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Assignment in Quantum Computing
To Dr Norhan
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Deutsch–Jozsa Algorithm (3-Qubit) Using Qiskit

This experiment implements the **Deutsch–Jozsa algorithm** for a 3-qubit input function
f:{0,1}3→{0,1}f:{0,1}3→{0,1}.
The goal is to distinguish between:

- **Constant functions** (all outputs are either 0 or 1),
- **Balanced functions** (half the outputs are 0, half are 1),

using **only one oracle query**, demonstrating the power of quantum parallelism.

The experiment was performed on:

1. **Aer Simulator** (ideal, noiseless),
2. **A real IBM Quantum backend** using **Qiskit Runtime** and **SamplerV2**.

2. Oracle Design

2.1 Constant Oracle

A constant oracle implements:

$f(x)=c \text{ where } c\in\{0,1\}$$f(x)=c \text{ where } c\in\{0,1\}$

Quantum implementation:

- If $c=0$$c=0$: do nothing.
- If $c=1$$c=1$: apply **X** on the output ancilla qubit.

This ensures:

$|x\rangle|y\rangle\rightarrow|x\rangle|y\oplus c\rangle$$|x\rangle|y\rangle\rightarrow|x\rangle|y\oplus c\rangle$

2.2 Balanced Oracle – Parity (XOR) Function

The chosen balanced function was:

$f(x0,x1,x2)=x0\oplus x1\oplus x2$$f(x0,x1,x2)=x0\oplus x1\oplus x2$

This function is **balanced** because exactly half the 8 possible inputs produce 1.

Quantum implementation:

- Use **three CNOT gates**, each controlling the output qubit.

$$|x0x1x2\rangle|y\rangle\rightarrow|x0x1x2\rangle|y\oplus x0\oplus x1\oplus x2\rangle$$$$|x0x1x2\rangle|y\rangle\rightarrow|x0x1x2\rangle|y\oplus x0\oplus x1\oplus x2\rangle$$

3. Complete Deutsch–Jozsa Circuit

Steps:

1. Prepare ∣0⟩⊗3∣1⟩∣0⟩⊗3∣1⟩.
2. Apply Hadamards to all qubits.
3. Apply oracle (constant or balanced).
4. Apply Hadamards to **input qubits only**.
5. Measure the input register.

Expected outcomes:

- **Constant oracle → output = `000`**
- **Balanced oracle → output ≠ `000`**

4. Simulation Results (Aer Simulator)

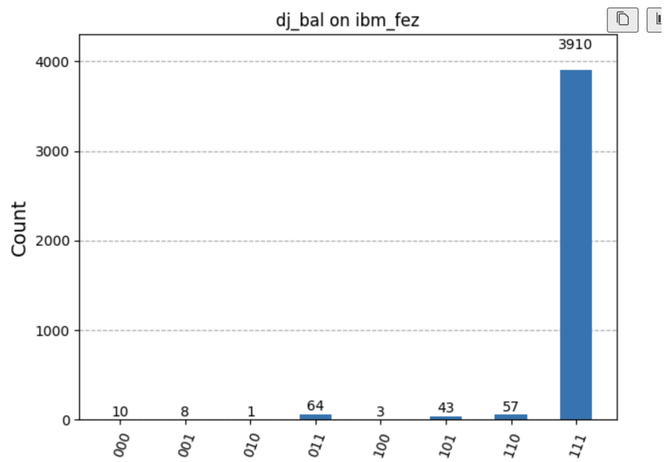
The simulator gives the ideal, noise-free output.

- **Constant oracle simulated counts:**
  - **Dominant output → 000**
- **Balanced oracle simulated counts:**
  - **000 suppressed → other bitstrings dominate**

6. Real Hardware Experiment

Real Hardware Measurement Counts

```
Submitting job to IBM Quantum…
Job ID: d4qv4rrher1c73bbni10
```



Interpretation:

The real backend produced a strong peak at 111, exactly matching the theoretical output of the balanced parity oracle.

Other states appear with low probability due to hardware noise.

Since 000 is almost absent, the hardware correctly identifies the function as balanced.