2.

$$\text{Entropy}(S) = \sum_{i=1}^{C} -P_i \log_2 P_i$$

[C = no. of class labels
P_i = Proportion of i^{th} class]

Target attribute = Output.

$$\text{Entropy}(5L, 5D) = \frac{-5}{10}\log_2^{5/10} \frac{5}{10}\log_2^{5/10}$$

$$= 1$$

$$\text{Gain}(S, A) = \text{Entropy}(S) - \sum_{V \in \text{values}(A)} \frac{|S_v|}{|S|} \text{Entropy}(S_v)$$

Values(weight) = {H, U}

$S = [5L, 5D)$

$S_H = [3L, 2D)$

$S_U = [2L, 3D]$

$$\text{Gain}(S, \text{weight}) = \text{Entropy}(S) - \sum_{V \in \{H,U\}} \frac{|S_v|}{|S|} \text{Entropy}(S_v)$$

$$= \text{Entropy}(S) - \frac{5}{10}\text{Entropy}(S_H) - \frac{5}{10}\text{Entropy}(S_W)$$

$$= 1 - \frac{5}{10}\left[-\frac{3}{5}\log_2^{3/5} - \frac{2}{5}\log_2^{2/5}\right] - \frac{5}{10}\left[-\frac{2}{5}\log_2^{2/5} - \frac{3}{5}\log_2^{3/5}\right]$$

$$= 1 - 5/10\,[0.442 + 0.528] - 5/10\,[0.442 + 0.528]$$

$$= 1 - 0.97 = 0.03$$

$$\boxed{\text{Gain}(S, \text{weight}) = 0.03}$$

values(eye color) : {A, V}

$S = [5L, 5D]$

$S_A = [1L, 4D]$

$S_V = [4L, 1D]$

$$\text{Gain}(S, \text{eyecolor}) = \text{Entropy}(S) - \sum_{V \in \{A,V\}} \frac{|S_V|}{|S|}\text{Entropy}(S_V)$$

$$= \text{Entropy}(S) - \frac{5}{10}\text{Entropy}(S_A) - \frac{5}{10}\text{Entropy}(S_V)$$

$$= 1 - \frac{5}{10}\left[\frac{-1}{5}\log_2^{1/5} - \frac{4}{5}\log_2^{4/5}\right]$$

$$- \frac{5}{10}\left[\frac{-4}{5}\log_2^{4/5} - \frac{1}{5}\log_2^{1/5}\right]$$

$$= 1 - \frac{5}{10}\left[0.464 + 0.257\right]$$

$$- \frac{5}{10}\left[0.257 + 0.464\right]$$

$$= 1 - 0.721$$

$$= 0.279.$$

Gain(S, eyecolor) = 0.279

Values(Numeyes) = {2, 3, 4}

S = [5L, 5D]

S_2 = (3L, 0D)

S_3 = (2L, 2D)

S_4 = (0L, 3D)

$$\text{Gain}(S, \text{Numeyes}) = \text{Entropy}(S) - \sum_{v \in \{2,3,4\}} \frac{|S_v|}{|S|}\text{Entropy}(S_v)$$

$$= 1 - \left[\frac{3}{10}\text{Entropy}(S_2) + \frac{4}{10}\text{Entropy}(S_3) + \frac{3}{10}\text{Entropy}(S_4)\right]$$

$$= 1 - \left[0 + 1 + 0\right] = 0$$

Gain (S, Numeyes) = 0

As Gain (S, eyecolor) is maximum, eyecolor is the root node of the DT