



**India's Most Comprehensive & the Most Relevant  
Test Series designed according to the latest pattern of exams!**



JEE MAIN



JEE ADV.



BITSAT



WBJEE



MHT CET

*and many more...*

**Click here to join Test Series for 2022**

It's time for you to crack upcoming IIT JEE Main & Advanced and other competitive exams with India's Most Trusted Online Test Series. **Many questions at JEE Main 2021 were same/similar to the ones asked in our test series.** That's the power of our test series!

**Trusted by thousands of students  
& their parents across the nation**

*Our result in JEE Main 2021*

**150+**

Got **99+ percentile** (overall)

**301**

Got **99+ percentile** in one or more subjects

**85%**

Improved their score by **25 percentile**

**89%**

Felt **overall confident** after the test series

**Click here to join Test Series for 2022**



# FREE Question Bank & Previous Year Questions for



JEE MAIN



JEE ADV.



BITSAT



WBJEE

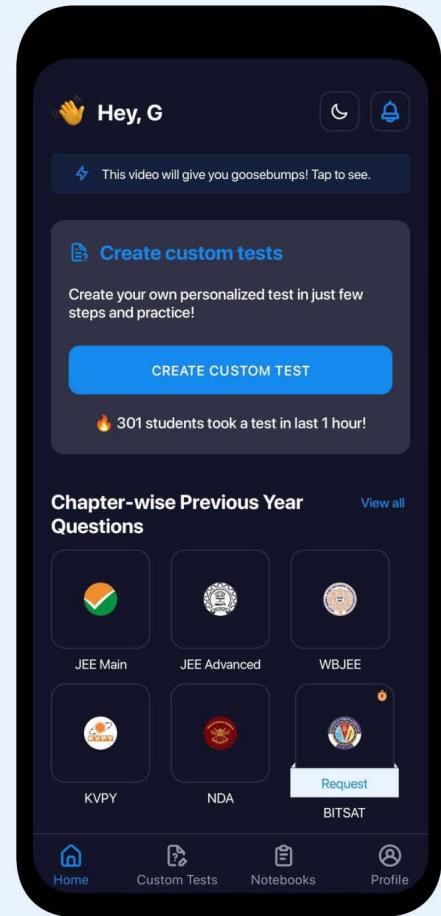


MHT CET

and many more...



Click on this button



## 😎 Why download MARKS?

- 📚 Chapter-wise PYQ of JEE Main, JEE Advanced, NEET, AIIMS, BITSAT, WBJEE, MHT CET etc.
- 📚 Chapter-wise NTA Abhyas questions
- ⌚ Daily practice challenge and goal completion
- 🔖 Bookmark important questions and add them to your notebooks
- ✍ Create unlimited Custom Tests

And all this for **FREE**. Yes, **FREE!** So what are you waiting for, download MARKS now.

4.8

Rating on Google Play

30,000+

Students using daily

1,00,000+

Questions available



Click on this button

# COMPLEX NUMBERS

## COMPLEX NUMBERS

If ‘a’, ‘b’ are two real numbers, then a number of the form  $a + ib$  is called a complex number

Set of complex Numbers : The set of all complex numbers is denoted by C.

$$\text{i.e. } C = \{a + ib \mid a, b \in \mathbb{R}\}$$

Equality of Complex Numbers : Two complex numbers  $z_1 = a_1 + ib_1$  and  $z_2 = a_2 + ib_2$  are equal if  $a_1 = a_2$  and  $b_1 = b_2$  i.e.  $\text{Re}(z_1) = \text{Re}(z_2)$  and  $\text{Im}(z_1) = \text{Im}(z_2)$

## FUNDAMENTAL OPERATIONS ON COMPLEX NUMBERS

ADDITION : Let  $z_1 = a_1 + ib_1$  and  $z_2 = a_2 + ib_2$  be two complex numbers. Then their sum  $z_1 + z_2$  is defined as the complex number  $(a_1 + a_2) + i(b_1 + b_2)$

### Properties of addition of complex numbers

(i) Addition is commutative : For any two complex numbers  $z_1$  and  $z_2$ , we have

$$z_1 + z_2 = z_2 + z_1$$

(ii) Addition is associative : For any three complex numbers  $z_1, z_2, z_3$  we have

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$$

(iii) Existence of additive identity : The complex number  $0 = 0 + i0$  is the identity element for addition i.e.

$$z + 0 = z = 0 + z \text{ for all } z \in C$$

(iv) Existence of additive inverse : For every complex number  $z$  there exists  $-z$  such that

$$z + (-z) = 0 = (-z) + z$$

The complex number  $-z$  is called the additive inverse of  $z$ .

Subtraction : Let  $z_1 = a_1 + ib_1$  and  $z_2 = a_2 + ib_2$  be two complex numbers. Then the subtraction of  $z_2$  from  $z_1$  is denoted by  $z_1 - z_2$  and is defined as the addition of  $z_1$  and  $-z_2$ .

Thus,

$$z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$$

Multiplication : Let  $z_1 = a_1 + ib_1$  and  $z_2 = a_2 + ib_2$  be two complex numbers. Then, the multiplication of  $z_1$  with  $z_2$  is denoted by  $z_1 z_2$  and is defined as the complex number

$$(a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1)$$

### Properties of Multiplication :

(i) Multiplication is commutative. For any two complex numbers  $z_1$  and  $z_2$ , we have

$$z_1 z_2 = z_2 z_1$$

(ii) Multiplication is associative : For any three complex numbers  $z_1, z_2, z_3$  we have

$$(z_1 z_2) z_3 = z_1 (z_2 z_3)$$

(iii) Existence of identity element for multiplication. The complex number  $1 = 1 + i0$  is the identity element for multiplication i.e. for every complex number  $z$ , we have

$$z \cdot 1 = z$$

(iv) Existence of multiplicative inverse : Corresponding to every non-zero complex number  $z = a + ib$  there exists a complex number  $z_1 = x + iy$  such that

$$z \cdot z_1 = 1 \Rightarrow z_1 = \frac{1}{z}$$

The complex number  $z_1$  is called the multiplicative inverse or reciprocal of  $z$  and is given by

$$z_1 = \frac{a}{a^2 + b^2} + \frac{i(-b)}{a^2 + b^2}$$

- (v) Multiplication of complex numbers is distributive over addition of complex numbers : For any three complex numbers  $z_1, z_2, z_3$  we have

$$(i) \quad z_1(z_2 + z_3) = z_1z_2 + z_1z_3 \quad (\text{Left distributivity})$$

$$(ii) \quad (z_2 + z_3)z_1 = z_2z_1 + z_3z_1 \quad (\text{Right distributivity})$$

Division : The division of a complex number  $z_1$  by a non-zero complex number  $z_2$  is defined as the multiplication

of  $z_1$  by the multiplicative inverse of  $z_2$  and is denoted by  $\frac{z_1}{z_2}$ .

$$\text{Thus, } \frac{z_1}{z_2} = z_1 \cdot z_2^{-1} = z_1 \cdot \left( \frac{1}{z_2} \right)$$

Conjugate : Let  $z = a + ib$  be a complex number. Then the conjugate of  $z$  is denoted by  $\bar{z}$  and is equal to  $a - ib$ .

$$\text{Thus, } z = a + ib \Rightarrow \bar{z} = a - ib$$

### Properties of Conjugate :

If  $z, z_1, z_2$  are complex numbers, then

$$(i) \quad z + \bar{z} = 2 \operatorname{Re}(z)$$

$$(ii) \quad z - \bar{z} = 2 \operatorname{Im}(z)$$

$$(iii) \quad z = \bar{z} \Leftrightarrow z \text{ is purely real}$$

$$(iv) \quad z + \bar{z} = 0 \Rightarrow z \text{ is purely imaginary.}$$

$$(v) \quad z\bar{z} = \{\operatorname{Re}(z)\}^2 + \{\operatorname{Im}(z)\}^2$$

$$(vi) \quad \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$(vii) \quad \overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$$

$$(ix) \quad \overline{z_1 z_2} = \overline{z_1} \overline{z_2}$$

$$(x) \quad \left( \frac{z_1}{z_2} \right) = \frac{\overline{z}_1}{\overline{z}_2}, z_2 \neq 0$$

$$(xi) \quad (\bar{z}) = z$$

### MODULUS OF A COMPLEX NUMBER

Definition : the modulus of a complex number  $z = a + ib$  is denoted by  $|z|$  and is defined as

$$|z| = \sqrt{a^2 + b^2} = \sqrt{\{\operatorname{Re}(z)\}^2 + \{\operatorname{Im}(z)\}^2}$$

- The multiplicative inverse of a non-zero complex number  $z$  is same as its reciprocal and is given by

$$\frac{\operatorname{Re}(z)}{|z|^2} + i \frac{(-\operatorname{Im}(z))}{|z|^2} = \frac{\bar{z}}{|z|^2}$$

If  $b$  is positive

$$\text{then } \therefore \sqrt{a+ib} = \pm \left[ \sqrt{\frac{1}{2}\{\sqrt{a^2+b^2} + a\}} + i\sqrt{\frac{1}{2}\{\sqrt{a^2+b^2} - a\}} \right]$$

If  $b$  is negative then

$$\sqrt{a+ib} = \pm \left[ \sqrt{\frac{1}{2}\{|z| + \operatorname{Re}(z)\}} - i\sqrt{\frac{1}{2}\{|z| - \operatorname{Re}(z)\}} \right]$$

### Argument or (amplitude) of a Complex Number

- (i) If  $x$  and  $y$  both are positive, then the argument of  $z = x + iy$  is the acute angle given by  $\tan^{-1} \left| \frac{y}{x} \right|$
- (ii)  $x < 0$  and  $y > 0$ , then the argument of  $z = x + iy$  is  $\pi - \alpha$ , where  $\alpha$  is the acute angle given by  $\tan^{-1} |y/x|$ .
- (iii) If  $x < 0$  and  $y < 0$  then the argument of  $z = x + iy$  is  $\alpha - \pi$  where  $\alpha$  is the acute angle given by  $\tan \alpha = \left| \frac{y}{x} \right|$ .
- (iv) If  $x > 0$  and  $y < 0$ , then the argument of  $z = x + iy$  is  $-\alpha$  where  $\alpha$  is the acute angle given by  $\tan \alpha = \left| \frac{y}{x} \right|$

### Polar or Trigonometrical Form of a Complex Number

Let  $z = x + iy$  be a complex number represented by a point  $P(x, y)$  in the Argand plane. Then, by the geometrical representation of  $z = x + iy$ , we have

$$\Rightarrow z = r(\cos \theta + i \sin \theta), \text{ where } r = |z| \text{ and } \theta = \arg(z)$$

This form of  $z$  is called a polar form of  $z$ .

### EULERIAN FORM OF A COMPLEX NUMBER

$$e^{i\theta} = \cos \theta + i \sin \theta \text{ and } e^{-i\theta} = \cos \theta - i \sin \theta$$

### Properties of Argument of $z$

- (i)  $\arg(\bar{z}) = -\arg(z)$
- (ii)  $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
- (iii)  $\arg(z_1 \bar{z}_2) = \arg(z_1) - \arg(z_2)$
- (iv)  $\arg(z_1 / z_2) = \arg(z_1) - \arg(z_2)$
- (v)  $\arg(z^n) = n \arg z$ .
- (vi)  $|z_1 + z_2| = |z_1 - z_2| \Rightarrow \arg z_1 - \arg z_2 = \pi/2$
- (vii)  $|z_1 + z_2| = |z_1| + |z_2| \Rightarrow \arg z_1 = \arg z_2$
- (viii) If  $\arg z = 0$ , then  $z$  is purely real
- (ix) If  $\arg z = \pm \pi/2$ , then  $z$  is purely imaginary

### Properties of Modulus of $z$

- (i)  $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2| \cos(\theta_1 - \theta_2)$ .  
or  
 $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1 \bar{z}_2)$

(ii)  $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$   
or

$$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\operatorname{Re}(z_1 \bar{z}_2)$$

(iii)  $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

(iv)  $|z_1 + z_2| = |z_1 - z_2| \Rightarrow \arg(z_1) - \arg(z_2) = \frac{\pi}{2}$

(v)  $|z_1 + z_2| = |z_1| + |z_2| \Leftrightarrow \arg(z_1) = \arg(z_2)$

(vi)  $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 \Leftrightarrow \frac{z_1}{z_2}$  is purely imaginary.

(vii)  $|z_1 + z_2| \leq |z_1| + |z_2|$

(viii)  $|z_1 - z_2| \leq |z_1| + |z_2|$

(ix)  $|z_1 + z_2| \geq |z_1| - |z_2|$

(x)  $|z_1 - z_2| \geq \|z_1| - |z_2\|$

(xi)  $|z_1 z_2| = |z_1| |z_2|$

(xii)  $|z^n| = |z|^n$

(xiii)  $|z|^2 = z \bar{z}$

(xiv)  $|z| = |\bar{z}| = |-z| = |-z|$

Distance Between Two Points : If  $z_1$  and  $z_2$  are the affixes of points P and Q respectively in the argand plane, then

$$PQ = |z_2 - z_1|$$

Section Formula : Let  $z_1$  and  $z_2$  be the affixes of two points P and Q respectively in the argand plane. Then, the affix of a point R dividing PQ internally in the ratio m : n is

$$\frac{mz_2 + nz_1}{m+n} \text{ but if R is external point, then affix of R is } \frac{mz_2 - nz_1}{m-n}$$

Mid Point Formula :

If R be the mid-point then affix of R is  $\frac{z_1 + z_2}{2}$

- If  $z_1, z_2, z_3$  are affixes of the vertices of a triangle, then the affix of its centroid is

$$\frac{z_1 + z_2 + z_3}{3}$$

- The equation of the perpendicular bisector of the line segment joining points having affixes  $z_1$  and  $z_2$  is

$$z(\bar{z}_1 - \bar{z}_2) + \bar{z}(z_1 - z_2) = |z_1|^2 - |z_2|^2$$

- The equation of a circle whose centre is at point having affix  $z_0$  and radius R is

$$|z - z_0| = R$$

- Note :
- If the centre of the circle is at the origin and radius R, then its equation is  $|z| = R$ .
  - General Equation of circle is

$$z \bar{z} + a \bar{z} + \bar{a} z + b = 0 \text{ where } b \in \mathbb{R} \text{ and } a \text{ is complex number}$$

represents a circle having centre at ' $-a$ '

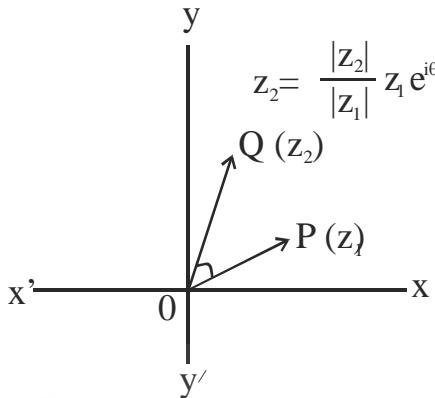
and radius =  $\sqrt{|a|^2 - b} = \sqrt{a \bar{a} - b}$

## COMPLEX NUMBER AS A ROTATING ARROW IN THE ARGAND PLANE

To obtain the point representing  $ze^{i\alpha}$  we rotate  $\overrightarrow{OP}$  through angle  $\alpha$  in anticlockwise sense. Thus, multiplication by  $e^{i\alpha}$  to  $z$  rotates the vector  $\overrightarrow{OP}$  in anticlockwise sense through an angle  $\alpha$ .

Let  $z_1$  and  $z_2$  be two complex numbers represented by points P and Q in the argand plane such that  $\angle POQ = \theta$ .

Then,  $z_1 e^{i\theta}$  is a vector of magnitude  $|z_1| = OP$  along  $\overrightarrow{OQ}$  and  $\frac{z_1 e^{i\theta}}{|z_1|}$  is a unit vector along  $\overrightarrow{OQ}$ .

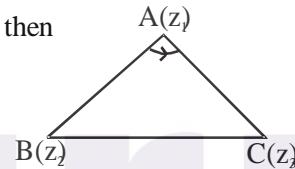


## SOME IMPORTANT RESULTS

I. If  $z_1, z_2, z_3$  are the affixes of the points A, B and C in the Argand plane, then

$$(i) \quad \angle BAC = \arg \left( \frac{z_3 - z_1}{z_2 - z_1} \right)$$

$$(ii) \quad \angle BAC = \arg \frac{z_3 - z_1}{z_2 - z_1} = \frac{|z_3 - z_1|}{|z_2 - z_1|} (\cos \alpha + i \sin \alpha), \text{ where } \alpha = \angle BAC.$$



If  $z_1, z_2, z_3$  and  $z_4$  are the affixes of the points A, B, C and D respectively in the Argand plane. Then AB is inclined to CD at the angle,

$$\arg \left( \frac{z_2 - z_1}{z_4 - z_3} \right)$$

(iii) The equation of the circle having  $z_1$  and  $z_2$  as the end points of a diameter is

$$(z - z_1)(\bar{z} - \bar{z}_2) + (\bar{z} - \bar{z}_1)(z - z_2) = 0$$

## DE-MOIVRE'S THEOREM

STATEMENT :

(i) If  $n \in \mathbb{Z}$  (the set of integers), then

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

(ii) If  $n \in \mathbb{Q}$  (the set of rational numbers), then  $\cos n\theta + i \sin n\theta$  is one of the values of  $(\cos \theta + i \sin \theta)^n$ .

$$(iii) \quad \frac{1}{\cos \theta + i \sin \theta} = \cos \theta - i \sin \theta$$

$$(iv) \quad (\cos \theta_1 + i \sin \theta_1)(\cos \theta_2 + i \sin \theta_2) = \cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)$$

## $n^{\text{th}}$ ROOTS OF UNITY

nth roots of unity are :  $\alpha^0 = 1, \alpha, \alpha^2, \alpha^3, \dots, \alpha^{n-1}$  where  $\alpha = e^{i2\pi/n} = \cos 2\pi/n + i \sin 2\pi/n$

## PROPERTIES OF $n^{\text{th}}$ ROOTS OF UNITY

Property 1 : nth roots of unity form a G.P. with common ratio  $e^{i2\pi/n}$

Property 2 : Sum of the nth roots of unity is always zero.

Property 3 : Sum of pth powers of nth roots of unity is zero, if p is not a multiple of n.

Property 4 : Sum of pth powers of nth roots of unity is n, if p is a multiple of n.

Property 5 : Product of nth roots of unity is  $(-1)^{n-1}$

Property 6 : nth roots of unity lie on the unit circle  $|z| = 1$  and divide its circumference into n equal parts.

## \* PROPERTIES OF CUBE ROOTS OF UNITY AND SOME USEFUL RESULTS RELATED TO THEM

(i) Cube roots of unity are  $1, \omega, \omega^2$  where

$$\omega = -\frac{1}{2} \pm i \frac{\sqrt{3}}{2}, \omega^2 = -\frac{1}{2} - i \frac{\sqrt{3}}{2}$$

(ii)  $\arg(\omega) = 2\pi/3$  and

(iii) Cube roots of  $-1$  are  $-1, -\omega, -\omega^2$

(iv)  $1 + \omega + \omega^2 = 0$

(v)  $\omega^3 = 1$

\* Four fourth roots of unity are  $-1, 1, -i, i$

\*  $\log(\alpha + i\beta) = \frac{1}{2}\log(\alpha^2 + \beta^2) + i\tan^{-1}\frac{\beta}{\alpha}$

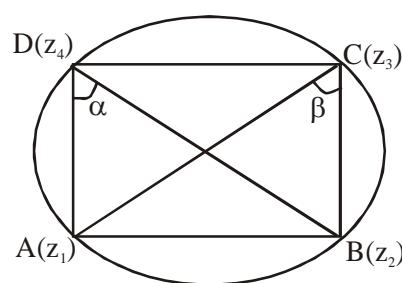
\* Condition for points  $A(z_1), B(z_2), C(z_3), D(z_4)$  to be concyclic :

$$\alpha = \beta$$

$$\Rightarrow \arg\left(\frac{z_2 - z_4}{z_1 - z_4}\right) = \arg\left(\frac{z_2 - z_3}{z_1 - z_3}\right)$$

$$\Rightarrow \arg\left(\frac{z_2 - z_4}{z_1 - z_4} \times \frac{z_1 - z_3}{z_2 - z_3}\right) = 0$$

$$\Rightarrow \left( \frac{(z_2 - z_4)(z_1 - z_3)}{(z_1 - z_4)(z_2 - z_3)} \right) \text{ is purely real.}$$

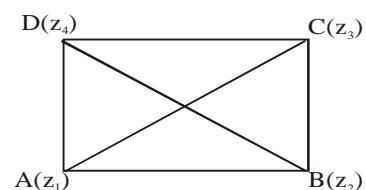


Condition (s) for four points  $A(z_1), B(z_2), C(z_3)$  and  $D(z_4)$  to represent vertices of a

(I) Parallelogram :

(i) The diagonals AC and BD must bisect each other

$$\Leftrightarrow \frac{1}{2}(z_1 + z_3) = \frac{1}{2}(z_2 + z_4)$$



$$\Leftrightarrow z_1 + z_3 = z_2 + z_4$$

(ii) Rhombus :

(a) The diagonals AC and BD bisect each other.

$$\Leftrightarrow z_1 + z_3 = z_2 + z_4$$

and (b) a pair of two adjacent sides are equal i.e.  $AD = AB$ .

$$\Leftrightarrow |z_4 - z_1| = |z_2 - z_1|$$

(iii) Square :

(a) The diagonals AC and BD bisect each other

$$\Leftrightarrow z_1 + z_3 = z_2 + z_4$$

(b) a pair of adjacent sides are equal

$$AD = AB$$

$$\Leftrightarrow |z_4 - z_1| = |z_2 - z_1|$$

(c) The two diagonals are equal

$$AC = BD \quad \Leftrightarrow |z_3 - z_1| = |z_4 - z_2|$$

(iv) Rectangle :

(a) The diagonals AC and BD bisect each other

$$z_1 + z_3 = z_2 - z_4$$

(b) The diagonals AC and BD are equal

$$\Leftrightarrow |z_3 + z_1| = |z_4 - z_2|$$

(v) Incentre : I(z) of the  $\Delta ABC$  is given by

$$z = \frac{az_1 + bz_2 + cz_3}{a + b + c}$$

(vi) Circumcentre (z) of the  $\Delta ABC$  is given by

$$z = \frac{z_1(\sin 2A) + z_2(\sin 2B) + z_3(\sin 2C)}{\sin 2A + \sin 2B + \sin 2C}$$

(v) Orthocentre (z) of the  $\Delta ABC$  is given by

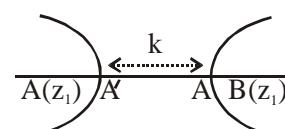
$$z = \frac{(a \sec A)z_1(b \sec B)z_2 + (c \sec C)z_3}{a \sec A + b \sec B + c \sec C}$$

(vi) Area of triangle ABC with vertices  $A(z_1)$ ,  $B(z_2)$ ,  $C(z_3)$  is given by

$$\Delta = \frac{1}{4} \text{ modulus of } \begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$$

(vi) Equation of line passing through  $A(z_1)$  and  $B(z_2)$  is

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$



(vii) General equation of a line is

$$\bar{a}z + a\bar{z} + b = 0, \text{ where } a \text{ is a complex number and } b \text{ is a real number.}$$

(viii) Complex slope of a line joining points  $A(z_1)$  and  $B(z_2)$  is

$$\text{given by } w = \frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$$

(ix) Two lines with complex slopes  $w_1$  and  $w_2$  are parallel if  $w_1 = w_2$  and perpendicular if  $w_1 + w_2 = w$

### Length of perpendicular from a point to a line

Length of perpendicular of a point  $A(\omega)$  from the line  $\bar{a}z + a\bar{z} + b = 0$

$$p = \frac{|a\omega + a\bar{\omega} + b|}{2|a|}$$

### Recognizing some loci by Inspection :

(i) If  $z_1$  and  $z_2$  are two fixed points, then

$|z - z_1| = |z - z_2|$  represent perpendicular bisector of the line segment joining  $A(z_1)$  and  $B(z_2)$ .

(ii) If  $z_1$  and  $z_2$  are two fixed points and  $k > 0$ ,  $k \neq 1$  is a real number then  $\frac{|z - z_1|}{|z - z_2|} = k$  represents a circle. For

$k = 1$  it represents perpendicular bisector of the segment joining  $A(z_1)$  and  $B(z_2)$

(iii) Let  $z_1$  and  $z_2$  be two fixed points and  $k$  be a positive real number.

(a) If  $k > |z_1 - z_2|$  then  $|z - z_1| + |z - z_2| = k$  represents an ellipse with foci at  $A(z_1)$  and  $B(z_2)$  and length of major axis =  $k = CD$ .

(b) If  $k = |z_1 - z_2|$  represents the line segment joining  $z_1$  and  $z_2$ .

(c) If  $k < |z_1 - z_2|$  then  $|z - z_1| + |z - z_2| = k$  does not represent any curve in the argand plane.

(iv) Let  $z_1$  and  $z_2$  be two fixed points,  $k$  be a positive real number.

(a) If  $k < |z_1 - z_2|$ , then  $|z - z_1| - |z - z_2| = k$  represents a hyperbola with foci at  $A(z_1)$  and  $B(z_2)$ .

(b) If  $k = (z_1 - z_2)$ , then

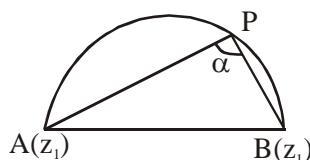
$$(z - z_1) - (z - z_2) = k$$

represents the straight line joining  $A(z_1)$  and  $B(z_2)$  excluding the segment AB.

(v) If  $z_1$  and  $z_2$  are two fixed points, then  $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$  represents a circle with  $z_1$  and  $z_2$  as extremities of a diameter.

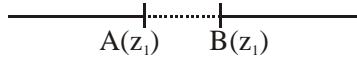
(vi) Let  $z_1$  and  $z_2$  be two fixed points and  $\alpha$  be a real number such that  $0 \leq \alpha \leq \pi$  then (a) If  $0 < \alpha < \pi$  and

$\alpha \neq \frac{\pi}{2}$  then  $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha$  represents a segment of the circle passing through  $A(z_1)$  and  $B(z_2)$



- (b) If  $\alpha_2 = \pi/2$ , then  $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha = \frac{\pi}{2}$  represents a circle with diameter as the segment joining  $A(z_1)$  and  $B(z_2)$ .

- (c) if  $\alpha = \pi$  then  $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha$  represents the straight line joining  $A(z_1)$  and  $B(z_2)$  but excluding the segment AB.



- (d) If  $\alpha = 0$ , then  $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha (= 0)$



represents the segment joining  $A(z_1)$  and  $B(z_2)$

