MIDTERM ASSESSMENT

UNIVERSITY OF LONDON

CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathemaitcs

INSTRUCTIONS TO CANDIDATES:

This assignment consists of five questions. You should answer all five questions.

Full marks will be awarded for complete answers to a total of Five questions. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Question 1

(a) The following shows the results of a survey of the types of exercises taken for a group of 100 people:

- 48 swim
- 65 run
- 60 cycle
- 40 run and swim
- 35 run and cycle
- 30 swim and cycle
- 25 run, swim and cycle

Let C be the set of people cycling, R be the set of people running and S be the set of people swimming.

i. Draw a Venn Diagram to show this information.
ii. Find $|C \cup R \cup S|$
iii. Find $|\overline{C \cup R \cup S}|$
iv. Find $|C \oplus R \oplus S|$
v. Find $|\overline{C \oplus R}|$

[10]

(b) Let A and B be two sets. Determine which of the following statements are true and which are false. Prove each statement that is true and give a counter-example for each statement that is false.

i. $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$
ii. $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

[6]

(c) Let A and B be any two sets. Construct an algebraic proof to that:

$$(A \cup \overline{B}) - B = (A - B) \cup \overline{B}$$

[4]

Question 2

(a) Find the Domain and the range of the following functions:

i. $f_1(x) = \frac{1}{\log(2-x)}$

ii. $f_2(x) = 2 - \sqrt{(-3x+2)}$

iii. $f_3(x) = -3^x + 1$

[6]

(b) Let $f : x \to \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}$, where $\mathbb{R}$ and $x > \frac{1}{2}$

i. Show that $f(x) = \frac{1}{2x-1}$.

ii. Find f^{-1}.

iii. Let $g(x) = \ln(x+1)$. Find the solution to $(f \ o \ g)(x) = \frac{1}{7}$, giving your answer in terms of e

[6]

(c) Given a function $F : \mathcal{P}(\{a,b,c\}) \to \{0,1,2,3\}$ is defined by $F(A) = |A|$ for all $A \in \mathcal{P}(\{a,b,c\})$.

i. Is F a one-to-one function? Prove or give a counter-example.

ii. Is F an onto function? Prove or give a counter-example.

[4]

(d) Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is onto then g is also onto. [4]

Question 3

(a) Let p, q, r and s be four propositions. Assuming that p and r are false and that q and s are true, find the truth value of each of the following propositions:

i. $((p \wedge \neg q) \rightarrow (q \wedge r)) \rightarrow (s \vee \neg q)$

ii. $((p \vee q) \wedge (q \vee s)) \rightarrow ((\neg r \vee p) \wedge (q \vee s))$

[4]

(b) Let h, w and s be three propositions defined as follows: h means ' John is healthy ' , w means ' John is wealthy ' whereas s means ' John is wise '

Express each of the three following compound propositions symbolically by using h, w and s, and appropriate logical symbols.

i. *'John is healthy and wealthy but not wise'*

ii. *' John is neither healthy nor wealthy, but he is wise*

iii. *'John is either healthy or wealthy, but not both '*

[6]

(c) Give the contrapositive, the converse and the inverse of each of the following statement:

i. $\forall x \in \mathbb{R},$ if $x > 2$ then $x^2 > 4$

ii. $\forall x \in \mathbb{R},$ if $x(x+2) > 0$ then $x > 0$ or $x < -2$

[6]

(d) A tautology is a proposition that is always true. Let p, q and r be three propositions, show that $(p \rightarrow (q \vee r) \Leftrightarrow ((p \wedge \neg q) \rightarrow r)$ is a tautology. [4]

Question 4

(a) indicate which of the following statements are true and which are false. Justify your answer.

i. $\forall x \in \mathbb{Z}^+, \exists y \in \mathbb{Z}^+$ such that $x = y + 1$

ii. $\exists x \in \mathbb{Z}^+, \forall y \in \mathbb{Z}^+$ such that $x = y + 1$

iii. $\exists x \in \mathbb{Z}^+, \exists y \in \mathbb{Z}^+$ such that $x = y + 1$

[6]

(b) Given the following argument:

" The football game will be cancelled only If it rains "

"it rained, therefore the football game was cancelled"

Assume p means " it rains" whereas q means "football game cancelled"

i. Translate this argument to a symbolic form.

ii. Construct the truth table.

iii. Determine if this argument is a valid argument or not.

[6]

(c) Say whether or not the following argument is a valid argument. Explain your answer.

(a) Successful candidates for this job must have either a Master's degree or five years of work experience
(b) Johnny has a Master's degree
(c)Johnny got the job

(d) $\therefore$ Johnny does not have five years of work experience [4]

(d) Let $P(x)$ and $Q(x)$ be two predicates and suppose D is the the domain of x. For the statement forms in each pair, determine whether they have the same truth value for every choice of $p(x)$, $Q(x)$ and D, or not.

i. $\exists x \in D, (P(x) \wedge Q(x))$, and $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$

ii. $\forall x \in D, (P(x) \vee Q(x))$, and $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$

[4]

Question 5

(a) Use the rules of boolean algebra to simplify the following boolean expressions:

i. $\overline{ab}(\overline{a}+b)(\bar{b}+b)$

ii. $\overline{a}(a+b)+(b+aa)(a+\overline{b})$

[6]

(b) Use the duality principle to find out the dual of the following equation:

$$ab + c\overline{d} = (a+c)(a+\overline{d})(b+c)(b+\overline{d})$$

[2]

(c) The lights in a classroom are controlled by two switches, x and y, so that flipping any one of the switches turns the light OFF if it was ON and turns the light ON if it was OF. Let $x = 1$ when the first switch is closed and $x = 0$ when it is open. Let $y = 1$ when the second switch is closed and $y = 0$ when it open. Let $F(x, y)$ be output of circuit with $F(x, y) = 1$ when the light is ON and $F(x, y) = 0$ when the light is OFF.

i. Give the truth table for this circuit.

ii. Give a boolean expression for $F(x, y)$.

iii. Draw a circuit to represent this circuit.

[6]

(d) Given the following sum-of-production boolean expression:

$$F(A,B,C,D) = \overline{AB}CD+\overline{A}BCD+ABCD+A\overline{B}CD+AB\overline{CD}+AB\overline{C}D+ABC\overline{D}$$

i. Build K-map for $F(A, B, C, D)$. [2]

ii. Use K-map in the previous questions and find a minimisation of $F(A, B, C, D)$. [4]

END OF PAPER

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