

Midterm of FCS

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Basic step:

when $n = 1, n(n^2 + 5) = 6$, which is divided by 6.

Hypothesis step:

Assuming $n = k, \forall k \in Z^+, k(k^2 + 5)$ is divided by 6.

Inductive step:

We want to show that $n = k + 1$ such that $(k + 1)((k + 1)^2 + 5)$ is also divided by 6, when $k \in Z^+$.

$$(k + 1)((k + 1)^2 + 5)$$

$$= (k + 1)(k^2 + 2k + 6)$$

$$= k(k + 1)(k + 2) + 6(k + 1)$$

(1) $6(k + 1)$ is divided by 6.

$$(2) \frac{k(k+1)(k+2)}{6} = 1^2 + 2^2 + \dots + k^2 \in Z^+,$$

so $k(k + 1)(k + 2) = 6(1^2 + 2^2 + \dots + k^2)$, which is also divided by 6.

Conclusion: $\forall n \in Z^+, 6$ divides $n(n^2 + 5)$.