Notes on Goldstein's Classical Mechanics, 2nd Edition

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Chapter 1

Elementary Principles

1.1 Mechanics of a Particle

We let $\boldsymbol{r}$ be the radius vector from the origin of a whatever coordinate system we're adopting to a particle. This particle has velocity $\boldsymbol{v}$, given by

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt},$$

and its linear momentum is expressed as

$$\boldsymbol{p} = m\boldsymbol{v}.$$

The dynamics of a particle is expressed by Newton's 2nd law, namely that the motion of a particle is described by the 2nd-order differential equation

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = m\frac{d^2\boldsymbol{r}}{dt^2} = m\boldsymbol{a}$$

as long as our force does not depend on any higher-order derivatives. A system in which this law holds is called *Gallilean* or *inertial*.

Much of mechanics is explained by "conservation theorems." There will be much explanation of these theorems and their relationships to coordinate symmetries in the notes to follow. For now, we express the principles:

Conservation of Linear Momentum of a Particle: if the total force acting on a particle $\boldsymbol{F}$ is zero, then $\dot{\boldsymbol{p}} = 0$ and so $\boldsymbol{p}$ is conserved.

The angular counterpart to momentum about a point O is denoted by $\boldsymbol{L}$ and defined

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p},$$

where $\boldsymbol{r}$ is the distance between the particle and the point O, and $\boldsymbol{p}$ is still the linear momentum of the particle as measured from a frame co-resting with O. Take note of the noncommutativity of the cross product. The corresponding analogue for force in an angular setting is called torque, and denoted

$$\boldsymbol{N} = \boldsymbol{r} \times \boldsymbol{F} = \boldsymbol{r} \times m\frac{d\boldsymbol{v}}{dt} = \frac{d\boldsymbol{L}}{dt}.$$

We have a corresponding conservation property of angular momentum.

Conservation of Angular Momentum of a Particle: if the total torque of a particle is zero, then $\dot{\boldsymbol{L}} = 0$ and angular momentum $\boldsymbol{L}$ is conserved. // //

We now consider the work done by an external force on a particle traversing a path from point 1 to point 2. This quantity is defined by the integral:

$$W_{12} = \int_1^2 \boldsymbol{F} \cdot d\boldsymbol{s}.$$

Assuming a constant mass, this can be reduced to the following –

$$\int \boldsymbol{F} \cdot d\boldsymbol{s} = m \int \frac{d\boldsymbol{v}}{dt} \cdot \boldsymbol{v} dt = \frac{m}{2} \int \frac{d}{dt}(v^2) dt$$

so

$$W_{12} = \frac{m}{2}(v_2^2 - v_1^2).$$

With $T = \frac{1}{2}mv^2$, we can write $W_{12} = T_2 - T_1$.

If the work done by a force is path-independent, we call the force *conservative.* If we imagine taking a point from one point to another and back again, path-independence implies that the total work done by the force will be zero:

$$\oint \boldsymbol{F} \cdot d\boldsymbol{s} = 0 = -\int \nabla V(\boldsymbol{r}) d\boldsymbol{r}.$$

A necessary and sufficient condition for a force to be conservative is that it is expressible as the gradient of a scalar potential field V.