# SUMMER WORKSHOP IN MATHEMATICS

(SWIM@KSOM - 2025)

## Analysis

(Problem Sheet 5)

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1. Prove that convergent sequences are bounded.
2. Prove that convergent sequences are Cauchy.
3. Prove that Cauchy sequences are bounded.
4. Prove that Cauchy sequences are convergent.
5. Consider the sequence $\{x_n\}$ defined as $x_1 = 0, x_2 = 1$ and

   $$x_{n+2} = \frac{x_{n+1} + x_n}{2}.$$

   Evaluate $\lim_{n\to\infty} x_n$.
6. Check whether $\left(1 + \frac{1}{n}\right)^n$ converges or not?
7. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.
8. Prove that the series $\sum_{n=0}^{\infty} \frac{1}{n!}$ converges to an irrational number.