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lyapunovCtrl

Creates the ODE function for a two link planar arm tracking a cubic polynomial trajectory by lyapunov-based control.

```
function [ dx ] = lyapunovCtrl( t, x, a1, a2)
```

Constants and Variables

Set the parameters for the arm:

```
I1=10;  I2 = 10; m1=5; r1=.5; m2=5; r2=.5; l1=1; l2=1;
g=9.8;
```

Calculate the parameters in the dynamic model:

```
a = I1+I2+m1*r1^2+ m2*(l1^2+ r2^2);
b = m2*l1*r2;
d = I2+ m2*r2^2;
```

Trajectory Generation

Note *x* is in the form of *q1*, *q2*, *q1_dot*, *q2_dot*:

Cubic polynomials:

```
vec_t = [1; t; t^2; t^3];
theta_d = [a1'*vec_t; a2'*vec_t];
```

Calculate the velocity and acceleration in both *theta 1* and *theta 2*:

```
a1_vel = [a1(2), 2*a1(3), 3*a1(4), 0];
a1_acc = [2*a1(3), 6*a1(4),0,0 ];
a2_vel = [a2(2), 2*a2(3), 3*a2(4), 0];
a2_acc = [2*a2(3), 6*a2(4),0,0 ];
```

Calculate the desired trajectory (assuming 3rd order polynomials for trajectories):

```
dtheta_d =[a1_vel*vec_t; a2_vel* vec_t];
ddtheta_d =[a1_acc*vec_t; a2_acc* vec_t];
theta = x(1:2,1);
theta_dot = x(3:4,1);
```

Planar Arm Dynamics

Calculate the parameters in the dynamic model:

```
a = I1+I2*t+m1*r1^2+ m2*(l1^2+ r2^2);
b = m2*l1*r2;
d = I2+ m2*r2^2;
```

Calculate the actual dynamic model of the system:

```
Mmat = [a+2*b*cos(x(2)), d+b*cos(x(2));  d+b*cos(x(2)), d];
Cmat = [-b*sin(x(2))*x(4), -b*sin(x(2))*(x(3)+x(4));
 b*sin(x(2))*x(3),0];
Gmat =  [m1*g*r1*cos(x(1))+m2*g*(l1*cos(x(1))+r2*cos(x(1)+x(2)));
    m2*g*r2*cos(x(1)+x(2))];
invM = inv(Mmat);
invMC = invM*Cmat;
```

Lyapunov-Based Controller

Set the *kv* gain constant (positive definite diagonal matrix):

```
kd = [25 0; ...
      0 25];
```

Set the *capital_lambda* constant (positive definite square matrix):

```
capital_lambda = [10 0; ...
                  0 10];
```

Calculate the tracking errors, *e* and *e_dot*:

```
e = theta - theta_d;
e_dot = theta_dot - dtheta_d;
```

Calculate *si_dot* and *si_dot_dot*:

```
si_dot = dtheta_d - capital_lambda*e;
si_dot_dot = ddtheta_d - capital_lambda*e_dot;
```

Calculate *sigma*:

```
I = eye(2,2);
% sigma = I*e_dot + capital_lambda*e;
sigma = theta_dot - si_dot;
```

Calculate the controller, *u*:

```
u = zeros(2,1);
u = Mmat*si_dot_dot + Cmat*si_dot + Gmat - kd*sigma;
```

Calculate the accleration values:

```
theta_dot_dot = zeros(2,1);
% theta_dot_dot = sigma_dot - si_dot_dot
```

```
theta_dot_dot = invM*( u - Cmat*theta_dot - Gmat);
```

Outputs

Initialize the output of the function, *dx*:

```
dx = zeros(4,1);
```

Final outputs:

```
dx(1) = x(3,1);
dx(2) = x(4,1);
dx(3) = theta_dot_dot(1);
dx(4) = theta_dot_dot(2);
```

```
end
```

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