SUMMER WORKSHOP IN MATHEMATICS

(SWIM@KSOM - 2025)

Introduction to University Mathematics

(Problem Sheet 1)

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1. Consider the infinite sum

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

Find S (Hint: Compute $2S$). Do the same technique for the series

$$1 + 2 + 3 + \dots$$

Is everything fine!!!

2. (Interchanging Limits) Can we interchange limit always as below (provided the limit exist)?

$$\lim_{x \to 0} \lim_{y \to 0} f(x, y) = \lim_{y \to 0} \lim_{x \to 0} f(x, y)$$

3. (Limit and Integral) Do the following equality holds always?

$$\lim_{n \to \infty} \int f_n = \int \lim_{n \to \infty} f_n$$

4. Prove the following properties addition of natural numbers (defined through Peano's axioms).
   (a) $n + 0 = n = 0$
   (b) $n + s(m) = s(n + m)$
   (c) Commutativity: $n + m = m + n$
   (d) Associativity: $(a + b) + c = a + (b + c)$

5. Prove the properties of ordering of natural numbers:
   (a) Reflexive: $a \geq a$ for all $a \in \mathbb{N}$
   (b) Antisymmetric: $a \geq b$ and $b \geq a$ implies $a = b$ for all $a, b \in \mathbb{N}$
   (c) Transitive: $a \geq b$ and $b \geq c$ implies $a \geq c$ for all $a, b, c \in \mathbb{N}$
   (d) $a \geq b$ then $a + c \geq b + c$ for all $a, b, c \in \mathbb{N}$