Midterm of FCS

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(a) $((R \longrightarrow S) \longrightarrow R) \longrightarrow R$
$= ((\neg R \vee S) \longrightarrow R) \longrightarrow R$
$= ((R \wedge \neg S) \vee R) \longrightarrow R$
$= (\neg(R \wedge \neg S) \wedge \neg R) \vee R$
$= ((\neg R \vee S) \wedge \neg R) \vee R$
$= ((\neg R \wedge \neg R) \vee (S \wedge \neg R)) \vee R$
$= (\neg R \vee (S \wedge \neg R)) \vee R$
$= \neg R \vee R \vee (S \wedge \neg R)$
$= T \vee (S \wedge \neg R)$
$= T$
(b) $(P \longleftrightarrow Q)$
$\equiv (P \longrightarrow Q) \wedge (Q \longrightarrow P)$
$\equiv (\neg P \vee Q) \wedge (\neg Q \vee P)$
$\equiv ((\neg P \vee Q) \wedge \neg Q) \vee ((\neg P \vee Q) \wedge P)$
$\equiv ((\neg P \wedge \neg Q) \vee (Q \wedge \neg Q)) \vee ((\neg P \wedge P) \vee (Q \wedge P))$
$\equiv (\neg P \wedge \neg Q) \vee (Q \wedge P)$
$\equiv (P \vee Q) \longrightarrow (P \wedge Q)$...(1)
and we can easily get that $(P \wedge Q) \longrightarrow (P \vee Q)$ is a tatuology by the following steps:
$(P \wedge Q) \longrightarrow (P \vee Q)$
$= \neg P \vee \neg Q \vee (P \vee Q)$
$= (\neg P \vee P) \vee (\neg Q \vee Q)$
$= T \vee T$
$= T$...(2)
Thus,According to (1)(2),we can conclude that
$(P \wedge Q) \longleftrightarrow (P \vee Q) \equiv (P \longleftrightarrow Q)$

(c)

$\neg(\forall x, P(x) \wedge [\exists x : Q(x) \wedge \neg R(x)])$
$= \exists x, \neg P(x) \vee \neg[\exists x : (Q(x) \wedge \neg R(x)]$
$= \exists x, \neg P(x) \vee [\forall x : \neg Q(x) \vee R(x)]$

(d)

$(P \longrightarrow Q) \longrightarrow R$