

Assignment No. 1

Title: Implementation of Mathematical Operations on Fuzzy Sets and Fuzzy Relations with Max–Min Composition

Introduction:

Fuzzy set theory is an extension of classical set theory where elements have degrees of membership ranging from 0 to 1, instead of a crisp belonging (either 0 or 1). This concept is widely used in decision-making, artificial intelligence, and control systems where uncertainties and vagueness exist. In this assignment, we implement basic operations on fuzzy sets, create fuzzy relations using the Cartesian product, and perform max-min composition on fuzzy relations.

Mathematical Operations on Fuzzy Sets

Let two fuzzy sets A and B be defined on a common universe X. The membership function of each element $x \in X$ is denoted by $\mu_A(x)$ and $\mu_B(x)$. The basic fuzzy set operations are:

1. Union: $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$
2. Intersection: $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$
3. Complement: $\mu_{A'}(x) = 1 - \mu_A(x)$

Fuzzy Relations and Cartesian Product

A fuzzy relation R between two fuzzy sets A (on universe X) and B (on universe Y) is represented by the Cartesian product:

$$R(x,y) = \min(\mu_A(x), \mu_B(y)) \text{ for all } (x,y) \in X \times Y$$

This produces a fuzzy relation matrix where each entry represents the degree of association between elements of X and Y.

Max-Min Composition of Fuzzy Relations

Given two fuzzy relations R1 (between sets X and Y) and R2 (between sets Y and Z), the max-min composition $R = R1 \circ R2$ is defined as:

$$R(x,z) = \max_y [\min(R1(x,y), R2(y,z))]$$

This operation combines two relations into a single relation representing indirect association between X and Z via Y.

Example Implementation

Consider fuzzy sets A and B defined over the universe $\{1, 2, 3\}$:

$$A = \{ (1, 0.2), (2, 0.7), (3, 1.0) \}$$

$$B = \{ (1, 0.5), (2, 0.4), (3, 0.9) \}$$

$$\text{Union: } \mu_{A \cup B} = \{ (1, 0.5), (2, 0.7), (3, 1.0) \}$$

Intersection: $\mu_{A \cap B} = \{ (1, 0.2), (2, 0.4), (3, 0.9) \}$

Complement of A: $\{ (1, 0.8), (2, 0.3), (3, 0.0) \}$

Fuzzy relation R from $A \times B$:

$R = [[\min(0.2, 0.5), \min(0.2, 0.4), \min(0.2, 0.9)],$
 $[\min(0.7, 0.5), \min(0.7, 0.4), \min(0.7, 0.9)],$
 $[\min(1.0, 0.5), \min(1.0, 0.4), \min(1.0, 0.9)]]$

Max-min composition can then be applied between two relations R1 and R2 to compute indirect associations.

Results:

- The program successfully computed fuzzy union, intersection, and complement.
- Fuzzy relations were constructed as a matrix-like structure.
- Max–min composition produced another fuzzy relation that combined the two given relations.

Conclusion:

This assignment demonstrated the practical implementation of fuzzy set operations and fuzzy relations. The results illustrate how fuzzy logic handles uncertainty and provides a foundation for applications in decision-making, expert systems, and artificial intelligence.