# Computer-assisted Billiard Self-Training Using Intelligent Glasses

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Abstract—Self-training plays an important role in sports exercise. However, if not under the instruction of a coach, it would be ineffective for most amateurs or inexperienced players to exercise on their own. Therefore, establishing computer-assisted training systems for sports exercise is a recently emerging topic. In this paper, we propose a billiard self-training system, which aims at improving billiard players' performance by utilizing intelligent glasses as a wearable camera and displayer. The proposed system is able to automatically analyze user-captured images of the billiard table from multiple views and display the ball configurations on a virtual top-view table. Enriched visual presentation can be provided to give the practitioner a further sight into the game. The experiments conducted on sixteen sets of different ball configurations show promising results.

Keywords—sports training; ball localization; intelligent glasses; wearable device; multi-view image processing

### I. INTRODUCTION

Computer-assisted sports training attracts considerable research interests due to its commercial benefits and practical applications in sports exercise. Numerous research works on sports video analysis have been proposed, especially for the topics of event detection [1]-[3] and tactic analysis [4]-[6]. From the perspective of professional coaches and players, rapid development of computer vision and video processing technologies leads to the proliferation of automatic/semiautomatic systems to assist players in improving their performance. Most amateurs and inexperienced players spend additional time exercising on their own in addition to taking the regular training courses given by a coach or instructor. However, it would be ineffective for most amateurs and inexperienced players during self-training if not under good instruction. Therefore, establishing automatic systems for self-training is an important issue in the research fields of computer-assisted sports training.

In this paper, we take billiards as our research subject and propose an intelligent glasses-based billiard self-training system, which aims at improving billiard players' performance. As a social pastime or a professional contest, billiards becomes one of the popular sports. Most existing works focus on analyzing video footage in order to give a 3D representation of the games or to detect specific events for further usage. Jiang et al. [7] propose an information-theoretical scheme not only to evaluate the reconstruction quality, but also to automatically optimize and reduce the errors of automated 3D modeling from broadcast billiard videos. Höferlin et al. [8] utilize video visualization to provide players and coaches with a quantifiable and comparable summary for a single shot. Chou et al. [9] propose a billiard tutoring system for aiming suggestion by analyzing broadcasting nine-ball videos.

Other research works design robots capable of automatically evaluating the table layout and executing shots on a real table. Nierhoff et al. [10] present a robotic system, which addresses the issues of the perception of relevant environment information, shot actions planning, and execution efficiency. Landry et al. [11] design a robotic pool to simulate the physics, execute the shots and analyze successive shots.

However, billiards involves complex blend of physical dexterity in addition to analytic geometry and strategy, which makes inexperienced players thirst for instruction of shot planning while exercising on their own. Thus, plenty of research works are devoted to constructing automatic/semi-automatic systems to provide inexperienced players with the instructions in better understanding. Shih et al. [12] present a system for strategy planning by calculating the best sequence of shots given a starting cue-ball position. Unlike the previous studies using only the RGB camera, Sousa et al. [13] exploit a Kinect 2 sensor hung on the ceiling to locate and track each objects using depth information. The trajectories of the cueball after collision are directly projected on the billiard table with a projector.

Most of the aforementioned research works on billiards are conducted on videos captured by a camera deployed right above the table, which, however, is not so practicable for general users. Thus, Jebara et al. [14] propose a wearable-based augmented reality system for billiards. The system locates the table, balls, and pockets through a video camera near user's eye with the probabilistic color model and symmetry operation. Further, all the trajectories to finish a shot on the scene are evaluated and ranked. The trajectory

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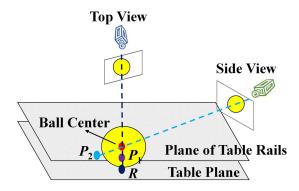


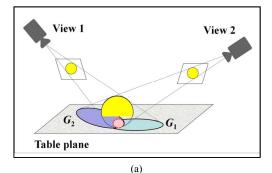
Fig. 1. Illustration of ball center projection from the top view and the side view.

with the lowest difficulty is rendered on the head-mounted displayer to assist users in aiming and strategy planning. However, users cannot see the real scene through the head-mounted displayer, which makes the users feel like playing a video game, not training.

It is our ultimate goal to develop a billiards guidance system enabling a user to see the possible direction and power of shooting a ball via intelligent glasses for strategy planning. To that end, ball localization as proposed in this work is essential. Besides, in billiards, ball configurations play a major role in strategic planning since the current ball locations have much impact on subsequent shots. To precisely analyze and execute subsequent shots, the accuracy of ball localization should be highly concerned.

However, most state-of-the-art works on billiard video analysis pay little attention to ball localization accuracy, and estimate the ball locations by simply transforming the ball center in the image to the table plane with the homography matrix obtained from the table rails. This scheme is workable only if the camera is deployed right above the table, because the table plane is not at the same plane with the table rails. As shown in Figure 1, the ball center projection from the top view, i.e.,  $P_1$ , on the plane of table rails can correctly indicate the ball's location, i.e., R on the table plane since  $P_1R$  and table plane are nearly perpendicular. On the contrary, the ball center projection from the side view, i.e.,  $P_2$ , on the plane of table rails cannot be regarded as the ball's location. Moreover, the balls arranged closely are harder to locate accurately from only one single viewpoint due to the occlusion problem.

To address the issues already outlined, we propose a multi-view ball localization scheme for billiards to extract and visualize ball configurations utilizing the intelligent glasses as a wearable camera and displayer. Our proposed multi-view localization approach is inspired by Liu et al. [15], who propose a vanishing point-based line sampling scheme and project the line samples to generate possible people locations from multiple views. Our proposed system can achieve high localization accuracy while the intelligent glasses enable the players not only to see the real scene but



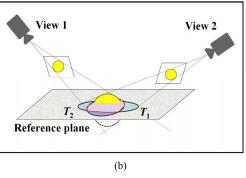


Fig. 2. Foreground projections on (a) the table plane and (b) the reference plane.

also to watch the ball configurations from a third-person perspective on a mini displayer.

One main concept of our work is that a foreground region, i.e., a ball, in a camera view can be regarded as a projection on the reference plane and the projections from multiple camera views will meet at where the ball touches that plane. When utilizing the intelligent glasses as a wearable camera, the ball projections on the table plane from multiple views will meet at the ball's bottom location, as shown in Figure 2(a), wherein the  $G_1$  and  $G_2$  are the table projections from View 1 and View 2, respectively. However, compared with ball center regions, extracted bottom regions may be less intact and are more easily impaired after some noise removal processing, such as morphological operations.

Hence, we employ a reference plane at the same height with the table rails. As illustrated in Figure 2(b), the intersection of the projections on the reference plane, i.e.,  $T_1$  and  $T_2$ , can be utilized to indicate the ball's location since the center part of a ball is nearly perpendicular to the table plane.

The rest of this paper is organized as follows. The framework of our proposed system is described in Section II. The processing modules of color-based table region extraction, camera calibration, and multi-view ball localization are elaborated in Sections III, IV and V, respectively. Then, Section VI presents the experimental results. Finally, Section VII concludes this paper.

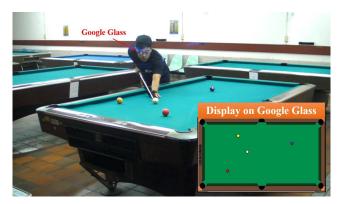


Fig. 3. Application scenario of our proposed billiard self-training system.

#### II. SYSTEM OVERVIEW

The application scenario of our proposed billiard self-training system is shown in Figure 3. In the proposed system, the ego-camera on the intelligent glasses does not keep recording video frames and estimating ball positions. Instead, before each shot, the user wearing the camera is requested to capture three images containing the full table from different viewpoints for subsequent analysis. Analyzing the three images, the system estimates ball positions and visualizes the ball configuration on a virtual top-view table only once for each shot.

The schematic flowchart of the proposed framework is illustrated in Figure 4. In the module of Color-based Table Region Extraction, the table region in each image is extracted by analyzing the distribution of dominant color and the positions of table rails. The obtained table region can reduce the processing area of the subsequent analysis, increasing the computational efficiency. The Camera Calibration module uses the intersections of table rails as corresponding points to calculate the projection matrix mapping coordinates on a reference plane to image points or vice versa. Then we fuse foreground regions on the reference plane projected from multiple views for ball candidate validation in the Multi-View Ball Localization module. Finally, the ball configurations can be mapped to a virtual top-view table for enriched visual presentation and displayed via the intelligent glasses, which will give the players a further sight into the game.

#### III. COLOR-BASED TABLE REGION EXTRACTION

Standard billiard table is mainly made up of one dominant color, as shown in Figure 5(a). Therefore, we can estimate the dominant color of the user-captured images to locate the table region. We first convert each image to HSV color space and calculate the H-histogram to obtain the dominant H value. As shown in Figure 5(b), we can obtain the dominant color mask  $M_D$  by extracting the area with the dominant H value and eliminating the glares on the periphery of table by removing the pixels with low saturation values. However, the contour of  $M_D$  may be incomplete due to the

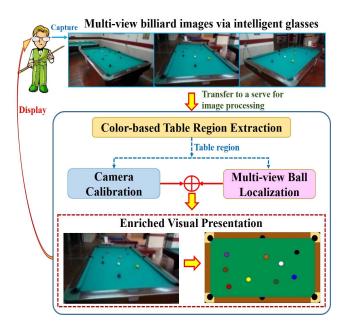


Fig. 4. Schematic flowchart of the proposed framework.

shadows of table rails. Hence, we refine the contour of  $M_D$  and obtain the mask of the table region  $M_T$  by radially calculating and retaining the most distant pixels from the centroid of  $M_D$  within a predefined region, as shown in Figure 5(c).

## IV. CAMERA CALIBRATION

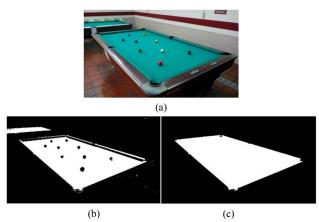
The positions of balls on the billiard table are semantically important because the information about ball configurations tells much about the subsequent shot planning. For semantic reasoning, it is required to obtain the positions of balls on the table in the real world. Camera calibration is performed to find the mapping between the image plane and the reference plane [16]. A homographic plane-to-plane mapping  $\mathbf{w} = \mathbf{H}\mathbf{p}$  is estimated, transforming from a position  $\mathbf{p} = (x, y, 1)^T$  in the image coordinates to a position  $\mathbf{w} = (u, v, 1)^T$  in the reference plane coordinate system by

$$\begin{pmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$
(1)

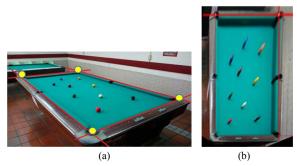
where  $\mathbf{H}$  is a 3×3 homographic image-to-plane transformation matrix. This is an eight-parameter perspective transformation since homogeneous coordinates are scaling invariant. To compute  $\mathbf{H}$ , we need at least four corresponding points.

#### A. Table Rail Detection

We choose the plane formed by the contour of  $M_T$  as our reference plane where the ball touches at center regions to extract corresponding points. In fact, detecting lines is more



**Fig. 5.** Table region extraction. (a) Table view images. (b)  $M_D$ : dominant color mask. (c)  $M_T$ : mask of the table region.



**Fig. 6.** Homographic Matrix Computation. (a) Detected rails (in red) and four corner pockets (in yellow). (b) Table image transformed from (a) on the reference plane.

robust than locating the accurate positions of specific points since the billiard table does not have obvious point features. For this reason, the intersections of lines are utilized to establish point-correspondences. A standard Hough transform is applied to the contour of  $M_T$  for estimating the line parameters and obtaining four table rails. The parameter space  $(\theta, d)$  is used to represent the line, where  $\theta$  is the angle between the line normal and the horizontal axis, and d is the distance of the line to the origin. For all  $(\theta, d)$ , an accumulator matrix is constructed and sampled at a resolution of one degree for  $\theta$  and one pixel for d. Since a line in (x, y) space corresponds to a point in  $(\theta, d)$  space, line candidates can be determined by extracting the local maxima in the accumulator matrix.

### B. Homographic Matrix Computation

Furthermore, four corner pockets can be extracted by computing the intersection points from each pair of table rails, as shown in Figure 6(a), wherein the yellow circles represent the four intersection points. With the four corresponding points obtained, i.e.,  $(x_1, y_1)$ ,  $(x_2, y_2)$ ,  $(x_3, y_3)$  and  $(x_4, y_4)$  in the image coordinates and  $(u_1, v_1)$ ,  $(u_2, v_2)$ ,  $(u_3, v_3)$  and  $(u_4, v_4)$  in reference plane coordinates, we can rewrite Eq. (1) as

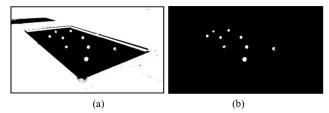


Fig. 7. Ball mask extraction. (a)  $M_N$ : non-dominant color mask. (b)  $M_B$ : ball mask within the extracted table region.

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -u_1x_1 & -u_1y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -v_1x_1 & -v_1y_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -u_2x_2 & -u_2y_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -v_2x_2 & -v_2y_2 \\ \vdots \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -u_4x_n & -u_4y_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -v_4x_n & -v_4y_4 \end{pmatrix} \begin{pmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ \vdots \\ u_4 \\ v_4 \end{pmatrix}. \tag{2}$$

After solving this equation system, the transformation matrix **H** can be computed. Figure 6(b) demonstrates the table image transformed from Figure 6(a) on the reference plane.

The processing step of camera calibration needs to be conducted for each user-captured image. For subsequent multi-view ball localization, we should align each user-captured table image. The first captured table image is set as a reference, and the remaining table images are projected onto the same reference plane to find the correct mapping. For each of the remaining image, the mapping with the largest number of intersection objects is retained and the obtained homographic transformation matrix is utilized for subsequent multi-view ball localization scheme.

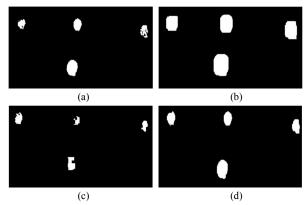
### V. MULTI-VIEW BALL LOCALIZATION

Since ball configurations reveal informative clues in the billiard games, automatic ball localization is a significant component in our proposed system. For consecutive shot analysis, it is required to obtain the ball locations on the virtual top-view table. This section describes how to extract and validate ball locations from multi-view images, and how to visualize the real-world ball configurations using homographic transformation and color information.

#### A. Ball Candidate Extraction

To extract ball candidates, we first produce a non-dominant color mask  $M_N$  by performing NOT operation on  $M_D$ , as shown in Figure 7(a). A ball mask  $M_B$  can be obtained by performing AND operation on  $M_N$  and  $M_T$ , followed by morphological operations for noise removal, as shown in Figure 7(b).

However, the extracted ball candidates in  $M_B$  may be incomplete due to the imposed lighting, as shown in Figure 8(a). To address this issue, a ball-shape refinement algorithm is proposed. Dilation operation is performed on  $M_B$  to extend the pixels of the ball candidates and to generate resultant



**Fig. 8.** Ball-shape refinement. (a)  $M_B$ : ball mask within the extracted table region. (b)  $I_D$ : image after applying dilation operation to  $M_B$ . (c)  $I_F$ : image after applying flood-fill algorithm to  $I_D$  utilizing RGB color information. (d)  $I_B$ : Refined ball candidate image.

image  $I_D$ , as shown in Figure 8(b). Then the flood-fill algorithm is applied to  $I_D$  for determining the connected regions with similar colors  $I_F$ , as shown in Figure 8(c). Finally, we combine  $I_F$  with  $M_B$  and perform the median filter to refine the ball shape to obtain the refined ball candidate image  $I_B$ , as shown in Figure 8(d).

### B. Ball Location Identification

For the foreground regions in each view belong to the same ball, they will have an intersection region after being projected on a reference plane, and the intersection region is where the ball touch that plane. To predict potential ball locations, the extracted foregrounds are projected from multiple camera views onto a reference plane by Eq. (1). Then, we propose a multi-view foreground fusion scheme to predict potential ball locations at the regions intersected by foreground projections from at least k camera views.

We project the extracted foreground regions from Kcamera views (K = 3 in our experiments) to the reference plane and display the result from a top-view, as shown in Figure 9(a), wherein different colors represent the projections from different views. Then, a binary mask  $B_i$  for each camera view  $V_i$  is utilized to indicate which pixels in the mask contain the foreground regions. A computationally efficient bitwiseoperation scheme is also applied to identify candidate ball locations. Let  $A_i^r$  be the result of bitwise AND operation on r binary masks (r = 2 in our experiments) chosen from the total K ones, where i = 1 to  $\binom{K}{r}$  for all combinations. Then, a fusion image F can be obtained by performing bitwise ORoperation on all  $A_i^r$ . As a result, the pixels with nonzero values in F, which implies that their corresponding pixels on the reference plane are intersected by the foregrounds projected from at least r camera views, are identified as ball locations, as shown in Figure 9(b).

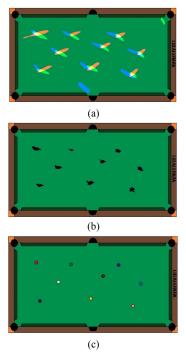


Fig. 9. Multi-view ball localization. (a) Projection from three different views. (b) Fusion image after bitwise-operation scheme. (c) Ball configuration visualization.

## C. Ball Configuration Visualization

For each extracted ball locations, we calculate the RGB color histogram from the corresponding pixels in the original captured images. Then, we extract each peak value from each channel as the final RGB color values of a ball and visualize all balls on a virtual top-view table, as shown in the Figure 8(c). Furthermore, the resulting image of ball configuration can be rendered directly on the intelligent glasses. By watching the ball configurations from a third-person perspective on the intelligent glasses, the player can conveniently learn and makes great progress on playing billiards.

### VI. EXPERIMENTAL RESULTS

To demonstrate the effectiveness of the proposed multiview ball localization approach, we conduct the experiments on 16 sets of different ball configurations. Each set is composed of images containing full table scene captured from three different views using the camera on the intelligent glasses (Google Glass is used as our development platform), as shown in Figure 10. All the captured images are normalized to the size of  $800 \times 450$  for computational efficiency and are sent to the server (a laptop) for processing. The resulting images containing the ball configurations on a virtual top-view table are sent back and displayed via the intelligent glasses. Figure 11 demonstrates the specifications of the proposed system, including a laptop for processing, a



Fig. 10. Two example sets for experiments and each set is composed of images containing full table scene captured from three different views using camera on intelligent glasses. (a) Nine balls. (b) Seven balls.



Fig. 11. The specifications of the proposed system.

DV recorder for ground truth capturing, and the intelligent glasses for image capturing and display.

In the 16 sets of ball configurations, the ground-truth ball locations are manually measured with the real-world coordinates. A location estimation is regarded as correct if it is at a distance d < 2.5 cm (the radius of a ball is about 2.85 cm) from a ground-truth location; otherwise, the estimation is regarded as false. A ground truth matching no estimation is regarded as a miss. The results of ball localization are presented in Table I, wherein the "precision" and "recall" are defined by Eq. (3) and Eq. (4).

$$precision = \frac{\#correct}{\#correct + \#false}$$

$$recall = \frac{\#correct}{\#correct + \#miss}$$
(3)

$$recall = \frac{\text{#correct}}{\text{#correct} + \text{#miss}}$$
 (4)

Over 82% balls can be correctly located, and the precision and recall rates are 82.86% and 82.86%, respectively. The effectiveness of ball localization facilitates the subsequent shot analysis. Based on observations, most of

TABLE I. RESULT OF BALL LOCALIZATION

# Balls in all sets	105
# Correctly located balls	87
# False alarms	18
Precision (%)	82.86
Recall (%)	82.86

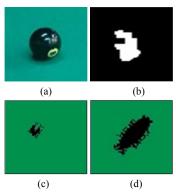
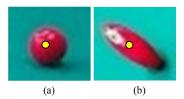


Fig. 12. Detection miss of ball location due to imposed lights. (a) The black ball in the original image. (b) The extracted ball candidate. (c) Projection of the incomplete ball shape. (d) Projection of the complete ball candidate.

the detection misses are caused by incomplete ball segmentation from captured images. Take Figure 12(a) as an example. The extracted foreground region of the black ball is broken due to the light refection, as illustrated in Figure 12(b). The projection of the ball candidate on the reference plane is much smaller (Figure 12(c)) compared to the projection of complete ball candidate (Figure 12(d)), which influences the performance of ball detection. In addition to precision and recall rates, the processing time of the proposed system is also evaluated. In our scenario, the user is requested to capture three images containing the full table from different viewpoints before each shot. The system estimates ball positions and visualizes the ball configuration on a virtual top-view table only once for each shot by analyzing the three images, which takes an acceptable processing time of about 7s. Since a billiards player usually moves around the table to see the ball configurations from different viewpoints, the use of our system would be desirable.

To demonstrate the advantage of our multi-view ball localization scheme, two single view approaches which are often adopted in existing billiards video analysis systems are implemented and compared: (i) Method-I projecting the ball centroids (see Figure 13(a)) on the reference plane as ball locations, as used in [9, 12, 14]; (ii) Method-II directly selecting the centroids of ball projections (see Figure 13(b)) as ball locations. Figure 14 illustrates the comparisons among Method-I, Method-II, and our approach on the average distances between the ground-truth ball locations and the estimated ones. The proposed multi-view ball localization approach achieves a much better performance than the other two methods. The estimated ball locations of our proposed approach are very close to the ground-truth locations



**Fig. 13.** Ball localization method utilizing only a single view. (a) Method-I: projecting the ball centroids on the reference plane as ball locations. (b) Method-II: selecting the centroids of ball projection as ball locations.

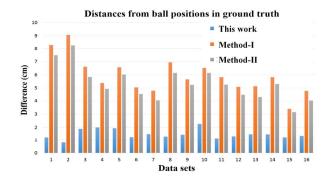


Fig. 14. The comparisons among Method-I, Method-II, and our approach on the average distances between the ground-truth ball locations and the estimated ones.

(distance ≤ 2.5 cm); however, in both Method-I and Method-II, the estimated ball locations are over 3 cm distant from the ground-truth locations. Method-I directly transforms the centroids of ball candidates in a side-view image to the reference plane, which may give birth to inaccurate calibration results since the selected points in the image is not located at the reference plane. For example, as shown in Figure 1,  $|\overline{P_1P_2}|$  is the error between the ground-truth location, i.e.,  $P_1$ , and the estimated ball location, i.e.,  $P_2$ . Furthermore, the errors of Method-II are caused by localizing using only a single view. To address these issues, our proposed multi-view approach locates balls by integrating projected foregrounds from multi-view images so that the balls can be well located. Some results of multi-view ball localization are presented in Figure 15, where the left column shows user-captured images from one of the multiple camera views and the right column demonstrates the corresponding results of ball configuration visualization.

### VII. CONCLUSION

Computer-assisted training for sports exercise is a recently emerging topic. It is our ultimate goal to develop a billiards guidance system enabling a user to see the possible direction and power of shooting a ball via intelligent glasses for strategy planning. To that end, ball localization as proposed in this work is essential. In billiards, current distribution of the balls on the table reveals informative



Fig. 15. Demonstration of multi-view ball localization. (a) One of the multiple camera views. (b) Ball configuration visualization.

tactical clues. In other words, ball configuration analysis is the key factor in success in subsequent shot planning for billiard players. In this paper, we propose a billiard training system, which aims at providing the user/practitioner with ball configurations on the table using intelligent glasses as a wearable camera and displayer. The proposed system provides accurate, reliable, and well-visualized balls on a virtual top-view table for more easily understanding of ball configurations. Besides, it takes an acceptable processing time of about 7s for the system to locate balls and visualize the ball configurations on a virtual top-view table. It is our belief that the preliminary work presented in the paper will inspire manifold applications of subsequent analysis in billiards.

There is still much future work beyond the current preliminary system in this paper. At present, the proposed system enables the player not only to see the real scene but also to watch the ball configurations from a third-person perspective on a mini displayer. How to further utilize the obtained ball configurations for self-training such as strategy planning is our future work. For example, aiming angle selection and cue-ball reposition control are two significant issues in success in billiards. It requires much effort to learn

the analytic geometry and improve the shot mechanics for an inexperienced billiard practitioner. Hence, we are working on enhancing the billiard self-training system by further considering the direction of cue-stick. We will also attempt predicting and generating trajectories of both the cue-ball and the object-ball after collision based on the cue-stick direction and characteristics of physics. To help practitioners improve their skills, the rendered top-view table with the predicted trajectories will be visualized and displayed via intelligent glasses afterwards.

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