



**India's Most Comprehensive & the Most Relevant
Test Series designed according to the latest pattern of exams!**



JEE MAIN



JEE ADV.



BITSAT



WBJEE



MHT CET

and many more...

Click here to join Test Series for 2022

It's time for you to crack upcoming IIT JEE Main & Advanced and other competitive exams with India's Most Trusted Online Test Series. **Many questions at JEE Main 2021 were same/similar to the ones asked in our test series.** That's the power of our test series!

**Trusted by thousands of students
& their parents across the nation**

Our result in JEE Main 2021

150+

Got **99+ percentile** (overall)

301

Got **99+ percentile** in one or more subjects

85%

Improved their score by **25 percentile**

89%

Felt **overall confident** after the test series

Click here to join Test Series for 2022



FREE Question Bank & Previous Year Questions for



JEE MAIN



JEE ADV.



BITSAT



WBJEE



MHT CET

and many more...



Click on this button



😎 Why download MARKS?

- 📚 Chapter-wise PYQ of JEE Main, JEE Advanced, NEET, AIIMS, BITSAT, WBJEE, MHT CET etc.
- 📚 Chapter-wise NTA Abhyas questions
- ⌚ Daily practice challenge and goal completion
- 🔖 Bookmark important questions and add them to your notebooks
- ✍ Create unlimited Custom Tests

And all this for **FREE**. Yes, **FREE!** So what are you waiting for, download MARKS now.

4.8

Rating on Google Play

30,000+

Students using daily

1,00,000+

Questions available



Click on this button

INVERSE TRIGONOMETRIC FUNCTIONS

INVERSE TRIGONOMETRIC FUNCTIONS

If $\sin \theta = x$ is a trigonometrical equation, then the value of θ which satisfies this equation is denoted by $\sin^{-1} x$ and it is read as ‘sine inverse x ’. It is called inverse function of sine. Similarly inverse functions of other trigonometrical functions are defined. Hence inverse functions of trigonometrical functions are defined as follows-

$$\begin{array}{lll} \sin^{-1} x = \theta & \Leftrightarrow & \sin \theta = x \\ \cos^{-1} x = \theta & \Leftrightarrow & \cos \theta = x \\ \tan^{-1} x = \theta & \Leftrightarrow & \tan \theta = x \\ \cot^{-1} x = \theta & \Leftrightarrow & \cot \theta = x \\ \sec^{-1} x = \theta & \Leftrightarrow & \sec \theta = x \\ \operatorname{cosec}^{-1} x = \theta & \Leftrightarrow & \operatorname{cosec} \theta = x \end{array}$$

DOMAIN AND RANGE OF INVERSE FUNCTIONS

As we know that in direct trigonometric functions, we are given the angle and we calculate the trigonometric ratio (sine, cosine, etc.) or the value at that angle. Also to many values of the angle the value of trigonometric ratio is same e.g., $\tan \theta = 1$ for $\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$, etc.

Inverse trigonometry deals with obtaining the angles, given the value of a trigonometric ratio. In inverse trigonometry some restrictions have been imposed on the angles, and these are based on the principle values of the angles.

The inverse of sine function is defined as $\sin^{-1} x = \theta$ or $\arcsin x = \theta$, where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ e.g.,

$\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$ and nothing else, although $\sin \frac{5\pi}{6}, \sin \frac{13\pi}{6}$ etc. are also equal to $\frac{1}{2}$, $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = \frac{\pi}{3}$ only. Note that

$$\sin^{-1} x \neq \frac{1}{\sin x} \quad (\text{Q } \frac{1}{\sin x} = (\sin x)^{-1})$$

We list below the definitions of all inverse trigonometric functions with their respective domains and ranges.

Function	Domain (permitted value of x)	Range (permitted value of y)
(i) $y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$
(ii) $y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
(iii) $y = \tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$
(iv) $y = \cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$
(v) $y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2} \right) \cup \left(\frac{\pi}{2}, \pi \right]$
(vi) $y = \operatorname{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$ [1]	$\left[-\frac{\pi}{2}, 0 \right) \cup \left(0, \frac{\pi}{2} \right]$

e.g., $\cos^{-1} \left(-\frac{1}{2} \right) = \frac{2\pi}{3}$; $\tan^{-1} (-1) = -\frac{\pi}{4}$; $\text{cosec}^{-1} (2) = \frac{\pi}{6}$; $\sec^{-1}(-1) = \pi$; $\cot^{-1} (-\sqrt{3}) = \frac{5\pi}{6}$, etc.

PROPERTIES OF INVERSE FUNCTIONS

(i)	$\sin(\sin^{-1} x) = x$	$-1 \leq x \leq 1$
	$\cos(\cos^{-1} x) = x$	$-1 \leq x \leq 1$
	$\tan(\tan^{-1} x) = x$	$-\infty < x < \infty$
	$\cot(\cot^{-1} x) = x$	$-\infty < x < \infty$
	$\sec(\sec^{-1} x) = x$	$x \leq -1 \text{ or } x \geq 1$
	$\text{cosec}(\text{cosec}^{-1} x) = x$	$x \leq -1 \text{ or } x \geq 1$
(ii)	$\sin^{-1}(\sin \theta) = \theta$	only if $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
	$\cos^{-1}(\cos \theta) = \theta$	only if $0 \leq \theta \leq \pi$
	$\tan^{-1}(\tan \theta) = \theta$	only if $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
	$\cot^{-1}(\cot \theta) = \theta$	only if $0 < \theta < \pi$
	$\sec^{-1}(\sec \theta) = \theta$	only if $0 \leq \theta < \frac{\pi}{2} \text{ or } \frac{\pi}{2} < \theta \leq \pi$
	$\text{cosec}^{-1}(\text{cosec} \theta) = \theta$	only if $-\frac{\pi}{2} \leq \theta < 0 \text{ or } 0 < \theta \leq \frac{\pi}{2}$
	e.g., $\tan^{-1} \left(\tan \frac{5\pi}{6} \right) \neq \frac{5\pi}{6}$	
	$\tan^{-1} \left(\tan \frac{5\pi}{6} \right) = \tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) = -\frac{\pi}{6}$	
(iii)	$\sin^{-1}(-x) = -\sin^{-1} x$	$-1 \leq x \leq 1$
	$\cos^{-1}(-x) = \pi - \cos^{-1} x$	$-1 \leq x \leq 1$
	$\tan^{-1}(-x) = -\tan^{-1} x$	$-\infty < x < \infty$
	$\cot^{-1}(-x) = \pi - \cot^{-1} x$	$-\infty < x < \infty$
(iv)	$\sin^{-1} x = \text{cosec}^{-1} \left(\frac{1}{x} \right)$	$-1 \leq x < 0 \text{ or } 0 < x \leq 1$
	$\cos^{-1} x = \sec^{-1} \left(\frac{1}{x} \right)$	$-1 \leq x < 0 \text{ or } 0 < x \leq 1$
	$\tan^{-1} x = \cot^{-1} \left(\frac{1}{x} \right)$	only if $x > 0$

because range of these two functions are different.

$$\text{If } x < 0, \tan^{-1} x = -\pi + \cot^{-1} \left(\frac{1}{x} \right)$$

$$(v) \quad \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad -1 \leq x \leq 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \quad -\infty < x < \infty$$

$$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2} \quad x \leq -1 \text{ or } x \geq 1$$

SUM AND DIFFERENCE FORMULAE

$$(i) \quad \sin^{-1} x + \sin^{-1} y = \sin^{-1} (x \sqrt{1-y^2} + y \sqrt{1-x^2}) \quad \text{if } xy < 0 \text{ or } x^2 + y^2 \leq 1$$

$$= \begin{cases} \pi - \sin^{-1}(x \sqrt{1-y^2} - y \sqrt{1-x^2}), & \text{if } x > 0, y > 0, x^2 + y^2 > 1 \\ -\pi - \sin^{-1}(x \sqrt{1-y^2} + y \sqrt{1-x^2}), & \text{if } x < 0, y < 0 \text{ and } x^2 + y^2 > 1 \end{cases}$$

$$\sin^{-1} x - \sin^{-1} y = \sin^{-1} (x \sqrt{1-y^2} - y \sqrt{1-x^2}) \quad \text{if } xy \geq 0 \text{ and } x^2 + y^2 \leq 1$$

$$(ii) \quad \cos^{-1} x + \cos^{-1} y = \cos^{-1} (xy - \sqrt{1-x^2} \sqrt{1-y^2}), \quad \text{if } x + y \neq 0$$

$$= 2\pi - \cos^{-1} (xy - \sqrt{1-x^2} \sqrt{1-y^2}), \quad \text{if } x + y < 0$$

$$(iii) \quad \cos^{-1} x - \cos^{-1} y = \begin{cases} \cos^{-1}(xy + \sqrt{1-x^2} \sqrt{1-y^2}), & \text{if } x \geq 0, y \geq 0, x \leq y \\ -\cos^{-1}(xy + \sqrt{1-x^2} \sqrt{1-y^2}), & \text{if } x \geq 0, y \geq 0, x > y \end{cases}$$

$$(iv) \quad \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \quad \text{if } x, y > 0 \text{ and } xy < 1$$

$$= \pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right) \quad \text{if } x, y > 0 \text{ and } xy > 1$$

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right) \quad \text{if } x, y > 0$$

$$(v) \quad \tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \left[\frac{x+y+z-xyz}{1-xy-yz-zx} \right]$$

SOME IMPORTANT RESULTS

$$(vi) \quad 2\sin^{-1} x = \sin^{-1} 2x \sqrt{1-x^2}$$

$$(vii) \quad 2\cos^{-1} x = \cos^{-1} (2x^2 - 1)$$

$$(viii) \quad 2\tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2} = \sin^{-1} \frac{2x}{1+x^2} = \cos^{-1} \frac{1-x^2}{1+x^2}$$

$$(ix) \quad 3\sin^{-1} x = \sin^{-1} (3x - 4x^3)$$

$$(x) \quad 3\cos^{-1} x = \cos^{-1} (4x^3 - 3x)$$

$$(xi) \quad 3\tan^{-1} x = \tan^{-1} \frac{3x-x^3}{1-3x^2}$$

MISCELLANEOUS RESULTS

$$(i) \quad \tan^{-1} \left[\frac{x}{\sqrt{a^2 - x^2}} \right] = \sin^{-1} \left(\frac{x}{a} \right)$$

$$(ii) \quad \tan^{-1} \left[\frac{3a^2x - x^3}{a(a^2 - 3x^2)} \right] = 3 \tan^{-1} \left(\frac{x}{a} \right)$$

$$(iii) \quad \tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$$

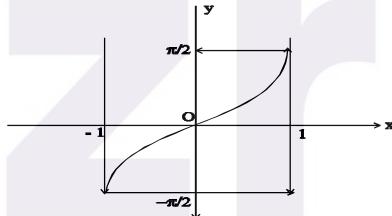
$$(iv) \quad \sin^{-1}(x) = \cos^{-1}(\sqrt{1-x^2}) = \tan^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) = \cot^{-1} \left(\frac{\sqrt{1-x^2}}{x} \right) = \sec^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right) = \cosec^{-1} \left(\frac{1}{x} \right)$$

$$(v) \quad \cos^{-1} x = \sin^{-1} \left(\sqrt{1-x^2} \right) = \tan^{-1} \left(\frac{\sqrt{1-x^2}}{x} \right) = \cot^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) = \sec^{-1} \left(\frac{1}{x} \right) = \cosec^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right)$$

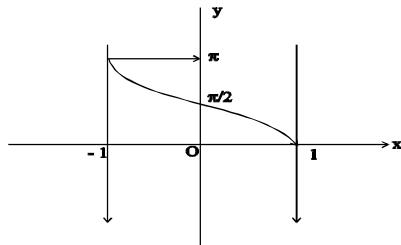
$$(vi) \quad \tan^{-1} x = \sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right) = \cos^{-1} \left(\frac{1}{\sqrt{1+x^2}} \right) = \cot^{-1} \left(\frac{1}{x} \right) = \sec^{-1} \left(\sqrt{1+x^2} \right) = \cosec^{-1} \left(\frac{\sqrt{1+x^2}}{x} \right)$$

GRAPHS

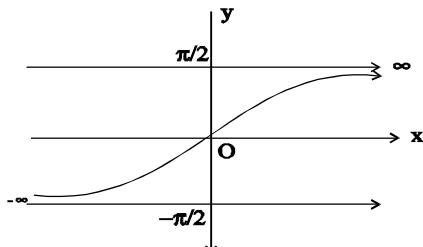
$$(1) \quad y = \sin^{-1} x, |x| \leq 1, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$



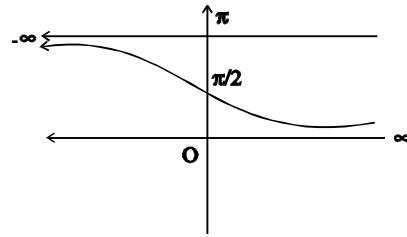
$$(2) \quad y = \cos^{-1} x, |x| \leq 1, y \in [0, \pi]$$



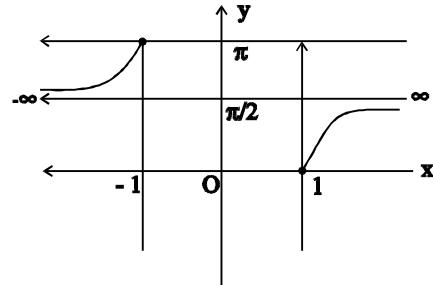
$$(3) \quad y = \tan^{-1} x, x \in \mathbb{R}, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$



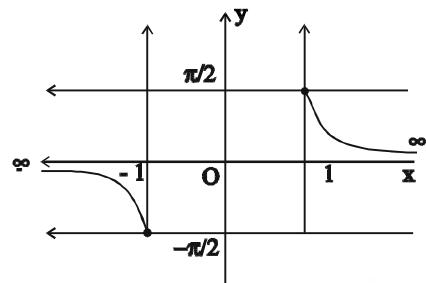
(4) $y = \cot^{-1} x, x \in \mathbb{R}, y \in [0, \pi]$



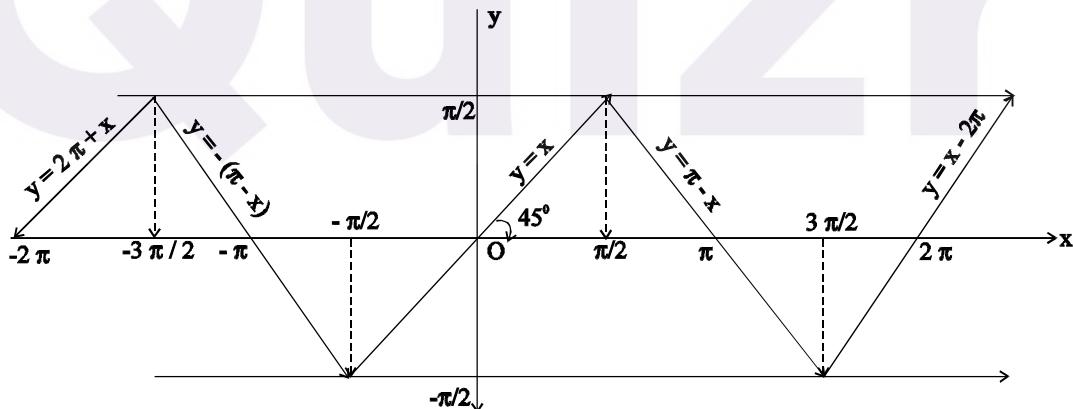
(5) $y = \sec^{-1} x, |x| \geq 1, y \in [0, \frac{\pi}{2}] \cup [\frac{\pi}{2}, \pi]$



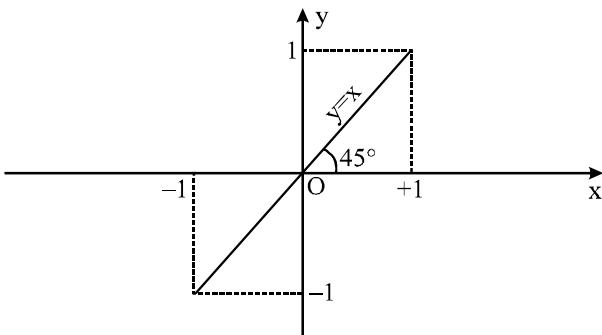
(6) $y = \operatorname{cosec}^{-1} x, |x| \geq 1, y \in [-\frac{\pi}{2}, 0] \cup [0, \frac{\pi}{2}]$



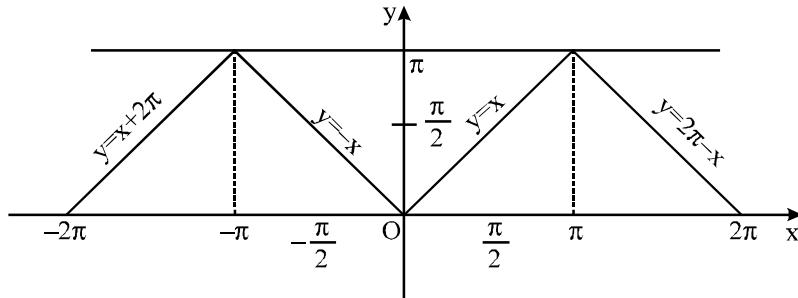
(i) $y = \sin^{-1}(\sin x) = x, x \in \mathbb{R}, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], y$ is periodic with period 2π



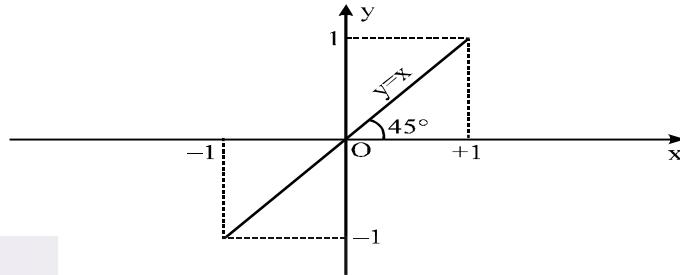
(ii) $y = \sin(\sin^{-1} x) = x, x \in [-1, 1], y \in [-1, 1]$



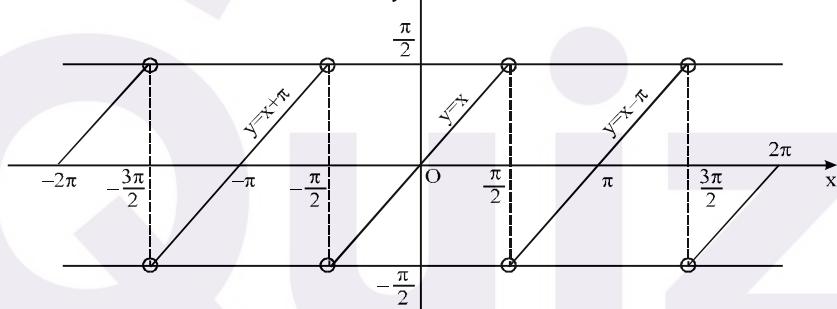
(i) $y = \cos^{-1}(\cos x) = x$, $x \in \mathbb{R}$, $y \in [0, \pi]$, y is periodic with period 2π



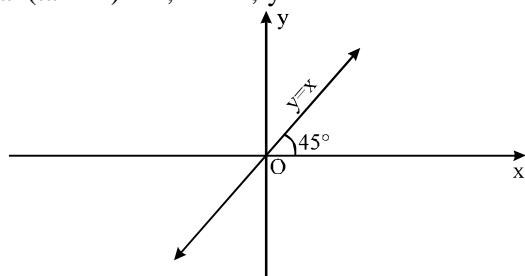
(ii) $y = \cos(\cos^{-1} x) = x$, $x \in [-1, 1]$, $y \in [-1, 1]$



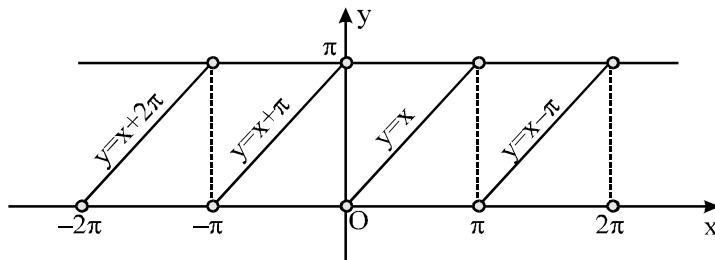
4.3 (i) $y = \tan^{-1}(\tan x) = x$, $x \in \mathbb{R} - \left\{ (2n-1)\frac{\pi}{2} \mid n \in \mathbb{Z} \right\}$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$, y is periodic with period π



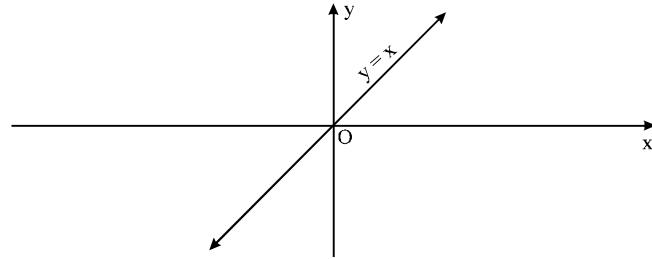
(ii) $y = \tan(\tan^{-1} x) = x$, $x \in \mathbb{R}$, $y \in \mathbb{R}$



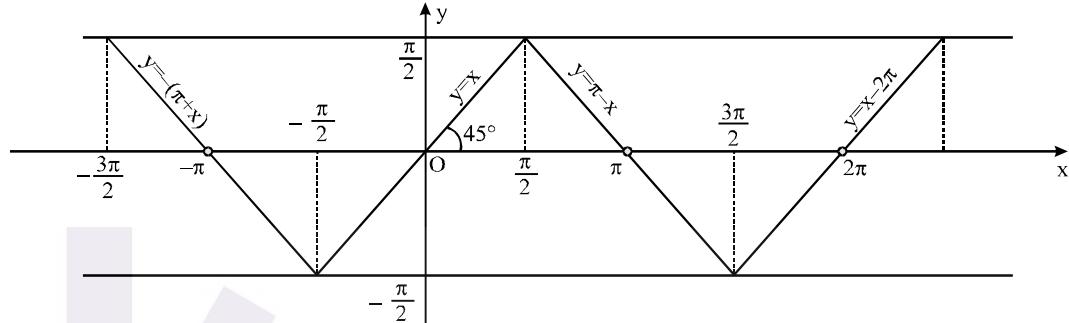
(i) $y = \cot^{-1}(\cot x) = x$, $x \in \mathbb{R} - \{n\pi\}$, $y \in [0, \pi]$, periodic with π



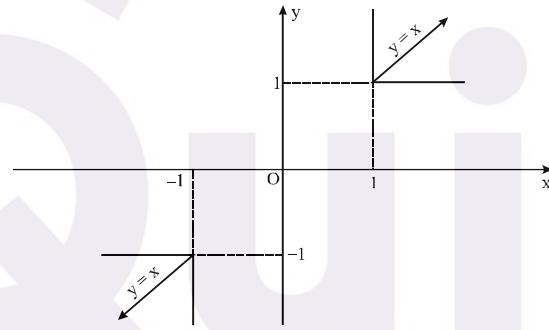
(ii) $y = \cot(\cot^{-1} x) = x$, $x \in \mathbb{R}$, $y \in \mathbb{R}$



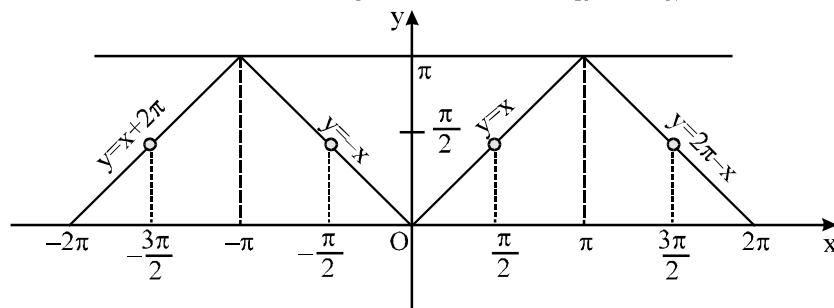
(i) $y = \operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$, $x \in \mathbb{R} - \{n\pi, n \in \mathbb{Z}\}$, $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$, y is periodic with period 2π



(ii) $y = \operatorname{cosec}(\operatorname{cosec}^{-1} x) = x$, $|x| \geq 1$, $|y| \geq 1$



(i) $y = \sec^{-1}(\sec x) = x$, $x \in \mathbb{R} - \left(2n - \frac{\pi}{2}\right)$, $n \in \mathbb{Z}$, $y \in \left[0, \frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}, \pi\right]$, y is periodic with period 2π



(ii) $y = \sec(\sec^{-1} x) = x$, $|x| \geq 1$; $|y| \geq 1$

