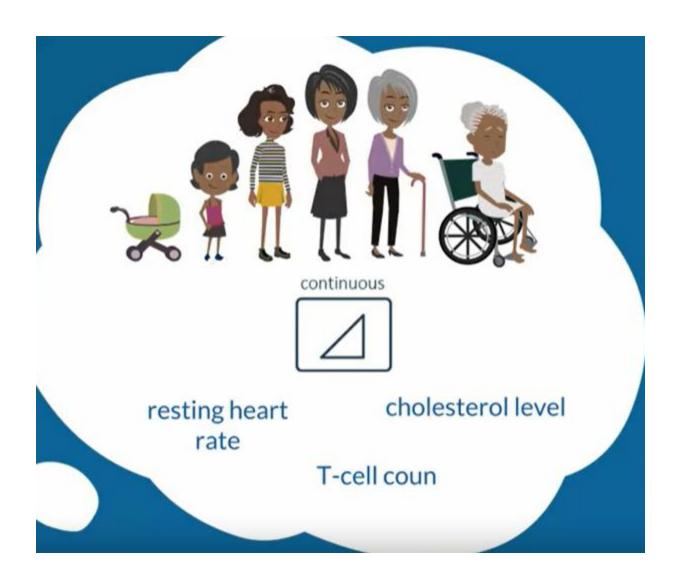
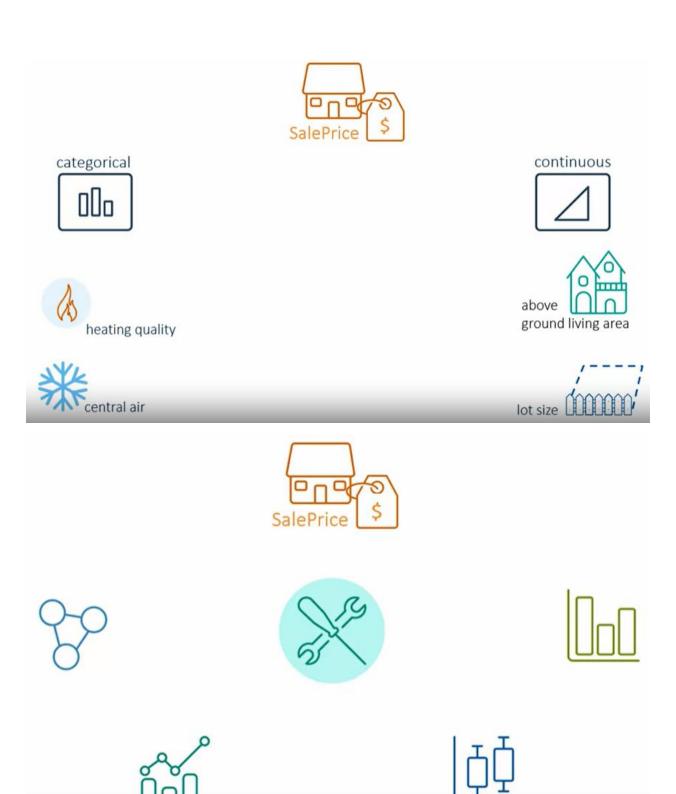
SBA: Statistical Business Analyst with SAS

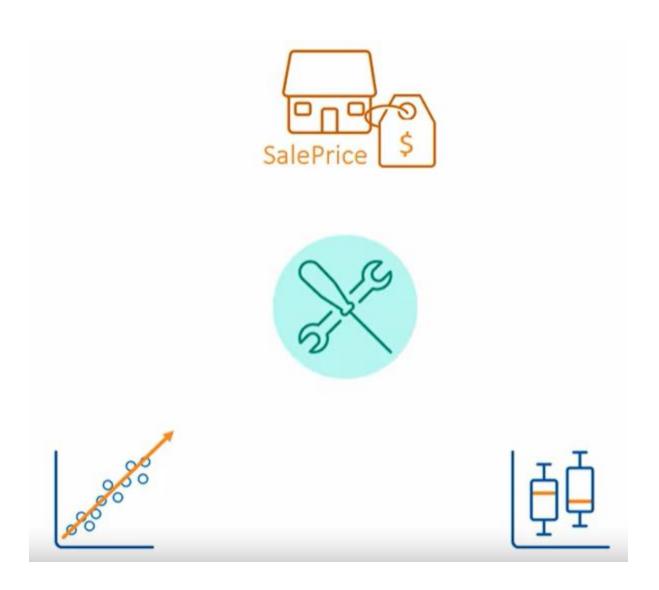
SBA1: Introduction to Statistical Analysis: Hypothesis Testing

### **ANOVA Overview**



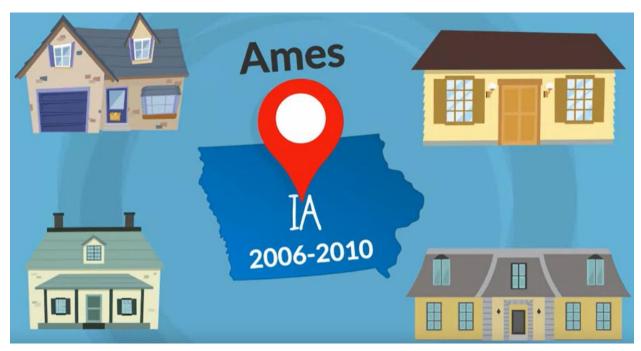






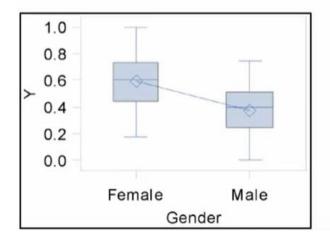


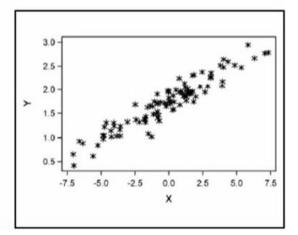
### **ANOVA Scenario**

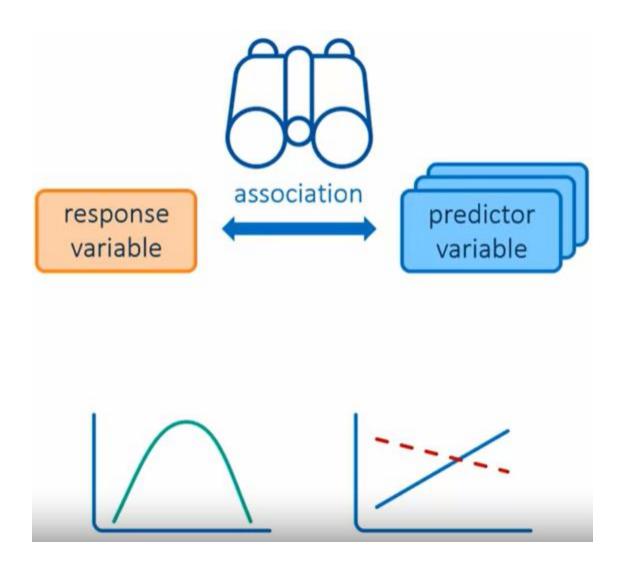




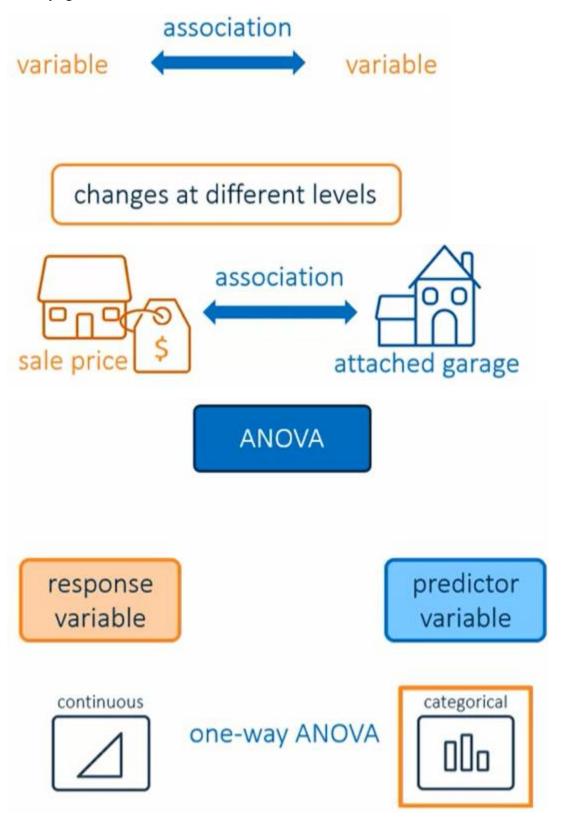


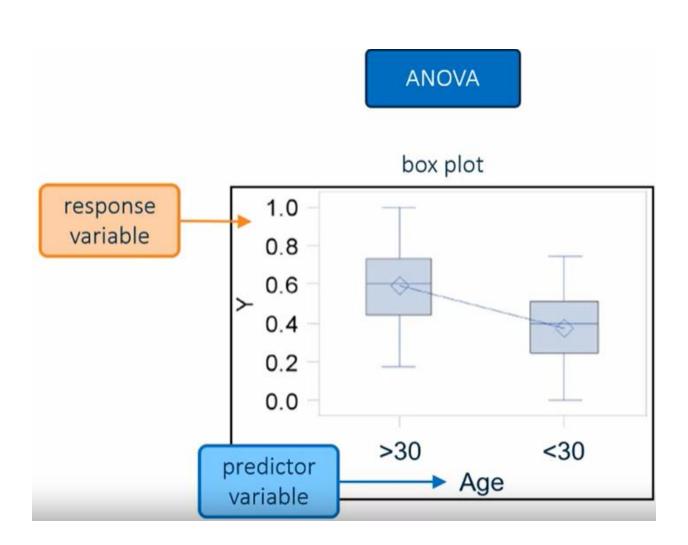


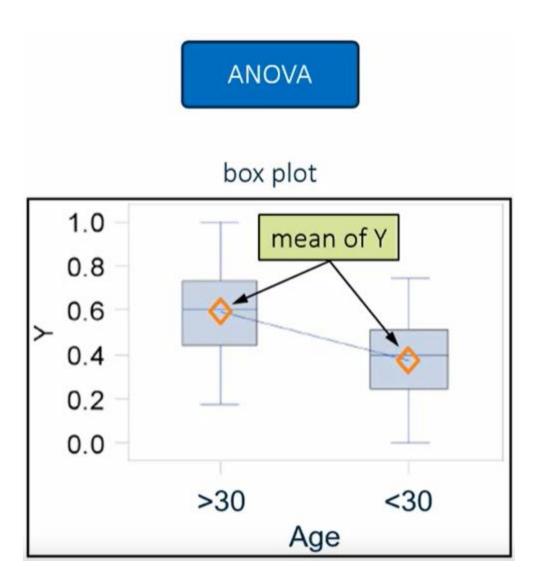




### **Identifying Associations in ANOVA with Box Plots**

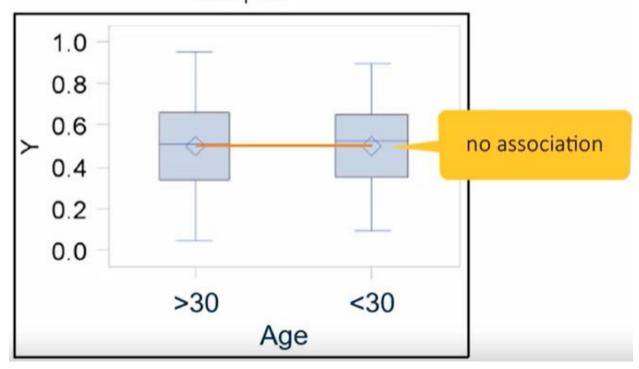


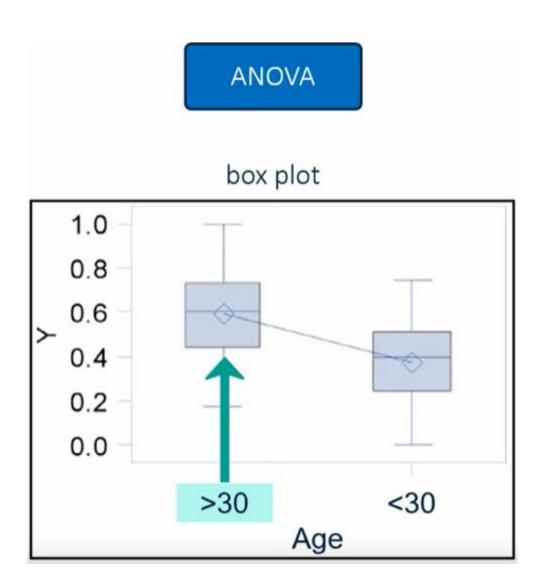




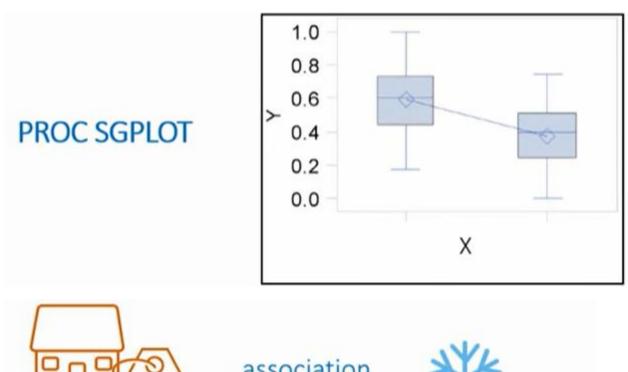


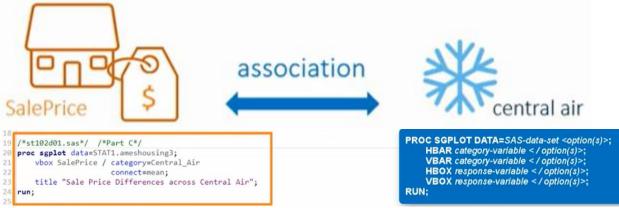
# box plot

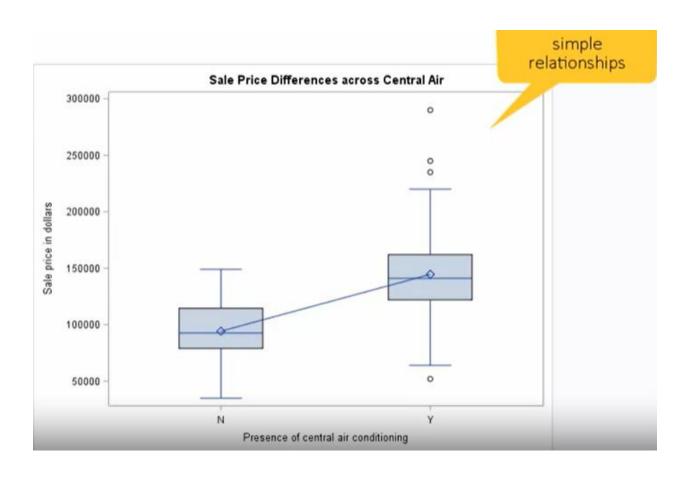




### **Demo Exploring Associations Using PROC SGPLOT**







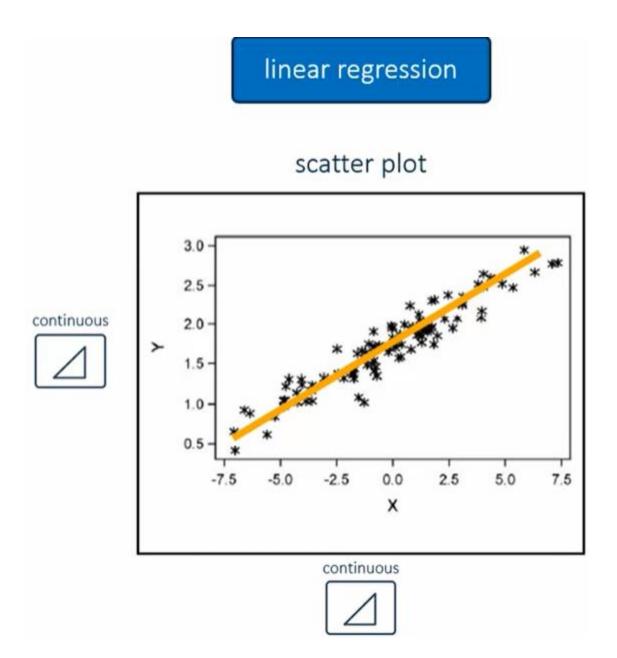
### **Identifying Associations in Linear Regression with Scatter Plots**





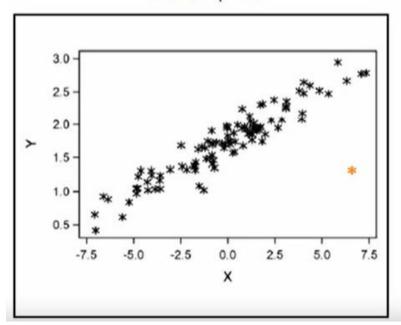




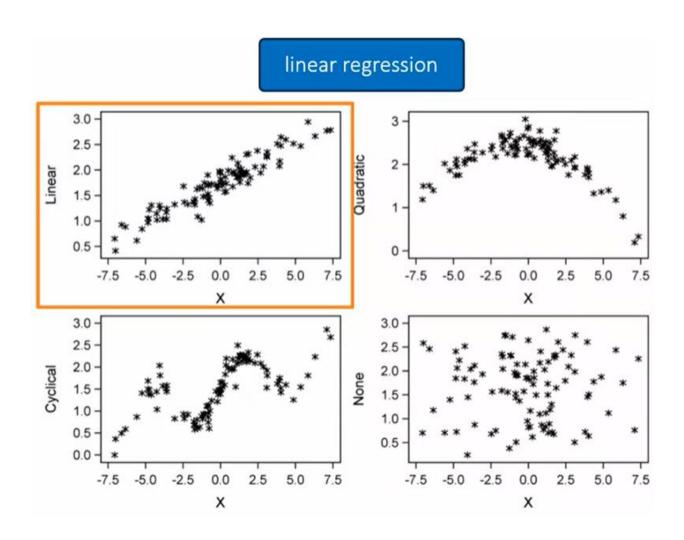


# linear regression

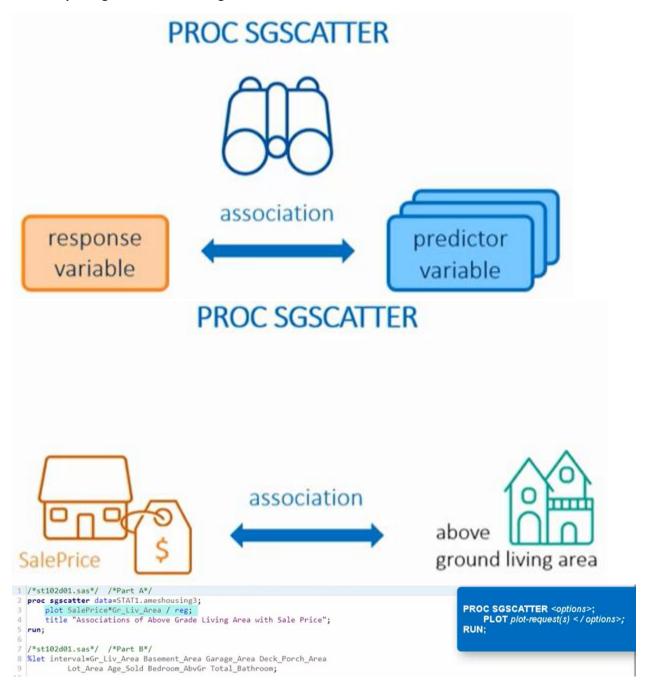
## scatter plot

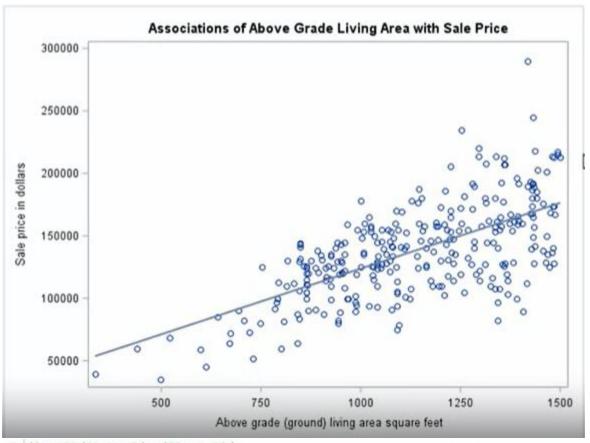


outliers
identify trends
range of values
communicate results



### **Demo Exploring Associations Using PROC SGSCATTER**

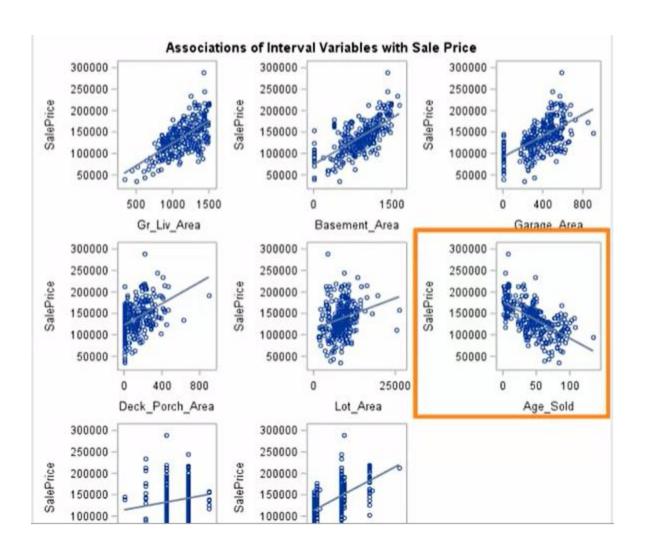




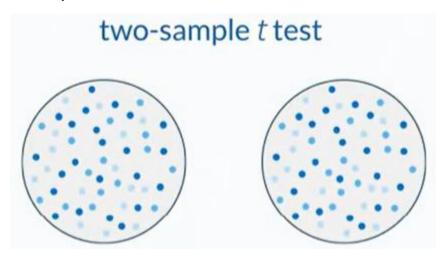
```
/*st102d01.sas*/ /*Part B*/
%let interval=Gr_Liv_Area Basement_Area Garage_Area Deck_Porch_Area
Lot_Area Age_Sold Bedroom_AbvGr Total_Bathroom;

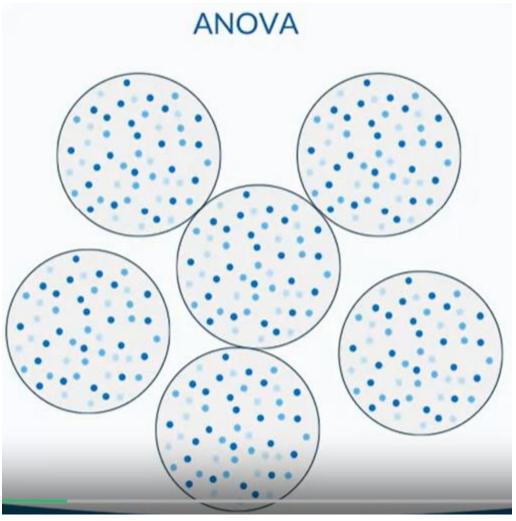
/*PROC SGSCATTER is used to explore relationships among continuous variables*/
/*using scatter plots*/
options nolabel;

proc sgscatter data=STAT1.ameshousing3;
plot SalePrice*(&interval) / reg;
title "Associations of Interval Variables with Sale Price";
run;
```



## One-Way ANOVA

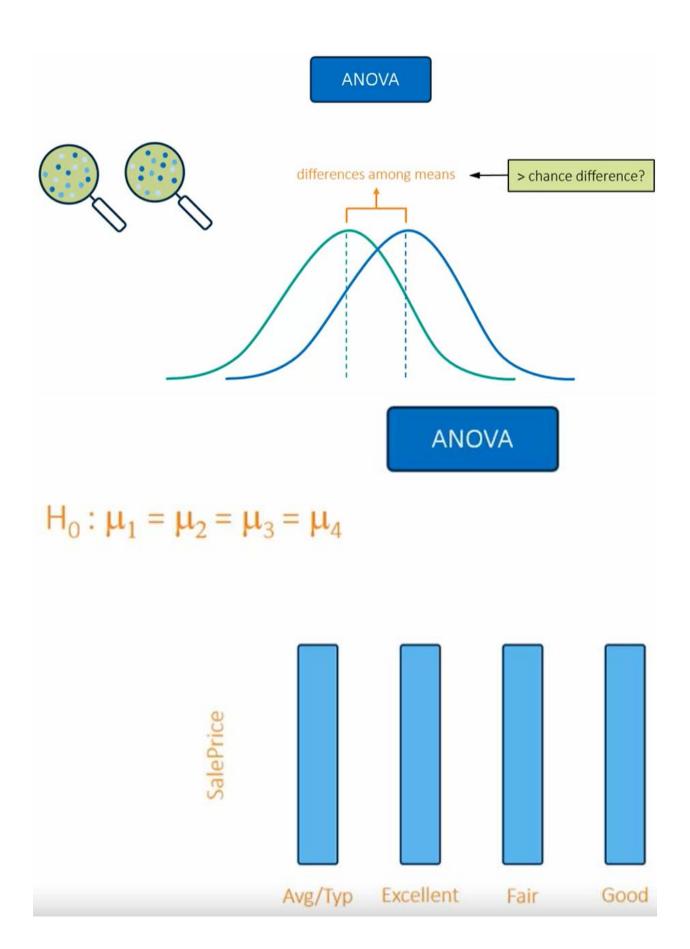






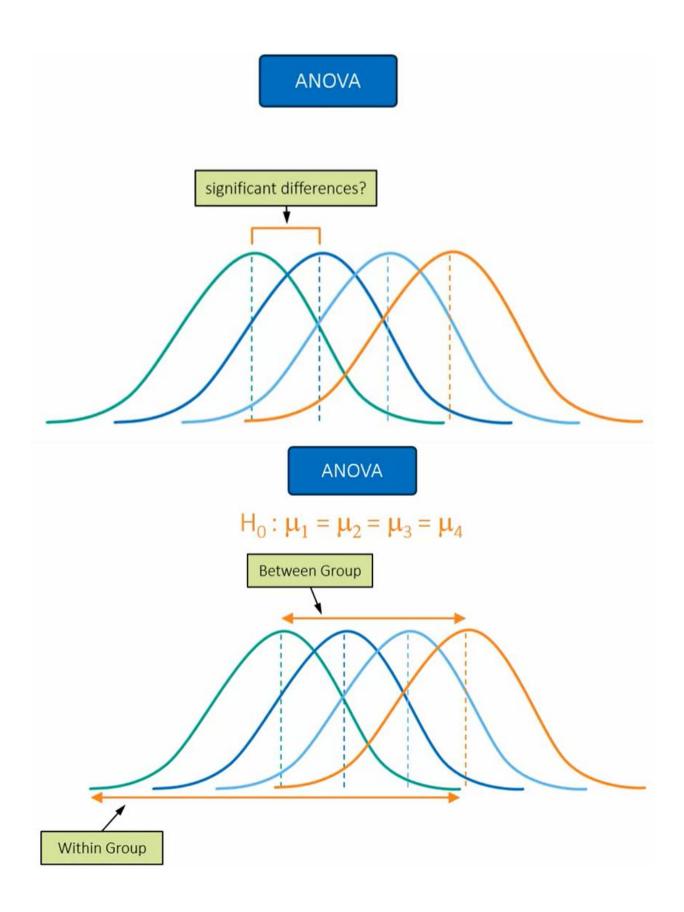


The ANOVA Hypothesis



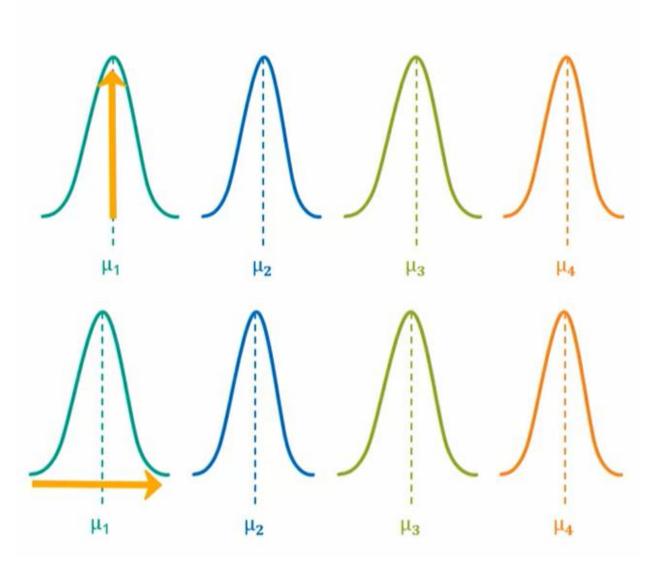


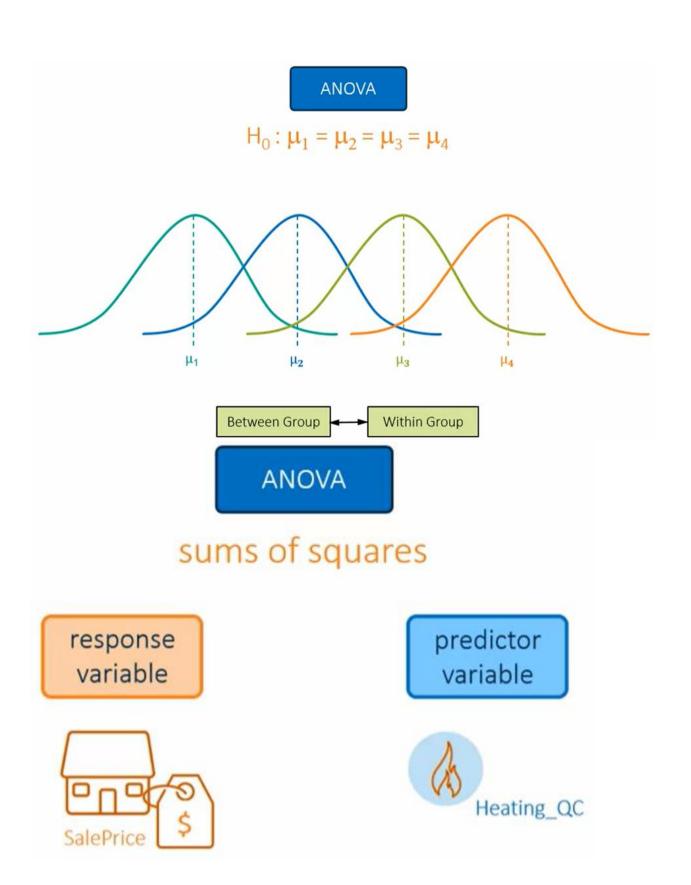
Partitioning Variability in ANOVA





$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$$





# ANOVA

**Total Variation** 

Total Sum of Squares  $(SS_T)$ 

 $\sum \sum \left(Y_{ij} - \overline{\overline{Y}}\right)^2$ 

Between Group

Variation

Model Sum of Squares

 $(SS_M)$ 

 $\sum n_i \left(\overline{\overline{Y}}_i - \overline{\overline{\overline{Y}}}\right)^2$ 

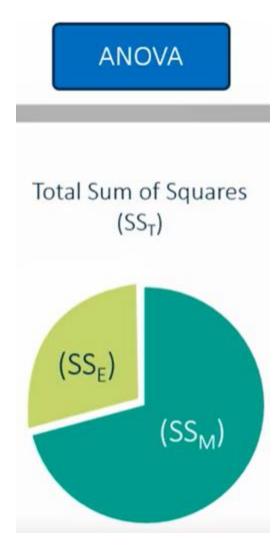
Within Group

Variation

Error Sum of Squares

 $(SS_E)$ 

 $\sum\sum \left(Y_{ij}-\overline{Y_i}\right)^2$ 

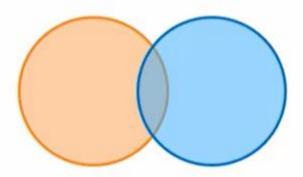


**Coefficient of Determination** 

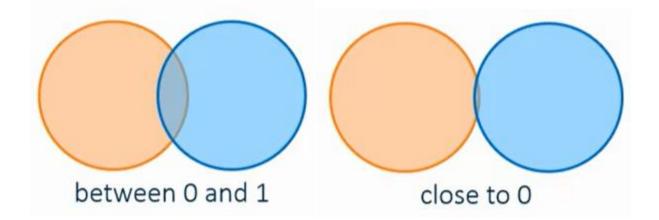
 $R^2$ 

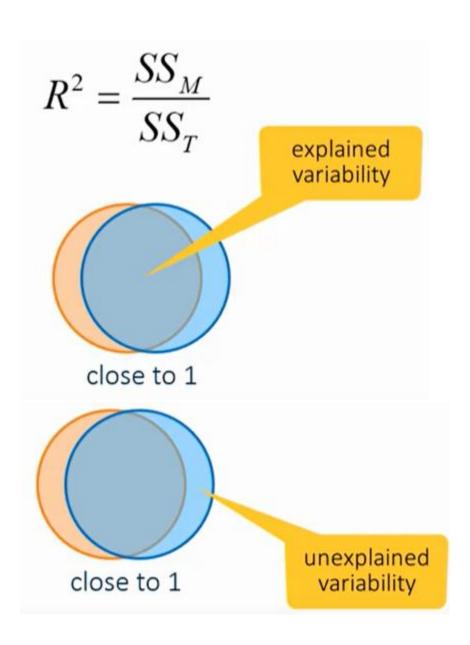
response variable

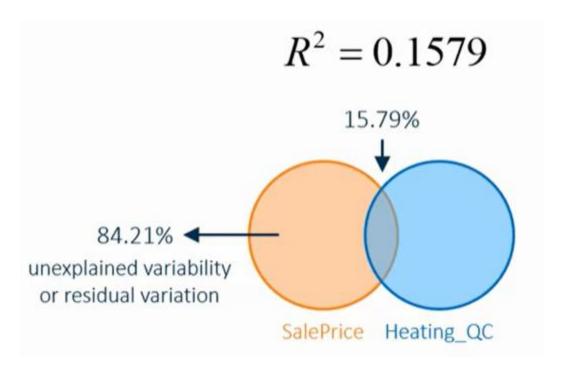
predictor variable



$$R^2 = \frac{SS_M}{SS_T}$$







### **F Statistic and Critical Values**

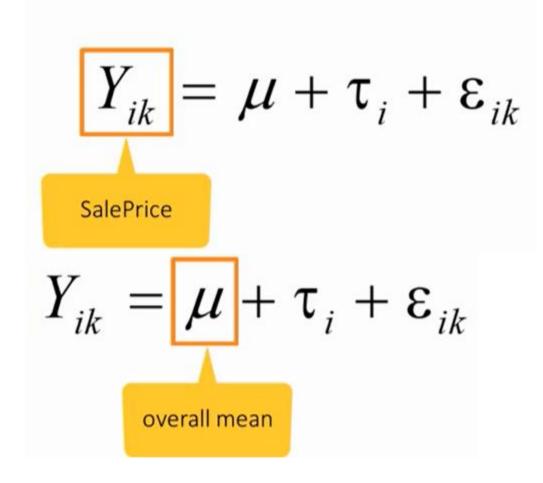
analysis of variance table

### The ANOVA Procedure

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	66835556221	22278518740	18.50	<.0001
Error	296	356387963289	1204013389.5		
Corrected Total	299	423223519511			

estimate of model variance

## **ANOVA Model**



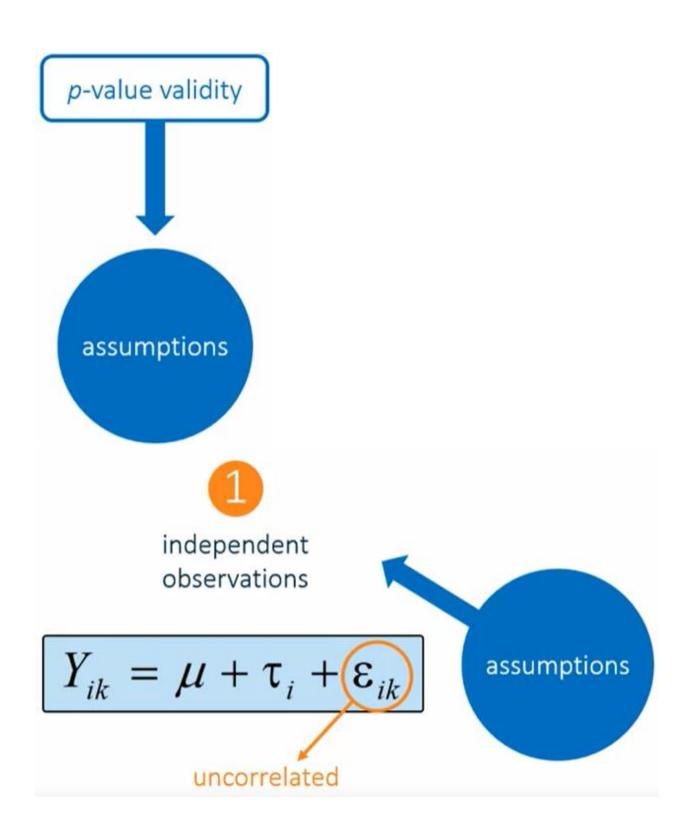
$$Y_{ik} = \mu + \tau_i + \epsilon_{ik}$$

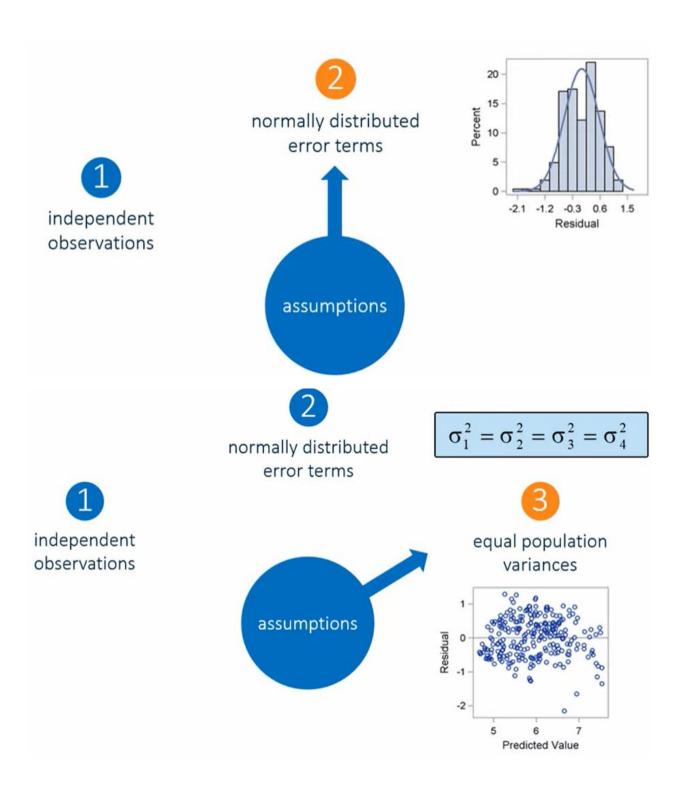
$$Y_{ik} = \mu + \tau_i + \epsilon_{ik}$$

$$Y_{ik} = \mu + \tau_i + \epsilon_{ik}$$
unaccounted for variation

### Observed - Predicted = Residual

Observation	Heating_QC	Observed	Predicted	Residual
1	Ex	213500.0000	154919.1869	58580.8131
2	Ex	191500.0000	154919.1869	36580.8131
3	TA	115000.0000	130573.5294	-15573.5294
4	Ex	160000.0000	154919.1869	5080.8131
5	Ex	180000.0000	154919.1869	25080.8131
6	TA	125000.0000	130573.5294	-5573.5294
7	TA	206000.0000	130573.5294	75426.4706
8	Gd	159000.0000	130844.0862	28155.9138
9	TA	180500.0000	130573.5294	49926.4706
10	Gd	142125.0000	130844.0862	11280.9138





# **PROC GLM**

analysis of variance





# **PROC GLM**

- analysis of variance
  - p-values
  - confidence intervals

# Residual plots





# normally distributed error terms



# independent observations

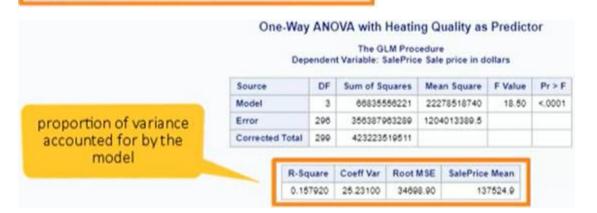


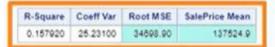
equal population variances

Levene's test of homogeneity



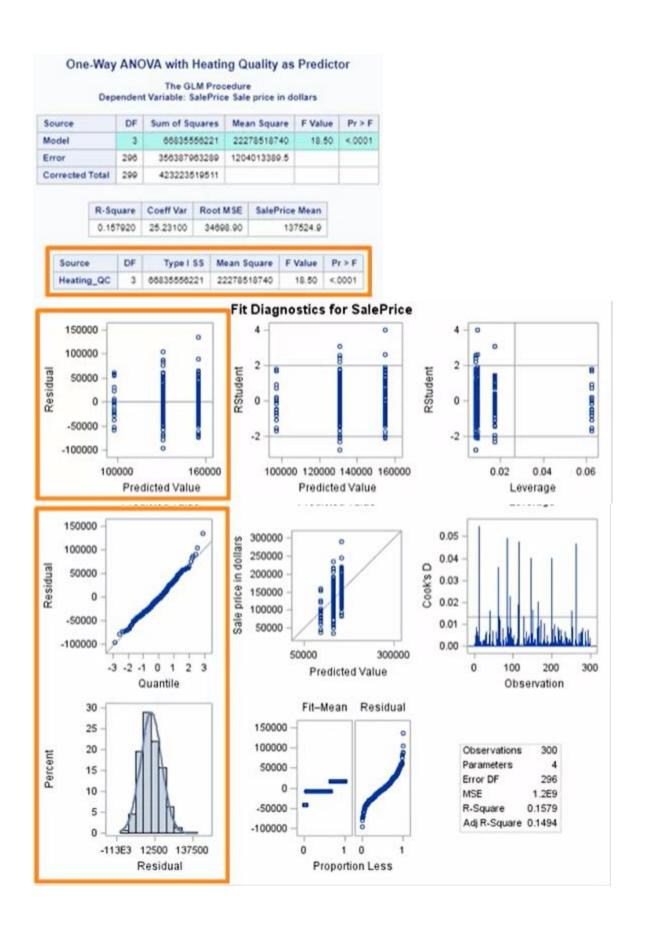
### Pr > F Source Sum of Squares Mean Square F Value 22278518740 Model 3 88835588221 18.50 <.0001 296 1204013389.5 356387963289 Error Corrected Total 200 423223519511

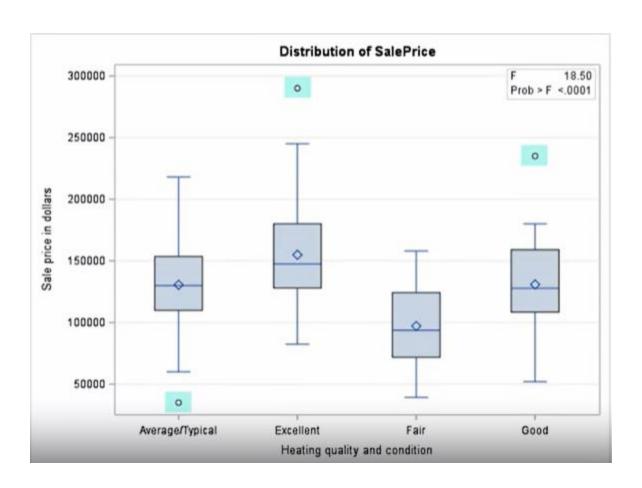


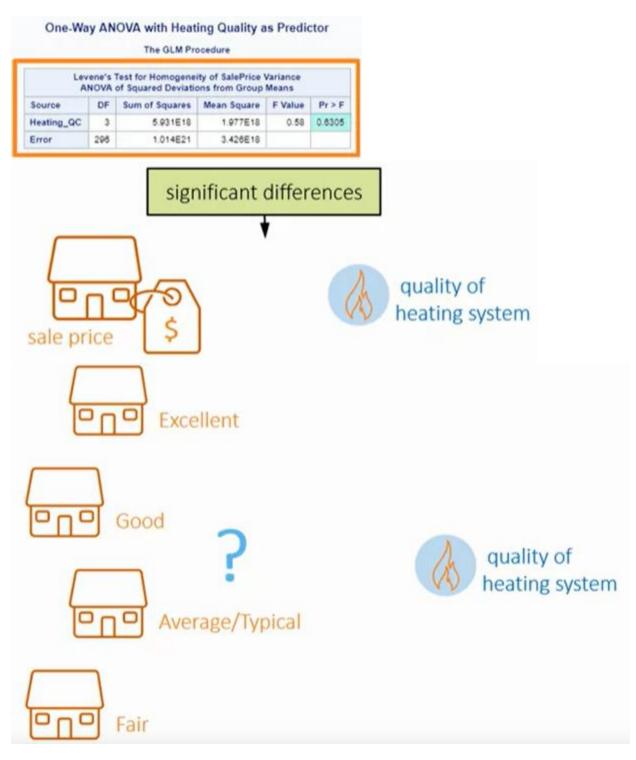


Root MSE/SalePrice Mean \* 100

August 15, 2021 Suhaimi William Chan Page | 42







/\*st102d02.sas\*/

ods graphics;

proc glm data=STAT1.ameshousing3 plots=diagnostics;

```
class Heating_QC;
model SalePrice=Heating_QC;
means Heating_QC / hovtest=levene;
format Heating_QC $Heating_QC.;
title "One-Way ANOVA with Heating Quality as Predictor";
run;
quit;
title;
```

### One-Way ANOVA with Heating Quality as Predictor

### The GLM Procedure

Class Level Information				
Class Levels		Values		
Heating_QC	4	Average/Typical Excellent Fair Good		

Number of Observations Read	300
Number of Observations Used	300

### One-Way ANOVA with Heating Quality as Predictor

The GLM Procedure

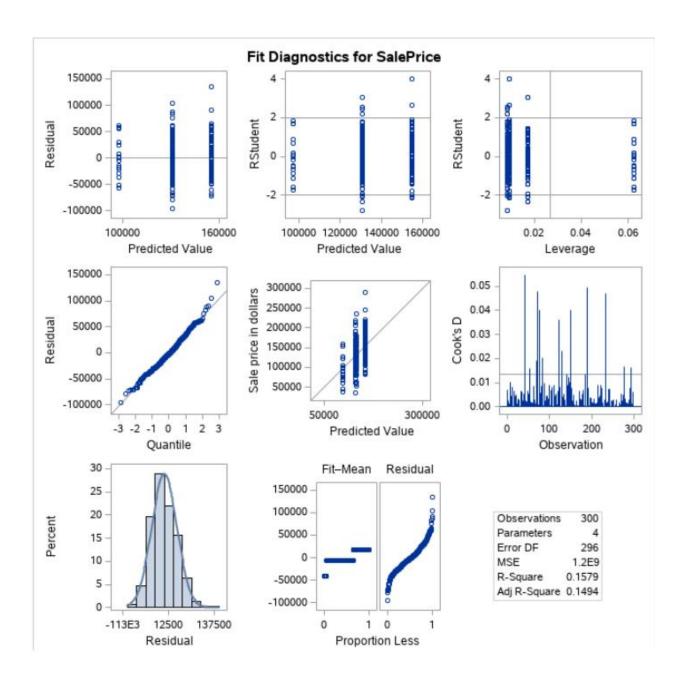
Dependent Variable: SalePrice Sale price in dollars

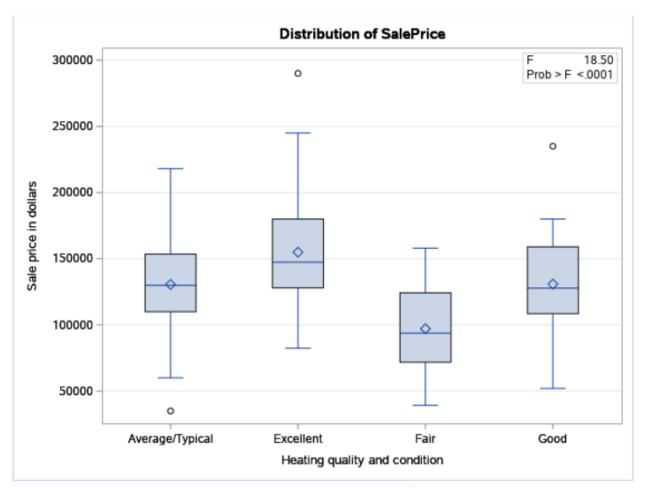
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	66835556221	22278518740	18.50	<.0001
Error	296	356387963289	1204013389.5		
Corrected Total	299	423223519511			

R-Square	Coeff Var	Root MSE	SalePrice Mean
0.157920	25.23100	34698.90	137524.9

Source	DF	Type I SS	Mean Square	F Value	Pr > F	
Heating_QC	3	66835556221	22278518740	18.50	<.0001	

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Heating_QC	3	66835556221	22278518740	18.50	<.0001





# One-Way ANOVA with Heating Quality as Predictor

The GLM Procedure

Levene's Test for Homogeneity of SalePrice Variance ANOVA of Squared Deviations from Group Means						
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F	
Heating_QC	3	5.931E18	1.977E18	0.58	0.6305	
Error	296	1.014E21	3.426E18			



Level of		SalePrice		
Heating_QC	N	Mean	Std Dev	
Average/Typical	119	130573.529	32177.4508	
Excellent	107	154919.187	36822.8795	
Fair	16	97118.750	37423.5437	
Good	58	130844.086	34912.5027	

# **Practice - Performing a One-Way ANOVA**

### **TOTAL POINTS 2**

1.

Question 1

Montana Gourmet Garlic is a company that uses organic methods to grow garlic. It specializes in hardneck varieties. Knowing a little about experimental methods, the owners design an experiment to test whether growth of the garlic is affected by the type of fertilizer. They limit the experimentation

to a Rocambole variety named Spanish Roja, and test three organic fertilizers and one chemical fertilizer (as a control). They "blind" themselves to the fertilizer by using containers with numbers 1 through 4. (In other words, they design the experiment in such a way that they do not know which fertilizer is in which container.) One acre of farmland is set aside for the experiment. The land is divided into 32 beds, and they randomly assign fertilizers to the beds. At harvest, they calculate the average weight of garlic bulbs in each of the beds. The data are in the **stat1.garlic** data set.

Consider an experiment to study four types of fertilizer, labeled 1, 2, 3, and 4. One fertilizer is chemical and the rest are organic. You want to see whether the average weights of the garlic bulbs are significantly different for plants in beds that use different fertilizers.

- 1. Test the hypothesis that the means are equal. Use PROC MEANS to generate descriptive statistics for the four groups, and use PROC SGPLOT to produce box plots of bulb weight for the four groups. Submit the code and view the results.
- 2. Which fertilizer has the highest mean? **Note**: for your answer, type the fertilizer number (1, 2, 3, or 4).

```
/*st102s01.sas*/ /*Part A*/
proc means data=STAT1.Garlic;

var BulbWt;

class Fertilizer;

title 'Descriptive Statistics of BulbWt by Fertilizer';

run;

proc sgplot data=STAT1.Garlic;

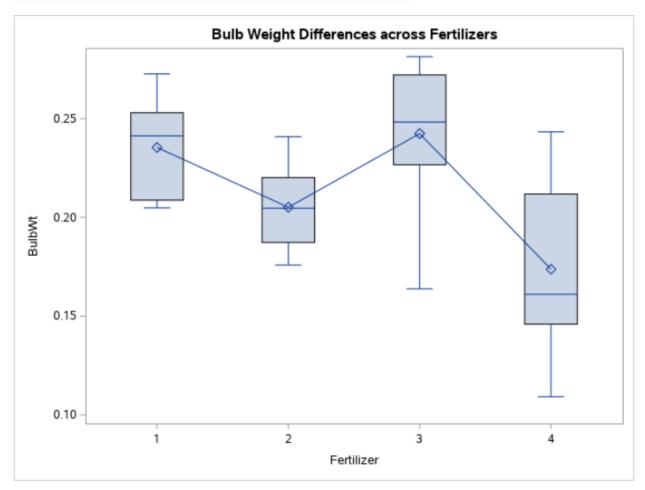
vbox BulbWt / category=Fertilizer

connect=mean;

title "Bulb Weight Differences across Fertilizers";

run;
```

De	Descriptive Statistics of BulbWt by Fertilizer							
	The MEANS Procedure							
	Analysis Variable : BulbWt							
Fertilizer	N Obs	N	Mean	Std Dev	Minimum	Maximum		
1	8	8	0.2353998	0.0254092	0.2047856	0.2726395		
2	8	8	0.2051141	0.0222098	0.1758361	0.2408676		
3	8	8	0.2424075	0.0386855	0.1638284	0.2813780		
4	8	8	0.1737649	0.0444702	0.1092144	0.2433058		



Perform a one-way ANOVA using PROC GLM. Be sure to check that the assumptions of the analysis method that you choose are met. Submit the code and view the results.

What conclusions can you reach at this point in your analysis?

Fertilizer 3 has the highest mean, 0.2424075, although its mean is quite close to fertilizers 1 and 2.

```
/*st102s01.sas*/ /*Part B*/
ods graphics;

proc glm data=STAT1.Garlic plots=diagnostics;
class Fertilizer;
model BulbWt=Fertilizer;
means Fertilizer / hovtest=levene;
title "One-Way ANOVA with Fertilizer as Predictor";
run;
quit;
title;
```

### One-Way ANOVA with Fertilizer as Predictor

The GLM Procedure

Class Level Information					
Class	Levels	Values			
Fertilizer	4	1234			

Number of Observations Read 32 Number of Observations Used 32

### One-Way ANOVA with Fertilizer as Predictor

The GLM Procedure

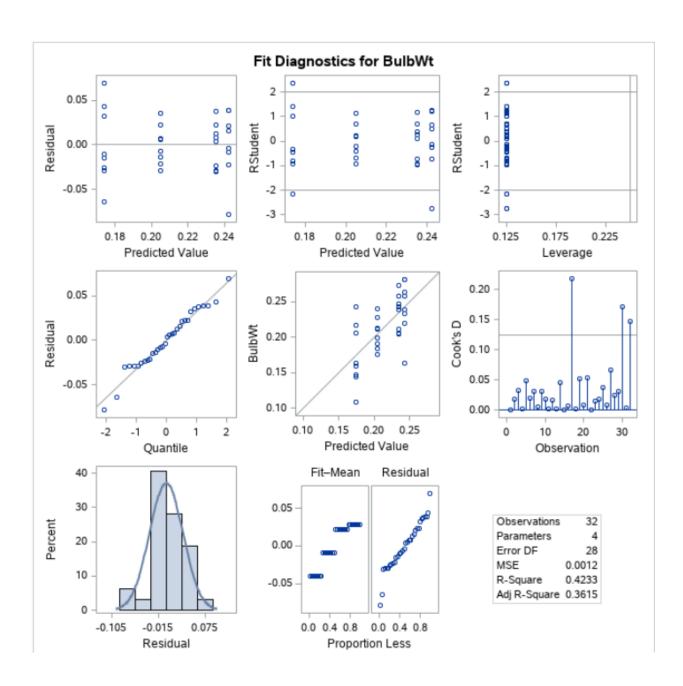
Dependent Variable: BulbWt

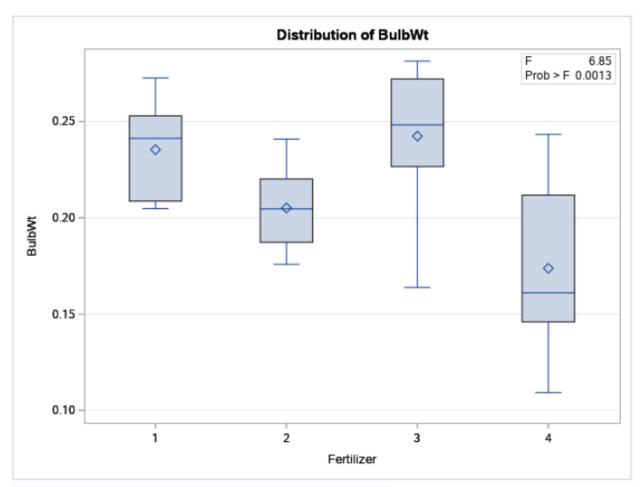
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	0.02370114	0.00790038	6.85	0.0013
Error	28	0.03229141	0.00115326		
Corrected Total	31	0.05599255			

R-Square	Coeff Var	Root MSE	BulbWt Mean
0.423291	15.85633	0.033960	0.214172

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Fertilizer	3	0.02370114	0.00790038	6.85	0.0013

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Fertilizer	3	0.02370114	0.00790038	6.85	0.0013





# One-Way ANOVA with Fertilizer as Predictor

The GLM Procedure

Levene's Test for Homogeneity of BulbWt Variance ANOVA of Squared Deviations from Group Means							
Source DF Sum		Sum of Squares	Mean Square	F Value	Pr > F		
Fertilizer	Fertilizer 3		3.043E-6	1.54	0.2257		
Error 28 0.0		0.000055	1.974E-6				

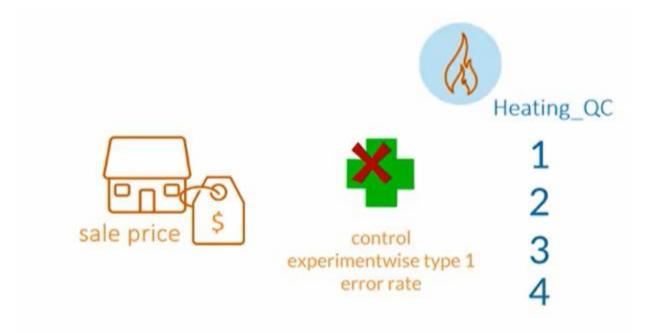
# Distribution of BulbWt 0.25 0.15 0.10 Fertilizer

One-Way ANOVA with Fertilizer as Predictor

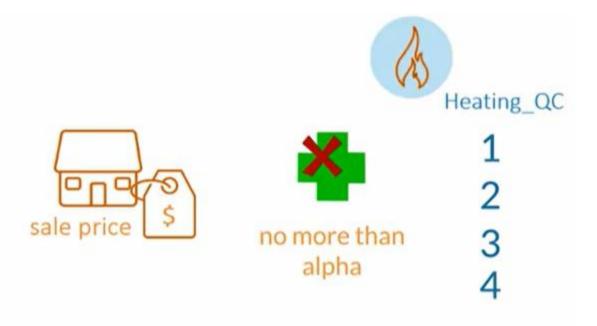
Level of		Bull	bWt
Fertilizer	N	Mean	Std Dev
1	8	0.23539981	0.02540915
2	8	0.20511406	0.02220977
3	8	0.24240747	0.03868547
4	8	0.17376488	0.04447015

The overall *F* value from the analysis of variance table is associated with a *p*-value of 0.0013. Presuming that all assumptions of the model are valid, you know that at least one treatment mean is different from one other treatment mean. At this point, you don't know which means are significantly different from one another. Both the histogram and Q-Q plot show that the residuals seem relatively normally distributed (one assumption for ANOVA). The Levene's Test for Homogeneity of Variance table shows a *p*-value greater than alpha. Therefore, do not reject the hypothesis of homogeneity of variances (equal variances across fertilizer types). This assumption for ANOVA is met.

### **ANOVA Post Hoc Tests**

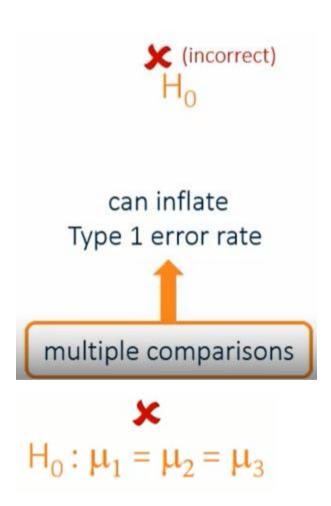


# post hoc tests - multiple-comparison procedures -



post hoc tests - multiple-comparison procedures -

**Multiple Comparison Methods** 



? 
$$H_0: \mu_1 = \mu_2$$
 
$$Pairwise$$
 
$$multiple \ comparisons$$
 
$$\alpha = 0.05$$
 
$$\alpha = 0.05$$
 
$$\alpha = 0.05$$
 
$$M_0: \mu_1 = \mu_2$$
 
$$M_0: \mu_1 = \mu_3$$
 
$$M_0: \mu_2 = \mu_3$$
 
$$M_0: \mu_2 = \mu_3$$
 
$$M_0: \mu_2 = \mu_3$$
 
$$M_0: \mu_1 = \mu_2$$
 
$$M_0: \mu_1 = \mu_3$$
 
$$M_0: \mu_2 = \mu_3$$
 
$$M_0: \mu_2 = \mu_3$$
 
$$M_0: \mu_2 = \mu_3$$

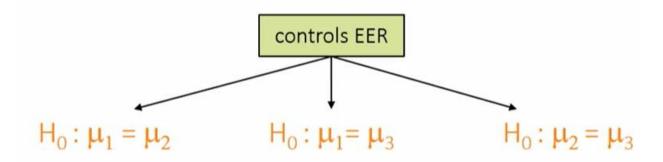
$$\mathrm{EER} = 1 - (1 - \alpha)^{nc}$$

Comparisonwise Error Rate	Number of Comparisons	Experimentwise Error Rate
0.05	1	0.05
0.05	3	0.14
0.05	6	0.26
0.05	10	0.40

### Tukey's and Dunnett's Multiple Comparison Methods

Tukey method

# honest significant difference test



pairwise comparisons

# overcorrects

# Tukey method

# honest significant difference test

 $EER \le \alpha$ 

$$H_0: \mu_1 = \mu_2$$

$$H_0: \mu_1 = \mu_3$$

$$H_0: \mu_2 = \mu_3$$

pairwise comparisons

# Dunnett's method

# specialized multiple comparison test











# other methods

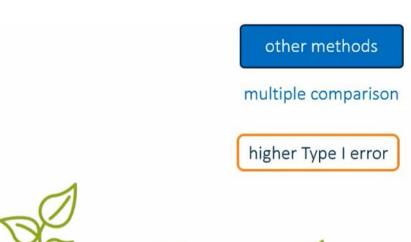
# multiple comparison

**EER** control varies

other methods

# multiple comparison

Actual Decision	H <sub>o</sub> is true	H <sub>o</sub> is false	
Fail to reject H <sub>0</sub>	Correct	Type II error p(Type II  H <sub>a</sub> )=β	
Reject H <sub>0</sub>	Type I error $p(Type I   H_0) = \alpha$	Correct (1 – β)=Power	





other methods

multiple comparison

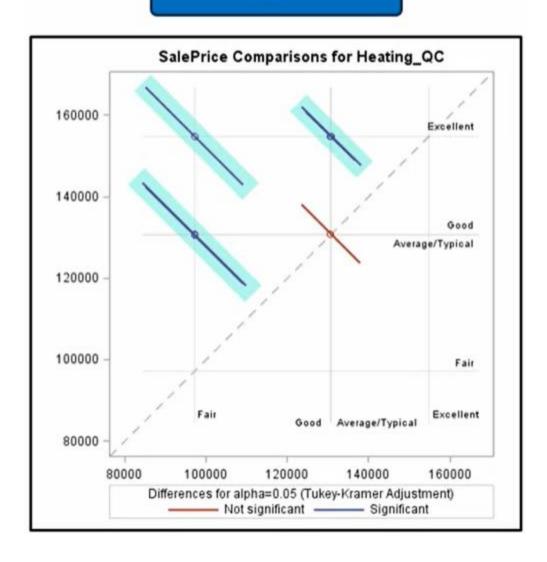
lower Type I error



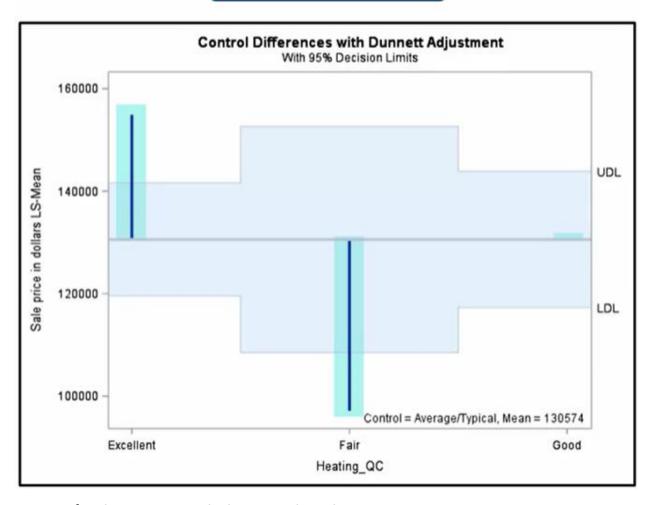
**Diffograms and Control Plots** 



# diffogram

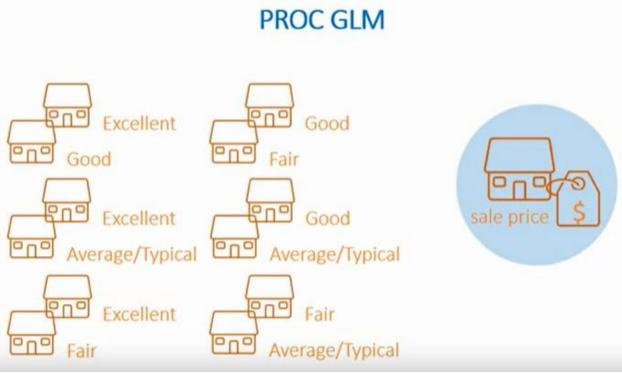


# control plot



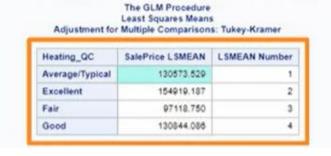
**Demo Performing a Post Hoc Pairwise Comparing Using PROC GLM** 



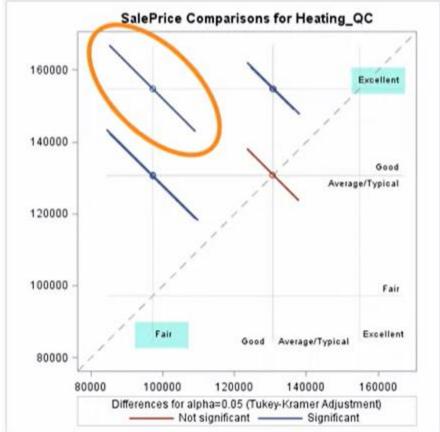


```
1 /*st102d03.sas*/
  2 ods graphics;
  4 ods select lsmeans diff diffplot controlplot;
  5 proc glm data=STAT1.ameshousing3
6 plots(only)=(diffplot(center) controlplot);
7 class Heating_QC;
                                                                                                 PROC GLM DATA=SAS-data-set <options>;
                                                                                                       CLASS variable(s);
MODEL dependent-variables=independent-effects </options>;
MEANS effects </options>;
         model SalePrice=Heating_QC;
lsmeans Heating_QC / pdiff=all
         adjust=tukey;
lsmeans Heating_QC / pdiff=control('Average/Typical')
 10
                                     adjust=dunnett;
         format Heating_QC $Heating_QC.;
title "Post-Hoc Analysis of ANOVA - Heating Quality as Predictor";
14
 15 run;
 16 quit;
18 title;
1 /*st102d03.sas*/
  2 ods graphics;
 4 ods select 1smeans diff diffplot controlplot;
   proc glm data=STAT1.ameshousing3
                                                                                                PROC GLM DATA=SAS-data-set <options>;
              plots(only)=(diffplot(center) controlplot);
                                                                                                     CLASS variable(s);
MODEL dependent-variables=independent-effects </options>;
LSMEANS effects </options>;
         class Heating_QC;
model SalePrice=Heating_QC;
        1smeans Heating QC / pdiff=all
        adjust=tukey;
lsmeans Heating_QC / pdiff=control('Average/Typical')
10
                                   adjust=dunnett;
       format Heating_QC $Heating_QC.;
title "Post-Hoc Analysis of ANOVA - Heating Quality as Predictor";
14
15 run;
16 quit;
18 title;
```

### Post-Hoc Analysis of ANOVA - Heating Quality as Predictor

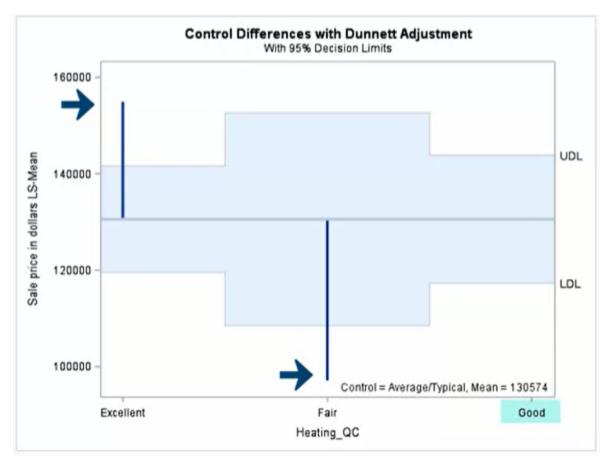






Post-Hoc Analysis of ANOVA - Heating Quality as Predictor

		H0:L\$Mean=Control
Heating_QC	SalePrice LSMEAN	Pr > [t]
Average/Typical	130573.529	
Excellent	154919.187	<.0001
Fair	97118.750	0.0010
Good	130844,088	0.9999



/\*st102d02.sas\*/

ods graphics;

```
proc glm data=STAT1.ameshousing3 plots=diagnostics;
  class Heating_QC;
  model SalePrice=Heating_QC;
    means Heating_QC / hovtest=levene;
  format Heating_QC $Heating_QC.;
  title "One-Way ANOVA with Heating Quality as Predictor";
run;
quit;
title;
```

### One-Way ANOVA with Heating Quality as Predictor

### The GLM Procedure

Class Level Information				
Class	Levels	Values		
Heating_QC	4	Average/Typical Excellent Fair Good		

Number of Observations Read	300
Number of Observations Used	300

### One-Way ANOVA with Heating Quality as Predictor

### The GLM Procedure

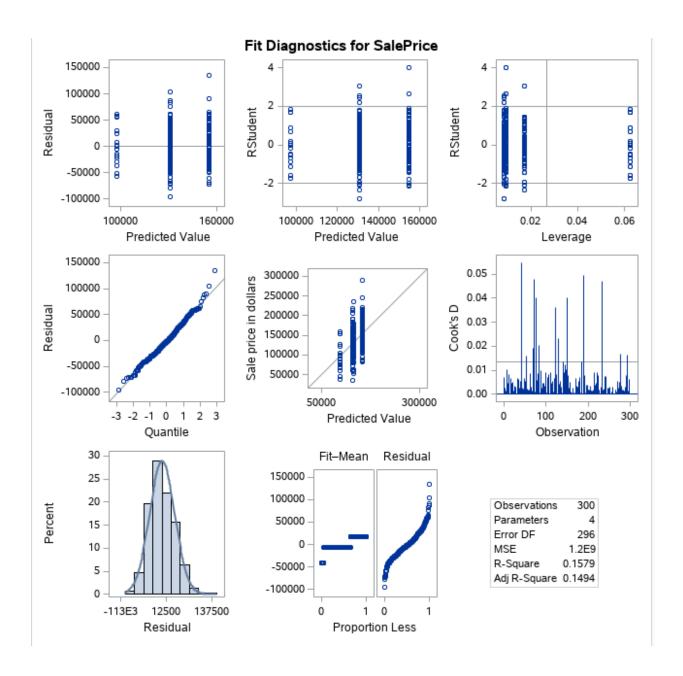
### Dependent Variable: SalePrice Sale price in dollars

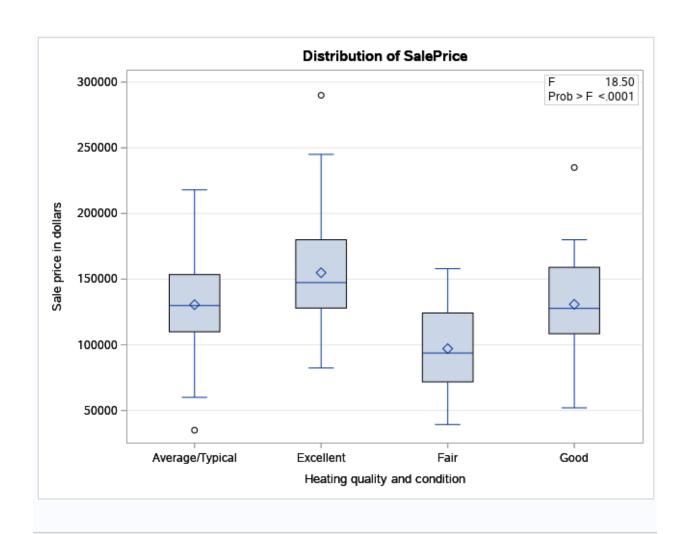
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	66835556221	22278518740	18.50	<.0001
Error	296	356387963289	1204013389.5		
Corrected Total	299	423223519511			

R-Square	Coeff Var	Root MSE	SalePrice Mean
0.157920	25.23100	34698.90	137524.9

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Heating_QC	3	66835556221	22278518740	18.50	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Heating_QC	3	66835556221	22278518740	18.50	<.0001

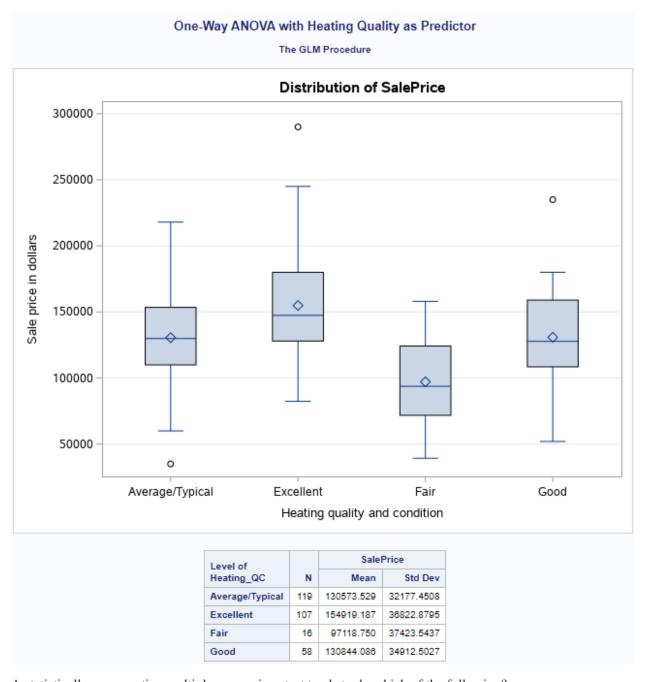




# One-Way ANOVA with Heating Quality as Predictor

The GLM Procedure

	Levene's Test for Homogeneity of SalePrice Variance ANOVA of Squared Deviations from Group Means						
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F		
Heating_QC	3	5.931E18	1.977E18	0.58	0.6305		
Error	296	1.014E21	3.426E18				



A statistically conservative multiple comparison test tends to do which of the following?

With a statistically conservative multiple comparison method, such as the Tukey or Dunnett method, you control for the EER, so there's a tendency to find fewer significant differences than might otherwise be found. When you make no adjustments for multiple comparisons, you are likely to find more significant differences than might otherwise be found.

Compared to a multiple comparisons test that controls the experimentwise error rate, what characteristics will a multiple comparisons test that only controls the comparisonwise error rate tend to have?

If only the comparisonwise error rate is controlled, the overall risk of a Type I error across all the comparisons is increased (and therefore the risk of Type II error is decreased), so the test might find more significant differences than would otherwise be found.

# Practice - Using PROC GLM to Perform Post Hoc Parwise Comparisons

### Question 1

Consider the analysis of the **garlic** data set. In the previous exercise, you used PROC GLM to perform one-way ANOVA, and found that there was a statistically significant difference among mean garlic bulb weights for the different fertilizers. Now, perform a post hoc test to look at the individual differences among means.

- 1. Use PROC GLM to conduct pairwise comparisons with an experimentwise error rate of  $\alpha$ =0.05. (Use the Tukey adjustment.) Submit the code and view the results.
- 2. Which types of fertilizer are significantly different?

The Tukey comparisons show significant differences between fertilizers 3 and 4 (p=0.0020) and 1 and 4 (p=0.0058).

Question 2

- 1. Use level 4 (the chemical fertilizer) as the control group and perform a Dunnett's comparison with the organic fertilizers to see whether they affected the average weights of garlic bulbs differently from the control fertilizer.
- 2. Which types of fertilizer are significantly different?

The Dunnett comparisons show the same pairs as significantly different, but with smaller p-values than with the Tukey comparisons (3 versus 4 p=0.0011, 1 versus 4 p=0.0031). This is due to the fact that the Tukey adjustment is for more pairwise comparisons than the Dunnett adjustment.

```
/*st102s02.sas*/
ods graphics;

ods select Ismeans diff diffplot controlplot;
proc glm data=STAT1.Garlic
    plots(only)=(diffplot(center) controlplot);
class Fertilizer;
model BulbWt=Fertilizer;
Tukey: Ismeans Fertilizer / pdiff=all
```

### adjust=tukey;

**Dunnett:Ismeans Fertilizer / pdiff=control('4')** 

adjust=dunnett;

No\_Adjust: Ismeans Fertilizer / pdiff=all adjust=t;

title "Post-Hoc Analysis of ANOVA - Fertilizer as Predictor";

run;

quit;

title;

### Post-Hoc Analysis of ANOVA - Fertilizer as Predictor

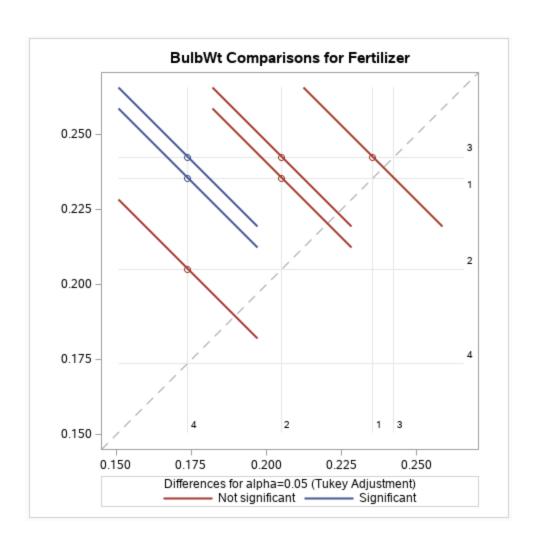
The GLM Procedure Least Squares Means Adjustment for Multiple Comparisons: Tukey

Fertilizer	BulbWt LSMEAN	LSMEAN Number
1	0.23539981	1
2	0.20511406	2
3	0.24240747	3
4	0.17376488	4

Least Squares Means for effect Fertilizer Pr > |t| for H0: L SMean(i)=L SMean(j)

Dependent Variable: RulbWt

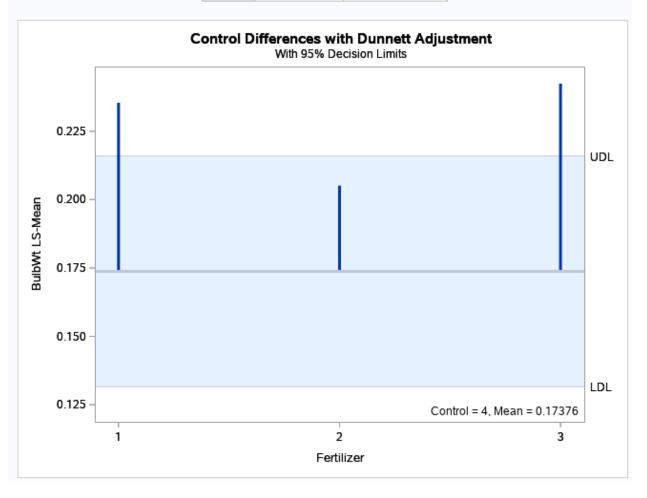
Dependent Variable: BulbWt								
i/j	1	2	3	4				
1		0.3021	0.9758	0.0058				
2	0.3021		0.1490	0.2738				
3	0.9758	0.1490		0.0020				
4	0.0058	0.2738	0.0020					



# Post-Hoc Analysis of ANOVA - Fertilizer as Predictor

The GLM Procedure Least Squares Means Adjustment for Multiple Comparisons: Dunnett

		H0:LSMean=Control	
Fertilizer	BulbWt LSMEAN	Pr >  t	
1	0.23539981	0.0031	
2	0.20511406	0.1801	
3	0.24240747	0.0011	
4	0.17376488		



t-Hoc	Ana	lysis o	f ANOV	A - Ferti	ilizer as	Predicto
			e GLM Pro st Square			
Fer	rtilize	r Bulb\	Nt LSMEA	N LSM	EAN Numb	per
1			0.2353998	31		1
2			0.2051140	06		2
3			0.24240747			3
4			0.17376488		4	
		>  t  for	es Means t H0: LSMe dent Varia	an(i)=LSN	lean(j)	
	i/j	1	2	3	4	
	1		0.0853	0.6830	0.0011	
	2	0.0853		0.0365	0.0755	
	3	0.6830	0.0365		0.0004	
	4	0.0011	0.0755	0.0004		

