}y)(\overline{x + x.\bar{y}})$-idempotent law

$= \left(y(x + \bar{x})\right).\bar{x}$-distributive law and absorption law

$= y.\bar{x}$-complement law

So the simplication is $y.\bar{x}$