Université de Montpellier

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Denil Bouafia, Julia Pham Ba Nien, Marton Roux

2025 ICPC SWERC

21-23 Novembre 2025

Contest (1)

template.cpp — 26 lines

```
#include <bits/stdc++.h>
#include <bits/extc++.h>
#pragma GCC target("avx2")
#pragma GCC optimize ("O3")
#pragma GCC optimize ("unroll-loops")
using namespace std;
using namespace __gnu_pbds;
#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
#define endl '\n'
typedef long long ll;
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().
     count());

int main() {
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);
    freopen("input.in", "r", stdin);
    freopen("output.out", "w", stdout);
    sort(all(a)), a.resize(unique(all(a)) - a.begin());
    ll k = rng(); // random 64 bits integer
    random_device dev;
    mt19937 rng(dev());
    uniform_int_distribution<mt19937::result_type> dist6(1,6);
        // distribution in range [1, 6]
    cout << fixed << setprecision(6) << dist6(rng) << endl;
}
```

.bashrc — 3 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps = <>
```

hash.sh — 3 lines

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that C++ code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

hashpy.sh — 2 lines

```
# Usage : cat fichier.py | hashpy.sh
cat - | grep -vE '^\s*(#|$)' | tr -d '[:space:]' | md5sum | cut
     -c-6
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by = e \\ cx + dy = f \end{aligned} \Rightarrow \begin{aligned} x = \frac{ed - bf}{ad - bc} \\ y = \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Vieta's Formulas

For a polynomial of degree n:

$$\frac{-b}{a} = \text{sum of roots}, \quad (-1)^n \cdot \frac{k}{a} = \text{product of roots}.$$

Here: - k is the coefficient of the constant term, - a is the leading coefficient, - b is the coefficient of the term of degree $n-1$.

2.3 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.4 Trigonometry

$$\begin{aligned}
\sin(v + w) &= \sin v \cos w + \cos v \sin w \\
\cos(v + w) &= \cos v \cos w - \sin v \sin w \\
\tan(v + w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\
\sin v + \sin w &= 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2} \\
\cos v + \cos w &= 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}
\end{aligned}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$\begin{aligned}
a \cos x + b \sin x &= r \cos(x - \phi) \\
a \sin x + b \cos x &= r \sin(x + \phi)
\end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.5 Geometry

2.5.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b + c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin \alpha}{a} = \dfrac{\sin \beta}{b} = \dfrac{\sin \gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\dfrac{a + b}{a - b} = \dfrac{\tan \dfrac{\alpha + \beta}{2}}{\tan \dfrac{\alpha - \beta}{2}}$

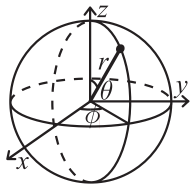
2.5.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

2.5.3 Spherical coordinates



$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\theta\sin\phi \qquad \theta = \mathrm{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r\cos\theta \qquad \phi = \mathrm{atan2}(y,x)$$

2.6 Sums

$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1}-c^a}{c-1}, c \neq 1$

$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$

$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

2.7 Additional Concepts

2.7.1 Divisors and Sum of Divisors

Number of Divisors: If $N = a^p \times b^q \times \cdots \times c^r$, the total number of divisors is:

$$(p+1)\times(q+1)\times\cdots\times(r+1).$$

Sum of Divisors: For $N = a^p \times b^q \times \cdots \times c^r$, the sum of divisors is:

$$\mathrm{SumDiv}(N) = \frac{a^{p+1}-1}{a-1}\cdot\frac{b^{q+1}-1}{b-1}\cdots\frac{c^{r+1}-1}{c-1}.$$

2.7.2 Sum of an Infinite Geometric Series

The sum of an infinite geometric series of the form:

$$1 + r + r^2 + r^3 + \ldots = \frac{1}{1-r}, \quad \text{for } |r| < 1.$$

2.7.3 Sum of First n Terms of an AP

The sum of the first n terms (S_n) is:
$S_n = \frac{n}{2}\cdot(2a + (n-1)d)$

Alternatively, if the last term (l) is known: $S_n = \frac{n}{2}\cdot(a+l)$ Here: - S_n = sum of the first n terms, - a = first term, - d = common difference, - n = number of terms, - l = last term.

2.8 Combinatorics

Combinations

Without Repetition: C_n^k With Repetition: $K_n^k = \frac{(n+k-1)!}{k!\cdot(n-1)!}$

Arrangements

Without Repetition: $A_n^k = \frac{n!}{(n-k)!}$ With Repetition: n^k

Distributing Identical Objects into Boxes

With empty boxes allowed: C_{n+k-1}^k With no empty box: C_{n-1}^{k-1}

2.8.1 Coefficients binomiaux

$$\binom{n}{k} = \begin{cases} 1 & \text{si } k = 0 \text{ ou } k = n, \\ \binom{n-1}{k-1} + \binom{n-1}{k} & \text{si } 1 \le k \le n-1. \end{cases}$$

Somme sur n: $\sum_{n=p}^{q}\binom{n}{k} = \binom{q+1}{k+1} - \binom{p}{k+1}$

2.8.2 Principe d'inclusion-exclusion

$|\bigcup_{i\in I} A_i| = \sum_{\substack{J\subseteq I \\ J\neq\emptyset}} (-1)^{|J|-1}|\bigcap_{j\in J} A_j|$

2.9 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear: Expectation is linear:
$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$
For independent X and Y,
$V(aX + bY) = a^2V(X) + b^2V(Y)$

2.9.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n,p)$, $n = 1, 2, \ldots, 0 \le p \le 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \le p \le 1$.

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \ldots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

2.9.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a,b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Data structures (3)

IndexedSet.h

Usage:
```
indexed_set<pii> s;
s.insert({x[i], i});
s.erase({x[i-k], i-k});
s.find_by_order((k-1)/2)->first;
s.order_of_key(7) //pos el would have
```
Time: $\mathcal{O}(\log N)$

<bits/extc++.h> 9559d2, 4 lines

```
using namespace __gnu_pbds;
template <typename T>
using indexed_set =
    tree<T, null_type, less<T>, rb_tree_tag,
         tree_order_statistics_node_update>;
```

UnionFind.h

7aa27c, 14 lines

```
struct UF {
  vi e;
  UF(int n) : e(n, -1) {}
  bool sameSet(int a, int b) { return find(a) == find(b); }
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x]); }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

RMQ.h

Description: min(V[a], V[a + 1], ... V[b - 1])
Usage:
```
RMQ rmq(values);
rmq.query(inclusive, exclusive);
```
Time: $\mathcal{O}(|V|\log|V| + Q)$

510c32, 16 lines

```
template<class T>
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
      jmp.emplace_back(sz(V) - pw * 2 + 1);
      rep(j,0,sz(jmp[k]))
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    }
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
```

```
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};
```

SegmentTree.py

4e1e77, 35 lines

```
def update(p, val):
    p += N
    seg[p] = val
    while p > 1:
        seg[p>>1] = seg[p] + seg[p^1]
        p >>= 1
def query(l, r): # [l, r)
    res = 0
    l += N; r += N
    while l < r:
        if l&1: res += seg[l]; l += 1
        if r&1: r -= 1; res += seg[r]
        l >>= 1; r >>= 1
    return res
N = 1 << n.bit_length()
seg = [0] * (2 * N)
for i in range(n):
    seg[N + i] = a[i]
for k in range(N - 1, 0, -1):
    seg[k] = seg[k<<1] + seg[k<<1|1]

# supprimer ligne -> for k in range(N - 1, 0, -1):
def update(l, r, val): # [l, r)
    l += N; r += N
    while l < r:
        if l&1: seg[l] += val; l += 1
        if r&1: r -= 1; seg[r] += val
        l >>= 1; r >>= 1
def query(p):
    res = 0
    p += N
    while p > 0:
        res += seg[p]
        p >>= 1
    return res
```

Fenwick.py

5f8270, 32 lines

```
def update(pos, v):
    while pos < len(T):
        T[pos] += v
        pos |= pos + 1
def _query(pos):
    r = 0
    while pos:
        r += T[pos-1]
        pos &= pos-1
    return r
def query(a, b): # [l, r)
    return _query(b) - _query(a)
T = list(X)
for i in range(n):
    j = i | (i+1)
    if j < n: T[j] += T[i]

def update_(i, val):
    i = i + 1
    while i < len(T):
        T[i] += val
        i += i & (-i)
def update(l, r, val): # [l, r)
    update_(l, val)
    update_(r, -val)  # T length n+1
def query(i):
    res = 0
    i = i + 1
    while i > 0:
        res += T[i]
        i -= i & (-i)
    return res
```

Fenwick2D.h

Description: 0-indexed FenwickTree2D<int> fenw(r, c); fenw.Modify(i, j, +1); fenw.Query(ib+1, jb+1)-fenw.Query(ia, jb+1)-fenw.Query(ib+1, ja)+fenw.Query(ia, ja);
Time: $\mathcal{O}(\log^2 N)$.

a7497c, 33 lines

```
template <typename T>
class FenwickTree2D {
 public:
  vector<vector<T>> fenw;
  int n, m;
  FenwickTree2D() : n(0), m(0) {}
  FenwickTree2D(int n_, int m_) : n(n_), m(m_) {
    fenw.resize(n);
    for (int i = 0; i < n; i++) {
      fenw[i].resize(m);
    }
  }
  void Modify(int i, int j, T v) {
    assert(0 <= i && i < n && 0 <= j && j < m);
    int x = i;
    while (x < n) {
      int y = j;
      while (y < m) {fenw[x][y] += v; y |= y + 1;}
      x |= x + 1;
    }
  }
  T Query(int i, int j) {
    assert(0 <= i && i <= n && 0 <= j && j <= m);
    T v{};
    int x = i;
    while (x > 0) {
      int y = j;
      while (y > 0) {v += fenw[x - 1][y - 1]; y &= y - 1;}
      x &= x - 1;
    }
    return v;
  }
};
```

PersistentSegmentTree.h

Description: Each modification of the Segment Tree we will receive a new root vertex. To quickly jump between two different versions of the Segment Tree, store this roots in an array. To use a specific version of the Segment Tree we simply call the query using the appropriate root vertex.
Time: $\mathcal{O}(\log N)$.

c4782a, 29 lines

```
struct Vertex {
    Vertex *l, *r;
    int sum;
    Vertex(int val) : l(nullptr), r(nullptr), sum(val) {}
    Vertex(Vertex *l, Vertex *r) : l(l), r(r), sum(0) {
        if (l) sum += l->sum;
        if (r) sum += r->sum;
    }
};
Vertex* build(int a[], int tl, int tr) {
    if (tl == tr) return new Vertex(a[tl]);
    int tm = (tl + tr) / 2;
    return new Vertex(build(a, tl, tm), build(a, tm+1, tr));
}
int get_sum(Vertex* v, int tl, int tr, int l, int r) {
    if (l > r) return 0;
    if (l == tl && tr == r) return v->sum;
    int tm = (tl + tr) / 2;
    return get_sum(v->l, tl, tm, l, min(r, tm))
         + get_sum(v->r, tm+1, tr, max(l, tm+1), r);
}
Vertex* update(Vertex* v, int tl, int tr, int pos, int new_val)
     {
    if (tl == tr) return new Vertex(new_val);
    int tm = (tl + tr) / 2;
    if (pos <= tm)
        return new Vertex(update(v->l, tl, tm, pos, new_val), v
             ->r);
    else
        return new Vertex(v->l, update(v->r, tm+1, tr, pos,
             new_val));
}
```

ImplicitTreap.h

Description: Implicit Treap 0-indexed. Updates and queries [l, r). Example of use : IT treap; treap.insert(position, value); treap.query<int>(l, r+1, f()); treap.upd(l, r+1, f()); treap.get_vector(); for(auto x : treap.result_vector)
Time: $\mathcal{O}(\log N)$

d6e901, 73 lines

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
     count());
typedef struct item * pitem;
struct item {
    int prior,val,sum=0,add=0,size=1,rev=0;
    pitem l=nullptr, r=nullptr;
    item(int val) : val(val), prior(rng()) {}};
int size(pitem p){return p?p->size:0;}
int sum(pitem p){return p?p->sum:0;}
void push(pitem t){
  if(!t){return;}
  if(t->add){
    t->val+=t->add; t->sum+=t->size*t->add;
    if(t->l){t->l->add+=t->add;}
    if(t->r){t->r->add+=t->add;}
    t->add=0;}
  if(t->rev){
      t->rev=0; swap(t->l, t->r);
      if(t->l)t->l->rev^=1;
      if(t->r)t->r->rev^=1;}}
void pull(pitem t){
    if(!t){return;}
    push(t->l),push(t->r);
    t->size=size(t->l)+size(t->r)+1;
    t->sum=t->val+sum(t->l)+sum(t->r);}
void merge(pitem &t,pitem l,pitem r){
    push(l),push(r);
  if (!l||!r) {t=l?l:r;} else
    if(l->prior>r->prior){merge(l->r,l->r,r),t=l;}
    else{merge(r->l,l,r->l),t=r;}
  pull(t);}
void split(pitem t,pitem &l,pitem &r,int val){
  if(!t)return void(l=r=nullptr);
    push(t);
  if(val>size(t->l)){
    split(t->r,t->r,r,val-size(t->l)-1), l = t;}
    else{split(t->l,l,t->l,val),r=t;}
  pull(t);}
function<void(pitem)> range_add(int v){
  return [v](pitem t) {t->add += v;};}
function<void(pitem)> reverse(){
  return [](pitem t) { t->rev ^= 1;};}
function<int(pitem)> range_sum() {
  // int return value
```

```
  return [](pitem t) {return t->sum;};}
struct IT {
  pitem root = nullptr;
    vector<int> result_vector;
  void insert(int i, int x) {
        pitem l, r; split(root, l, r, i);
        merge(l, l, new item(x)); merge(root, l, r);}
  void del(int i) {
    pitem l, r;
    split(root, l, r, i); split(r, root, r, 1);
    merge(root, l, r);}
  void upd(int l, int r, function<void(pitem)> f) { // [l, r)
    pitem a, b, c;  // a: [0, l); b: [l, r); c: [r, )
    split(root, a, b, l); split(b, b, c, r - l);
    if (b) { f(b); }
    merge(root, a, b); merge(root, root, c);}
  template<typename R>R query(int l,int r,function<R(pitem)>f){
    pitem a, b, c;  // a: [0, l); b: [l, r); c: [r, )
    split(root, a, b, l); split(b, b, c, r - l);
    assert(b); R x = f(b);
    merge(root, a, b); merge(root, root, c);
    return x;}
    void each(pitem t, const function<void(pitem)> &f) {
        if(!t) return;
        push(t); each(t->l, f); f(t); each(t->r, f);}
    void get_vector() {
        each(root,[this](pitem x){result_vector.push_back(x->
             val);});}
    void output() {
        each(root, [](pitem x){printf("(%d, %d) ", x->val, x->
             sum);});}
};
```

MergeSortTree.h

Description: usage: t[n+i] = x;

e4c722, 21 lines

```
const int N = 1<<18;
vi t[2*N];
int n;
void build() {
    for(int i=n-1; i>0; i--)
        merge(all(t[i<<1]), all(t[i<<1|1]), back_inserter(t[i])
             );
}
void modify(int p, int v) {
    for (t[p+=n]=v; p>1; p>>=1) t[p>>1]=t[p]+t[p^1];
}
int query(int l, int r, int c, int d) {
  int res = 0;
  for(l+=n, r+=n; l<r; l>>=1, r>>=1){
    if(l&1){
        res += (upper_bound(all(t[l]), d) - lower_bound(all(t[l
             ]), c));
        l++;}
    if(r&1){r--;
        res += (upper_bound(all(t[r]), d) - lower_bound(all(t[r
             ]), c));}
  }
  return res;
}
```

LineContainer.h

Description: Lines kx+m, max query at x.
Time: $\mathcal{O}(\log N)$

8ec1c7, 29 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};
struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

MinDeque.h

Description: push_front, push_back, getmin in $O(1)$.

936c82, 41 lines

```
struct minstack {
  stack<pair<int, int>> st;
  int getmin() {return st.top().second;}
  bool empty() {return st.empty();}
  int size() {return st.size();}
  void push(int x) {
    int mn = x;
    if (!empty()) mn = min(mn, getmin());
    st.push({x, mn});
  }
  void pop() {st.pop();}
  int top() {return st.top().first;}
  void swap(minstack &x) {st.swap(x.st);}
};

struct mindeque {
  minstack l, r, t;
  void rebalance() {
    bool f = false;
    if (r.empty()) {f = true; l.swap(r);}
    int sz = r.size() / 2;
    while (sz--) {t.push(r.top()); r.pop();}
    while (!r.empty()) {l.push(r.top()); r.pop();}
    while (!t.empty()) {r.push(t.top()); t.pop();}
    if (f) l.swap(r);
  }
  int getmin() {
    if (l.empty()) return r.getmin();
    if (r.empty()) return l.getmin();
    return min(l.getmin(), r.getmin());
  }
  bool empty() {return l.empty() && r.empty();}
  int size() {return l.size() + r.size();}
  void push_front(int x) {l.push(x);}
  void push_back(int x) {r.push(x);}
  void pop_front() {if (l.empty()) rebalance(); l.pop();}
  void pop_back() {if (r.empty()) rebalance(); r.pop();}
  int front() {if (l.empty()) rebalance(); return l.top();}
  int back() {if (r.empty()) rebalance(); return r.top();}
  void swap(mindeque &x) {l.swap(x.l); r.swap(x.r);}
};
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback(). Usage : int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$

de4ad0, 21 lines

```
struct RollbackUF {
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

LazySegmentTree.h

Description: Segment tree with ability to add or set values of large intervals, and compute max of intervals. Can be changed to other things. Use with a bump allocator for better performance, and SmallPtr or implicit indices to save memory.
Usage: `Node* tr = new Node(v, 0, sz(v));`
Time: $\mathcal{O}(\log N)$.

"../various/BumpAllocator.h" 34ecf5, 50 lines

```
const int inf = 1e9;
struct Node {
  Node *l = 0, *r = 0;
  int lo, hi, mset = inf, madd = 0, val = -inf;
  Node(int lo,int hi):lo(lo),hi(hi){} // Large interval of -inf
  Node(vi& v, int lo, int hi) : lo(lo), hi(hi) {
    if (lo + 1 < hi) {
      int mid = lo + (hi - lo)/2;
      l = new Node(v, lo, mid); r = new Node(v, mid, hi);
      val = max(l->val, r->val);
    }
    else val = v[lo];
  }
  int query(int L, int R) {
    if (R <= lo || hi <= L) return -inf;
    if (L <= lo && hi <= R) return val;
    push();
    return max(l->query(L, R), r->query(L, R));
  }
  void set(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) mset = val = x, madd = 0;
    else {
      push(), l->set(L, R, x), r->set(L, R, x);
      val = max(l->val, r->val);
    }
  }
  void add(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) {
      if (mset != inf) mset += x;
      else madd += x;
```

```
      val += x;
    }
    else {
      push(), l->add(L, R, x), r->add(L, R, x);
      val = max(l->val, r->val);
    }
  }
  void push() {
    if (!l) {
      int mid = lo + (hi - lo)/2;
      l = new Node(lo, mid); r = new Node(mid, hi);
    }
    if (mset != inf)
      l->set(lo,hi,mset), r->set(lo,hi,mset), mset = inf;
    else if (madd)
      l->add(lo,hi,madd), r->add(lo,hi,madd), madd = 0;
  }
};
```

QueryTree.h

a32c35, 37 lines

```
struct QueryTree {
    vector<vector<query>> t;
    RollbackUF dsu;
    int T;
    QueryTree(int T, int n) : t(4*T+4), dsu(n), T(T) {}
    void add_to_tree(int v, int l, int r, int ul, int ur, query
         & q) {
        if (ul > ur) return;
        if (l == ul && r == ur) {
            t[v].push_back(q);
            return;
        }
        int m = (l + r) / 2;
        add_to_tree(2*v, l, m, ul, min(ur, m), q);
        add_to_tree(2*v+1, m+1, r, max(ul, m+1), ur, q);
    }
    void add_query(query q, int l, int r) {
        add_to_tree(1, 0, T-1, l, r, q);
    }
    void dfs(int v, int l, int r, vector<int>& ans) {
        int snapshot = dsu.time();
        for (query& q : t[v]) {
            dsu.join(q.v, q.u);
        }
        if (l == r) ans[l] = dsu.comps;
        else {
            int m = (l+r) / 2;
            dfs(2*v, l, m, ans);
            dfs(2*v+1, m+1, r, ans);
        }
        dsu.rollback(snapshot);
    }
    vector<int> solve() {
        vector<int> ans(T);
        dfs(1, 0, T-1, ans);
        return ans;
    }
};
```

Mo.h

Usage: `intervals [l, r)`

8ab63c, 49 lines

```
void add(int ind, int end); // add a[ind] (end = 0 or 1)
void del(int ind, int end); // remove a[ind]
int calc();                 // compute current answer

vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) {
    pii q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();
  }
  return res;
}

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0){
  int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;
  };
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) rep(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  }
  return res;
}
```

Numerical (4)

4.1 Polynomials and recurrences

PolyRoots.h

Description: Finds the real roots to a polynomial.

Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`

Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

e897c3, 40 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.

Time: $\mathcal{O}\left(n^2\right)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.

Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`

Time: $\mathcal{O}\left(N^2\right)$

0323c8, 27 lines

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}

vector<ll> berlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;
  ll b = 1;
  rep(i,0,n) { ++m;
```

```
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }
  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 22 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);
  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
  };
  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;
  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }
  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

FFT.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

f982eb, 63 lines

```
using C=complex<double>; const ll mod=998244353;
void fft(vector<C>&a){
  int n=sz(a),L=31-__builtin_clz(n);
  static vector<complex<long double>>R(2,1);
  static vector<C> rt(2,1);
  for(static int k=2;k<n;k*=2){
    R.resize(n);rt.resize(n);
    auto x=polar(1.0L,acos(-1.0L)/k);
    for(int i=k;i<2*k;i++)
      rt[i]=R[i]=i&1?R[i/2]*x:R[i/2];
  }
  vector<int> rev(n);
  for(int i=0;i<n;i++)
    rev[i]=(rev[i/2]|(i&1)<<L)/2;
  for(int i=0;i<n;i++)
    if(i<rev[i])
      swap(a[i],a[rev[i]]);
  for(int k=1;k<n;k*=2){
    for(int i=0;i<n;i+=2*k)
      for(int j=0;j<k;j++){
        auto x=(double*)&rt[j+k],y=(double*)&a[i+j+k];
        C z(x[0]*y[0]-x[1]*y[1],x[0]*y[1]+x[1]*y[0]);
        a[i+j+k]=a[i+j]-z;
        a[i+j]+=z;}}}
template<ll M>
vector<ll> convMod(const vector<ll>&a, const vector<ll>&b){
  if(a.empty() || b.empty()) return {};
  vector<ll> res(sz(a)+sz(b)+1);
  int B=32-__builtin_clz(sz(res)),n=1<<B,cut=int(sqrt(M));
  vector<C> L(n),R(n),outs(n),outl(n);
  for(int i=0;i<sz(a);i++)
    L[i]=C((int)a[i]/cut,(int)a[i]%cut);
  for(int i=0;i<sz(b);i++)
    R[i]=C((int)b[i]/cut,(int)b[i]%cut);
  fft(L),fft(R);
  for(int i=0;i<n;i++){
    int j=-i&(n-1);
    outl[j]=(L[i]+conj(L[j]))*R[i]/(2.0*n);
    outs[j]=(L[i]-conj(L[j]))*R[i]/(2.0*n)/1i;}
  fft(outl),fft(outs);
  for(int i=0;i<sz(res);i++){
    ll av=ll(real(outl[i])+.5),cv=ll(imag(outs[i])+.5);
    ll bv=ll(imag(outl[i])+.5)+ll(real(outs[i])+.5);
    res[i]=((av%M*cut+bv)%M*cut+cv)%M;}
  return res;}
vlli shift(vector<ll> &a, ll v){
  ll n=sz(a)-1;
  vlli f(n+1), g(n+1), i_fact(n+1);
  f[0]=a[0];
  g[n]=1;
  i_fact[0]=1;
  ll fact=1,potk=1;
  for(int i=1;i<n+1;i++){
    fact=fact*i%mod;
    f[i]=fact*a[i]%mod;
    potk=(potk*v%mod+mod)%mod;
    g[n-i]=((potk*inv(fact))%mod+mod)%mod;
    i_fact[i]=inv(fact);}
  auto p = convMod<mod>(f,g);
  vlli res(n+1);
  for(int i=0;i<n+1;i++)
    res[i]=(p[i+n]*i_fact[i]%mod+mod)%mod;
  return res;}
```

4.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is *eps*. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}(\log((b-a)/\epsilon))$

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
}
```

4.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$

bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$

3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2 m\right)$

44c9ab, 36 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
```

```
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }
  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2 m\right)$

fa2d7a, 33 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }
  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}\left(n^3\right)$

ebfff6, 32 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;
  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }
  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }
  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

Number theory (5)

5.1 Modular arithmetic

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in `__gcd` instead. If a and b are coprime, then x is the inverse of $a \pmod{b}$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set `mod` to some number first and then you can use the structure.

"euclid.h" 724c2e, 19 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
    bool operator==(Mod b) const { return x == b.x; }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\le$ mod and that mod is a prime.

6f684f, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

b83e45, 8 lines

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

bbbd8f, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h"   60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h"   d8d98d, 18 lines

```
ull pollard(ull n) {
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  auto f = [&](ull x) { return modmul(x, x, n) + i; };
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

5.2 Divisibility

CRT.py

Description: a, n = values, modulos For non-coprime moduli : crtNonCoprime

3e205a, 27 lines

```
def crt(a, n):
    x = 0
    p = 1
    for ni in n: p *= ni
    for ai, ni in zip(a, n):
        xi = p // ni
        try: inv = pow(xi, -1, ni)
        except ValueError: return None
        x += ai * xi * inv
    return x % p

def crtNonCoprime(a, n):
    prime = {}
    for ai, ni in zip(a, n):
        for k, v in getPrimeFactors(ni).items():
            m = pow(k, v)
            aj, nj = prime.get(k, (ai % m, m))
            if aj != (ai % m) % nj:
                return None
            if nj > m:
                continue
            prime[k] = (ai % m, m)
    newa, newn = [], []
    for k, (ai, ni) in prime.items():
        newa.append(ai)
        newn.append(ni)
    return crt(newa, newn)
```

5.2.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

Bezout.py

Description: bezout(a, b) calcule une solution à l'équation ax + by = pgcd(a, b) solve(a, b, n) calcule (x, y) tq ax + by = n

c63cf4, 22 lines

```
def bezout(a, b):
    px, py = 1, 0
    x, y = 0, 1
    while b != 0:
        a, (q, b) = b, divmod(a, b)
        px, x = x, px - q * x
        py, y = y, py - q * y
# pgcd, x, y
    return a, px, py

def solve(c1, a, c2, b, n):
    g, x, y = bezout(a, b)
    if n%g: return -1
    # to get to ax + by = n
    x *= n // g
    y *= n // g
    # lo <= k <= hi to get x and y positive
    lo = -((x * g) // b)
    hi = (y * g) // a
    if lo > hi:
        return -1
    return x + (b * lo) // g, y - (a * lo) // g
```

PhiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1 - 1} ... (p_r - 1)p_r^{k_r - 1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p}\ \forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.3 Primality

Sieve.py

Description: Sieve of eratosthenes

235da5, 32 lines

```
maxn = 1000000
divisors = [[] for _ in range(maxn + 1)]
for i in range(1, maxn + 1):
    for j in range(i, maxn + 1, i):
        divisors[j].append(i)

spf = [-1] * (maxn + 2)
for i in range(2, maxn+1):
    if spf[i] == -1:
        for j in range(i+i, maxn+1, i):
            spf[j] = i

mobius = [0] * (maxn + 2)
mobius[1] = 1
for i in range(2, maxn + 1):
    if spf[i // spf[i]] == spf[i]:
        mobius[i] = 0
    else:
        mobius[i] = -1*mobius[i//spf[i]]

def getnpf(x):
    pfac = defaultdict(int)
    while spf[x] != -1:
        pfac[spf[x]] += 1
        x //= spf[x]
    if x != 1:
        pfac[x] += 1
    tot, pf = 1, 0
    for k, v in pfac.items():
        tot *= (v + 1)
        pf += 1
    return tot - pf
```

Divisors.py
b55362, 12 lines

```
def getDivisors(n):
    primeFactors = getPrimeFactors(n)
    res, pw = [], 1
    for factor in primeFactors:
        pw *= primeFactors[factor] + 1
    for i in range(pw):
        divisor = 1
        for factor in primeFactors:
            divisor *= factor ** (i % (primeFactors[factor] +
                1))
            i //= primeFactors[factor] + 1
        res.append(divisor)
    return res
```

Totient.py
Description: Counts the positive integers up to n that are relatively prime to n
0922a5, 8 lines

```
def totient(n):
    res = 1
    for p, a in pfac(n).items():
        res *= pow(p, a - 1) * (p - 1)
    return res

def sum_coprimes(n):
    return (totient(n) * n) // 2
```

5.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.5 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.6 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.7 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1 2 3 4 5 6 7 8 9 10
$n!$	1 2 6 24 120 720 5040 40320 362880 3628800
n	11 12 13 14 15 16 17
$n!$	4.0e7 4.8e8 6.2e9 8.7e10 1.3e12 2.1e13 3.6e14
n	20 25 30 40 50 100 150 171
$n!$	2e18 2e25 3e32 8e47 3e64 9e157 6e262 >DBL_MAX

IntPerm.h
Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.
Time: $\mathcal{O}(n)$
044568, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
    use |= 1 << x;                    // (note: minus, not ~!)
  return r;
}
```

6.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.1.3 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Partitions.py
Description: Génère partitions de n
7090ac, 16 lines

```
def generateur_partitions(n):
    def partition_suivante(a, k, N):
        b = a[:k] + [a[k]-1]
        q, r = divmod((N+1), b[k])
        b = b + q*[b[k]]
        if r!=0: b.append(r)
        while b[k]!=1:
            k+=1
            if k==len(b): break
        return (b, k-1, len(b) - k)
    p = [n]
    yield p
    k, N = (-1, 1) if n==1 else (0, 0)
    while k >= 0:
        p, k, N = partition_suivante(p, k, N)
        yield p
```

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \; p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0 1 2 3 4 5 6 7 8 9 20 50 100
$p(n)$	1 1 2 3 5 7 11 15 22 30 627 ~2e5 ~2e8

6.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$

6.2.3 Binomials

multinomial.h
Description: Count the number of ways to divide $k_1 + k_2 + \cdots + k_n$ objects into n groups of sizes $k_1, k_2, \ldots, k_n$.
Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \ldots k_n!}$.
a0a312, 5 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i]) c = c * ++m / (j+1);
  return c;
}
```

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able). $B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k), \; c(0,0) = 1$$
$$\textstyle\sum_{k=0}^n c(n,k) x^k = x(x+1) \ldots (x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

6.4 Computation

6.4.1 Catalan Balanced Sequences

Function: `catalan_bal`
Counts bracket sequences of length n with balance ≥ 0.
Formula:

$$\text{catalan\_bal}(n,s,e) = \begin{cases} 0, (n+s+e) \mod 2 \neq 0 \text{ or } s < 0 \text{ or } e < 0, \\ \binom{n}{\frac{n+e-s}{2}} - \binom{n}{\frac{n-e-s-2}{2}}, \text{otherwise.} \end{cases}$$

6.4.2 Grid Path Calculations

Grid Path: `path`
Paths from $(0,0)$ to (x,y):

$$\text{path}(x,y) = \binom{x+y}{x}.$$

Avoiding Low Barrier: `path_low`
Avoiding $y = x + b$, $b < 0$:

$$\text{path\_low}(x,y,b) = \begin{cases} 0, & b \geq 0, \\ \text{path}(x,y) - \text{path}(y-b, x+b), & b < 0. \end{cases}$$

Avoiding Upper Barrier: `path_up`
Avoiding $y = x + b$, $b > 0$:

$$\text{path\_up}(x,y,b) = \begin{cases} 0, & b \leq 0, \\ \text{path}(x,y) - \text{path}(y-b, x+b), & b > 0. \end{cases}$$

Alternating Constraints: `calc_LUL`
Paths from $(0,0)$ to (x,y) alternating constraints $y = x + b_1$ and $y = x + b_2$:
$\text{calc\_LUL}(x,y,b_1,b_2) =$

$$\begin{cases} 0, x < b_1 \text{ or } y < -b_1, \\ \text{path}(x-b_1, y+b_1) - \text{calc\_LUL}(y+b_1, x-b_1, -b_2, -b_1), \text{otherwise} \end{cases}$$

Two Barriers: `path_2`
Avoiding $y = x + b_1$ and $y = x + b_2$: $\text{path\_2}(x,y,b_1,b_2) = \text{path}(x,y) - \text{calc\_LUL}(x,y,b_1,b_2) - \text{calc\_LUL}(y,x,-b_2,-b_1)$

6.4.3 Gambler's Ruin Problem

Right Boundary: `gambler_ruin_right`
Probability of reaching R starting at L:

$\text{gambler\_ruin\_right}(L,R,p_{\text{right}}) =$

$$\begin{cases} \frac{L}{L+R}, & p_{\text{right}} = 0.5, \\ 1, & p_{\text{right}} = 1, \\ 0, & p_{\text{right}} = 0, \\ \frac{1-v^L}{1-v^{L+R}}, & \text{otherwise, } v = \frac{1-p_{\text{right}}}{p_{\text{right}}}. \end{cases}$$

Left Boundary: `gambler_ruin_left`
Probability of reaching 0:

$$\text{gambler\_ruin\_left}(L,R,p_{\text{left}}) = 1 - \text{gambler\_ruin\_right}(L,R,1-p_{\text{left}}).$$

Dynamic programming (7)

TargetSum.py
Description: Equivalent to bitset<1000001> dp; dp.set(0); for(int j = 0; j < n; j++) dp —= dp << s[j];
Time: $\mathcal{O}(N \cdot MAX)$
25de18, 6 lines

```
MAX = 1000001
dp = [0] * MAX
dp[0] = 1
for x in a:
    for j in range(MAX-1, x-1, -1):
        dp[j] |= dp[j-x]
```

BitmaskDP.py
Description: GospersHack(k,n) génère toutes les combinaisons possibles de k bits parmi n bits
Time: X
f969ed, 18 lines

```
def GospersHack(k,n):
    if k==0: yield 0
    cur=(1<<k)-1
    while 0<cur<1<<n:
        yield cur
        lb=cur&-cur
        r=cur+lb
        cur=(r^cur)>>lb.bit_length()+1|r
dp = [1<<59] * (1<<N)
dp[0] = 0
for mask in range(1, 1<<N):
    for col1 in range(N):
        if mask>>col1&1 == 0: continue
        c = 0
        for col2 in range(N):
            if mask>>col2&1: continue
            c += cost[col1][col2]
        dp[mask] = min(dp[mask], dp[mask^(1<<col1)]+c)
```

GrayCodeDP.py
Description: Gray code in $O(2^n)$ instead of $O(n * 2^n)$.
Usage: `g = m^(m>>1)`
`rev_g(int g) { int n = 0; for (; g; g >>= 1) n ^= g; }`

LIS.py
Description: Compute LIS
Time: $\mathcal{O}(N \log N)$
1e1871, 15 lines

```
def longest_increasing_subsequence(lst):
    idxs, nums = [], []
    for n in lst:
        idx = bisect.bisect_left(nums, n)
        if idx == len(nums):nums.append(n)
        else:nums[idx] = n
        idxs.append(idx)
    ct = len(nums) - 1
    ret, ret_idx = [], []
    for i in range(len(lst)-1,-1,-1):
        if idxs[i] == ct:
            ret.append(lst[i])
```

```
        ret_idx.append(i); ct-=1
    ret.reverse(); ret_idx.reverse()
    return [ret, ret_idx]
```

TSP.py

Description: Also retrieve path

Time: $\mathcal{O}\left(N^2 \cdot 2^N\right)$

327cdc, 25 lines

```
def tsp_dp(n, dist):
    dp = [[-1] * (1<<n) for _ in range(n)]
    _next = [[-1] * (1<<n) for _ in range(n)]
    def F(i, mask):
        mask ^= (1<<i)
        if mask == 0:
            return dist[i][0]
        if dp[i][mask] != -1:
            return dp[i][mask]
        dp[i][mask] = 1<<59
        for bit in range(n):
            if mask & (1<<bit):
                new = F(bit, mask) + dist[i][bit]
                if new < dp[i][mask]:
                    dp[i][mask] = new
                    _next[i][mask] = bit
        return dp[i][mask]
    F(0, (1 << n) - 1)
    path, node, mask = [], 0, (1 << n) - 1
    while node != -1:
        mask ^= (1 << node)
        path.append(node)
        node = _next[node][mask]
    path.append(0)
    return path
```

StockSpan.py

Description: Max Reactangle Area in Histogram

Time: $\mathcal{O}(N)$

62ca62, 16 lines

```
def compute_extension(a):
    extension, ms = [0], [0]
    for i in range(1, len(a)):
        while ms and a[i] <= a[ms[-1]]: ms.pop()
        if ms: extension.append(i - ms[-1] - 1)
        else: extension.append(i)
        ms.append(i)
    return extension
n = int(input())
a = list(map(int,input().split()))
left_extension = compute_extension(a)
right_extension = compute_extension(a[::-1])[::-1]
cover = [a[i] * (r+l+1) for i, (l,r) in
         enumerate(zip(left_extension, right_extension))]
print(*left_extension); print(*right_extension)
print(*cover); print(max(cover))
```

CYK.py

Description: Cocke–Younger–Kasami, n length of parsed string, $|G|$ size of the CNF grammar.

Time: $\mathcal{O}\left(n^3 \cdot |G|\right)$

30cdf9, 19 lines

```
s = input(); n = len(s)
dp = [[[1<<59 * k for _ in range(n)] for _ in range(n)]
for i in range(n):
    dp[i][i][chars.find(s[i])] = 0
for span in range(1, n):
    for l in range(n - span):
        # dp[l][l+span][z] = min| dp[l][l+d][x] + dp[l+d+1][l+
            span][y] + cost(x,y) if x+y -> z
        for d in range(span):
            for x in range(k):
                for y in range(k):
                    c, z = rule[x][y]
                    dp[l][l+span][z] = min(dp[l][l+span][z],
                        dp[l][l+d][x] + dp[l+d+1][l+span][y] +
                            c)
bestcost = float('inf')
best = None
for char in chars:
    if bestcost > dp[0][n-1][chars.find(char)]:
        bestcost = dp[0][n-1][chars.find(char)]
        best = char
```

DigitDP.py

a10279, 21 lines

```
def calc(pos, k, und, st, t1, t2=-1):
    if pos == len(num):
        if not st: return 0
        if t2 == -1: return int(k >= 20)
        return int(k == 20)
    state = (pos, k, und, st)
    if state in dp: return dp[state]
    res = 0
    for d in range(10):
        ddiff = int(num[pos])
        if not und and d > ddiff: break
        is_und = und or (d < ddiff)
        is_st = st or (d > 0)
        if is_st and -1 != t2 != d and t1 != d: continue
        new_k = k
        if is_st:
            new_k += (d == t1)
            new_k -= (d != t1)
        res += calc(pos+1, new_k, is_und, is_st, t1, t2)
    dp[state] = res
    return res
```

Interleaving.py

Description: Test if string C is an interleaving of strings a and b

Time: $\mathcal{O}\left(N^2\right)$

897224, 11 lines

```
if len(a) + len(b) != len(c): print('NO') & exit()
dp = [[0] * (len(b)+1) for _ in range(len(a)+1)]
for i in range(len(a)+1):
    for j in range(len(b)+1):
        if i == 0 and j == 0: dp[i][j] = 1
        elif i == 0 and b[j-1] == c[j-1]: dp[i][j]=dp[i][j-1]
        elif j == 0 and a[i-1] == c[i-1]: dp[i][j]=dp[i-1][j]
        else:
            if a[i-1] == c[i+j-1]: dp[i][j] = dp[i-1][j]
            if b[j-1] == c[i+j-1]: dp[i][j] |= dp[i][j-1]
print('YES' if dp[len(a)][len(b)] else 'NO')
```

LCSubsequence.py

Description: Longest Common Subsequence of two strings. Reconstruct it from the dp table

Time: $\mathcal{O}\left(N^2\right)$

d9bf6c, 15 lines

```
dp = [[0] * (len(a)+1) for _ in range(len(b)+1)]
for i in range(1, len(b)+1):
    for j in range(1, len(a)+1):
        if b[i-1] == a[j-1]: dp[i][j] = dp[i-1][j-1]+1
        else: dp[i][j] = max(dp[i-1][j], dp[i][j-1])
print(dp[len(b)][len(a)])
# n types, k occurences for each
occ = [[] for _ in range(n+1)]
for i, x in enumerate(b):
    occ[x].append(i)
T = [0] * (n*k+1)
for i, x in enumerate(a, 1):
    for j in occ[x][::-1]:
        update(j, query(j) + 1)
print(query(n*k+1))
```

TreeKnapsack.py

Usage: `tin[u] = début de visite`
`order.append(u)`
`tout[v] = fin de visite du subtree root en v`

6ca69b, 11 lines

```
dp = [[0] * (w+1) for _ in range(n+1)]
for v in order[::-1]:
    i, o = tin[v], tout[v]+1
    wi, vi = weights[v], values[v]
    for j in range(w+1):
        # use child
        if j-wi >= 0:
            dp[i][j] = max(dp[i][j], dp[i+1][j-wi] + vi)
        # or skip subtree of child
        dp[i][j] = max(dp[i][j], dp[o][j])
print(dp[0][w])
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.

Time: $\mathcal{O}(N \max(w_i))$

b20ccc, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}
```

FastSubsetSum.h

Description: On traite les poids du plus petit au plus grand en maintenant une liste initialement vide W'. S'il y a occ occurrences du plus petit poids w : - Si occ est impair : ajouter $(occ-1)/2$ occurrences de $2w$ aux poids, et une occurrence de w à W'. - Si occ est pair : ajouter $(occ-2)/2$ occurrences de $2w$ aux poids, et deux occurrences de w à W'.

Time: $\mathcal{O}\left(C\sqrt{C}\right)$, où $\sum_{i=1}^{N} w_i = C$.

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

Time: $\mathcal{O}\left(N^2\right)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \leq k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.

Time: $\mathcal{O}((N + (hi - lo)) \log N)$ d38d2b, 17 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }
  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

AlienTrick.h

Description: return the maximum sum along with the number of subarrays used if creating a subarray penalizes the sum by "lmb" and there is no limit to the number of subarrays you can create a0a3e5, 18 lines

```
auto solve = [&](ll lmb) {
  pair<ll, ll> dp[n][2];
  dp[0][0] = {0, 0};
  dp[0][1] = {a[0]-lmb, 1};
  for (int i = 1; i < n; i++) {
    dp[i][0] = max(dp[i-1][0], dp[i-1][1]);
    dp[i][1] =
      max({dp[i-1][0].first + a[i]-lmb, dp[i-1][0].second + 1},
        {dp[i-1][1].first + a[i], dp[i-1][1].second});
  }
  return max(dp[n-1][0], dp[n-1][1]);
};
ll lo = 0, hi = 1e18;
while (lo < hi) {
  ll mid = (lo + hi + 1) / 2;
  solve(mid).second >= k ? lo = mid : hi = mid-1;
}
cout << solve(lo).first + lo * k << endl;
```

ConvexHullTrick.h

c89e0e, 12 lines

```
double slope(int i, int j) { return (double)(dp[i] - dp[j]) / (
    a[i].x - a[j].x); }
deque<ll> q;
q.push_back(0);
for (int i = 1; i <= n; i++) {
  while (q.size() > 1 && slope(q[0], q[1]) >= a[i].y) q.
      pop_front();
  ll j = q.front();
  dp[i] = max(dp[i - 1], a[i].x * a[i].y - a[i].a + dp[j] - a[j
      ].x * a[i].y);
  while (q.size() > 1 && slope(q[q.size() - 2], q.back()) <=
      slope(q.back(), i))
    q.pop_back();
  q.push_back(i);
}
cout << dp[n];
```

Graph (8)

8.1 Fundamentals

Bellman-Ford

$d_k[v]$ = shortest length from source to v using at most k edges

$$d(v) = \min_u d(u) + w_{uv}$$

Floyd-Warshall

$d_k[u][v]$ = shortest length between u and v using only nodes $< k$

$$d(u, v) = \min_w d(u, w) + d(w, v)$$

System of weighted contraints

$A - B \leq w$: add edge $B \to A$ with weight w. Add edge $(0, \text{vtx}, 0)$ for each vertex. Negative cycle: no solution. Otherwise, solution: $-\min(D)$.

Dijkstra.h

Description: Dijkstra for dense graphs.
Time: $\mathcal{O}(V^2)$ cdb8d8, 19 lines

```
void dijkstra(int s, vector<int> & d, vector<int> & p) {
    int n = adj.size();
    d.assign(n, INF);
    p.assign(n, -1);
    vector<bool> u(n, false);
    d[s] = 0;
    for (int i = 0; i < n; i++) {
        int v = -1;
        for (int j = 0; j < n; j++) {
            if (!u[j] && (v == -1 || d[j] < d[v])) v = j;
        }
        if (d[v] == INF) break;
        u[v] = true;
        for (auto edge : adj[v]) {
            int to = edge.first;
            int len = edge.second;
            if (d[v] + len < d[to]) {
              d[to] = d[v] + len; p[to] = v;
            }}}}
```

BellmanFord.py

Description: Bellman-Ford
Usage: edges 1-indexed
Time: $\mathcal{O}(VE)$ 5d1684, 15 lines

```
def bellmanford(n, edges):
    dist = [float('inf')] * (n + 1)
    prec = [-1] * (n + 1)
    for _ in range(n):
        x = -1
        for u, v, w in edges:
            if dist[u] + w < dist[v]:
                prec[v] = u; dist[v] = dist[u] + w; x = v
    if x == -1: return []
    for _ in range(n): x = prec[x]
    end, path = x, [x]
    while prec[x] != end:
        x = prec[x]
        path.append(x)
    return (path+[end])[::-1]
```

Dials.h

Description: Dial's algorithm, Faster than dijkstra for graphs with weights <= 10
Time: $\mathcal{O}(V \cdot lim + E)$. 77655f, 15 lines

```
void dials(int st, vector<vpii> adj, int lim=10){
    int n = sz(adj);
    vector<int> dist(n, -1);
    vector<vector<int>> Qs(lim + 1);
    dist[st] = 0; Qs[0].push_back(st);
    for (int d = 0, mx = 0; d <= mx; d++) {
        for (auto& Q = Qs[d % (lim + 1)]; Q.size();) {
            int cur = Q.back(); Q.pop_back();
            if (dist[cur] != d) continue;
            for (const auto& [nxt, cost] : adj[cur]) {
                if (dist[nxt] == -1 || dist[nxt] > d + cost) {
                    dist[nxt] = d + cost;
                    Qs[dist[nxt] % (lim + 1)].push_back(nxt);
                    mx = max(mx, dist[nxt]);}}}}
}
```

Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

8.2 Network flow

dinic.h

Description: Flow algorithm with complexity $O(VE \log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching. f6e693, 38 lines

```
struct Dinic {
  struct Edge {
    int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, 0LL); } // if you need flows
  };
  vector<int> lvl, ptr, q;
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
  void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].push_back({b, int((adj[b]).size()), c, c});
    adj[b].push_back({a, int((adj[a]).size()) - 1, rcap, rcap})
        ;}
  ll dfs(int v, int t, ll f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < int((adj[v]).size()); i++) {
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (ll p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;}}
    return 0;}
  ll calc(int s, int t) {
    ll flow = 0; q[0] = s;
      for(int L=0; L<31; L++){ // 'int L=30' maybe faster for
          random data
        do {
          lvl = ptr = vector<int>(int((q).size()));
```

```
                int qi = 0, qe = lvl[s] = 1;
                while (qi < qe && !lvl[t]) {
                    int v = q[qi++];
                    for (Edge e : adj[v])
                        if (!lvl[e.to] && e.c >> (30 - L))
                            q[qe++] = e.to, lvl[e.to] = lvl[v]
                                 + 1;
                }
                while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
            } while (lvl[t]);}
    return flow;}
  bool leftOfMinCut(int a) { return lvl[a] != 0; }
};
```

dinic.py

Description: Dinic Maximum flow, fonctionne si arcs bidirectionnels à capacité partagée
Time: $\mathcal{O}\left(V^2 * E\right)$ 1dadab, 31 lines

```
def dinic(source, target, graph):
    def _dinic_step(lev, u, t, limit):
        if limit <= 0: return 0
        if u == t: return limit
        val = 0
        for v in graph[u]:
            residuel = graph[u][v] - flow[u][v]
            if lev[v] == lev[u] + 1 and residuel > 0:
                z = min(limit, residuel)
                aug = _dinic_step(lev, v, t, z)
                flow[u][v] += aug; flow[v][u] -= aug
                val += aug; limit -= aug
        if val == 0: lev[u] = None
        return val
    n, Q, total = len(graph), deque(), 0
    flow = [[0] * n for _ in range(n)]
    while True:
        Q.appendleft(source)
        lev = [None] * n; lev[source] = 0
        while Q:
            u = Q.pop()
            for v in graph[u]:
                if lev[v] is None and graph[u][v] > flow[u][v]:
                    lev[v] = lev[u] + 1; Q.appendleft(v)
        if lev[target] is None: break
        UB = sum(graph[source][v] for v in graph[source]) -
             total
        total += _dinic_step(lev, source, target, UB)
    return flow, total
def add_edge(u, v, c):
    if v not in graph[u]: graph[u][v] = 0; graph[v][u] = 0
    graph[u][v] += c
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$ 8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}};
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i};
  rep(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio. queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();
      rep(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
}
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
Time: $\mathcal{O}(V)$ Flow Computations e2b333, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
  vector<Edge> tree;
  vi par(N);
  rep(i,1,N) {
    Dinic D(N); // Dinic also works
    for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
    tree.push_back({i, par[i], D.calc(i, par[i])});
    rep(j,i+1,N)
      if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
  }
  return tree;
}
```

MinCostMaxFlow.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi. 3ce322, 69 lines

```
const ll INF = numeric_limits<ll>::max() / 4;
struct MCMF {
  struct edge {
    int from, to, rev;
    ll cap, cost, flow;
  };
  int N;
  vector<vector<edge>> ed;
  vi seen;
  vector<ll> dist, pi;
  vector<edge*> par;
  MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N), par(N) {}
  void addEdge(int from, int to, ll cap, ll cost) {
    if (from == to) return;
    ed[from].push_back(edge{ from,to,(int)(ed[to]).size(),cap,
         cost,0 });
    ed[to].push_back(edge{ to,from,(int)(ed[from]).size()-1,0,-
         cost,0 });
  }
  void path(int s) {
    fill(seen.begin(), seen.end(), 0);
    fill(dist.begin(), dist.end(), INF);
    dist[s] = 0; ll di;
    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({ 0, s });
    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (edge& e : ed[s]) if (!seen[e.to]) {
        ll val = di - pi[e.to] + e.cost;
        if (e.cap - e.flow > 0 && val < dist[e.to]) {
          dist[e.to] = val;
          par[e.to] = &e;
          if (its[e.to] == q.end())
            its[e.to] = q.push({ -dist[e.to], e.to });
          else
            q.modify(its[e.to], { -dist[e.to], e.to });
        }
      }
    }
    for(int i=0; i<N; i++) pi[i] = min(pi[i] + dist[i], INF);
  }
  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (edge* x = par[t]; x; x = par[x->from])
        fl = min(fl, x->cap - x->flow);

      totflow += fl;
      for (edge* x = par[t]; x; x = par[x->from]) {
        x->flow += fl;
        ed[x->to][x->rev].flow -= fl;
      }
    }
    for(int i=0; i<N; i++) for(edge& e : ed[i]) totcost += e.
         cost * e.flow;
    return {totflow, totcost/2};
  }
  // If some costs can be negative, call this before maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(pi.begin(), pi.end(), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      for(int i=0; i<N; i++) if (pi[i] != INF)
        for (edge& e : ed[i]) if (e.cap)
          if ((v = pi[i] + e.cost) < pi[e.to])
            pi[e.to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

CirculationDemands.py

Description: Circulation with lowerbounds d(e) <= f(e) <= c(e)
Time: X 965054, 18 lines

```
graph = [{} for _ in range(N+2)]
isum, osum, edges = [0]*N, [0]*N, []
for _ in range(M):
    a, b, d, c = map(int,input().split())
    add_edge(graph, a, b, c-d)
    isum[b]+=d; osum[a]+=d
s, t = N, N+1
indemands = outdemands = 0
for v in range(N):
    need = isum[v] - osum[v]
    if need < 0:
        add_edge(graph, v, t, -need); outdemands -= need
    if need > 0:
        add_edge(graph, s, v, need); indemands += need
if indemands != outdemands: print("NON")
else:
    _, mf = dinic(s, t, graph)
    print("OUI" if mf == indemands else "NON")
```

Flow with demands

We make the following changes in the network. We add a new source s' and a new sink t', a new edge from the source s' to every other vertex, a new edge for every vertex to the sink t', and one edge from t to s. Additionally, we define the new capacity function c' as:

$c'((s',v)) = \sum_{u \in V} d((u,v))$ for each edge (s',v),
$c'((v,t')) = \sum_{w \in V} d((v,w))$ for each edge (v,t'),
$c'((u,v)) =$
$c((u,v)) - d((u,v))$ for each edge (u,v) in the old network,
$c'((t,s)) = \infty$.

If the new network has a saturating flow (a flow where each edge outgoing from s' is completely filled, which is equivalent to every edge incoming to t' being completely filled), then the network with demands has a valid flow, and the actual flow can be easily reconstructed from the new network. Otherwise, there doesn't exist a flow that satisfies all conditions. Since a saturating flow has to be a maximum flow, it can be found by any maximum flow algorithm.

$s - t$ coupe minimum pour graphe planaire

Dans le graphe *dual*, chaque cellule est un sommet, et il existe deux sommets supplémentaires s' et t'. Le sommet s' représente le bord gauche et haut de la grille et le sommet t' le bord droit et bas. Dans ce nouveau graphe, deux sommets sont reliés s'ils sont séparés par une arête du graphe primal. Le poids des arêtes est le même. Tout $s' - t'$ chemin dans le graphe dual de longueur w correspond à une $s - t$ coupe dans le graphe primal de même valeur w et vice-versa.

8.3 Matching

HopcroftKarp.h

Time: $\mathcal{O}\left(E\sqrt{V}\right)$

151d57, 34 lines

```cpp
struct HopcroftKarp {
  vector<int> g, l, r;
  int ans;
  HopcroftKarp(int n, int m, const vector<pii> &e)
      : g(e.size()), l(n, -1), r(m, -1), ans(0) {
    vector<int> deg(n + 1);
    for (int i=0; i<(int)e.size(); i++) deg[e[i].first]++;
    for (int i = 1; i <= n; i++) deg[i] += deg[i - 1];
    for (int i=0; i<(int)e.size(); i++) g[--deg[e[i].first]] =
        e[i].second;
    vector<int> a, p, q(n);
    for (;;) {
      a.assign(n, -1), p.assign(n, -1);
      int t = 0;
      for (int i = 0; i < n; i++)
        if (l[i] == -1) q[t++] = a[i] = p[i] = i;
      bool match = false;
      for (int i = 0; i < t; i++) {
        int x = q[i];
        if (~l[a[x]]) continue;
        for (int j = deg[x]; j < deg[x + 1]; j++) {
          int y = g[j];
          if (r[y] == -1) {
            while (~y) r[y] = x, swap(l[x], y), x = p[x];
            match = true, ans++;
            break;
          }
          if (p[r[y]] == -1)
            q[t++] = y = r[y], p[y] = x, a[y] = a[x];
        }
      }
      if (!match) break;
    }
  }
};
```

MinimumVertexCover.h

6a77a3, 28 lines

```cpp
vi cover(vector<pii>& ed, int n, int m) {
    vvi g(n);
    for(auto[u, v]: ed) g[u].pb(v);
  vi match(m, -1);
    HopcroftKarp hk(n, m, ed);
    int res = hk.ans;
    for(int i=0; i<n; i++){
        if(~hk.l[i]){
            match[hk.l[i]] = i; // mp(i, hk.l[i]);
        }
    }
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

Dilworth.h

Description: In a DAG, the size of a minimum general path cover equals the size of a maximum antichain. Min Node-disjoint path cover -> N - MCBM (edge u-v si edge de u à v). Min General path cover -> N - MCBM (modifié: edge u-v si chemin de u à v)

Usage: Produire un graphe biparti H(V-, V+, E) avec V-, V+ étant des copies de V, et (u-, v+) appartient à E ssi (u, v) appartient à A.

ec9170, 21 lines

```cpp
vi ml(n, -1), mr(n, -1);
for (int i=0; i<n; i++) {
    if (~hk.l[i]){
        ml[i] = hk.l[i];
        mr[hk.l[i]] = i;
    }
}
vi part(n);
vvi chains;
for(int v=0; v<n; v++){
    if(ml[v] != -1) continue;
    int u = v;
    vi chain;
    while (u != -1){
        chain.pb(u+1);
        part[u] = 1;
        u = mr[u];
    }
    reverse(chain.begin(), chain.end());
    chains.pb(chain);
}
```

KuhnMunkres.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.

Time: $\mathcal{O}\left(N^2M\right)$

1e0fe9, 31 lines

```cpp
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

GaleShapley.py

Description: Mariages stables.

Usage: men et women numérotés de 0 à n - 1. Le tableau men contient pour chaque homme la liste des femmes par préférence décroissante. Même chose pour women

Time: $\mathcal{O}\left(V^2\right)$

cc837e, 18 lines

```python
def gale_shapley(men, women):
    n = len(men)
    assert n == len(women)
    suiv, mari = [0] * n, [None] * n
    rank = [[0] * n for j in range(n)]  # build rank
    for j in range(n):
        for r in range(n): rank[j][women[j][r]] = r
    singles = deque(range(n))  # all men are single
    while singles:             # and get in the queue
        i = singles.popleft()
        j = men[i][suiv[i]]; suiv[i] += 1
        if mari[j] is None: mari[j] = i
        elif rank[j][mari[j]] < rank[j][i]:
            singles.append(i)
        else:
            singles.put(mari[j])  # divorce mari[j]
            mari[j] = i
    return mari
```

8.4 DFS algorithms

SCC.py

Description: SCC
Time: $\mathcal{O}(N)$

a890c0, 22 lines

```
def tarjan(G):
    n = len(G)
    SCC, S, P = [], [], []
    Q, st = list(range(n)), [0] * n
    while Q:
        node = Q.pop()
        if node < 0:
            d = st[~node] - 1
            if P[-1] > d:
                SCC.append(S[d:])
                del S[d:]; P.pop()
                for v in SCC[-1]:
                    st[v] = -1
        elif st[node] > 0:
            while P[-1] > st[node]:P.pop()
        elif st[node] == 0:
            S.append(node)
            P.append(len(S))
            st[node] = len(S)
            Q.append(~node)
            Q.extend(G[node])
    return SCC
```

BiconnectedComponents.py

Description: Returns BCC

75d6ee, 16 lines

```
def find_BCC(G):
    n = len(G)
    dep = [0] * n
    Q = [0]
    d = 0
    while Q:
        node = Q.pop()
        if node < 0:
            node = ~node
            d -= 1
        elif not dep[node]:
            d = dep[node] = d + 1
            Q.append(~node)
            Q += G[node]
    G2 = [[v for v in G[u] if dep[u] + 1 >= dep[v] != dep[u] -
        1] for u in range(n)]
    return find_SCC(G2)
```

BiconnectedComponents.h

Description: Returns block-cut tree

c2f1ac, 53 lines

```
vvi biconnected_components(int n, int m, vvpii &g, vi &
    is_cutpoint, vi &is_bridge, vi &id) {
  vvi comps;
  vi stk, num(n), low(n);
  is_cutpoint.resize(n);
    is_bridge.resize(m);
  function<void(int, int, int &)> dfs = [&](int node, int par,
      int &time) {
    num[node] = low[node] = ++time;
    stk.pb(node);
    for (auto [son, i] : g[node]) {
      if (son == par) continue;
      if (num[son]) {
        low[node] = min(low[node], num[son]);
      } else {
        dfs(son, node, time);
        low[node] = min(low[node], low[son]);
        if (low[son] >= num[node]) {
          is_cutpoint[node] = (num[node] > 1 || num[son] > 2);
          comps.pb({node});
          while (comps.back().back() != son) {
            comps.back().pb(stk.back());
            stk.pop_back();
          }
        }
              is_bridge[i] = (low[son] > num[node]);
      }
    }
  };
  int time = 0;
    dfs(0, -1, time);
  id.resize(n);
  function<vvi()> build_tree = [&]() {
    vvi t(1);
    int node_id = 0;
    for (int node = 0; node < n; node++) {
      if (!is_cutpoint[node]) continue;
          id[node] = node_id++;
          t.pb({});
    }
    for (auto &comp : comps) {
      int node = node_id++;
      t.pb({});
      for (int u : comp)
        if (!is_cutpoint[u]) {
          id[u] = node;
        } else {
          t[node].pb(id[u]);
          t[id[u]].pb(node);
        }
    }
    return t;
  };
  return build_tree();
}
```

2sat.py

Description: Solve 2SAT
Time: $\mathcal{O}(N)$

90402d, 21 lines

```
class TwoSat:
    def __init__(self, n):
        self.n = n
        self.graph = [[] for _ in range(2*n)]
    def _imply(self, x, y):
        self.graph[x].append(y if y >= 0 else 2*self.n+y)
    def either(self, x, y):
        self._imply(~x, y)
        self._imply(~y, x)
    def set(self, x):
        self._imply(~x, x)
    def implies(self, x, y):
        self.either(~x, y)
    def solve(self):
        SCC = tarjan(self.graph)
        order = [0] * (2 * self.n)
        for i, comp in enumerate(SCC):
            for x in comp: order[x] = i
        for i in range(self.n):
            if order[i] == order[~i]: return False, None
        return True,[+(order[i] > order[~i])for i in range(self
            .n)]
```

EulerTour.py

Description: Check connectivity - Undirected graph : connected and all nodes have even deg, or only 2 have odd degree - Directed graph : At most one node has out_i - in_i = 1 and at most one node has in_i - out_i = 1 - Multigraph : Mind case with only one node
Time: $\mathcal{O}(N)$

f294be, 29 lines

```
def eulerian_tour(m, s, graph):
    # also works with a multigraph if directed
    P, Q = [], [s]
    while Q:
        u = Q[-1]
        if len(graph[u]):
            v = graph[u].pop()
            # graph[v].discard(u) # if undir
            Q.append(v)
        else:
            P.append(Q.pop())
    return P[::-1] if len(P)==m+1 else -1

def eulerian_tour_undir_multigraph(m, s, graph, rem):
    P, Q = [], [s]
    while Q:
        u = Q[-1]
        while len(graph[u]):
            if rem[u][len(graph[u])-1]:
                graph[u].pop()
            else: break
        if len(graph[u]):
            rem[u][len(graph[u])-1] = True
            v, j = graph[u].pop()
            rem[v][j] = True
            Q.append(v)
        else:
            P.append(Q.pop())
    return P[::-1] if len(P)==m+1 else -1
```

TopoSort.py

Description: Smallest perm
Time: $\mathcal{O}(N \log N)$

bf5396, 13 lines

```
revgraph, outdeg = [[] for _ in range(n)], [0] * n
for _ in range(m):
    a, b = map(int,input().split())
    revgraph[b].append(a); outdeg[a] += 1
Q = [-node for node in range(n) if outdeg[node] == 0]
heapify(Q); order = []
while Q:
    node = -heappop(Q)
    order.append(node + 1)
    for neigh in revgraph[node]:
        outdeg[neigh] -= 1
        if outdeg[neigh] == 0: heappush(Q, -neigh)
print(*order[::-1])
```

DfsTree.py

Description: Compute DFS-Tree
Time: $\mathcal{O}(N)$

063486, 19 lines

```
if cut_nodes_edges(g): print(0) & exit()
ans = [set() for _ in range(n)]
seen = [False]*n; seen[0]=True; d = [-1]*n
@bootstrap
def dfs(p, v):
    d[v] = d[p]+1
    for u in g[v]:
        if u == p: continue
        if seen[u]:
            # back edge
            if d[v] < d[u]: ans[u].add(v)
```

```
            else: ans[v].add(u)
        else:
            seen[u]=True; ans[v].add(u)  # dfs tree edge
            (yield dfs(v, u))
    yield
dfs(0, 0)
for v in range(n):
    for u in ans[v]: print(v+1, u+1)
```

8.5 General

ChromaticNumber.py

Description: Minimum Clique Cover <=> Coloring of Complement Graph
Constraints : N <= 20, No multiples or self edge
Time: $\mathcal{O}\left(2^N\right)$

d7362d, 18 lines

```
def chromatic_number(n:int, edges:list[tuple[int,int]])->int:
    edge = [0] * n
    for uv in edges:
        u, v = uv; edge[u]|=1<<v; edge[v]|=1<<u
    dp = [0] * (1 << n); dp[0] = 1; cur = [0] * (1 << n)
    for bit in range(1, 1 << n):
        v = (~bit & (bit-1)).bit_count()
        dp[bit] = dp[bit^(1<<v)]+dp[(bit^(1<<v))&(~edge[v])]
    for bit in range(1 << n):
        if (n - bit.bit_count()) & 1: cur[bit] = -1
        else: cur[bit] = 1
    for k in range(1, n):
        tmp = 0
        for bit in range(1 << n):
            cur[bit] *= dp[bit]; tmp += cur[bit]
        if tmp != 0: res = k; break
    else: res = n
    return res
```

MIS.py

Description: Complement of Minimum Vertex Cover. Max clique of the complement graph
Time: Works for N < 40

4e62e9, 31 lines

```
def maximum_independent_set(n: int, edges: list[tuple[int, int
    ]])\
      -> tuple[int, list[int]]:
    adj = [0] * n
    for u, v in edges:
        if u > v: u, v = v, u
        adj[u] |= 1 << v
    dp = {0: 0}
    for i in range(n):
        nex = dict()
        for S, val in dp.items():
            if S >> i & 1 == 0:
                S1 = S | adj[i]
                nv = (val + (1 << n)) | (1 << i)
                if S1 in nex:
                    if nex[S1] < nv:
                        nex[S1] = nv
                else:
                    nex[S1] = nv
            S2 = S & ~(1 << i)
            if S2 in nex:
                if nex[S2] < val:
                    nex[S2] = val
            else:
                nex[S2] = val
        dp = nex
    size, bitset = divmod(dp[0], 1 << n)
    res = []
    for i in range(n):
        if bitset >> i & 1:
            res.append(i)
    return size, res
```

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$

e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  }
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
  return ret;
}
```

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

Trees (9)

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S|\log|S|)$

"LCA.h" 46e6b7, 20 lines

```
vpii compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.time));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.lca(a, b));
  }
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpii ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
  }
  return ret;
}
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.
Time: $\mathcal{O}(N\log N+Q)$

"../data-structures/RMQ.h" 0f62fb, 19 lines

```
struct LCA {
  int T = 0;
  vi time, path, ret;
  RMQ<int> rmq;
  LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1), ret)) {}
  void dfs(vector<vi>& C, int v, int par) {
    time[v] = T++;
    for (int y : C[v]) if (y != par) {
      path.push_back(v), ret.push_back(time[v]);
      dfs(C, y, v);
    }
  }
  int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)];
  }
  //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
};
```

BinaryLifting.py

35b1b8, 22 lines

```
up = [up]
wei = [wei]
for _ in range(1, LOG):
    wei.append([max(wei[-1][v], wei[-1][up[-1][v]]) for v in
         range(n)])
    up.append([up[-1][up[-1][v]] for v in range(n)])

def lca(u, v):
    if dep[u] > dep[v]: u, v = v, u
    k = dep[v] - dep[u]
    maxw = 0
    for i in range(LOG):
        if k>>i&1:
```

```
            maxw = max(maxw, wei[i][v])
            v = up[i][v]
    if v == u: return maxw
    for i in range(LOG-1, -1, -1):
        if up[i][u] != up[i][v]:
            maxw = max(maxw, wei[i][u], wei[i][v])
            u, v = up[i][u], up[i][v]
    maxw = max(maxw, wei[0][u], wei[0][v])
    # return up[0][u]
    return maxw
```

SubtreeQueries.py

Description: For subtree queries with Fenwick
Usage: `update(index[u], x - val[index[u]]); query(index[u], index[u] + sz[u])`
Time: $\mathcal{O}(N)$

51f638, 24 lines

```
prev = [None] * n
sz, ts = [1] * n, [-1] * n
ts[0] = 0
Q, order = [0], []
while Q:
    v = Q[-1]
    if ts[v] == len(adj[v]):
        for u in adj[v]:
            if u == prev[v]: continue
            sz[v] += sz[u]
        order.append(Q.pop())
    else:
        u = adj[v][ts[v]]
        if u == prev[v]:
            ts[v] += 1
            if ts[v] == len(adj[v]):continue
            u = adj[v][ts[v]]
        ts[v] += 1; ts[u] = 0; prev[u] = v
        Q.append(u)
order.reverse()
index = [None] * n
for i, e in enumerate(order):
    index[e] = i
val = [val[i] for i in order]
```

SmallToLarge.h

3ef92f, 19 lines

```
void merge(set<int>& s, set<int>& t) {
  if (s.size() < t.size()) s.swap(t);
  for (auto x : t) s.insert(x);
  t = {};
}
function<void(int,int)> dfs = [&](int v, int par) {
    for(int u : adj[v]){
        if(u == par) continue;
        dfs(u, v);
        merge(vset[v], vset[u]);
    }
    vset[v].insert(p[v]);
    for(auto T : query[v]){
        auto [i, l, r] = T;
        auto it = vset[v].lower_bound(l);
        ans[i] = it != vset[v].end() && *it <= r;
    }
};
dfs(1, 0);
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.
Time: $\mathcal{O}\left((\log N)^2\right)$

03139d, 33 lines

```
template <bool VALS_EDGES> struct HLD {
  int N, tim = 0; vector<vi> adj;
  vi par, siz, rt, pos; Node *tree;
  HLD(vector<vi> adj_)
    : N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1),
      rt(N),pos(N),tree(new Node(0, N)){ dfsSz(0); dfsHld(0); }
  void dfsSz(int v) {
    if (par[v] != -1) adj[v].erase(find(all(adj[v]), par[v]));
    for (int& u : adj[v]) {
      par[u] = v; dfsSz(u); siz[v] += siz[u];
      if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);}}
  void dfsHld(int v) {
    pos[v] = tim++;
    for (int u : adj[v]) {
      rt[u] = (u == adj[v][0] ? rt[v] : u); dfsHld(u);
    }}
  template <class B> void process(int u, int v, B op) {
    for (; rt[u] != rt[v]; v = par[rt[v]]) {
      if (pos[rt[u]] > pos[rt[v]]) swap(u, v);
      op(pos[rt[v]], pos[v] + 1);
    }
    if (pos[u] > pos[v]) swap(u, v);
    op(pos[u] + VALS_EDGES, pos[v] + 1);}
  void modifyPath(int u, int v, int val) {
    process(u, v, [&](int l, int r) { tree->add(l, r, val); });
        }
  int queryPath(int u, int v) { // Modify depending on problem
    int res = -1e9;
    process(u, v, [&](int l, int r) {
            res = max(res, tree->query(l, r));});
    return res;}
  int querySubtree(int v) { // modifySubtree is similar
    return tree->query(pos[v] + VALS_EDGES, pos[v] + siz[v]);}
};
```

CentroidDecomp.h

Description: Centroid Decomposition is a divide and conquer technique for trees. Works by repeated splitting the tree and each of the resulting subgraphs at the centroid, producing $\mathcal{O}(\log N)$ layers of subgraphs.
Time: $\mathcal{O}(\log N)$

a05201, 45 lines

```
vvi adj; vi subtree_size;
vi min_dist; vector<bool> is_removed;
vvpii ancestors;
int get_subtree_size(int node, int parent = -1) {
  subtree_size[node] = 1;
  for (int child : adj[node]) {
    if (child == parent || is_removed[child]) { continue; }
    subtree_size[node] += get_subtree_size(child, node);}
  return subtree_size[node];
}
int get_centroid(int node, int tree_size, int parent = -1) {
  for (int child : adj[node]) {
    if (child == parent || is_removed[child]) { continue; }
    if (subtree_size[child] * 2 > tree_size) {
      return get_centroid(child, tree_size, node);}}
  return node;
}
void get_dists(int node, int centroid, int parent = -1, int
     cur_dist = 1) {
  for (int child : adj[node]) {
    if (child == parent || is_removed[child]) { continue; }
    cur_dist++;
    get_dists(child, centroid, node, cur_dist);
    cur_dist--;
  }
  ancestors[node].push_back({centroid, cur_dist});}
void build_centroid_decomp(int node = 0) {
  int centroid = get_centroid(node, get_subtree_size(node));
  for (int child : adj[centroid]) {
    if (is_removed[child]) { continue; }
    get_dists(child, centroid, centroid);}
  is_removed[centroid] = true;
  for (int child : adj[centroid]) {
    if (is_removed[child]) { continue; }
    build_centroid_decomp(child);}
}
void paint(int node) {
  for (auto &[ancestor, dist] : ancestors[node]) {
    min_dist[ancestor] = min(min_dist[ancestor], dist);}
  min_dist[node] = 0;}
void query(int node) {
  int ans = min_dist[node];
  for (auto &[ancestor, dist] : ancestors[node]) {
    if (!dist) continue;
    ans = min(ans, dist + min_dist[ancestor]);}
  cout << ans << "\n";}
```

KruskalReconstructionTree.h

Description: Decomposing Kruskal's algorithm to solve problems about min/max edge weights.

bb1e13, 19 lines

```
UF dsu(n+m);
vi minw(m);
vvi adj(n+m);
vi c_edge(n+m); iota(all(c_edge), 0);
rep(e,0,m){
    deg[u] ^= 1; deg[v] ^= 1;
    minw[e] = w;
    int ru = dsu.find(u);
    int rv = dsu.find(v);
    if(ru != rv){
        dsu.join(ru, rv);
        adj[n+e].pb(c_edge[ru]);
        adj[n+e].pb(c_edge[rv]);
        c_edge[dsu.find(ru)] = n+e;
    } else {
        adj[n+e].pb(c_edge[ru]);
        c_edge[ru] = n+e;
    }
}
```

Prim.py

Description: Min spanning tree for dense graphs
Time: $\mathcal{O}(N \log N)$

eedd2a, 14 lines

```
def prim(graph):
    visited = [False] * n
    distmin = [float('inf')] * n
    heap, total = [(0, 0)], 0
    while heap:
        d, i = heappop(heap)
        if visited[i]: continue
        visited[i] = True; total += d
        for j, w in graph[i]:
            if visited[j]: continue
            if d + w < distmin[j]:
                distmin[j] = d + w
                heappush(heap, (d + w, j))
```

```
  return total
```

Geometry (10)

10.1 Geometric primitives

Point.h

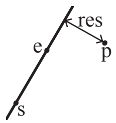
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

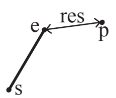


"Point.h" f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



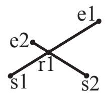
Usage:
```
Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
```

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



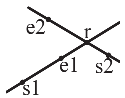
Usage:
```
vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
```

"Point.h", "OnSegment.h" 9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage:
```
auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```

"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage:
```
bool left = sideOf(p1,p2,q)==1;
```

"Point.h" 3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h

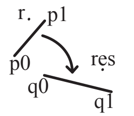
Description: Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:



Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

"Point.h" 03a306, 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d, 5 lines

```
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage:
```
vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively
oriented triangles with vertices at 0 and i
```

0f0602, 35 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
```

```
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

10.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 9 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h};
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

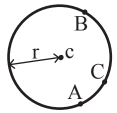
"../../content/geometry/Point.h" a1ee63, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

10.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.py

efc017, 5 lines

```
def area(p):
    A = 0
    for i in range(len(p)):
        A += p[i-1][0]*p[i][1]-p[i][0]*p[i-1][1]
    return abs(A/2)
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

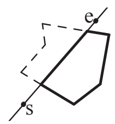
"Point.h" 9706dc, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" f2b7d4, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
```

```
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

PolygonUnion.h

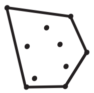
Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}\left(N^2\right)$, where N is the total number of points

"Point.h", "sideOf.h" 3931c6, 33 lines

```
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
    P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
    vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
    rep(j,0,sz(poly)) if (i != j) {
      rep(u,0,sz(poly[j])) {
        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
        if (sc != sd) {
          double sa = C.cross(D, A), sb = C.cross(D, B);
          if (min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
        } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
          segs.emplace_back(rat(C - A, B - A), 1);
          segs.emplace_back(rat(D - A, B - A), -1);
        }
      }
    }
    sort(all(segs));
    for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
    double sum = 0;
    int cnt = segs[0].second;
    rep(j,1,sz(segs)) {
      if (!cnt) sum += segs[j].first - segs[j - 1].first;
      cnt += segs[j].second;
    }
    ret += A.cross(B) * sum;
  }
  return ret / 2;
}
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: $\mathcal{O}(n \log n)$



"Point.h" 01732e, 17 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector <P> pt) {
    int n = sz(pt);
    sort(all(pt), [&](P a, P b) {
        return a.x == b.x ? a.y < b.y : a.x < b.x;
    });
    vector<P> ans = {pt[0]};
    for (int t : {0, 1}) {
        int m = sz(ans);
        for (int i = 1; i < n; ++i) {
            while (sz(ans) > m && ans[sz(ans)-2].cross(ans.back
                (), pt[i]) < 0)
                ans.pop_back();
            ans.pb(pt[i]);}
        reverse(all(pt));}
    ans.pop_back();
    return ans;
}
```

ConvexHull.py

Description: Attention aux points colliénaires verticalement
Time: $\mathcal{O}(N \log N)$

7bbfcc, 14 lines

```
eps = 1e-7
def left_turn(a, b, c):
    return (a[0] - c[0]) * (b[1] - c[1]) - (a[1] - c[1]) * (b
        [0] - c[0]) > 0 + eps
def convex_hull(S):
    S.sort()
    top, bot = [], []
    for p in S:
        while len(top) >= 2 and not left_turn(p, top[-1], top
            [-2]):
            top.pop()
        top.append(p)
        while len(bot) >= 2 and not left_turn(bot[-2], bot[-1],
            p):
            bot.pop()
        bot.append(p)
    return bot[:-1] + top[:0:-1]
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

10.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
```

```
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

ChebyshevDist.h

Description: Chebyshev distance Given points (x, y) and you need to calculate the Manhattan distances between them, instead of using $|x1 - x2| + |y1 - y2|$ you can first convert all points (x, y) into $(x + y, x - y)$ and the distances will become $max(|x1 - x2|, |y1 - y2|)$ (x, y, z) to the point $(x + y + z, x + y - z, x - y + z, -x + y + z)$

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"Point.h" df6f59, 23 lines

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
  vi id(sz(ps));
  iota(all(id), 0);
  vector<array<int, 3>> edges;
  rep(k,0,4) {
    sort(all(id), [&](int i, int j) {
        return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
        int j = it->second;
        P d = ps[i] - ps[j];
        if (d.y > d.x) break;
        edges.push_back({d.y + d.x, i, j});
      }
      sweep[-ps[i].y] = i;
    }
    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p.y);
  }
  return edges;
}
```

10.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" 5b45fc, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

10.6 Misc.

FacesPlanarGraph.py

Description: Compute faces delimited by segments, then sum of squared areas of faces
Time: $\mathcal{O}(N \log N)$

37cfea, 50 lines

```
from math import atan2, fsum
from collections import defaultdict, namedtuple
Point = namedtuple("p", "x y")
def shoelace(poly):
    fw = sum(poly[i - 1].x * poly[i].y for i in range(len(poly)
        ))
    poly = list(reversed(poly))
    bw = sum(poly[i - 1].x * poly[i].y for i in range(len(poly)
        ))
    return abs(fw - bw) / 2.0
def sort_ccw(p, points):
    return sorted(points, key=lambda point: atan2(point.y - p.y
        , point.x - p.x))
def find_face(neighbors, u, v):
    face, current, previous = [], v, u
    face.append(previous)
    while True:
        face.append(current)
        current_neighbors = neighbors[current]
        index = current_neighbors.index(previous)
        next_index = (index + 1) % len(current_neighbors)
        next_vertex = current_neighbors[next_index]
        if next_vertex == u: break
        previous, current = current, next_vertex
    face.append(u)
    return face
def find_outer_edge(coord):
    leftmost = min(coord, key=lambda p: (p.x, p.y))
    N_leftmost = coord[leftmost]
    sorted_neighbors = sorted(
```

```
        N_leftmost, key=lambda p: atan2(p.y-leftmost.y, p.x-
            leftmost.x))
    u = sorted_neighbors[0]
    return (leftmost, u)
n, coord = int(input()), defaultdict(list)
for l in range(n):
    x1, y1, x2, y2 = map(float, input().split())
    p1, p2 = Point(x1, y1), Point(x2, y2)
    coord[p1].append(p2); coord[p2].append(p1)
for p in coord:coord[p] = sort_ccw(p, coord[p])
seen = set()
p, q = find_outer_edge(coord)
outer = find_face(coord, p, q)
for i in range(len(outer)-1):seen.add((outer[i],outer[i+1%len(
    outer)]))
areas = []
for p in coord:
    for q in coord[p]:
        if (p, q) in seen: continue
        seen.add((p, q))
        face = find_face(coord, p, q)
        areas.append(shoelace(face) ** 2)
        for i in range(len(face) - 1):
            seen.add((face[i], face[i + 1 % len(face)]))
print(fsum(areas))
```

HalfPlaneIntersec.h

Description: Compute half-plane intersection
Time: $\mathcal{O}(N \log N)$

a31d0a, 42 lines

```
typedef Point<double> P;
const long double eps = 1e-9, inf = 1e9;
struct Halfplane {
    // 'p' is a passing point of the line and 'pq' is the
        direction vector of the line.
    P p, pq; long double angle;
    Halfplane() {}
    Halfplane(const P& a, const P& b) : p(a), pq(b - a) { angle
        = atan2l(pq.y, pq.x); }
  bool operator < (const Halfplane& e) const { return angle < e
      .angle; }
    // Check if point 'r' is outside this half-plane. Every
        half-plane allows the region to the LEFT of its line.
    bool out(const P& r) { return pq.cross(r - p) < -eps; }
    // Intersection point of the lines of two half-planes. It
        is assumed they're never parallel.
    friend P inter(const Halfplane& s, const Halfplane& t) {
        long double alpha = (t.p - s.p).cross(t.pq) / s.pq.
            cross(t.pq);
        return s.p + (s.pq * alpha);}};
vector<P> hp_intersect(vector<Halfplane>& H) {
    P box[4] = {P(inf,inf), P(-inf,inf), P(-inf,-inf), P(inf,-
        inf)};
    for(int i = 0; i<4; i++) {
        Halfplane aux(box[i], box[(i+1) % 4]); H.push_back(aux)
            ;}
    sort(H.begin(), H.end());
    deque<Halfplane> dq;
    int len = 0;
    for(int i = 0; i < int(H.size()); i++) {
        while (len > 1 && H[i].out(inter(dq[len-1], dq[len-2]))
            ) {
            dq.pop_back(); --len;}
        while (len > 1 && H[i].out(inter(dq[0], dq[1]))) {
            dq.pop_front(); --len;}
        if (len > 0 && fabsl(H[i].pq.cross(dq[len-1].pq)) < eps
            ) {
            if (H[i].pq.dot(dq[len-1].pq) < 0.0) return vector<
                P>();
            if (H[i].out(dq[len-1].p)) {
                dq.pop_back(); --len;}
            else continue;
        }
        dq.push_back(H[i]); ++len;}
    while (len > 2 && dq[0].out(inter(dq[len-1], dq[len-2]))) {
        dq.pop_back(); --len; }
    while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))) {
        dq.pop_front(); --len; }
    if (len < 3) return vector<P>();
    vector<P> ret(len);
    for(int i = 0; i+1 < len; i++) ret[i]=inter(dq[i], dq[i+1])
        ;
    ret.back() = inter(dq[len-1], dq[0]);
    return ret;}
```

Strings (11)

Trie.py

Description: size 10^6 for 10^5 words, $5*10^5$ seems to work too
Time: $\mathcal{O}(N)$

171a35, 16 lines

```
trie, j = [-1]*int(27*1e6), 27
#insert list of words
for w in words:
    i = 0
    for char in w:
        c = ord(char) - 97
        if trie[i+c]==-1: trie[i+c]=j; j+=27
        i = trie[i + c]
    trie[i + 26] = 1
#check if w in trie
i = 0
for char in w:
    c = ord(char) - 97
    i = trie[i + c]
    if i == -1:  break
    if trie[i + 26] == 1: pass# this is a valid word
```

Z.py

Description: Z[i] is length of the longest substr starting from str[i] which is also a prefix of str[0..n-1] (i + Z[i] == n) => prefix suffix Si recherche de motif, concaténer motif devant la string if i + Z[i] == n and n % (i+1) == 0 : period = max(period, i+1)
Time: $\mathcal{O}(N)$

f8780f, 5 lines

```
Z, l, r = [0] * n, 0, 0
for i in range(1, n):
    if i <= r: Z[i] = min(Z[i-1],r-i+1)
    while i+Z[i]<n and s[Z[i]]==s[i + Z[i]]: Z[i] += 1
    if i + Z[i] - 1 > r: l, r = i, i + Z[i] - 1
```

Manacher.py

Description: Preprocess the string to insert '#' between each character to handle even-length palindromes
Time: $\mathcal{O}(N)$

fd0df0, 14 lines

```
def longest_pal_substr(s):
    t = '#'.join('^{}$'.format(s))
    n = len(t); p = [0] * n
    center, right = 0, 0
    for i in range(1, n - 1):
        if i < right:
            mirror = 2 * center - i
            p[i] = min(right - i, p[mirror])
        while t[i+p[i]+1] == t[i-p[i]-1]: p[i] += 1
        if i + p[i] > right: center, right = i, i + p[i]
    max_len = max(p)
    center_idx = p.index(max_len)
    start_idx = (center_idx-max_len)>>1
    return s[start_idx:start_idx+max_len]
```

StringFactoring.py

Description: ababc -> 2(ab)c
Time: $\mathcal{O}(N^3)$

76994d, 18 lines

```
def prefix(s):
    n, pi = len(s), [0]*n
    for i in range(1,n):
        j = pi[i - 1]
        while j > 0 and s[i] != s[j]: j=pi[j-1]
        if s[i] == s[j]: j += 1
        pi[i] = j
    return pi[-1]
dp = [[701] * (n+1) for _ in range(n+1)]
for l in range(1, n + 1):
    for i in range(n - l + 1):
        j = i + l - 1
        if i == j: dp[i][j] = 1; continue
        for k in range(i, j):
            dp[i][j] = min(dp[i][j],dp[i][k]+dp[k+1][j])
        pref = prefix(s[i : i + l])
        if l % (l - pref) == 0:
            dp[i][j] = min(dp[i][j], dp[i][i+l-pref-1])
```

RollingHash.py

Description: Rolling hashes, use M=1e9+7, P1=31, P2=37
Time: $\mathcal{O}(N)$

d3e8e3, 9 lines

```
def create_string_hash(string, n, p, mod):
    hash_p, powers = [0] * (n + 1), [1] * (n + 1)
    for i in range(1, n + 1): powers[i] = (powers[i - 1] * p) %
        mod
    for i in range(n): hash_p[i+1]=(hash_p[i]*p+(ord(string[i])
        -ord("a")+1))%mod
    return hash_p, powers

def get_hash(hash_p, powers, mod, i, j):
    # Returns the hash of the substring s[i:j] (j excluded)
    return (hash_p[j] - hash_p[i] * powers[j-i]) % mod
```

SuffixArray.py

Description: Compute SuffixArray and LCP Suffix Array and LCP (longest common prefix), LCS (longest common substr), LRS (longest repeadted substr), Unique Substrings
Time: $\mathcal{O}(N)$

6181a8, 114 lines

```
def gen_sufarr_lcp(words):
    def radix_sort(p_, c_):
        count = [0 for _ in range(len(p_))]
        for x in p_: count[c_[x]] += 1
        new_p = p_.copy()
        pos = [0 for _ in range(len(p_))]
        for i in range(1, len(p_)): pos[i]=pos[i-1]+count[i-1]
        for x in p_: new_p[pos[c_[x]]]=x; pos[c_[x]]+=1
        return new_p
    s_, sep, sidx = [], 0, []
    for idx, word in enumerate(words):
        for char in word:
            s_.append(len(words)+ord(char)); sidx.append(idx)
        s_.append(sep); sidx.append(-1); sep += 1
    n = len(s_)
    a = [(s_[i], i) for i in range(n)]
    a.sort(key=lambda x: x[0])
    sufarr = [a[i][1] for i in range(n)]
    c = [0] * n; c[sufarr[0]] = 0
    for i in range(1, n):
```

```
        if a[i][0] == a[i-1][0]: c[sufarr[i]]=c[sufarr[i-1]]
        else: c[sufarr[i]] = c[sufarr[i-1]]+1
    k = 1
    while (1 << k) <= 2*n:
        sufarr = [(sufarr[i]-(1<<(k-1)))%n for i in range(n)]
        sufarr = radix_sort(sufarr, c)
        c_new = [0] * n
        prev = (c[sufarr[0]], c[(sufarr[0]+(1<<(k-1)))%n])
        for i in range(1, n):
            curr = (c[sufarr[i]], c[(sufarr[i]+(1<<(k-1)))%n])
            if prev == curr: c_new[sufarr[i]]=c_new[sufarr[i
                -1]]
            else: c_new[sufarr[i]]=c_new[sufarr[i-1]]+1
            prev = curr
        c = c_new; k += 1
        if c_new[sufarr[n-1]] == n - 1: break
    # sufarr is done generating
    lcp, k = [0] * (n-1), 0
    for i in range(n-1):
        pi = c[i]; j = sufarr[pi - 1]
        while s_[i+k] == s_[j+k]: k += 1
        lcp[pi-1] = k; k = max(k-1, 0)
    suf2s = [sidx[sufarr[i]] for i in range(len(words), n)]
    sufarr = sufarr[len(words):]; lcp = lcp[len(words)-1:]
    return sufarr, lcp, suf2s, s_

def longest_common_substring(S, T):
    L,A=len(S), S+"!"+T
    sa = suffix_array(A)
    lcp = lcp_array(A, sa)
    ans, best = [0]*4, 0
    for i in range(len(A)-1):
        if (sa[i]<L)==(sa[i+1]<L): continue
        if lcp[i]>best:
            best=lcp[i]
            if sa[i]<L:
                ans=[sa[i], sa[i]+best,
                     sa[i+1]-L-1, sa[i+1]-L-1+best]
            else:
                ans=[sa[i+1], sa[i+1]+best,
                     sa[i]-L-1, sa[i]-L-1+best]
    print(*ans)

def longest_common_substring(strings, k):
    # longest common substring such that k strings shares it
    n = len(strings)
    if n == 1:return len(strings[0]),[strings[0]]
    assert k > 1
    sufarr, lcp, suf2s, text = gen_sufarr_lcp(strings)
    st = SparseTable(lcp)
    l, sn = 0, [0] * n
    sn[suf2s[0]] += 1
    zeros, lcs, start_idx = n-1, 0, []
    for r in range(1, len(lcp)):
        if sn[suf2s[r]] == 0:zeros -= 1
        sn[suf2s[r]] += 1
        while (n - zeros) >= k:
            minimum = st.prod(l+1, r+1)
            if minimum > lcs:lcs = minimum; start_idx=[sufarr[r
                ]]
            elif minimum == lcs:start_idx.append(sufarr[r])
            sn[suf2s[l]] -= 1
            if sn[suf2s[l]] == 0:zeros += 1
            l += 1
    if lcs == 0: return 0, []
    substrings = sorted(''.join(chr(char - n) for char in text[
        st:st+lcs])
                        for st in start_idx)
    last, res = None, []
    for subs in substrings:
        if subs != last: res.append(subs)
        last = subs
    return lcs, res

def longest_repeated_substring(s):
    # overlap is OK : 'abrabra' -> 'abra'
    sufarr, lcp, _, _ = gen_sufarr_lcp([s])
    lrs = max(lcp)
    if lrs == 0: return 0, []
    substrings = []
    for i in range(len(lcp)):
        if lcp[i] == lrs:substrings.append(s[sufarr[i]:sufarr[i
            ]+lrs])
    substrings.sort()
    last, res = None, []
    for subs in substrings:
        if subs != last: res.append(subs)
        last = subs
    return lrs, res

def count_unique_substrings(s):
    _, lcp, _, _ = gen_sufarr_lcp([s])
    # return len(set(substrings of s))
    return (len(s)*(len(s)+1)>>1) - sum(lcp)

def count_repeated_substrings(s):
    _, lcp, _, _ = gen_sufarr_lcp([s])
    return sum(max(lcp[i] - lcp[i-1], 0) for i in range(1, len(
        lcp)))
```

MinRotation.py

Description: Duval
Time: $\mathcal{O}(N)$

0beec2, 9 lines

```
def minRotation(s):
    a,N=0,len(s)
    s+=s
    for b in range(N):
        for k in range(N):
            if a+k==b or s[a+k]<s[b+k]:
                b+=max(0,k-1);break
            if s[a+k]>s[b+k]:a=b;break
    return a
```

RK2d.h

Description: RabinKarp rolling hashes in 2d
Usage: RK2d(pattern, masterpiece, 31)
Time: $\mathcal{O}(N \cdot M)$

b5f2a2, 29 lines

```
int RK2d(vector<string>&p, vector<string>&m, ll A) {
    int hp=sz(p),wp=sz(p[0]),hm=sz(m),wm=sz(m[0]);
    ll iA=Mod(A).invert(A).x;
    Mod rh=0;
    for(int i=0;i<hp;i++)
        for(int j=0;j<wp;j++)
            rh=rh+Mod(p[i][j])*(Mod(A)^(wp*i+j));
    vector<vector<Mod>>h1d(hm,vector<Mod>());
    for(int i=0;i<hm;i++) {
        Mod ch=0;
        for(int j=0;j<wp;j++)ch=ch+Mod(m[i][j])*(Mod(A)^j);
        h1d[i].push_back(ch);
        for(int j=wp;j<wm;j++) {
            ch=(ch-Mod(m[i][j-wp]))*Mod(iA)+\
            Mod(m[i][j])*(Mod(A)^(wp-1));
            h1d[i].push_back(ch);}}
    vector<vector<Mod>>h2d(hm-hp+1,vector<Mod>(wm-wp+1,Mod(0)))
        ;
    for(int j=wp;j<=wm;j++) {
        Mod ch=0;
        for(int i=0;i<hp;i++)ch=ch+h1d[i][j-wp]*(Mod(A)^(i*wp))
            ;
        h2d[0][j-wp]=ch;}
    for(int i=hp;i<hm;i++)for(int j=wp;j<=wm;j++)
        h2d[i-hp+1][j-wp]=(h2d[i-hp][j-wp]-h1d[i-hp][j-wp])*\
        (Mod(iA)^wp)+h1d[i][j-wp]*(Mod(A)^(wp*(hp-1)));
    int mt=0;
    for(int i=0;i<=hm-hp;i++)
        for(int j=0;j<=wm-wp;j++)if(h2d[i][j]==rh)mt++;
    return mt;
}
```

AhoCorasick.h

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

f35677, 66 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'}; // change this!
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1, nmatches = 0;
    Node(int v) { memset(next, v, sizeof(next)); }
  };
  vector<Node> N;
  vi backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    for (char c : s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) : N(1, -1) {
    rep(i,0,sz(pat)) insert(pat[i], i);
    N[0].back = sz(N);
    N.emplace_back(0);

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      rep(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[ed].start])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  vi find(string word) {
```

```
    int n = 0;
    vi res; // ll count = 0;
    for (char c : word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vi> findAll(vector<string>& pat, string word) {
    vi r = find(word);
    vector<vi> res(sz(word));
    rep(i,0,sz(word)) {
      int ind = r[i];
      while (ind != -1) {
        res[i - sz(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

WPM.h

Description: Wildcard Pattern Matching Given strings S and T consisting of lowercase English letters and asterisks (*) wildcards, returns W where W[i] = 1 iff $S[i : i + |T|]$ and T are matched, 0 otherwise S : abc*b*a***a, T : *b*a, W : 10111011
Time: approx. $\mathcal{O}(N)$

<tr2/dynamic_bitset> af2bfd, 24 lines

```
using namespace std::tr2;
void WPM(string s, string t){
    int n = s.length(), m = t.length();
    vector<dynamic_bitset<>> b(26, dynamic_bitset<>(n));
    for(int i = 0; i < n; i ++){
        if(s[i] != '*')
            b[s[i] - 'a'][i] = true;
        else
            for(int c = 0; c < 26; c ++)
                b[c][i] = true;
    }
    vector<int> shift(26, 0);
    dynamic_bitset<> good(n);
    good.set();
    for(int i = 0; i < m; i ++){
        if(t[i] == '*') continue;
        b[t[i] - 'a'] >>= (i - shift[t[i] - 'a']);
        shift[t[i] - 'a'] = i;
        good &= (b[t[i] - 'a']);
    }
    for(int i = 0; i < n - m + 1; i ++)
        cout << good[i];
    cout << endl;
}
```

Various (12)

Grundy.py

Description: Computes the Minimum EXcluded for a given state. To convert to iterative version
Time: $\mathcal{O}(N \cdot M)$, N number of states, M average number of moves per state.

c4e4e9, 14 lines

```
n = 1000
mex = [None] * (n+1)
mex[0] = 0; mex[1] = 1
def grundyValue(n):
    if mex[n] is not None:
        return mex[n]
    excluded = {grundyValue(n-2)}
    for i in range(2, n):
        excluded.add(grundyValue(i-2) ^ grundyValue(n-i-1))
    res = 0
    while res in excluded:
        res += 1
    mex[n] = res
    return res
```

Floyd.py

Description: Cycle detection
Time: $\mathcal{O}(N + M)$

f8986c, 13 lines

```
def floyd(x0, f):
    t, h = f(x0), f(f(x0))
    while t != h:
        t, h = f(t), f(f(h))
    start, t = 0, x0
    while t != h:
        t, h = f(t), f(h)
        start += 1
    period, h = 1, f(t)
    while t != h:
        h = f(h)
        period += 1
    return start, period
```

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

edce47, 22 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}
void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

9e9d8d, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b - a))$

9155b4, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

continuousSearch.py

aac02c, 13 lines

```
def continuousTernSearch(a, b, f):
    while b-a > 1e-9:
        mid1 = (2*a+b)/3
        mid2 = (a+2*b)/3
        if f(mid1) < f(mid2): b = mid2
        else: a = mid1
    return a
def continuousBinSearch(a, b, f):
    for _ in range(64):
        mid = (a+b)/2
        if f(mid): b = mid
        else: a = mid
    return a
```

parallelBinarySearch.h

e2912a, 26 lines

```
vpii qq(q);
vi ql(q, 0), qr(q, m);
for(int i=0; i<q; i++){
    int u, v; cin >> u >> v;
    qq[i] = {--u, --v};
}
while(1){
    vvi queries(m+1);
    bool added = false;
    for(int i=0; i<q; i++){
        if(ql[i] < qr[i]){
            queries[(ql[i]+qr[i])>>1].pb(i);
            added = true;
        }
    }
    if(!added) break;
```

```
    UF dsu(n);
    for(int i=0; i<m; i++){
        for(int qi: queries[i]){
            auto[x, y] = qq[qi];
            if(dsu.find(x) == dsu.find(y)) qr[qi] = i;
            else ql[qi] = i+1;
        }
        dsu.join(edges[i].first, edges[i].second);
    }
}
```

12.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.