

Problem 9 starts as:

$$\frac{d}{dx} k(x) \frac{du}{dx} = f(x), \quad x \in (a, b)$$

Which can be expanded to:

$$k(x) \frac{d^2 u}{dx^2} + \frac{dk}{dx} \frac{du}{dx} = f(x)$$

Which can be expressed as:

$$k u'' + k' u' = f(x)$$

In centered finite difference (for x_j), where $k(x_j) = k_j$ and $U(x_j) = U_j$:

$$\frac{k_j}{h^2} (U_{j-1} - 2U_j + U_{j+1}) + \frac{(k_{j+1} - k_{j-1})}{2h} (U_{j+1} - U_{j-1}) \approx f_j$$

The [A] matrix then is a tridiagonal matrix, where the main diagonal is:

$$\frac{-2 k_j}{h^2}$$

The superdiagonal is:

$$\frac{k_j}{h^2} + \frac{(k_{j+1} - k_{j-1})}{2h}$$

The subdiagonal is:

$$\frac{k_j}{h^2} - \frac{(k_{j+1} - k_{j-1})}{2h}$$

And finally, the f_j vector is modified by:

$$f_0 \rightarrow f_0 - u_a \left(\frac{k_1}{h^2} - \frac{(k_2 - k_0)}{2h} \right)$$

and

$$f_{n-1} \rightarrow f_{n-1} - u_b \left(\frac{k_{n-1}}{h^2} + \frac{(k_n - k_{n-2})}{2h} \right)$$