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Tangents and Normals

1. GEOMETRICAL INTERPRETATION OF THE DERIVATIVE

Let $y = f(x)$ be a given function. Its derivative $f'(x)$ or $\frac{dy}{dx}$ is equal to the trigonometrical tangent of the angle which tangent to the graph of the function at the point (x, y) makes with the positive direction of x -axis. Therefore $\frac{dy}{dx}$ is the slope of the tangent.

Thus
$$f'(x) = \frac{dy}{dx} = \tan \Psi$$

Hence at any point of a curve $y = f(x)$

- (i) Inclination of the tangent (with x -axis) = $\tan^{-1}\left(\frac{dy}{dx}\right)$
- (ii) Slope of the tangent = $\frac{dy}{dx}$
- (iii) Slope of the normal = $-\frac{1}{\left(\frac{dy}{dx}\right)} = -\left(\frac{dx}{dy}\right)$
- (iv) Slope of the tangent at (x_1, y_1) is denoted by $\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$
- (v) Slope of the normal at (x_1, y_1) is denoted by $\left(-\frac{dx}{dy}\right)_{(x_1, y_1)}$

2. EQUATION OF TANGENT

- (i) The equation of tangent to the curve $y = f(x)$ at (x_1, y_1) is $(Y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (X - x_1)$
Slope of tangent = $\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

- (ii) If a tangent is parallel to the axis of x then $\Psi = 0$

$$\therefore \frac{dy}{dx} = \tan \Psi = \tan 0 = 0 \Rightarrow \boxed{\frac{dy}{dx} = 0}$$

- (iii) If the equation of the curve be given in the parametric form say $x = f(t)$ and $y = g(t)$, then

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) / \left(\frac{dx}{dt}\right) = \frac{g'(t)}{f'(t)}$$

The equation of tangent at any point 't' on the curve is given by
$$\boxed{y - g(t) = \frac{g'(t)}{f'(t)}(x - f(t))}$$

(v) If the tangent is perpendicular to the axis of x , then $\Psi = \frac{\pi}{2}$

$$\therefore \frac{dy}{dx} = \tan \Psi = \tan \frac{\pi}{2} = \infty \Rightarrow \boxed{\frac{dx}{dy} = 0}$$

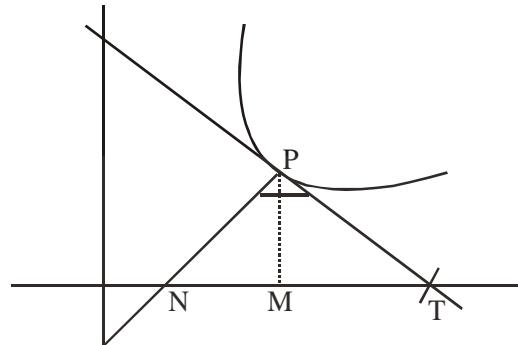
(vi) Q The value of Ψ always lie in $(-\pi, \pi]$

(vii) If the tangent at any point on the curve is equally inclined to both the axes then $\left(\frac{dy}{dx}\right) = \pm 1$

4. LENGTH OF THE TANGENT

$$PT = y \cosec \Psi = y \sqrt{1 + \cot^2 \Psi}$$

$$= \left| y \sqrt{1 + \left(\frac{dx}{dy} \right)^2} \right| \quad \text{or} \quad \left| \frac{y \sqrt{1 + (dy/dx)^2}}{(dy/dx)} \right|$$



5. LENGTH OF SUB-TANGENT

$$TM = y \cot \Psi = \frac{y}{\tan \Psi} = \left| y \frac{dx}{dy} \right| \quad \text{or} \quad \frac{y}{(dy/dx)}$$

6. EQUATION OF NORMAL

The equation of normal

(i) at the point $P(x_1, y_1)$ on the curve $Y = f(x)$ is

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx} \right)_{(x_1, y_1)}} (x - x_1)$$

$$\boxed{y - y_1 = -\left(\frac{dx}{dy} \right)_{(x_1, y_1)} (x - x_1)}$$

$$\text{Slope of normal} = -\frac{1}{\text{slope of tangent}} = -\frac{1}{\left(\frac{dy}{dx} \right)}$$

(ii) If the normal is parallel to the axis of y , then $\Rightarrow \Psi = 0$

$$\therefore \frac{dy}{dx} = \tan \Psi = \tan 0 = 0$$

(iii) If the normal is parallel to the axis of x , then $\therefore \frac{dx}{dy} = 0$

7. LENGTH OF NORMAL

$$PN = y \sec \Psi = y \sqrt{1 + \tan^2 \Psi} = \left| y \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \right|$$

8. LENGTH OF SUB-NORMAL

$$MN = y \tan \Psi = \left| y \frac{dy}{dx} \right|$$

9. ANGLE OF INTERSECTION OF TWO CURVES

If two curves $y = f_1(x)$ and $y = f_2(x)$ intersect at a point p, the angle between their tangents at p is defined as the angle between these two curves at p. But slopes of tangents at P are $\left(\frac{dy}{dx} \right)_1$ and $\left(\frac{dy}{dx} \right)_2$

So at p their angle of intersection ψ is given by

$$\tan \psi = \left| \frac{\left(\frac{dy}{dx} \right)_1 - \left(\frac{dy}{dx} \right)_2}{1 + \left(\frac{dy}{dx} \right)_1 \left(\frac{dy}{dx} \right)_2} \right| \quad \text{or} \quad \boxed{\tan \psi = \pm \frac{m_1 - m_2}{1 + m_1 m_2}}$$

1. If two curves cut perpendicular then $\psi = \frac{\pi}{2}$

$$\left(\frac{dy}{dx} \right)_1 \left(\frac{dy}{dx} \right)_2 = -1 \text{ or } m_1 m_2 = -1$$

2. If two curves are parallel $\psi = 0^\circ$

$$\left(\frac{dy}{dx} \right)_1 = \left(\frac{dy}{dx} \right)_2 \text{ or } m_1 = m_2$$

10. ROLLE'S THEOREM

If a function $f(x)$ is defined on $[a,b]$ satisfying

- (i) f is continuous on $[a,b]$
- (ii) f is differentiable on (a,b)
- (iii) $f(a) = f(b)$ then there exists $c \in (a,b)$; Such that $f'(c) = 0$

11. LANGRAGE'S MEAN VALUE THEOREM

If a function $f(x)$ is defined on $[a,b]$ satisfying

- (i) f is continuous on $[a,b]$

- (ii) f is differentiable on (a,b) then there exists $c \in (a,b)$ Such that $f'(c) = \frac{f(b) - f(a)}{b - a}$