**Topics: Normal distribution, Functions of Random Variables**

1. **The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
2. **0.3875**
3. **0.2676**
4. **0.5**
5. **0.6987**

Solution:

Since the worker plans to start his work 10 minutes after the car is dropped off, therefore, the remaining time to work is 60min – 10min = 50min.

Suppose if the service manager is possible to meet his commitment within 50minutes, then probability would be

P(X<=50) Where, X = time taken to complete work

Standard normal variable is given by

Z =

Where, X = 50min

= 45min

*σ* = 8 min

Z = = 0.625

P(Z<=0.625) = 0.7324 (From Z table) = 73.24%

Since the service manager cannot meet his commitment within 50minutes, therefore, 100 – 73.24 = 26.76%

Therefore, option B = 0.2676 is the correct answer.

1. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.**
2. **More employees at the processing center are older than 44 than between 38 and 44.**
3. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

Solution:

1. Standard normal variable is given by

Z =

* To find Z score for 44

Where, X = 44, = 38, *σ* = 6

Z = = 1 = 0.8413 (From Z table) = 84.13%

* People percentage above 44 age, 100 – 84.13 = 15.87%

Therefore, people above 44 age out of 400 = 63

* To find Z score for 38

Where, X = 38, = 38, *σ* = 6

Z = = 0 = 0.5000 (From Z table) = 50%

People percentage between 38 and 44 age is given by,

84.13 – 50 = 34.13%

It means people between 38 and 44 age out of 400 is 137

Therefore, employees at the processing center are older than 44 than between 38 and 44 – STATEMENT IS FALSE

Solution:

1. To find Z score for 30

Where, X = 30, = 38, *σ* = 6

Z = = - 1.33 = 0.0915 (From Z table) = 9.15%

Therefore, people less than 30 age out of 400 = 36

Therefore, A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - STATEMENT IS TRUE

1. **If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

Solution:

*X1* ~ *N*(μ, σ2) then 2X1 ~ *N*(2μ, 4σ2) and

*X*2 ~ *N*(μ, σ2) then *X*1 + *X*2 ~ *N*(2μ, 4σ2)

So there is no difference between the 2\*X1 and X1+X2. These both are having same normal distribution and mean increased to 2times original mean and standard deviation is 4times original standard deviation

**Note: Please refer Code mentioned in the Jupyter notebook**

1. **Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**
2. **90.5, 105.9**
3. **80.2, 119.8**
4. **22, 78**
5. **48.5, 151.5**
6. **90.1, 109.9**

Solution:

Identify symmetric values for the standard normal distribution such that the area enclosed is .99

we have to excluded area of .005 in each of the left and right tails. Hence, find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

a = -(20\*2.576) + 100= 48.5

b = (20\*2.576)+100= 151.5

Two values symmetric about mean for the given standard normal distribution are [48.5,151.5]

**Note: Code is also mentioned in the Jupyter notebook**

1. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**
2. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**
3. **Specify the 5th percentile of profit (in Rupees) for the company**
4. **Which of the two divisions has a larger probability of making a loss in a given year?**

Solution:

1. Range is Rs (99.00810347848784, 980.9918965215122) in Millions
2. 5th percentile of profit (in Million Rupees) is 170.0

* Probability of Division 1 making a loss P(X<0)

0.0477903522728147

* Probability of Division 2 making a loss P(X<0)

0.040059156863817086

**Note: Please refer Code mentioned in the Jupyter notebook**