

SUMMER WORKSHOP IN MATHEMATICS

(SWIM@KSOM - 2025)

Analysis

(Problem Sheet 2)

1. Does there exist $\alpha, \beta \in \mathbb{R} \setminus \mathbb{Q}$ such that $\alpha^\beta \in \mathbb{Q}$
2. Prove that convergent sequences of real numbers is bounded.
3. Suppose that $a_n \rightarrow 0$ and $|x_n - x| \leq a_n$, then show that $x_n \rightarrow x$.
4. Evaluate the following limits and prove the convergence:
 - (a) $\sqrt{4n^2 + n} - 2n$
 - (b) $\lim_{n \rightarrow \infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$
 - (c) $\lim_{n \rightarrow \infty} \frac{n^2}{n!}$
5. **Squeeze Theorem:** Suppose $\{x_n\}, \{y_n\}, \{z_n\}$ be sequences of real numbers such that $x_n \leq y_n \leq z_n$ for all $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} z_n = a$. Prove that $\lim_{n \rightarrow \infty} y_n = a$.
6. Evaluate the following limits:
 - (a) $\frac{\sin n}{n}$
 - (b) $\frac{\cos n}{n^2}$
7. Consider the sequence $\{x_n\}$ defined as $x_1 = 0, x_2 = 1$ and

$$x_{n+2} = \frac{x_{n+1} + x_n}{2}.$$

Evaluate $\lim_{n \rightarrow \infty} x_n$.