Ranking using Pairwise Preferences

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Summary:

We have implemented the Rank centrality [1], Rank aggregation via NNM [2], and SVM rank aggregation [3] algorithms. We have run these algorithms on Synthetic Dataset(BTL Model) and on Real Dataset (SOC)[4]. We have compared these algorithms on the basis of two measures which are Pairwise Disagreement Error and DL_1 error(as described in Rank centrality [1]). The dimension along which we have considered these measures is k the number of times a pair is compared in the dataset. We have also measured the empirical time complexity of each of the algorithms as we vary n the number of items. It can be seen from the results that Rank Centrality perfoms better than NNM and SVM Rank Aggregation on PDE metric for BTL Dataset. NNM performs slightly better than SVM and also Rank Centrality is more robust to missing values as compared to NNM and SVM Rank Aggregation.

Problem Statement:

To study ranking from pairwise preferences where there is a set of n items and one is given pairwise comparisons among these items, for example pairwise preferences on candidates in an election, results of a sequence of pairwise games etc.To implement and compare the Rank centrality [1], Rank aggregation via NNM [2], and SVM rank aggregation [3] algorithms on the number of pairwise comparisons each algorithm requires for learning an optimal ranking, for various pairwise comparison models.

DataSets Used:

BTL Dataset: When comparing pairs of items from n items of interest, represented as $[n] = 1, ..., n$, the Bradley-Terry-Luce model assumes that there is a weight or score $w_i \in R_+$ associated with each item $i \in [n]$. The outcome of a comparison for pair of items i and j is determined only by the corresponding weights w_i and w_j.
Let Y_{ij}^l denote the outcome of the l-th comparison of the pair i and j, such that $Y_{ij}^l = 1$ if j is preferred over i and 0 otherwise. Then, according to the BTL model,
$Y_{ij}^l = 1$ with probability $\frac{w_i}{w_i+w_j}$.
$Y_{ij}^l = 0$ otherwise.

Real Dataset: SOC Strict Order Complete Dataset[4] which contains rankings given by n users for a set of items.

Measures under Consideration:

Pairwise Disagreement Error: Given the ground truth matrix P we define the expected pairwise disagreement error of a permutation $\sigma \in S_n$ (set of all permutation of n items) as
$err = \sum_{i \neq j} P_{ij} 1_{\sigma(i)<\sigma(j)}$

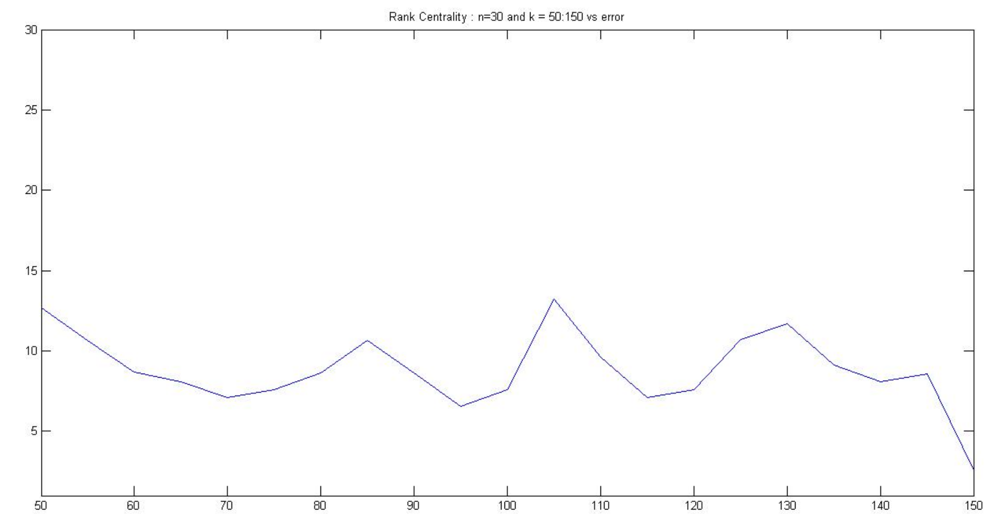
DL_1 Error: The DL_1 error[1] is defined as:
Let σ_{GT} be the ranking we get by applying our choice of rank aggregation algorithm to the complete dataset, and σ_{Sample} be the ranking we get from sampled dataset. To measure the resulting error in the ranking, the following metric is used:
$DL_1(\sigma_{GT}\ ,\ \sigma_{Sample}\ ) = \frac{1}{n} \sum_i |\sigma_{GT}(i)$ - $\sigma_{Sample}(i)|$

Results:

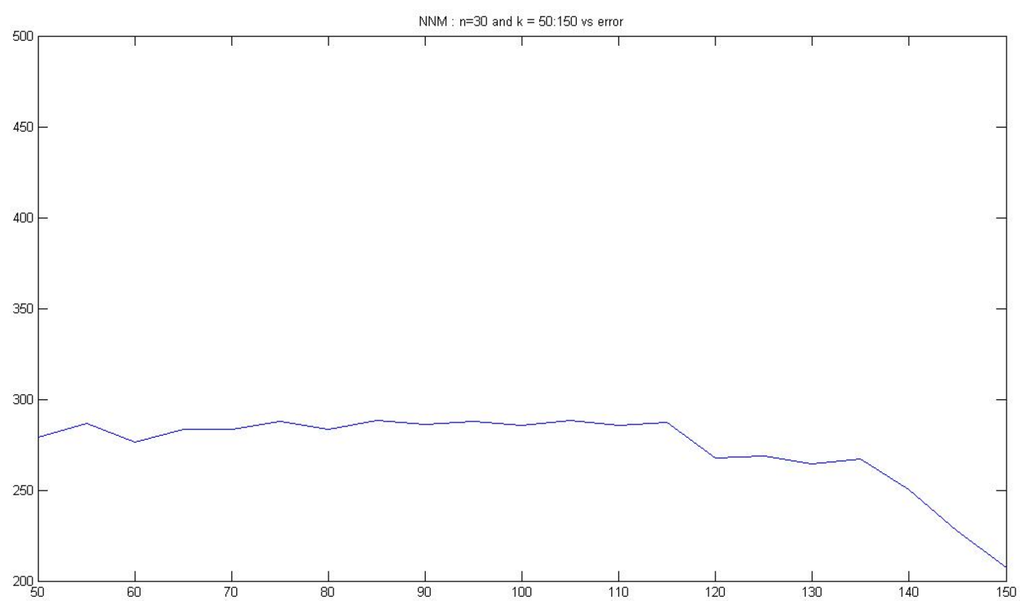
PairWise Disagreement error (BTL Dataset):

PDE versus number of comparisons per pair(k)

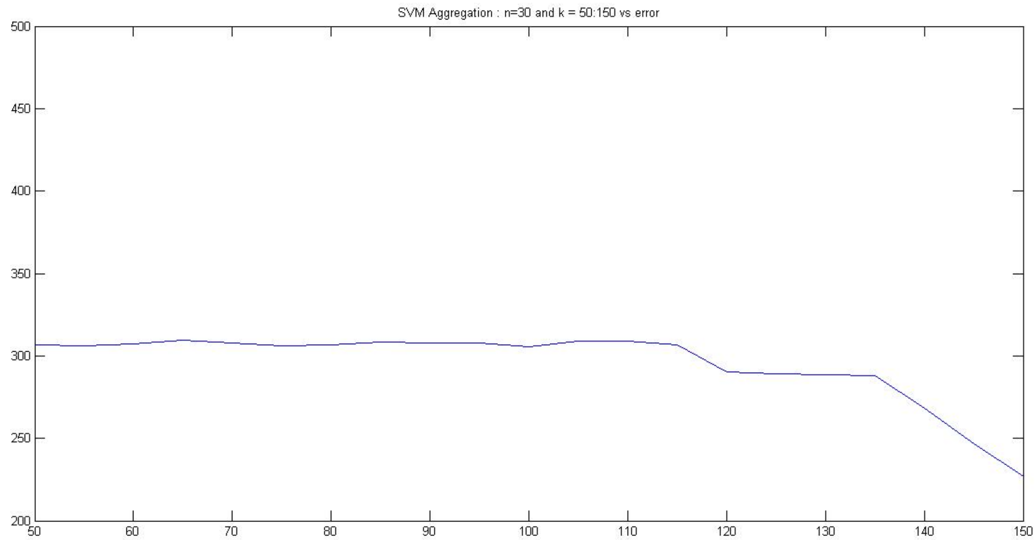
n = 30
Rank Centrality:



Rank Aggregation via NNM:



SVM Rank-Aggregation:

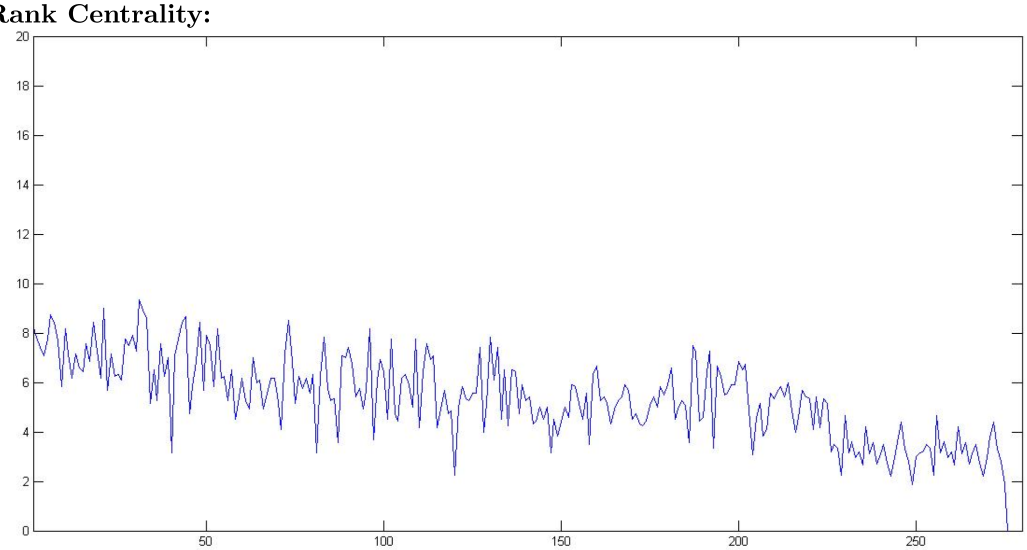


DL_1 error(BTL Dataset):

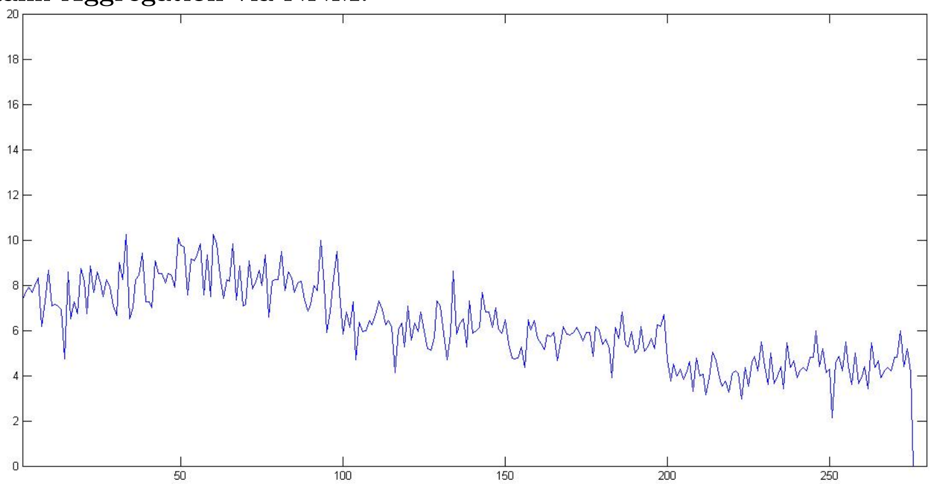
n = 24, k=64

DL_1 versus the number of samples:

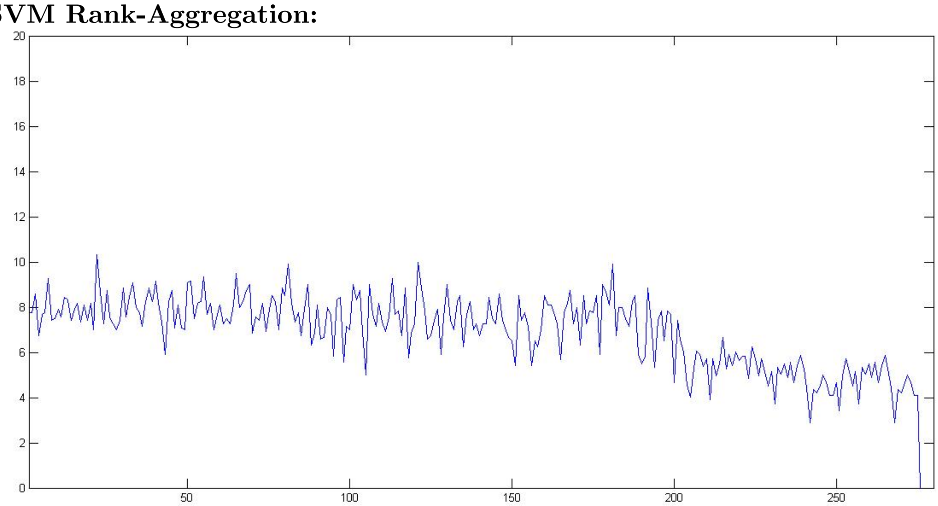
Rank Centrality:



Rank Aggregation via NNM:

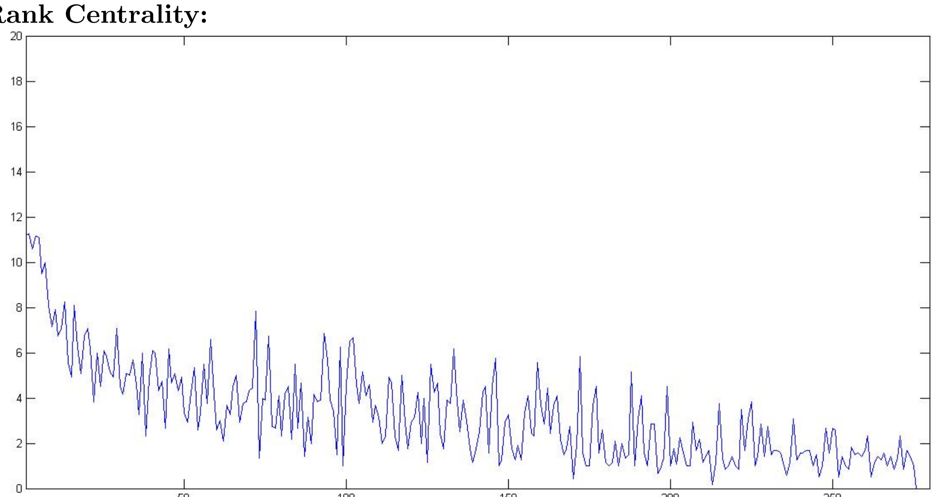


SVM Rank-Aggregation:

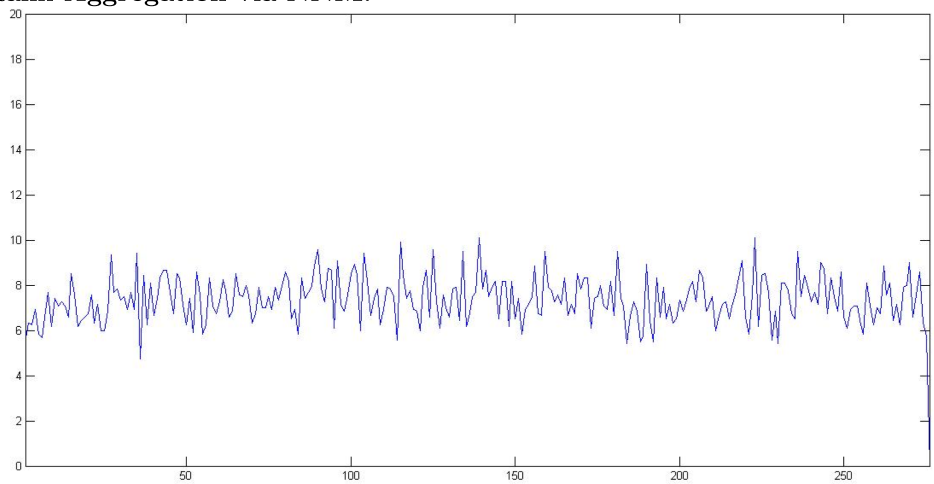


SOC Dataset

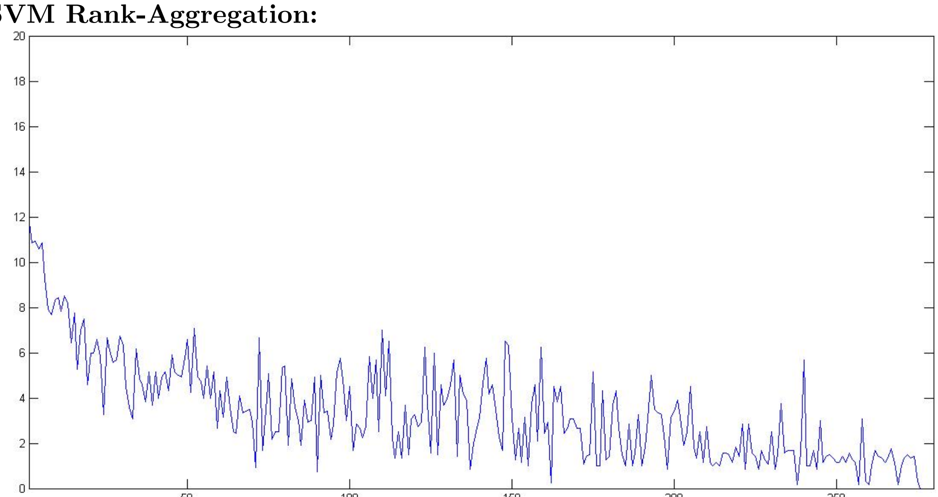
Rank Centrality:



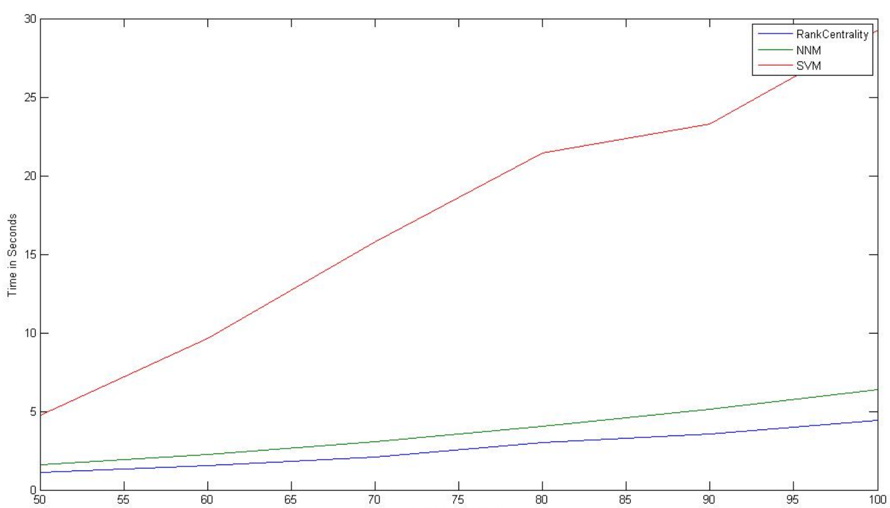
Rank Aggregation via NNM:



SVM Rank-Aggregation:



Empirical Time estimation:



Deductions:

1. It can be seen from the results that Rank Centrality perfoms better than NNM and SVM-Aggregation on PDE metric for BTL Dataset. NNM performs slightly better than SVM.

2. We can see from DL_1 error on BTL and SOC Dataset that Rank Centrality is more robust to missing values as compared to NNM and SVM Rank Aggregation.

3. The Emperical time taken by each of the algorithms increases as we increase the number of items n. Rank Centrality and NNM are more time efficient than SVM Aggregation.

References

[1] Sahand Negahban, Sewoong Oh, and Devavrat Shah. Rank centrality: Ranking from pairwise compar- isons. arXiv preprint arXiv:1209.1688, 2012.

[2] David F Gleich and Lek-heng Lim. Rank aggregation via nuclear norm minimization. In Proceedings of the 17th ACM SIGKDD international conference on Knowledge discovery and data mining ACM, 2011.

[3] Arun Rajkumar and Shivani Agarwal. A statistical convergence perspective of algorithms for rank aggregation from pairwise data. In Proceedings of the 31st International Conference on Machine Learning, 2014.

[4] (strict ordering complete list)www.preflib.org/data/packs/index.phpsoi