# Circuits Lab 1

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February 2025

## 1 Experiment 1: Resistance Measurement

We selected a nominally 11kΩ resistor for this experiment. When measured with the Keithly Source Unit we recorded a value of 10.90kΩ. Using the SMU we obtained the Current Voltage Characteristic Curve (Figure 1).

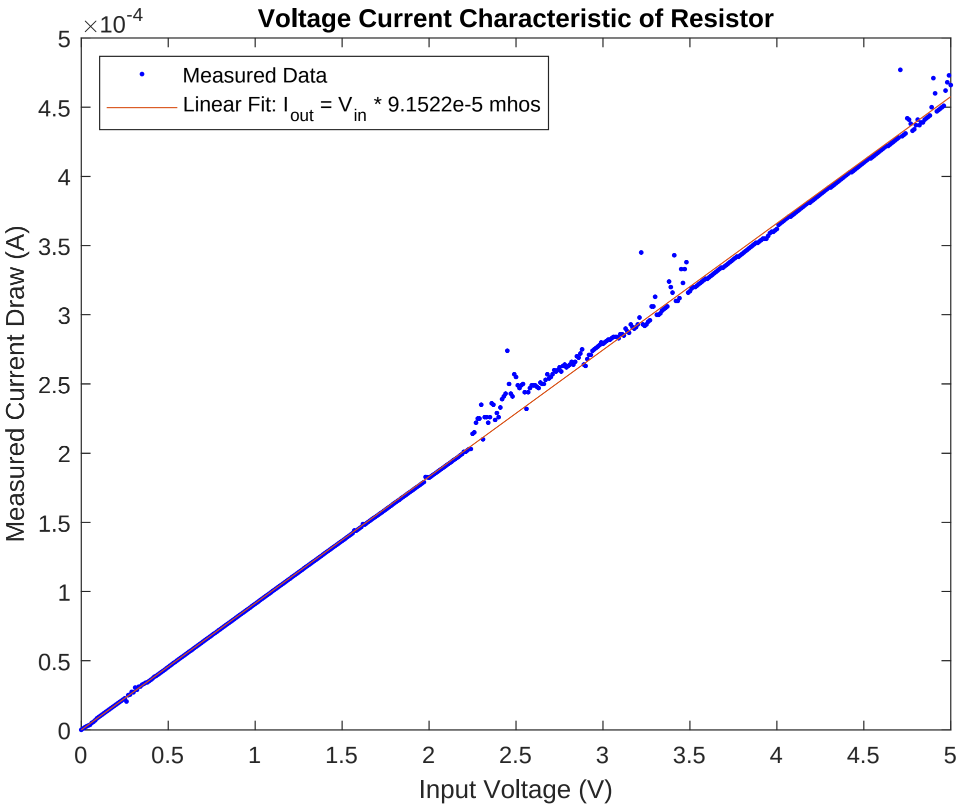


Figure 1: Voltage Current Characteristic obtained by sweeping input voltages from 0 to 5 volts from one SMU channel and measuring the current drawn by the resistor via the other SMU channel.

When fitted linearly, the transfer curve has a slope of 9.1522e-05 mhos. To determine a resistance we can rearrange ohms law:

$$V = IR \tag{1}$$

$$I = V * 1/R \tag{2}$$

Therefore the inverse of our fitted slope yields the measured resistance:

$$R = 1/(9.1522 \times 10^{-5} \text{ mhos}) = 10.926\text{k}\Omega$$

These three resistance values are fairly close in value, with the two experimental values having a percent difference of .23%. Both measurements are below the nominal resistance by .67% (Via SMU) and .9% (Via Keethly). The deviation from the nominal resistance is well within the manufacturer tolerance of 2%. One possible explanation for the discrepancy is the data collected via the SMU, which measured several outliers in the 2-3V input voltage range. This was consistent over multiple runs, suggesting some part of the SMU design results in inconsistent measurements in certain voltage ranges. The Keethly on the other hand, obtains a measurement by sending a single test current and provides a single measurement to interpret. While the difference between the Keethly and SMU measurements is small, we believe that the Keethly is the more accurate as the measurement doesn't include many outliers due to the SMU sweeping voltages.

One potential explanation for why the Keethly has trouble measuring voltages just above and below 3.3V and below 5V is that the internal voltage measurement circuits may be switching reference voltages to prevent hitting a rail. A brief examination of the ADS129 IC used in the measuring circuit on the SMU suggests this is plausible, with the functional block diagram shows some switching occurring in the amplification circuitry but further examination of the measuring device would be required to understand the drawbacks of utilizing it. Beyond the specifics, the Keethly is designed to be used as a high-precision measurement device for commercial use, so we would in general place higher faith in its measurements than a more home-brewed instrumentation package.

# 2 Experiment 2: Resistive Voltage Division

To prepare for creating our voltage divider, current divider, and R-2R ladder circuit, we obtained two Bourns resistor array chips and measured each of the 16 resistors using the same Keithley 2400 Source Meter as earlier. The theoretical value for each resistor in the array is 10k ohms, while the measured values are recorded in Table 1.

| **Resistor #** | **Array 1 Resistances (kΩ)** | **Array 2 Resistances (kΩ)** |
|---|---|---|
| Resistor 1 | 9.9932 | 9.9789 |
| Resistor 2 | 9.9723 | 9.9549 |
| Resistor 3 | 9.9548 | 9.9573 |
| Resistor 4 | 9.9717 | 9.9628 |
| Resistor 5 | 9.9563 | 9.9550 |
| Resistor 6 | 9.9452 | 9.9605 |
| Resistor 7 | 9.9482 | 9.9461 |
| Resistor 8 | 9.9612 | 9.9729 |

Table 1: A table with the resistances of each of the eight resistros in our two Bourns resistor array chips. Ideal resistance is 10kΩ with a ±2% tolerance.

Within both chips, the resistors perform within the expected ±2% tolerance. If we compare to our worse-case scenario, a resistor would be 2% above or below the 10kΩ resistance; given that 2% of $10,000\Omega$ is $200\Omega$, this would mean the worst-case scenario for the resistors would be 9.8kΩ or 10.2kΩ. As all of our resistors are between 9.94kΩ and 10.00kΩ, they are all within tolerance.

Between chips, we see very similar shifts in tolerance. In fact, in every resistor we measured, we found a slight tolerance error that brought the resistance slightly down from it's ideal resistance. Although this may not have a large effect over a single resistor, since each of the resistors measure below 10kΩ (compared to a mix of above and below 10kΩ), a longer chain of these resistors will demonstrate more severe error.

Next, we used the first of our Bourns arrays to create a voltage divider by connecting two resistors in series (Figure 2). Since both resistors are ideally 10kΩ, the ideal voltage ratio is 0.5. We can test this by applying varying input voltages across the divider and measuring the voltage output. By taking the ratio of output voltage to input voltage, or the slope of the graph of output over input, we can measure the ratio of the voltage divider (Figure 3).

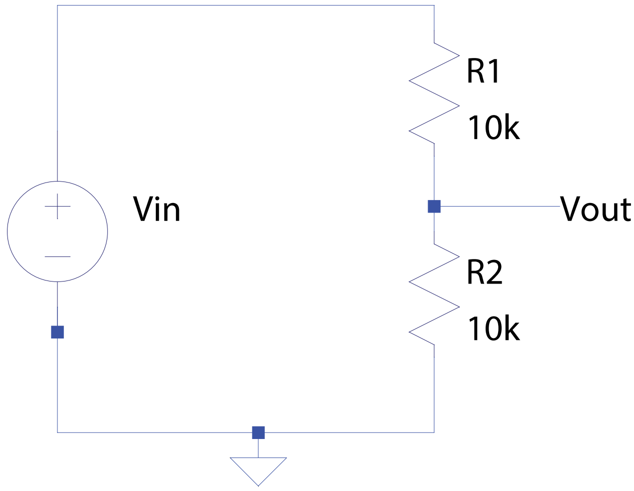


Figure 2: A 0.5 ratio voltage divider circuit created using the first two 10kΩ 2% solid film resistors from Bourns Array 1. Schematic created using LTSpice.

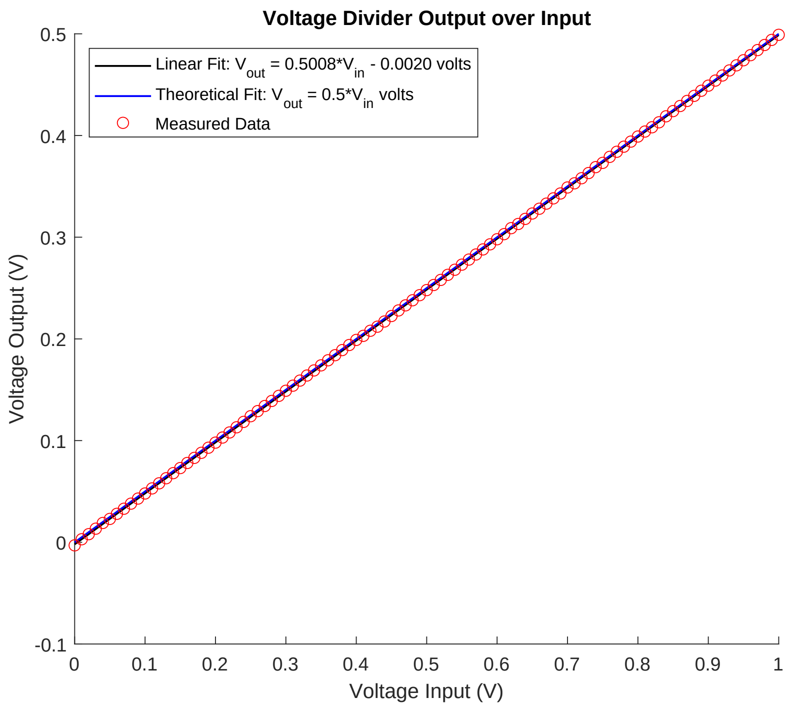


Figure 3: Measured Voltage Output (V) over Voltage Input (V) across a 0.5 ratio voltage divider created using two 10kΩ 2% solid film resistors. Obtained by sweeping input voltages from 0 to 1 volt from one SMU channel and measuring the voltage across R2 via the other SMU channel.

Our measured divider ratio (0.5008) compares well to our ideal ratio (0.5000), with only a 0.16% error due to tolerances. This discrepancy is consistent with our measured voltages from the Bourns array: Ratio = 9.9932kΩ/(9.9932kΩ + 9.9723kΩ) = 0.50053. Compared to our measured divider ratio, there is only a 0.05% inconsistency.

# 3 Experiment 3: Resistive Current Division

We constructed a 2-way Resistive Current Divider using 2 of the nominally 10kΩ resistors to form a divider with a theoretical divider ratio of 1/2. When built from imperfect components we observed the following current transfer characteristics.

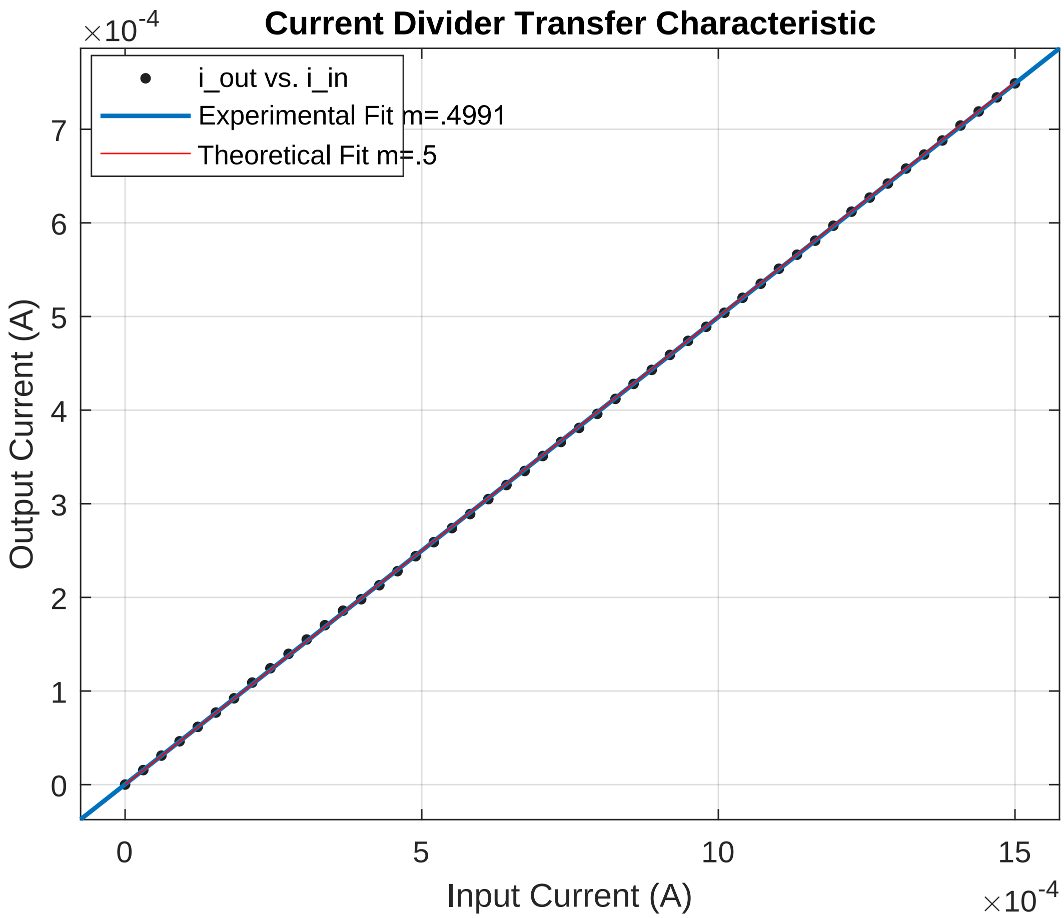


Figure 4: Current Divider Transfer Characteristic obtained using SMU and two 2% solid film resistors

The fitted slope of the data indicates a divider ratio of .4991 a difference of .18%. The divider was made out of R1 and R2 on chip one, which had measured Resistances of 9.9932kΩ and 9.9723kΩ respectively. When the divider ratio is calculated from measured instead of theoretical resistor values a divider ratio of .49947 is found. This leaves a measurement error of .007%. This supports the discrepancy between theoretical and experimental divider ratios being a result of resistance mismatch in the resistor array. The remaining measurement error is insignificant when the resistor tolerance is accounted for.

# 4 Experiment 4: R-2R Ladder Network

For our fourth experiment, we created a R-2R ladder network using parallel resistors as described in the prelab. We used 13 of the 16 resistors in our Bourns resistor array chips to build a ladder network with four branches, which simplifies to the R-2R network shown as in Figure 5.

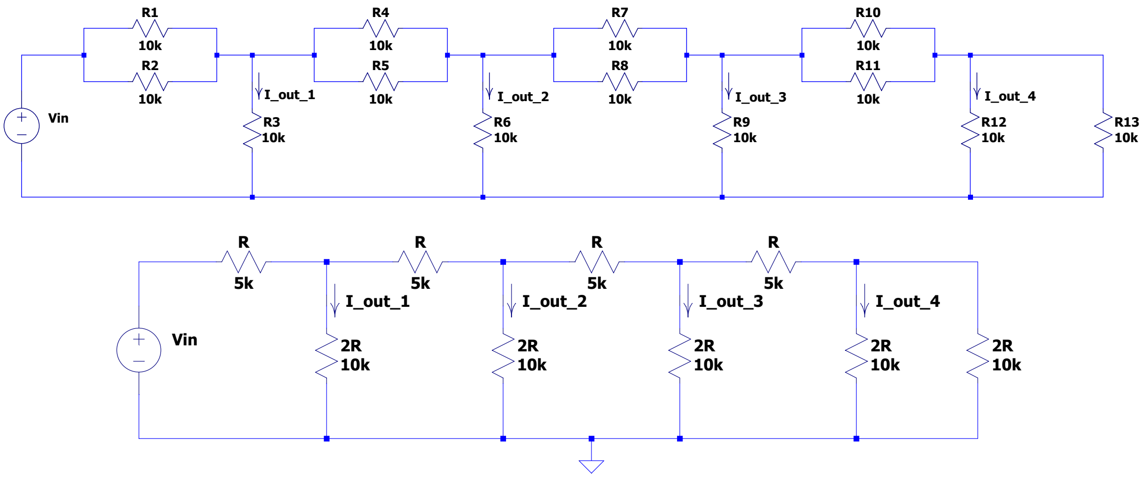


Figure 5: Ladder network built using 10kΩ resistors from the Bourns resistor chip. Current was measured at each of the I out points.

In order to experimentally test our circuit, we measured current at each of the different branches (at the I out points denoted in Figure 5). We measured these currents with input voltages of 1V and 2V.

| Bridge Measured | Current at 1V (A) | Current at 2V (A) |
|---|---|---|
| Iout 1 | $3.37 * 10^{-5}$ | $6.09 * 10^{-5}$ |
| Iout 2 | $1.69 * 10^{-5}$ | $3.35 * 10^{-5}$ |
| Iout 3 | $8.45 * 10^{-6}$ | $1.68 * 10^{-5}$ |
| Iout 4 | $4.29 * 10^{-6}$ | $8.44 * 10^{-6}$ |

Table 2: A table with the currents taken at each of the points specified in Figure 5 at 1V and 2V of input voltage. Current reading was taken using SMU with one channel supplying voltage and the other channel measuring current.

Now that we have our measured values, we need to calculate our theoretical values. We know from the prelab that the current through a branch of an R-2R circuit can be determined by resistance $R$, voltage $V$, and branch number $n$ according to the formula

$$I_{out} = \frac{V_{in}}{2^{n+1}R}.$$

Our ideal R is two 10kΩ resistors in parallel, or an equivalent voltage of 5kΩ. Our V is 1V or 2V. Given V and R, we can compute the theoretical currents in table 3.

| Bridge Measured | Theoretical Current at 1V (A) | Theoretical Current at 2V (A) |
|---|---|---|
| Iout 1 | $5.00 * 10^{-5}$ | $1.00 * 10^{-4}$ |
| Iout 2 | $2.50 * 10^{-5}$ | $5.00 * 10^{-5}$ |
| Iout 3 | $1.25 * 10^{-5}$ | $2.50 * 10^{-5}$ |
| Iout 4 | $6.125 * 10^{-6}$ | $1.25 * 10^{-5}$ |

Table 3: A table with the theoretical ideal taken at each of the points specified in Figure 5 at 1V and 2V of input voltage.

Now that we have both measured and theoretical values, we can plot these as a function of branch number to see how they compare to each other as well as change over time (Figure 6).

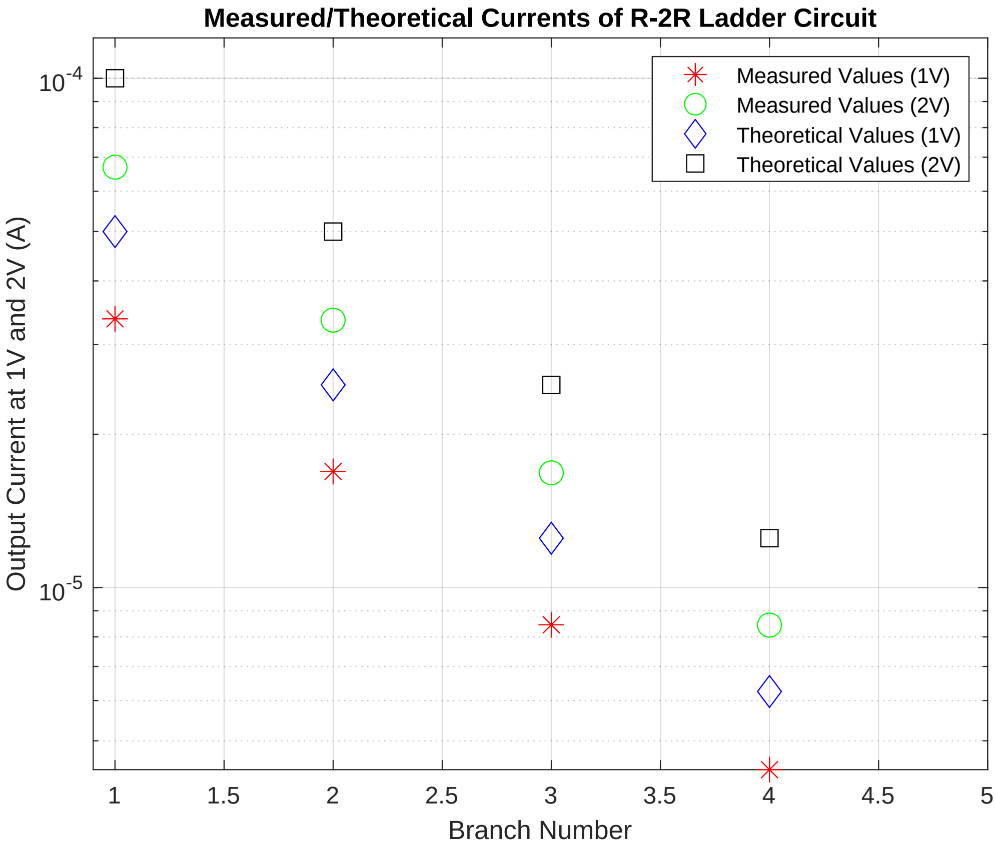


Figure 6: Measured and Theoretical Currents of the R-2R Ladder Circuit shown in Figure 5. Measured currents were taken using SMU.

As expected, the branch number and current output behavior linearly on a semilog plot. This is because the relationship between branch number and current is an exponential decay relationship defined by $I \propto (\frac{1}{2})^n$. On a semilog plot, the exponential becomes a negative linear slope.

However, what was unexpected was how different our theoretical values were to our measured values. After triple checking the circuit, we received similar results and concluded that the error must not be in the circuit. Using MATLAB's cftool, we can find the slopes of the semilog graph to be:

Measured Values (1V): m = -2.15e-05; Theoretical Values (1V): m = -3.21e-05;
Measured Values (2V): m = -4.29e-05; Theoretical Values (2V): m = -6.41e-05.

We can see that the slopes for 2V are correctly twice as steep as the slopes for 1V. However, the measured slopes are 39.5522% difference for 1V and 39.6262% difference for 2V. Because these measured differences are so close, we concluded that there was a consistent error in measuring our values; most likely, that our circuit had an error in it. After reinspecting our circuit, we could not find any error, leaving the reason for the differences unclear.