Question 2

a.

Applying the Master theorem, in this case, a = 3, b = 2, d = 0.
Since $\log_2 3 > 0$, we can conclude that T(n)=O($n^{\log_2 3}$).

b.

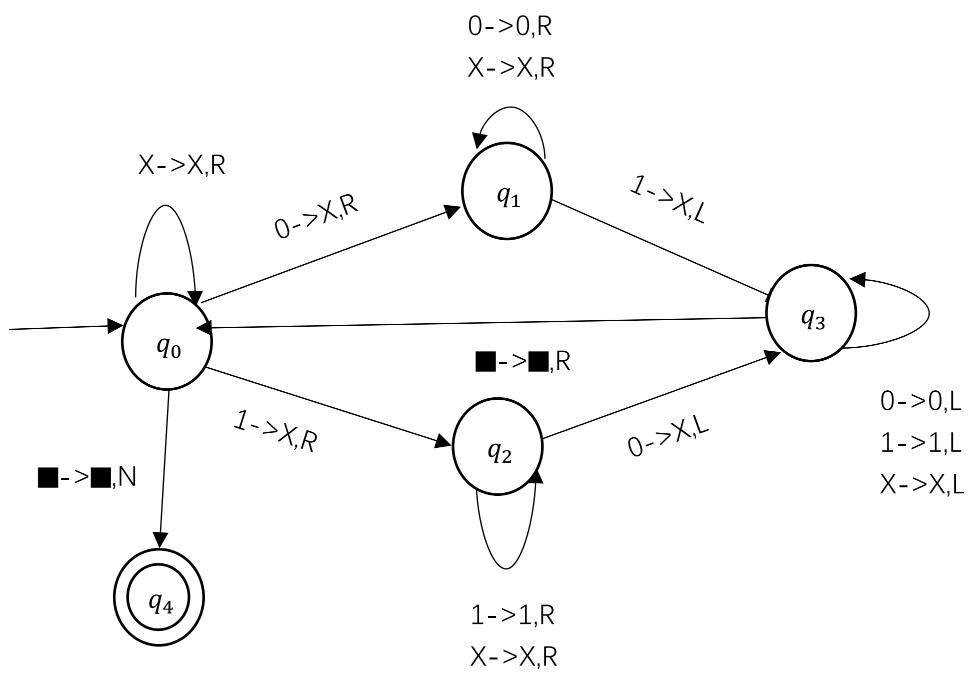
i.

$$S \to L11|11R|L11R$$
$$L \to \varepsilon|0|L0|L10$$
$$R \to \varepsilon|0|0R|01R$$

ii.

$$S \to L|R|LR|L11|11R|L11R$$
$$L \to \varepsilon|0|L0|L10$$
$$R \to \varepsilon|0|0R|01R$$

c.



d.

step 1: Divide the array into two equal halves: [5,7,12,23], [8,1,2,4]

step 2: Divide each sub-array into two equal halves: [5,7], [12,23], [8,1], [2,4]

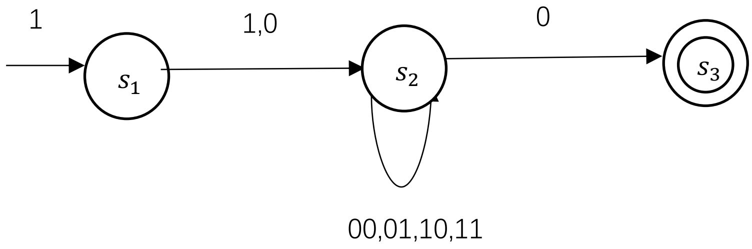
step 3: Divide each sub-array into two equal halves, and now they become arrays of unit length:[5], [7], [12], [23], [8], [1], [2], [4]

step 4: merge [5] and [7] to [5, 7] since they are already in the ascending order; merge [12] and [23] to [12, 23] since they are already in the ascending order; merge [8] and [1] to [1, 8] since 1<8; merge [2] and [4] to [2, 4] since they are already in the ascending order.

Step 5: merge [5,7] and [12,23] to [5,7,12,23] since they are already in the ascending order; merge [1,8] and [2,4] to [1,2,4,8] with merge function.

Step 6: merge [5,7,12,23] and [1,2,4,8] to [1,2,4,5,7,8,12,23] with merge function.

e.



Question 3

a.

i.

after one iteration by using bubble sort, we get A= [3,11,12,2,7,4,1,17]

after the second parsing, A= [3,11,2,7,4,1,12,17]

ii.

A= [7,5,3,1,2,4,6,8]

Because the pivot happens to be the smallest element in the list for each sort.

iii.

A= [1,2,3,4,5,6,7,8]

Because the list is already in the correct order, so it will only take once for each sort.

b.

i.

$$S = A|B$$
$$A = 001|0A1|0A$$
$$B = 011|0B1|B1$$

ii.

$$L = \varepsilon|0L1|1L0|01L|10L|L01|L10$$

c.

1)for f(n)=100n+nlogn+5, the asymptotic function is g(n)= nlogn

there exist c=101, k=100, such that f(x)≦c*g(x) for all x>k.

Proof: c*g(x)=101xlogx.

c*g(x)-f(x)=101xlogx-100x-xlogx-5=100x(logx-1)-5, since logx>2 when x >100, so 100x(logx-1)-5 > 100*100*(2-1)-5 =9995 > 0

so we conclude that g(n)=nlogn is the asymptotic function.

2)for f(n)=$n^{0.5}$+10logn+40, the asymptotic function is g(n)= $n^{0.5}$

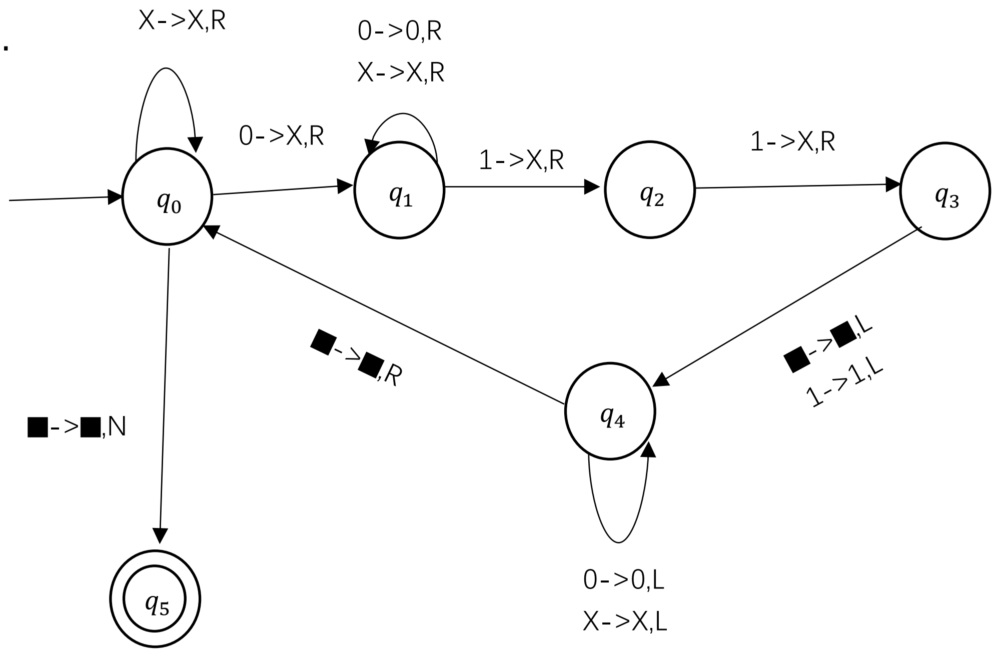
There exist c=11, k=100,such that f(x) ≦c*g(x) for all x>k.

Proof:c*g(x)=11 $x^{0.5}$

c*g(x)-f(x)= 10$x^{0.5}$-10logx+40=10($x^{0.5}$-logx)+40, since $x^{0.5}$>10 when x>100,and logx > 2 when x>100,and $x^{0.5}$ is growing faster than logx, so 10($x^{0.5}$-logx)+40>10*(10-2)+40=120>0

so we conclude that g(n)= $n^{0.5}$ is the asymptotic function.

d.



e.

if there exists zero 0, then the expression should be 1^+
if there exists one 0, then the expression should be 1^+01^* U 1^*01^+
if there exists two 0, then the expression should be $1^+01^*01^*$ U $1^*01^+01^*$ U $1^*01^*01^+$
In all, the regular expression will be 1^+ U 1^+01^* U 1^*01^+ U $1^+01^*01^*$ U $1^*01^+01^*$ U $1^*01^*01^+$