# Approximations

## Linear Approximations of basic functions near 0

$$
\begin{aligned}
(1+x)^r &\approx 1+rx \\
\sin(x) \approx \sin(0)+\cos(0)x &= x \\
\cos(x) \approx cos(0)-sin(0)x &= 1 \\
e^x \approx e^0+e^0x &= 1+x \\
\ln(1+x) \approx \ln(1+0)+\frac{1}{1+0}x &= x
\end{aligned}
$$

## Best fit quadratic

- The best fit quadratic or best fit parabola to a function $f(x)$ at the point $x=0$ is the quadratic function $q(x)$ whose value agree with the value of $f$ at $x=0$, and whose first and second derivatives agree with the first and second derivatives of $f$ at $x=0$, i.e.:

$$
\begin{gathered}
f(0)=q(0) \\
f'(0)=q'(0) \\
f''(0)=q''(0)
\end{gathered}
$$

## Quadratic Approximation

- The **quadratic approximation** near $x=a$ is the **best fit parabola** to $f$ at the point $x=a$.

  The formula for the quadratic approximation of a function $f$ near a point $x=a$ is:

$$f(x) \approx f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2$$

  When $a=0$, this quadratic approximation becomes

$$f(x) \approx f(0)+f'(0)x+\frac{f''(0)}{2}x^2$$

# Big-O notation

- A function $f(x)$ is on the order $x^n$ near $x = 0$, which is denoted using big "0" notation as $f(x) = O(x^n)$ near $x = 0$, if $|f(x)| \leq kx^n$

# Newton's Method

- Given a function $f(x)$, find $x$ such that $f(x) = 0$.

  i. Make a good guess $x_0$.

  ii. Call $x_1$ the $x$-intercept of the tangent line through $(x_0, f(x_0))$. It has the formula

  $$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

  iii. Repeat. The general formula is

  $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

  for $n = 0, 1, 2, \ldots.$