Série 2) ARTHUR FREEMAN.

Ex 1)

a) $|\Omega| = 67$, A = "lisent FR"

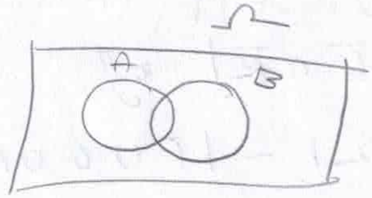
B = " " GE "

C = "Lisent FR $\wedge$ GE"

$P(A) = \frac{|A|}{|\Omega|} \simeq 0,70$

$P(B) = \frac{35}{|\Omega|} \simeq 0,52$

$P(A \cap B) = \frac{23}{|\Omega|} \simeq 0,34$.



$P(\bar{A} \cap \bar{B})$

$\bar{A} \cap \bar{B} = \Omega \setminus A \cup B$

$P(\Omega \setminus A \cup B) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

Via Poincaré :

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\simeq 0,88$,

Donc, $P(\overline{A \cup B}) = 0,12$.

$12\%\ 67 \simeq 8,04$ étudiants.

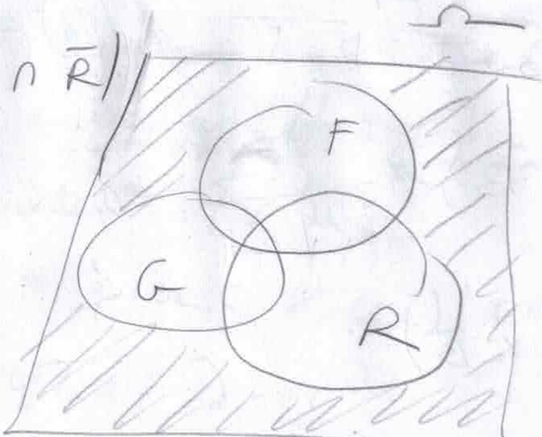
Ex 1) $\quad (\Omega, \mathcal{P}(\Omega), \mathbb{P})$

b) $|\Omega| = 67$, $|F| = 47$, $|G| = 35$, $|F \cap G| = 23$,
$|R| = 20$,

$P(F \cap R) = \dfrac{12}{67}$, $P(G \cap R) = \dfrac{11}{67}$

$P(G \cap F \cap R) = \dfrac{5}{67}$

Calculez $|F \cap \overline{G} \cap \overline{R}|$



$|F \cap \overline{G} \cap \overline{R}|$

$= |\Omega| - |F \cup G \cup R|$

$= \underbrace{|\Omega|}_{67} - \Big[\underbrace{|F|}_{=47} + \underbrace{|G|}_{35} + \underbrace{|R|}_{20} - \underbrace{|F \cap G|}_{23} - \underbrace{|F \cap R|}_{=12} - \underbrace{|G \cap R|}_{11}$
$+ \underbrace{|F \cap G \cap R|}_{5}\Big]$

$= 6$

x 2) $\Omega = \{\{R,A\}, \{R,B\}, \{R,C\}, \{\bar{R},A\}, \{\bar{R},B\}, \{\bar{R},C\}\}$

$\{\bar{R},\bar{A}\}, \{\bar{R},\bar{B}\}, \{\bar{R},\bar{C}\}$.

A) Refuge loin, B) cache proche, C) quitte pays

$P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{1}{4}$

$P(\underbrace{\text{retrouver}}_{=R} \mid B) = 0,9$, $P(R \mid C) = 0,5$

$P(R \mid A) = 1$.

Calculez $P(R \cap B)$, $P(R \cap A)$, $P(R \cap C)$

et calculez $P(\bar{R})$.

$P(R \mid B) = \frac{P(R \cap B)}{P(B)} \Rightarrow P(R \cap B) = P(R \mid B)\,P(B)$

$\simeq 0,9 \cdot \frac{1}{2} = 0,45$.

$P(R \cap A) = P(R \mid A) \cdot P(A) \simeq 1/4$

$P(R \cap C) = P(R \mid C) \cdot P(C) \simeq \frac{1}{2} \cdot \frac{1}{4} = 1/8$

$P(\bar{R}) = 1 - P(R)$

Mais la formule de proba totales dit que :

$P(R) = \sum_m P(\overset{=R}{B} \mid A_m) \cdot P(A_m) = P(R \cap B) + P(R \cap A) + P(R \cap C)$

$= 0,825$,

$P(\bar{R}) = 0,175$. Le terroriste est mal barré.

Ex 3) Soit $\mu \in [0,1]$ μ = mort

$P(M) = \mu$. Soit $0 \leq a \leq 1$.

E_n = " parmi n personnes ayant contracté maladie, au moins l'une d'elle meurt "

$V_n = \overline{E_n}$ = " parmi n personnes ay. cont. m. personne ne meurt.

$P(E_n) = 1 - |V_n|$.

Pour chaque personne, on a deux possibilités soit elle vit soit elle meurt. $\Omega = \{V, M\}^n$

$= \{(x_1, \dots, x_n) \mid x_i \in \{V, M\}\}$

$V_n = \{(x_1, \dots, x_n) \mid \forall i \in [\![1, n]\!], x_i = V\}$.

$P(V_n) = \binom{n}{n} \cdot \underbrace{(1-\mu)^n}_{P_{vivant}} \cdot \mu^{n-n} = (1-\mu)^n$

même logique que pour les dés.

$P(E_n) = 1-(1-\mu)^n \leq a \Leftrightarrow (1-\mu)^n \geq 1-a$

$\Leftrightarrow n \cdot \log(1-\mu) \geq \log(1-a)$

$\Leftrightarrow n \leq \dfrac{\log(1-a)}{\log(1-\mu)}$ Si $a = 1/2$, $n = 22$.

Si $a = 95/100$, $n = 98$.