

SUMMER WORKSHOP IN MATHEMATICS

(SWIM@KSOM - 2025)

Introduction to University Mathematics

(Problem Sheet 2)

1. **Ordered Field:** A ordered field F is an ordered set such that for all $x, y, z \in F$

1. $x + y < x + z$ whenever $y < z$
2. $xy > 0$ whenever $x > 0, y > 0$

Prove that the following are true in an ordered field.

- (a) $x > 0 \implies -x < 0$
 - (b) $xy < xz$ whenever $x > 0$ and $y < z$
 - (c) $xy > xz$ whenever $x > 0$ and $y > z$
 - (d) $x^2 > 0$ for all $x \neq 0$
 - (e) $1 > 0$
 - (f) $0 < \frac{1}{y} < \frac{1}{x}$ whenever $0 < x < y$
 - (g) $x < \frac{x+y}{2} < y$
2. Prove that the field of complex numbers cannot be an ordered field.
3. Prove that $\inf S = -\sup\{-s : s \in S\}$
4. Prove that $\sup(A \cup B) = \max\{\sup A, \sup B\}$
5. Suppose S is an ordered set with supremum property, then prove that S has infimum property also.
6. **Archimedean Property:** Prove that Natural Numbers are unbounded.
7. **Density Property:** Prove that between any two real numbers there is a rational number.
8. Prove that between any two real numbers there is an irrational number.