Circuits Lab 2

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Experiment 1: Bipolar Transistor Characteristics

For this experiment, we characterized the behavior of the 2N3904 npn bipolar junction transistor (BJT) by measuring its base current and emitter current under a range of base voltages. The collector current is determined indirectly from the base and emitter currents. The collector is connected to a 5V power supply. The emitter is grounded.

A voltage source connected to the base is swept over a range that causes the emitter current to fluctuate over a range of roughly 10nA (10^{-8} A) to 10mA (10^{-2} A). One channel of the SMU measures the base current as the base voltage is being swept. The other channel of the SMU measures the emitter current. The experiment aims to study the transistor's current gain ($\beta = \frac{I_C}{I_B}$) and its behavior in the active region.

To begin, we can derive the collector voltage from the relation: $I_e = I_c + I_b$ amps, where $I_c = I_e - I_b$ amps. Now, we have the measured values of both I_b and I_c. To derive the theoretical values of the collector current, we can use the relation: $I_c = I_s e^{\frac{V_{be}}{U_T}}$ amps. We used the same U_T and I_s values that we derived in the previous lab ($U_T = 0.0267366096$ volts, $I_s = e^{-32.1846}$ amps). Because the emitter voltage is fixed at zero, V_{be}, which is equal to $V_b - V_e$, is equal to V_b. Given that we have the measured values of the base voltage, we can create a theoretical value for the collector current.

The theoretical base current can be derived from the following relation: $I_b = \frac{I_c}{\beta} = \frac{I_s}{\beta} e^{\frac{V_{be}}{U_T}}$ amps. The beta value can be determined as the ratio of the collector current to the base current. By averaging the ratio between the two values across different input base voltages, we determined a beta value of $\beta = 103.5373$. Now, we have a complete theoretical approximation of the base current too.

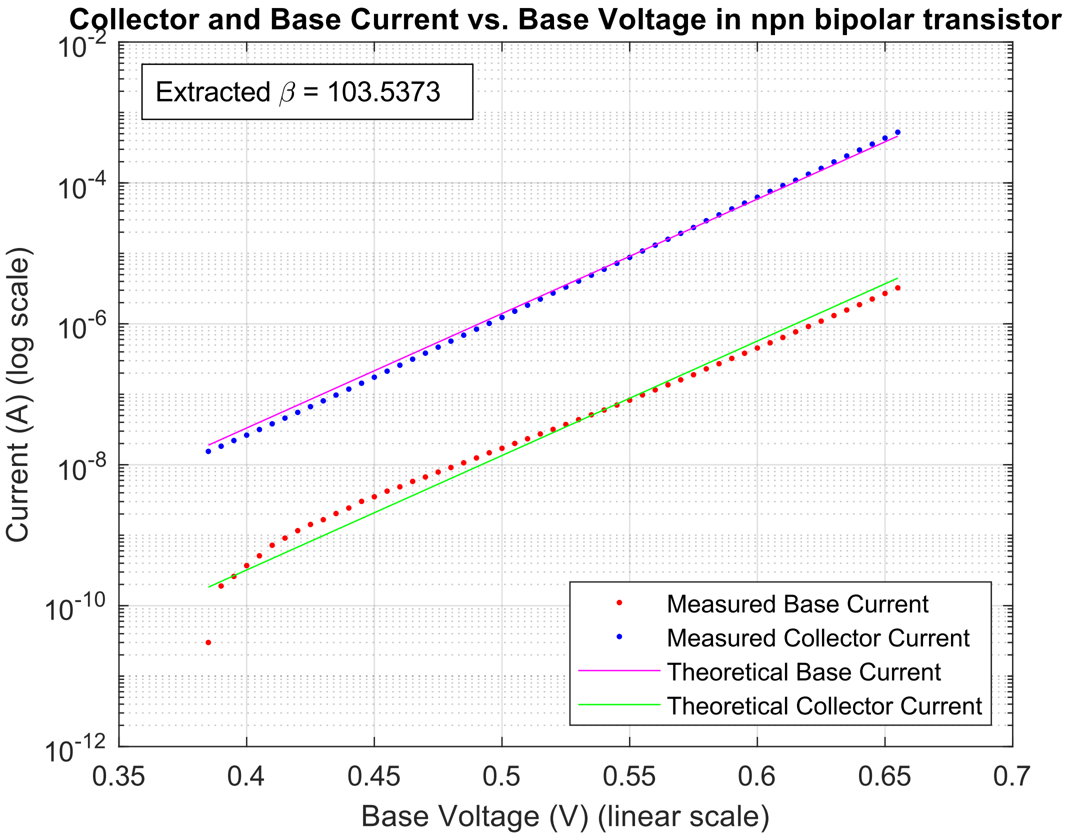

Figure 1: A semilog plot showing the measured and theoretical base and collector currents versus the base voltage of the npn BJT.

The theoretical and measured current values for the base and collector current match pretty well. Since the measured and theoretical values align, we can assume that the collector current also follows an exponential relationship with the base voltage.

As stated above, the current gain, otherwise known as beta, is defined as the ratio of the collector current to the base current, and can be calculated simply by dividing the collector current by the base current.

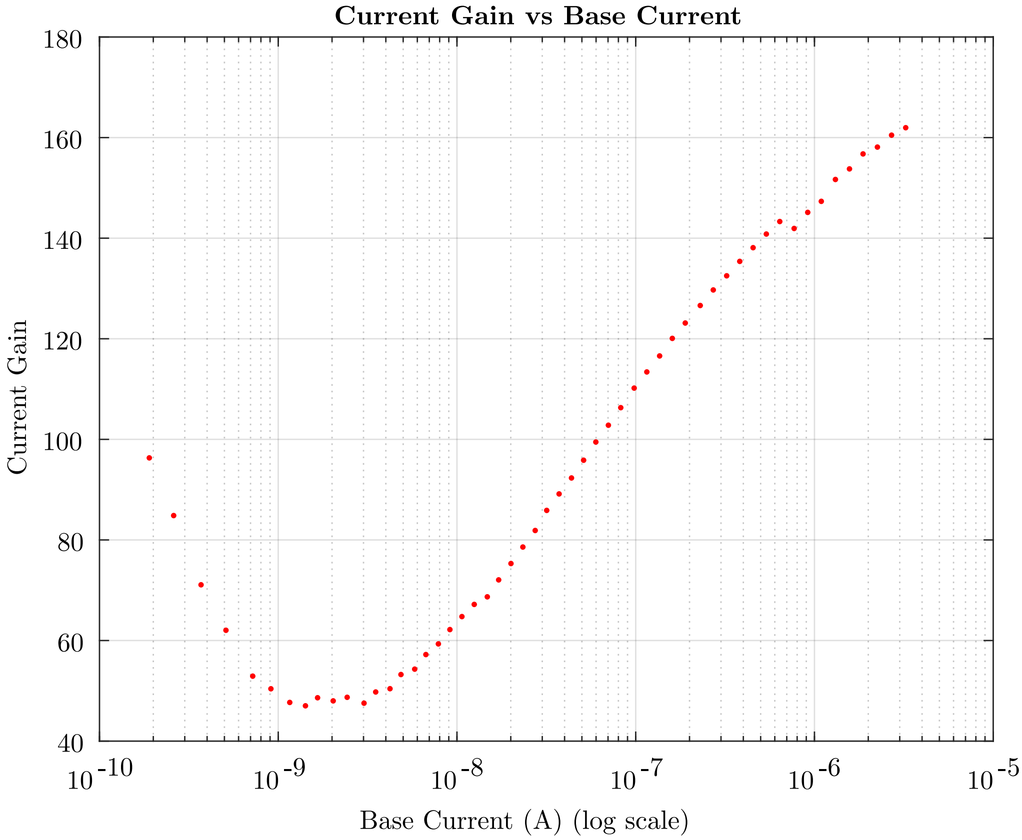

Figure 2: A semilog plot of the current gain's relationship to the base current of the npn BJT.

The current gain is clearly not constant with the base current, as there is a clear increase and fluctuation in the current gain as the base current increases. The decrease in gain at lower currents is due to recombination in the base-emitter depletion region. However, at more moderate currents, under stable conditions (stable temperature, active region), we can assume it to be constant.

We can extract the incremental base resistance of the base current-voltage characteristic by differentiating the base voltage with respect to the base current: $r_b = \frac{\delta V_b}{\delta I_b}$ ohms. The theoretical approximation of the incremental base resistance can also be derived as the ratio between the thermal voltage U_T and the base current: $r_{b_{\text{theoretical}}} = \frac{U_T}{I_b}$ ohms.

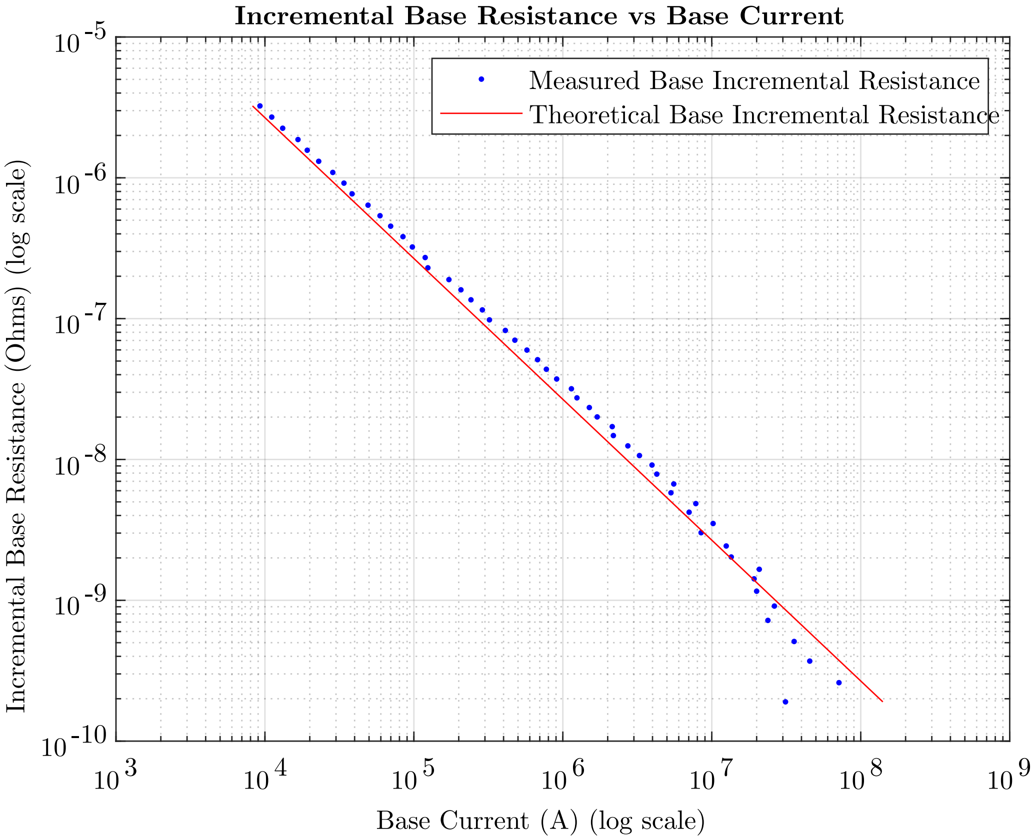

Figure 3: A log-log plot of the incremental base resistance versus the base current of the npn BJT.

The theoretical fit presents a good match with the measured data.

We can also derive the incremental transconductance gain, g_m, from the collector current-base voltage transfer characteristic, as the incremental transconductance gain is given by differentiating the collector current with respect to the base voltage, $g_m = \frac{\delta I_c}{\delta V_b}$ mhos. Additionally, we can derive a theoretical approximation of the incremental transconductance gain as the ratio between the collector current and the thermal voltage: $g_{m_{\text{theoretical}}} = \frac{I_c}{U_T}$ mhos.

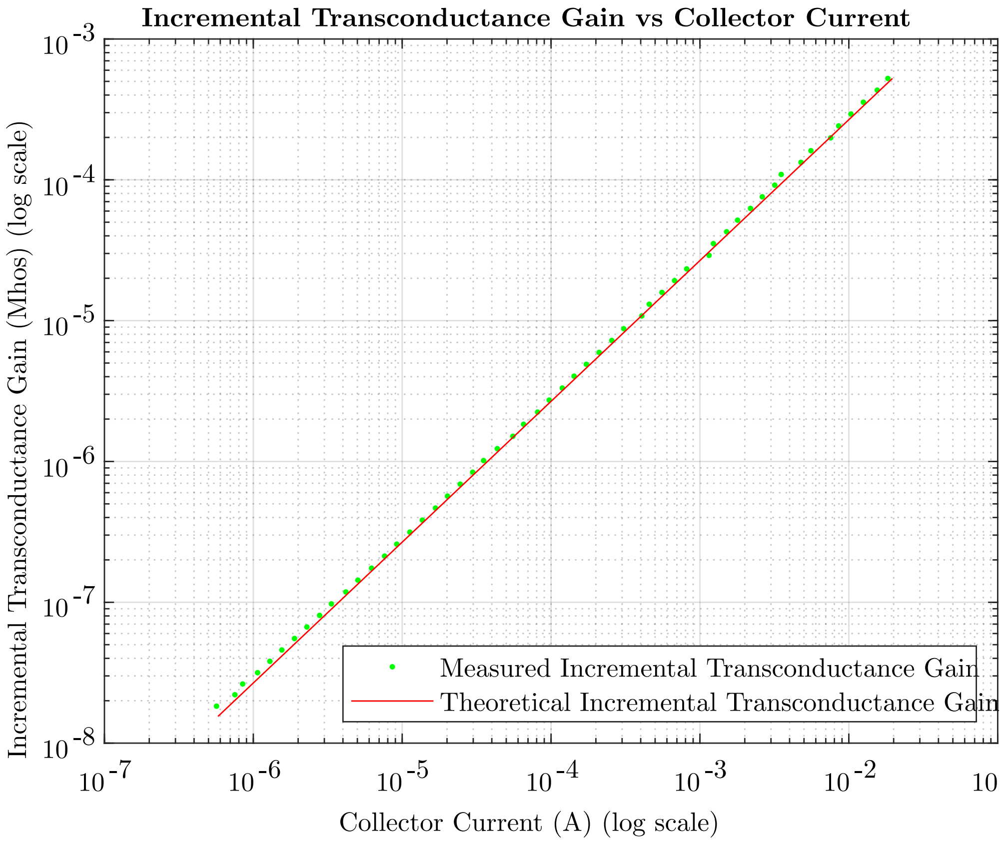

Figure 4: A log-log plot of the incremental transconductance gain versus the collector current of the npn BJT.

The theoretical fit of the incremental transconductance gain also presents a good match with the measured data.

Experiment 2: Emitter-Degenerated Bipolar Characteristics

For the next part of our experiment, we obtained three resistors of magnitude 3kΩ, 30kΩ, 300kΩ. We put each of these resistors, in turn, between the emitter of our 2N3904 and ground. This creates an emitter follower, a simple circuit where the emitter voltage (output) "follows" the base voltage (input).

We then used channel one of our SMU to measure the base current, sweeping the base voltage from 0V to +5V. We used channel two of the SMU to measure the current flowing through the emitter resistor, which is also the emitter current.

To examine the relationship between the input and output of the emitter follower, we can plot the collector current for as a function of base voltage for each of our resistors as shown in **Figure 5**. Additionally, we will compare this characteristic to the characteristic found in experiment one. As in experiment one, we found our collector current by subtracting our emitter current from our base current from the formula:

$$(I_e = I_c + I_b) \text{ amps} \quad \rightarrow \quad (I_c = I_e - I_b) \text{ amps}$$

Our theoretical fit/measured data for the transistor with no resistor are reused from Experiment 1.

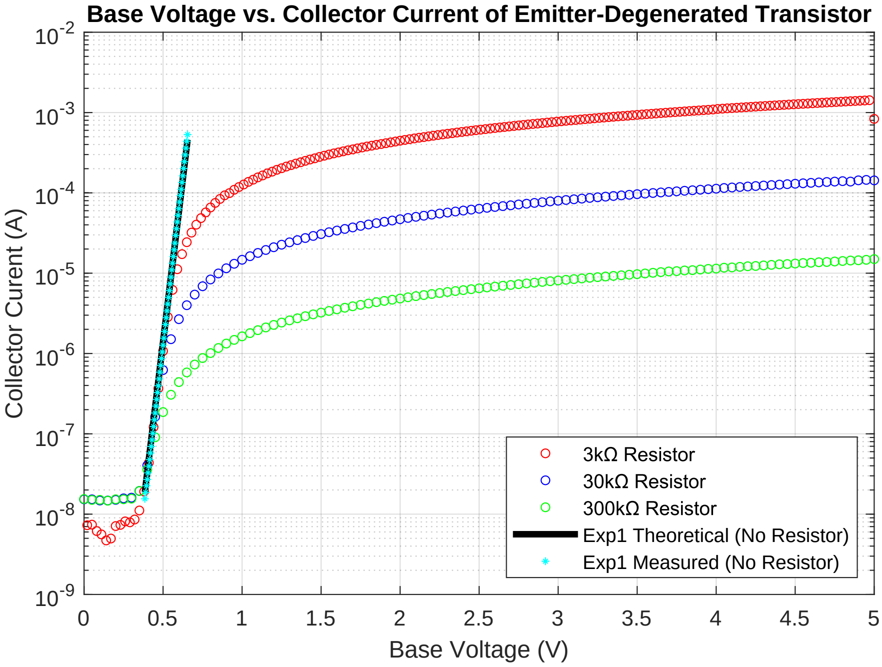

Figure 5: A semilog plot showing measured and theoretical collector currents versus base voltages of an emitter follower circuit with resistor values of 3kΩ, 30kΩ, 300kΩ, and the measured/theoretical fit of the npn bipolar transistor with no resistor as found in Experiment.

Additionally, for each of our resistors, we can make a linear plot of collector current as a function of base voltage, along with an appropriate theoretical fit (**Figures 6, 7, and 8**). We can find this theoretical fit using forumals derived in the prelab and Ohm's law:

$$I_c = \frac{(V_b - V_{on})}{R} \text{ amps}; \quad V_{on} = U_T * ln(\frac{I_{on}}{I_s}) \text{ volts}; \quad I_{on} = \frac{U_T}{R}\alpha \text{ amps}$$

We use the same U_T and I_s values as derived in Lab 2 ($U_T = 0.0267366096$ volts, $I_s = e^{-32.1846}$ amps). We use α to be $\frac{\beta}{\beta+1} = 0.990$, with $\beta = 103.537$ as derived in Experiment 1.

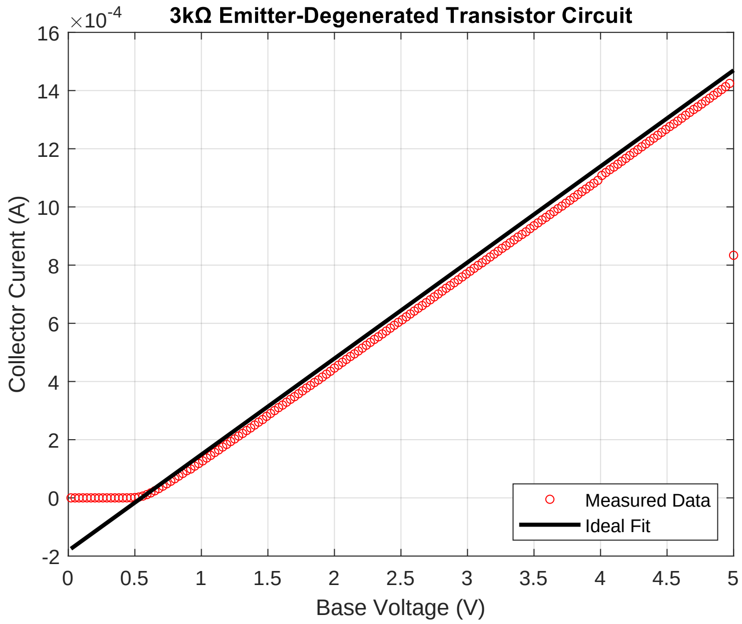

Figure 6: A linear plot of collector current as a function of base voltage, swept from 0V to 5V with ideal fit for our Emitter-Degenerated Transistor circuit with 3kΩ resistor.

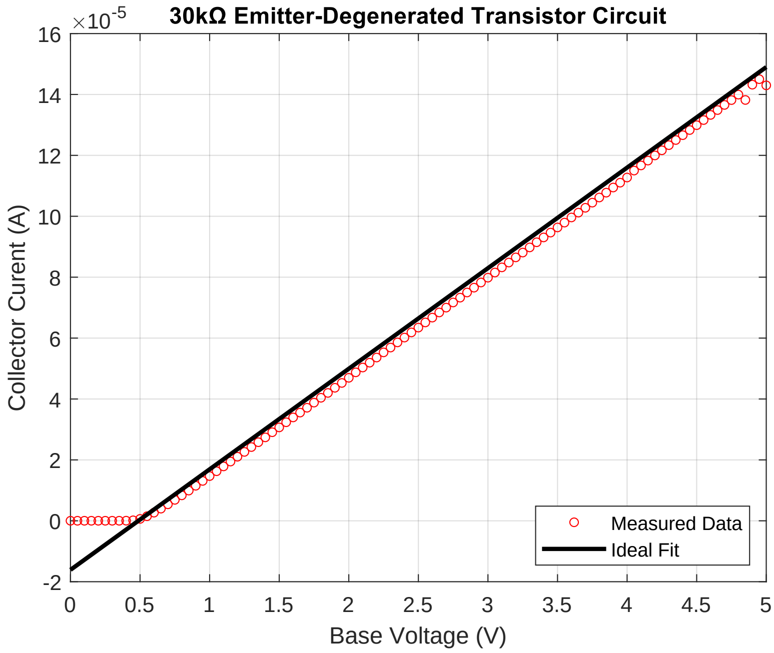

Figure 7: A linear plot of collector current as a function of base voltage, swept from 0V to 5V with ideal fit for our Emitter-Degenerated Transistor circuit with 30kΩ resistor.

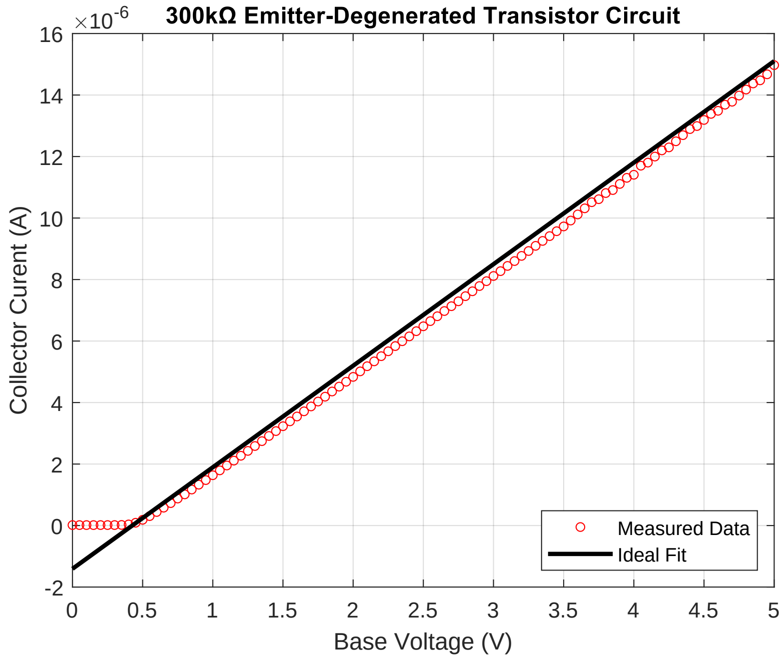

Figure 8: A linear plot of collector current as a function of base voltage, swept from 0V to 5V with ideal fit for our Emitter-Degenerated Transistor circuit with 300kΩ resistor.

In all cases, the fit parameters for our theoretical plots match our measured data in the appropriate regimes. We can see that the Emitter-Degenerated Transistors follow the same curve initially, increasing the collector current exponentially with the base voltage. However, after reaching roughly 0.5V, the collector current begins to increase linearly with base voltage rather than exponentially. Additionally, after 0.5V, the current varies by a decade due to the difference in resistances.

To further analyze the Emitter-Degenerated Transistor, we can extract the incremental resistance of the base terminal, Rb, for each resistor, from the base current–voltage characteristics.

We can calculate our measured incremental resistance by taking $r_b = \frac{\delta V_b}{\delta I_b}$ ohms. To do this, we use the diff command in MATLAB (r_b = diff(Vb)./diff(Ib) ohms). Our theoretical incremental resistance can be found using the formula derived in the prelab: $r_b = \frac{U_T}{I_b} + R(\beta + 1)$ ohms.

We can compare the theoretical and measured incremental resistances against each other as functions of base current, as shown in **Figure 9**.

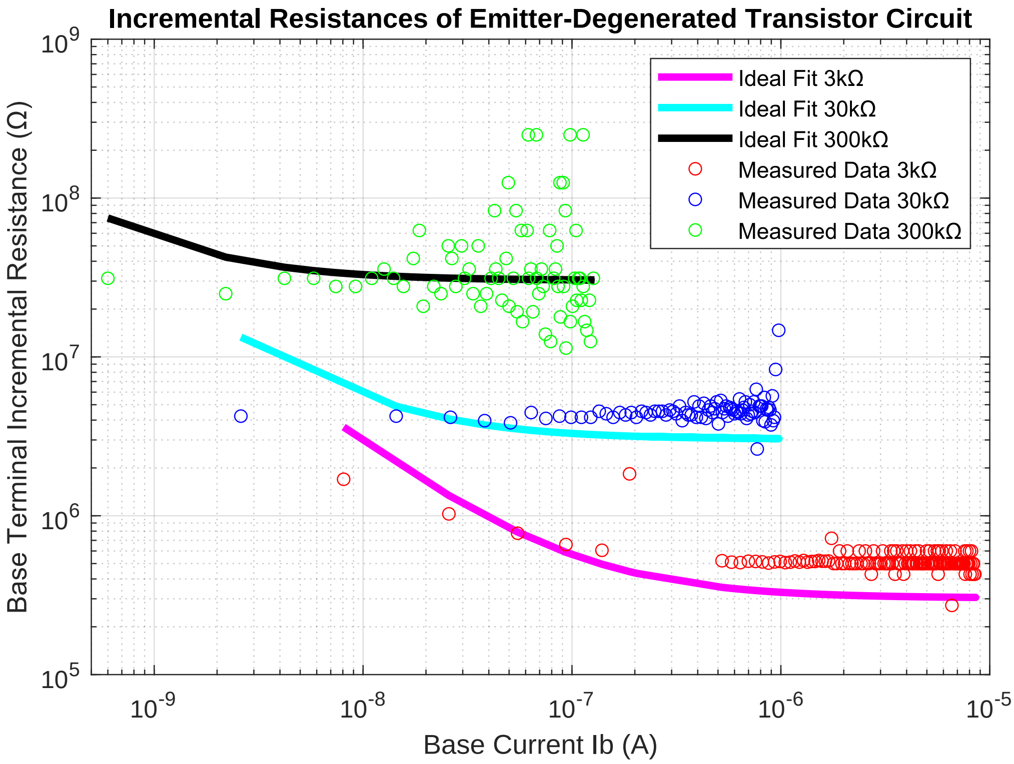

Figure 9: A loglog plot of measured and theoretical incremental resistances as as functions base current. Base current was measured as we swept base voltage from 0V to 5V.

The theoretical fits for the incremental resistances roughly match the measured data, but are noticeably far off from perfect. The 3kΩ resistor circuit has very concentrated data, the 30kΩ circuit has relatively concentrated data, and our 300kΩ circuit is significantly dispersed over nearly two decades. However, the best fit seems to be opposite: the 300kΩ circuit has the most centered fit, then the 30kΩ circuit, and the 3kΩ circuit has the worst fit.

We believe this is due to the fact that with a higher resistor value, the voltage drop across emitter resistor is much larger, which both increases the stability of the operating point (since a higher resistance will cause voltage to vary less significantly with change in current) and increases the dynamic range of the incremental resistance ($r_b = \frac{U_T}{I_b} + R(\beta + 1)$ ohms, so a smaller I_b leads to more variation due to U_T), leading to more stable but more dispersed data.

Similarly, we can extract the incremental transconductance gain of the transistor with emitter degeneration for each resistor from the collector characteristics. We can calculate our measured incremental transconductance gain by taking $G_m = \frac{\delta I_c}{\delta V_b}$ mhos, again using the diff command in MATLAB. Our theoretical transconductance gain can be found using the formula derived in the prelab: $G_m = \frac{I_c}{U_T + I_c R\alpha}$ mhos.

We can compare the theoretical and measured transconductance gains against each other as functions of collector current, as shown in **Figure 10**.

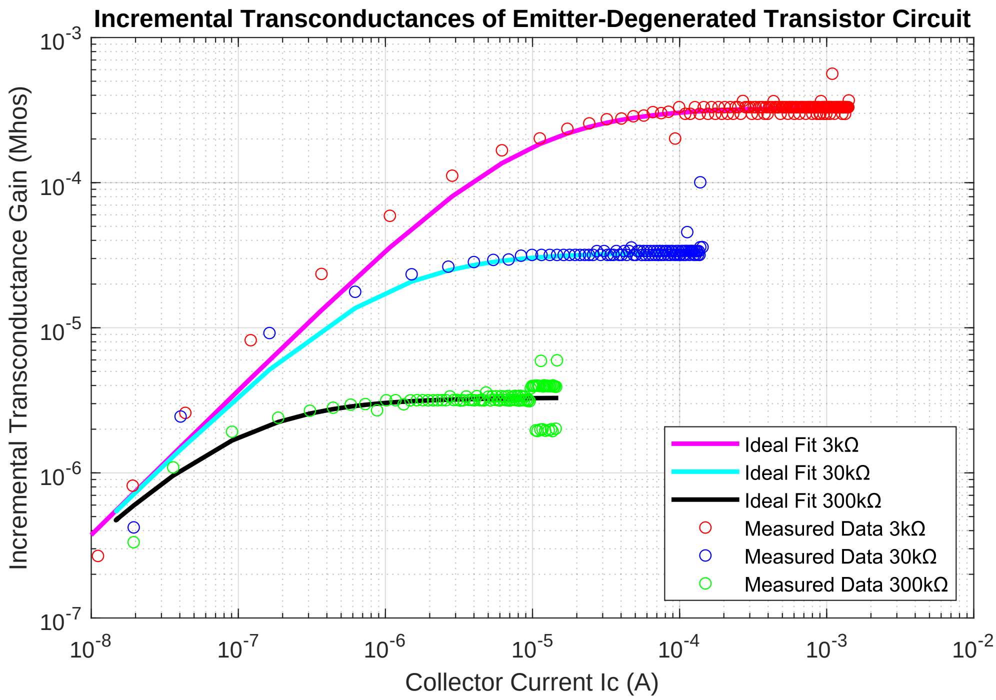

Figure 10: A loglog plot of measured and theoretical transconductance gains as as functions of collector current. Collector current was measured as we swept base voltage from 0V to 5V.

Our theoretical and measured incremental transconductance gains match much better than our incremental resistances. While there is slight variation and a few outliers, the overall data matches very well with our predictions.

Experiment 3: Follower Voltage Transfer Characteristics

The circuit that we investigated in Experiment 2 is often used as a simple voltage buffer, which is commonly called an emitter-follower. In this context, the transistor's base voltage is the circuit's input and the transistor's emitter voltage is its output.

We can make a plot showing the emitter-follower's voltage transfer characteristic (VTC), which is a plot of Vout as a function of Vin, along with a theoretical fit. We will our circuit with a 30kΩ resistor for this experiment. In this configuration, the emitter voltage follows the base voltage's lead, hence its name.

In order to create the characteristic, we measured the emitter voltage as we swept and measured the base voltage from 0V to 5V. We created our theoretical fit by integrating over $\delta V_e = \frac{RI_c}{U_T\alpha + RI_c}\delta V_b$ volts, as shown in **Figure 11**, using the MATLAB command trapz to compute a close estimate of the definite integral.

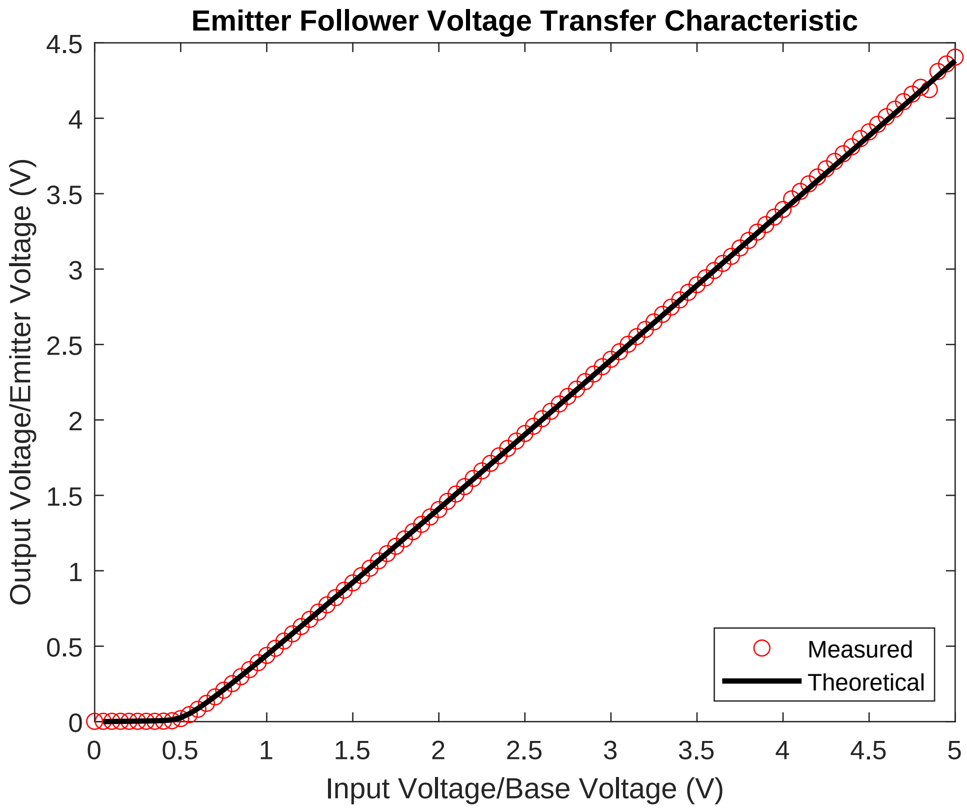

Figure 11: A linear plot of measured and theoretical Voltage Transfer Characteristics for the emitter-follower with a 30kΩ resistor.

Our theoretical and measured transfer functions match almost exactly.

We can extract the incremental voltage gain of the emitter follower by examining the slope of output voltage/input voltage. Using MATLAB's curve fitter tool, we extracted the value of the slope between 0.5 volts and 5 volts to be 0.989. This matches our alpha value of $\alpha = \frac{\beta}{\beta+1} = \frac{103.54}{104.54} = 0.990$.

We can approximate the difference between Vin and Vout after 0.5 volts to be a set offset of Von. We can mathematically see this by looking at our equations to derive Emitter Voltage as a function of Base Voltage:

$$I_c = \frac{(V_b - V_{on})}{R} \text{ amps} \rightarrow I_c = I_e \cdot \alpha = \frac{V_e}{R} \cdot \alpha \text{ amps} \rightarrow \frac{V_e}{R} \cdot \alpha = \frac{(V_b - V_{on})}{R} \rightarrow V_e = \frac{V_b - V_{on}}{\alpha} \text{ volts}$$

Therefore, we can see that difference between Vin and Vout is a set difference of Von (given that we take alpha to be 1). This is caused by base-emitter voltage V_{on}, which is required to turn on the transistor, and the voltage drop across the emitter degeneration resistor R, which introduces additional feedback and stabilization.

Experiment 4: Inverter Voltage Transfer Characteristics

This experiment involved building and analyzing a simple inverting voltage amplifier, also known as a common-emitter amplifier, using a 2N3904 NPN transistor. The circuit is configured as a simple inverting voltage amplifier, where the base voltage serves as the input and the collector voltage as the output with a resistor in series. The emitter voltage is also grounded with a resistor. One channel of the SMU monitors the input voltage, and the other the output voltage. The circuit acts as an inverting amplifier - an increase in the base voltage leads to a decrease in the collector voltage. The experiment explores the effect of varying the collector resistor on the amplifier's behavior.

We can derive a theoretical approximation for the inverting amplifier portion of the VTC: $V_c = V_{cc} - I_c R_c$ volts, where the current across the collector can be derived as $I_c = \frac{V_e}{R_e}$ amps, and the voltage across the emitter can be derived from the difference between the base voltage and the base-emitter junction voltage: $V_e = V_b - V_{be} volts$. Given that the power source attached to the transistor is a 5V DC power source:

$$V_c = 5 - \frac{R_c}{R_e}(V_b - V_{be}) \quad \text{volts}$$

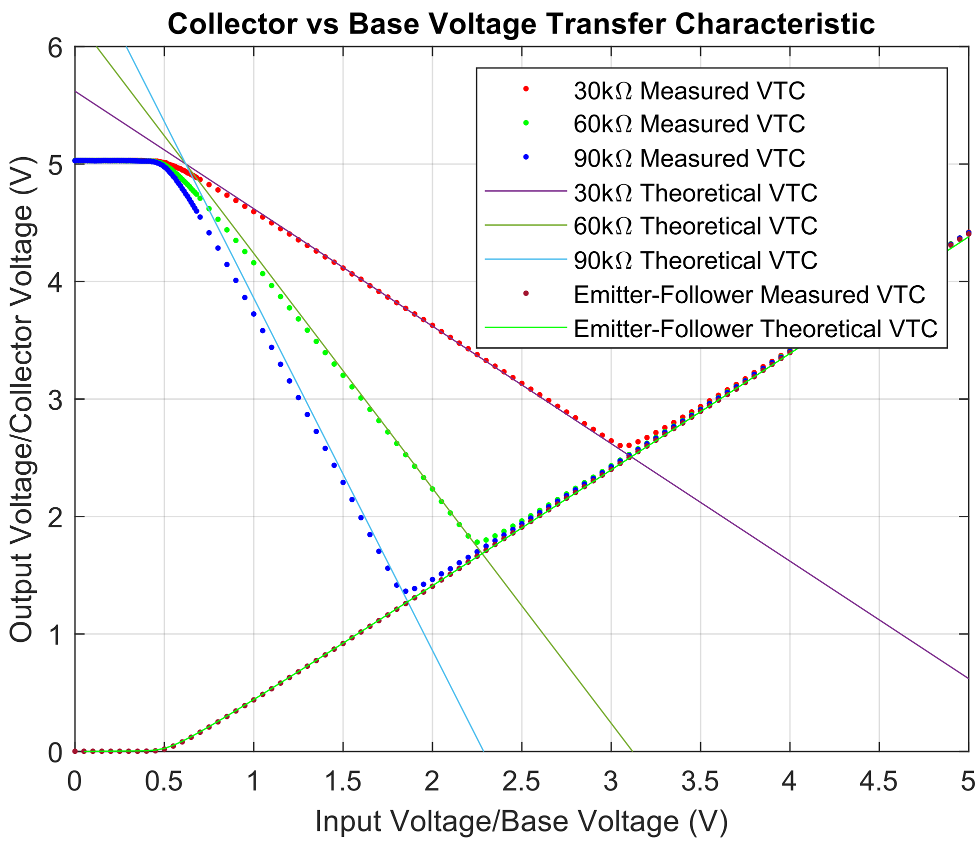

Figure 12: A plot of the voltage transfer characteristics of the collector and base voltages of a simple inverting voltage amplifier, overlaid with theoretical voltage transfer characteristics and the measured and theoretical emitter-follower voltage transfer characteristics derived in the previous experiment.

For the 30kΩ collector resistor, the amplifier has an incremental voltage gain of around -1. The amplifier with the 60kΩ collector resistor has an incremental voltage gain of around -2. Finally, the amplifier with the 90kΩ collector resistor has an incremental voltage gain of around -3. The incremental voltage gain is determined by a ratio of the collector and emitter resistors: $-\frac{R_c}{R_e}$. Given the equation for the theoretical fit, that makes sense.

This experiment demonstrates the characteristics and operation of a common-emitter amplifier using a 2N3904 transistor. By varying the collector resistor ($30k\Omega$, $60k\Omega$, and $90k\Omega$), the effect on the amplifier's gain and output characteristics is apparent.