CS 239 Quantum Programming Winter 2021

Midterm Exam

Feb 9, 2021: 90 minutes. On the first page you submit, write your name and UCLA id and whether you took the exam at 2pm or 9pm PST. Submit a single pdf file to CCLE.

Each of questions 1–10 is worth 5 points, and each of questions 11-15 is worth 10 points.

1. After measurement of a qubit, can we continue quantum computing with that qubit? Justify your answer.

2. When we teleport a single qubit, what do we physically transmit?

3. If f is either a constant function or a balanced function from bitstrings of length 8 to bits, how many times will we have to run the Deutsch-Jozsa circuit for f to determine whether f is constant or balanced? Justify your answer.

4. If f is a function from bitstrings of length 16 to bitstrings of length 16, how many times will we have to run Simon's circuit for f to determine the solution to Simon's problem with probability at least 98 percent? Justify your answer.

5. If f is a function from bits to bits, and $f(x) = x$, and we run the Bernstein-Vazirani circuit a single time, what is the probability that we will measure 1? Justify your answer.

6. Calculate the following tensor product.

$$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

7. Calculate the following tensor product.

$$\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \right) \otimes \left(\frac{4}{5}|0\rangle + \frac{3}{5}|1\rangle \right)$$

8. What state do we get if we apply $(X \otimes H)\ CNOT$ to the following state?

$$\frac{3}{5}\ |01\rangle + \frac{4}{5}\ |10\rangle$$

9. For the following state, suppose we measure the left qubit in the standard basis and get 0. Show the resulting state. Justify your answer.

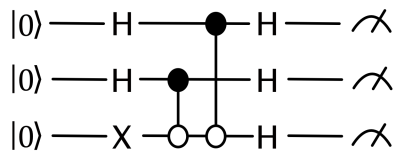
$$\frac{3}{5\sqrt{2}}|00\rangle - \frac{4}{5\sqrt{2}}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$$

10. Consider the following state.

$$\frac{3}{5\sqrt{2}}|00\rangle + \frac{4}{5\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

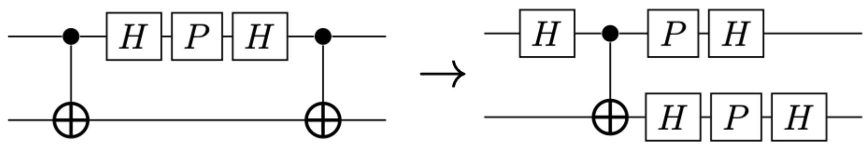
Suppose we measure the left qubit. What is the probability of getting 0, and if that happens, what is the state of the other qubit? Also, suppose we measure the right qubit. What is the probability of getting 1, and if that happens, what is the state of the other qubit?

11. Consider the following circuit with three qubits.



Suppose that at the end, we measure all three qubits in the standard basis. What is the probability that we will get 000 ? Justify your answer.

12. Let $P = R_z(\frac{\pi}{2})$. Phrase the following diagram as an equation and prove that it is correct, or show that it is wrong.



13. Show, step by step, that the Deutsch-Jozsa algorithm works for the case of f, where $f(0) = 1$ and $f(1) = 0$.

14. For the case of $n = 3$ and a function f where

$$\begin{array}{lllll} f(000) & = & f(111) & = & 110 \\ f(001) & = & f(110) & = & 101 \end{array} \qquad \begin{array}{lllll} f(100) & = & f(011) & = & 011 \\ f(101) & = & f(010) & = & 111 \end{array}$$

give two different examples of equations that the first step of Simon's algorithm may produce. Explain what those equations mean. Show how producing a sufficient number of equations can lead to solving Simon's problem. Justify your answer. What is the solution to Simon's problem in this case?

15. Show, step-by-step, that Grover's algorithm works for the case of 2 qubits and a function f where $f(10) = 1$ and $f(00) = f(01) = f(11) = 0$.