

**Part A – Mini-Essay oriented questions**

- 1) **Critical reflection question (required):** at the beginning of our course, we discussed heuristics and decision biases. Since 2020, the world is in the grip of a pandemic that has changed the decision-making environment profoundly and, on any levels, all the way from governments, businesses to private life. Reflect on the role of heuristics and decision-biases in this altered environment.
- 2) Explain the concepts of bounded rationality and satisficing as proposed by Simon.
- 3) Feasibility Region:
  - a. Explain what the feasibility region in an LP model is.
  - b. Explain how it changes when an integer constraint is introduced.
  - c. Explain how it changes if RHS values of binding constraints change.
- 4) Describe the anchoring and framing biases and give examples of each of them, other than those provided through the decision-making example.
- 5) Explain the concept of subjective utility. Then describe the shape of the utility curve for a risk averse, risk neutral and risk taking individual and explain the reason for the different shapes.

**Part B – Application oriented questions**

1. The ABC company produces two products. The total profit achieved from these products is described by the following equation:

$$\text{Total Profit} = -0.2 * X_1^2 - 0.3 * X_2^2 + 8 * X_1 + 12 * X_2 + 1500$$

where  $X_1$  = thousands of units of product 1  
 $X_2$  = thousands of units of product 2

Every 1000 units of  $X_1$  requires one hour of time in the shipping dept and every 1000 units of  $X_2$  requires 45 minutes in the shipping dept. Each unit of each product requires 2 pounds of a special ingredient of which 64000 pounds are available. Additionally, 70 hours of shipping labor are available. Demand for  $X_1$  and  $X_2$  is unlimited.

Formulate an NLP model for this problem. Implement your model using Solver.

2. “Paints for the planet” produces 2 types of paints, outdoor and indoor. The outdoor line sells for \$40 a gallon, with production costs of \$25, the indoor line for \$28, with production costs of \$15. Each gallon of paint goes through two processing steps: pigment mixing and final paint mix. The table below summarizes the hours needed. There are 2,400 hours of pigment mixing time available, and 1,500 hours of final paint mix. The company wants to maximize profit, while also minimizing waste water usage. A gallon of outdoor paint requires 45 gallons of water during production, while indoor requires 30. The company set itself a limit of not producing more than 15,000 gallons of waste water per production run. Furthermore, the marketing team has decided that demand for the outdoor line is at least 100 gallons. How many of each type of paint should “Paints for the planet” produce in order to maximize profit and keep its environmental promise?

Formulate the ILP model, implement it in Excel and solve it. Discuss the results for the constraints (you do not have to create a sensitivity report to do so).

Process step	Outdoor	Indoor
Pigment mixing	2	3
Final Paint mix	4	1

3. Carol is the owner of Carol’s Cuisine (a French restaurant). She wants to study the growth of her business using simulation. She is interested in simulating the number of customers and the amount ordered by customers each month. She currently serves 800 customers per month and feels this can vary uniformly between a decrease of as much as 5% and an increase of up to 8% (calculated per month, i.e. the second month uses the value of the first month as base). The bill for each customer is a normally distributed random variable with a mean of \$50 and a standard deviation of \$15. The average order has been increasing steadily over the years and the owner expects the mean order will increase by 1% per month. Formulate a simulation, computing the mean total revenue and the standard deviation in one year.

4. Mary Smart tries to improve the management of her student budget. She collected data, and found out the following: Based on various student jobs she has income between \$600 and \$750 per month, probabilities are given in the table below. Her expenses also vary, between \$550 and \$800 per month, please see table below. Her starting balance for the current month is \$800. What will her average balance be after one year? What is the probability that she is running out of funds during any month of the year? Formulate a simulation, computing the mean end of year balance and probability of a negative month.

Monthly Income	Probability
\$600.00	0.4
\$650.00	0.3
\$700.00	0.2
\$750.00	0.1

Monthly Expenses	Probability
\$550.00	0.1
\$600.00	0.3
\$700.00	0.3
\$800.00	0.3

## Answers:

### Part A

#### Answer 1

Heuristics is the nature of taking decisions quickly without following a complex thinking process. Decision bias is nothing but cognitive bias, which occurs with wrong interpretation. Due to sudden cloud of pandemic all over the world, everyone, in one or other situation have experienced decision-making biases. While there were few situations where decisions needed to be taken quickly on maintaining health conditions, some were situations needed to be taken on long run sustainability. From worldwide leaders to individuals, all have made biased decisions in predicting the future conditions.

People under government were under perception that more harm would occur by failing to take right actions, which is called omission bias. Also, this omission bias is reflected on some people failing to take vaccines though they knew that vaccines would shorten the risk of death.

Most of interpretation happened on financial crisis, that huge financial recession would sink the world and there would be food shortages all over the world. This can be categorized under availability biases, to think what comes easily to our mind.

Many countries during Covid second wave, rejected to lock down, on a tendency that lock down would result nothing in controlling the pandemic situations other than financial halt. This comes under overconfidence bias, since even during second wave, the death rate has increased more than the first which anyway lead to lockdown.

On current reference scale, assuming a loss in the scale such as future employment opportunities during pandemic time comes under status quo bias.

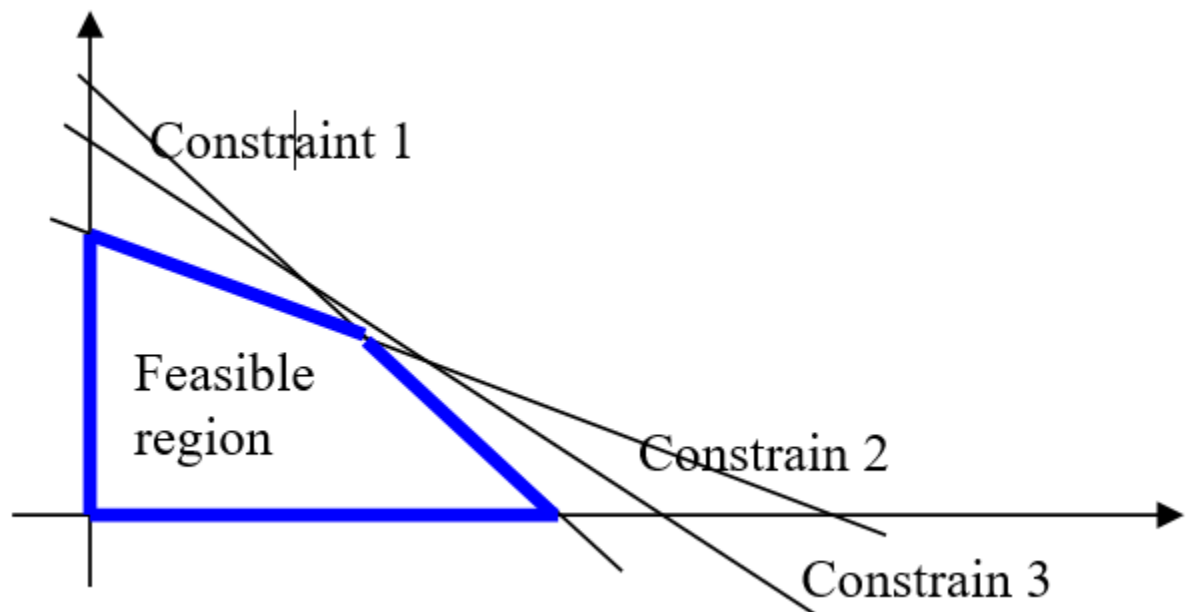
**Reference:**

Joseph Zaccardi, How Bias Affected Our Decisions During the Pandemic, Family Business Magazine.

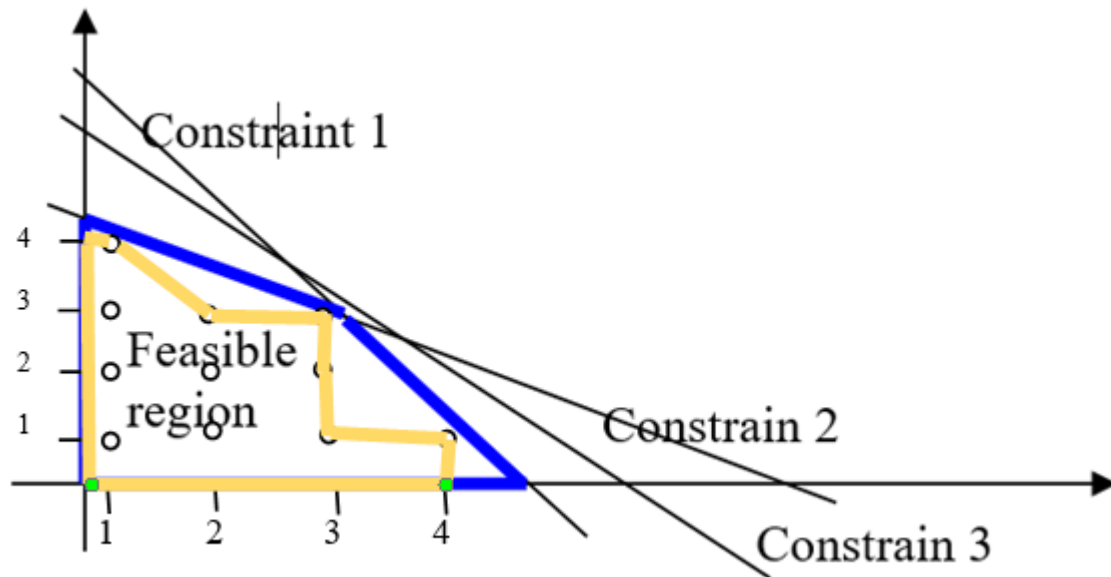
Scott D. Halpern, Robert D Troug, Franklin G miller (2020) Cognitive Bias and Public Health Policy During the COVID 19 Pandemic.

**Answer 3**

a) In a linear programming problem, the area under which solution set exists, satisfying all the constraints is called as a feasible region. Technically, it is the area bounded by the constraints in which the decision variables are assumed to exist. The best point in this feasible region for a proposed maximizing or minimizing objective function, gives the optimal solution. Additionally, there are no optimal solutions acceptable outside the feasible region.



b) When decision variables only take integer values, by introducing integer constraint, a linear programming model becomes integer linear programming model. Even in ILP, the solution set is bounded by the constraints, except that the feasible region decreases as the solution set is restricted to give only integer values.



The above figure shows that how the feasibility region decreases when integer constraint is introduced. While blue bordered area shows the feasible region for a linear programming model, the region bordered under yellow gives the feasible region for integer linear programming model, taking only integer feasible solutions (marked with small circles in the figure). The solution set bounded between feasible region of LP model and ILP model is called optimal relaxed solution.

c) Constraints with zero slack in the optimal solution to a linear programming problem are called binding constraints. Slack in an answer report denotes the criticality of the constraint; zero slack indicates that the constraint is more significant and sensitive in the model. Hence a binding constraint is a constraint which holds a sensitive parameter in changing the solution to the problem. The change in RHS values of binding constraint affects the change in feasible region. This is quantified by shadow price in the sensitivity report. For a positive shadow price, a unit increase in RHS value of binding constraint results in an increase in optimal objective function value, which means an increase in feasible region. Similarly, for a negative shadow price, unit increase in RHS of binding constraint results in a decrease in optimal objective function value, resulting in a reduction of feasible region. Hence there is a proportional change in feasible region with respect to RHS of binding constraint with positive or negative shadow price, assuming all other constraints remain constant.

#### Answer 4

The decision-making problems- anchoring and framing are the effects associated with errors in human judgement. Both of these cognitive biases are psychological terms which change the way of deciding. The human tendency to get influenced or to get relied on first piece of information or on a pre-existing notion when making a decision is described as anchoring bias. The terms “first piece of information”, “pre-existing notion” or “first impression” acts as anchors in influencing future decisions. In simple terms, it is the tendency of a regular person to go with a decision anchored to already existing perception. For example, when a person with good conduct makes a mistake, people usually forgive the person with a pre-existing good impression on that person. When another person with bad conduct commits the same mistake, the chance of forgiving this

person is very low irrespective of whether this person had committed the mistake or not. This type of making cognitive judgement anchored to the conduct of the person is called anchoring effect or anchoring bias.

Similarly, the human tendency of making a decision gets influenced by the way information is framed or presented, called framing bias or framing effect. Considering the above example, the chance of forgiving both the persons for committing the same mistake may be relied on the way the situation is framed on other's perception, called framing effect. Considering another example, while buying milk in a shop, let say one type of milk is 95% fatless and another type with 5% fat milk. Chance of buying 95% of fatless is higher than 5% milk with fat. The way both the milk types are framed on customers perception affects the sales. Though both the milk qualities describe same, the customer's judgement while choosing one from two is called framing effect or framing bias as the perception of advertising the products influence the sales.

### References

Lijie., Haimengo., Linhai., (2020). Influence of the framing effect, anchoring effect, and knowledge on consumer attitude and purchase. *Frontiers in Psychology*; <https://www.frontiersin.org/articles/10.3389/fpsyg.2020.02022/full>

<https://thedecisionlab.com/biases/framing-effect/>

## Part B

### Problem 1

**Identifying decision variables:** As given,  $x_1$  is thousands of units of product 1 and  $x_2$  is thousands of units of product 2

$\Rightarrow x_1$  and  $x_2$  are the decision variables

**Optimizing (Objective function):** total profit =  $-0.2 \cdot x_1^2 - 0.3 \cdot x_2^2 + 8 \cdot x_1 + 12 \cdot x_2 + 1500$

**Identifying Constraints:**

Given that every 1000 units of  $x_1$  and  $x_2$  requires 1 and 0.75 (45 minutes) hours of time in shipping dept for an available shipping labor of 70 hours

$\Rightarrow x_1 + 0.75 \cdot x_2 \leq 70$

Also, each unit of  $x_1$  and  $x_2$  requires 2 pounds of special ingredient on a total availability of 64,000 pounds

$\Rightarrow 2000x_1 + 2000x_2 \leq 64000$

The bounds being  $x_1, x_2 \geq 0$

The above model is executed in solver and the following result is obtained

ABC Company				
	Product 1	Product 2		
Thousands of units of Products	15.2	16.8		
Total Profit	1692.32			
Constraints				bound
Time for labor in shipping dept (hours)	1	0.75	27.8	70
Availability of special ingredient (pound)	2000	2000	64000	64000

The model gives the result of producing 15.2 thousand units (15200 units) of product 1 and 16.8 thousand units (16,800 units) of product 2 for a maximum total profit of 1692.32

## Problem 2

**Identifying decision variables:** Let  $x_1$  be number of outdoor and  $x_2$  be number indoor type of paints produced by “Paints for the planet”

=>  $x_1, x_2$  are the decision variables

**Optimizing:** Maximizing the profit and minimizing waste water usage

Given that selling price for  $x_1, x_2$  is \$40 and \$28 with cost price of \$25 and \$15 respectively

=> Profit =  $(40-25)x_1 + (28-15)x_2$

=> Maximizing  $15x_1 + 13x_2$

**Constraints:** from table, for pigment mixing, it requires 2 hours for outdoor and 3 hours for indoor on total available time of 2400 hours

=>  $2x_1 + 3x_2 \leq 2400$

Similarly, for final paint mix it requires 4 hours for outdoor and an hour for indoor on total available time of 1500 hours

=>  $4x_1 + x_2 \leq 1500$

Also given that a gallon of  $x_1, x_2$  require 45, 30 gallons of water respectively on a total limit of producing not more than 15,000 gallons of waste water per production run

=>  $45x_1 + 30x_2 \leq 15,000$

Demand for  $x_1$  is at least 100 gallons

=> the bound constraints are  $x_1 \geq 100$  and  $x_2 \geq 0$

The following result of producing 100 outdoor paints and 350 indoor paints is required for maximum profit of \$6050

Paint for the planets				
	Outdoor	Indoor		
No. of paints	100	350		
Selling price	40	28		
Cost price	25	15		
Profit	15	13	6050	
Constraints				
Pigment mixing hours	2	3	1250	2400
Final paint mix hours	4	1	750	1500
Water usage in gallons	45	30	15000	15000

Therefore, from the model execution, it requires 1250 hours for mixing pigments out of 2400 available hours and 750 hours for final paint mix out of 1500 available hours. It clearly shows that for maximizing the profit it requires half of total available time for pigment mixing and final paint mix.

Also, the water usage of \$15,00 gallons is completely utilized.

Moreover, the constraint that marketing team has decided plays an important role because when the model is run by changing this constraint to a normal bound of  $x_1 \geq 0$ , then the result obtained is not to produce any outdoor paints although the profit is increased to 6500

Paint for the planets				
	Outdoor	Indoor		
No. of paints	0	500		
Selling price	40	28		
Cost price	25	15		
Profit	15	13	6500	
Constraints				
Pigment mixing hours	2	3	1500	2400
Final paint mix hours	4	1	500	1500
Water usage in gallons	45	30	15000	15000



Hence the constraint  $x_1 \geq 100$  holds a sensitive position and is important in obtaining the optimal solution.

### Problem 3

Carol is currently serving 800 customers per month, uniformly varying with decrease of 5% and increase of up to 8%. The customers served for upcoming months is calculated from the formula

$$\Rightarrow \text{Round}(800 * \text{psiuniform}(1-5\%, 1+8\%), 0)$$

The bill for each customer is normally distributed with mean of \$50 and standard deviation of \$15, with mean increase of 1% in average order

Average bill is calculated by the formula,

$$\Rightarrow \text{psinormal}(50 * (1+1\%)^{\text{month}}, 15)$$

From the above formulae, the monthly revenue is calculated by product of customers served and average bill. The bill for an year is simulated as in the figure below and mean and standard deviation are calculated.

	Number of customer		800		Max decrease	5%	Max increase	8%	Uniform
	Mean of bill		\$50		Monthly Increase	1%	Std dev	\$15	Normal
Month	Customers Served	Average Bill	Monthly Revenue						
1	838	\$63.38	\$53,108.55						
2	844	\$42.51	\$35,874.97						
3	909	\$47.32	\$43,013.20						
4	978	\$61.07	\$59,726.50						
5	1011	\$35.65	\$36,042.65						
6	1046	\$71.32	\$74,595.71						
7	1102	\$70.16	\$77,314.53						
8	1162	\$73.79	\$85,742.76						
9	1188	\$49.13	\$58,365.77						
10	1179	\$71.51	\$84,313.17						
11	1220	\$63.30	\$77,223.59						
12	1182	\$45.07	\$53,276.56						
			\$738,597.95	Total Revenue					
			\$569,018.61	Mean of Rotal Revenue					
			\$65,966.52	Std Dev of Total Revenue					

### Problem 4

Below figure gives the solution obtained after simulating

Starting balance for current month (say1)				\$800	Monthly Income		Monthly Expenses	
					\$600	0.4	\$550	0.1
					\$650	0.3	\$600	0.3
					\$700	0.2	\$700	0.3
					\$750	0.1	\$800	0.3
Month	Monthly income	Monthly expenses	Total monthly balance					
1	\$1,450	600	\$850					
2	\$1,450	600	\$850					
3	\$1,600	700	\$900					
4	\$1,500	600	\$900					
5	\$1,500	800	\$700					
6	\$1,400	700	\$700					
7	\$1,350	700	\$650					
8	\$1,350	700	\$650					
9	\$1,250	800	\$450					
10	\$1,100	800	\$300					
11	\$1,000	700	\$300					
12	\$900	600	\$300					
			\$7,550	Total balance				
			\$6,181	Average balance after one year				
			\$0	Month with running out of funds				
			\$0	Probability of negative month				
			0.229	Mean of end of year balance				