D.G.E -HR.SEC. EXAMINATION MAR - 2017

EXAM ROLL NUMBER **893891**

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PERMANENT REG.NO : **1710883891**

SUBJECT : **041 MATHEMATICS (ENG)**

(C) APPLIED FOR : SCAN COPY

D.G.E -HR.SEC. EXAMINATION MAR - 2017

(B) (GADBFIHFHDF)

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SUB CODE : 041
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SUBJECT : **041 MATHEMATICS (ENG)**

Marks already Awarded	Marks after Retotalling / Revaluation	+/-	Marks in Difference

Designation	Signature
Examiner 1	
Examiner 2	
Examiner 3	

J.D (Ret . / Rev)

J.D (H.S)

Director

AE : 511

Bundle No: 6 | 3 | 0

மதிப்பெண்கள் பக்கம் / Marking Page

Total Marks 1 | 6 | 1

13.4.17

அரசுத் தேர்வுகள் துறை
DEPARTMENT OF GOVERNMENT EXAMINATIONS

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HSE

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Qn Booklet Series
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A	
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விடைத்தாள் திருத்துவோர் நிறைவு செய்ய வேண்டியவை
FOR THE USE OF EXAMINERS ONLY

வினாவாரியாக மொத்தம் Questionwise Total										பக்கவாரியாக மொத்தம் Pagewise Total			
வினா எண் Q.No.	மதிப் பெண்கள் Marks	வினா எண் Q.No.	மதிப் பெண்கள் Marks	வினா எண் Q.No.	மதிப் பெண்கள் Marks	வினா எண் Q.No.	மதிப் பெண்கள் Marks	வினா எண் Q.No.	மதிப் பெண்கள் Marks	பக்க எண் Page No.	மதிப் பெண்கள் Marks	பக்க எண் Page No.	மதிப் பெண்கள் Marks
1	1	21	1	41	6	61	10	81		1	11	21	
2	1	22	1	42	3	62	10	82		2	26	22	
3	1	23	1	43	6	63		83		3	11	23	
4	1	24	1	44	6	64	5	84		4	4	24	
5	1	25	1	45	2	65		85		5	5	25	
6	0	26	1	46	6	66		86		6	10	26	
7	1	27	1	47		67	10	87		7	10	27	
8	1	28	1	48		68	4	88		8	10	28	
9	1	29	1	49		69		89		9	10	29	
10	1	30	1	50		70	0	90	—	10	10	30	
11	1	31	1	51	6	71		91		11	0	31	
12	1	32	1	52	6	72		92		12	10	32	
13	1	33	1	53	3	73		93		13	9	33	
14	1	34	1	54	0	74		94		14	6	34	
15	0	35	1	55		75		95		15	8	35	
16	1	36	1	56	10	76		96		16	6	36	
17	1	37	1	57	10	77		97		17	6	37	
18	1	38	1	58	10	78		98		18	9	38	
19	1	39	1	59	10	79		99		19	0	39	
20	1	40	1	60		80		100		20		40	
மொத்தம் TOTAL	18	மொத்தம் TOTAL	20	மொத்தம் TOTAL	84	மொத்தம் TOTAL	39	மொத்தம் TOTAL		மொத்தம் TOTAL	161	மொத்தம் TOTAL	

வினாவாரியாக ஒட்டு மொத்தம்
Question-wise Grand Total: 161

பக்கவாரியாக ஒட்டு மொத்தம்
Page-wise Grand Total: 161

AE: [signature] 13/4/17

SO: [signature] 13/4/17 AN
SO:27

CE: [signature] 13/4

GCP--388-3--33,95,000 Cps.--13-8-16 (Acer-5)

AE 511

தேர்வு எழுதுபவர் செய்யக்கூடியவை மற்றும் செய்யக்கூடாதவை
Do's & Dont's for Candidates

1. முகப்புச்சீட்டில் உரிய இடத்தில் கையொப்பமிட வேண்டும்.
   Put your signature in the Top sheet in the appropriate place.
2. விடைத்தாளில் ஒரு பக்கத்திற்கு 20 முதல் 25 வரிகள் வரை எழுதவேண்டும்.
   Write 20 to 25 lines in a page.
3. விடைத்தாளின் இருபுறத்திலும் எழுத வேண்டும்.
   Write answers in both sides of paper.
4. செய்முறைகள் யாவும் விடைத்தாளின் பகுதியில் இடம் பெறவேண்டும்.
   All rough works must be done on the lower part of the page:
5. சரியான வினா எண் தவறாமல் எழுத வேண்டும். வினா எண் எழுதாத மற்றும் தவறான வினா எண்கள் குறிப்பிடப்பட்டு எழுதப்பட்ட விடைகள் மதிப்பீடு செய்யப்படமாட்டாது.
   Write the question numbers without fail. Answers without question numbers and wrong question numbers will not be valued.
6. இரு விடைகளுக்கிடையே இடைவெளி விட்டு எழுத வேண்டும்.
   Leave space between two answers.
7. வினாத்தாளின் வரிசை (A or B) மதிப்பெண்கள் பக்கத்தில் குறிக்கப்படல் வேண்டும்
   Question paper booklet series (A or B) should be mentioned in the Marking Page
8. விடைத்தாளில் நீலம்/கருப்புமை கொண்ட பேனாவால் விடைகளை தெளிவாக எழுத வேண்டும்.
   Answers must be legibly written either in Blue or Black ink pen.
9. விடைத்தாளில் எழுதாத பக்கங்களில் குறுக்குக்கோடு இடவேண்டும்.
   Cross the unwritten pages.

1. வினாத்தாளில் எந்தவித குறியீடும் இடக் கூடாது.
   No marking in the question paper.
2. விடைத்தாளை சேதப்படுத்தக் கூடாது.
   Don't damage the answer paper.
3. விடைத்தாளில் எந்த ஒரு பக்கத்திலும் தேர்வு எண்/பெயர் எழுதக்கூடாது.
   Don't write name, Register Number in any page of the answer book.
4. வண்ணக்கலர் கொண்ட பேனா/ பென்சில் எதையும் பயன்படுத்தக் கூடாது.
   Don't write with sketch / colour pencils.
5. விடைத்தாள் கோட்டின் இடது ஓரத்தில் எழுதக்கூடாது.
   Don't write on the left margin.
6. விடைத்தாள் புத்தகத்தின் எந்த தாளையும் கிழிக்கவோ/நீக்கவோ கூடாது.
   Don't tare / remove any page from the answer book.

விளாா எண். Qn.No.

MATHEMATICS

PART - A

1. 1) (6 , 6)

2. 2) $x = ce^{-my}$

3. 2) $y = \pm \frac{5i}{6} x$

4. 3) ω^{n-k}

5. (4) $\frac{1}{2}$

6. 3) of order 1 and degree ~~6~~

7. 1) (0 , 0 , −4)

8. 1) $(-\frac{1}{2}, -8)$

9. 4) $(25, \frac{1}{5})$

10. 4) Two rows can have same number of zeros before the first non-zero entry.

11. 2) an asymptote parallel to y-axis.

12. 2) 2/3

11

வினா எண். Qn.No.	
13.	3) i , iii , iv
14.	3) $\frac{dw}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$
15.	1) 1
16.	1) straight line $x = 1/4$
17.	(3) $f''(x_0) = 0$
18.	1) $-t_2$
19.	4) $\frac{1}{5\sqrt{2\pi}}$
20.	1) $[\vec{r} - \vec{a}, \vec{u}, \vec{v}] = 0$
21.	3) $\frac{5\sqrt{3}}{2}$
22.	2) [5]
23.	3) 2
24.	1) 9
25.	1) $k^3 \det(A)$

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26. 2) $\frac{1}{3}$

27. 4) $\vec{r} = (\vec{i} + 3\vec{j} + 5\vec{k}) + t(\vec{i} + 5\vec{j} + 3/2\vec{k})$

28. 2) $\begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$

29. 4) 5

30. 2) $\cos x$

31. 2) $\left(\log \frac{\Delta_2}{\Delta_1}, \log \frac{\Delta_3}{\Delta_1}\right)$

32. 1) Gradient of velocity / time graph.

33. 4) b : a

34. 2) $(-\pi, \pi]$

35. 2) 2π

36. 2) $f(x) \geq 0$

37. 4) 60°

38. 1) $\frac{dy}{dx} = \lambda y$

39. 2) 0

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40. 4) $\frac{1}{n}\cos^{n-1}x\sin x+\left(\frac{n-1}{n}\right)I_{n-2}$

PART-C

67. Let T be the temperature of coffee.

$T=100^\circ C$

S be the room temperature

$S=15^\circ C$.

$$\frac{dT}{dt}\propto(T-S)$$

It $\frac{dT}{dt}=k(T-S)$

Integrate,

$$\int\frac{dT}{dt}=\int k(T-S)$$

$$\int\frac{dT}{T-S}=\int k\,dt$$

$$\log(T-S)=kt+c$$

$$T-S=e^{kt+c}$$

$$T-S=c\cdot e^{kt}$$

$$\therefore \boxed{T=ce^{kt}+S}$$

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Temperature (T)	Time (t)
100	0
60	5
?	10

When $T = 100^\circ c$, $t = 0$, $s = 15^\circ c$

$$T = ce^{kt} + s$$

$$100 = ce^{0} + 15$$

$$\therefore \boxed{c = 85}$$

When $T = 60^\circ c$, $t = 5$, $s = 15^\circ c$

$$60 = ce^{k(5)} + 15$$

$$45 = 85 \cdot e^{5k}$$

$$\therefore \boxed{e^{5k} = 45/85}$$

When $T = ?$, $t = 10$, ~~$s = 15^\circ c$~~

$$T = ce^{10k} + 15$$

$$= 85\left(\frac{45}{85}\right)^{2} + 15$$

$$= \cancel{85} \times \left(\frac{\cancel{45}^{9} \times 45}{\cancel{85}_{1} \times \cancel{85}_{17}}\right) + 15$$

9×45 ... $\frac{17 \times 8}{136}$ 5 40

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$$= \frac{\cancel{405}^{23.82}}{\cancel{17}} + 15$$

$$= 23.82 + 15$$

$$T = 38.82^\circ C$$

$\therefore$ Temperature after a further interval of 5 minutes is $T = 38.82^\circ c$.

68. G be a group under matrix multiplication.

Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $A = \begin{pmatrix} w & 0 \\ 0 & w^2 \end{pmatrix}$,

$B = \begin{pmatrix} w^2 & 0 \\ 0 & w \end{pmatrix}$, $C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & w^2 \\ w & 0 \end{pmatrix}$,

$E = \begin{pmatrix} 0 & w \\ w^2 & 0 \end{pmatrix}$

$\therefore G = \{I, A, B, C, D, E\}$

The Cayley's Table is

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I	A	B	C	D	E
A	A	B	C	D	E
B	B	~~B~~ I	E	C	D
C	C	D	A	B	B
D	D	E	~~B~~ A	~~B~~ I	~~A~~ A
E	E	C	B	~~I~~ A	~~A~~ I

(i) CLOSURE AXIOM:

All the 36 elements in the cayley's table belongs to G.

Closure Axiom is True.

(ii) ASSOCIATIVE AXIOM:

Associative is always true under Matrix multiplication.

∴ Associative axiom is true.

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எண்.
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(iii) IDENTITY AXIOM:

I is the identity element

$I \in G$

$\therefore$ Identity axiom is true.

(iv) INVERSE AXIOM:

The inverse of A is A

Inverse of B is B

Inverse of C is

Inverse of D is

Inverse of E is

$\therefore$ Inverse axiom is true.

$\therefore (G, *)$ forms a group

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64. Given,

$$U = \frac{x}{y^2} - \frac{y}{x^2}$$

To Verify:

$$\frac{\partial^2 U}{\partial x \partial y} = \frac{\partial^2 U}{\partial y \partial x}$$

Let $L.H.S = \frac{\partial^2 U}{\partial x \partial y}$

$$\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left(\frac{x}{y^2} - \frac{y}{x^2}\right)$$

$$= \left(\frac{-x}{y^3} - \frac{1}{x^2}\right)$$

$$\frac{\partial^2 U}{\partial x \partial y} = \frac{\partial^2 U}{\partial x}\left(\frac{-x}{y^3} - \frac{1}{x^2}\right)$$

$$= \left(\frac{-1}{y^3} + \frac{1}{x^3}\right)$$

$$= \left(\frac{1}{x^3} - \frac{1}{y^3}\right)$$

$$\therefore \frac{\partial^2 U}{\partial x \partial y} = \left(\frac{1}{x^3} - \frac{1}{y^3}\right) \quad \text{--- (1)}$$

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Let R.H.S $= \frac{\partial^2 u}{\partial y \partial x}$

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{y^2} - \frac{y}{x^2}\right)$

$= \left(\frac{1}{y^2} + \frac{y}{x^3}\right)$

$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial u}{\partial y}\left(\frac{1}{y^2} + \cancel{\frac{y}{x^3}}\right)$

$= \left(\frac{-1}{y^3} + \frac{1}{x^3}\right)$

$= \left(\frac{1}{x^3} - \frac{1}{y^3}\right)$

$\therefore \frac{\partial^2 u}{\partial y \partial x} = \left(\frac{1}{x^3} - \frac{1}{y^3}\right)$ — (2)

From (1) and (2),

$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

Hence verified.

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56. Given,

$$x+y+2z=4, \quad -①$$

$$2x+2y+4z=8, \quad -②$$

$$3x+3y+6z=12 \quad -③$$

using determinant method,

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{vmatrix}$$

$$= 1(12-12) - 1(12-12) + 2(6-6)$$

$$= 1(0) - 1(0) + 2(0)$$

$$\Delta = 0$$

$$\Delta x = \begin{vmatrix} 4 & 1 & 2 \\ 8 & 2 & 4 \\ 12 & 3 & 6 \end{vmatrix}$$

$$= 4(12-12) - 1(48-48) + 2(24-24)$$

$$\Delta x = 0$$

$$\Delta y = \begin{vmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ 3 & 12 & 6 \end{vmatrix}$$

10

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$$= 1(48-48) - 4(12-12) + 2(24-24)$$

$\Delta y = 0$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 2 & 8 \\ 3 & 3 & 12 \end{vmatrix}$$

$\Delta z = 0$ $\quad \because [C_1 = C_2]$

Here,

$\Delta = 0,$

$\Delta x = 0,$

$\Delta y = 0, \ \Delta z = 0.$

All the minors of $\Delta, \Delta x, \Delta y,$ $\Delta z = 0.$

$$\left(\begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix} = 6 - 6 = 0\right)$$

(Also, atleast one non-zero element is present $2 \neq 0$)

$\therefore$ The system is consistent and it reduces to a single equation.

Let $y = k_1, \ z = k_2.$

Put in (1),

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$x + y + 2z = 4$

$x + k_1 + 2k_2 = 4$

$x = 4 - k_1 - 2k_2$

$\therefore$ The solution set is

$(4 - k_1 - 2k_2, k_1, k_2)$

where $k_1, k_2 \in R$

58. Given,

$\vec{OA} = \vec{a} = 3\vec{i} + 4\vec{j} + 2\vec{k}$,

$\vec{OB} = \vec{b} = 2\vec{i} - 2\vec{j} - \vec{k}$, $\vec{OC} = \vec{c} = 7\vec{i} + \vec{k}$.

The plane is in three-points form.

(i) VECTOR EQUATION:

Let $\vec{a} = 3\vec{i} + 4\vec{j} + 2\vec{k}$,

$\vec{b} = 2\vec{i} - 2\vec{j} - \vec{k}$,

$\vec{c} = 7\vec{i} + \vec{k}$.

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$\therefore$ The vector Equation is

$$\vec{r} = (1-s-t)\vec{a} + s\vec{b} + t\vec{c}.$$

$$\vec{r} = (1-s-t)\, 3\vec{i}+4\vec{j}+2\vec{k} + s(2\vec{i}-2\vec{j}-\vec{k}) + t(7\vec{i}+\vec{k})$$

where s, t are scalars.

(ii) CARTESIAN EQUATION:

Let $(x_1, y_1, z_1) = (3, 4, 2)$

$(x_2, y_2, z_2) = (2, -2, -1)$

$(x_3, y_3, z_3) = (7, 0, 1)$

$\therefore$ The cartesian Equation is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-3 & y-4 & z-2 \\ 2-3 & -2-4 & -1-2 \\ 7-3 & 0-4 & 1-2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-3 & y-4 & z-2 \\ -1 & -6 & -3 \\ 4 & -4 & -1 \end{vmatrix} = 0$$

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$$x-3(6-12) - (y-4)(1+12) + z-2(4+24) = 0$$

$$x-3(-6) - y+4(13) + z-2(28) = 0$$

$$-6x+18-13y+52+28z-56 = 0.$$

$$-6x-13y+28z+60-56 = 0$$

$$-6x-13y+28z+14 = 0.$$

$(\div) - \Rightarrow$ $6x+13y-28z-14=0$ is the required cartesian Equation of a plane.

162.

Given,

One of the asymptote is

$$x+2y-5=0$$

Another asymptote is $2x-y+k_1=0$

The points are $(6,0)$ and $(-3,0)$

The combined equation of an asymptote is

$$(x+2y-5)(2x-y+k_1)=0.$$

$\frac{4\times 13}{52}$ $\frac{18}{52}$ $\overline{70}$ $\frac{86}{56}$ $\overline{14}$

10

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Equation of a rectangular hyperbola is

$(x+2y-5)(2x-y+k_1)+k_2=0$

(since, the combined equation of asymptotes and hyperbola differ only by a constant term)

point $(6,0)$ passes through the equation,

$(6+2(0)-5)(2(6)-0+k_1)+k_2=0$

$1(12+k_1)+k_2=0$

$k_1+k_2=-12$ —①

point $(-3,0)$ passes through the same equation,

$(-3+2(0)-5)(2(-3)-0+k_1)+k_2=0$

$-8(-6+k_1)+k_2=0$

$48-8k_1+k_2=0$

$-8k_1+k_2=-48$ —②

~~① × 8 ⟹ $8k_1+8k_2=-96$~~

~~② ⟹ $-8k_1+k_2=-48$~~

~~$9k_2=$~~

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① $\Rightarrow$ $k_1 + k_2 = -12$

② $\Rightarrow$ $-8k_1 + k_2 = -48$

(+) (+) (+)

$9k_1 = 36$

$\therefore \boxed{k_1 = 4}$

$k_1 + k_2 = -12$

$4 + k_2 = -12$

$\therefore \boxed{k_2 = -16}$

$\therefore$ The required equation of R·H is

$(x+2y-5)(2x-y+4) - 16 = 0$

10

59. Given,

$x^4 - x^3 + x^2 - x + 1 = 0$

$\Rightarrow$ $1 - x + x^2 - x^3 + x^4 = 0$

It in in G·P,

$r = \frac{t_2}{t_1}$

$= \frac{-x}{1} \Rightarrow r = -x$

$a = 1$

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$\therefore S_n = \left(\frac{1-r^n}{1-r}\right)$

$= \left(\frac{1-x^5}{1-x}\right)$

Here $x \neq 1$

$\frac{1-x^5}{1-x} \neq +1$

$1-(x^5) = 0$

$(x^5) = 1$

$x = (1)^{1/5}$

$= (\cos\pi + i\sin\pi)^{1/5}$

$\because (1 = \cos\pi + i\sin\pi)$

Add $2k\pi$,

$x = [\cos(\pi + 2k\pi) + i\sin(\pi + 2k\pi)]^{1/5}$

$= [\cos\pi(2k+1) + i\sin\pi(2k+1)]^{1/5}$

Apply De-Moivre's theorem,

$= \left[\cos\frac{\pi}{5}(2k+1) + i\sin\frac{\pi}{5}(2k+1)\right]$

$x = cis\,\frac{\pi}{5}(2k+1)$

where $k = 0, 1, 2, 3, \ldots$

Put $k = 0$, $x = \operatorname{cis} \pi/5$

$k = 1$, $x = \operatorname{cis} 3\pi/5$

$k = 2$, $x = \operatorname{cis} 5\pi/5 = 0$

$\because (\operatorname{cis} \pi = 1)$

[$\therefore$ ($k = 2$) is not possible]

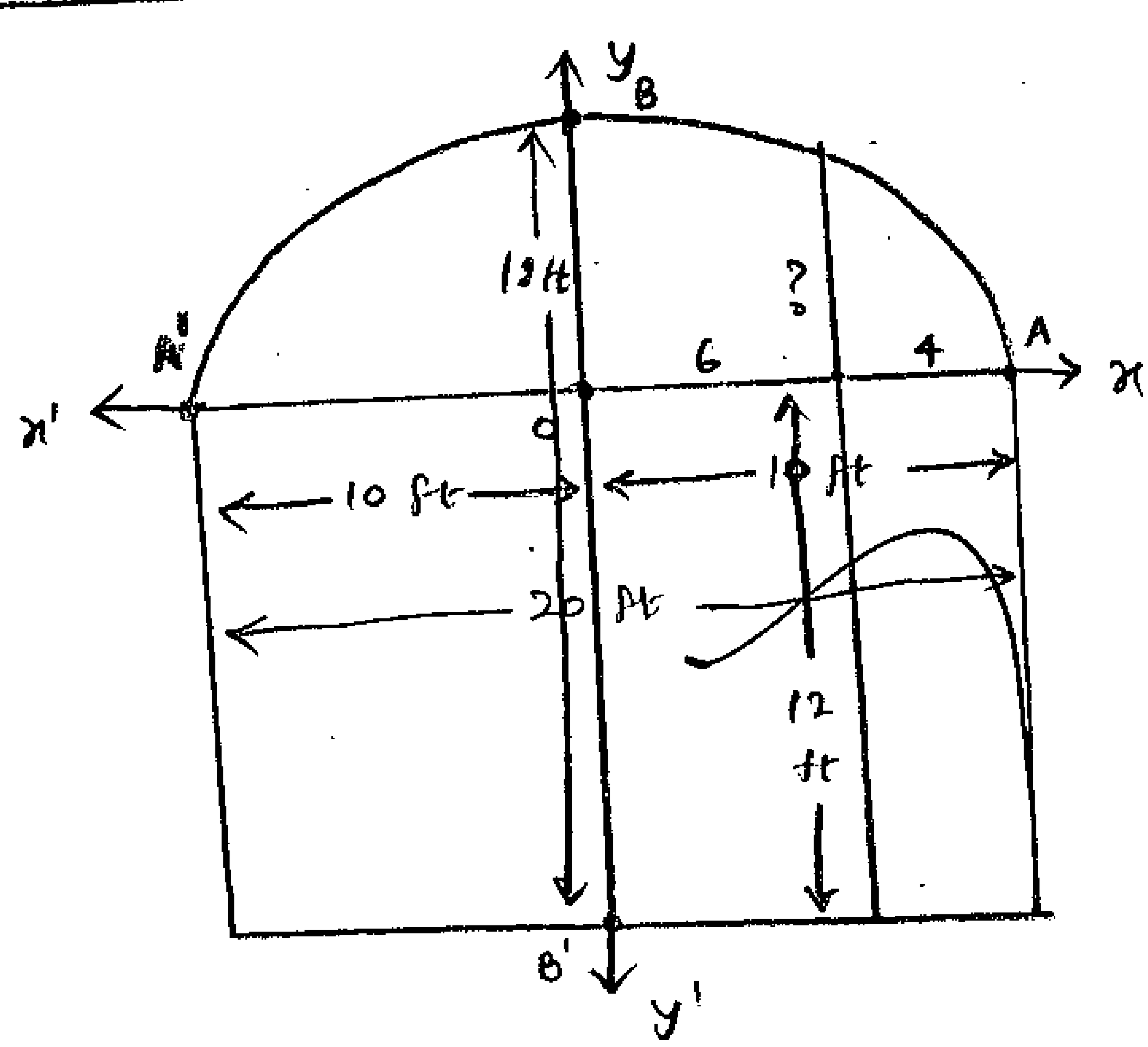
$k = 3$, $x = \operatorname{cis} 7\pi/5$

$k = 4$, $x = \operatorname{cis} 9\pi/5$

$k = 5$, $x = \operatorname{cis} 11\pi/5$

$\therefore$ The roots of $x^4 - x^3 + x^2 - x + 1 = 0$ are ($\operatorname{cis} \pi/5$, $\operatorname{cis} 3\pi/5$, $\operatorname{cis} 7\pi/5$, $\operatorname{cis} 9\pi/5$, $\operatorname{cis} 11\pi/5$)

61.



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Given,

A semi-ellipse is a ceiling in a hallway.

Let $AA' = 20$ ft

$BB' = 18$ ft

Let us consider,

Equation of a semi-ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Here $2a = 20 \Rightarrow a = 10$

$b = 6$.

$$\therefore \frac{x^2}{10^2} + \frac{y^2}{6^2} = 1$$ is the equation.

The point $(6, y_1)$ passes through the equation.

$$\frac{6^2}{10^2} + \frac{y_1^2}{6^2} = 1$$

$$\frac{36}{100} + \frac{y_1^2}{36} = 1$$

$$\frac{y_1^2}{36} = 1 - \frac{36}{100}$$

$$y_1^2 = \frac{100-36}{100} \times 36$$

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$$= \frac{64}{\cancel{100}\ \cancel{50}\ 25} \times \cancel{36}\ \cancel{18}\ 9$$

$$= \frac{576}{25}$$

$$y_1^2 = 23.04$$

$$\therefore y_1 = \sqrt{23.04}$$

$$\boxed{y_1 = 4.8}$$

$\therefore$ Total Height of ceiling 4 ft from either wall $= 12 + 4.8$

$$y_1 = 16.8 \text{ ft}$$

$\therefore$ Height of ceiling from 4 ft of either wall is 16.8 ft

64×9

576

25

5

$$25 \overline{)576}\ 23.04$$

50

76

75

100

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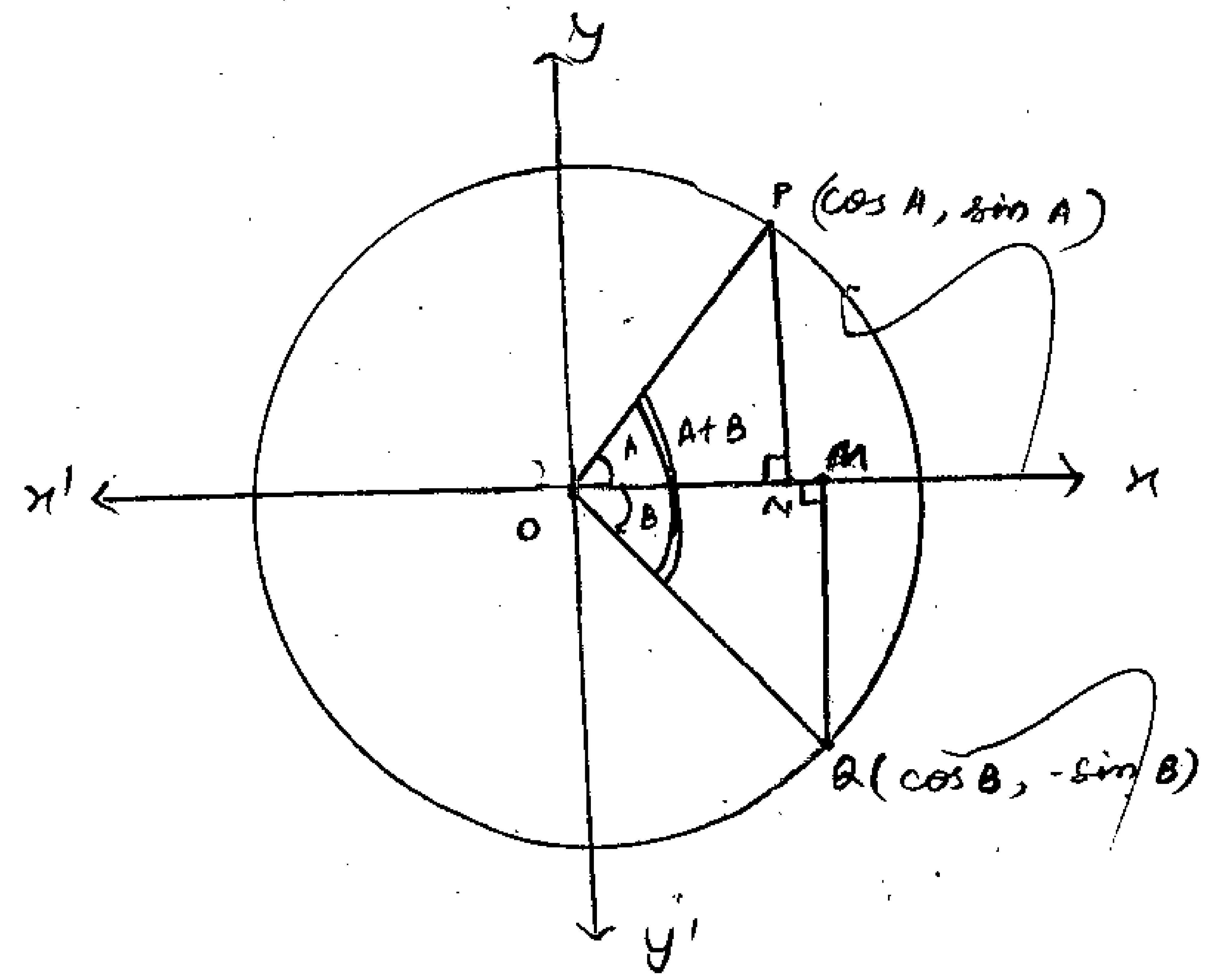
57.



Let P and Q be two points on the unit circle.

To prove:

$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

These two points makes the angles A and B on the unit circle with centre O.

$$\angle POQ = \angle POx + \angle QOx$$

$$\angle POQ = A + B.$$

Clearly, the co-ordinates of P and Q are $(\cos A, \sin A)$ and $(\cos B, \sin B)$

x and y axes have $\vec{i}$ and $\vec{j}$ vectors respectively.

$\therefore \quad \vec{OP} = \cos A\,\vec{i} + \sin A\,\vec{j}$

$\vec{OQ} = \cos B\,\vec{i} + \sin B(-\vec{j})$

$= \cos B\,\vec{i} + (-\sin B\,\vec{j})$

$\therefore \quad \cos(A+B) = \vec{OP} \cdot \vec{OQ}$

$$\vec{OP} \cdot \vec{OQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos A & \sin A & 0 \\ \cos B & -\sin B & 0 \end{vmatrix}$$

$= \vec{i}\,(0) - \vec{j}\,(0) + \vec{k}\,($

$\vec{OP} \cdot \vec{OQ} = (\cos A\,\vec{i} + \sin A\,\vec{j}) \cdot (\cos B\,\vec{i} - \sin B\,\vec{j})$

$= (\cos A \cos B - \cancel{\sin A} \sin B)$

$\therefore \quad \cos(A+B) = \cos A \cos B - \sin A \sin B$

Hence Proved

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PART - II

41. Given;

$$\rho(A) = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & -1 & 0 \\ 2 & -3 & 0 & -1 \\ 0 & 1 & 2 & 1 \end{pmatrix} \quad [\because R_1 \leftrightarrow R_3]$$

$$\sim \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & -5 & 2 & -1 \\ 0 & 1 & 2 & 1 \end{pmatrix} \quad [\because R_2 \to R_2 - 2R_1]$$

$$\rho(A) \sim \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & -5 & 2 & -1 \\ 0 & 0 & 8 & 6 \end{pmatrix} \quad \because [R_3 \to 5R_3 + R_2]$$

$\therefore$ It is in Echelon form.

$\therefore \rho(A) = 3$.

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42. Given,

Let $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$A^{-1} = ?$

$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$

$|A| = \begin{vmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{vmatrix}$

$= 3(2-0) - 1(-2-0) - 1(4+2)$

$= \cancel{6} + 2 - \cancel{6}$

$|A| = 2 \neq 0$.

$\therefore A^{-1}$ exists

adj $\cdot$ A = cofactor of A

$$= \begin{pmatrix} \begin{vmatrix} -2 & 0 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} \end{pmatrix}$$

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$$= \begin{bmatrix} (2-0) & -(-2-0) & (4+2) \\ -(-1+2) & (-3+1) & (6-1) \\ (0+2) & -(0+2) & (-6-2) \end{bmatrix}$$

$$A_c = \begin{pmatrix} 2 & 2 & 6 \\ 1 & -2 & 5 \\ 2 & -2 & -8 \end{pmatrix}$$

$\therefore$ Transpose of A_c,

$$= \begin{pmatrix} 2 & 1 & 2 \\ 2 & -2 & -2 \\ 6 & 5 & -8 \end{pmatrix}$$

$$\therefore \text{adj} \cdot A = \begin{pmatrix} 2 & 1 & 2 \\ 2 & -2 & -2 \\ 6 & 5 & -8 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{2} \begin{pmatrix} 2 & 1 & 2 \\ 2 & -2 & -2 \\ 6 & 5 & -8 \end{pmatrix}$$

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44. (i) Given,

$\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$

$\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$

To show:

$\vec{a} - \vec{d} = \vec{b} - \vec{c}$ are parallel

$(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$

$\therefore [\vec{a} \times \vec{b} = 0$ for parallel$]$

$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) - (\vec{d} \times \vec{b}) + (\vec{d} \times \vec{c}) = 0$

$\cancel{(\vec{a} \times \vec{b})} - \cancel{(\vec{a} \times \vec{c})} + \cancel{(\vec{b} \times \vec{d})} - \cancel{(\vec{c} \times \vec{d})} = 0$

$0 = 0$

$\therefore [\vec{a} \times \vec{b} = \vec{c} \times \vec{d}, \ \vec{a} \times \vec{c} = \vec{b} \times \vec{d}]$

$\therefore \vec{a} \times \vec{c}$ and $\vec{b} - \vec{c}$ are parallel

Hence, showed.

(ii) Given,

$a_1 = 2, -3, 1$

$a_2 = 3, 1, -2$

$a_2 - a_1 = 1, 4, -3$

$r = |a_2 - a_1| = \sqrt{1^2 + 4^2 + (-3)^2}$

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$= \sqrt{1+16+9}$

$r = \sqrt{26}$

$\therefore$ Direction cosines is

$= \frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{-3}{\sqrt{26}}$.

45. Given,

$\alpha = -\sqrt{2} + i$

$\beta = -\sqrt{2} - i$

To find :

$\alpha^2 + \beta^2 - \alpha\beta = ?$

$\Rightarrow (-\sqrt{2}+i)^2 + (-\sqrt{2}-i)^2 - (-\sqrt{2}+i)(-\sqrt{2}-i)$

$\Rightarrow [(-\sqrt{2})^2 + i^2 + 2(-\sqrt{2})(i)] + [(-\sqrt{2})^2 + (-i)^2 + 2(-\sqrt{2})(-i)]$

$-(2 + \cancel{\sqrt{2}i} - \cancel{\sqrt{2}i} - i^2)$

$\Rightarrow (2 - 1 - 2\sqrt{2}i) + (2 + 1 + 2\sqrt{2}i) - (2+1)$

$(1 - 2\sqrt{2}i) + (3 + 2\sqrt{2}i) - 3$

$4 - \cancel{2\sqrt{2}i} + \cancel{2\sqrt{2}i} - 3$

$= 1$

$\therefore \alpha^2 + \beta^2 - \alpha\beta = 1$

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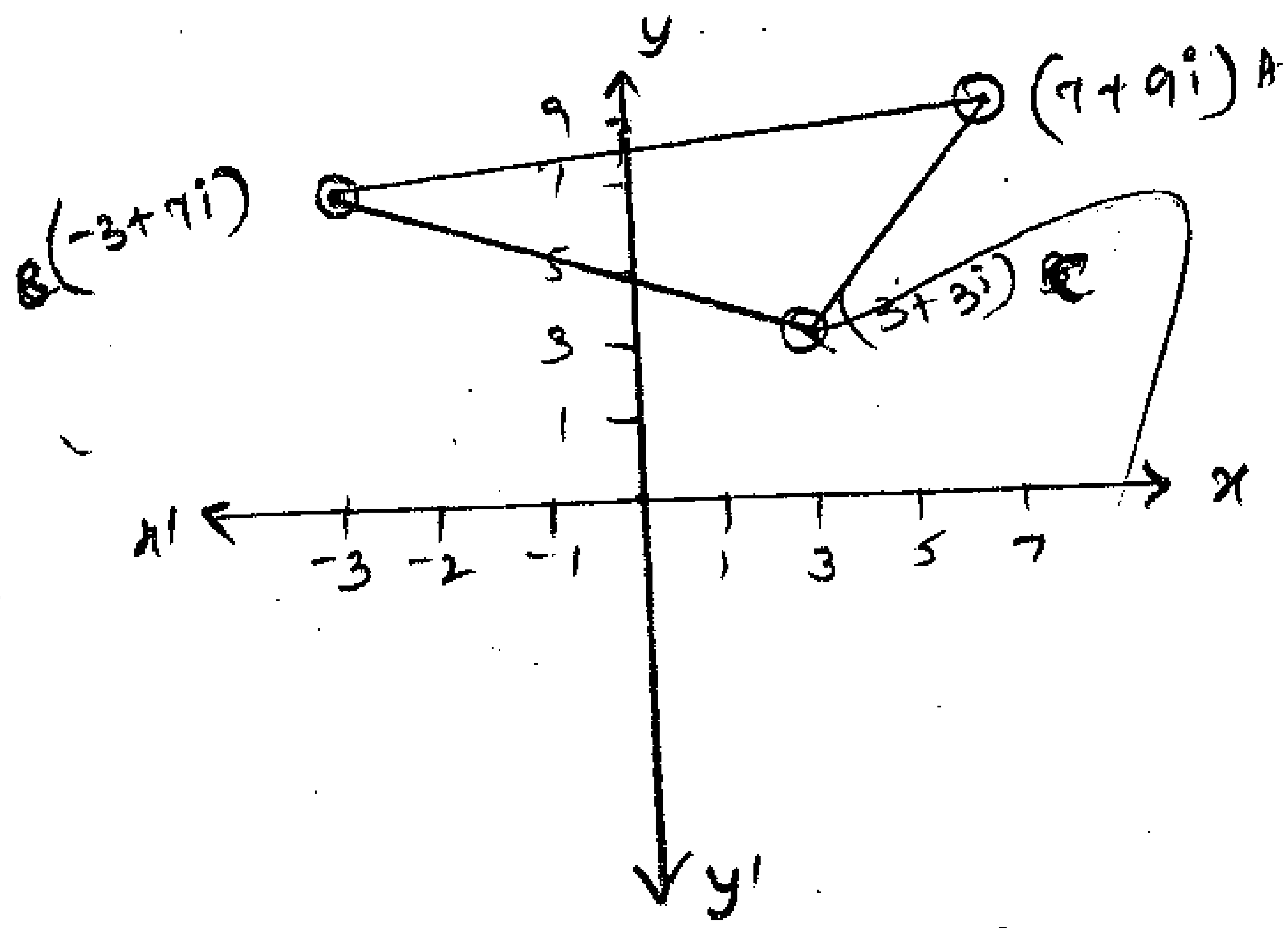
46.



The points are $(7+9i)$ A , $(3+3i)$ C and $(-3+7i)$ B.

To show :

points form a right angled triangle.

$|AB| = |A - B|$

$= |7+9i - 3 - 3i|$

$= |4+6i|$

$= \sqrt{4^2 + 6^2 i^2}$

$= \sqrt{16 - 36}$

$= \sqrt{-20}$

$|BC| = |B - C|$

$= |3+3i + 3 - 7i|$

$= |6 - 4i|$

$= \sqrt{36 + 16}$

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$|BC| = \sqrt{52}$

$|CA| = |C - A|$

$= |-3 + 7i - 7 - 9i|$

$= |-11 - 2i|$

$= \sqrt{121 + 4}$

$= \sqrt{125}$

$|AB| = |A - B|$

$= |7 + 9i + 3 - 7i|$

$= |10 + 2i|$

$= \sqrt{100 + 4}$

$|AB| = \sqrt{104}$

$|BC| = |B - C|$

$= |-3 + 7i - 3 - 3i|$

$= |-6 + 4i|$

$= \sqrt{36 + 16}$

$|BC| = \sqrt{52}$

$|CA| = |C - A|$

$= |3 + 3i - 7 - 9i|$

$= |-4 - 6i|$

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$= \sqrt{16+36}$

$|CA| = \sqrt{52}$

By Pythogoreas theorem,

$(|AB|)^2 = (|BC|)^2 + (|CA|)^2$

$(\sqrt{104})^2 = (\sqrt{52})^2 + (\sqrt{52})^2$

$104 = 104$

$\therefore$ They form a right angled triangle.

Hence showed.

51.

p	q	$\sim q$	$p \vee (\sim q)$	$q \vee (p \vee (\sim q))$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

$\Downarrow$ ①

From ①, the statement is Tautology

[$\because$ All entries are 'T's]

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52.

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \vee (\sim r)$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

54. Given,

$p = 3/\cancel{6}, \quad q = 3/\cancel{6} \Rightarrow p = 1/2, \; q = 1/2$

Mean = ? Variance = ?

$P(\text{No odd}) = \cancel{pp}\; qq$

$= \frac{1}{2} \cdot \frac{1}{2}$

$= 1$

$P(1 \text{ odd}) = pq + qp$

$= 2(pq)$

$= 2\left(\frac{1}{2} \cdot \frac{1}{2}\right)$

$= 2(1) \Rightarrow 2$

$P(2 \text{ odd}) = PP$

$= \frac{1}{2} \cdot \frac{1}{2}$

$= 1$

x	0	1	2
$P(X=x)$	1	2	1

Mean $= \sum_{\infty}^{-\infty} x_i p_i$

$= 0(1) + 1(2) + 2(1)$

$= 2 + 2$

$= 4$

variance $= \sum_{-\infty}^{\infty} x_i^2 p_i$

$= 0^2(1) + 1^2(2) + 2^2(1)$

$= 2 + 4$

$= 6$

$\therefore$ Mean $= 4$, Variance $= 6$.

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43. Given,

$(x_1, y_1, z_1) = (1, 1, -1)$

$(x_2, y_2, z_2) = (-1, 0, 1)$

It is of form,

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-1}{-2} = \frac{y-1}{-1} = \frac{z+1}{2}$$

It lies in xy plane, $z = 0$.

$$\frac{x-1}{-2} = \frac{y-1}{-1} = \frac{1}{2}$$

Compare,

$$\frac{x-1}{-\cancel{2}} = \frac{1}{\cancel{2}}$$

$$x - 1 = -1$$

$$x = 0$$

$$\frac{y-1}{-1} = \frac{1}{2}$$

$$y - 1 = \frac{-1}{2}$$

$$y = \frac{-1}{2} + 1$$

$$= \frac{-1+2}{2}$$

$$y = 1/2$$

$\therefore$ The point of intersection is

$(0, 1/2, 0)$

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55. a) Given,

centre, $c = (2, 5)$

directrix $= \frac{2a}{e} = 15$

foci $= 2ae = 20$

$\therefore ae = 10$

Transverse axis is parallel to y-axis

$\therefore$ Equation of hyperbola is

$$\frac{(\cancel{y}-k)^2}{a^2} - \frac{(\cancel{x}-h)^2}{b^2} = 1$$

$$\frac{(y-5)^2}{a^2} - \frac{(x-2)^2}{b^2} = 1$$

Here $a = 10/e$

$2a/e = 15$

$$2\,\frac{\frac{10}{e}}{e} = 15$$

$$2\,\frac{10}{e^2} = 15$$

$$\frac{20}{e^2} = 15$$

$$\frac{1}{e^2} = \frac{20}{15}$$

$$\therefore \boxed{e = \sqrt{3}/_{2}}$$

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$ae = 10$ Here $e = 10/a$

$a\left(\frac{\sqrt{3}}{2}\right) = 10$ $\cancel{2\frac{a}{e} = 15}$

$a = \frac{20}{\sqrt{3}}$ $\cancel{2\frac{a}{5 10/a} = 15}$

$b^2 = a^2(e^2 - 1)$ $\cancel{\frac{a^2}{5} = 15}$

$= \frac{20^2}{3}\left(\frac{3}{4} - 1\right)$ $\cancel{a^2 = 20}$

$= \frac{20^2}{3}\left(\frac{-1}{4}\right)$ $\cancel{\boxed{a = 2\sqrt{5}}}$

$b^2 = \frac{401}{12}$

$\cancel{ae = 10}$

$\cancel{2\sqrt{5}\, e = 10}$

$\cancel{e = \sqrt{5}}$

$\therefore$ Equation of Hyperbola is

$$\frac{x^2}{20^2/\sqrt{3}^2} - \frac{y^2}{401/12} = 1.$$

$$\frac{3x^2}{400} - \frac{\cancel{144}\,12\, y^2}{401} = 1$$

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Part - C

70. b) Given,

$$(x+y)^2 \frac{dy}{dx} = a^2$$

It is of form,

$$\frac{dy}{dx} + Py = Q$$

$(\div)\ (x+y)^2$,

$$\frac{dy}{dx} + \frac{1}{(x+y)^2} = \frac{a^2}{(x+y)^2}$$

Here $P = \frac{1}{(x+y)^2}$, $Q = \frac{a^2}{(x+y)^2}$

$$P.I \text{ is } = e^{\int p\,dx}$$

$$= e^{\int \frac{1}{(x+y)^2} \cdot dx}$$

$$P.I = e^{\frac{-1}{(x+y)^3}(1+y)}$$

composite Function, C.F

$$y \{P.I\} = \int Q \cdot (I.F)\, dx + c$$

~~y. e^f~~

$$y \cdot e^{-y/(x+y)^3} = \int \frac{a^2}{(x+y)^2} e^{-y/(x+y)^3} \cdot dx + c$$

0

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$$y \cdot e^{-y/(x+y)^3} = a^2\left(\frac{\cancel{+}+y}{(x+y)^3} \cdot e^{-y/(x+y)^3}\right) + c$$

$$= a^2\left(\frac{+y\cancel{-}}{(x+y)^3}\, e^{-y/(x+y)^3}\right) + c$$

$$\therefore (x+y)^2\frac{dy}{dx} = a^2 \text{ is}$$

$$y \cdot e^{-y/(x+y)^3} = a^2\left(\frac{-y\cancel{-}}{(x+y)^3} \cdot e^{-y/(x+y)^3}\right) + c$$