Assignment 3: Bayesian Inference, Temporal State Estimation and Decision Making under Uncertainty

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Problem 1:

a

The probability that all five of the Boolean variables are simultaneously true is:
$P(A) = 0.2$
$P(B) = 0.5$
$P(C) = 0.8$
$P(D \mid A \wedge B) = 0.1$
$P(E \mid B \wedge C) = 0.3$
$P(A \wedge B) = 0.1$
$P(A \wedge B \wedge C) = 0.08$
$P(A \wedge B \wedge C) \times P(D \mid A \wedge B) = 0.008$
$P(A \wedge B \wedge C) \times P(D \mid A \wedge B) \times P(E \mid B \wedge C) = 0.0024$

b

The probability that all five of the Boolean variables are simultaneously false is:
$P(\neg A) = 0.8$
$P(\neg B) = 0.5$
$P(\neg C) = 0.2$
$P(\neg D \mid \neg A \wedge \neg B) = 0.1$
$P(\neg E \mid \neg B \wedge \neg C) = 0.8$
$P(\neg A \wedge \neg B) = 0.4$
$P(\neg A \wedge \neg B \wedge \neg C) = 0.08$
$P(\neg A \wedge \neg B \wedge \neg C) \times P(\neg D \mid \neg A \wedge \neg B) = 0.008$
$P(\neg A \wedge \neg B \wedge \neg C) \times P(\neg D \mid \neg A \wedge \neg B) \times P(\neg E \mid \neg B \wedge \neg C) = 0.0064$

c

$P(\neg A) = 0.8$
$P(D \wedge B) = 0.7$
$P(D \wedge B \mid \neg A) = 0.6$
$P(\neg A \mid D \wedge B) = \frac{0.8*0.6}{0.7} = 0.686$

Problem 2:

a

Query: $P(Burglary \mid JohnCalls = true, MaryCalls = true)$
Variable Elimination
Query expression:
$P(B \mid j, m) = \alpha f_1(B) * \sum_e f_2(E) * \sum_a f_3(A, B, E) * f_4(A) * f_5 * (A)$
$f_6(B, E) = \sum_a f_3(A, B, E) * f_4(A) * f_5(A)$
$= (f_3(a, B, E) * f_4(a) * f_5(a)) + (f_3(\neg a, B, E) * f_4(\neg a) * f_5(\neg a))$
$P(B \mid j, m) = \alpha f_1(B) * \sum_e f_2(E) * f_6(B, E)$
$f_7(B) = \sum_e f_2(E) * f_6(B, E)$
$= f_2(e) * f_6(B, E) + f_2(\neg e) * f_6(B, \neg e)$
$P(B \mid j, m) = \alpha f_1(B) * f_7(B)$

b

Variable Elimination Algorithm - Arithmetic Operations Performed
Additions: 1
Multiplications: 5
Divisions: 1
Tree Enumeration Algorithm - Operations Performed
Additions: 3
Multiplications: 9
Divisions: 1

c

If a Bayesian network has the form of a chain, the complexity of computing $P(X_1 \mid X_n = true)$ using enumeration is $O(n)$ because every single X would need to be used in the calculation.
Computing the complexity with variable elimination is also $O(n)$ because no variables would be eliminated. Since the Bayesian network is a chain, there would be no eliminated variables and the complexity would be exactly the same as tree enumeration.

Problem 3:

a

b

c

Problem 4:

a

b

c

d

Problem 5 - Programming Component:

a

b

c - Generating Ground Truth Data

d - Filtering and Viterbi Algorithms in Large Maps

e

f

g

h - Computational Approximations