

Midterm of FCS

Swen Chan

July 2023

1

$$\begin{aligned} \text{(a)} & ((R \longrightarrow S) \longrightarrow R) \longrightarrow R \\ &= ((\neg R \vee S) \longrightarrow R) \longrightarrow R \\ &= ((R \wedge \neg S) \vee R) \longrightarrow R \\ &= (\neg(R \wedge \neg S) \wedge \neg R) \vee R \\ &= ((\neg R \vee S) \wedge \neg R) \vee R \\ &= ((\neg R \wedge \neg R) \vee (S \wedge \neg R)) \vee R \\ &= (\neg R \vee (S \wedge \neg R)) \vee R \\ &= \neg R \vee R \vee (S \wedge \neg R) \\ &= T \vee (S \wedge \neg R) \\ &= T \end{aligned}$$

$$\begin{aligned} \text{(b)} & (P \longleftrightarrow Q) \\ &\equiv (P \longrightarrow Q) \wedge (Q \longrightarrow P) \\ &\equiv (\neg P \vee Q) \wedge (\neg Q \vee P) \\ &\equiv ((\neg P \vee Q) \wedge \neg Q) \vee ((\neg P \vee Q) \wedge P) \\ &\equiv ((\neg P \wedge \neg Q) \vee (Q \wedge \neg Q)) \vee ((\neg P \wedge P) \vee (Q \wedge P)) \\ &\equiv (\neg P \wedge \neg Q) \vee (Q \wedge P) \\ &\equiv (P \vee Q) \longrightarrow (P \wedge Q) \dots (1) \end{aligned}$$

and we can easily get that $(P \wedge Q) \longrightarrow (P \vee Q)$ is a tautology by the following steps:

$$\begin{aligned} & (P \wedge Q) \longrightarrow (P \vee Q) \\ &= \neg P \vee \neg Q \vee (P \vee Q) \\ &= (\neg P \vee P) \vee (\neg Q \vee Q) \\ &= T \vee T \\ &= T \dots (2) \end{aligned}$$

Thus, According to (1)(2), we can conclude that $(P \wedge Q) \longleftrightarrow (P \vee Q) \equiv (P \longleftrightarrow Q)$

(c)

$$\begin{aligned} & \neg(\forall x, P(x) \wedge [\exists x : Q(x) \wedge \neg R(x)]) \\ &= \exists x, \neg P(x) \vee \neg[\exists x : (Q(x) \wedge \neg R(x))] \\ &= \exists x, \neg P(x) \vee [\forall x : \neg Q(x) \vee R(x)] \end{aligned}$$

(d)

$$(P \longrightarrow Q) \longrightarrow R$$