Sets

Georg Cantor:

"*We understand a* "*set*" *as a combination in some whole M of definite well distinctive things m from our observation or thought. These things are called* "*elements*" *of the set M.*"

Informally, **a set is a collection of well discernible objects, which is itself apprehended as a single object**.

$A = \{a, b\}$

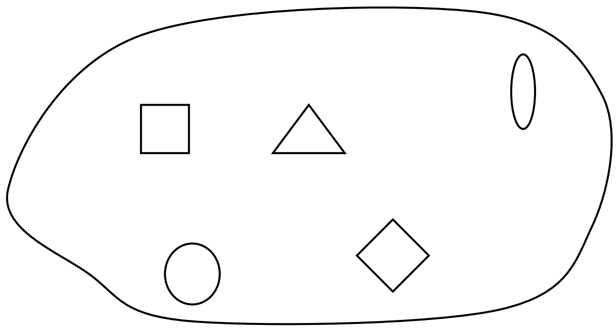
$a \in C$ the element *a* belongs to the set *C*

$d \notin C$ *d* does not belong to *C*

$A \subseteq C$ *A* is a subset of *C* (every element in *A* is in *C*)

$A \subset C$ A is a proper subset of *C* (every element in *A* is in *C* and there is an element in *C* that is not in *A*)

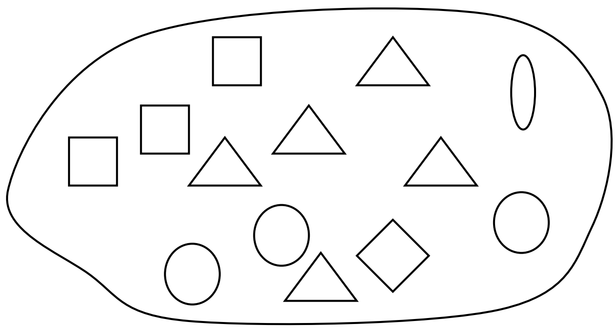
Sets



{1, 2, 3}

{a, b, c, d, e}

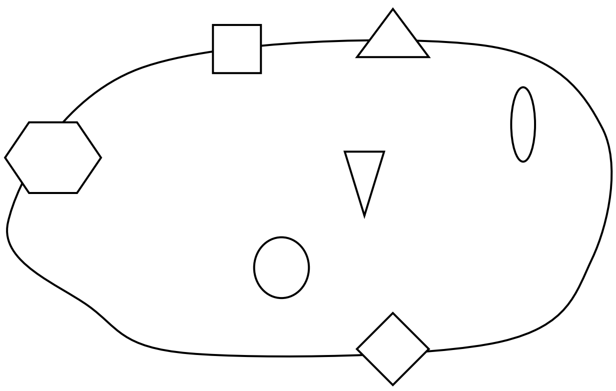
Multisets



{1, 1, 2, 2, 2 3, 3, 3, 3, 3}

{a, b, b, c, c, c, d, e, e}

Fuzzy sets



{(1, 0.5), (2, 0.4), (3, 1)}

{(a, 0.1), (b, 0.7), (c, 0.5)}

Being very important, the concept of *multiset* also called *m-set* has been rediscovered many times. Consequently, it has existed under a variety of names: a bag, heap, bunch, sample, weighted set, occurrence set, and fireset (finitely repeated element set). Knuth (1998) attributes the first study of multisets to the Indian mathematician Bhascara Acharya (circa 1150). The term *multiset*, according to Knuth (1981), was suggested by N.G. de Bruijn in private communication.

There are different notations for multisets. For instance, all following expressions denote one and the same multiset:

$$M = \{ a, b, b, b, c, c, c, c, c \}$$

$$M = [a, b, b, b, c, c, c, c, c]$$

$$M = [[a, b, b, b, c, c, c, c, c]]$$

$$M = [a, b, c]_{1,3,5}$$

$$M = [a^1, b^3, c^5]$$

$$M = [a1, b3, c5]$$

$$M = [a] + 3[b] + 5[c]$$

Definition 1. *A multiset is a collection that is like a set but can include identical or indistinguishable elements*.

$$M = (X, f, N)$$

Definition 2 (Zadeh, 1965). A *fuzzy set A in a set U* is defined by its a *membership function* μ_A: $U \to [0,1]$ of A where $\mu_A(x)$ is interpreted as the *degree of membership* in A of an element $x \in U$.

$$A = (U, \mu_A, [0,1])$$

Named sets as the most encompassing and fundamental mathematical construction include ordinary sets and all their generalizations, such as fuzzy sets and multisets, providing unified foundations for the whole mathematics. Functions, mappings, operations, relations, graphs, multigraphs, operators, fiber bundles, morphisms, functors, enumerations (numberings) and many other mathematical structures are named sets. Moreover, all mathematical structures are built of named sets.

A *named set* (also called a *fundamental triad*) has the following graphic representation

$$\text{Entity 1} \xrightarrow{\text{connection}} \text{Entity 2} \qquad (1)$$

or

$$\text{Essence 1} \xrightarrow{\text{correspondence}} \text{Essence 2} \qquad (2)$$

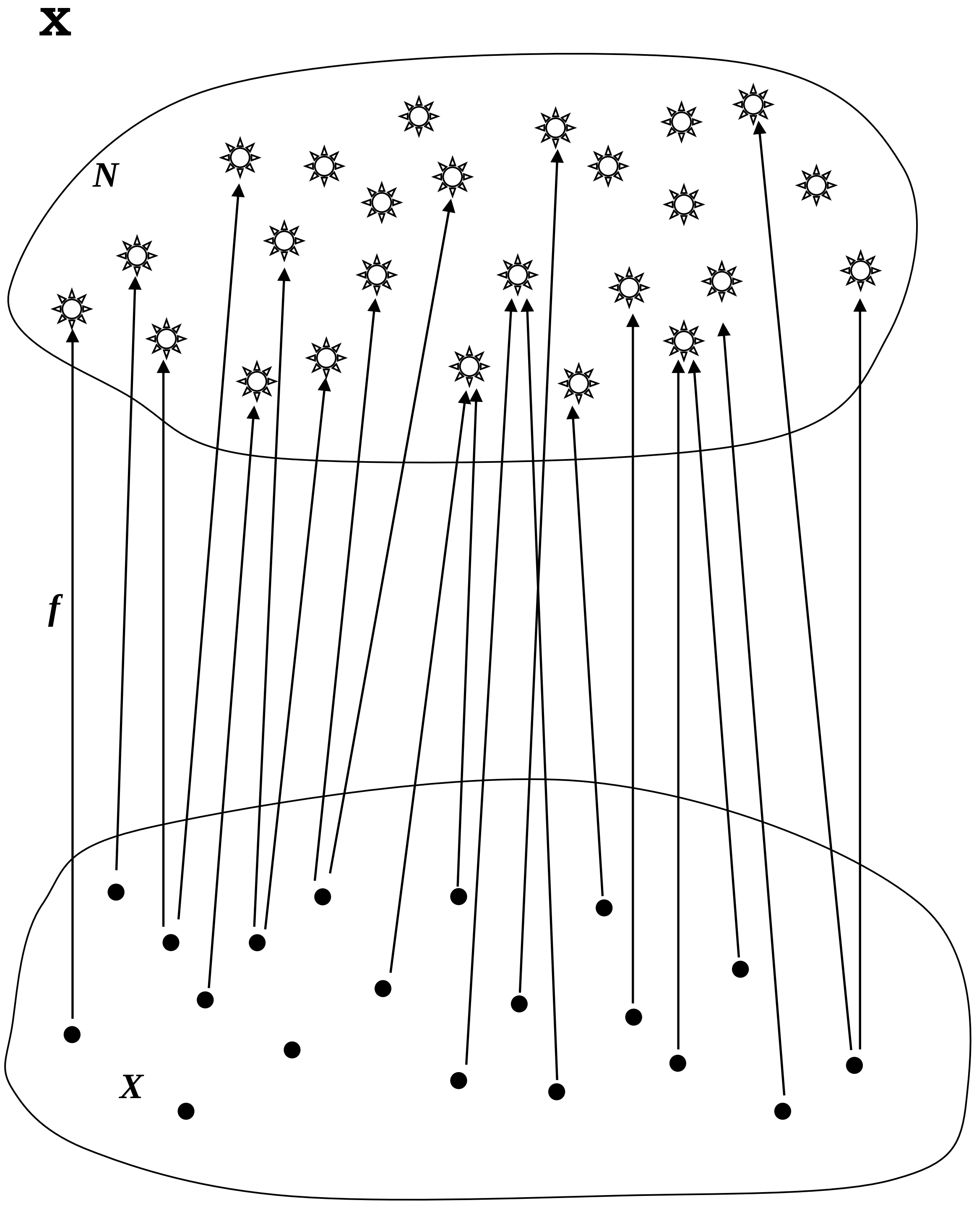
In the fundamental triad (named set) (1) or (2), Entity 1 (Essence 1) is called the *support*, the Entity 2 (Essence 2) is called the *reflector* (also called the *set* or *component of names*) and the connection (correspondence) between Entity 1 (Essence 1) and connection (correspondence) is called the *reflection* (also called the *naming correspondence*) of the fundamental triad (1) (respectively, (2)).

In the symbolic form, a *named set* (also called a *fundamental triad*) **X** is a triad (X, f, I) where X is the *support* of **X** and is denoted by S(**X**), I is the *component of names* (also called *set of names* or *reflector*) of **X** and is denoted by N(**X**), and f is the *naming correspondence* (also called *reflection*) of the named set **X** and is denoted by n(**X**). The most popular type of named sets is a named set **X** = (X, f, I) in which X and I are sets and f consists of connections between their elements. When these connections are set theoretical, i.e., each connection is represented by a pair (x, a) where x is an element from X and a is its name from I, we have a *set theoretical named set*, which is binary relation.

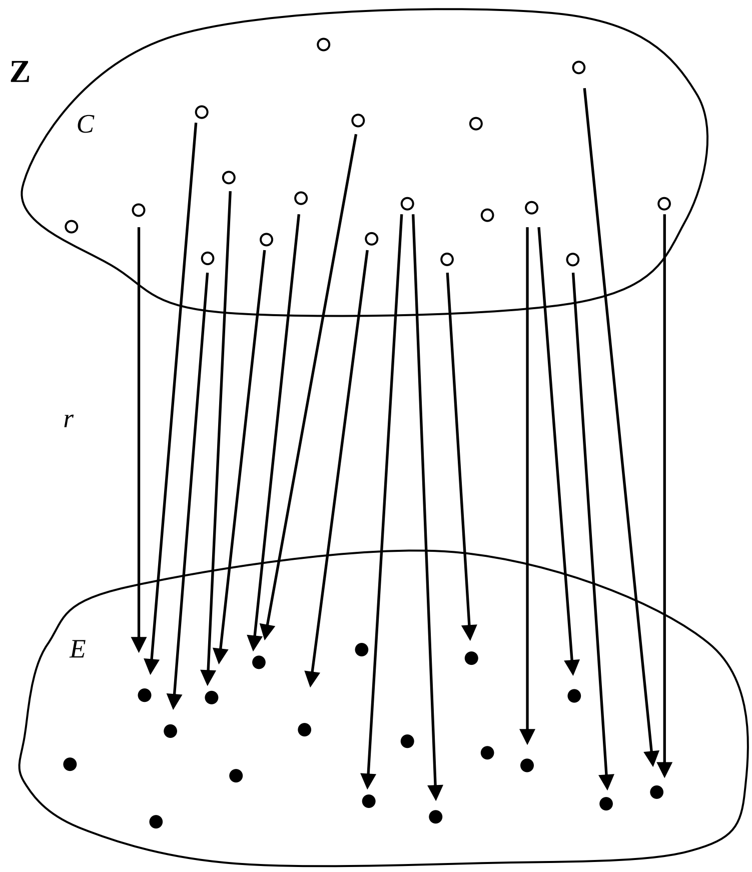
Two model examples of a named set:

X is a group of people, N is a set of their names and f is the connection between people and their names.

X is a collection of Internet resources, N is a set of their Internet names and f is the connection between resources and their names.



A set-theoretical named set $\mathbf{X} = (X, f, N)$



A set-theoretical named set $\mathbf{Z} = (C, r, E)$

Two model examples of a named set:

X is a group of people, *N* is a set of their names and *f* is the connection between people and their names.

X is a collection of Internet resources, *N* is a set of their Internet names and *f* is the connection between resources and their names.

Any operation of naming creates a named set.

If there are names in some system, then there are named sets in or related to this system.

Applications of named sets in computer science, computer and network technology and programming

1. Naming systems and in particular, intentional naming systems

INS (Intentional Naming System) is a new naming system intended for naming and discovering a variety of resources in future networks of devices and services.

2. Naming schemas and in particular, intentional naming schemas

3. The Domain Name System (DNS)

4. Layered Naming Architecture for the Internet

5. Location-independent naming

6. Labeled database management

7. Naming and binding of objects

8. Named graphs

9. Named parameters

10. Content routers for high-speed name-based operations

11. Name-oriented networking or named data networking (NDN)

A. Naming is a fundamental issue of growing importance in distributed systems. As the number of directly accessible systems and resources grows, it becomes increasingly difficult to discover the (names of) objects of interest.

   In an *intentional naming* and *resolution architecture*, applications describe their intent and specify *what* they are looking for but not *where* it is situated. This shifts the burden of resolving ‘what is desired’ to ‘where it is’ from the user to the network infrastructure. It also allows applications to communicate seamlessly with end-nodes, despite changes in the mapping from name to end-node addresses during the session.

B. Some important results in theoretical computer science are based on *nameability* of properties within a given naming scheme for partial functions. Such a naming scheme can, but need not be, an executable formalism. A programming language is an example of an executable naming scheme, where the program text names the partial function it implements. Halting is an example of a property that is not nameable in that naming scheme.

C. Many of the Internet’s problems are related to names. There are many empirical ideas of the further development of naming on the Internet. However, there are no theoretical foundations for efficient development of such systems.

D. Naming is one of the most important and most frequently overlooked areas of computer science.

E. Objects, patterns, Wiki and XP, all are systems of names.

F. Network developers have been considering the design of naming systems for almost 25 years, but still do not give naming systems their due in the engineering of network systems.

G. In computer programming, named parameters, pass-by-name, or keyword arguments refer to a computer language's support for function calls that clearly state the name of each parameter within the function call.

H. Named Data Networking (NDN) (related to Content-Centric Networking (CCN), content-based networking, data-oriented networking or information-centric networking) is a Future Internet Architecture inspired by years of empirical research into network usage and a growing awareness of unsolved problems in contemporary internet architectures.