SUMMER WORKSHOP IN MATHEMATICS

(SWIM@KSOM - 2025)

Analysis

(Problem Sheet 5)

- 1. Prove that convergent sequences are Cauchy.
- 2. Prove that Cauchy sequences are bounded.
- 3. Prove that Cauchy sequences are convergent.
- 4. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.
- 5. Prove that the series $\sum_{n=0}^{\infty} \frac{1}{n!}$ converges to an irrational number
- 6. Let $f,g:\mathbb{R}\to\mathbb{R}$ be continuous at x_0 . Prove the following are continuous at x_0
 - (a) f + g.
 - (b) fg.
 - (c) f/g provided $g(x_0) \neq 0$.
 - (d) $f \circ g$