# Production and Costs

In the previous chapter, we have discussed the behaviour of the consumers. In this chapter as well as in the next, we shall examine the behaviour of a producer. A producer or a firm acquires different inputs like labour, machines, land, raw materials, etc. Combining these inputs, it produces output. This is called the process of production. In order to acquire inputs, it has to pay for them. That is the cost of production. Once the output has been produced, the firm sells it in the market and earns revenue. The revenue that it earns net of cost is the profit of the firm. We assume here that the objective of a firm is to maximise its profit. A firm looking at its cost structure and the market price of output decides to produce an amount of output such that its profit reaches the maximum.

In this chapter, we study different aspects of the production function of a firm. We discuss here – the relationship between inputs and output, the contribution of a variable input in the production process and different properties of production function. Then we look at the cost structure of the firm. We discuss the cost function and its various aspects. We learn about the properties of the short run and the long run cost curves.

3.1 PRODUCTION FUNCTION

The production function of a firm is a relationship between inputs used and output produced by the firm. For various quantities of inputs used, it gives the maximum quantity of output that can be produced.

Consider a manufacturer who produces shoes. She employs two workers – worker 1 and worker 2, two machines – machine 1 and machine 2, and 10 kilograms of raw materials. Worker 1 is good in operating machine 1 and worker 2 is good in operating machine 2. If worker 1 uses machine 1 and worker 2 uses machine 2, then with 10 kilograms of raw materials, they can produce 10 pairs of shoes. However, if worker 1 uses machine 2 and worker 2 uses machine 1, which they are not good at operating, with the same 10 kilograms of raw materials, they will end up producing only 8 pairs of shoes. So with efficient use of inputs, 10 pairs of shoes can be produced whereas an inefficient use results in production

Input Combination

Output

A Firm Effort
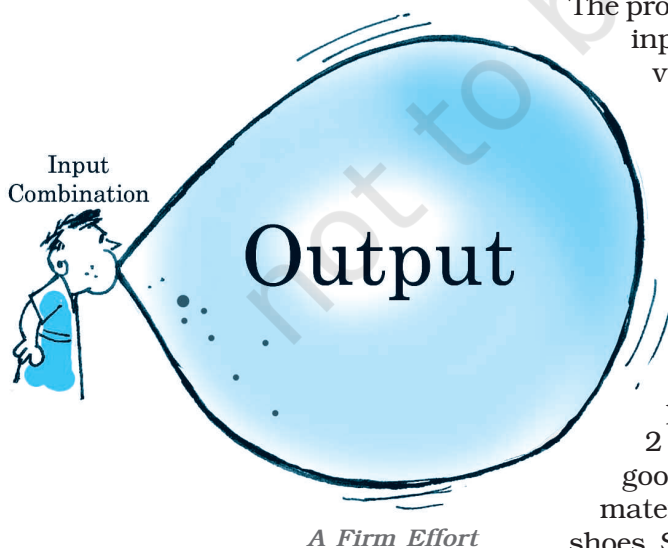
of 8 pairs of shoes. Production function considers only the efficient use of inputs. It says that worker 1, worker 2, machine 1, machine 2 and 10 kilograms of raw materials together can produce 10 pairs of shoes which is the maximum possible output for this input combination.

A production function is defined for a given technology. It is the technological knowledge that determines the maximum levels of output that can be produced using different combinations of inputs. If the technology improves, the maximum levels of output obtainable for different input combinations increase. We then have a new production function.

The inputs that a firm uses in the production process are called factors of production. In order to produce output, a firm may require any number of different inputs. However, for the time being, here we consider a firm that produces output using only two factors of production – factor 1 and factor 2. Our production function, therefore, tells us what maximum quantity of output can be produced by using different combinations of these two factors.

We may write the production function as

$$q = f(x_1, x_2) \tag{3.1}$$

It says that by using x_1 amount of factor 1 and x_2 amount of factor 2, we can at most produce q amount of the commodity.

Table 3.1: **Production Function**

| Factor		x_2						
		0	1	2	3	4	5	6
	0	0	0	0	0	0	0	0
	1	0	1	3	7	10	12	13
	2	0	3	10	18	24	29	33
x_1	3	0	7	18	30	40	46	50
	4	0	10	24	40	50	56	57
	5	0	12	29	46	56	58	59
	6	0	13	33	50	57	59	60

A numerical example of production function is given in Table 3.1. The left column shows the amount of factor 1 and the top row shows the amount of factor 2. As we move to the right along any row, factor 2 increases and as we move down along any column, factor 1 increases. For different values of the two factors, the table shows the corresponding output levels. For example, with 1 unit of factor 1 and 1 unit of factor 2, the firm can produce at most 1 unit of output; with 2 units of factor 1 and 2 units of factor 2, it can produce at most 10 units of output; with 3 units of factor 1 and 2 units of factor 2, it can produce at most 18 units of output and so on.

Isoquant

In Chapter 2, we have learnt about indifference curves. Here, we introduce a similar concept known as isoquant. It is just an alternative way of representing the production function. Consider a production function with two inputs factor 1 and factor 2. An isoquant is the set of all possible combinations of the two inputs that yield the same maximum possible level of output. Each isoquant represents a particular level of output and is labelled with that amount of output.
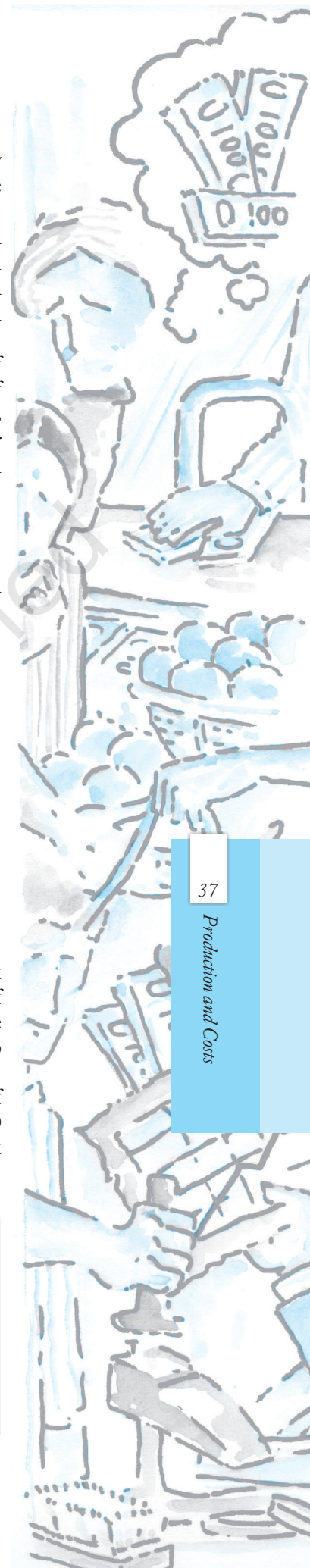
In the diagram, we have three isoquants for the three output levels, namely $q = q_1$, $q = q_2$ and $q = q_3$ in the inputs plane. Two input combinations (x_1', x_2'') and (x_1'', x_2') give us the same level of output q_1. If we fix factor 2 at x_2' and increase factor 1 to x_1''', output increases and we reach a higher isoquant, $q = q_2$. When marginal products are positive, with greater amount of one input, the same level of output can be produced by using lesser amount of the other. Therefore, isoquants are negatively sloped.
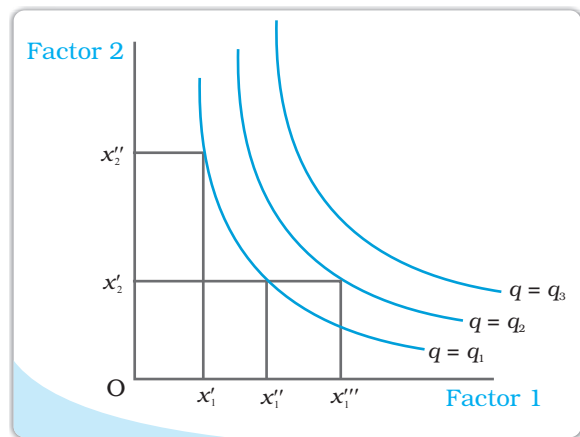
In our example, both the inputs are necessary for the production. If any of the inputs becomes zero, there will be no production. With both inputs positive, output will be positive. As we increase the amount of any input, output increases.

3.2 THE SHORT RUN AND THE LONG RUN

Before we begin with any further analysis, it is important to discuss two concepts– the **short run** and the **long run**.

In the **short run**, a firm cannot vary all the inputs. One of the factors – factor 1 or factor 2 – cannot be varied, and therefore, remain fixed in the short run. In order to vary the output level, the firm can vary only the other factor. The factor that remains fixed is called the **fixed input** whereas the other factor which the firm can vary is called the **variable input**.

Consider the example represented through Table 3.1. Suppose, in the short run, factor 2 remains fixed at 5 units. Then the corresponding column shows the different levels of output that the firm may produce using different quantities of factor 1 in the short run.

In the **long run**, all factors of production can be varied. A firm in order to produce different levels of output in the long run may vary both the inputs simultaneously. So, in the long run, there is no fixed input.

For any particular production process, long run generally refers to a longer time period than the short run. For different production processes, the long run periods may be different. It is not advisable to define short run and long run in terms of say, days, months or years. We define a period as long run or short run simply by looking at whether all the inputs can be varied or not.

3.3 TOTAL PRODUCT, AVERAGE PRODUCT AND MARGINAL PRODUCT

3.3.1 Total Product

Suppose we vary a single input and keep all other inputs constant. Then for different levels of employment of that input, we get different levels of output from the production function. This relationship between the variable input and output, keeping all other inputs constant, is often referred to as **Total Product** (TP) of the variable input.

In our production function, if we keep factor 2 constant, say, at the value $\bar{x}_2$ and vary factor 1, then for each value of x_1, we get a value of q for that particular $\bar{x}_2$. We write it in the following way

$$q = f(x_1; \bar{x}_2) \qquad (3.2)$$

This is the total product function of factor 1.

Let us again look at Table 3.1. Suppose factor 2 is fixed at 4 units. Now in the Table 3.1, we look at the column where factor 2 takes the value 4. As we move down along the column, we get the output values for different values of factor 1. This is the total product of factor 1 schedule with $x_2 = 4$. At $x_1 = 0$, the TP is 0, at $x_1 = 1$, TP is 10 units of output, at $x_1 = 2$, TP is 24 units of output and so on. This is also sometimes called **total return** to or **total physical product** of the variable input.

Once we have defined total product, it will be useful to define the concepts of average product (AP) and marginal product (MP). They are useful in order to describe the contribution of the variable input to the production process.

3.3.2 Average Product

Average product is defined as the output per unit of variable input. We calculate it as

$$AP_1 = \frac{TP}{x_1} = \frac{f(x_1 : \bar{x}_2)}{x_1} \qquad (3.3)$$

Table 3.2 gives us a numerical example of average product of factor 1. In Table 3.1, we have already seen the total product of factor 1 for $x_2 = 4$. In Table 3.2 we reproduce the total product schedule and extend the table to show the corresponding values of average product and marginal product. The first column shows the amount of factor 1 and in the fourth column we get the corresponding average product value. It shows that at 1 unit of factor 1, AP_1 is 10 units of output, at 2 units of factor 1, AP_1 is 12 units of output and so on.
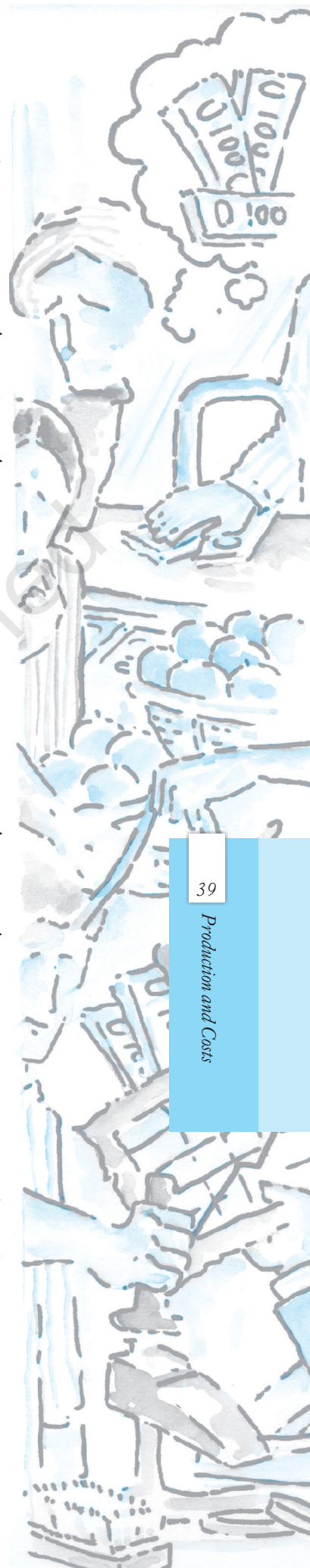
3.3.3 Marginal Product

Marginal product of an input is defined as the change in output per unit of change in the input when all other inputs are held constant. When factor 2 is held constant, the marginal product of factor 1 is

$$MP_1 = \frac{change\ in\ output}{change\ in\ input}$$

$$= \frac{\Delta q}{\Delta x_1} \qquad (3.4)$$

where Δ represents the change of the variable.

If the input changes by discrete units, the marginal product can be defined in the following way. Suppose, factor 2 is fixed at $\bar{x}_2$. With $\bar{x}_2$ amount of factor 2, let, according to the total product curve, x_1 units of factor 1 produce 20 units of the output and $x_1 - 1$ units of factor 1 produce 15 units of the output. We say that the marginal product of the x_1th unit of factor 1 is

$$MP_1 = f(x_1; \bar{x}_2) - f(x_1 - 1; \bar{x}_2) \qquad (3.5)$$

$$= (\text{TP at } x_1 \text{ units}) - (\text{TP at } x_1 - 1 \text{ unit})$$

$$= (20 - 15) \text{ units of output}$$

$$= 5 \text{ units of output}$$

Since inputs cannot take negative values, marginal product is undefined at zero level of input employment. Marginal products are additions to total product. For any level of employment of an input, the sum of marginal products of every unit of that input up to that level gives the total product of that input at that employment level. So total product is the sum of marginal products.

Average product of an input at any level of employment is the average of all marginal products up to that level. Average and marginal products are often referred to as average and marginal returns, respectively, to the variable input.

In the example represented through Table 3.1, if we keep factor 2 constant say, at 4 units, we get a total product schedule. From the total product, we then derive the marginal product and average product of factor 1. The third column of Table 3.2 shows that at zero unit of factor 1, MP_1 is undefined. At $x_1 = 1$, Mp_1 is 10 units of output, at $x_1 = 2$, MP_1 is 14 units of output and so on.

Table 3.2: **Total Product, Marginal product and Average product**

Factor 1	*TP*	*MP_1*	*AP_1*
0	0	-	-
1	10	10	10
2	24	14	12
3	40	16	13.33
4	50	10	12.5
5	56	6	11.2
6	57	1	9.5

3.4 THE LAW OF DIMINISHING MARGINAL PRODUCT AND THE LAW OF VARIABLE PROPORTIONS

The **law of diminishing marginal product** says that if we keep increasing the employment of an input, with other inputs fixed, eventually a point will be reached after which the resulting addition to output (i.e., marginal product of that input) will start falling.

A somewhat related concept with the law of diminishing marginal product is the **law of variable proportions**. It says that the marginal product of a factor input initially rises with its employment level. But after reaching a certain level of employment, it starts falling.

The reason behind the law of diminishing returns or the law of variable proportion is the following. As we hold one factor input fixed and keep increasing the other, the factor proportions change. Initially, as we increase the amount of the variable input, the factor proportions become more and more suitable for the production and marginal product increases. But after a certain level of employment, the production process becomes too crowded with the variable input and the factor proportions become less and less suitable for the production. It is from this point that the marginal product of the variable input starts falling.

Let us look at Table 3.2 again. With factor 2 fixed at 4 units, the table shows us the TP, MP_1 and AP_1 for different values of factor 1. We see that up to the employment level of 3 units of factor 1, its marginal product increases. Then it starts falling.

3.5 SHAPES OF TOTAL PRODUCT, MARGINAL PRODUCT AND AVERAGE PRODUCT CURVES

An increase in the amount of one of the inputs keeping all other inputs constant generally results in an increase in output. Table 3.2 shows how the total product changes as the amount of factor 1 increases. The total product curve in the input-output plane is a positively sloped curve. Figure 3.1 shows the shape of the total product curve for a typical firm.

We measure units of factor 1 along the horizontal axis and output along the vertical axis. With x_1 units of factor 1, the firm can at most produce q_1 units of output.

According to the law of variable proportions, the marginal product of an input initially rises and then after a certain level of employment, it starts falling. The MP curve in the input-output plane, therefore, looks like an inverse 'U'-shaped curve.

Let us now see what the AP curve looks like. For the first unit of the variable input, one can easily check that the MP and the AP are same. Now as we increase the amount of input, the MP rises. AP being the average of marginal products, also rises, but rises less than MP. Then, after a point, the MP starts falling. However, as long as the value of MP remains higher than the value of the prevailing AP, the latter continues to rise. Once MP has fallen sufficiently, its value becomes less than the prevailing AP and the latter also starts falling. So AP curve is also inverse 'U'-shaped.

As long as the AP increases, it must be the case that MP is greater than AP. Otherwise, AP cannot rise. Similarly, when AP falls, MP has to be less than AP. It, therefore, follows that MP curve cuts AP curve from above at its maximum.

Figure 3.2 shows the shapes of AP and MP curves for a typical firm.

The AP of factor 1 is maximum at x_1. To the left of x_1, AP is rising and MP is greater than AP. To the right of x_1, AP is falling and MP is less than AP.
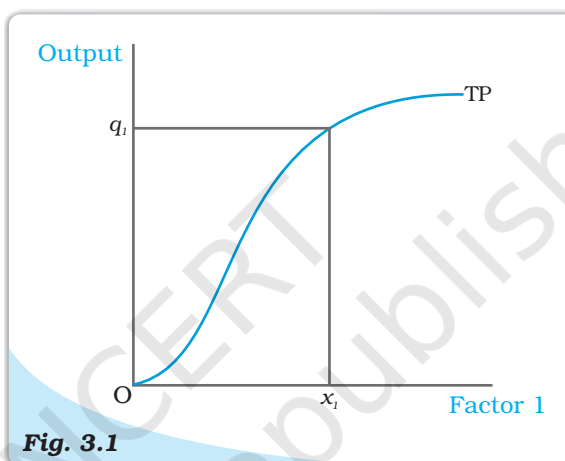

Fig. 3.1

Total Product. This is a total product curve for factor 1. When all other inputs are held constant, it shows the different output levels obtainable from different amounts of factor 1.
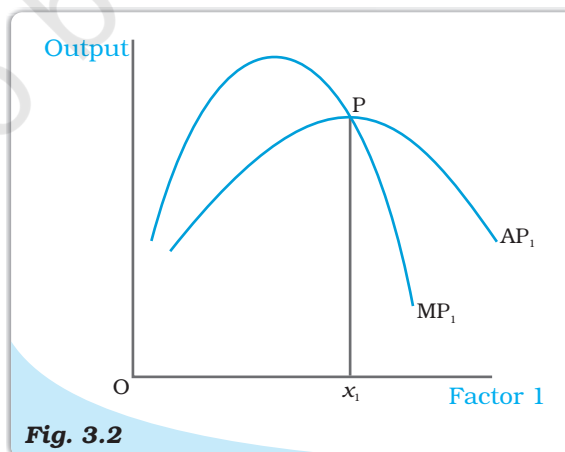

Fig. 3.2

Average and Marginal Product. These are average and marginal product curves of factor 1.

3.6 RETURNS TO SCALE

So far we looked at various aspects of production function when a single input varied and others remained fixed. Now we shall see what happens when all inputs vary simultaneously.

Constant returns to scale (CRS) is a property of production function that holds when a proportional increase in all inputs results in an increase in output by the same proportion.

Increasing returns to scale (IRS) holds when a proportional increase in all inputs results in an increase in output by more than the proportion.

Decreasing returns to scale (DRS) holds when a proportional increase in all inputs results in an increase in output by less than the proportion.

For example, suppose in a production process, all inputs get doubled. As a result, if the output gets doubled, the production function exhibits CRS. If output is less than doubled, the DRS holds, and if it is more than doubled, the IRS holds.

Returns to Scale

Consider a production function

$$q = f(x_1, x_2)$$

where the firm produces q amount of output using x_1 amount of factor 1 and x_2 amount of factor 2. Now suppose the firm decides to increase the employment level of both the factors t ($t > 1$) times. Mathematically, we can say that the production function exhibits constant returns to scale if we have,

$$f(tx_1, tx_2) = t.f(x_1, x_2)$$

ie the new output level $f(tx_1, tx_2)$ is exactly t times the previous output level $f(x_1, x_2)$.

Similarly, the production function exhibits increasing returns to scale if,

$$f(tx_1, tx_2) > t.f(x_1, x_2).$$

It exhibits decreasing returns to scale if,

$$f(tx_1, tx_2) < t.f(x_1, x_2).$$

3.7 COSTS

In order to produce output, the firm needs to employ inputs. But a given level of output, typically, can be produced in many ways. There can be more than one input combinations with which a firm can produce a desired level of output. In Table 3.1, we can see that 50 units of output can be produced by three different input combinations – ($x_1 = 6$, $x_2 = 3$), ($x_1 = 4$, $x_2 = 4$) and ($x_1 = 3$, $x_2 = 6$). The question is which input combination will the firm choose? With the input prices given, it will choose that combination of inputs which is least expensive. So, for every level of output, the firm chooses the least cost input combination. This output-cost relationship is the cost function of the firm.

Cobb-Douglas Production Function

Consider a production function

$$q = x_1^{\alpha} x_2^{\beta}$$

where α and β are constants. The firm produces q amount of output using x_1 amount of factor 1 and x_2 amount of factor 2. This is called a **Cobb-Douglas production function**. Suppose with $x_1 = \overline{x}_1$ and $x_2 = \overline{x}_2$, we have q_0 units of output, i.e.

$$q_0 = \overline{x}_1^{\alpha} \overline{x}_2^{\beta}$$

If we increase both the inputs t ($t > 1$) times, we get the new output

$$q_1 = (t\overline{x}_1)^{\alpha}(t\overline{x}_2)^{\beta}$$
$$= t^{\alpha+\beta} \overline{x}_1^{\alpha} \overline{x}_2^{\beta}$$

When $\alpha + \beta = 1$, we have $q_1 = tq_0$. That is, the output increases t times. So the production function exhibits CRS. Similarly, when $\alpha + \beta > 1$, the production function exhibits IRS. When $\alpha + \beta < 1$ the production function exhibits DRS.

3.7.1 Short Run Costs

We have previously discussed the short run and the long run. In the short run, some of the factors of production cannot be varied, and therefore, remain fixed. The cost that a firm incurs to employ these fixed inputs is called the total fixed cost (TFC). Whatever amount of output the firm produces, this cost remains fixed for the firm. To produce any required level of output, the firm, in the short run, can adjust only variable inputs. Accordingly, the cost that a firm incurs to employ these variable inputs is called the total variable cost (TVC). Adding the fixed and the variable costs, we get the total cost (TC) of a firm

$$TC = TVC + TFC \qquad (3.6)$$

In order to increase the production of output, the firm needs to employ more of the variable inputs. As a result, total variable cost and total cost will increase. Therefore, as output increases, total variable cost and total cost increase.

In Table 3.3, we have an example of cost function of a typical firm. The first column shows different levels of output. For all levels of output, the total fixed cost is Rs 20. Total variable cost increases as output increases. With output zero, TVC is zero. For 1 unit of output, TVC is Rs 10; for 2 units of output, TVC is Rs 18 and so on. In the fourth column, we obtain the total cost (TC) as the sum of the corresponding values in second column (TFC) and third column (TVC). At zero level of output, TC is just the fixed cost, and hence, equal to Rs 20. For 1 unit of output, total cost is Rs 30; for 2 units of output, the TC is Rs 38 and so on.

The **short run average cost** (SAC) incurred by the firm is defined as the total cost per unit of output. We calculate it as

$$SAC = \frac{TC}{q} \qquad (3.7)$$

In Table 3.3, we get the SAC-column by dividing the values of the fourth column by the corresponding values of the first column. At zero output, SAC is undefined. For the first unit, SAC is Rs 30; for 2 units of output, SAC is Rs 19 and so on.
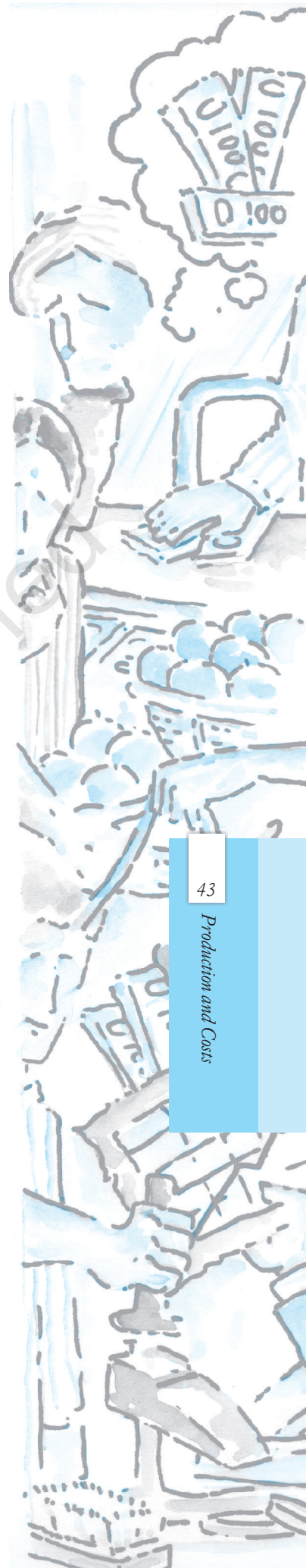
Similarly, the average variable cost (AVC) is defined as the total variable cost per unit of output. We calculate it as

$$AVC = \frac{TVC}{q} \tag{3.8}$$

Also, average fixed cost (AFC) is

$$AFC = \frac{TFC}{q} \tag{3.9}$$

Clearly,

$$SAC = AVC + AFC \tag{3.10}$$

In Table 3.3, we get the AFC-column by dividing the values of the second column by the corresponding values of the first column. Similarly, we get the AVC-column by dividing the values of the third column by the corresponding values of the first column. At zero level of output, both AFC and AVC are undefined. For the first unit of output, AFC is Rs 20 and AVC is Rs 10. Adding them, we get the SAC equal to Rs 30.

The short run marginal cost (SMC) is defined as the change in total cost per unit of change in output

$$SMC = \frac{change\ in\ total\ \cos t}{change\ in\ output} = \frac{\Delta TC}{\Delta q} \tag{3.11}$$

where Δ represents the change of the variable.

If output changes in discrete units, we may define the marginal cost in the following way. Let the cost of production for q_1 units and $q_1 - 1$ units of output be Rs 20 and Rs 15 respectively. Then the marginal cost that the firm incurs for producing q_1th unit of output is

$$MC = (TC\ \text{at}\ q_1) - (TC\ \text{at}\ q_1 - 1) \tag{3.12}$$
$$= \text{Rs } 20 - \text{Rs } 15 = \text{Rs } 5$$

Just like the case of marginal product, marginal cost also is undefined at zero level of output. It is important to note here that in the short run, fixed cost cannot be changed. When we change the level of output, whatever change occurs to total cost is entirely due to the change in total variable cost. So in the short

Table 3.3: **Various Concepts of Costs**

Output (units)	TFC (Rs)	TVC (Rs)	TC (Rs)	AFC (Rs)	AVC (Rs)	SAC (Rs)	SMC (Rs)
0	20	0	20	–	–	–	–
1	20	10	30	20	10	30	10
2	20	18	38	10	9	19	8
3	20	24	44	6.67	8	14.67	6
4	20	29	49	5	7.25	12.25	5
5	20	33	53	4	6.6	10.6	4
6	20	39	59	3.33	6.5	9.83	6
7	20	47	67	2.86	6.7	9.57	8
8	20	60	80	2.5	7.5	10	13
9	20	75	95	2.22	8.33	10.55	15
10	20	95	115	2	9.5	11.5	20

run, marginal cost is the increase in TVC due to increase in production of one extra unit of output. For any level of output, the sum of marginal costs up to that level gives us the total variable cost at that level. One may wish to check this from the example represented through Table 3.3. Average variable cost at some level of output is therefore, the average of all marginal costs up to that level. In Table 3.3, we see that when the output is zero, SMC is undefined. For the first unit of output, SMC is Rs 10; for the second unit, the SMC is Rs 8 and so on.

Shapes of the Short Run Cost Curves

Now let us see what these short run cost curves look like in the output-cost plane.

Previously, we have discussed that in order to increase the production of output the firm needs to employ more of the variable inputs. This results in an increase in total variable cost, and hence, an increase in total cost. Therefore, as output increases, total variable cost and total cost increase. Total fixed cost, however, is independent of the amount of output produced and remains constant for all levels of production.

Figure 3.3 illustrates the shapes of total fixed cost, total variable cost and total cost curves for a typical firm. TFC is a constant which takes the value c_1 and does not change with the change in output. It is, therefore, a horizontal straight line cutting the cost axis at the point c_1. At q_1, TVC is c_2 and TC is c_3.

AFC is the ratio of TFC to q. TFC is a constant. Therefore, as q increases, AFC decreases. When output is very close to zero, AFC is arbitrarily large, and as output moves towards infinity, AFC moves towards zero. AFC curve is, in fact, a rectangular hyperbola. If we multiply any value q of output with its corresponding AFC, we always get a constant, namely TFC.

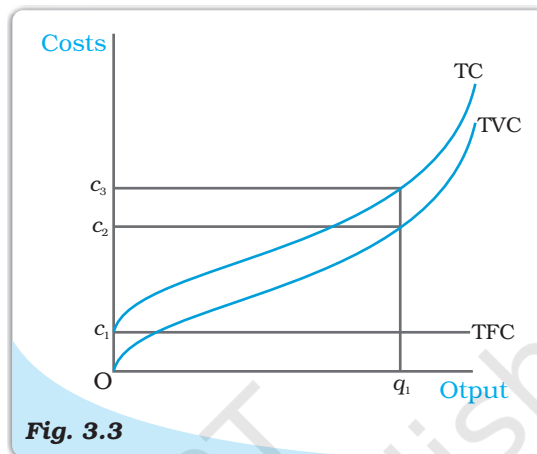Figure 3.4 shows the shape of average fixed cost curve for a typical

Fig. 3.3

Costs. These are total fixed cost (TFC), total variable cost (TVC) and total cost (TC) curves for a firm. Total cost is the vertical sum of total fixed cost and total variable cost.
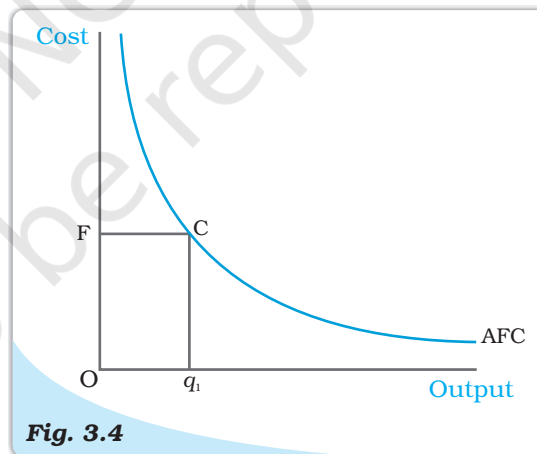

Fig. 3.4

Average Fixed Cost. The average fixed cost curve is a rectangular hyperbola. The area of the rectangle $OFCq_1$ *gives us the total fixed cost.*

firm. We measure output along the horizontal axis and AFC along the vertical axis. At q_1 level of output, we get the corresponding average fixed cost at *F*. The TFC can be calculated as

$$TFC = AFC \times \text{quantity}$$
$$= OF \times Oq_1$$
$$= \text{the area of the rectangle } OFCq_1$$

We can also calculate AFC from TFC curve. In Figure 3.5, the horizontal straight line cutting the vertical axis at F is the TFC curve. At q_0 level of output, total fixed cost is equal to OF. At q_0, the corresponding point on the TFC curve is A. Let the angle $\angle AOq_0$ be θ. The AFC at q_0 is

$$AFC = \frac{TFC}{quantity}$$

$$= \frac{Aq_0}{Oq_0} = \tan\theta$$

Let us now look at the SMC curve. Marginal cost is the additional cost that a firm incurs to produce one extra unit of output. According to the law of variable proportions, initially, the marginal product of a factor increases as employment increases, and then after a certain point, it decreases. This means initially to produce every next unit of output, the requirement of the factor becomes less and less, and then after a certain point, it becomes greater and greater. As a result, with the factor price given, initially the SMC falls, and then after a certain point, it rises. SMC curve is, therefore, 'U'-shaped.

At zero level of output, SMC is undefined. When output is discrete, the TVC at a particular level of output is the sum of all marginal costs up to that level. When output is perfectly divisible, the TVC at a particular level of output is given by the area under the SMC curve up to that level.

Now, what does the AVC curve look like? For the first unit of output, it is easy to check that SMC and AVC are the same. So both SMC and AVC curves start from the same point. Then, as output increases, SMC falls. AVC being the average of marginal costs, also falls, but falls less than SMC. Then, after a point, SMC starts rising. AVC, however, continues to fall as long as the value of SMC remains less than the prevailing value of AVC. Once the SMC has risen sufficiently, its value becomes greater than the value of AVC. The AVC then starts rising. The AVC curve is therefore 'U'-shaped.

As long as AVC is falling, SMC must be less than the AVC and as AVC, rises, SMC must be greater than the AVC. So the SMC curve cuts the AVC curve from below at the minimum point of AVC.

In Figure 3.6 we measure output along the horizontal axis and AVC along the vertical axis. At q_0 level of output, AVC is equal to OV. The total variable cost at q_0 is

$$TVC = AVC \quad quantity$$

$$= OV \quad Oq_0$$

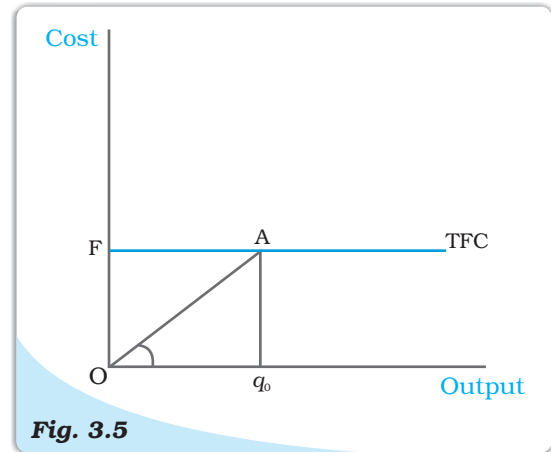$$= \text{the area of the rectangle } OVBq_0.$$

Fig. 3.5

The Total Fixed Cost Curve. The slope of the angle $\angle AOq_0$ gives us the average fixed cost at q_0.
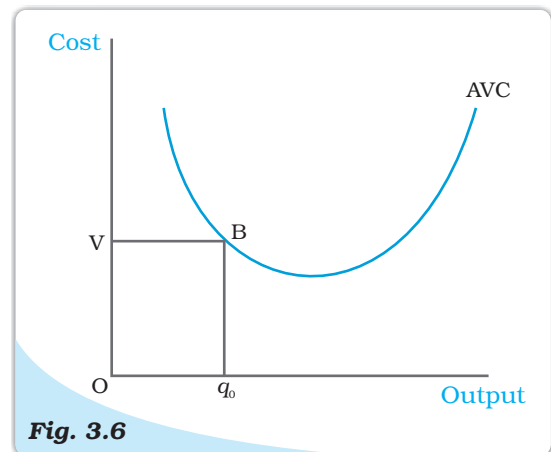
Fig. 3.6

The Average Variable Cost Curve. The area of the rectangle $OVBq_0$ gives us the total variable cost at q_0.

In Figure 3.7, we measure output along the horizontal axis and TVC along the vertical axis. At q_0 level of output, OV is the total variable cost. Let the angle $\angle EOq_0$ be equal to θ. Then, at q_0, the AVC can be calculated as

$$AVC = \frac{TVC}{output}$$

$$= \frac{Eq_0}{Oq_0} = \tan\theta$$

Let us now look at SAC. SAC is the sum of AVC and AFC. Initially, both AVC and AFC decrease as output increases. Therefore, SAC initially falls. After a certain level of

Fig. 3.7

The Total Variable Cost Curve. The slope of the angle $\angle EOq_0$ gives us the average variable cost at q_0.

output production, AVC starts rising. Now AVC and AFC are moving in opposite direction. Here, initially the fall in AFC is greater than the rise in AVC and SAC is still falling. But, after a certain level of production, rise in AVC overrides the fall in AFC. From this point onwards, SAC is rising. SAC curve is therefore 'U'-shaped.

It lies above the AVC curve with the vertical difference being equal to the value of AFC. The minimum point of SAC curve lies to the right of the minimum point of AVC curve.

Similar to the case of AVC and SMC, here too as long as SAC is falling, SMC is less than the SAC and when SAC is rising, SMC is greater than the SAC. SMC curve cuts the SAC curve from below at the minimum point of SAC.

Figure 3.8 shows the shapes of short run marginal cost, average variable cost and short run average cost curves for a typical firm. AVC reaches its minimum at q_1 units of output. To the left of q_1, AVC is falling and SMC is less than AVC. To the right of q_1, AVC is rising and SMC is greater than AVC. SMC curve cuts the AVC curve at 'P' which is the minimum point of AVC curve. The minimum point of SAC curve is 'S' which corresponds to the output q_2. It is the intersection point between SMC and SAC curves. To the left of q_2, SAC is falling and SMC is less than SAC. To the right of q_2, SAC is rising and SMC is greater than SAC.

Fig. 3.8

Short Run Costs. Short run marginal cost, average variable cost and average cost curves.

3.7.2 Long Run Costs

In the long run, all inputs are variable. The total cost and the total variable cost therefore, coincide in the long run. **Long run average cost** (LRAC) is defined as cost per unit of output, i.e.
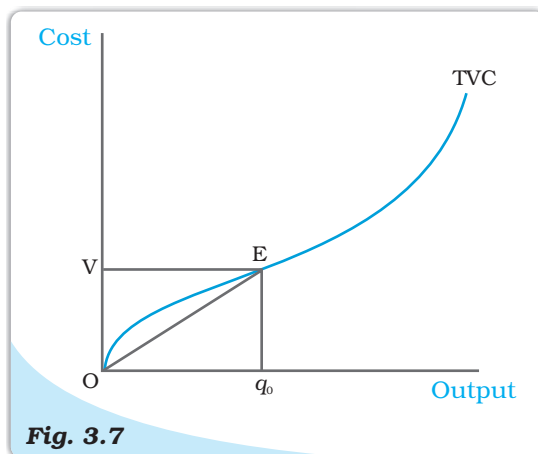
$$LRAC = \frac{TC}{q} \tag{3.13}$$

Long run marginal cost (LRMC) is the change in total cost per unit of change in output. When output changes in discrete units, then, if we increase production from $q_1 - 1$ to q_1 units of output, the marginal cost of producing q_1^{th} unit will be measured as

$$LRMC = (TC \text{ at } q_1 \text{ units}) - (TC \text{ at } q_1 - 1 \text{ units}) \tag{3.14}$$

Just like the short run, in the long run, the sum of all marginal costs up to some output level gives us the total cost at that level.

Shapes of the Long Run Cost Curves

We have previously discussed the returns to scales. Now let us see their implications for the shape of LRAC.

IRS implies that if we increase all the inputs by a certain proportion, output increases by more than that proportion. In other words, to increase output by a certain proportion, inputs need to be increased by less than that proportion. With the input prices given, cost also increases by a lesser proportion. For example, suppose we want to double the output. To do that, inputs need to be increased by less than double. The cost that the firm incurs to hire those inputs therefore also need to be increased by less than double. What is happening to the average cost here? It must be the case that as long as IRS operates, average cost falls as the firm increases output.

DRS implies that if we want to increase the output by a certain proportion, inputs need to be increased by more than that proportion. As a result, cost also increases by more than that proportion. So, as long as DRS operates, the average cost must be rising as the firm increases output.

CRS implies a proportional increase in inputs resulting in a proportional increase in output. So the average cost remains constant as long as CRS operates.

It is argued that in a typical firm IRS is observed at the initial level of production. This is then followed by the CRS and then by the DRS. Accordingly, the LRAC curve is a 'U'-shaped curve. Its downward sloping part corresponds to IRS and upward rising part corresponds to DRS. At the minimum point of the LRAC curve, CRS is observed.

Let us check how the LRMC curve looks like. For the first unit of output, both LRMC and LRAC are the same. Then, as output increases, LRAC initially falls, and then, after a certain point, it rises. As long as average cost is falling, marginal cost must be less than the average cost. When the average cost is rising, marginal cost must be greater than the average cost. LRMC curve is therefore a 'U'-shaped curve. It cuts the LRAC curve from below at the minimum point of the LRAC. Figure 3.9 shows the shapes of the long run marginal cost and the long run average cost curves for a typical firm.

LRAC reaches its minimum at q_1. To the left of q_1, LRAC is falling and LRMC is less than the LRAC curve. To the right of q_1, LRAC is rising and LRMC is higher than LRAC.
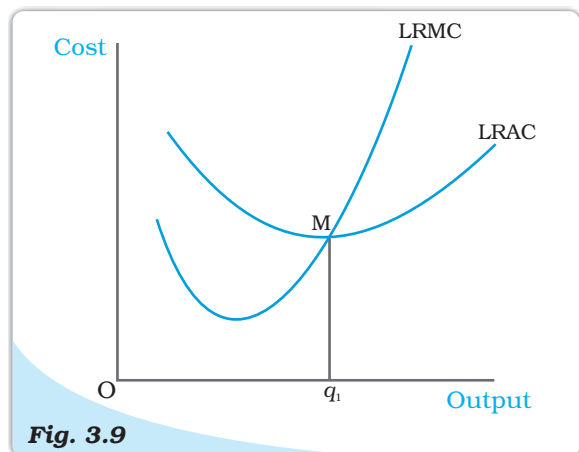

Fig. 3.9

Long Run Costs. Long run marginal cost and average cost curves.

Summary

- For different combinations of inputs, the production function shows the maximum quantity of output that can be produced.
- In the short run, some inputs cannot be varied. In the long run, all inputs can be varied.
- Total product is the relationship between a variable input and output when all other inputs are held constant.
- For any level of employment of an input, the sum of marginal products of every unit of that input up to that level gives the total product of that input at that employment level.
- Both the marginal product and the average product curves are inverse 'U'-shaped. The marginal product curve cuts the average product curve from above at the maximum point of average product curve.
- In order to produce output, the firm chooses least cost input combinations.
- Total cost is the sum of total variable cost and the total fixed cost.
- Average cost is the sum of average variable cost and average fixed cost.
- Average fixed cost curve is downward sloping.
- Short run marginal cost, average variable cost and short run average cost curves are 'U'-shaped.
- SMC curve cuts the AVC curve from below at the minimum point of AVC.
- SMC curve cuts the SAC curve from below at the minimum point of SAC.
- In the short run, for any level of output, sum of marginal costs up to that level gives us the total variable cost. The area under the SMC curve up to any level of output gives us the total variable cost up to that level.
- Both LRAC and LRMC curves are 'U' shaped.
- LRMC curve cuts the LRAC curve from below at the minimum point of LRAC.

Key Concepts

Production function

Long run

Marginal product

Law of diminishing marginal product
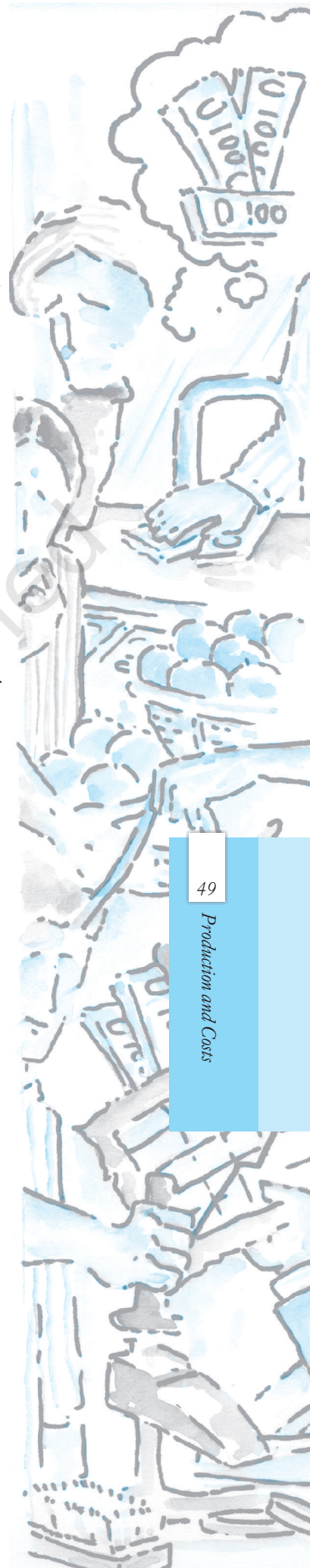
Cost function

Short run

Total product

Average product

Law of variable proportions

Returns to scale

Marginal cost, Average cost

Exercises

1. Explain the concept of a production function.
2. What is the total product of an input?
3. What is the average product of an input?
4. What is the marginal product of an input?
5. Explain the relationship between the marginal products and the total product of an input.
6. Explain the concepts of the short run and the long run.
7. What is the law of diminishing marginal product?
8. What is the law of variable proportions?
9. When does a production function satisfy constant returns to scale?
10. When does a production function satisfy increasing returns to scale?

11. When does a production function satisfy decreasing returns to scale?

12. Briefly explain the concept of the cost function.

13. What are the total fixed cost, total variable cost and total cost of a firm? How are they related?

14. What are the average fixed cost, average variable cost and average cost of a firm? How are they related?

15. Can there be some fixed cost in the long run? If not, why?

16. What does the average fixed cost curve look like? Why does it look so?

17. What do the short run marginal cost, average variable cost and short run average cost curves look like?

18. Why does the SMC curve cut the AVC curve at the minimum point of the AVC curve?

19. At which point does the SMC curve cut the SAC curve? Give reason in support of your answer.

20. Why is the short run marginal cost curve 'U'-shaped?

21. What do the long run marginal cost and the average cost curves look like?

22. The following table gives the total product schedule of labour. Find the corresponding average product and marginal product schedules of labour.

L	TP_L
0	0
1	15
2	35
3	50
4	40
5	48

23. The following table gives the average product schedule of labour. Find the total product and marginal product schedules. It is given that the total product is zero at zero level of labour employment.

L	AP_L
1	2
2	3
3	4
4	4.25
5	4
6	3.5

24. The following table gives the marginal product schedule of labour. It is also given that total product of labour is zero at zero level of employment. Calculate the total and average product schedules of labour.

L	MP_L
1	3
2	5
3	7
4	5
5	3
6	1

25. The following table shows the total cost schedule of a firm. What is the total fixed cost schedule of this firm? Calculate the TVC, AFC, AVC, SAC and SMC schedules of the firm.

Q	TC
0	10
1	30
2	45
3	55
4	70
5	90
6	120

26. The following table gives the total cost schedule of a firm. It is also given that the average fixed cost at 4 units of output is Rs 5. Find the TVC, TFC, AVC, AFC, SAC and SMC schedules of the firm for the corresponding values of output.

Q	TC
1	50
2	65
3	75
4	95
5	130
6	185

27. A firm's SMC schedule is shown in the following table. The total fixed cost of the firm is Rs 100. Find the TVC, TC, AVC and SAC schedules of the firm.

Q	TC
0	-
1	500
2	300
3	200
4	300
5	500
6	800

28. Let the production function of a firm be

$$Q = 5 L^{\frac{1}{2}} \ K^{\frac{1}{2}}$$

Find out the maximum possible output that the firm can produce with 100 units of L and 100 units of K.

29. Let the production function of a firm be

$$Q = 2L^2K^2$$

Find out the maximum possible output that the firm can produce with 5 units of L and 2 units of K. What is the maximum possible output that the firm can produce with zero unit of L and 10 units of K?

30. Find out the maximum possible output for a firm with zero unit of L and 10 units of K when its production function is

$$Q = 5L + 2K$$