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Quantitative Engineering Analysis II
Project 1: Bridge of Doom

I. Finding Linear and Angular Velocity:

I. We started with the parametric equation of the curve:

$$r(u) = 0.3960\cos(2.65(u + 1.4))\hat{i} - 0.99\sin(u + 1.4)\hat{j}\ ,\ u \in [0, 3.2]$$

II. To find linear velocity, we took the magnitude of r(u) with respect to u:

$$|dr/du| \quad \sqrt{\frac{27531009\left|\sin\left(\frac{53u}{20}+\frac{371}{100}\right)\right|^2}{25000000}+\frac{9801\left|\cos\left(u+\frac{7}{5}\right)\right|^2}{10000}}$$

III. To find angular velocity, we took the cross of $\hat{T}$ with respect to the neato (the B frame) and $d\hat{T}/dt$ with respect to the ground (the G frame). This gave us the angular velocity for the Neato with respect to the ground frame.

$$\hat{T}_B = \frac{\frac{dr}{dt}}{\left|\frac{dr}{dt}\right|}\begin{pmatrix}-\frac{53\sin\left(\frac{53u}{200}+\frac{371}{100}\right)}{\sigma_1} & -\frac{50\cos\left(\frac{u}{10}+\frac{7}{5}\right)}{\sigma_1} & 0\end{pmatrix} \text{ where } \quad \sigma_1 = \sqrt{2500\cos\left(\frac{u}{10}+\frac{7}{5}\right)^2+2809\sin\left(\frac{53u}{200}+\frac{371}{100}\right)^2}$$

$${}^G\omega^B = \hat{T}_B \times \frac{d\hat{T}_B}{dt}_G = -\frac{3869\cos\left(\frac{33u}{200}+\frac{231}{100}\right)+1749\cos\left(\frac{73u}{200}+\frac{511}{100}\right)}{22472\left|\sin\left(\frac{53u}{200}+\frac{371}{100}\right)\right|^2+20000\left|\cos\left(\frac{u}{10}+\frac{7}{5}\right)\right|^2}$$

IV. Lastly, we can combine the linear velocity and angular velocity to find the velocity of our left and right wheels with the formulas:

Left Velocity = Linear Velocity - Angular Velocity * (Distance between Wheels/2)
Right Velocity = Linear Velocity + Angular Velocity * (Distance between Wheels/2)

$$\text{Left Wheel Velocity} = \frac{23697625\cos\left(\frac{33u}{200}+\frac{231}{100}\right)+10712625\cos\left(\frac{73u}{200}+\frac{511}{100}\right)+792\,{\sigma_1}^{3/2}}{400000\,\sigma_1}$$

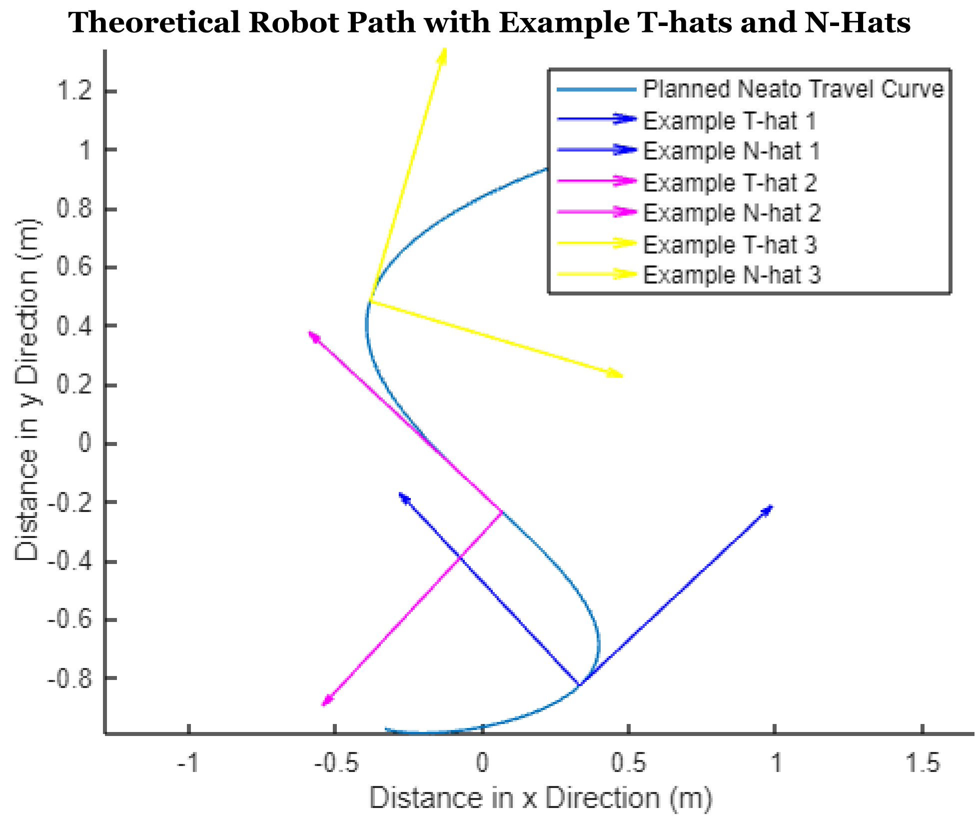
$$\text{Right Wheel Velocity} = -\frac{23697625\cos\left(\frac{33u}{200}+\frac{231}{100}\right)+10712625\cos\left(\frac{73u}{200}+\frac{511}{100}\right)-792\,{\sigma_1}^{3/2}}{400000\,\sigma_1}$$

$$\text{Where} \quad \sigma_1 = 2809\left|\sin\left(\frac{53u}{200}+\frac{371}{100}\right)\right|^2+2500\left|\cos\left(\frac{u}{10}+\frac{7}{5}\right)\right|^2$$

V. After analyzing our velocities, we chose u = t/10 in order to keep our velocities under 0.3 m/s. The final step was to run these velocities over a period of 32 seconds, $t \in [0, 32]$ or $u \in [0, 3.2]$.

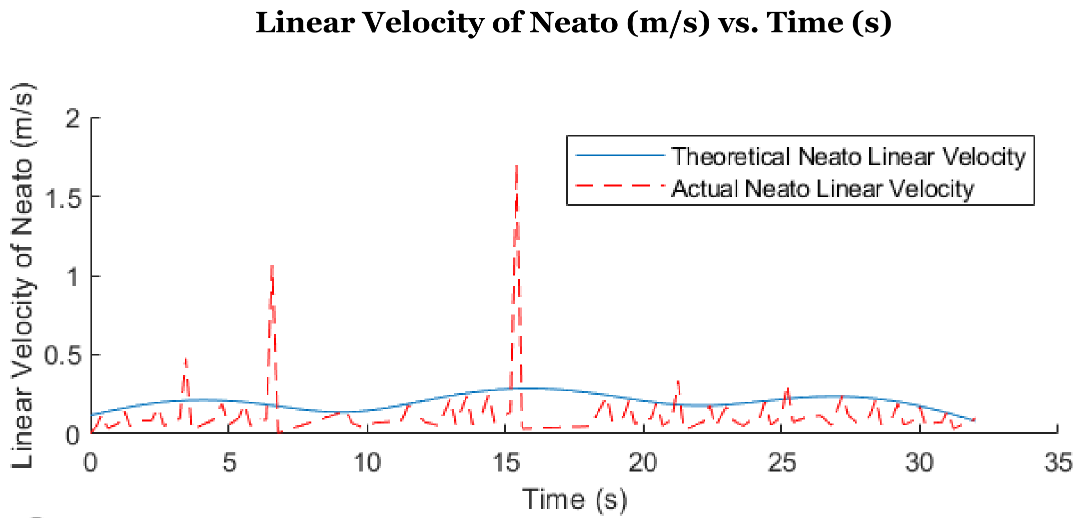
VI. As the robot went along the curve, constantly updating its wheel velocities, it recorded encoder data, which we used to calculate velocities. The difference in encoder distances between timesteps divided by the length of time between the recordings gives us velocity, which allows us to find the actual values for all of the above theoretical values.

II. Parametric Curve Plot:

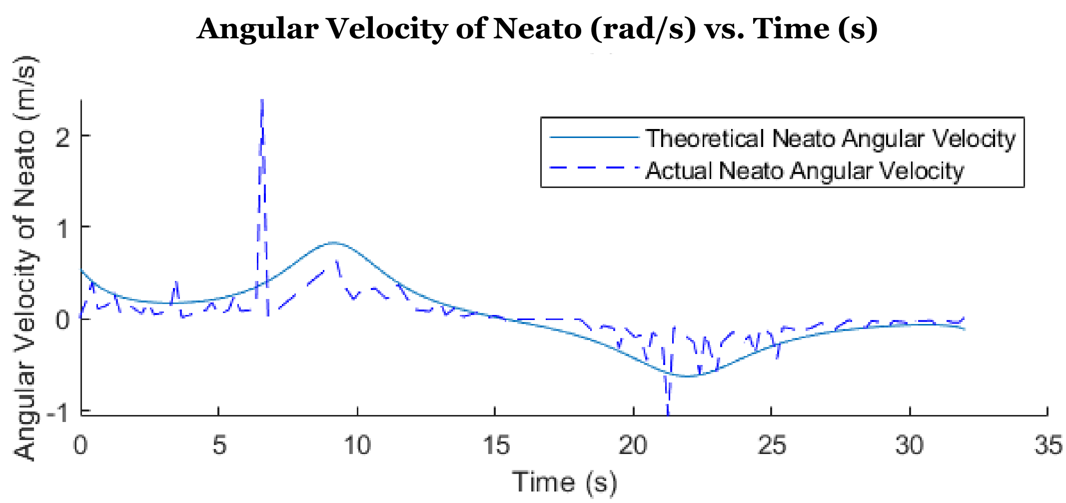


This is the theoretical travel curve of the Neato, along with the T-hat & N-hat vectors at three different points.

III. Graph of Linear and Angular Velocity as a Function of Time:



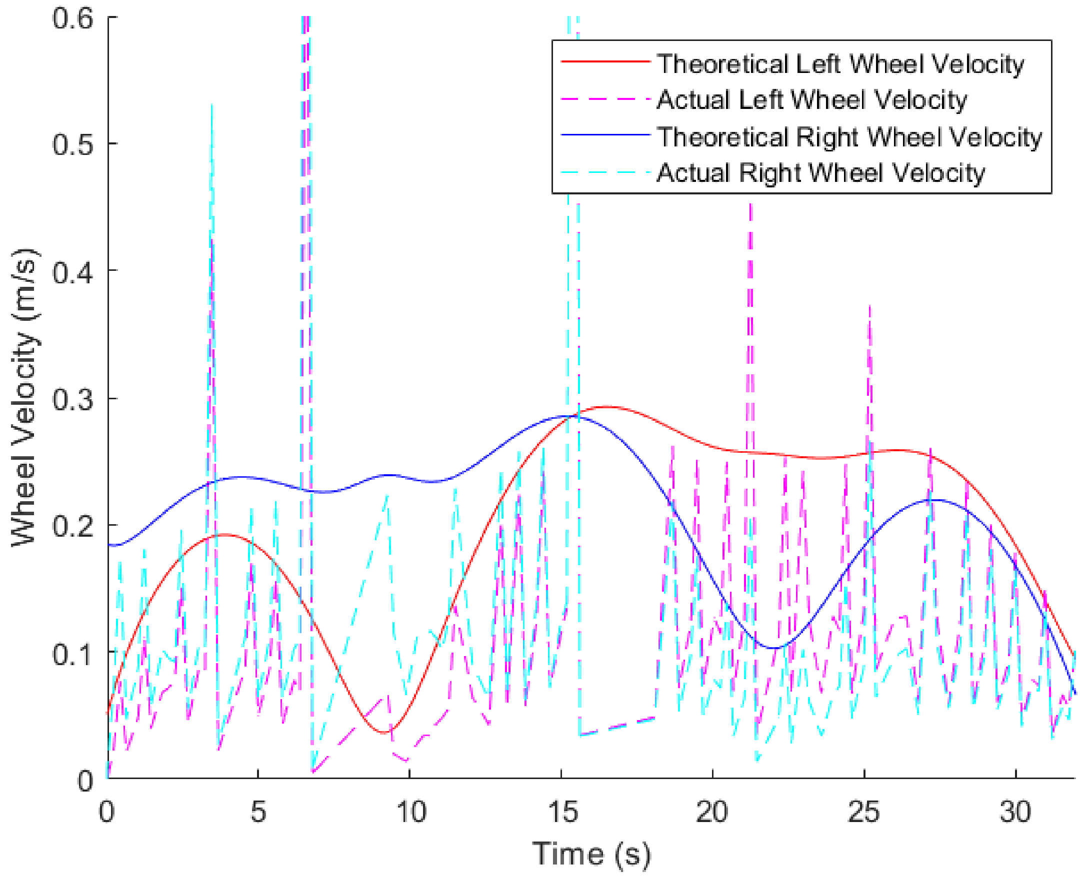
Linear Velocity of Neato vs. Time. The theoretical data was found using the derivative of position, and the actual data was found using the average difference in the encoder data over time.



Angular Velocity of Neato (rad/s) vs. Time (s). The theoretical data was found using the T-hat cross the derivative of T-hat with respect to time, and the actual data was found using the difference in the encoder data over time for the right wheel minus that of the left wheel, divided by distance between the wheels.

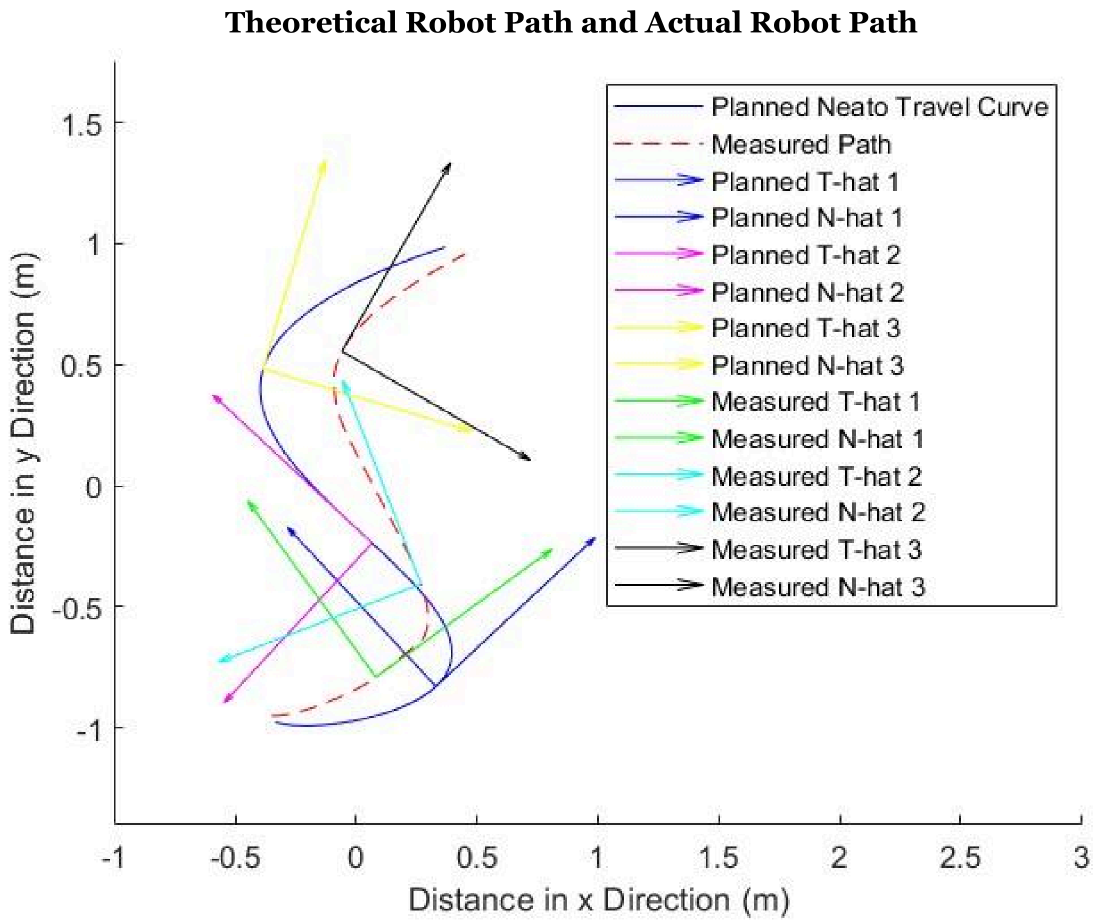
IV. Graph of Left and Right Wheel Velocity as a Function of Time

Left and Right Wheel Velocities vs. Time



Left and Right Wheel Velocities vs. Time. The theoretical data was found using the linear speed plus or minus half the distance between the wheels times angular speed, depending on if it were right or left wheel. The actual data was found using the difference in the encoder data over time for the right wheel or the left wheel.

V. Graph of Planned and Actual Path of Robot as a Function of Time



This is the theoretical path of the Neato, along with the actual path of the Neato. The theoretical path is found parametrically. The actual path of the Neato is found using the cumulative sum of the robot's position, changing at each timestep in the direction of T-hat by a distance of normal velocity times time elapsed.