Assignment No. 1

Title: Implementation of Mathematical Operations on Fuzzy Sets and Fuzzy Relations with Max–Min Composition

Introduction:

Fuzzy set theory is an extension of classical set theory where elements have degrees of membership ranging from 0 to 1, instead of a crisp belonging (either 0 or 1). This concept is widely used in decision-making, artificial intelligence, and control systems where uncertainties and vagueness exist. In this assignment, we implement basic operations on fuzzy sets, create fuzzy relations using the Cartesian product, and perform max-min composition on fuzzy relations.

Mathematical Operations on Fuzzy Sets

Let two fuzzy sets A and B be defined on a common universe X. The membership function of each element $x \in X$ is denoted by $\mu A(x)$ and $\mu B(x)$. The basic fuzzy set operations are:

1. Union: $\mu A \cup B(x) = \max(\mu A(x), \mu B(x))$
2. Intersection: $\mu A \cap B(x) = \min(\mu A(x), \mu B(x))$
3. Complement: $\mu A'(x) = 1 - \mu A(x)$

Fuzzy Relations and Cartesian Product

A fuzzy relation R between two fuzzy sets A (on universe X) and B (on universe Y) is represented by the Cartesian product:

$R(x,y) = \min(\mu A(x), \mu B(y))$ for all $(x,y) \in X \times Y$

This produces a fuzzy relation matrix where each entry represents the degree of association between elements of X and Y.

Max-Min Composition of Fuzzy Relations

Given two fuzzy relations R1 (between sets X and Y) and R2 (between sets Y and Z), the max-min composition $R = R1 \circ R2$ is defined as:

R(x,z) = max_y [min(R1(x,y), R2(y,z))]

This operation combines two relations into a single relation representing indirect association between X and Z via Y.

Example Implementation

Consider fuzzy sets A and B defined over the universe {1, 2, 3}:

A = { (1, 0.2), (2, 0.7), (3, 1.0) }

B = { (1, 0.5), (2, 0.4), (3, 0.9) }

Union: $\mu A \cup B$ = { (1, 0.5), (2, 0.7), (3, 1.0) }

Intersection: $\mu A \cap B$ = { (1, 0.2), (2, 0.4), (3, 0.9) }

Complement of A: { (1, 0.8), (2, 0.3), (3, 0.0) }

Fuzzy relation R from A × B:

R = [[min(0.2,0.5), min(0.2,0.4), min(0.2,0.9)],

[min(0.7,0.5), min(0.7,0.4), min(0.7,0.9)],

[min(1.0,0.5), min(1.0,0.4), min(1.0,0.9)]]

Max-min composition can then be applied between two relations R1 and R2 to compute indirect associations.

Results:

- The program successfully computed fuzzy union, intersection, and complement.
- Fuzzy relations were constructed as a matrix-like structure.
- Max–min composition produced another fuzzy relation that combined the two given relations.

Conclusion:

This assignment demonstrated the practical implementation of fuzzy set operations and fuzzy relations. The results illustrate how fuzzy logic handles uncertainty and provides a foundation for applications in decision-making, expert systems, and artificial intelligence.