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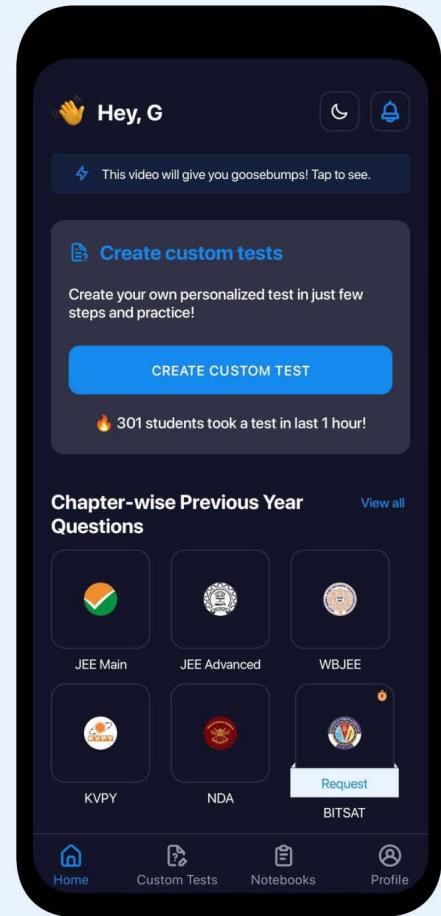


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QUADRATIC EQUATIONS & EXPRESSIONS

1. QUADRATIC EQUATION

An equation $ax^2 + bx + c = 0$ (where $a \neq 0$, and $a, b, c \in C$), is called a quadratic equation. Here a , b and c are called coefficients of the equation. This equation always has two roots. Let the roots be α and β .

$$\text{Then } \alpha + \beta = -\frac{b}{a} \quad (\text{sum of the roots}) \quad \alpha\beta = \frac{c}{a} \quad (\text{product of the roots})$$

The quantity $D = b^2 - 4ac$ is called discriminant of the equation.

$$\text{Roots of the equation are given by } x = \frac{-b \pm \sqrt{D}}{2a}$$

2. NATURE OF ROOTS

- (i) If $D > 0$, roots of the equation are real and distinct.
- (ii) If $D = 0$, roots of the equation are real and equal.
- (iii) If $D < 0$, roots of the equation are imaginary.
- (iv) If D is square of a rational number, roots of the equation are rational. (Provided a , b and c are rational numbers)
- (v) If $D > 0$ and D is not a perfect square, then roots are irrational

3. FORMATION OF AN EQUATION WITH GIVEN ROOTS

A quadratic equation whose roots are α and β is given by

$$(x - \alpha)(x - \beta) = 0$$

$$\Rightarrow x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e., $x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$

TRANSFORMATION OF AN EQAUTION

If α, β are roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are

- (i) $\frac{1}{\alpha}, \frac{1}{\beta}$ is $cx^2 + bx + a = 0$ (Replace x by $\frac{1}{x}$)
- (ii) $-\alpha, -\beta$ is $ax^2 - bx + c = 0$ (Replace x by $-x$)
- (iii) $k + \alpha, k + \beta$ is $a(x - k)^2 + b(x - k) + c = 0$ {Replace x by $(x - k)$ }
- (iv) α^n, β^n ($n \in N$) is $a(x^{1/n})^2 + b(x^{1/n}) + c = 0$ (Replace x by $x^{1/n}$)

(v) $\alpha^{1/n}, \beta^{1/n} (n \in N)$ is $a(x^n)^2 + b(x^n) + c = 0$ (Replace x by x^n)

(vi) $k\alpha, k\beta$ is $ax^2 + kbx + k^2c = 0$ (Replace x by $\frac{x}{k}$)

(vii) $\frac{\alpha}{k}, \frac{\beta}{k}$ is $k^2ax^2 + kbx + c = 0$ (Replace x by kx)

5. ROOTS IN SPECIAL CASES

$ax^2 + bx + c \equiv a(x - \alpha)(x - \beta)$ (If α and β are roots of the equation)

If $b^2 - 4ac > 0$	Then	
$a > 0, b > 0, c > 0$	$\alpha + \beta < 0, \alpha\beta > 0$	Both roots are negative
$a > 0, b > 0, c < 0$	$\alpha + \beta < 0, \alpha\beta < 0$	Roots are opposite in sign. Magnitude of negative root is more than the magnitude of positive root.
$a > 0, b < 0, c > 0$	$\alpha + \beta > 0, \alpha\beta > 0$	Both roots are positive
$a > 0, b < 0, c < 0$	$\alpha + \beta > 0, \alpha\beta < 0$	Roots are opposite in sign. Magnitude of positive root is more than the magnitude of negative root.

6. CONDITION FOR COMMON ROOT(S)

(i) Let $ax^2 + bx + c = 0$ and $dx^2 + ex + f = 0$ have a common root α (say). Then $a\alpha^2 + b\alpha + c = 0$ and $d\alpha^2 + e\alpha + f = 0$. Then

$$\Rightarrow (dc - af)^2 = (bf - ce)(ae - bd)$$

which is the required condition for the two equations to have a common root.

(ii) Condition for both the roots to be common is $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

8. GREATEST AND LEAST VALUES OF QUADRATIC EXPRESSION

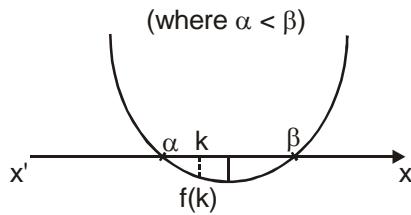
(i) If $a > 0$, then the quadratic expression $ax^2 + bx + c$ has least value $\frac{4ac - b^2}{4a}$ at $x = -\frac{b}{2a}$

(ii) If $a < 0$, then the quadratic expression $ax^2 + bx + c$ has greatest value $\frac{4ac - b^2}{4a}$ at $x = -\frac{b}{2a}$

10. LOCATION OF ROOTS (INTERVAL IN WHICH ROOTS LIE)

In some problems we want the roots α and β of the equation $ax^2 + bx + c = 0$ to lie in a given interval. For this we impose conditions on a , b and c . Since $a \neq 0$, we can take $f(x) = ax^2 + bx + c$.

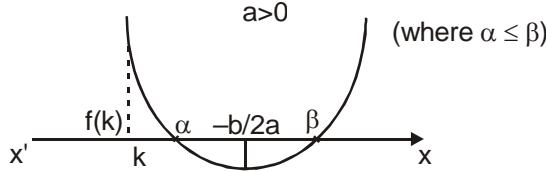
(i) A given number k will lie between the roots if $f(k) < 0$, $D > 0$.



In particular, the roots of the equation will be of opposite signs if 0 lies between the roots $\Rightarrow f(0) < 0$.

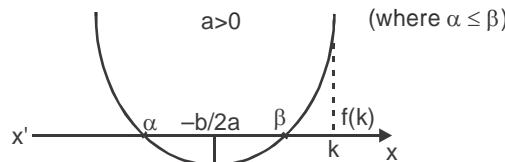
- (ii) Both the roots are greater than given number k if the following three conditions are satisfied

$$D \geq 0, -\frac{b}{2a} > k \text{ and } f(k) > 0$$



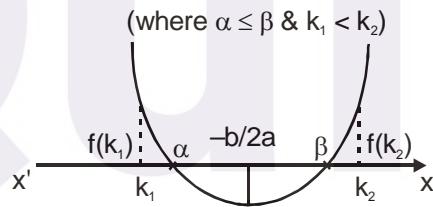
- (iii) Both the roots will be less than a given number k if the following conditions are satisfied:

$$D \geq 0, -\frac{b}{2a} < k \text{ and } f(k) > 0$$

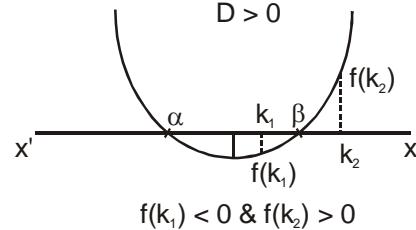
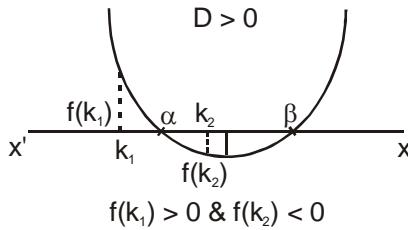


- (iv) Both the roots will lie in the given interval (k_1, k_2) if the following conditions are satisfied:

$$D \geq 0, k_1 < -\frac{b}{2a} < k_2 \text{ and } f(k_1) > 0, f(k_2) > 0$$



- (v) Exactly one of the root lies in the given interval (k_1, k_2) if $f(k_1)f(k_2) < 0$



11. THEORY OF EQUATIONS

- (i) If $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$, ($a_0, \dots, a_n \in \mathbb{R}$ and $a_n \neq 0$) then $p(x) = 0$ has n roots. (real /complex)
- (ii) A polynomial equation in x of odd degree has at least one real root (it has odd no. of real roots).
- (iii) If x_1, \dots, x_n are the roots of $p(x) = 0$, then $p(x)$ can be written in the form $p(x) \equiv a_n(x - x_1)(x - x_2) \dots (x - x_n)$.
- (iv) If α is a root of $p(x) = 0$, then $(x - \alpha)$ is a factor of $p(x)$ and vice - versa.

- (v) If x_1, \dots, x_n are the roots of $p(x) \equiv a_n x^n + \dots + a_0 = 0$, $a_n \neq 0$.

$$\text{Then } \sum_{i=1}^n x_i = -\frac{a_{n-1}}{a_n}, \quad \sum_{1 \leq i < j \leq n} x_i x_j = \frac{a_{n-2}}{a_n}, \quad x_1 x_2 \dots x_n = (-1)^n \frac{a_0}{a_n}.$$

- (vi) If equation $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$ has more than n distinct roots then $f(x)$ is identically zero.
- (vii) If $p(a)$ and $p(b)$ ($a < b$) are of opposite signs, then $p(x) = 0$ has odd number of real roots in (a, b) , i.e. it has at least one real root in (a, b) and if $p(a)$ and $p(b)$ are of same sign then $p(x) = 0$ has even number of real roots in (a, b) .
- (ix) If coefficients of $p(x)$ (polynomial in x written in descending order) have ' m ' changes in signs, then polynomial equation of n^{th} degree $p(x) = 0$ have at the most ' m ' positive real roots and if $p(-x)$ have ' t ' changes in sign, then $p(x) = 0$ have at most ' t ' negative real roots. By this we can find maximum number of real roots.
No. of complex roots = $n - (m + t)$ where $m + t < n$.
- (x) Imaginary roots of a quadratic equation always occur in conjugate pair.
- (xi) Irrational roots of a quadratic equation always occur in conjugate pair.

12. QUADRATIC EXPRESSION IN TWO VARIABLE

The general form of a quadratic expression in two variables x, y is

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c$$

The condition that this expression may be resolved into two linear rational factors is

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

or

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

- (i) If roots of quadratic equations $a_1 x^2 + b_1 x + c_1 = 0$ and $a_2 x^2 + b_2 x + c_2 = 0$ are in the same ratio

$$\left(\text{i.e. } \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} \right) \text{ then } \frac{b_1^2}{b_2^2} = \frac{a_1 c_1}{a_2 c_2}$$

- (ii) If one root is k times the other root of quadratic equation $a_1 x^2 + b_1 x + c_1 = 0$ then $\frac{(k+1)^2}{k} = \frac{b_1^2}{a_1 c_1}$