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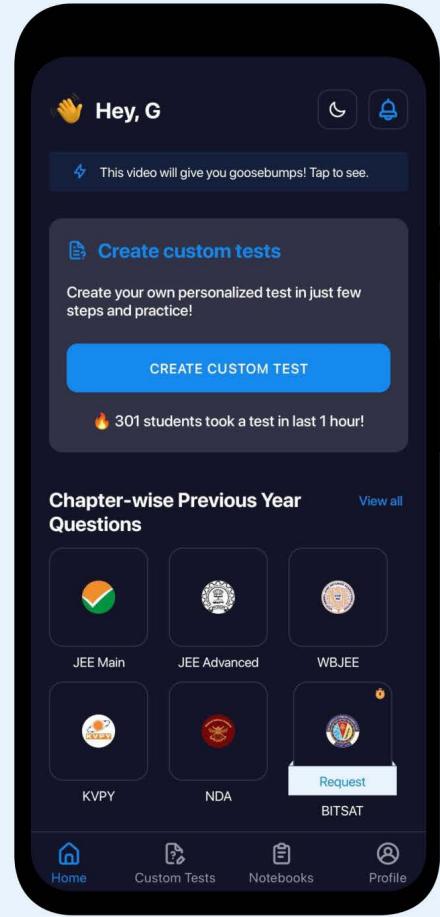


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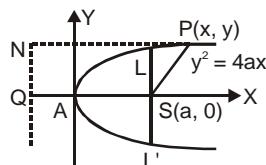
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# CONIC SECTION

## PARABOLA

### DEFINITION

A parabola is the locus of a point which moves in a plane such that its distance from a fixed point (called the focus) is equal to its distance from a fixed straight line (called the directrix).



Let S be the focus. QN be the directrix and P be any point on the parabola. Then by definition. PS = PN where PN is the length of the perpendicular from P on the directrix QN.

### TERMS RELATED TO PARABOLA

Axis : A straight line passes through the focus and perpendicular to the directrix is called the axis of parabola.

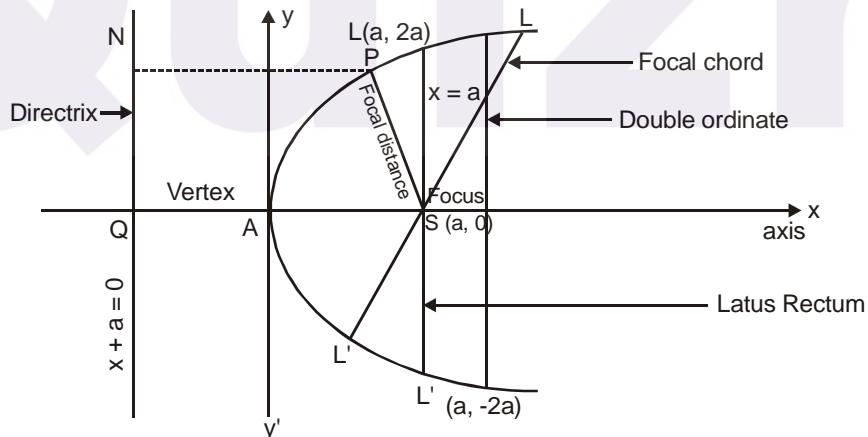
Vertex : The point of intersection of a parabola and its axis is called the vertex of the parabola.

The vertex is the middle point of the focus and the point of intersection of axis and directrix.

Eccentricity : If P be a point on the parabola and PN and PS are the distance from the directrix and focus S respectively then the ratio PS/PN is called the eccentricity of the parabola which is denoted by e.

By the definition for the parabola  $e = 1$ .

If  $e > 1 \Rightarrow$  hyperbola,  $e = 0 \Rightarrow$  circle,  $e < 1 \Rightarrow$  ellipse



### Latus Rectum

Let the given parabola be  $y^2 = 4ax$ . In the figure  $LSL'$  (a line through focus  $\perp$  to axis) is the latus rectum.

Also by definition,

$$LSL' = 2(\sqrt{4a \cdot a}) = 4a$$

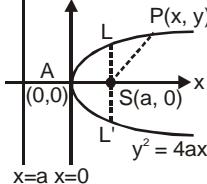
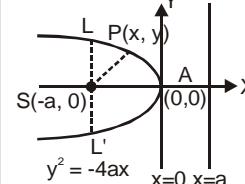
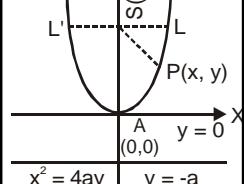
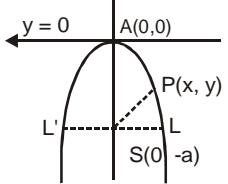
= double ordinate (Any chord of the parabola  $y^2 = 4ax$  which is  $\perp$  to its axis is called the double ordinate) through the focus S.

Note : Two parabolas are said to be equal when their latus recta are equal.

### Focal Chord

Any chord to the parabola which passes through the focus is called a focal chord of the parabola.

## FOUR STANDARD FORMS OF THE PARABOLA

Standard Equation	$y^2 = 4ax$ ( $a > 0$ )	$y^2 = -4ax$ ( $a > 0$ )	$x^2 = 4ay$ ( $a > 0$ )	$x^2 = -4ay$ ( $a > 0$ )
Shape of Parabola	 $y^2 = 4ax$	 $y^2 = -4ax$	 $x^2 = 4ay$	 $x^2 = -4ay$
Vertex	$A(0, 0)$	$A(0, 0)$	$A(0, 0)$	$A(0, 0)$
Focus	$S(a, 0)$	$S(-a, 0)$	$S(0, a)$	$S(0, -a)$
Equation of directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$
Extremities of latus rectum	$(a, \pm 2a)$	$(-a, \pm 2a)$	$(\pm 2a, a)$	$(\pm 2a, -a)$
Equation of latus rectum	$x = a$	$x = -a$	$y = a$	$y = -a$
Equation of tangents at vertex	$x = 0$	$x = 0$	$y = 0$	$y = 0$
Focal distance of a point $P(x, y)$	$x + a$	$x - a$	$y + a$	$y - a$
Parametric coordinates	$(at^2, 2at)$	$(-at^2, 2at)$	$(2at, at^2)$	$(2at, -at^2)$
Eccentricity (e)	1	1	1	1

## REDUCTION OF STANDARD EQUATION

If the equation of a parabola contains second degree term either in  $y$  or in  $x$  (but not in both) then it can be reduced into standard form. For this we change the given equation into the following forms-

$$(y - k)^2 = 4a(x - h) \text{ or } (x - p)^2 = 4b(y - q)$$

Then we compare from the following table for the results related to parabola.

## GENERAL EQUATION OF A PARABOLA

If  $(h, k)$  be the locus of a parabola and the equation of directrix is  $ax + by + c = 0$ , then its equation is given by

$$\sqrt{(x-h)^2 + (y-k)^2} = \frac{|ax+by+c|}{\sqrt{a^2+b^2}} \text{ which gives } (bx-ay)^2 + 2gx + 2fy + d = 0$$

where  $g, f, d$  are the constant.

Note

- The general equation of second degree  $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$  represents a parabola, if

(a)  $h^2 = ab$

(b)  $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$

## PARAMETRIC EQUATIONS OF A PARABOLA

Clearly  $x = at^2$ ,  $y = 2at$  satisfy the equation  $y^2 = 4ax$  for all real values of  $t$ . Hence the parametric equation of the parabola  $y^2 = 4ax$  are  $x = at^2$ ,  $y = 2at$ , where  $t$  is the parameter.

Also,  $(at^2, 2at)$  is a point on the parabola  $y^2 = 4ax$  for all real values of  $t$ . This point is also described as the point ' $t$ ' on the parabola.

## EQUATION OF A CHORD

- (i) The equation of chord joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  on the parabola  $y^2 = 4ax$  is

$$y(y_1 + y_2) = 4ax + y_1 y_2$$

- (ii) The equation of chord joining the points  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  is-

$$(y - 2at_1) = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2} (x - at_1^2)$$

$$\Rightarrow y - 2at_1 = \frac{2}{t_1 + t_2} (x - at_1^2)$$

$$y(t_1 + t_2) = 2(x + at_1 t_2)$$

- (iii) Length of the chord  $y = mx + c$  to the parabola  $y^2 = 4ax$  is given by  $\frac{4}{m^2} \sqrt{1+m^2} \sqrt{a(a-mc)}$ .

Condition for the Chord to be a Focal Chord

The chord joining the points  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  passes through focus provided  $t_1 t_2 = -1$ .

Length of Focal Chord

The length of a focal chord joining the points  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  is  $(t_2 - t_1)^2$ .

Note :

- The length of the focal chord through the point ' $t$ ' on the parabola  $y^2 = 4ax$  is  $a(t + 1/t)^2$
- The length of the chord joining two points ' $t_1$ ' and ' $t_2$ ' on the parabola  $y^2 = 4ax$  is

$$a(t_1 - t_2) \sqrt{(t_1 + t_2)^2 + 4}$$

## CONDITION FOR TANGENCY AND POINT OF CONTACT

The line  $y = mx + c$  touches the parabola  $y^2 = 4ax$  if  $c = \frac{a}{m}$  and the coordinates of the point of contact are

$$\left( \frac{a}{m^2}, \frac{2a}{m} \right).$$

Note

- The line  $y = mx + c$  touches parabola  $x^2 = 4ay$  if  $c = -am^2$
- The line  $x \cos \alpha + y \sin \alpha = p$  touches the parabola  $y^2 = 4ax$  if  $a \sin^2 \alpha + p \cos \alpha = 0$ .

## EQUATION OF TANGENT IN DIFFERENT FORMS

- (i) Point Form

The equation of the tangent to the parabola  $y^2 = 4ax$  at the point  $(x_1, y_1)$  is

$$yy_1 = 2a(x + x_1)$$

Note :

(ii) Parametric Form

The equation of the tangent to the parabola  $y^2 = 4ax$  at the point  $(at^2, 2at)$  is  

$$ty = x + at^2.$$

(iii) Slope Form

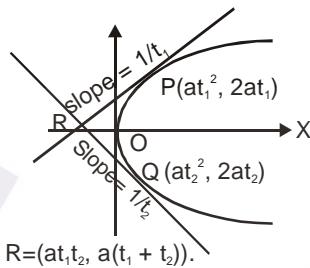
The equation of tangent to the parabola  $y^2 = 4ax$  in terms of slope 'm' is

$$y = mx + \frac{a}{m}.$$

The coordinate of the point of contact are  $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

## POINT OF INTERSECTION OF TANGENTS

The point of intersection of tangents drawn at two different points of contact P( $at_1^2, 2at_1$ ) and Q( $at_2^2, 2at_2$ ) on the parabola  $y^2 = 4ax$  is



Note :

- Angle between tangents at two points P( $at_1^2, 2at_1$ ) and Q( $at_2^2, 2at_2$ ) on the parabola  $y^2 = 4ax$  is  $\theta = \tan^{-1} \left| \frac{t_2 - t_1}{1 + t_1 t_2} \right|$
- The G.M. of the x-coordinates of P and Q (i.e.,  $\sqrt{at_1^2 \times at_2^2} = at_1 t_2$ ) is the x-coordinate of the point of intersection of tangents at P and Q on the parabola.
- The A.M. of the y-coordinates of P and Q (i.e.,  $\frac{2at_1 + 2at_2}{2} = a(t_1 + t_2)$ ) is the y-coordinate of the point of intersection of tangents at P and Q on the parabola.
- The orthocentre of the triangle formed by three tangents to the parabola lies on the directrix.

## EQUATIONS OF NORMAL IN DIFFERENT FORMS

(i) Point form

The equation of the normal to the parabola  $y^2 = 4ax$  at a point  $(x_1, y_1)$  is

$$y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

(ii) Parametric form

The equation of the normal to the parabola  $y^2 = 4ax$  at the point  $(at^2, 2at)$  is  

$$y + tx = 2at + at^3.$$

(iii) Slope form

The equation of normal to the parabola  $y^2 = 4ax$  in terms of slope 'm' is

$$y = mx - 2am - am^3$$

Note : The coordinates of the point of contact are  $(am^2 - 2am)$ .

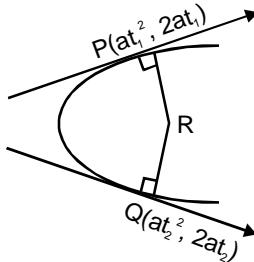
### Condition for Normality

The line  $y = mx + c$  is normal to the parabola

$$y^2 = 4ax \quad \text{if } c = -2am - am^3 \quad \text{and} \quad x^2 = 4ay \quad \text{if } c = 2a + \frac{a}{m^2}$$

### Point of Intersection of Normals

The point of intersection of normals drawn at two different points of contact  $P(at_1^2, 2at_1)$  and  $Q(at_2^2, 2at_2)$  on the parabola  $y^2 = 4ax$  is

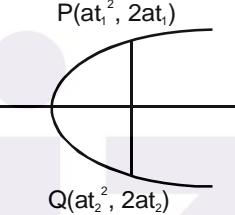


$$R \equiv [2a + a(t_1^2 + t_2^2 + t_1 t_2), -at_1 t_2 (t_1 + t_2)]$$

Note :

- If the normal at the point  $P(at_1^2, 2at_1)$  meets the parabola  $y^2 = 4ax$  again at  $Q(at_2^2, 2at_2)$ , then

$$t_2 = -t_1 - \frac{2}{t_1}$$



It is clear that  $PQ$  is normal to the parabola at  $P$  and not at  $Q$ .

- If the normals at the points  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  meet on the parabola  $y^2 = 4ax$ , then  $t_1 t_2 = 2$
- The normal at the extremities of the latus rectum of a parabola intersect at right angle on the axis of the parabola.

### Co-normal Points

Any three points on a parabola normals at which pass through a common point are called co-normal points

Note :

This implies that if three normals are drawn through a point  $(x_1, y_1)$  then their slopes are the roots of the cubic:  $y_1 = mx_1 - 2am - am^3$  which gives three values of  $m$ . Let these values are  $m_1, m_2, m_3$  then from the eq<sup>n</sup>.

$$\Rightarrow am^3 + (2a - x_1)m + y_1 = 0$$

- The sum of the slopes of the normals at co-normal points is zero, i.e.,  $m_1 + m_2 + m_3 = 0$ .

$$\text{and } m_1 m_2 + m_2 m_3 + m_3 m_1 = \frac{2a - x_1}{a} \quad \text{and } m_1 m_2 m_3 = -\frac{y_1}{a}$$

- The sum of the ordinates of the co-normal points is zero (i.e.,  $-2am_1 - 2am_2 - 2am_3 = -2a(m_1 + m_2 + m_3) = 0$ ).
- The centroid of the triangle formed by the co-normal points lies on the axis of the parabola

- The vertices of the triangle formed by the co-normal points are  $(am_1^2 - 2am_1)$ ,  $(am_2^2, -2am_2)$  and  $(am_3^2, -2am_3)$ . Thus, y-coordinate of the centroid becomes

$$\frac{-2a(m_1 + m_2 + m_3)}{3} = \frac{-2a}{3} \times 0 = 0.$$

i.e., centroid of triangle  $\left( \frac{am_1^2 + am_2^2 + am_3^2}{3}, \frac{2am_1 + 2am_2 + 2am_3}{3} \right) = \left( \frac{am_1^2 + am_2^2 + am_3^2}{3}, 0 \right)$

Hence, the centroid lies on the x-axis i.e., axis of the parabola.]

- If three normals drawn to any parabola  $y^2 = 4ax$  from a given point  $(h, k)$  be real, then  $h > 2a$ .

## POSITION OF A POINT & LINE W.R.T. A PARABOLA

- The point  $(x_1, y_1)$  lies outside, on or inside the parabola  $y^2 = 4ax$  according as  $y_1^2 - 4ax_1 >$ ,  $=$  or  $< 0$ , respectively.
- The line  $y = mx + c$  will intersect a parabola  $y^2 = 4ax$  in two real and different, coincident or imaginary point, according as  $a - mc >$ ,  $=$  or  $< 0$ .

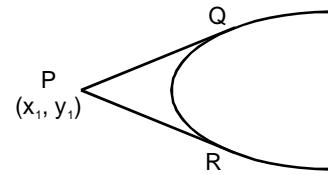
Number of tangents drawn from a point to a parabola

Two tangents can be drawn from a point to a parabola. The two tangents are real and distinct or coincident or imaginary according as the given point lies outside, on or inside the parabola.

## EQUATION OF THE PAIR OF TANGENTS

	CONDITION	POSITION	DIAGRAM	NO. OF COMMON TANGENTS
(i)	$C_1C_2 > r_1 + r_2$	do not intersect or one outside the other		4
(ii)	$C_1C_2 <  r_1 - r_2 $	one inside the other		0
(iii)	$C_1C_2 = r_1 + r_2$	external touch		3
(iv)	$C_1C_2 =  r_1 - r_2 $	internal touch		1
(v)	$ r_1 - r_2  < C_1C_2 < r_1 + r_2$	intersection at two real points		2

The equation of the pair of tangents drawn from a point  $P(x_1, y_1)$  to the parabola  $y^2 = 4ax$  is  $SS_1 = T^2$ .



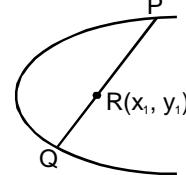
where  $S \equiv y^2 - 4ax$ ,  $S_1 \equiv y_1^2 - 4ax_1$  and  $T \equiv yy_1 - 2a(x + x_1)$

### LOCUS OF POINT OF INTERSECTION

The locus of point of intersection of tangent to the parabola  $y^2 = 4ax$  which are having an angle  $\theta$  between them given by  $y^2 - 4ax = (a + x)^2 \tan^2 \theta$

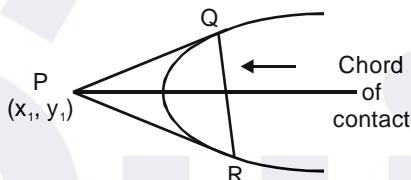
Note :

- If  $\theta = 0^\circ$  or p then locus is  $(y^2 - 4ax) = 0$  which is the given parabola.
- If  $\theta = 90^\circ$ , then locus is  $x + a = 0$  which is the directrix of the parabola.



### CHORD OF CONTACT

The equation of chord of contact of tangents drawn from a point  $P(x_1, y_1)$  to the parabola  $y^2 = 4ax$  is  $T = 0$  where  $T \equiv yy_1 - 2a(x + x_1)$ .



Note :

- The chord of contact joining the point of contact of two perpendicular tangents always passes through focus.
- Lengths of the chord of contact is  $\frac{1}{a} \sqrt{(y_1^2 - 4ax_1)(y_1^2 + 4a^2)}$
- Area of triangle formed by tangents drawn from  $(x_1, y_1)$  and their chord of contact is  $\frac{1}{2a} (y_1^2 - 4ax_1)^{3/2}$ .

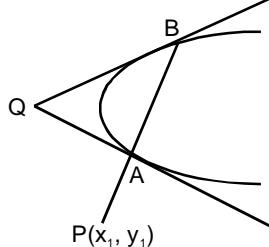
### CHORD WITH A GIVEN MID POINT

The equation of the chord of the parabola  $y^2 = 4ax$  with  $P(x_1, y_1)$  as its middle point is given by

$$T = S_1 \text{ where } T \equiv yy_1 - 2a(x + x_1) \text{ and } S_1 \equiv y_1^2 - 4ax.$$

### POLE AND POLAR

Let P be a given point. Let a line through P intersect the parabola at two points A and B. Let the tangents at A and B intersect at Q. The locus of point Q is a straight line called the polar of point P with respect to the parabola and the point P is called the pole of the polar.



### Equation of Polar of a Point

The polar of a point  $P(x_1, y_1)$  with respect to the parabola  $y^2 = 4ax$  is  $T = 0$  where  $T \equiv yy_1 - 2a(x + x_1)$ .

Coordinate of pole

The pole of the line  $lx + my + n = 0$  with respect to the parabola  $y^2 = 4ax$  is  $\left(\frac{n}{l}, -\frac{2am}{l}\right)$

### Conjugate points and conjugate lines

- (i) If the polar of  $P(x_1, y_1)$  passes through  $Q(x_2, y_2)$  then the polar of  $Q$  will pass through  $P$  and such points are said to be conjugate points.

So two points  $(x_1, y_1)$  and  $(x_2, y_2)$  are conjugate points with respect to parabola  $y^2 = 4ax$  if  $yy_1 = 2a(x_1 + x_2)$

- (ii) If the pole of a line  $ax + by + c = 0$  lies on the another line  $a_1x + b_1y + c_1 = 0$  then the pole of the second line will lie on the first and such line are said to be conjugate lines.

So two lines  $l_1x + m_1y + n_1 = 0$  and  $l_2x + m_2y + n_2 = 0$  are conjugate lines with respect to parabola  $y^2 = 4ax$  if  $(l_1n_2 + l_2n_1) = 2am_1m_2$

Note

- The polar of focus is directrix and pole of directrix is focus.
- The polars of all points on directrix always pass through a fixed point and this fixed point is focus.
- The pole of a focal chord lies on directrix and locus of poles of focal chord is a directrix.
- The chord of contact and polar of any point on the directrix always passes through focus.

### DIAMETER OF A PARABOLA

Diameter of a parabola is the locus of middle points of a series of its parallel chords.

The equation of the diameter bisecting chords of slope  $m$  of the parabola  $y^2 = 4ax$  is  $y = \frac{2a}{m}$ .

