

SwInBee 2018

Name:

Instructions

1. Duration: 1 hour.
2. No materials allowed besides pens and pencils. Paper will be supplied for rough working.
3. No partial marks awarded. This includes the “+ C” for indefinite integrals: if an appropriate constant is not included then you will get zero.
4. A prize of \$25 will be given for solution of the prize question, with the tie-breaker for multiple correct answers being the total number of points.

Integrals

1. $\int \sqrt{x} e^{x\sqrt{x}} dx = \frac{2}{3} e^{x^{3/2}} + C$

2. $\int \frac{1}{\sqrt{x}} \left(\ln \sqrt{x} + \frac{1}{x} \right) dx = \frac{-2(1+x) + x \ln x}{\sqrt{x}} + C$

3. $\int \sinh x \arctan(\sinh x) dx = \arctan(\sinh x) \cosh x - x + C$

4. $\int \frac{dx}{x^3 + x} = \ln x - \frac{1}{2} \ln(1 + x^2) + C$

5. $\int \frac{1 + e^x}{e^x - 1} dx = 2 \ln(1 - e^x) - x + C$

6. $\int e^x \cos(\cos(e^x)) \sin(e^x) dx = -\sin(\cos e^x) + C$

7. $\int \frac{x^3}{1 + x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} \ln(1 + x^2) + C$

8. $\int \ln \left(\frac{1+x}{1-x} \right) dx = \ln(1-x) + \ln(1+x) + x \ln \left(\frac{1+x}{1-x} \right) + C$

$$9. \int \frac{1 - \cos x}{\sin \frac{x}{2}} dx = -4 \cos \left(\frac{x}{2} \right) + C$$

$$10. \int \frac{x^2}{2} \ln \left(\frac{2}{x^2} \right) dx = \frac{1}{9}x^3 + \frac{1}{6}x^3 \ln \left(\frac{2}{x^2} \right) + C$$

$$11. \int \frac{dx}{x^2 - 1} = \frac{1}{2} \ln(1 - x) - \frac{1}{2} \ln(1 + x) + C$$

$$12. \int \frac{\sinh x \cos(\cosh x)}{\sin^2(\cosh x)} dx = -\frac{1}{\sin(\cosh x)} + C$$

$$13. \int \frac{dx}{\sqrt{7 - x^2}} = \arcsin \left(\frac{x}{\sqrt{7}} \right) + C$$

$$14. \int_{-2\pi}^{2\pi} (x^3 \cos 4x - (x^4 + x^2 + 1) \sin 3x) dx = 0$$

$$15. \int x^8 e^{x^3} dx = \frac{1}{3} e^{x^3} (2 - 2x^3 + x^6) + C$$

$$16. \int_0^{2018} |\sin(2018\pi x)| dx = \frac{4036}{\pi}$$

$$17. \int \frac{x^2 - 3x + 2}{(x + 1)^3} dx = \frac{2 + 5x}{(1 + x)^2} + \ln(1 + x) + C$$

$$18. \int (x(x(x(x(\dots)^{1/2})^{1/2})^{1/2})^{1/2})^{1/2} dx = \frac{1}{2}x^2 + C$$

$$19. \int \cos^8 x dx = \frac{35}{128}x + \frac{7}{32} \sin(2x) + \frac{7}{128} \sin(4x) + \frac{1}{96} \sin(6x) + \frac{1}{1024} \sin(8x) + C$$

$$20. \textbf{Prize question!} \quad \int_{-\infty}^{\infty} e^{-3x^2} dx = \sqrt{\frac{\pi}{3}}$$