UNIVERSITY OF LONDON

CM1025

BSc EXAMINATION

COMPUTER SCIENCE

Fundamentals of Computer Science

Release date: Wednesday 9 March 2022 at 12:00 midday British Summer Time

Submission date: Thursday 10 March 2022 by 12:00 midday British Summer Time

Time allowed: 24 hours to submit

INSTRUCTIONS TO CANDIDATES:

Section A of this assessment consists of a set of **TEN** Multiple Choice Questions (MCQs) which you will take separately from this paper. You should attempt to answer **ALL** the questions in Section A. The maximum mark for Section A is 20.

Section A will be completed online on the VLE. You may choose to access the MCQs at any time following the release of the paper, but once you have accessed the MCQs you must submit your answers before the deadline or within **4 hours** of starting whichever occurs first.

Section B of this assessment is an online assessment to be completed within the same 24-hour window as Section A. We anticipate that approximately **1 hour** is sufficient for you to answer Section B. Candidates must answer **TWO** out of the THREE questions in Section B. The maximum mark for Section B is 80.

You may use any calculator for any appropriate calculations, but you may not use computer software to obtain solutions. Credit will only be given if all workings are shown.

You should complete Section B of this paper and submit your answers as **one document**, if possible, in Microsoft Word or a PDF to the appropriate area on the VLE. Your answers must have your **candidate number** written clearly at the top of the page before you upload your work. Do not write your name anywhere in your answers.

SECTION A

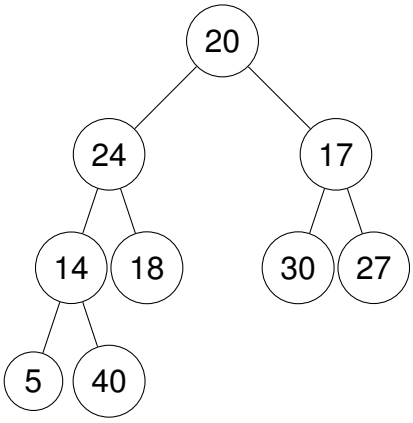
Candidates should answer the **TEN** Multiple Choice Questions (MCQs) quiz, **Question 1** in Section A on the VLE.

SECTION B

Candidates should answer any **TWO** questions from Section B.

Question 2

(a) Heapify the following tree, make every step clear. (Min heap) [6]



(b) Given $R = 1^*0^+1^+0^+1^+$ and $S = (1^+0^+)^*1^+$ where $\Sigma = \{0, 1\}$

- Give an example of a string that is neither in the language of R nor in S. [2]
- Give an example of a string that is in the language of S but not R. [2]
- Give an example of a string that is in the language of R but not S. [2]
- Give an example of a string that is in the language of R and S. [2]
- Design a regular expression that accepts the language of all binary strings with no occurrences of 111. [4]

(c) Answer the following for the context-free grammar

$$G:$$

$$S \to aS|Sb|a|b|\epsilon$$

- Give two non-empty strings that can be generated from G, show the derivations. [4]
- Give two strings that cannot be generated from the context-free grammar G. [2]
- Can string ba be generated by G? Justify your reasoning. [2]
- What is the language of G? [3]

(d) Use mathematical induction to prove that for all natural numbers $n > 0$, $4^n - 1$ is divisible by 3. State every step of the proof. [6]

(e) State the contrapositive and use it to prove that the following statement is true. If $n^2 + 2n - 1$ is divisible by 3 then n is divisible by 3. [5]

Question 3

(a) Write the negation of the following in their simplest form, state every rule you use:

- $p \wedge q \vee \neg p$ [4]
- $\forall x(P(x) \rightarrow \neg Q(x) \wedge \neg P(x))$ [4]

(b) Each student has a password, which is 6 characters long and each character is either a digit or a lowercase letter. Each password must start with a lowercase letter and have at least TWO digits. How many possible passwords are there? [6]

(c) Using the Master theorem write the time complexity of the following, make every step clear.

- $T(n) = 16T(n/2) + O(1)$ [4]
- $T(n) = 9T(n/3) + O(n^2)$ [4]

(d) Design a context-free grammar that accepts the language of all binary strings with at most one occurrence of 000. [8]

(e) Give a finite automaton that accepts all binary strings with at most one occurrence of 000. [7]

(f) Given $\Sigma_1 = \{a, b, c\}$, and $\Sigma_2 = \{1, 2\}$. List three strings of $\Sigma_2^2 \circ \Sigma_1^3$. [3]

Question 4

(a) Assume $S = \{$ NKem(N), Elena(E), Fatima(F)$\}$and $H = \{$Whittington(W), Royal Free(R), Highgate(H)$\}$. The list of the preference is as follows. Find the stable match using the Gale-Shapley algorithm. Show the steps of the algorithms clearly. [6]

NKem	H	R	W
Elena	W	R	H
Fatima	W	H	R

Whittington (W)	N	F	E
Royal Free(R)	F	N	E
Highgate(H)	F	E	N

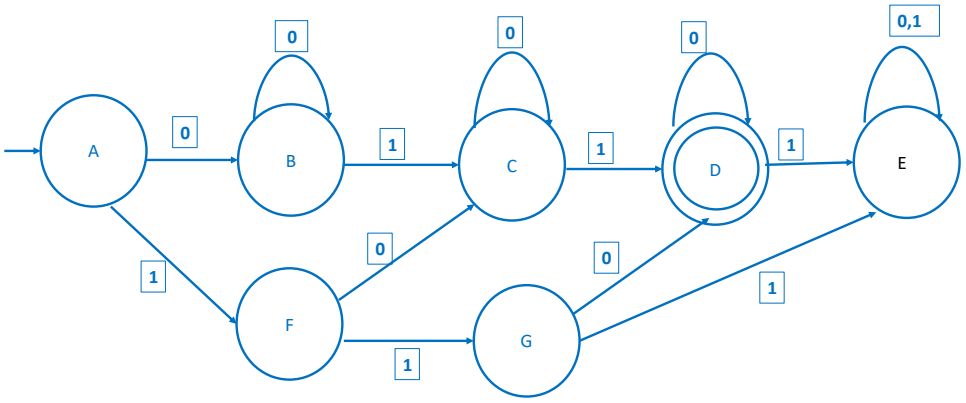
(b) A coin is flipped ten times where each flip comes up either heads or tails. In how many possible outcomes number of heads is a multiple of three? [5]

(c) Write the paths representing the parsing of the following input by the automaton depicted below, state if the input is accepted or rejected. [2]

i. 1101

ii. 010010

- Describe the language of this automaton in plain English. [4]
- Describe the language of this automaton using Regular expression. [3]



(d) Use the merge sort to sort the following list in ascending order. Show your work step by step.

4 19 6 15 10 3 8 11 1

[6]

(e) Write the asymptotic functions of the following. Prove your claim: if you claim $f(n) = O(g(n))$ you need to show there exist c, k such that $f(x) \leq c \cdot g(x)$ for all $x > k$.

- $h(n) = 2n + n \log n + 6$ [4]
- $l(n) = 3n + 2n^2$ [4]

(f) Design a Turing Machine that accepts all binary words in the form of $a^*b^*a^*$. [6]

.

END OF PAPER