Assignment 3: Bayesian Inference, Temporal State Estimation and Decision Making under Uncertainty

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April 11, 2017

Problem 1:

a

The probability that all five of the Boolean variables are simultaneously true is:
$P(A) = 0.2$
$P(B) = 0.5$
$P(C) = 0.8$
$P(D \mid A \wedge B) = 0.1$
$P(E \mid B \wedge C) = 0.3$
$P(A \wedge B) = 0.1$
$P(A \wedge B \wedge C) = 0.08$
$P(A \wedge B \wedge C) \times P(D \mid A \wedge B) = 0.008$
$P(A \wedge B \wedge C) \times P(D \mid A \wedge B) \times P(E \mid B \wedge C) = 0.0024$

b

The probability that all five of the Boolean variables are simultaneously false is:
$P(\neg A) = 0.8$
$P(\neg B) = 0.5$
$P(\neg C) = 0.2$
$P(\neg D \mid \neg A \wedge \neg B) = 0.1$
$P(\neg E \mid \neg B \wedge \neg C) = 0.8$
$P(\neg A \wedge \neg B) = 0.4$
$P(\neg A \wedge \neg B \wedge \neg C) = 0.08$
$P(\neg A \wedge \neg B \wedge \neg C) \times P(\neg D \mid \neg A \wedge \neg B) = 0.008$
$P(\neg A \wedge \neg B \wedge \neg C) \times P(\neg D \mid \neg A \wedge \neg B) \times P(\neg E \mid \neg B \wedge \neg C) = 0.0064$

c

$P(\neg A) = 0.8$
$P(D \wedge B) = 0.7$
$P(D \wedge B \mid \neg A) = 0.6$
$P(\neg A \mid D \wedge B) = \frac{0.8*0.6}{0.7} = 0.686$

Problem 2:

a

b

c

Problem 3:

a

Prove:

$$P(X \mid MB(X)) = \alpha P(X \mid U_1, ..., U_m) \prod_{Yi} P(Y_i \mid Z_i, ...)$$

Definition of Bayesian network:

$$(x_1, x_2, ..., x_n) = \prod_{i=1}^{n} P(x_i \mid parents(X_i))$$

via product rule:

$$(x_1, x_2, ..., x_n) = P(x_1 \mid x_{n-1}, ..., x_1) P(x_{n-1}, ..., x_1)$$

Definition of Markov Assumption:

$$P(X_t \mid X_{0:t-1}) = P(X_t \mid X_{t-1})$$

Using the Markov assumption on the Bayesian network lets all nodes be conditionally independent from the other nodes in the graph given the Markov blanket because X's children conditionally depend on X as well as the child's parents, and x depends on its parents.

b

$$P(Rain \mid Sprinkler = true \wedge WetGrass = true)$$

MCMC would solve this by trying fixing sprinkler and wet grass to true while testing rain by calculating the probability repeatedly randomly changing the non fixed variable values. In the case above, there would be 4 states to take into consideration. Cloudy=T/F, Rain=T/F.

c

Problem 4:

a

b

c

d

Problem 5 - Programming Component:

a

b

c - Generating Ground Truth Data

d - Filtering and Viterbi Algorithms in Large Maps

e

f

g

h - Computational Approximations