DISCOVERING THE FREQUENCY OF TIBETAN SINGING BOWLS

QUANTITATIVE ENGINEERING ANALYSIS 3 | FALL 2024

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01 Modeling Singing Bowls: How can we calculate the resonant vibration/frequency of a Tibetan Singing Bowl?

In order to help derive our model, we studied the papers "Tibetan Singing Bowls" by Denis Terwage and John Bush, and "In Vino Veritas: A Study of Wineglass Acoustics". The papers approximate the bowl as a cylindrical shell with a rigid base and open top, vibrating in the (2,0) mode (2 nodal meridians with 0 nodal parallels). This mode can often be achieved by rubbing the edge of the bowl with the striker.

Frequency content is an important aspect to study as it reveals the vibrational modes, resonant frequencies, and energy distribution, directly linking the system's physical behavior to its acoustic properties.

1. To calculate the frequency, can examine the oscillation of the rim of the singing bowl as a function of time

$$\Delta(t) = \Delta_0 \cos \omega t.$$

2. Using the law of conservation of energy, we can calculate the energy of the system to be the sum of the kinetic and potential energy in terms of the constants A and B

$$E = A\left(\frac{d\Delta}{dt}\right)^2 + B\Delta^2,$$

3. If we ignore damping, we can substitute in the oscillation of the rim as defined in step one and rearrange to solve for the resonant frequency in terms of A and B.

$$\omega^2 = B/A.$$

4. Next, we can calculate the values for A and B by solving for the kinetic and potential energies of the system in terms of the density, radius, thickness, and height of the bowls.

$$A = \frac{5\pi}{8}\rho_g a R \int_0^H [f(z)]^2\, dz,$$
$$B = \frac{3\pi Y a^3}{8R^3}\left[1 + \frac{4}{3}\left(\frac{R}{H}\right)^4\right]\int_0^H [f(z)]^2\, dz.$$

5. Lastly, we can substitute in A and B into the equation for natural frequency. This gives us our final equation for the natural angular velocity of singing bowls for the lowest mode (2,0).

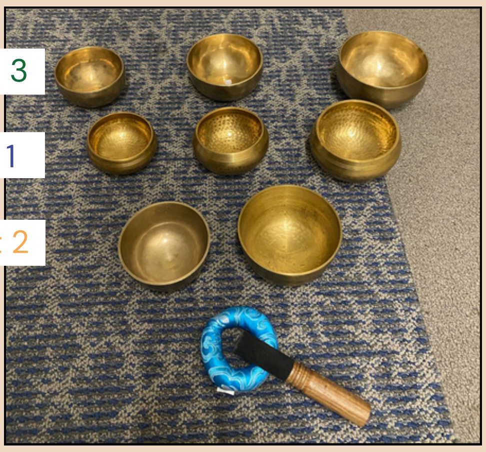
$$\nu_0 = \frac{1}{2\pi}\left(\frac{3Y}{5\rho_g}\right)^{1/2}\frac{a}{R^2}\left[1 + \frac{4}{3}\left(\frac{R}{H}\right)^4\right]^{1/2}$$

02 Data collection and analysis: How did our experimental data compare to our model?

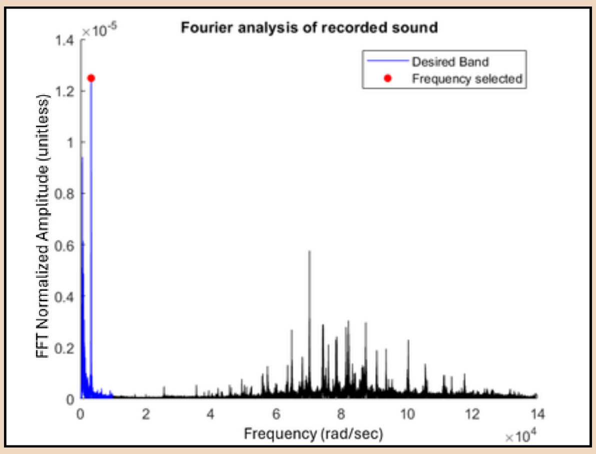
Set 3

Set 1

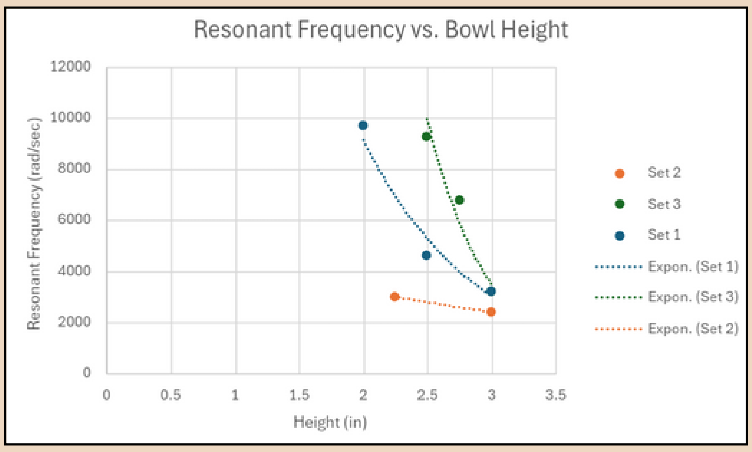
Set 2



We recorded the frequency of eight bowls across three different sets to test the accuracy of our equation. Each set of bowls varied in height but had roughly the same radius, thickness, and material



Here is an example DFT frequency plot of different audible frequencies of sound. This plot is from Set 1, Bowl 1 (biggest size).

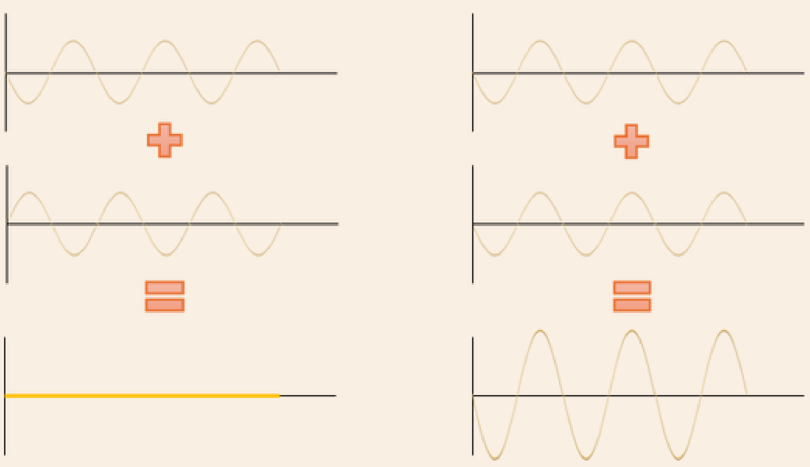


We found the most accurate correlation between height of the bowl and frequency. As the height of the bowl increased, the frequency decreased, roughly by a factor of (1/H^2) as predicted by our equation. We examined each set individually due to the wide differences between sets.

03 NODAL MERIDIANS????

When you hit a singing bowl filled partially with water, you can notice little droplets in specific areas of the bowl - the areas where these DO NOT form are known as **Nodal Meridians**.

Rubbing	Hitting
• Frequency (sound) from a continuous, uniform force • Fundamental Frequency - no phase shift in audible frequencies	• Multiple excited frequencies mix • Nodal Meridians form where there is **destructive interference** • Droplets form where there is **max displacement**



Destructive Interference

Max Displacement

Ring-driven and shell-driven vibrations also excite different nodal vibrations!