BDC5101: Assignment
Semester I, 2018/2019, NUS
Due on Oct 3, 2018

1. (6') Two students are discussing their term project, and they come out the first cut for their model as follows.

$$\begin{aligned} \min \quad & 3x_1^2 - 2x_2 \\ s.t. \quad & x_1 + \sqrt{A}x_2 \leq 1.5 \\ & 0 \leq |x_1| \leq 1 \\ & 0 \leq x_2 \leq 1 \\ & A \text{ is a value given}, x_1, x_2 \text{ are d.v.} \end{aligned}$$

the next day, when student A looks at this model again, he thinks this model does not seem to be a linear model and asks student B to verify. While busy in meeting, Student B asks student A which part of the model is not linear.

After checking the model again, student B realized he made a typo and few less realistic assumptions and revised the model as follows.

$$\begin{aligned} \min \quad & 3x_1 - 2x_2 \\ s.t. \quad & |x_1| + |x_2| \leq 1.5, \\ & 0 \leq |x_1| \leq 1, \\ & 0 \leq |x_2| \leq 1. \end{aligned}$$

Student A doubted it again, but this time student B confirmed there is no other mistake and told student A this is a LP model or at least can be converted into LP model.

(a) (1') Can you help student A to list out those part(s) of the original model that is(are) not linear, before student B revised his model of typo and less realistic assumptions?

(b) (3') Can you help student B put the revised model into general LP form? So that student A would be convinced

(c) (2') What is the optimal value of the model? what are the values of those d.v. when the model is optimal? and any constraint(s) is(are) tight/binding when the system is optimal? Hints you can solve this problem using PC

2. **Planning Problem:** (7') A company producing EV(Electric Vehicle) has a capacity of 50,000 EVs per month. The sales forecast in thousand of units are given in the table. It is possible to augment production by up to 50% through overtime working,but this increases the production cost for a EV from the usual \$32k to \$40k, due to additional staff cost and equipment rental. Currently there are 4,500 EVs in stock, and storage cost for any stock in inventory per unit per month is \$5,000.

Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**
50	25	25	42	55	67

(a) (3')Formulate a LP model to advise on how many EV is to be produced and stored during the next 6 month, so as to satisfy the demand and minimize the total cost.

(b) (2')Solve it using software and report the objective function value, production plan and screenshots of the model and solver results.

(c) (1')Due to a few good marketing campaigns, the sales team has adjusted their forecast as follows. What would you advise on the adjustment of the production planning? Anything critical to highlight?

Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**
60	85	85	100	55	87

(d) (1')In the real world, it is less likely there will be a very accurate estimate on the demand as shown in here, can you share some ideas how to tackle it?

3. (9') A graduate of this course joined a manufacturing company as a decision scientist, that mainly produced three types of products (A,B,C) based on a mix of three type of raw materials(M1,M2,M3). The profit for product A, B, C are 60, 30 and 20 per unit respectively. The decision scientist has built the following LP model for his boss who appreciates a lot the way of abstracting the problem and thinking mathematically .

$$
\begin{aligned}
\max \quad & 60x_1 + 30x_2 + 20x_3 \\
s.t. \quad & 8x_1 + 6x_2 + x_3 \leq 48 \\
& 4x_1 + 2x_2 + 1.5x_3 \leq 20 \\
& 2x_1 + 1.5x_2 + 0.5x_3 \leq 8 \\
& x_1, x_2, x_3 >= 0, d.v.
\end{aligned}
$$

(a) (1') From the model, can we tell the availability of raw materials (M1,M2,M3)? and the raw material mix to produce each type of the product? List out those info and your assumptions if any

(b) (1') Convert this LP into standard form and represent it into matrix form

(c) (2') Find any two basic feasible solutions and show how they are qualifying

(d) (2') If the existing LP is primal, find its dual. Explain, for the dual model, the meaning of each of the d.v. and constraints carry, in this problem context

(e) (1') If the primal model has a feasible solution (x1 = 1,x2 = 1, x3 = 1), what can we conclude on dual model's bound?

(f) (2') If there is another company wishing to buy all the raw materials from this company, what is the minimum price they shall offer as a whole, and also the detailed price offered to each raw material

4. (4') There is a manufacturer producing product A and B. Product A requires 2 unit of raw material L and 1 units of raw material W whereas product B need 3 and 2 units respectively for raw material L and W. Assume there are only 25 units of raw material L (cost $5 per unit) and 15 units of raw material W (cost $10 per unit). Product A could be sold at $23 per unit, and B could be sold at $40 per unit.

   (a) (2') Write a LP for this manufacturer to maximize its profit. Screenshot of the model and solution from software

   (b) (2') List all the shadow prices, and interpret how much this manufacturer is willing to pay for additional unit of raw material L and W

5. (4') Below is a collection of questions to check our understanding on LP and related stuffs.

   (a) (1') In general, is LP more difficult than IP (Integer Programming)? Explain the reasons for your answer

   (b) (1') Explain the difference between strong duality and weak duality

   (c) (1') Explain the concept of shadow price and how we use it

   (d) (1') What are the typical features we could leverage to assess difficulty of a model in general?