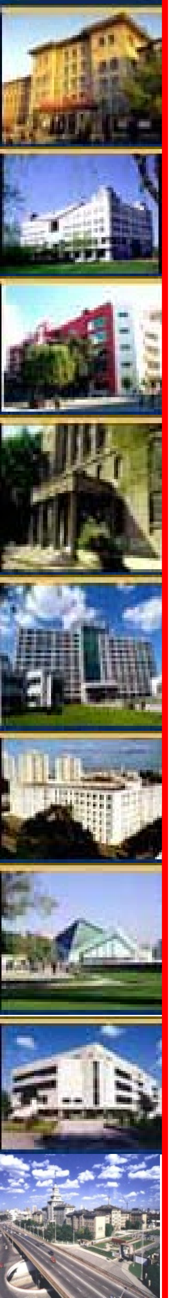

哈爾濱工業大學

机电学院本科课程

《电路IB》

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Electric Machine & Energy Conversion Lab

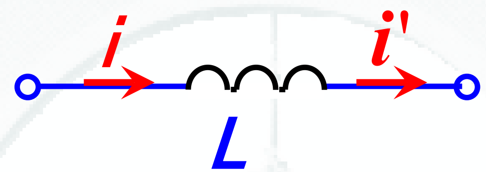
Love in Truth

总 目 录

10.1 二端口网络概念

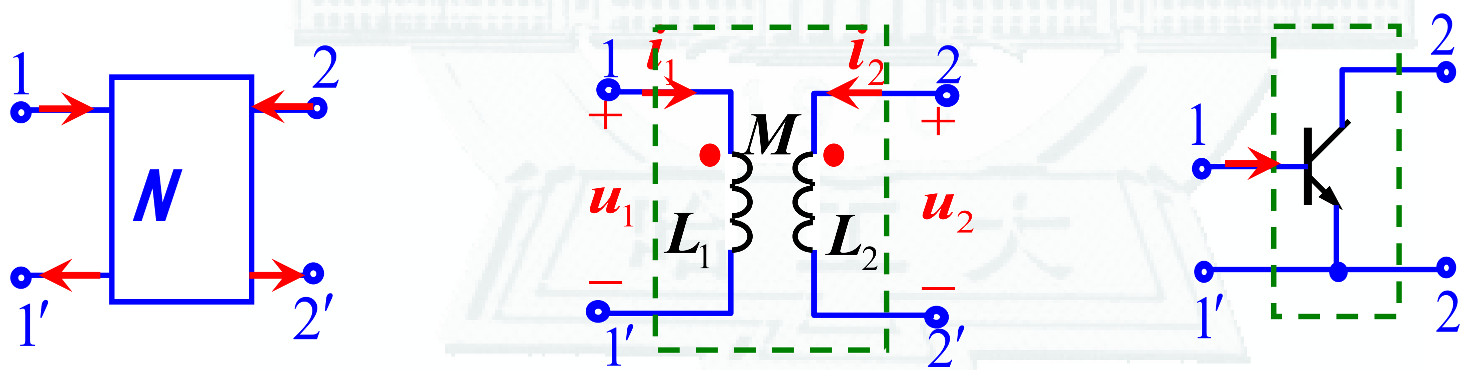
1．单端口网络：

特点： $i = i'$



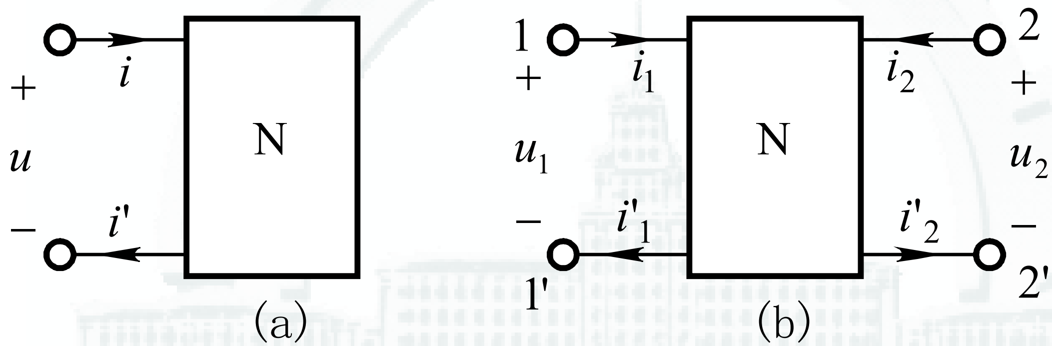
2．二端口网络：

1) 四端网络： 四个端钮，一对输入口、一对输出口。



2) 二端口网络

满足$i_1 = i'_1 , i_2 = i'_2$时的四端网络，称为二端口网络。



一端口　　　　二端口

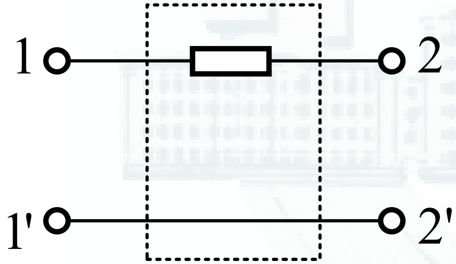
外部特性可用端口电压、电流方程来描述。而对网络内部电压、电流不必详细描述。

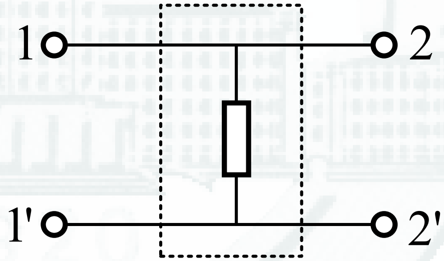
注意：

如果 $i_1 \neq i'_1 , i_2 \neq i'_2$ ，此四端网络就不能称为二端口网络。

10.1 二端口网络概念

本章只讨论线性无独立电源的二端口，即其中含有线性电阻、电容、自感、互感和线性受控电源，而不含独立电源；同时还假设其中所有电感和电容都处于零状态，即在复频域模型中不含附加电源。





第十章 二端口网络

10.1 二端口网络概念

10.2 二端口网络参数方程

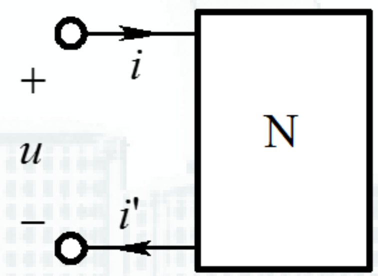
10.3 二端口网络等效电路

10.4 二端口网络与电源和负载的连接

10.5 二端口网络级联

1．单端口网络方程： 变量为端口电压和电流 $\dot{U}$、$\dot{I}$

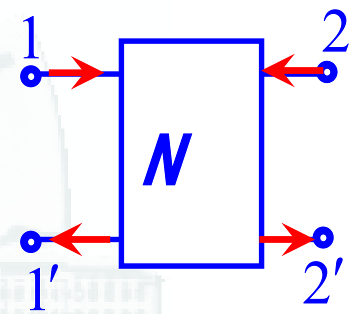
正弦稳态时，一个不含独立源的单端口网络方程依照其端口可表示为：



输入阻抗方程： $\dot{U}=Z\dot{I}$

输入导纳方程： $\dot{I}=Y\dot{U}$

2．二端口网络方程：变量为端口电压和电流 $\dot{U}_1$、$\dot{I}_1$、$\dot{U}_2$、$\dot{I}_2$ 正弦稳态时，可以用六组方程表征二端口网络端口变量的关系，即：



1） 短路导纳参数方程：Y 参数方程

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

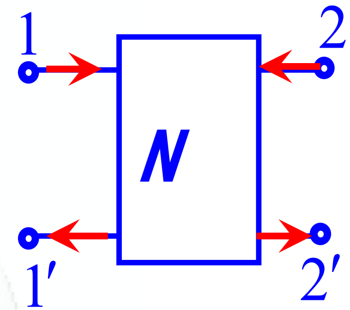
2） 开路阻抗参数方程：Z 参数方程

$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$

10.2 二端口网络参数方程

3） 传输参数方程：
A 参数方程

$$\begin{cases}\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}\left(-\dot{I}_2\right) \\ \dot{I}_1 = A_{21}\dot{U}_2 + A_{22}\left(-\dot{I}_2\right)\end{cases}$$



4） 逆传输参数方程：
B 参数方程

$$\begin{cases}\dot{U}_2 = B_{11}\dot{U}_1 + B_{12}(-\dot{I}_1) \\ \dot{I}_2 = B_{21}\dot{U}_1 + B_{22}(-\dot{I}_1)\end{cases}$$

5） 混合参数方程：
H 参数方程

$$\begin{cases}\dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2\end{cases}$$

6） 逆混合参数方程：
G 参数方程

$$\begin{cases}\dot{I}_1 = G_{11}\dot{U}_1 + G_{12}\dot{I}_2 \\ \dot{U}_2 = G_{21}\dot{U}_1 + G_{22}\dot{I}_2\end{cases}$$

参数矩阵是否互为逆矩阵？

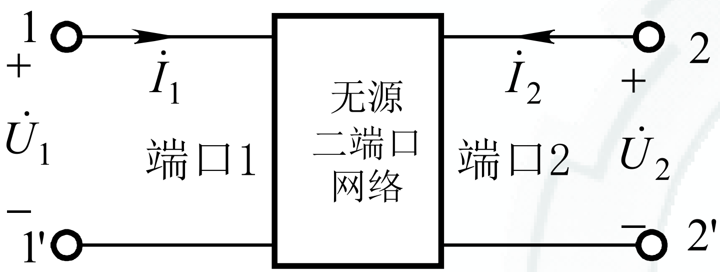
- 叠加原理解释二端口网络参数方程

以*Y*参数方程为例，如图根据叠加原理，对线性网络，响应可表示为激励的线性组合，故：

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

Y_{ij}与二端口网络的结构、元件参数及激励频率有关。

➢ 短路导纳参数（Y参数）



$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

$$Y_{11} = \left.\frac{\dot{I}_1}{\dot{U}_1}\right|_{\dot{U}_2=0}$$

端口2短路
端口1输入导纳

$$Y_{12} = \left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{U}_1=0}$$

端口1短路 端口2对
端口1转移导纳

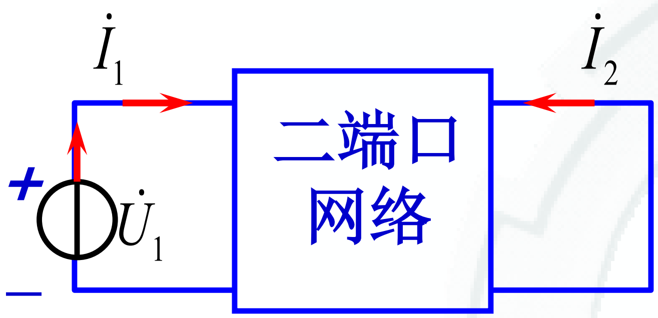
$$Y_{21} = \left.\frac{\dot{I}_2}{\dot{U}_1}\right|_{\dot{U}_2=0}$$

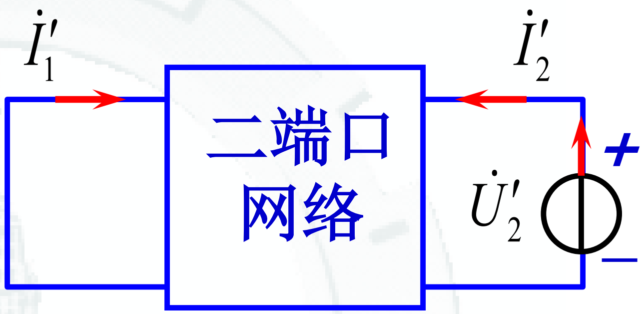
端口2短路 端口1对
端口2转移导纳

$$Y_{22} = \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{U}_1=0}$$

端口1短路
端口2输入导纳

1. 互易条件：





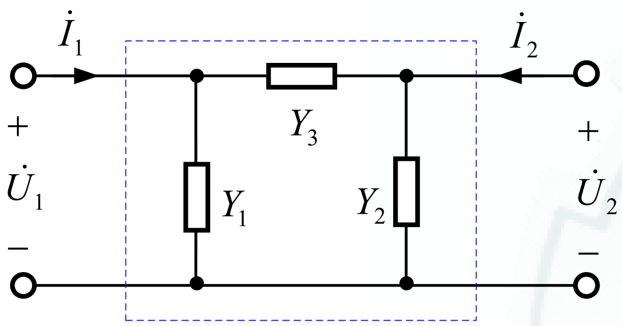
$$Y_{12}=Y_{21}$$

2. 对称条件：

将输入端口与输出端口对换之后，二端口网络的特性保持不变。

$$Y_{12}=Y_{21},\quad Y_{11}=Y_{22}$$

例：通过测试法确定下图Π形二端口网络的*Y*参数。



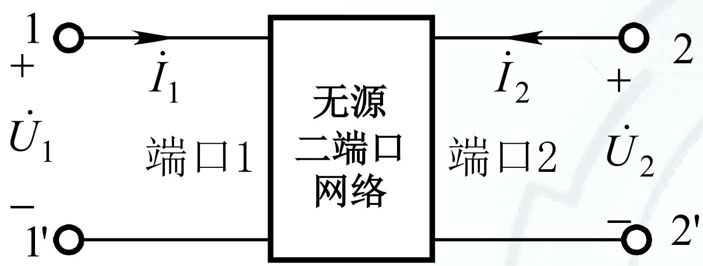
对称二端口 $\begin{cases} Y_{12} = Y_{21} \\ Y_{11} = Y_{22} \end{cases}$ 互易

【解】

$$Y_{11} = \frac{\dot{I}_1}{\dot{U}_1}\bigg|_{\dot{U}_2=0} = Y_1 + Y_3 \qquad Y_{12} = \frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{U}_1=0} = -Y_3$$

$$Y_{21} = \frac{\dot{I}_2}{\dot{U}_1}\bigg|_{\dot{U}_2=0} = -Y_3 \qquad Y_{22} = \frac{\dot{I}_2}{\dot{U}_2}\bigg|_{\dot{U}_1=0} = Y_2 + Y_3$$

➢ 开路阻抗参数（Z 参数）



$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$

$$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0}$$

端口2开路
端口1输入阻抗

$$Z_{12} = \left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

端口1开路 端口2对
端口1转移阻抗

$$Z_{21} = \left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0}$$

端口2开路 端口1
对端口2转移阻抗

$$Z_{22} = \left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

端口1开路
端口2输入阻抗

1. 互易条件：

$$\boldsymbol{Z} = \boldsymbol{Y}^{-1} = \frac{1}{\Delta_Y}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Z_{12} = Z_{21}$$

2. 对称条件：

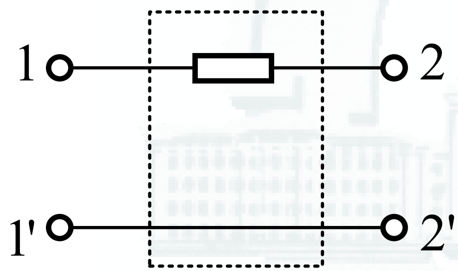
将输入端口与输出端口对换之后，二端口网络的特性保持不变。

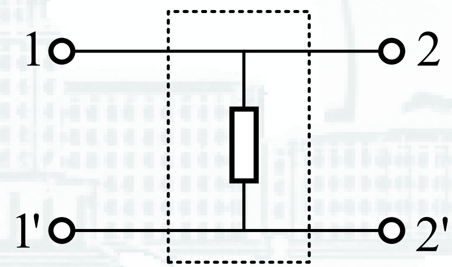
$$Z_{12} = Z_{21}, \quad Z_{11} = Z_{22}$$

10.2 二端口网络参数方程

对于一般二端口即有***Y***矩阵参数又有***Z***矩阵参数，并且

$$\boldsymbol{Z}=\boldsymbol{Y}^{-1}=\frac{1}{\Delta_Y}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}=\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$





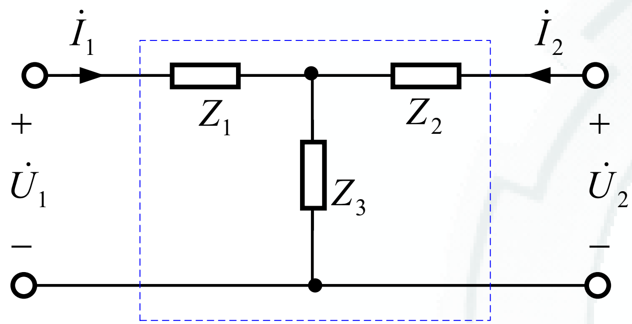
没有***Z***矩阵

没有***Y***矩阵

$$\boldsymbol{Y}=\begin{bmatrix} Y & -Y \\ -Y & Y \end{bmatrix}$$

$$\boldsymbol{Z}=\begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$$

例：通过测试法确定下图T形二端口网络的Z参数。



对称二端口

$$\begin{cases} Z_{12} = Z_{21} & \text{互易} \\ Z_{11} = Z_{22} \end{cases}$$

【解】

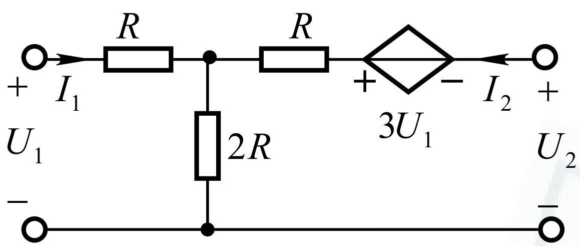
$$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0} = Z_1 + Z_3$$

$$Z_{12} = \left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0} = Z_3$$

$$Z_{21} = \left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0} = Z_3$$

$$Z_{22} = \left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0} = Z_2 + Z_3$$

例：求图示二端口网络的Z参数矩阵。



$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$

【解】 回路电流法

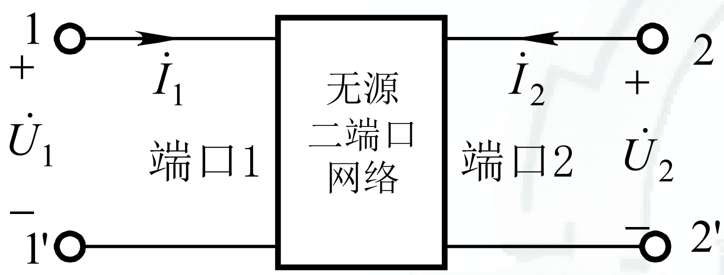
$$\begin{cases} U_1 = 3RI_1 + 2RI_2 \\ U_2 = -3U_1 + 3RI_2 + 2RI_1 \end{cases}$$

$$\begin{cases} U_1 = 3RI_1 + 2RI_2 \\ U_2 = -7RI_1 - 3RI_2 \end{cases} \qquad \boldsymbol{Z} = \begin{bmatrix} 3R & 2R \\ -7R & -3R \end{bmatrix}$$

非互易网络

➢ 传输参数（*A*参数）

一个端口的电流、电压与另一个端口的电流、电压之间的直接关系。



$$\begin{cases} \dot{U}_1 = A_{11}\dot{U}_2 + A_{12}\left(-\dot{I}_2\right) \\ \dot{I}_1 = A_{21}\dot{U}_2 + A_{22}\left(-\dot{I}_2\right) \end{cases}$$

$A_{11} = \left.\dfrac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0,}$ 开路电压比

$A_{12} = -\left.\dfrac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{U}_2=0,}$ 短路转移阻抗

$A_{21} = \left.\dfrac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0,}$ 开路转移导纳

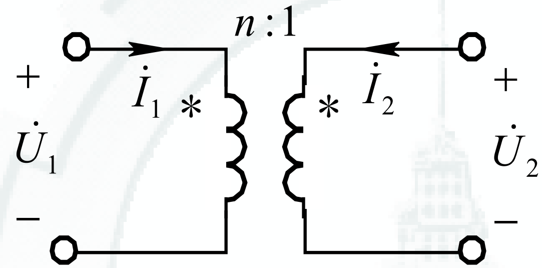
$A_{22} = -\left.\dfrac{\dot{I}_1}{\dot{I}_2}\right|_{\dot{U}_2=0,}$ 短路转移电流比

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases} \Rightarrow \begin{cases} \dot{I}_1 = \dfrac{A_{22}}{A_{12}}\dot{U}_1 + (A_{21} - \dfrac{A_{11}A_{22}}{A_{12}})\dot{U}_2 \\ \dot{I}_2 = -\dfrac{1}{A_{12}}\dot{U}_1 + \dfrac{A_{11}}{A_{12}}\dot{U}_2 \end{cases}$$

互易条件： $A_{11}A_{22} - A_{12}A_{21} = \dfrac{Y_{12}}{Y_{21}} = 1$

对称条件： $\begin{cases} A_{11}A_{22} - A_{12}A_{21} = \dfrac{Y_{12}}{Y_{21}} = 1 \\ A_{11} = A_{22} \end{cases}$

例：图示理想变压器。根据它的元件方程，写出传输参数矩阵.

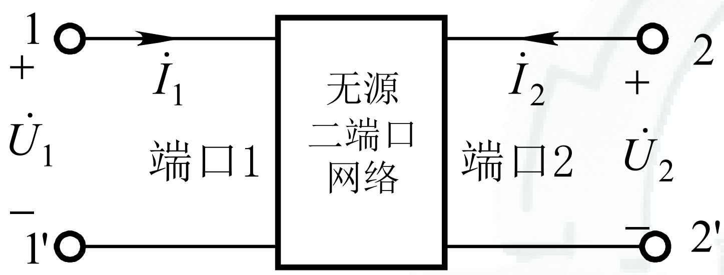


【解】 $\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix} \Rightarrow \boldsymbol{A} = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$

满足 $A_{11}A_{22} - A_{12}A_{21} = 1$ 的互易条件。但由此方程可见，理想变压器不存在阻抗参数和导纳参数。

➢ 混合参数（*H* 参数）

一个端口电压和另一个端口电流与另外电流、电压之间的直接关系。



$$\begin{cases} \dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2 \end{cases}$$

$$H_{11} = \frac{\dot{U}_1}{\dot{I}_1}\bigg|_{\dot{U}_2=0}$$ 端口2短路 端口1输入阻抗

$$H_{12} = \frac{\dot{U}_1}{\dot{U}_2}\bigg|_{\dot{I}_1=0}$$ 端口1开路 转移电压比

$$H_{21} = \frac{\dot{I}_2}{\dot{I}_1}\bigg|_{\dot{U}_2=0}$$ 端口2短路 转移电流比

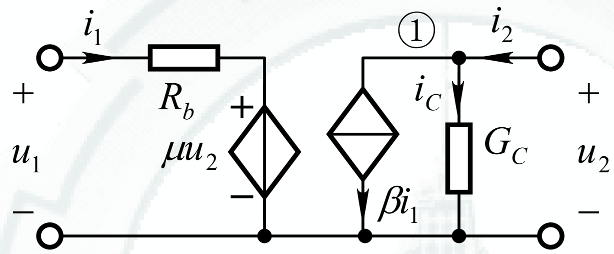
$$H_{22} = \frac{\dot{I}_2}{\dot{U}_2}\bigg|_{\dot{I}_1=0}$$ 端口1开路 端口2输入导纳

$$\begin{cases}\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2\end{cases} \Rightarrow \begin{cases}\dot{U}_1 = \dfrac{1}{Y_{11}}\dot{I}_1 - \dfrac{Y_{12}}{Y_{11}}\dot{U}_2 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 = \dfrac{Y_{21}}{Y_{11}}\dot{I}_1 + \dfrac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{11}}\dot{U}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2\end{cases}$$

互易条件： $H_{12} = -H_{21}$

对称条件： $\begin{cases} H_{12} = -H_{21} \\ H_{11}H_{22} - H_{12}H_{21} = 1 \end{cases}$

例：求图示半导体晶体管低频小信号等效电路的混合参数矩阵。



【解】对输入端口所在回路列KVL方程

$$u_1 = R_b i_1 + \mu u_2$$

对节点①列KCL方程

$$i_2 = \beta i_1 + i_C = \beta i_1 + G_C u_2$$

由此得混合参数矩阵

$$\boldsymbol{H} = \begin{bmatrix} R_b & \mu \\ \beta & G_C \end{bmatrix}$$

10.2 二端口网络参数方程

对比项	开路阻抗参数 Z	短路导纳参数Y
代数方程	$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$ $\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$	$\begin{cases}\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2\end{cases}$
矩阵方程	$\begin{bmatrix}\dot{U}_1 \\ \dot{U}_2\end{bmatrix} = \begin{bmatrix}Z_{11} & Z_{12} \\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1 \\ \dot{I}_2\end{bmatrix}$	$\begin{bmatrix}\dot{I}_1 \\ \dot{I}_2\end{bmatrix} = \begin{bmatrix}Y_{11} & Y_{12} \\ Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1 \\ \dot{U}_2\end{bmatrix}$
参数计算	$Z_{11} = \left.\dfrac{\dot{U}_1}{\dot{I}_1}\right\|_{\dot{I}_2=0}$ $Z_{12} = \left.\dfrac{\dot{U}_1}{\dot{I}_2}\right\|_{\dot{I}_1=0}$ $Z_{21} = \left.\dfrac{\dot{U}_2}{\dot{I}_1}\right\|_{\dot{I}_2=0}$ $Z_{22} = \left.\dfrac{\dot{U}_2}{\dot{I}_2}\right\|_{\dot{I}_1=0}$	$Y_{11} = \left.\dfrac{\dot{I}_1}{\dot{U}_1}\right\|_{\dot{U}_2=0}$ $Y_{12} = \left.\dfrac{\dot{I}_1}{\dot{U}_2}\right\|_{\dot{U}_1=0}$ $Y_{21} = \left.\dfrac{\dot{I}_2}{\dot{U}_1}\right\|_{\dot{U}_2=0}$ $Y_{22} = \left.\dfrac{\dot{I}_2}{\dot{U}_2}\right\|_{\dot{U}_1=0}$
互易对称条件	互易 $Z_{12} = Z_{21}$ 对称 $Z_{12} = Z_{21}$, $Z_{11} = Z_{22}$	互易 $Y_{12} = Y_{21}$ 对称 $Y_{12} = Y_{21}$, $Y_{11} = Y_{22}$

10.2 二端口网络参数方程

对比项	传输参数A	混合参数H
代数方程	$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$ $\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$	$\dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2$ $\dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2$
矩阵方程	$\begin{bmatrix}\dot{U}_1\\ \dot{I}_1\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_2\\ -\dot{I}_2\end{bmatrix}$	$\begin{bmatrix}\dot{U}_1\\ \dot{I}_2\end{bmatrix} = \begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1\\ \dot{U}_2\end{bmatrix}$
参数计算	$A_{11} = \dfrac{\dot{U}_1}{\dot{U}_2}\Big\vert_{\dot{I}_2=0}, A_{21} = \dfrac{\dot{I}_1}{\dot{U}_2}\Big\vert_{\dot{I}_2=0}$ $A_{12} = \dfrac{\dot{U}_1}{-\dot{I}_2}\Big\vert_{\dot{U}_2=0}, A_{22} = \dfrac{\dot{I}_1}{-\dot{I}_2}\Big\vert_{\dot{U}_2=0}$	$H_{11} = \dfrac{\dot{U}_1}{\dot{I}_1}\Big\vert_{\dot{U}_2=0}, H_{21} = \dfrac{\dot{I}_2}{\dot{I}_1}\Big\vert_{\dot{U}_2=0}$ $H_{12} = \dfrac{\dot{U}_1}{\dot{U}_2}\Big\vert_{\dot{I}_1=0}, H_{22} = \dfrac{\dot{I}_2}{\dot{U}_2}\Big\vert_{\dot{I}_1=0}$
互易对称条件	互易 $A_{11}A_{22} - A_{12}A_{21} = 1$ 对称 $A_{11}A_{22} - A_{12}A_{21} = 1,$ $A_{11} = A_{22}$	互易 $H_{12} = -H_{21}$ 对称$H_{12} = -H_{21},$ $H_{11}H_{22} - H_{12}H_{21} = 1$

第十章 二端口网络

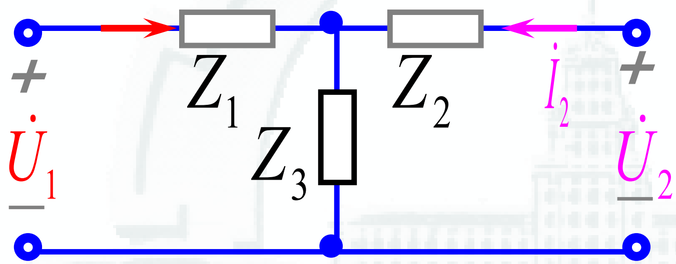
10.3 二端口网络等效电路

一个不含独立源的一端口网络，不管其内部电路如何复杂，从外部特性来看，总可以用一个阻抗(或导纳)来等效代替。同理，一个二端口网络亦可用一个简单的等效电路来代替。

二端口网络的等效电路与原网络必须具有相同的外部特性，即具有相同的网络方程及参数。

➢ 不含受控源的互易网络

若已知一个二端口网络的Z参数，等效电路和参数关系：



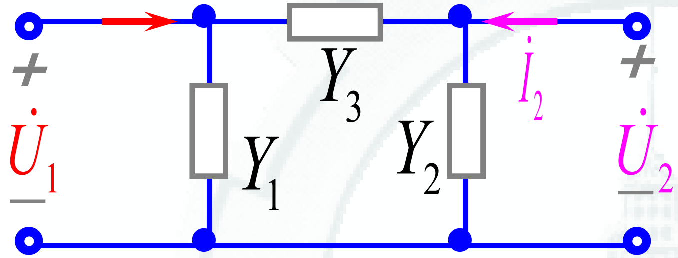
(a)

$$Z_{12}=Z_{21}$$

$$Z_{11}=Z_1+Z_3\,;Z_{12}=Z_{21}=Z_3\,;Z_{22}=Z_2+Z_3$$

$$\boxed{\Rightarrow Z_1=Z_{11}-Z_{12}\,;\;Z_2=Z_{22}-Z_{12}\,;\;Z_3=Z_{12}}$$

若已知一个二端口网络的*Y*参数，等效电路和参数关系：



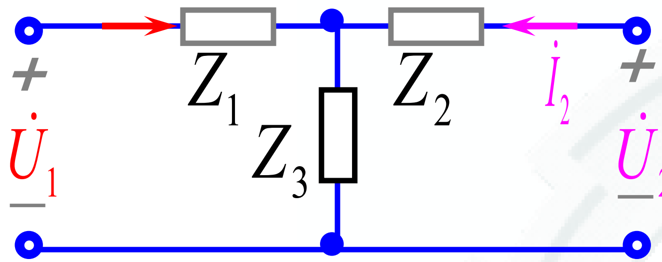
(b)

$$Y_{12}=Y_{21}$$

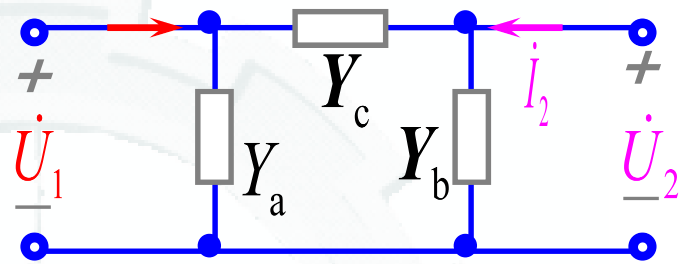
同理：对于(*b*)图，*π*型电路中各导纳值为：

$$Y_{11}=Y_1+Y_3\ ；\ Y_{12}=Y_{21}=-Y_3\ ；\ Y_{22}=Y_2+Y_3$$

$$\boxed{\Rightarrow Y_1=Y_{11}+Y_{12}\ ；\ Y_3=-Y_{21}=-Y_{12}\ ；\ Y_2=Y_{22}+Y_{21}}$$



(a)



(b)

若给定传输参数*A*，对于互易网络，得：

图(a) *T* 型：

$$\boxed{Z_1=\frac{A_{11}-1}{A_{21}}\;;\;Z_2=\frac{A_{22}-1}{A_{21}}\;;\;Z_3=\frac{1}{A_{21}}}$$

图(b) *π*型：

$$\boxed{Y_a=\frac{A_{22}-1}{A_{12}}\;;\;Y_b=\frac{A_{11}-1}{A_{12}}\;;\;Y_c=\frac{1}{A_{12}}}$$

如果二端口网络给定的是传输参数或混合参数，一般要将它们变换成阻抗参数或导纳参数，然后按上述方法求得T形或Π形等效电路。

对于对称二端口，因

$$Z_{11}=Z_{22} \qquad Y_{11}=Y_{22}$$

$$Z_1=Z_2 \qquad Y_1=Y_2$$

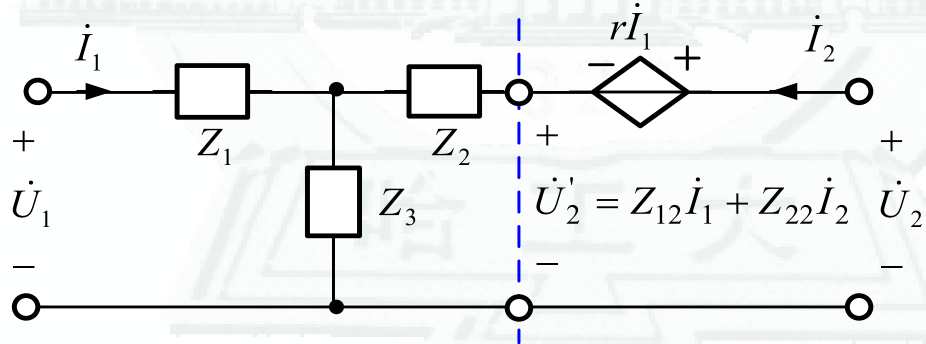
即它的T形和Π形等效电路也必定是对称的.

如果二端口是非互易的，则有4个独立参数。若给定二端口的Z参数为

$$\left.\begin{aligned}\dot{U}_1 &= Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 &= Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2\end{aligned}\right\}$$

$$\left.\begin{aligned}\dot{U}_1 &= Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ U_2 &= Z_{12}\dot{I}_1 + Z_{22}\dot{I}_2 + (Z_{21} - Z_{12})\ \dot{I}_1\end{aligned}\right\}$$

在方程中虚线的左侧仍是一个互易性二端口的表达式，可用上述T形电路来代替；而虚线右部分，则是一个电流控制电压源。



(a)

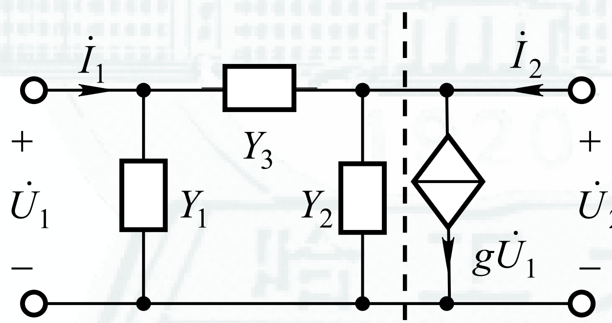
$$r = Z_{21} - Z_{12}$$

10.3 二端口网络等效电路

给定非互易二端口Y参数，在方程中虚线的左侧仍是一个互易性二端口的表达式，可用上述π型电路来代替；而虚线右部分，则是一个电压控制电流源。

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

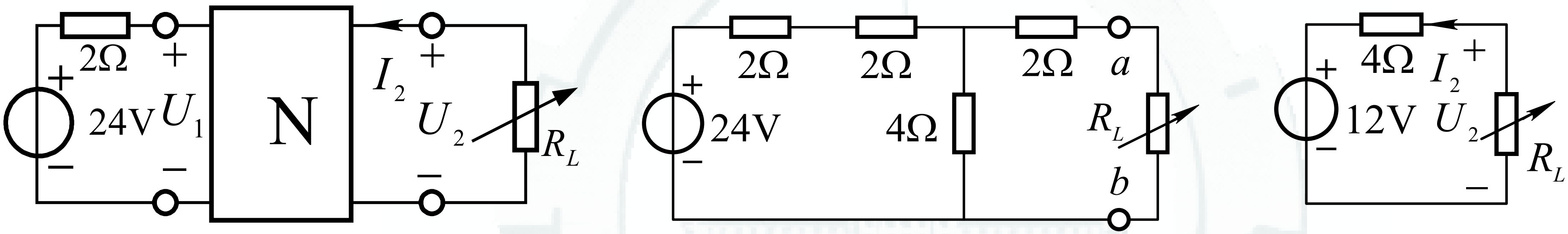
$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{12}\dot{U}_1 + Y_{22}\dot{U}_2 + (Y_{21} - Y_{12})\dot{U}_1 \end{cases}$$



(b)

$$g = Y_{21} - Y_{12}$$

例：图(a)所示二端口网络N的阻抗参数矩阵为 $Z=\begin{bmatrix}6 & 4\\4 & 6\end{bmatrix}\Omega$ ，求R_L何值时可获得最大功率，并求出此功率。



(a) (b) (c)

【解】 方法一，将二端口网络用T形电路等效，如图 (b)

$$U_{\mathrm{OC}}=\frac{4}{4+2+2}\times 24\mathrm{V}=12\mathrm{V}$$

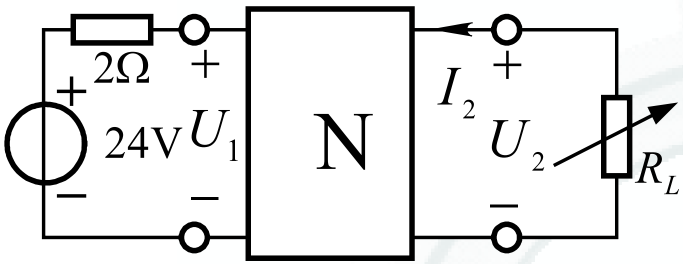
$$R_{\mathrm{i}}=\frac{1}{2}\times 4\Omega+2\Omega=4\Omega$$

戴维南等效电路如图(c)

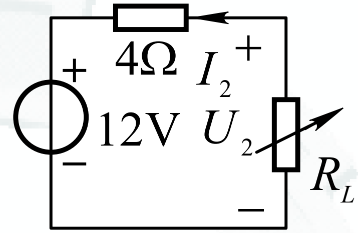
$$R_L=4\Omega$$

时它可获得最大功率

$$P_{\max}=\frac{U_{\mathrm{OC}}^2}{4R_{\mathrm{i}}}=\frac{12^2}{4\times 4}=9\mathrm{W}$$



(a)



(c)

方法二，由二端口参数和端口条件得出戴维南等效电路。

$$\boldsymbol{Z}=\begin{bmatrix}6 & 4\\ 4 & 6\end{bmatrix}\Omega$$

$$U_1=24\text{V}-2\Omega\times I_1=6\Omega\times I_1+4\Omega\times I_2 \longrightarrow I_1=3\text{A}-0.5I_2$$

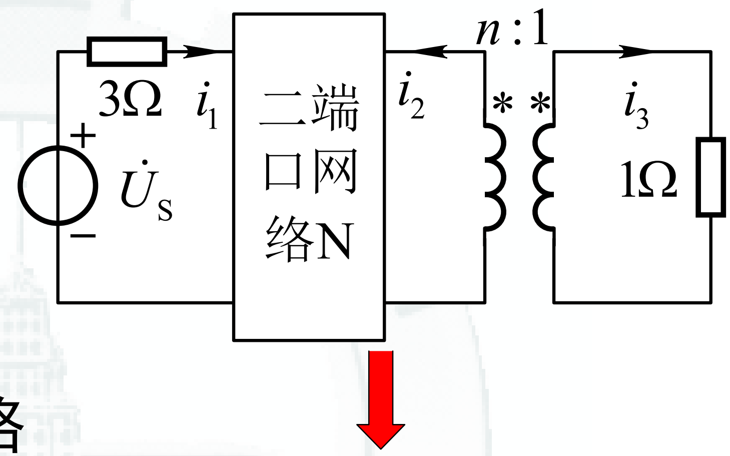
$$U_2=4\Omega\times I_1+6\Omega I_2 \longrightarrow U_2=12\text{V}+4\Omega I_2$$

例：已知$\dot{U}_S = 20\angle 0°\text{V}$　$n = 2$

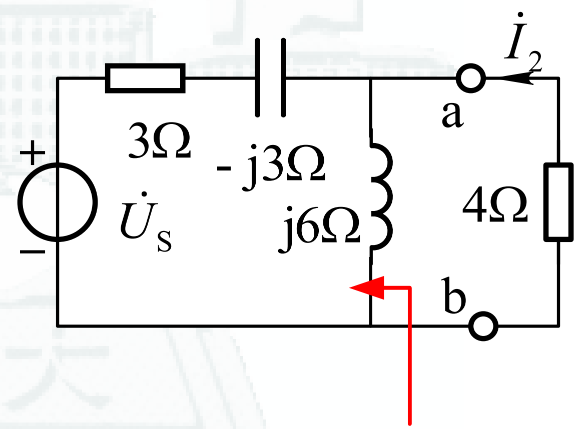
二端口网络N的阻抗参数矩阵

$$Z = \begin{bmatrix} \text{j}3 & \text{j}6 \\ \text{j}6 & \text{j}6 \end{bmatrix}\Omega$$

求电流 i_3



【解】　求ab端左侧的戴维南等效电路



$$\dot{U}_{OC} = \frac{\text{j}6}{\text{j}6 + 3 - \text{j}3} \times \dot{U}_S = 20\sqrt{2}\angle 45°\text{V}$$

$$Z_o = \frac{\text{j}6 \times (3 - \text{j}3)}{\text{j}6 + 3 - \text{j}3} = 6\Omega$$

$$\dot{I}_2 = -\frac{\dot{U}_{OC}}{4 + Z_o} = -\frac{20\sqrt{2}\angle 45°}{4 + 6} = -2\sqrt{2}\angle 45°\text{A}$$

$$\dot{I}_3 = n(-\dot{I}_2) = 4\sqrt{2}\angle 45°\text{A}$$

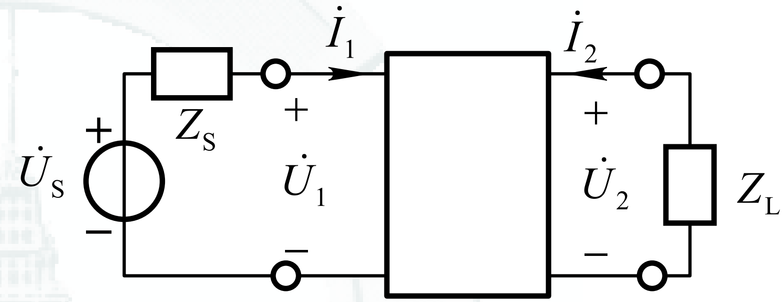
- 给定电路参数，求端口电压和电流

约束端口的方程有

1. 二端口参数方程

2. 电源支路方程

3. 负载支路方程



在负载上和参考方向相反

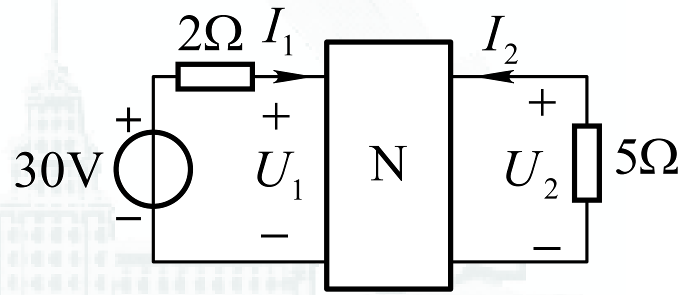
$$\left.\begin{aligned}
\dot{U}_1 &= A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2) \\
\dot{I}_1 &= A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2) \\
\dot{U}_1 &= \dot{U}_S - Z_S\dot{I}_1 \\
\dot{U}_2 &= -Z_L\dot{I}_2
\end{aligned}\right\}$$

联立求解 → 得到端口电压电流

例：已知二端口导纳参数矩阵

$$\mathbf{Y}=\begin{bmatrix}0.5 & -0.5\\ -0.3 & 0.4\end{bmatrix}\mathrm{S}$$

求端口电流 I_1 和 I_2



【解】 由Y参数可得

$$I_1=0.5U_1-0.5U_2$$

$$I_2=-0.3U_1+0.4U_2$$

由端口特性得

$$U_1=30-2I_1 \quad\Rightarrow\quad I_1=15-0.5U_1$$

$$I_2=-0.2U_2$$

$$15-0.5U_1=0.5U_1-0.5U_2$$

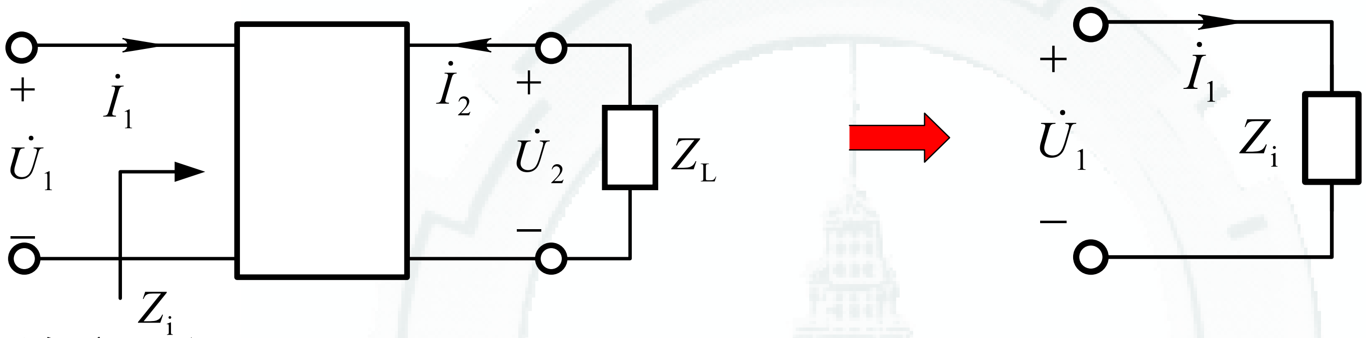
$$-0.2U_2=-0.3U_1+0.4U_2$$

解得 $U_1=20\mathrm{V}$ $\quad U_2=10\mathrm{V}$

$I_1=5\mathrm{A}$ $\quad I_2=-2\mathrm{A}$

10.4 二端口网络与电源和负载的连接

➢ 输入阻抗



- 给定*A*参数

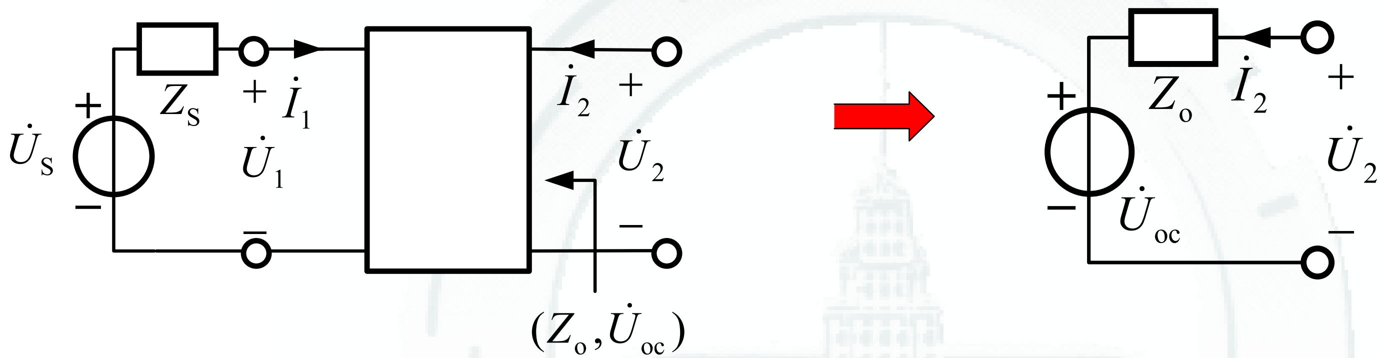
$$Z_i=\frac{\dot{U}_1}{\dot{I}_1}=\frac{A_{11}\dot{U}_2-A_{12}\dot{I}_2}{A_{21}\dot{U}_2-A_{22}\dot{I}_2}=\frac{A_{11}(-Z_L\dot{I}_2)-A_{12}\dot{I}_2}{A_{21}(-Z_L\dot{I}_2)-A_{22}\dot{I}_2}=\frac{A_{11}Z_L+A_{12}}{A_{21}Z_L+A_{22}}$$

- 给定*Z*参数

$$Z_i=\frac{\dot{U}_1}{\dot{I}_1}=Z_{11}-\frac{Z_{12}Z_{21}}{Z_L+Z_{22}}$$

输入阻抗不仅与二端口网络参数有关，且与负载阻抗有关；二端口具有变换阻抗的作用。

➢ 输出端的戴维南等效电路



$$\begin{cases} \dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2) \\ \dot{U}_1 = \dot{U}_S - Z_S\dot{I}_1 \end{cases} \qquad \dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$$

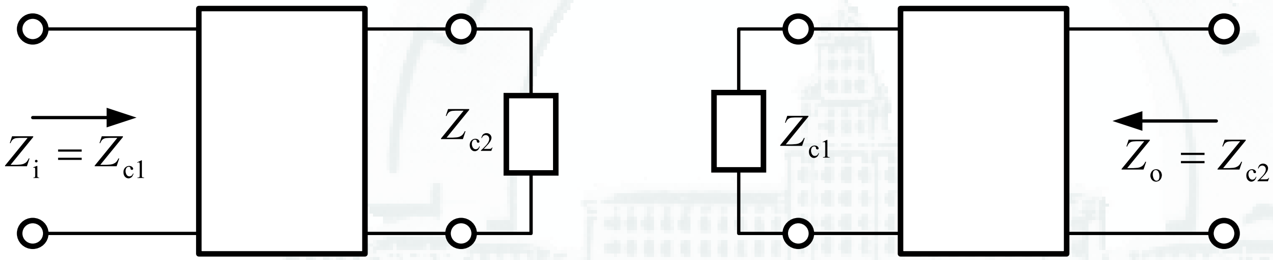
$$\Longrightarrow \dot{U}_2 = \frac{\dot{U}_S}{A_{21}Z_S + A_{11}} + \frac{A_{22}Z_S + A_{12}}{A_{21}Z_S + A_{11}}\dot{I}_2 = \dot{U}_{oc} + Z_o\dot{I}_2$$

$$\boxed{\dot{U}_{oc} = \frac{\dot{U}_S}{A_{21}Z_S + A_{11}} \qquad Z_o = \frac{\dot{U}_2}{\dot{I}_2}\Big|_{U_S=0} = \frac{A_{22}Z_S + A_{12}}{A_{21}Z_S + A_{11}}}$$

➢ 特性阻抗

接电源和负载的二端口网络要求 $Z_i = Z_S \quad Z_o = Z_L$

设给定二端口参数，设 $Z_S = Z_{c1} \quad Z_L = Z_{c2}$



$$Z_i = Z_{c1} = \frac{A_{11}Z_{c2} + A_{12}}{A_{21}Z_{c2} + A_{22}} \qquad Z_o = Z_{c2} = \frac{A_{22}Z_{c1} + A_{12}}{A_{21}Z_{c1} + A_{11}}$$

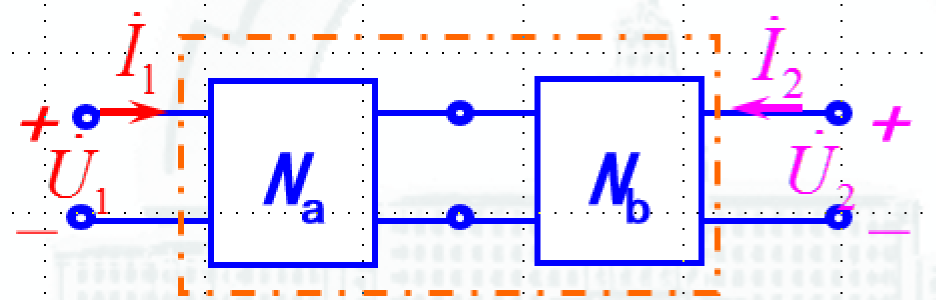
$$\Rightarrow \quad Z_{c1} = \sqrt{\frac{A_{11}A_{12}}{A_{21}A_{22}}} \qquad Z_{c2} = \sqrt{\frac{A_{22}A_{12}}{A_{11}A_{21}}}$$

Z_{c1} Z_{c2} 分别称为输入端口和输出端口的特性阻抗

当 $Z_S = Z_{c1} \quad Z_L = Z_{c2}$时，称二端口网络与电源和负载匹配连接

➢ 二端口网络的级联

1. 概念：一个二端口络的输出端与另一个网络的输入端相连的联接方式。

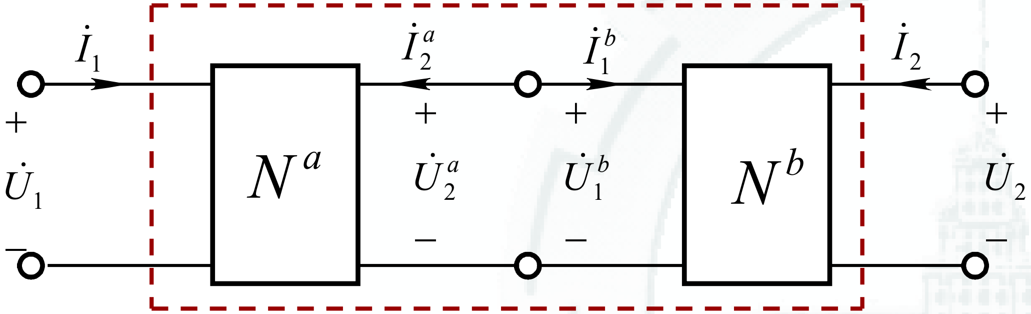


2. 结论：

级联后的复合二端口网络

$$\boxed{T = T_a \bullet T_b}$$

级联是第一个二端口的输出端口与第二个二端口的输入端口相联，如图所示



$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} A_{11}^a & A_{12}^a \\ A_{21}^a & A_{22}^a \end{bmatrix}\begin{bmatrix} A_{11}^b & A_{12}^b \\ A_{21}^b & A_{22}^b \end{bmatrix}$$

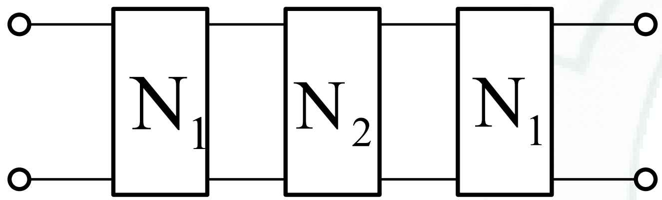
$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} A_{11}^a & A_{12}^a \\ A_{21}^a & A_{22}^a \end{bmatrix}\begin{bmatrix} \dot{U}_2^a \\ -\dot{I}_2^a \end{bmatrix}$$

$$\begin{bmatrix} \dot{U}_2^a \\ -\dot{I}_2^a \end{bmatrix} = \begin{bmatrix} \dot{U}_1^b \\ \dot{I}_1^b \end{bmatrix}$$

$$\begin{bmatrix} \dot{U}_1^b \\ \dot{I}_1^b \end{bmatrix} = \begin{bmatrix} A_{11}^b & A_{12}^b \\ A_{21}^b & A_{22}^b \end{bmatrix}\begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \boxed{\begin{bmatrix} A_{11}^a & A_{12}^a \\ A_{21}^a & A_{22}^a \end{bmatrix}\begin{bmatrix} A_{11}^b & A_{12}^b \\ A_{21}^b & A_{22}^b \end{bmatrix}}\begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

例：图示三个二端口网络级联，已知N_1的阻抗参数矩阵为$Z_1=\begin{bmatrix}6 & 2\\ 2 & 6\end{bmatrix}\Omega$，$N_2$的传输参数矩阵为$A_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$。求复合二端口网络传输参数。



【解】

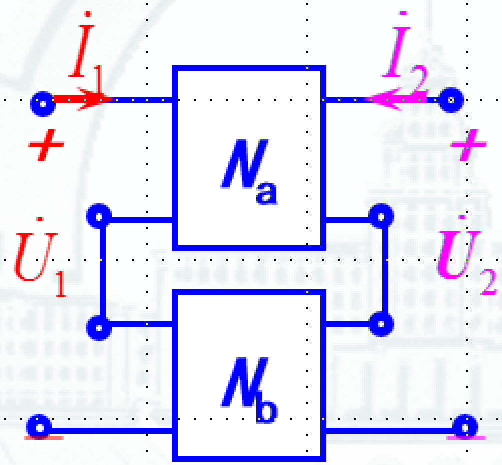
$$\begin{cases}u_1=6i_1+2i_2 & (1)\\ u_2=2i_1+6i_2 & (2)\end{cases}\Rightarrow\begin{cases}u_1=3u_2+16(-i_2)\\ i_1=0.5u_2+3(-i_2)\end{cases}$$

$$A_1=\begin{bmatrix}3 & 16\Omega\\ 0.5\text{S} & 3\end{bmatrix}$$

$$A=A_1A_2A_1=\begin{bmatrix}3 & 16\\ 0.5 & 3\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 16\\ 0.5 & 3\end{bmatrix}=\begin{bmatrix}17 & 96\\ 3 & 17\end{bmatrix}$$

➢ 二端口网络的串联

1. 概念： 两个二端口网络输入端口相互串联，输出端口也串联的联接方式。



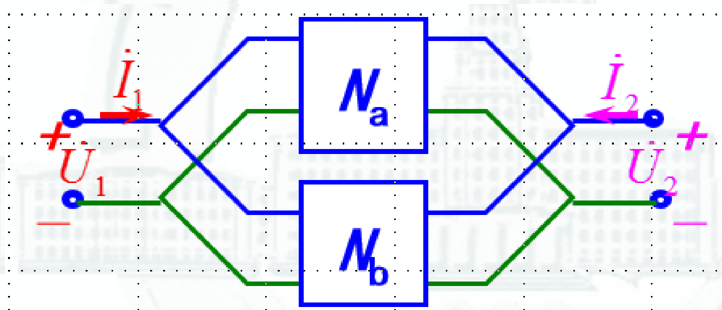
2. 结论：

串联后的复合二端口网络：

$$\boxed{Z = Z_a + Z_b}$$

➢ 二端口网络的并联

1．概念：两个二端口网络的输入端口并联，输出端口也并联的联接方式。



2．结论：

并联后的复合二端口网络：

$$\boxed{Y = Y_a + Y_b}$$

本章小结

- 二端口网络
  - 参数方程
    - 开路阻抗Z
短路导纳Y
参数方程
互易/对称条件
    - 传输A
混合H
参数方程
互易/对称条件
  - 等效电路
    - T型
Π型
互易网络
    - T型
Π型
非互易网络
  - 电源/负载
    - 确定二端口电压电流
    - 输入端口等效Z_i或
输出端口等效Z_o
戴维南等效电路
  - 二端口互联
    - 级联A
串联Z
并联Y

谢谢！

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