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1) <u>Problem Statement</u>

→ Modelling corona virus (COVID-19) outbreak for various countries.

→ Train a model to fit statistics for India.

→ The SIR (Susceptible, Infected, Removed/Recovered) model is used as a reference from lewuathe.com/covid-19-dynamics-with-sir-model.html.

Data: The dataset provided for this assignment had the fields:

- new daily cases
- new daily deaths
- total recoveries
- total infected
- total deaths

for a span of 94 days. Using this raw data we can calculate current active cases as (total infected - total deaths - total recoveries) and ~~po~~ susceptible population = N - total cases for each particular day. N is the population. Now; ~~sus~~ susceptible present, infected present and total recoveries serve as the gold standard SIR values. All the relevant plots were generated to get a proper feel of the data set. SIR is a good choice of a model for modelling such pandemics. Though N is supposed to be population size, putting N as 1.5 billion was not feasible as it makes everything else insignificant. Therefore, we set $N \sim 10^5$ and used the intuition of exposed population to infection λ as is a subset of original population.

By SIR Model,

$$\frac{dS}{dt} = -\frac{\beta SI}{N}, \quad \frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I, \quad \frac{dR}{dt} = \gamma I$$

We train our models to learn β and γ.

Now, let $\bar{S}, \bar{I}, \bar{R}$ be the computed SIR values using the model for time = 1 to d. d is the number of days i.e 94 in this case. We define a loss:

$\mathcal{L}(\beta, \gamma)$ = L2 norm of (infected present − $\bar{I}$). Denoting infected present as I.

$$\mathcal{L}(\beta, \gamma) = [\bar{I} - I]^T [\bar{I} - I]$$

$$\therefore \frac{\partial \mathcal{L}}{\partial \beta} = 2[\bar{I} - I] \frac{\partial \bar{I}}{\partial \beta} = 2[\bar{I} - I] \cdot \frac{\bar{I}(\beta + \Delta\beta) - \bar{I}(\beta)}{\Delta\beta}$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = 2[\bar{I} - I] \cdot \frac{\partial \bar{I}}{\partial \gamma} = 2[\bar{I} - I] \frac{\bar{I}(\gamma + \Delta\gamma) - \bar{I}(\gamma)}{\Delta\gamma}$$

- $\Delta\beta, \Delta\gamma$ are very small.
- We integrate over the range of t = 0 to 94 on the given differentials to generate all $\bar{I}$ values.
- Note, $\frac{\partial \bar{I}}{\partial \beta}$ cannot be computed analytically.

Following procedure gives a fairly decent approximation.

We have $w = [\beta, \gamma]^T$. Update for GD:

$$w^{(k+1)} \leftarrow w^{(k)} - \eta \, 2 \begin{bmatrix} (I-\bar{I}) \dfrac{\bar{I}(\beta+\Delta\beta) - \bar{I}(\beta)}{\Delta\beta} \\ (I-\bar{I}) \dfrac{\bar{I}(\gamma+\Delta\gamma) - \bar{I}(\gamma)}{\Delta\gamma} \end{bmatrix}$$

And for Newton's updates Hessian computation:

$$H = \begin{bmatrix} \dfrac{\partial^2 L}{\partial \beta^2} & \dfrac{\partial^2 L}{\partial\beta\partial\gamma} \\ \dfrac{\partial^2 L}{\partial\gamma\partial\beta} & \dfrac{\partial^2 L}{\partial\gamma^2} \end{bmatrix}$$

And $$\frac{\partial^2 L}{\partial \beta^2} = 2\left[(\bar{I} - I) \frac{\partial^2 \bar{I}}{\partial \beta^2} + \left(\frac{\partial I}{\partial \beta}\right)^2 \right]$$

where $$\frac{\partial^2 \bar{I}}{\partial \beta^2} = \frac{I(\beta + 2\Delta\beta) + I(\beta) - 2I(\beta+\Delta\beta)}{(\Delta\beta)^2}$$

$$\frac{\partial^2 L}{\partial\beta\partial\gamma} = \frac{\partial^2 L}{\partial\gamma\partial\beta} = 2\left[(\bar{I} - I) \frac{\partial^2 I}{\partial\gamma\partial\beta} + \left(\frac{\partial I}{\partial \beta}\right)\left(\frac{\partial I}{\partial \gamma}\right) \right]$$

where $$\frac{\partial^2 I}{\partial\beta\partial\gamma} = \frac{I(\beta+\Delta\beta, \gamma+\Delta\gamma) - I(\beta, \gamma+\Delta\gamma) - I(\beta+\Delta\beta, \gamma) + I(\beta, \gamma)}{(\Delta\beta)(\Delta\gamma)}$$

$$\frac{\partial^2 L}{\partial \gamma^2} = 2\left[(\bar{I} - I) \frac{\partial^2 \bar{I}}{\partial \gamma^2} + \left(\frac{\partial I}{\partial \gamma}\right)^2 \right]$$

where $$\frac{\partial^2 \bar{I}}{\partial \gamma^2} = \frac{I(\gamma + 2\Delta\gamma) + I(\gamma) - 2I(\gamma+\Delta\gamma)}{(\Delta\gamma)^2}$$

Newton's Update:

$$w^{(k+1)} \leftarrow w^{(k)} - H^{-1} \begin{bmatrix} (\bar{I} - I)[\bar{I}(\beta+\Delta\beta) - \bar{I}(\beta)] / \Delta\beta \\ (\bar{I} - I)[\bar{I}(\gamma+\Delta\gamma) - \bar{I}(\gamma)] / \Delta\gamma \end{bmatrix}$$