

COMPUTER VISION AND
PHOTOGRAMMETRY

MESHING

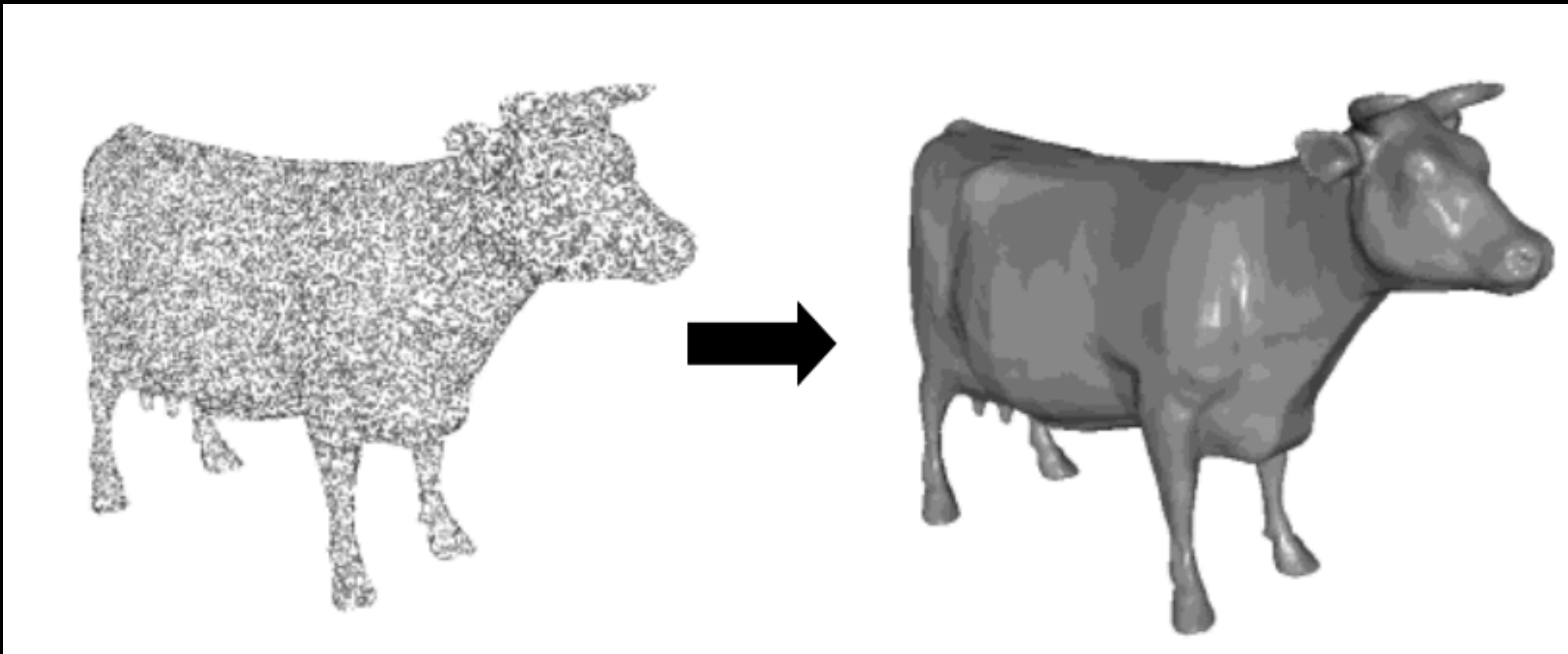
CONTENT

- ▶ Intro
- ▶ Explicit
- ▶ Implicit
 - ▶ Normal estimation
 - ▶ Radial Basis Functions
 - ▶ Others
 - ▶ Poisson Reconstruction
 - ▶ Marching Cubes

PROBLEM FORMULATION AND OVERVIEW

TEXT

FROM POINT CLOUDS TO MESHES



FROM POINT CLOUDS TO MESHES

- ▶ This is a common problem for scan data
- ▶ Range imaging
- ▶ Multi-view stereo

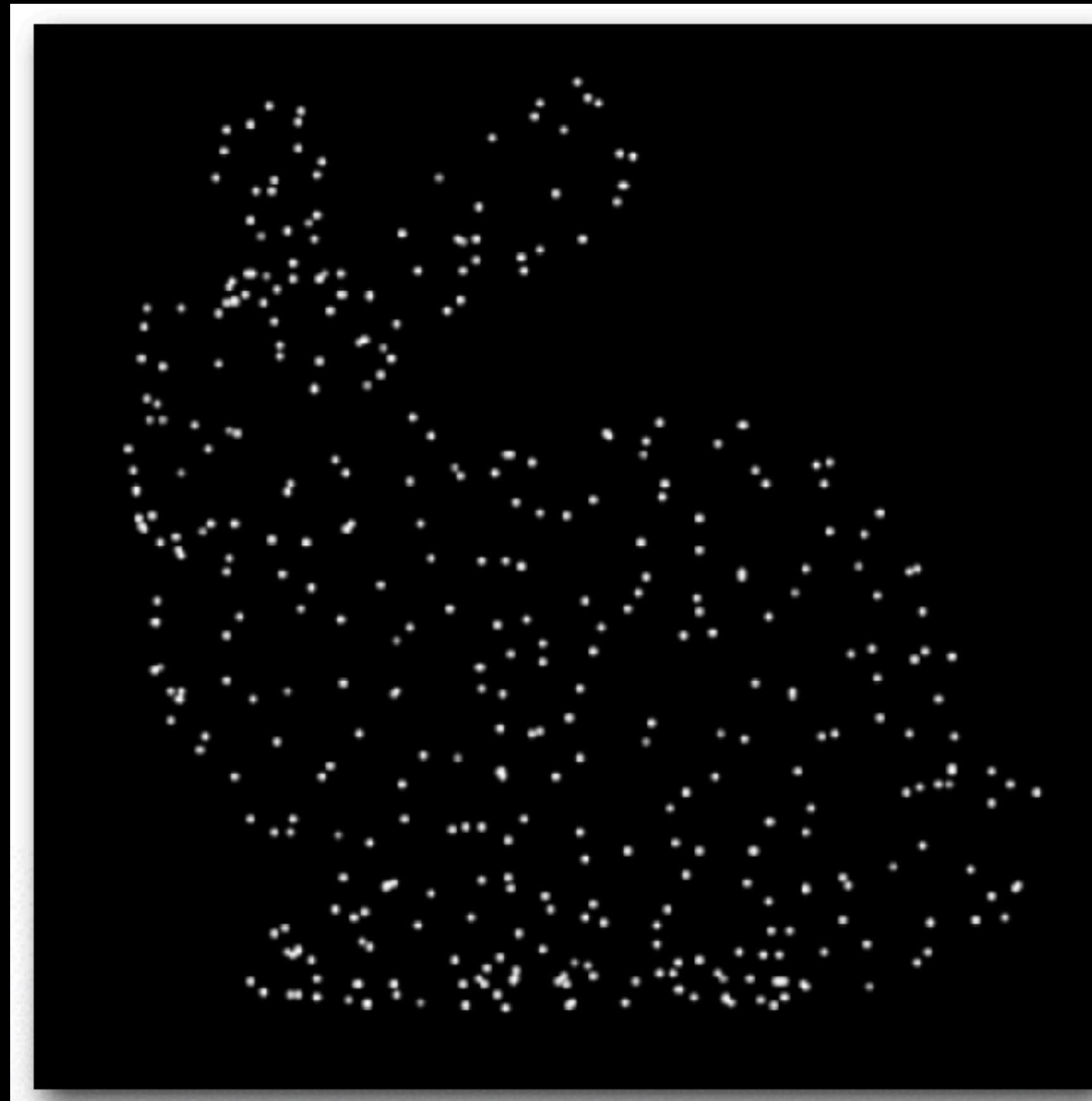
INPUT CAN BE

- ▶ Organized or not (for us not)
- ▶ Oriented (normal information) or not (for us not)
- ▶ Non-uniform/sparse
- ▶ Noisy



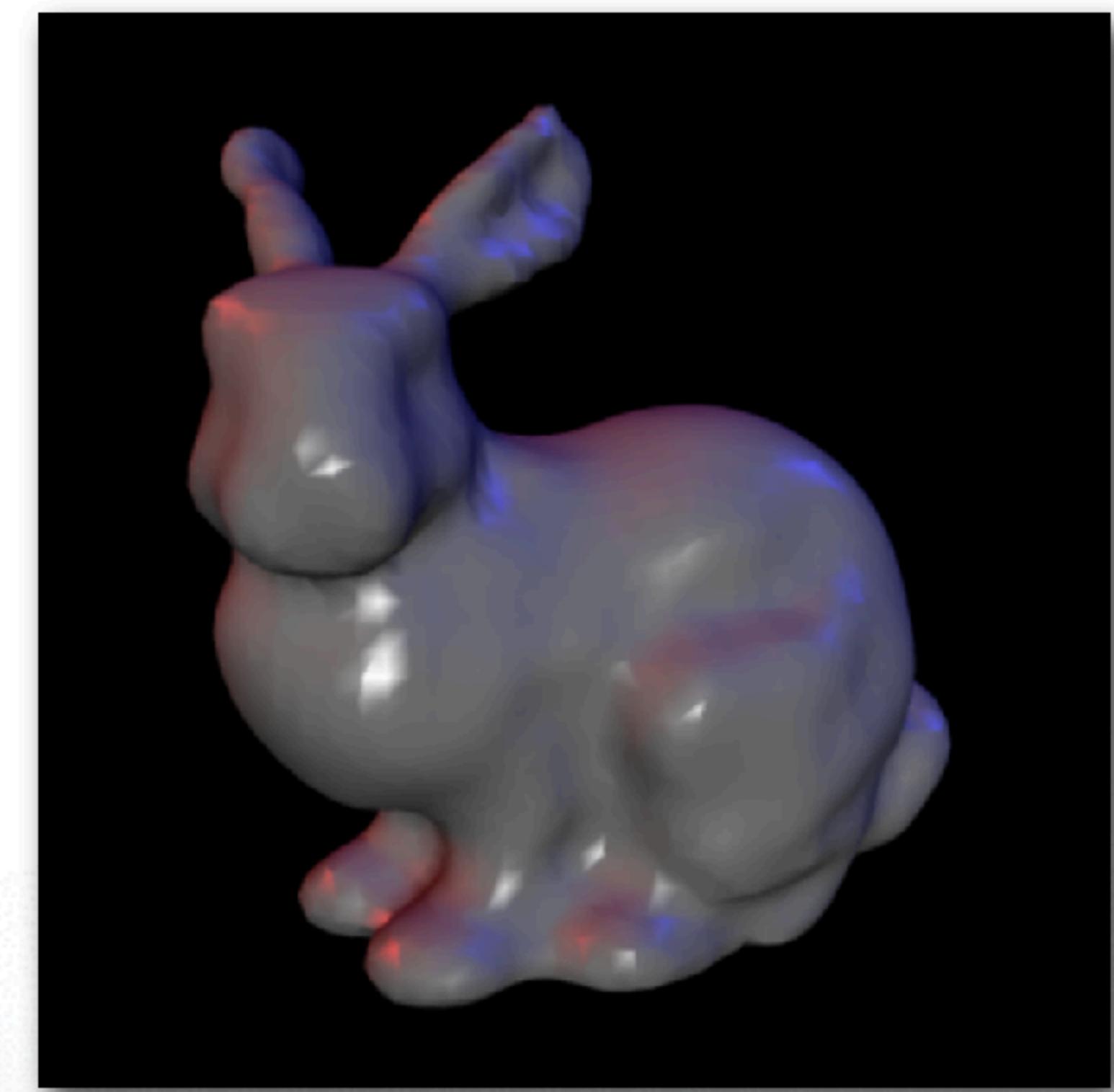
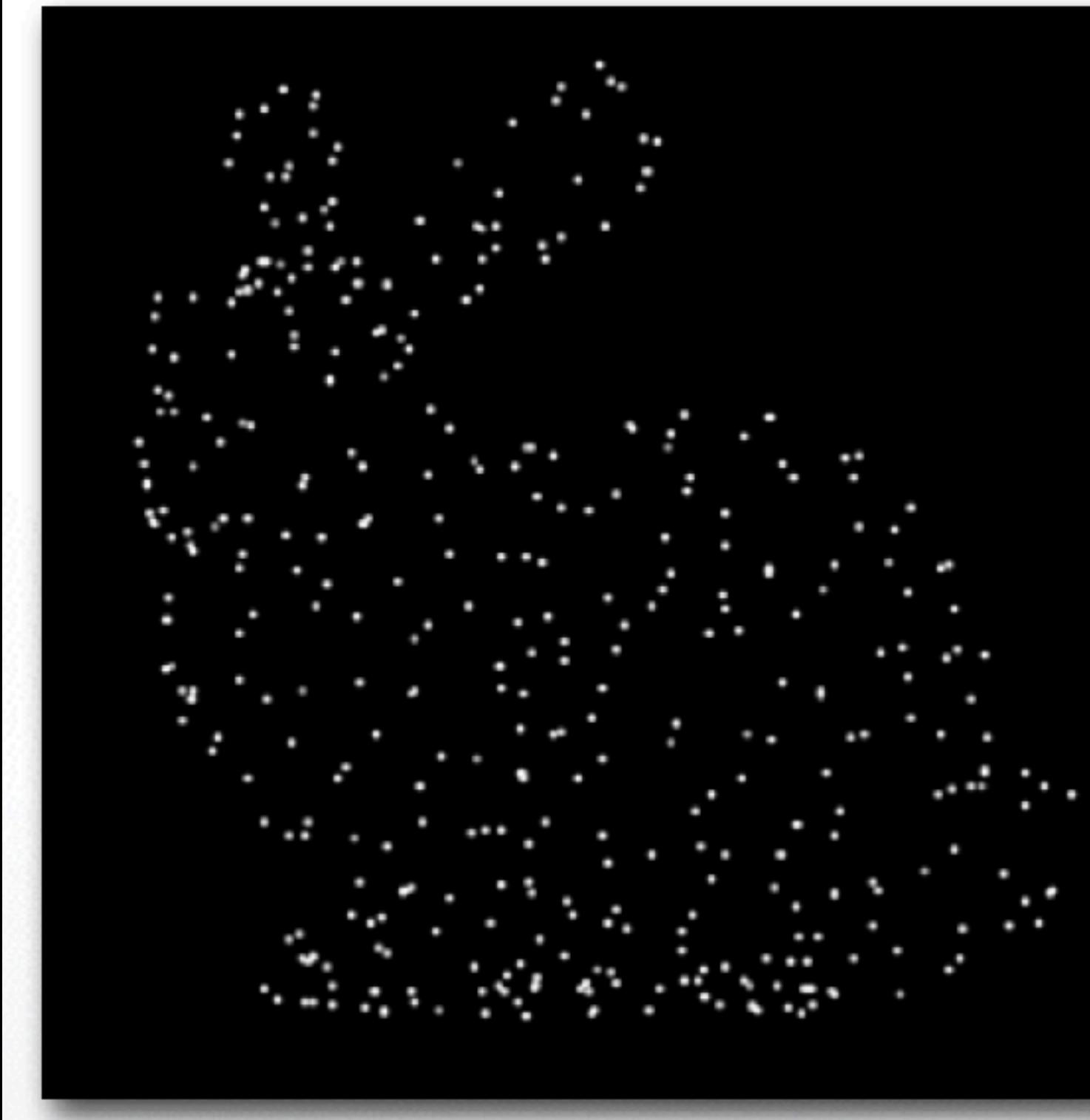
PROBLEM STATEMENT

Given a set of points $\mathcal{P} = \{\mathbf{p}_1, \dots, \mathbf{p}_n\}$ with $\mathbf{p}_i \in \mathbb{R}^3$



PROBLEM STATEMENT

Find a manifold surface $\mathcal{S} \subset \mathbb{R}^3$ which approximates \mathcal{P}



REGULAR / IRREGULAR

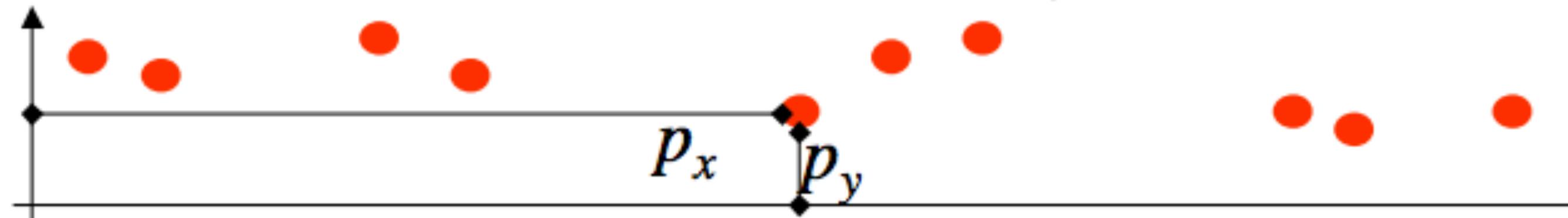
- Regular

- Requires to store only values



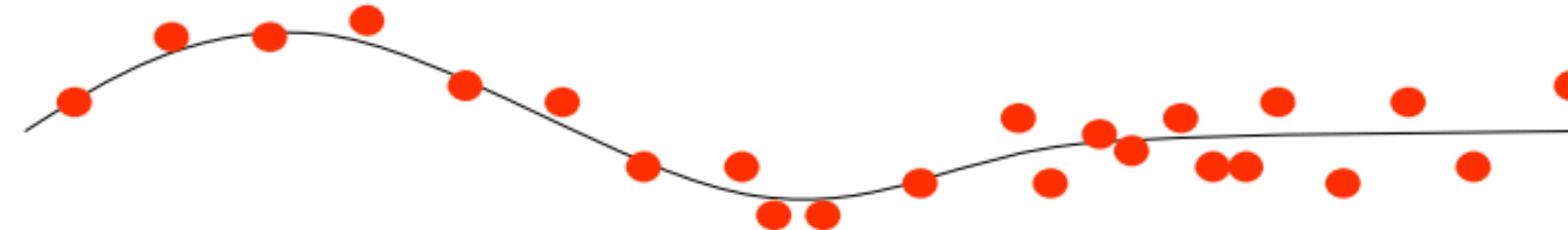
- Irregular

- Requires to store locations p_i

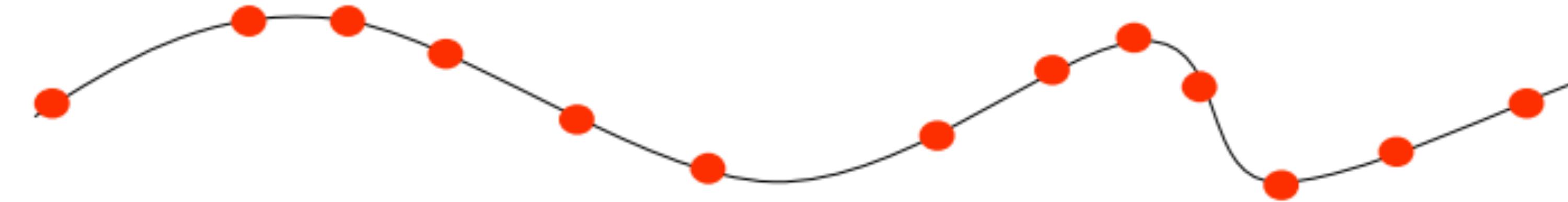


NOISY / PERFECT DATA

- Noisy data -> Approximation

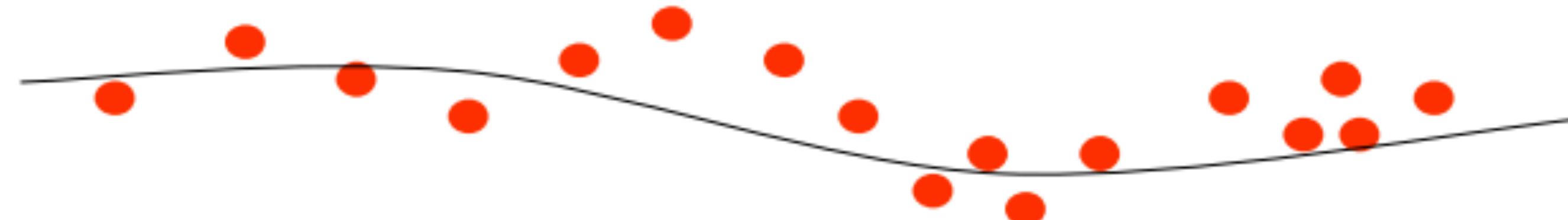


- Perfect data -> Interpolation

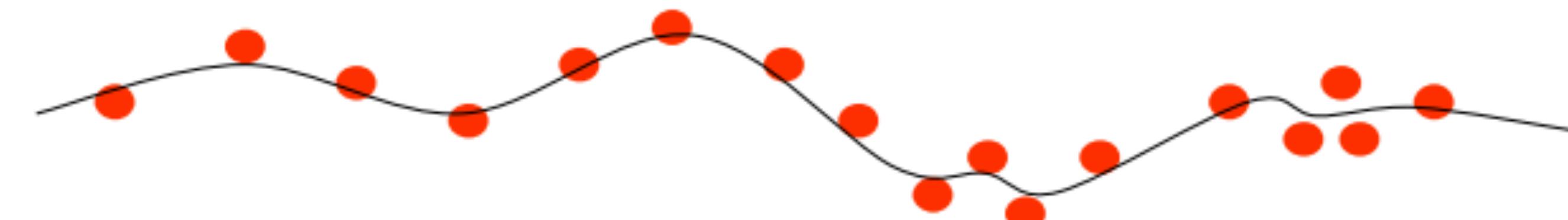


GLOBAL / LOCAL

- Global approximation



- Local approximation



- Locality comes at the expense of smoothness

TWO MAIN APPROACHES

- ▶ Explicit:
 - ▶ Local surface connectivity estimation
 - ▶ Point interpolation
- ▶ Implicit:
 - ▶ Signed distance function estimation
 - ▶ Mesh approximation

EXPLICIT RECONSTRUCTION

EXPLICIT RECONSTRUCTION

- ▶ Connect sample points by triangles
- ▶ Exact interpolation of sample points
- ▶ Bad for noisy or misaligned data
- ▶ Can lead to holes or non-manifold situations

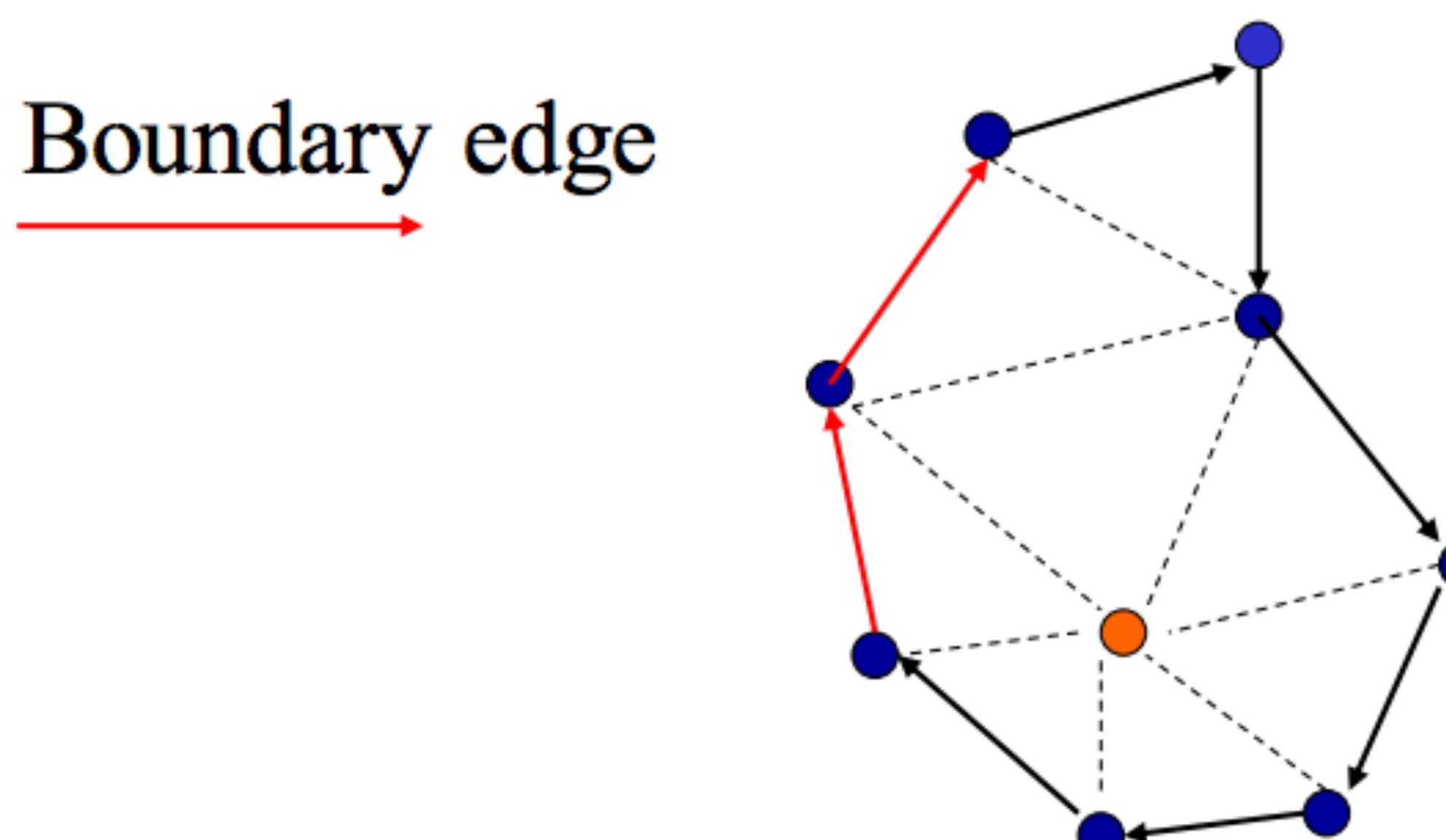
BALL PIVOTING

THE BALL PIVOTING ALGORITHM

- Pick a ball radius, roll ball around surface, connect what it hits



THE BALL PIVOTING ALGORITHM



Ball pivoting around active edge

Active edge

- Point on front
- Internal point

THE BALL PIVOTING ALGORITHM

Possible problems?



Undersampling



Small Concavities

CRUST

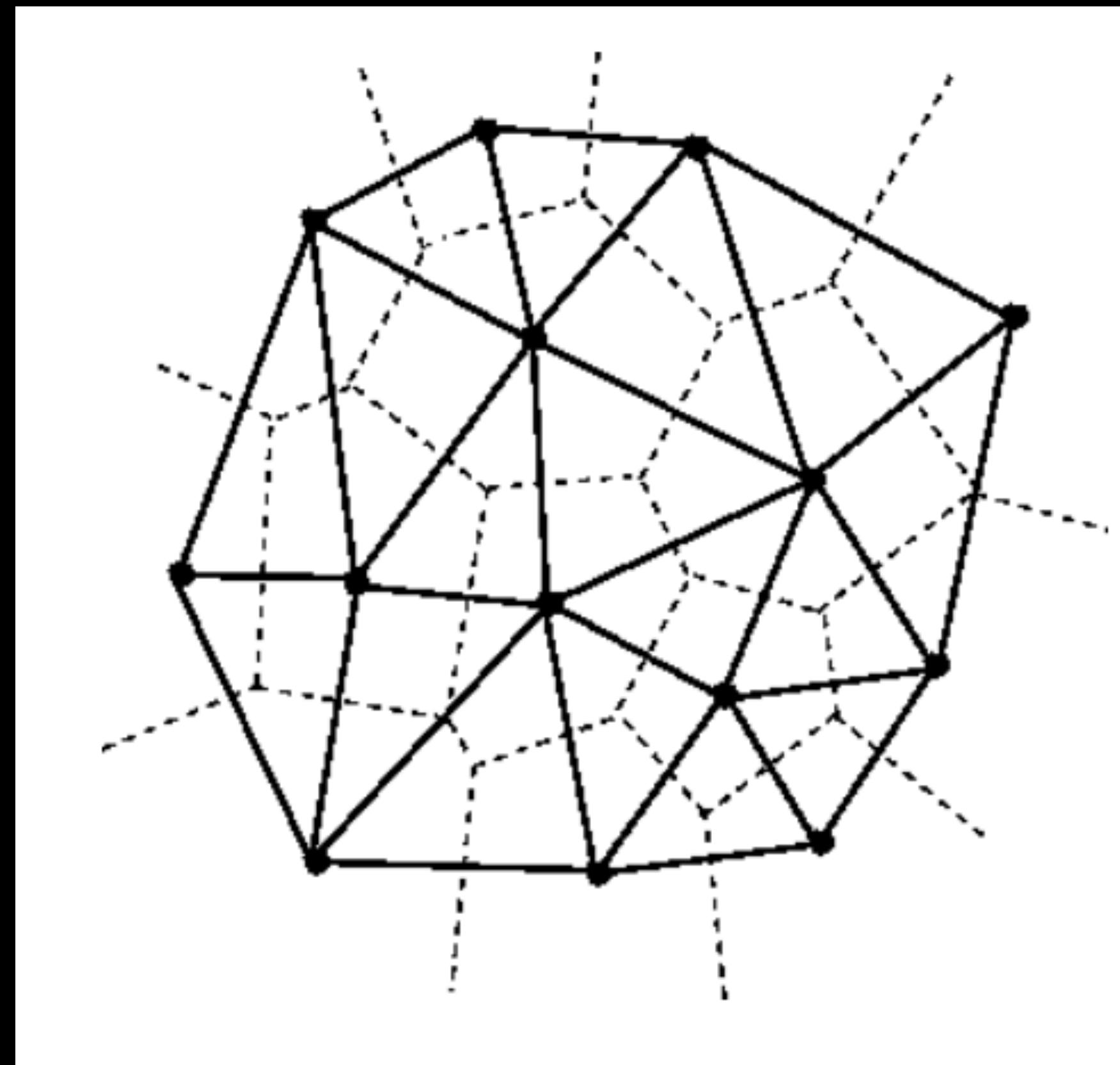
CRUST

- ▶ Aims to find adjacent surface without a parameter specifying feature sizes

TEXT

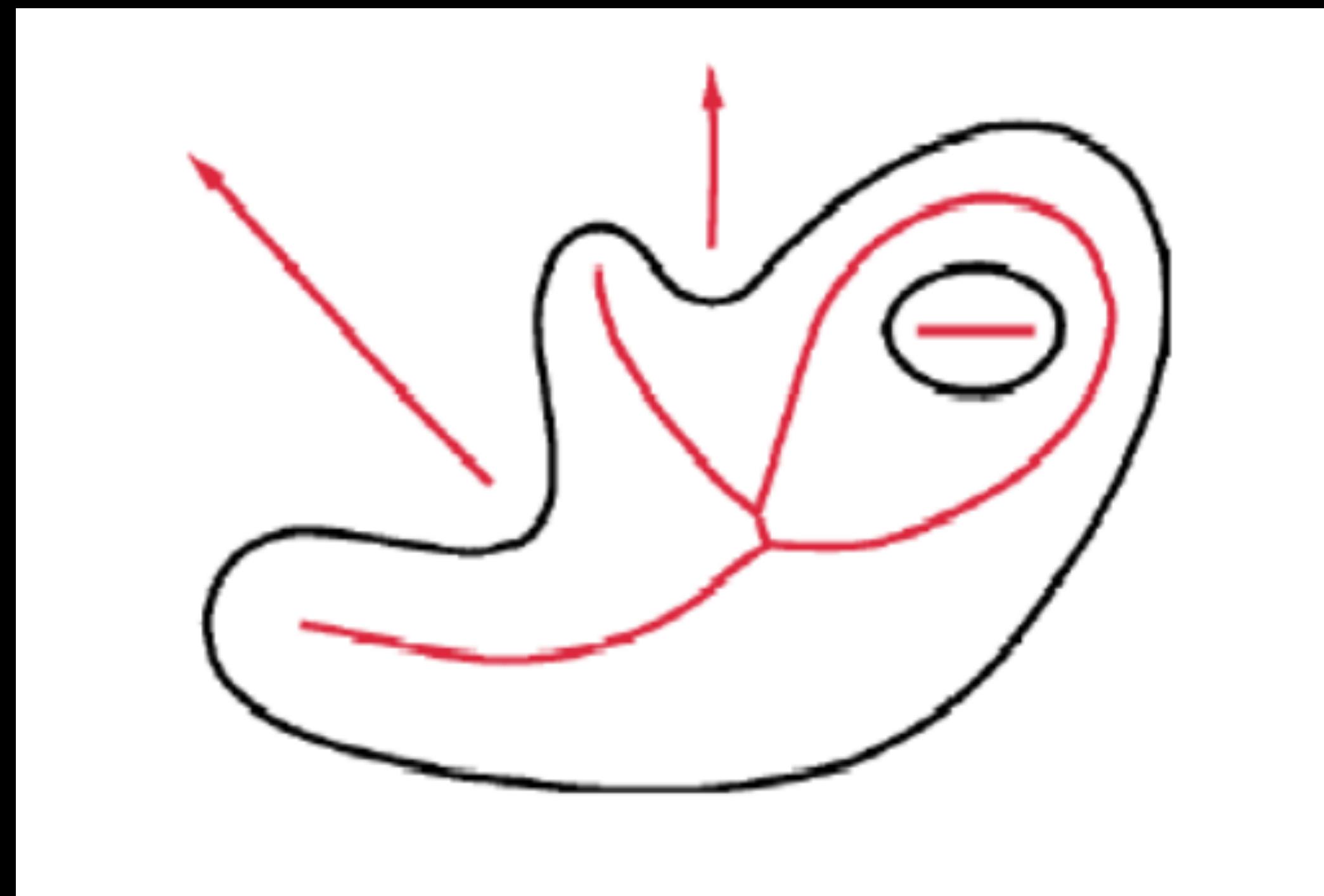
CRUST

- ▶ Delaunay Triangulation, Voronoi Diagram



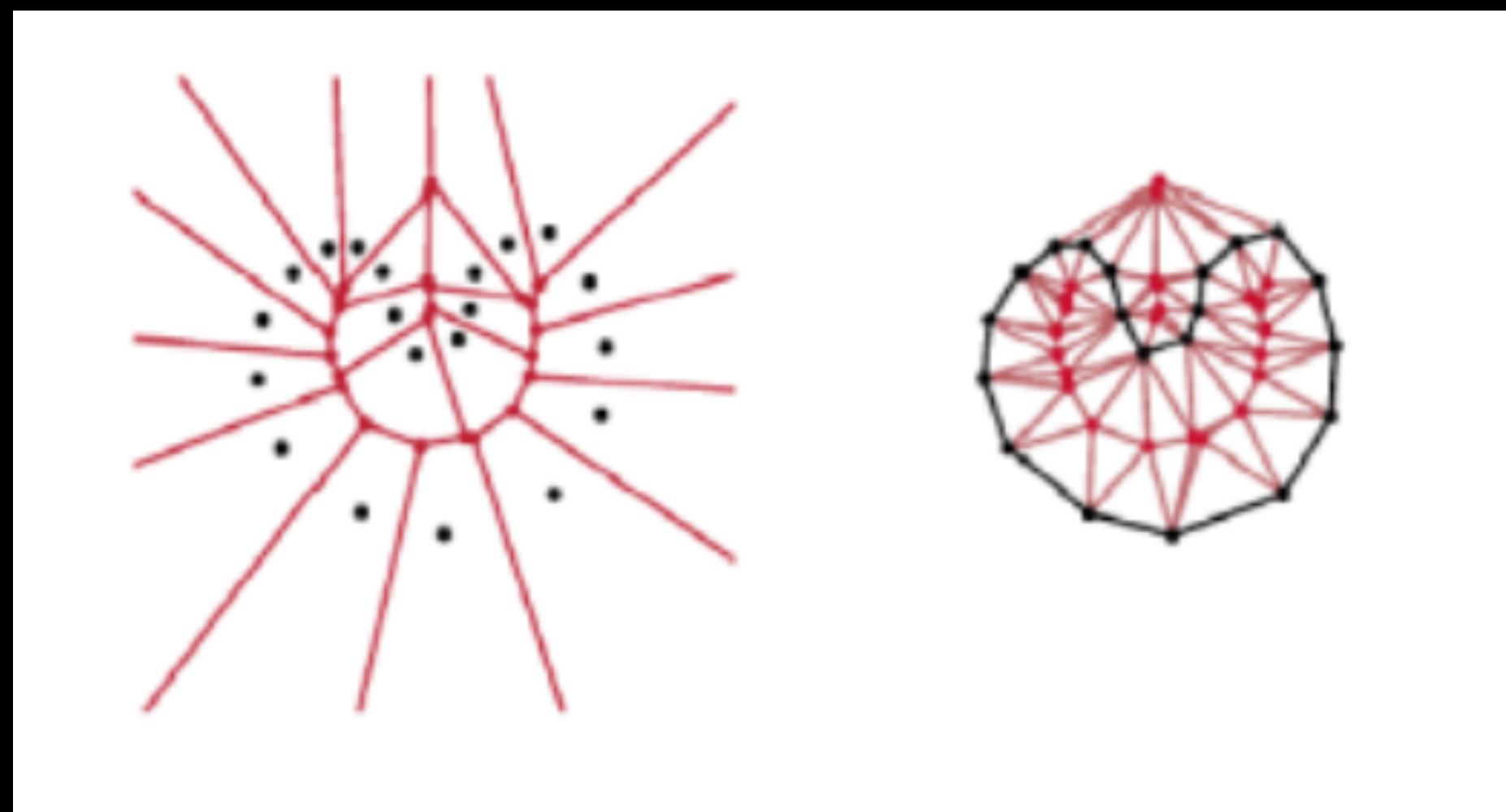
CRUST

- ▶ Medial Axis: of surface F is the closure of points that have more than one closest point in F .



CRUST

- ▶ The Voronoi Cells of a dense sampling are thin and long.
- ▶ The Medial Axis is the extension of Voronoi Diagram for continuous surfaces in the sense that the Voronoi Diagram of S Can be defined as the set of points with more than one closest point in S. (S = Sample Point Set)

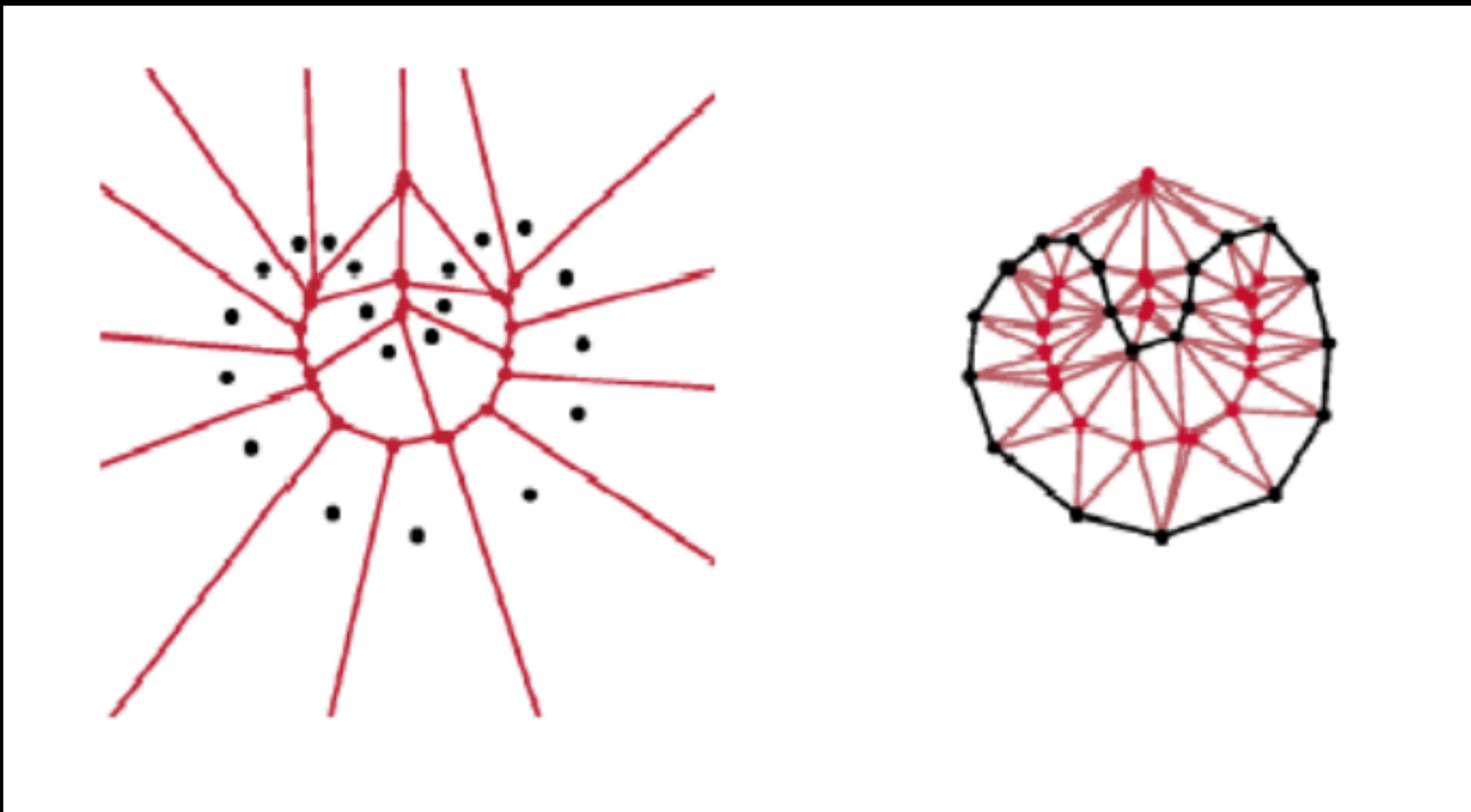


CRUST IN 2D

- ▶ Input : $P = \text{Set of sample points in the plane}$
- ▶ Output: $E = \text{Set of edges connecting points in } P$
- ▶ The Algorithm
 - ▶ Compute the Voronoi vertices of $P = V$
 - ▶ Calculate the Delaunay of $(P \cup V)$
 - ▶ Pick the edges (p,q) where both p,q are in P

TEXT

CRUST IN 2D: OUTPUT

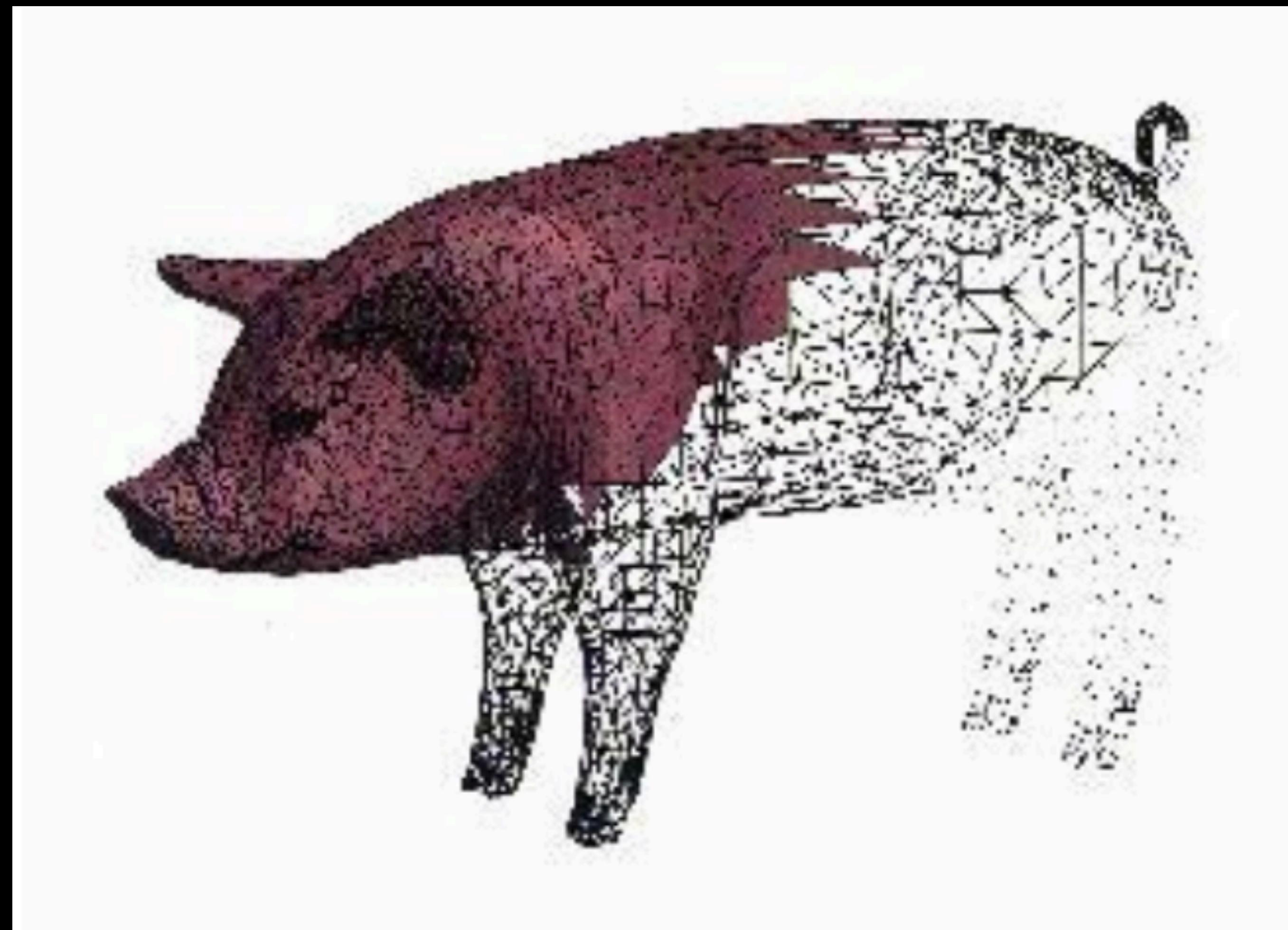


CRUST IN 3D

- ▶ <http://web.mit.edu/manoli/crust/www/sigcrust.pdf>
- ▶ <http://www.cgal.org/>

TEXT

CRUST: SAMPLE OUTPUT

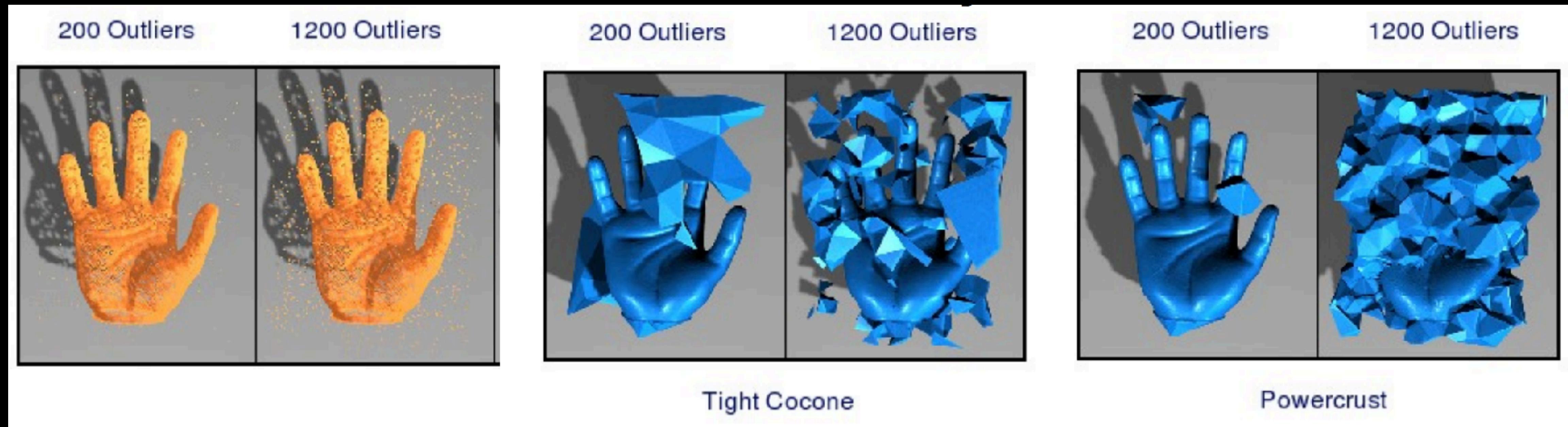


DELAUNAY TRIANGULATION

- ▶ More than 50% of existing methods are based on Delaunay triangulation and Voronoi diagram
- ▶ Output mesh size ~ input point cloud size
- ▶ Known and uniform sampling => very accurate results
- ▶ Noise or outliers => usually fail

TEXT

DELAUNAY TRIANGULATION



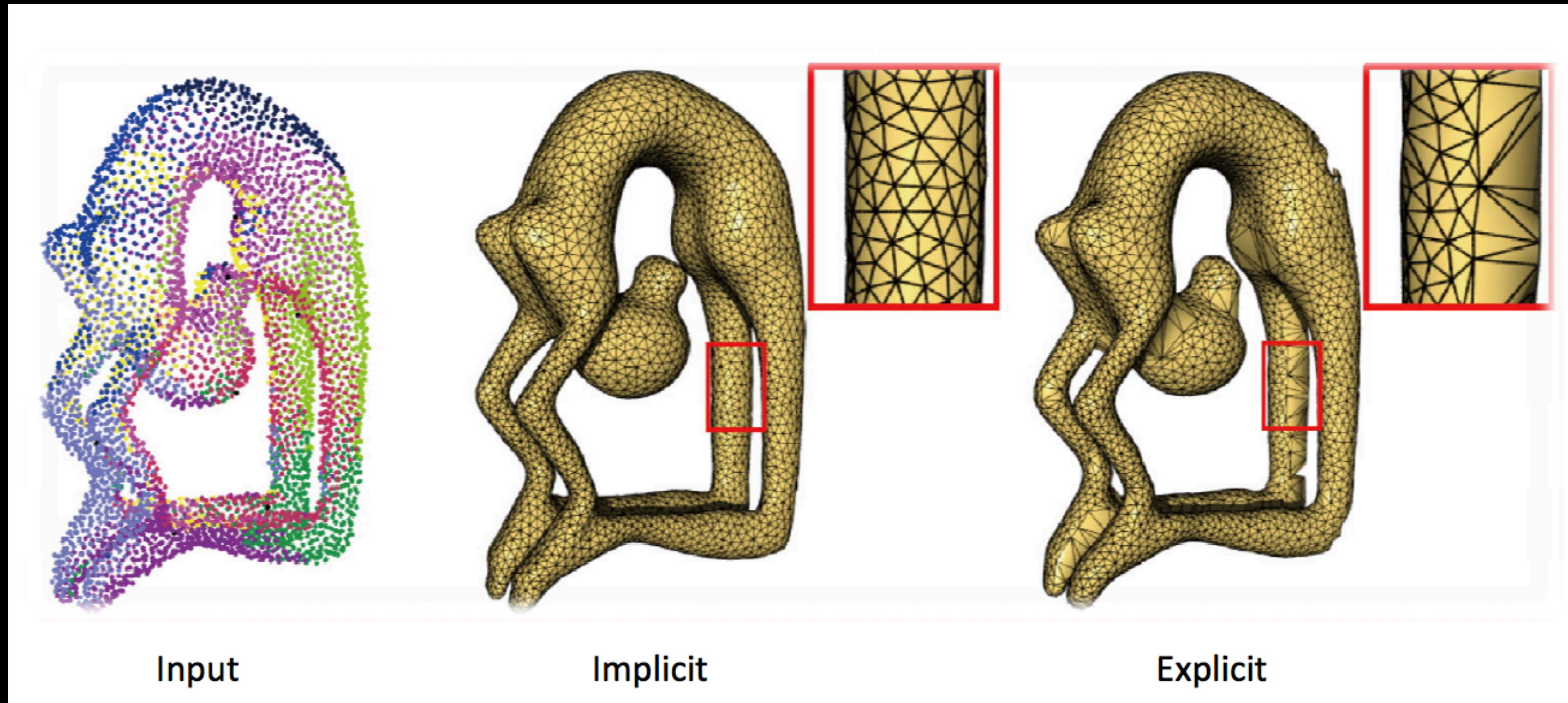
IMPLICIT RECONSTRUCTION

IMPLICIT SURFACE FITTING

- ▶ Idea:
 - ▶ 1. Define a smooth implicit surface that approximate the underlying real surface
 - ▶ 2. Project or generate points on this implicit surface
- ▶ Main issue: how to define the implicit surface ?
- ▶ Lots of possibilities: distance function, MLS, Radial Basis Functions (RBF), ...

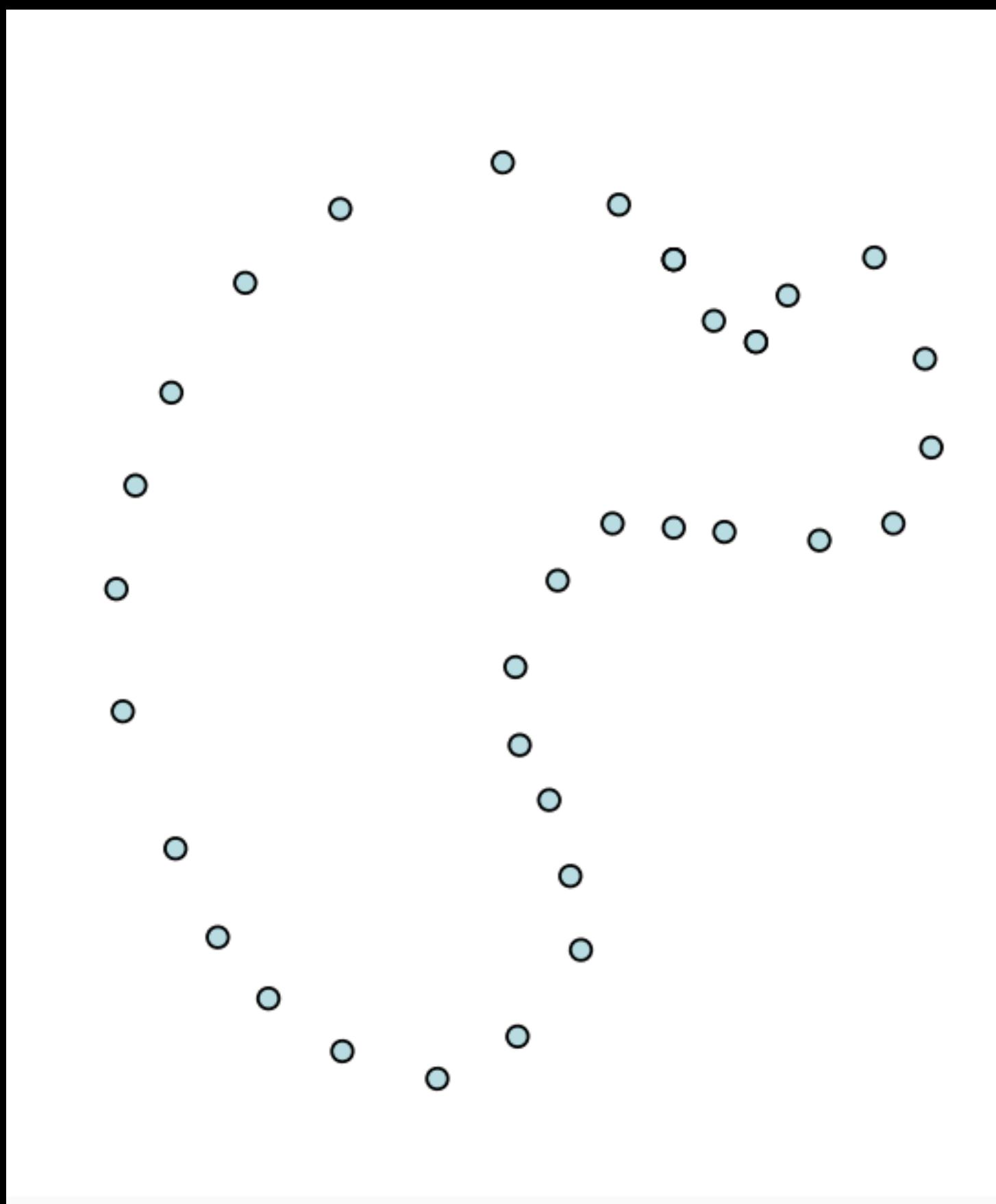
TEXT

IMPLICIT RECONSTRUCTION



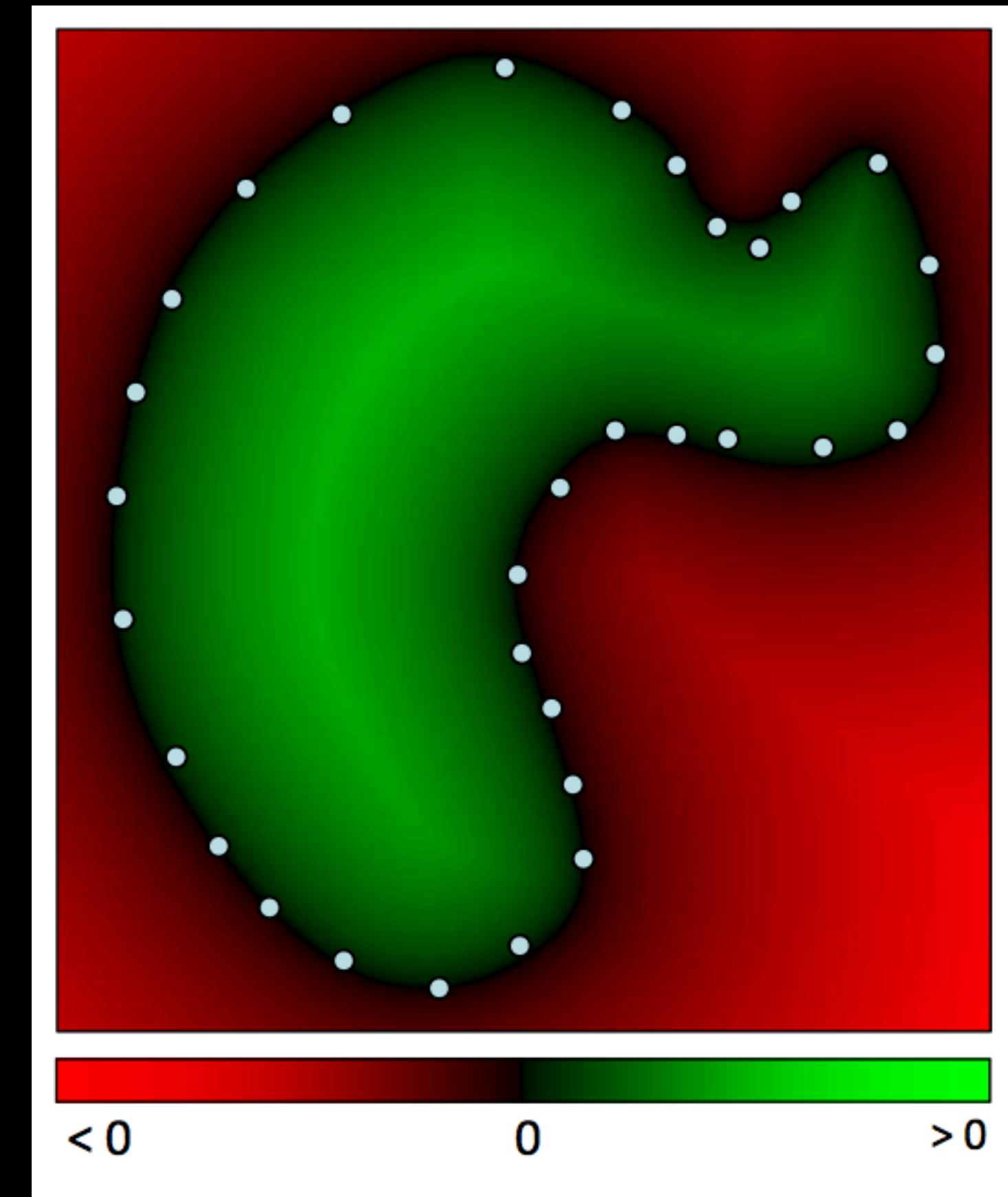
TEXT

IMPLICIT RECONSTRUCTION



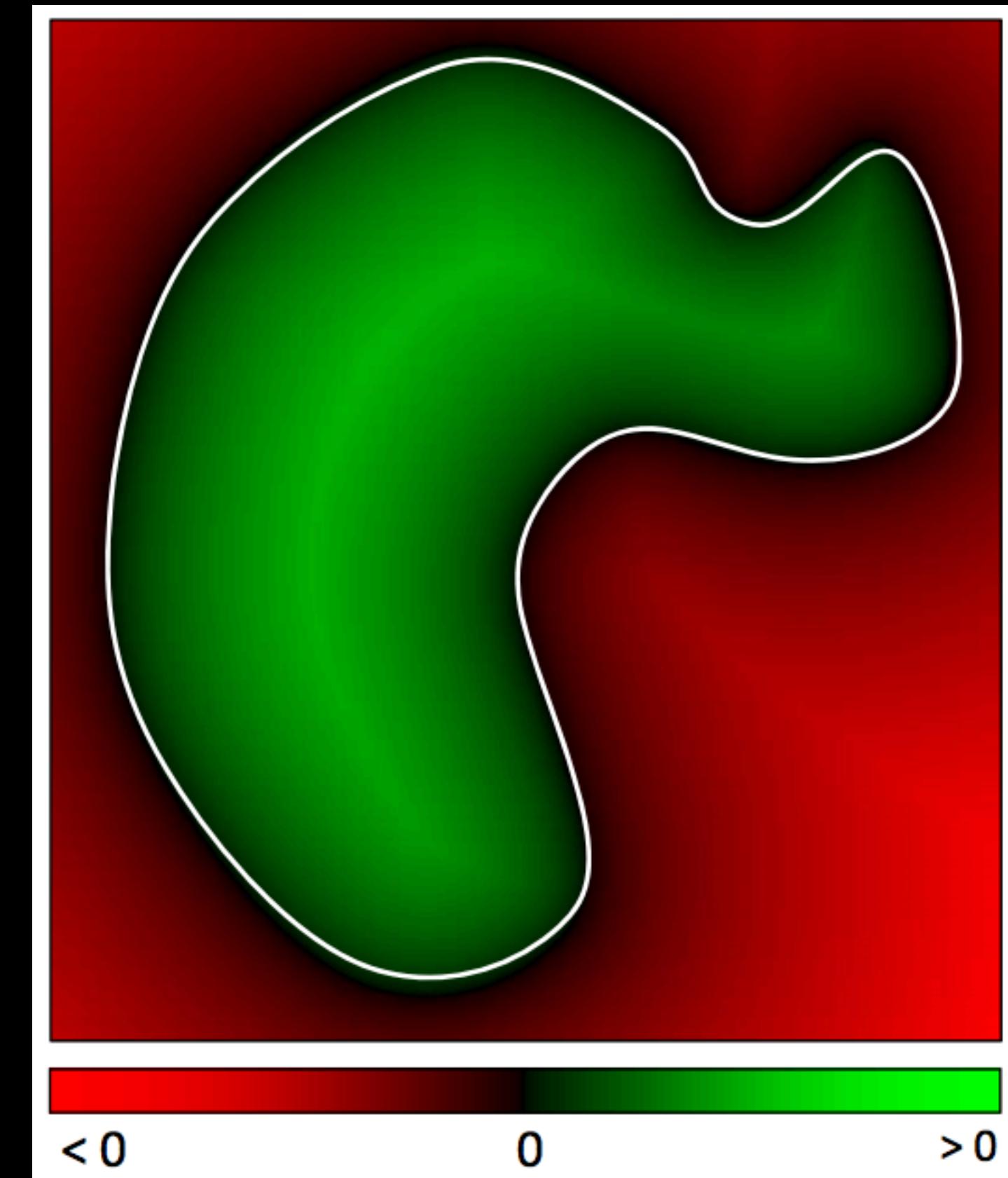
IMPLICIT RECONSTRUCTION

- ▶ Define a function
 - ▶ $f : \mathbb{R}^3 \rightarrow \mathbb{R}$
- ▶ with value < 0 outside the shape
and > 0 inside



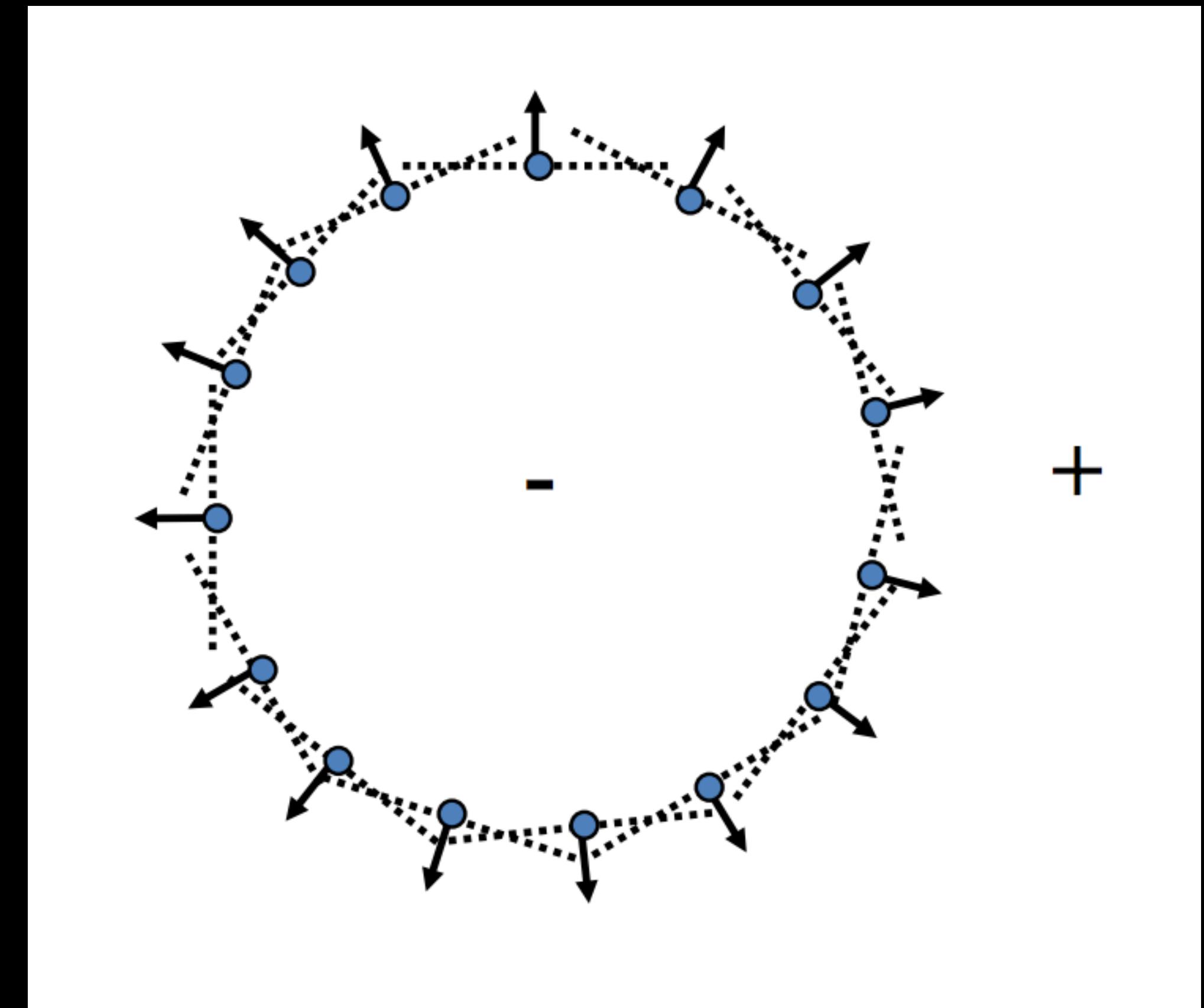
IMPLICIT RECONSTRUCTION

- ▶ Define a function
 - ▶ $f : \mathbb{R}^3 \rightarrow \mathbb{R}$
- ▶ with value < 0 outside the shape
and > 0 inside
- ▶ Extract the 0 set
 - ▶ $\{ x : f(x) = 0 \}$



SDF FROM POINTS AND NORMALS

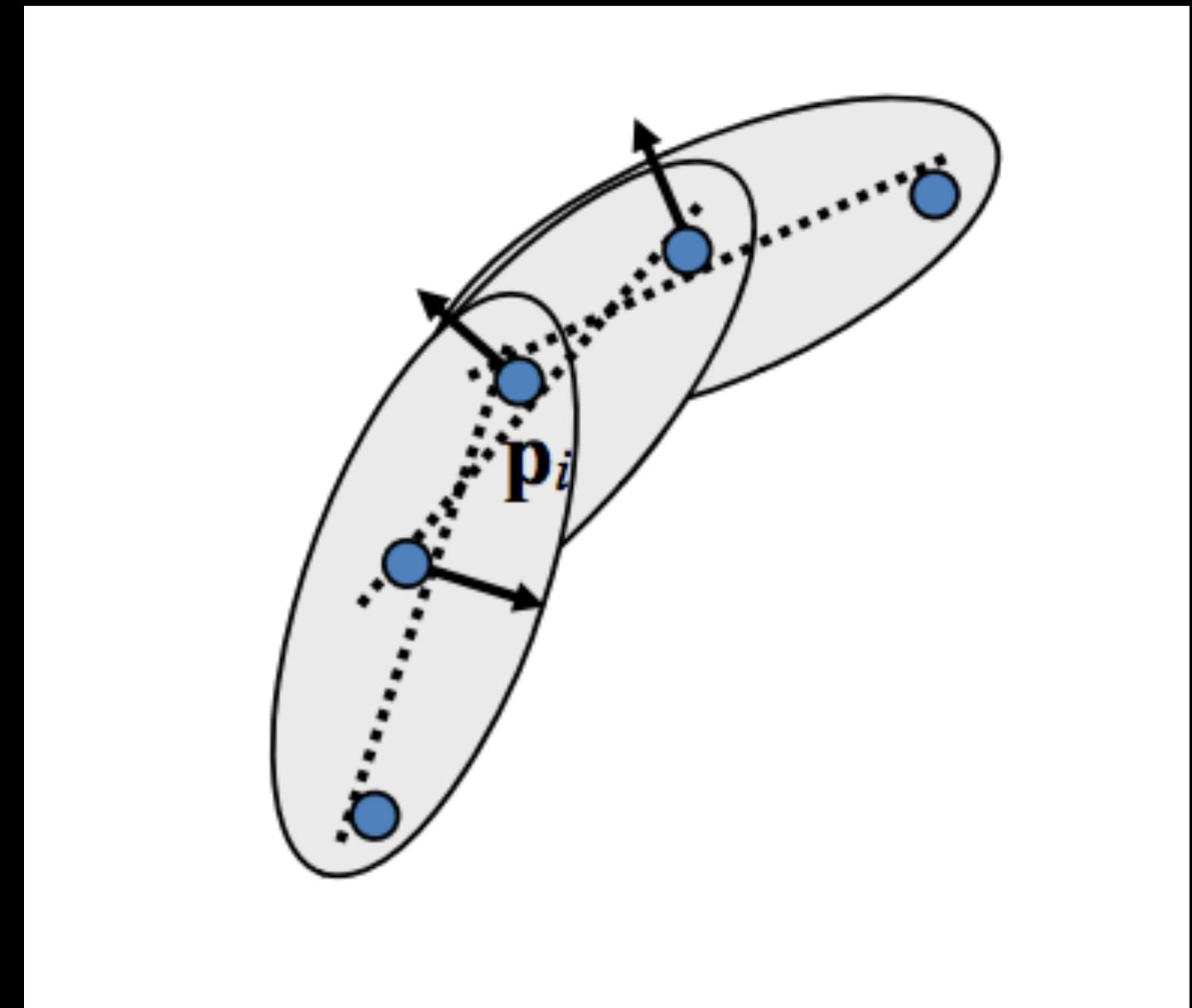
- ▶ Compute signed distance to the tangent plane of the closest point.
- ▶ Normals help to distinguish between inside and outside.
- ▶ How to get normals?



NORMAL ESTIMATION

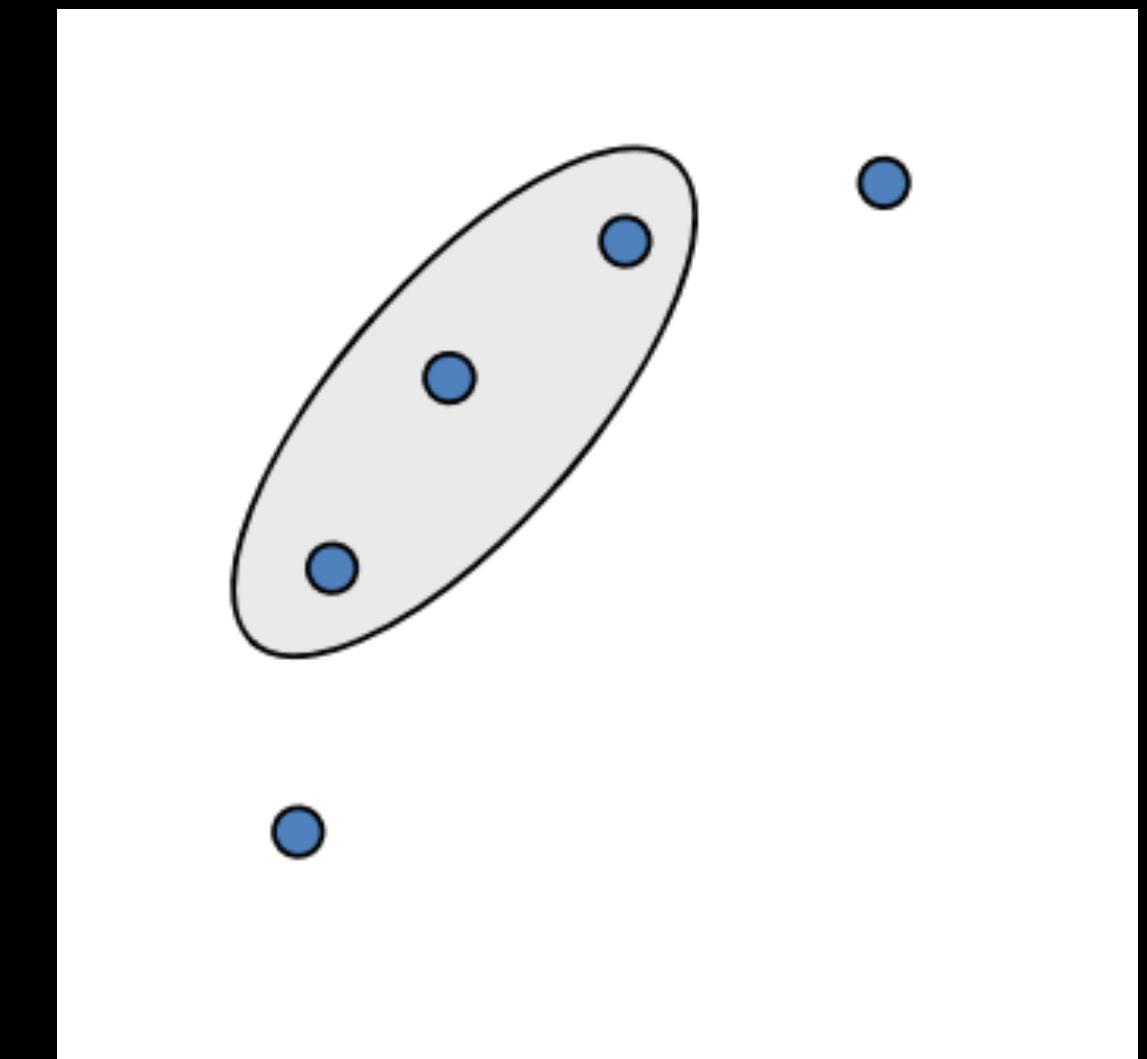
NORMAL ESTIMATION

- ▶ Find normal \mathbf{n} for each sample point \mathbf{p}
 - ▶ Examine local neighborhood for each point
 - ▶ Set of K nearest neighbors
 - ▶ Compute best approximating tangent plane
 - ▶ Covariance analysis
 - ▶ Determine normal orientation
 - ▶ Minimal Spanning Tree propagation



LOCAL NEIGHBORHOOD

- ▶ Find k nearest neighbors (kNN) of a point
 - ▶ Brute force: $O(n)$ complexity
 - ▶ Use BSP tree
 - ▶ Binary space partitioning tree
 - ▶ Recursively partition 3D space by planes
 - ▶ Tree should be balanced, put plane at median
 - ▶ $\log(n)$ tree levels, complexity $O(\log n)$



COMPUTE BEST APPROXIMATING TANGENT PLANE

- ▶ Plane center is barycenter of points

$$\mathbf{c} = \frac{1}{m} \sum_{i=1}^m \mathbf{p}_i$$

- ▶ Normal is eigenvector w.r.t. smallest eigenvalue of covariance matrix

$$\mathbf{C} = \sum_{i=1}^m (\mathbf{p}_i - \mathbf{c})(\mathbf{p}_i - \mathbf{c})^T$$

COMPUTE BEST APPROXIMATING TANGENT PLANE

- ▶ Principal Component Analysis (PCA)
- ▶ Extract points $\{q_i\}$ in neighborhood
- ▶ Compute covariance matrix M
- ▶ Analyze eigenvalues and eigenvectors of M (via SVD)
- ▶ Surface normal is estimated by eigenvector (principal axis) associated with smallest eigenvalue

$$\mathbf{M} = \frac{1}{n} \sum_{i=1}^n \begin{bmatrix} q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\ q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\ q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z \end{bmatrix}$$

Covariance Matrix

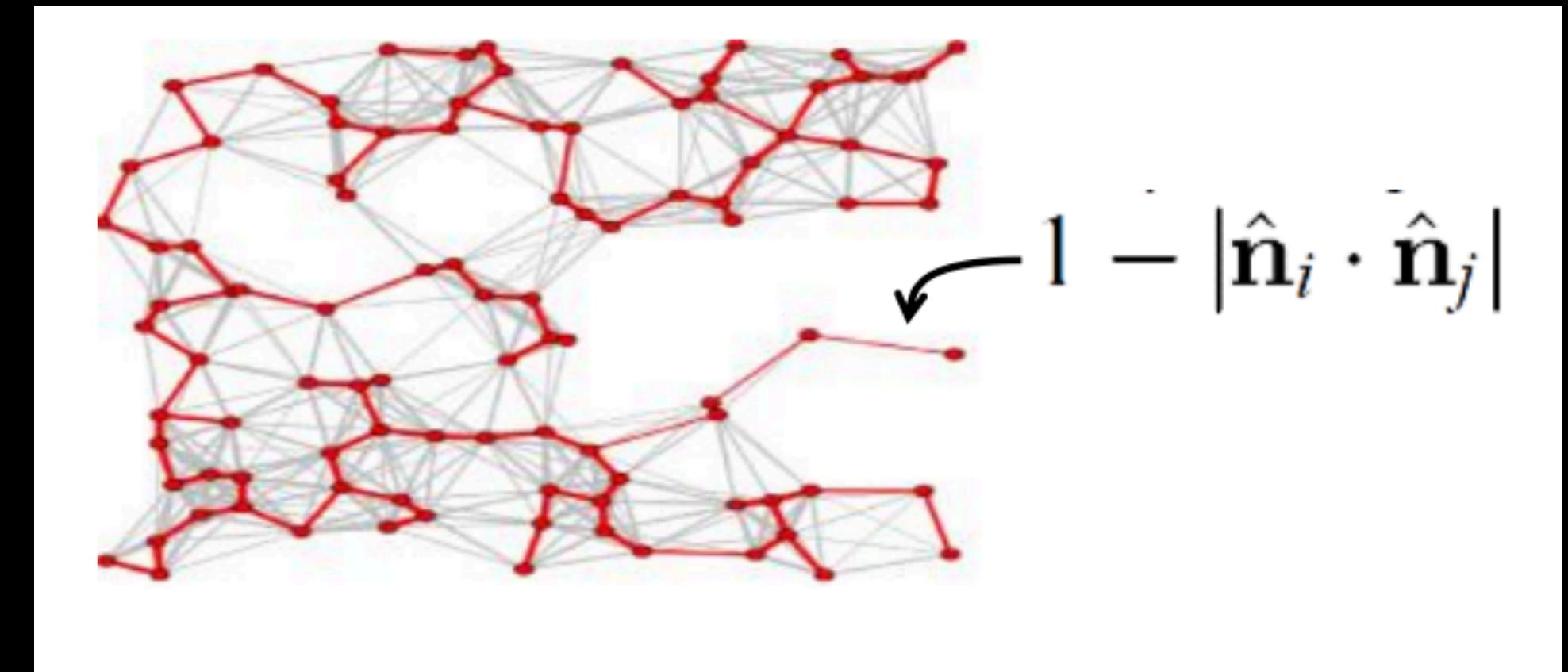
$$\mathbf{M} = \mathbf{U} \mathbf{S} \mathbf{U}^t$$

$$\mathbf{S} = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix}$$

Eigenvalues & Eigenvectors

NORMAL ORIENTATION

- ▶ Normals might not be consistent
- ▶ General Pointcloud
 - ▶ Traverse nearest neighbor graph flipping normals for consistency
 - ▶ Greedy propagation algorithm
 - ▶ (minimum spanning tree of normal similarity)
- ▶ In our case:
 - ▶ We can check that the normal of the point looks towards the camera where it's visible



HOPPE

HOPPE ET AL. SIGGRAPH 1992

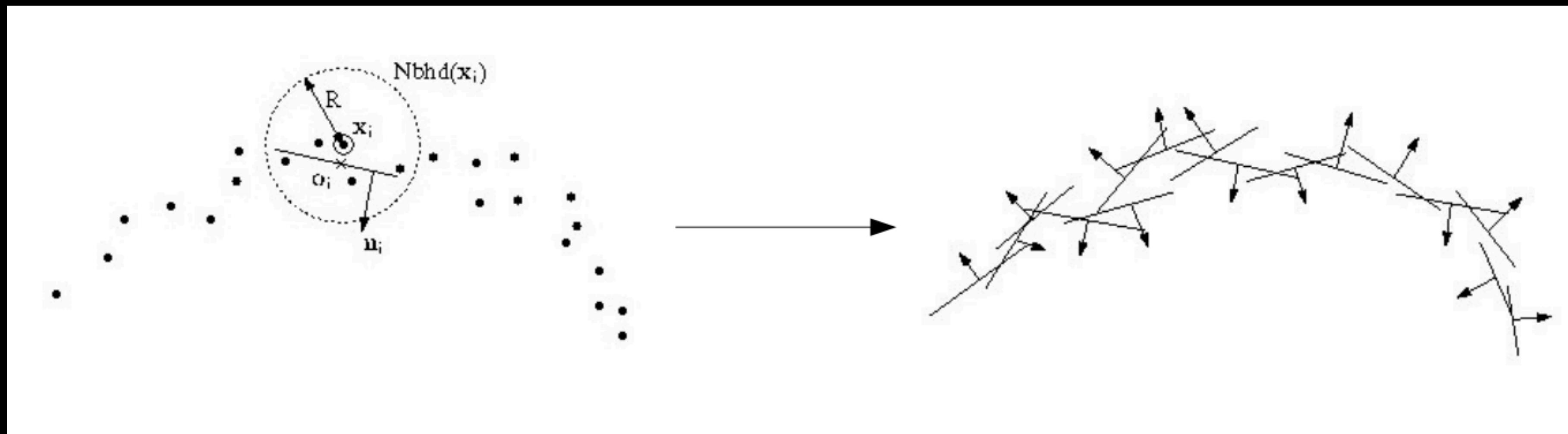
- ▶ “Surface reconstruction from unorganized points”
- ▶ Input:
 - ▶ No geometry (normals), topology or boundaries information: inferred from the point set
 - ▶ Sampling density and max error known
- ▶ Output: a meshed surface
 - ▶ Compact, connected, orientable 2-manifold
 - ▶ Not necessarily triangles

HOPPE ET AL. SIGGRAPH 1992

- ▶ 4 stages:
 - ▶ 1. Estimate tangent plane for each input point
 - ▶ Local linear approximation of the surface
 - ▶ 2. Establish consistent orientation for nearby planes
 - ▶ => consistent orientation for the whole surface
 - ▶ 3. Compute signed distances on a voxel grid
 - ▶ 4. Extract an isosurface
 - ▶ Distance function: $f \sim$ signed Euclidean distance to the input unknown surface

STAGE 1: TANGENT PLANE

- ▶ Nearest neighbors approach
 - ▶ $R = d+e$, d = sampling density, e = max error
 - ▶ Approximating plane found by a least square approximation



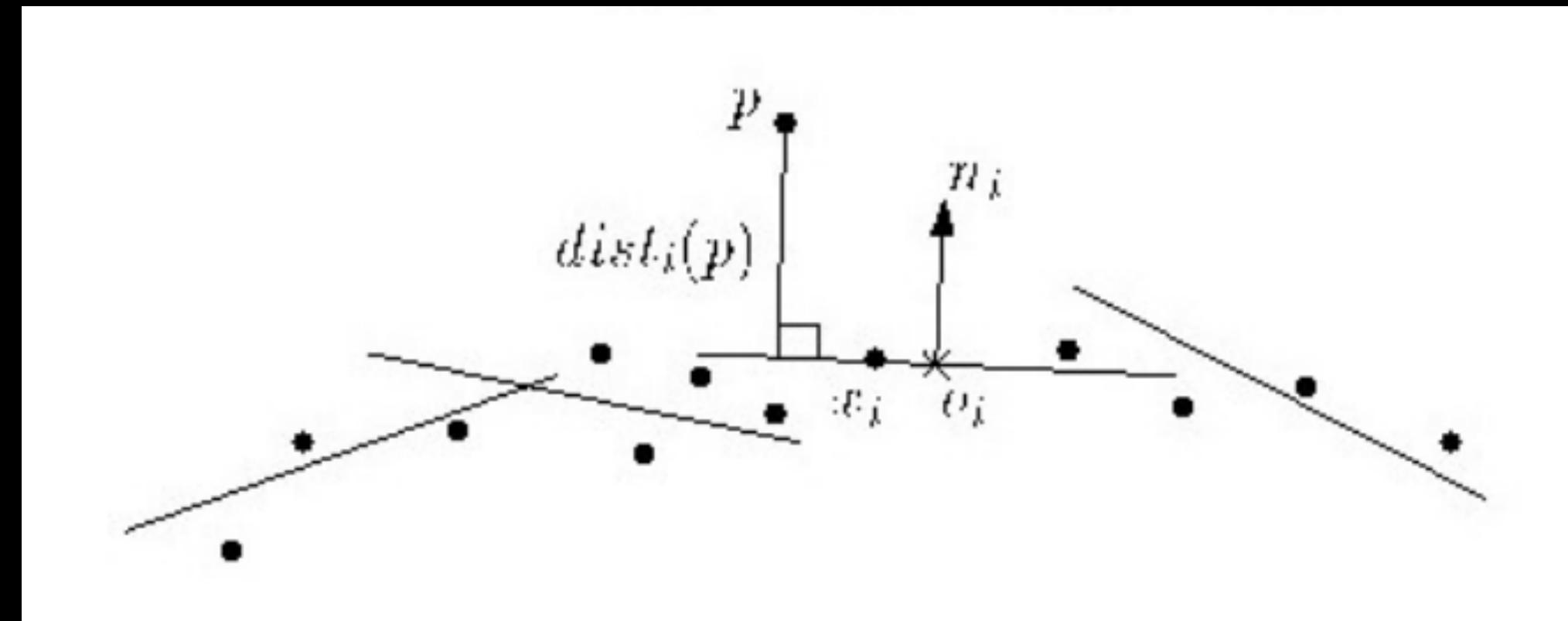
STAGE 2: ORIENTATION CONSISTENCY

- ▶ Graph optimization approach
 - ▶ One node/plane, one edge p_1-p_2 if centroids are close
 - ▶ Edge weight = scalar product of plane normals
 - ▶ Maximize total weight of the graph
 - ▶ NP-complete => a little more complicated than that

STAGE 3: SIGNED DISTANCES

- ▶ Signed distance from each point to the closest plane:

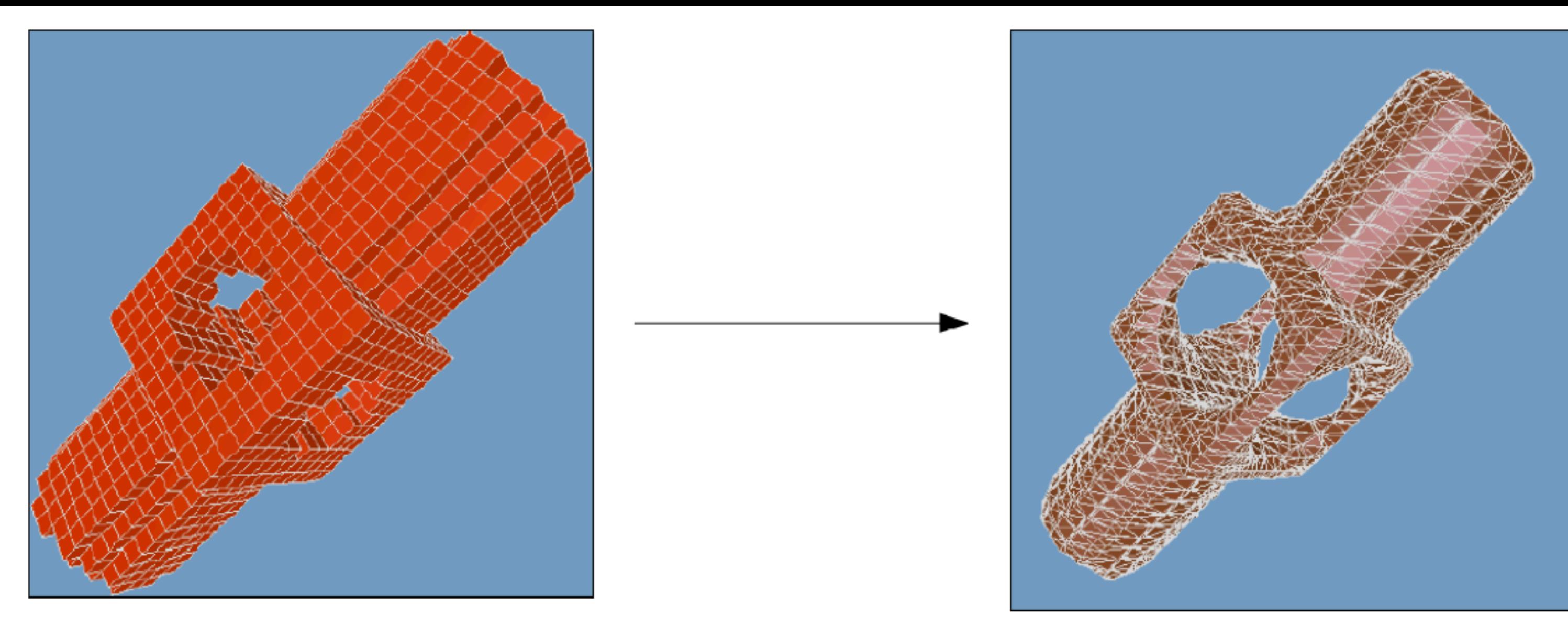
$$\text{dist}_i(\mathbf{p}) = (\mathbf{p} - \mathbf{o}_i) \cdot \mathbf{n}_i$$



- ▶ Surfaces with boundaries: points too far away are assigned an “undefined” distance
- ▶ Distance sampled at the vertices of a voxel grid

STAGE 4: ISOSURFACE EXTRACTION

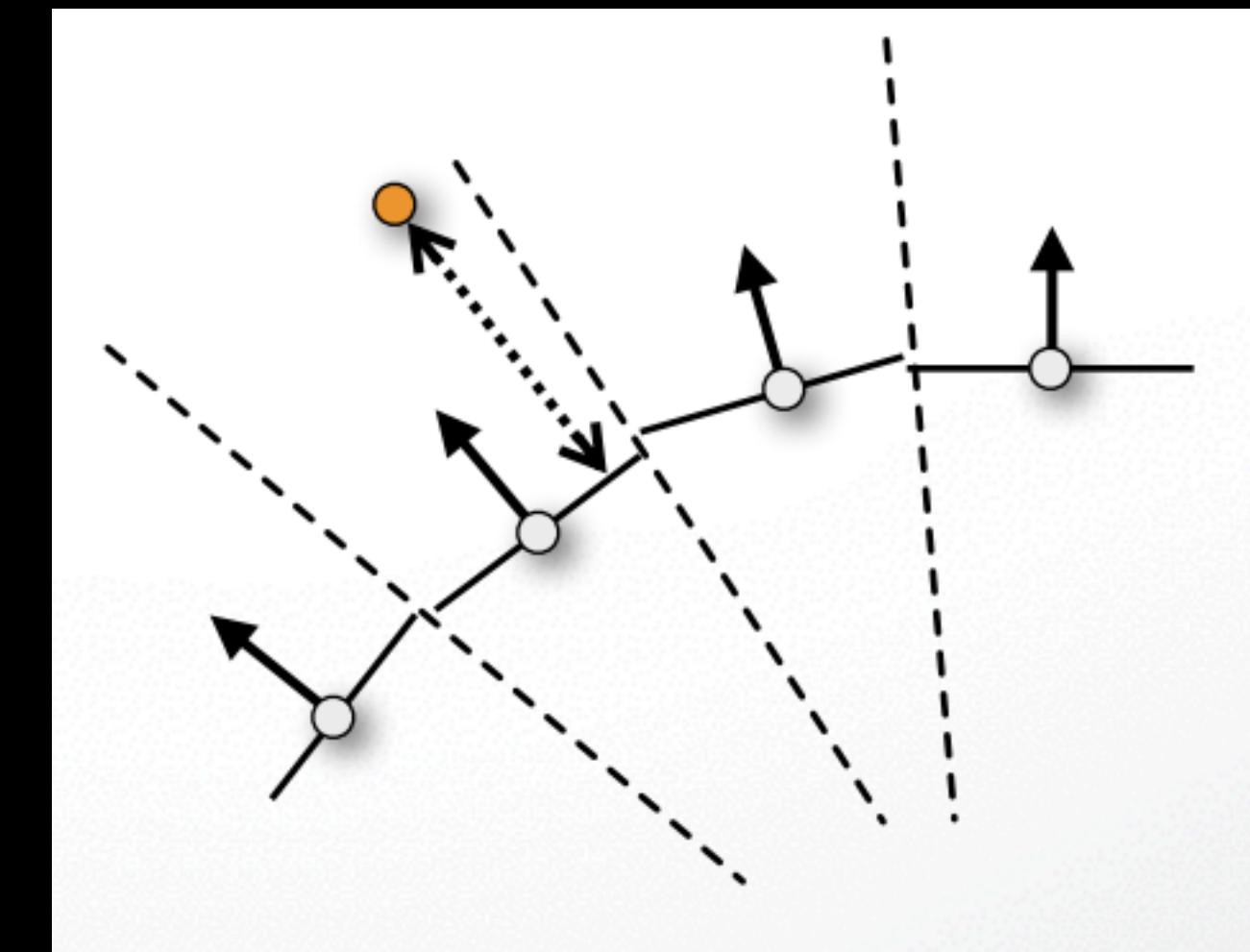
- ▶ Well-known Marching Cubes
- ▶ Lorensen & Cline, SIGGRAPH 1987
- ▶ When “undefined”: no triangle



SIGNED DISTANCE FUNCTION

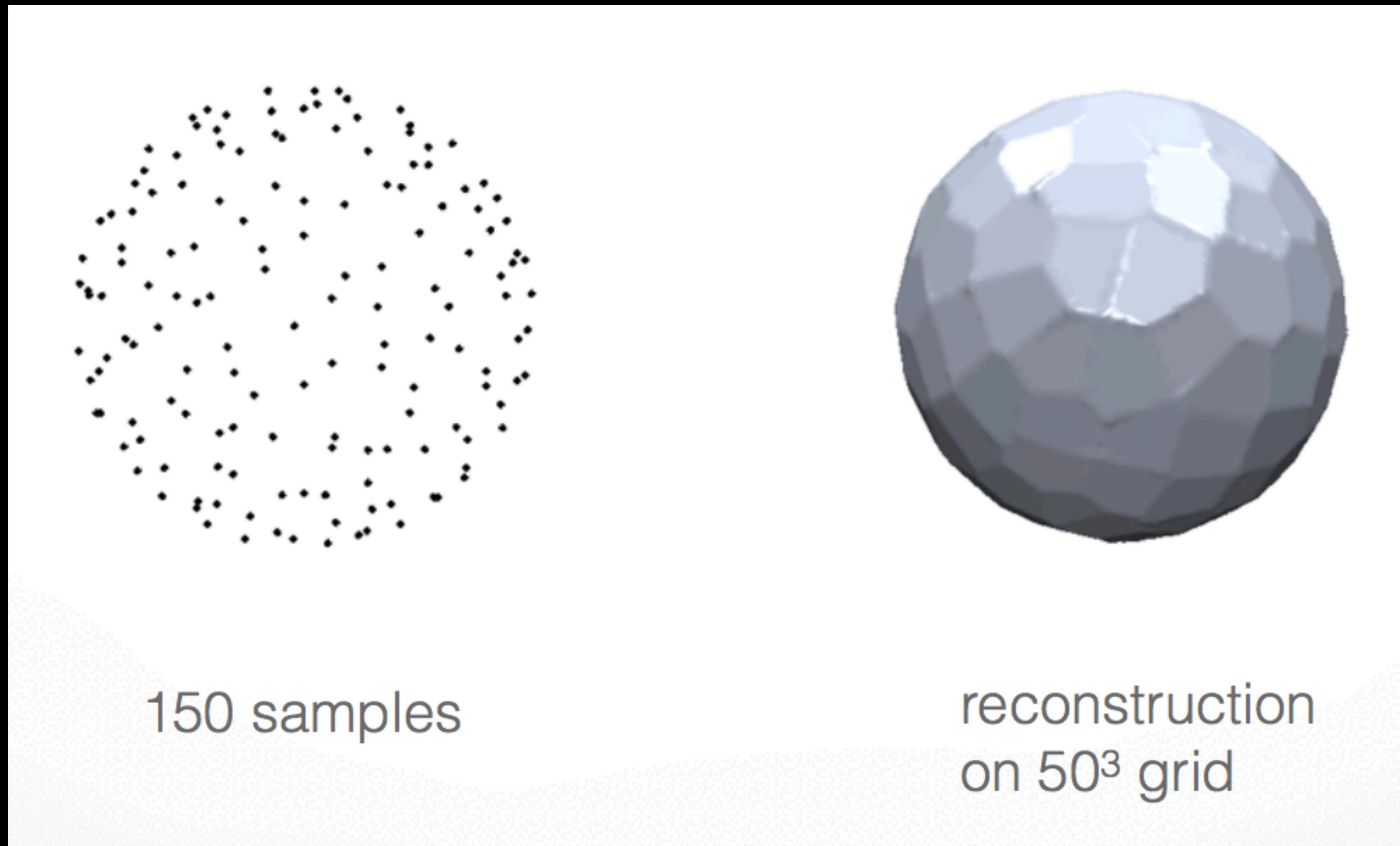
SIGNED DISTANCE FUNCTION

- ▶ Points + normals determine local tangent planes
- ▶ Use distance from closest point's tangent plane
- ▶ Linear approximation in Voronoi cell
- ▶ Simple and efficient, but...



TEXT

SIGNED DISTANCE FUNCTION

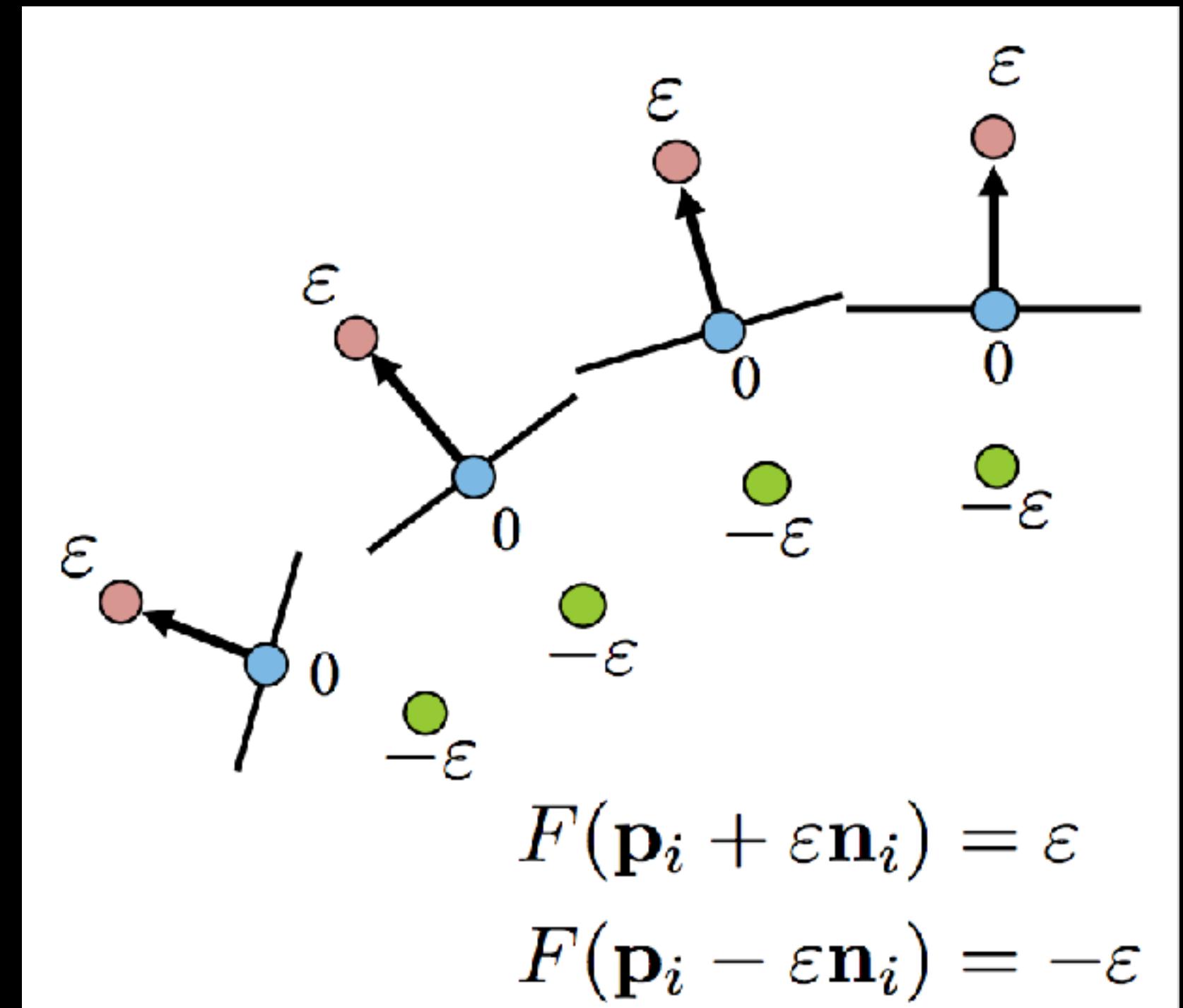


SMOOTH SDF

- ▶ Instead find a smooth formulation for F .
- ▶ Requiring:
- ▶ $F(p) = 0$ (interpolating)
- ▶ F is smooth.
- ▶ Avoid trivial solution $F \equiv 0$. (0 everywhere in space)

SMOOTH SDF

- ▶ Instead find a smooth formulation for F .
- ▶ Requiring:
- ▶ $F(\mathbf{p}) = 0$ (interpolating)
- ▶ F is smooth.
- ▶ Avoid trivial solution $F \equiv 0$. (0 everywhere in space)



RADIAL BASIS FUNCTION INTERPOLATION

- RBF: Weighted sum of shifted, smooth kernels

$$F(x) = \sum_{i=0}^{N-1} w_i \varphi(\|x - c_i\|)$$

Scalar weights
Unknowns

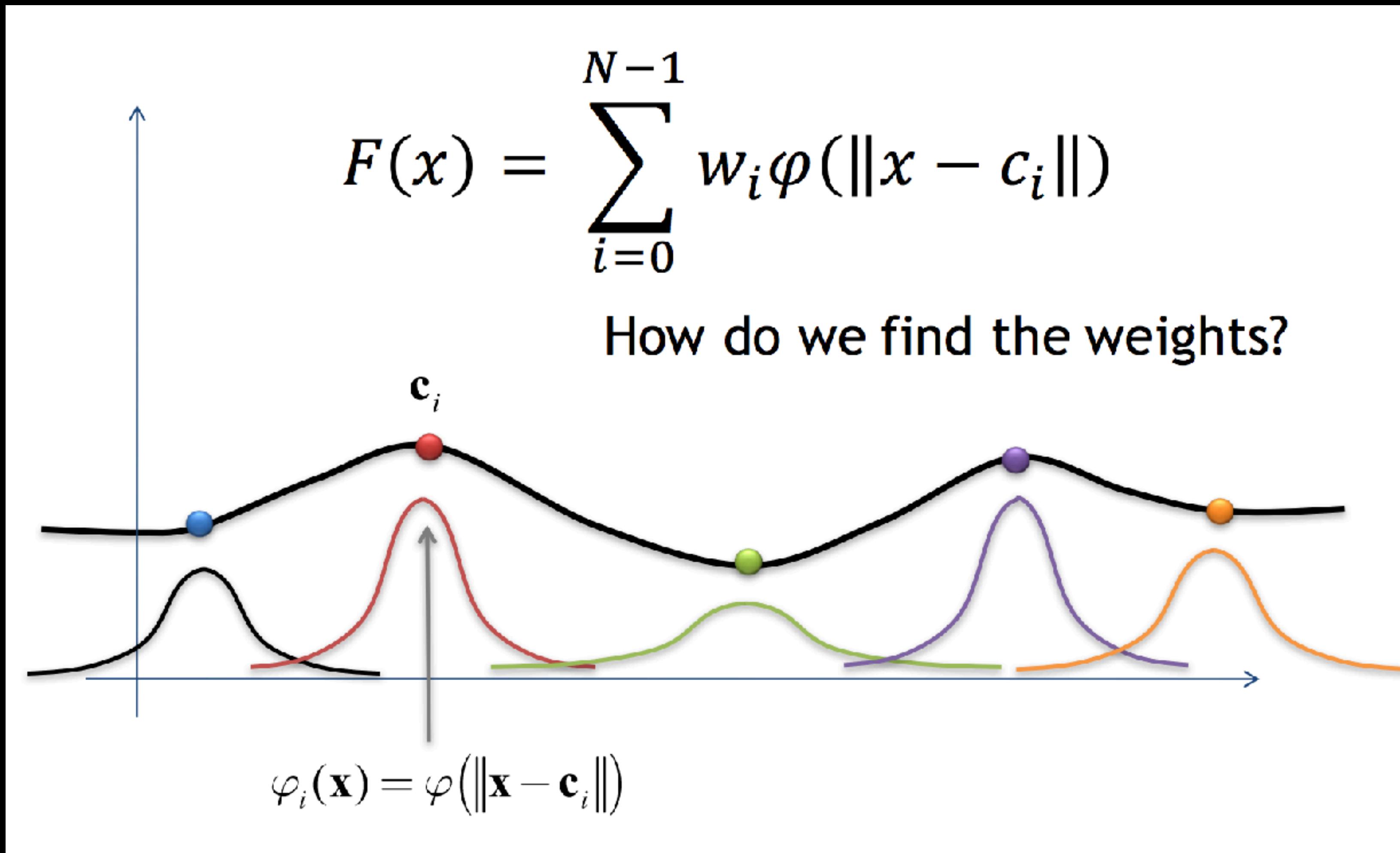
Smooth kernels
(basis functions)
centered at constrained
points.

For example:

$$\varphi(r) = r^3$$

$$N = 3n$$

RADIAL BASIS FUNCTION INTERPOLATION



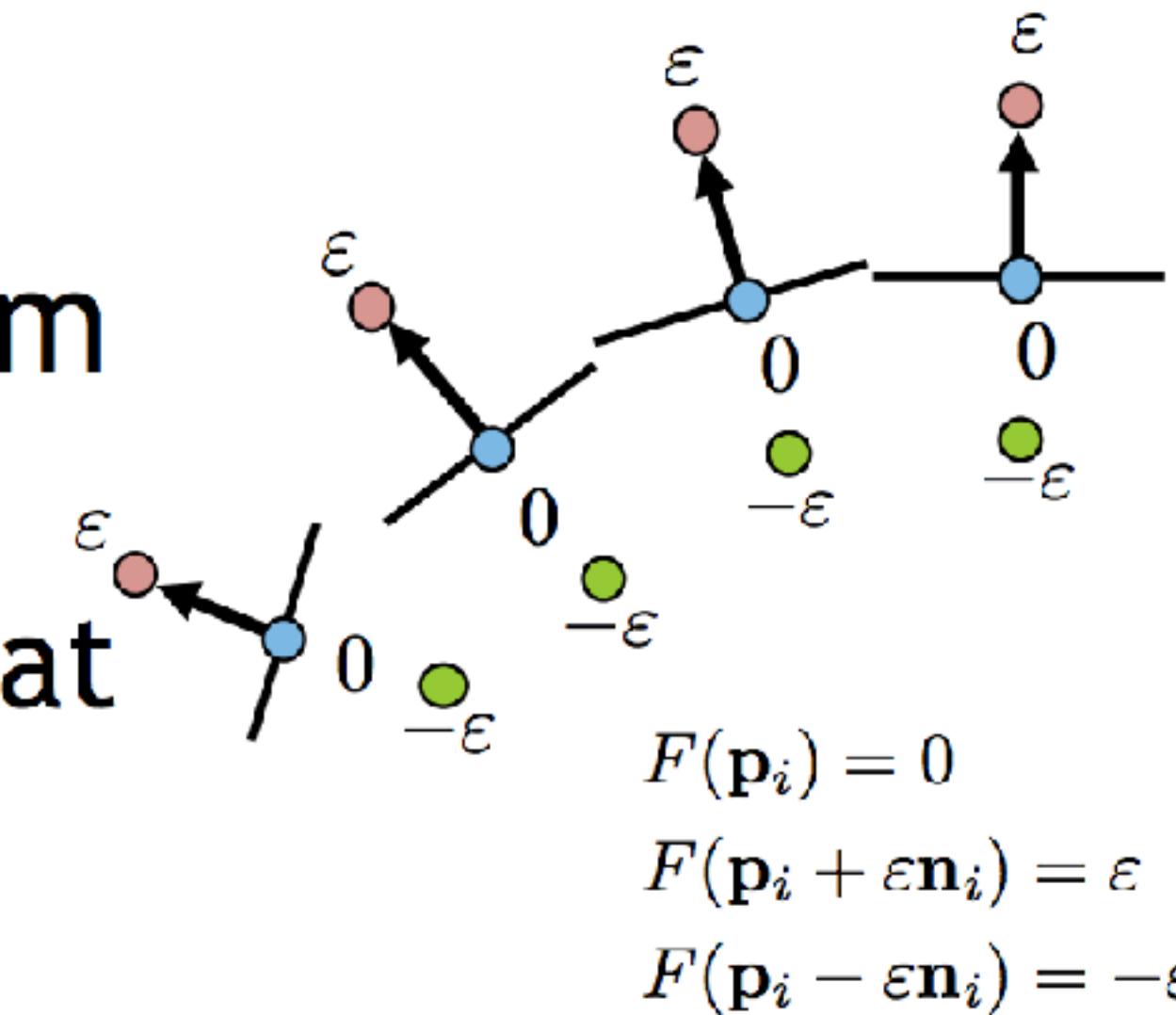
RADIAL BASIS FUNCTION INTERPOLATION

- ▶ Interpolate the constraints:

$$\{\mathbf{c}_{3i}, \mathbf{c}_{3i+1}, \mathbf{c}_{3i+2}\} = \{\mathbf{p}_i, \mathbf{p}_i + \varepsilon \mathbf{n}_i, \mathbf{p}_i - \varepsilon \mathbf{n}_i\}$$

$$\forall j = 0, \dots, N-1, \quad \sum_{i=0}^{N-1} w_i \varphi(\|\mathbf{c}_j - \mathbf{c}_i\|) = d_j$$

- In words: we know the desired function value (sum of RBF) at each center.
 - We solve for the weights that do that!



RADIAL BASIS FUNCTION INTERPOLATION

- ▶ Interpolate the constraints:

$$\{\mathbf{c}_{3i}, \mathbf{c}_{3i+1}, \mathbf{c}_{3i+2}\} = \{\mathbf{p}_i, \mathbf{p}_i + \varepsilon \mathbf{n}_i, \mathbf{p}_i - \varepsilon \mathbf{n}_i\}$$

- ▶ Symmetric linear system to get the weights:

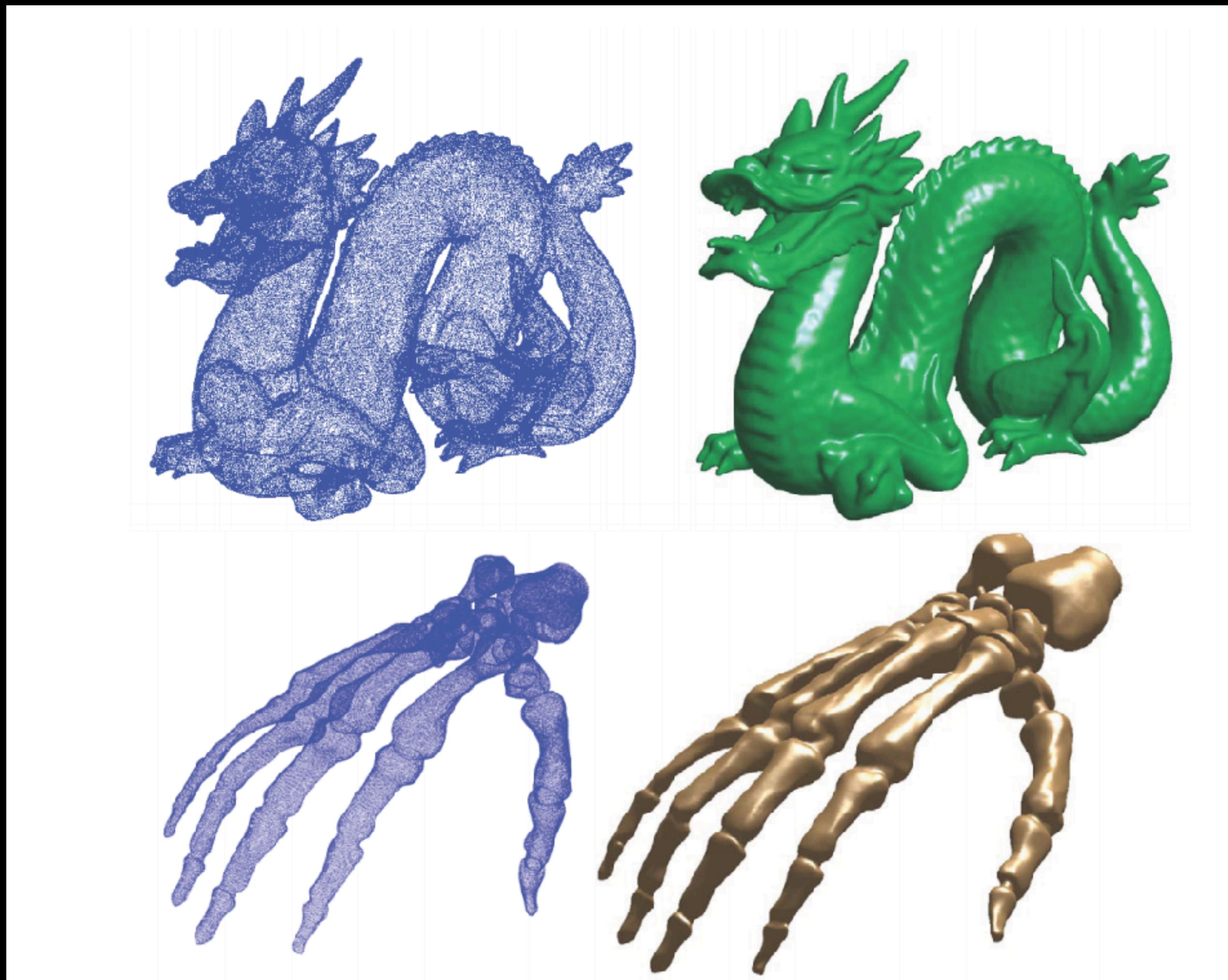
$$\begin{pmatrix} \varphi(\|\mathbf{c}_0 - \mathbf{c}_0\|) & \dots & \varphi(\|\mathbf{c}_0 - \mathbf{c}_{N-1}\|) \\ \vdots & \ddots & \vdots \\ \varphi(\|\mathbf{c}_{N-1} - \mathbf{c}_0\|) & \dots & \varphi(\|\mathbf{c}_{N-1} - \mathbf{c}_{N-1}\|) \end{pmatrix} \begin{pmatrix} w_0 \\ \vdots \\ w_{N-1} \end{pmatrix} = \begin{pmatrix} d_0 \\ \vdots \\ d_{N-1} \end{pmatrix}$$

RBF KERNELS

- ▶ Triharmonic: $\varphi(r) = r^3$
- ▶ Globally supported.
- ▶ Leads to dense symmetric linear system.
- ▶ C^2 smoothness.
- ▶ Works well for highly irregular sampling.

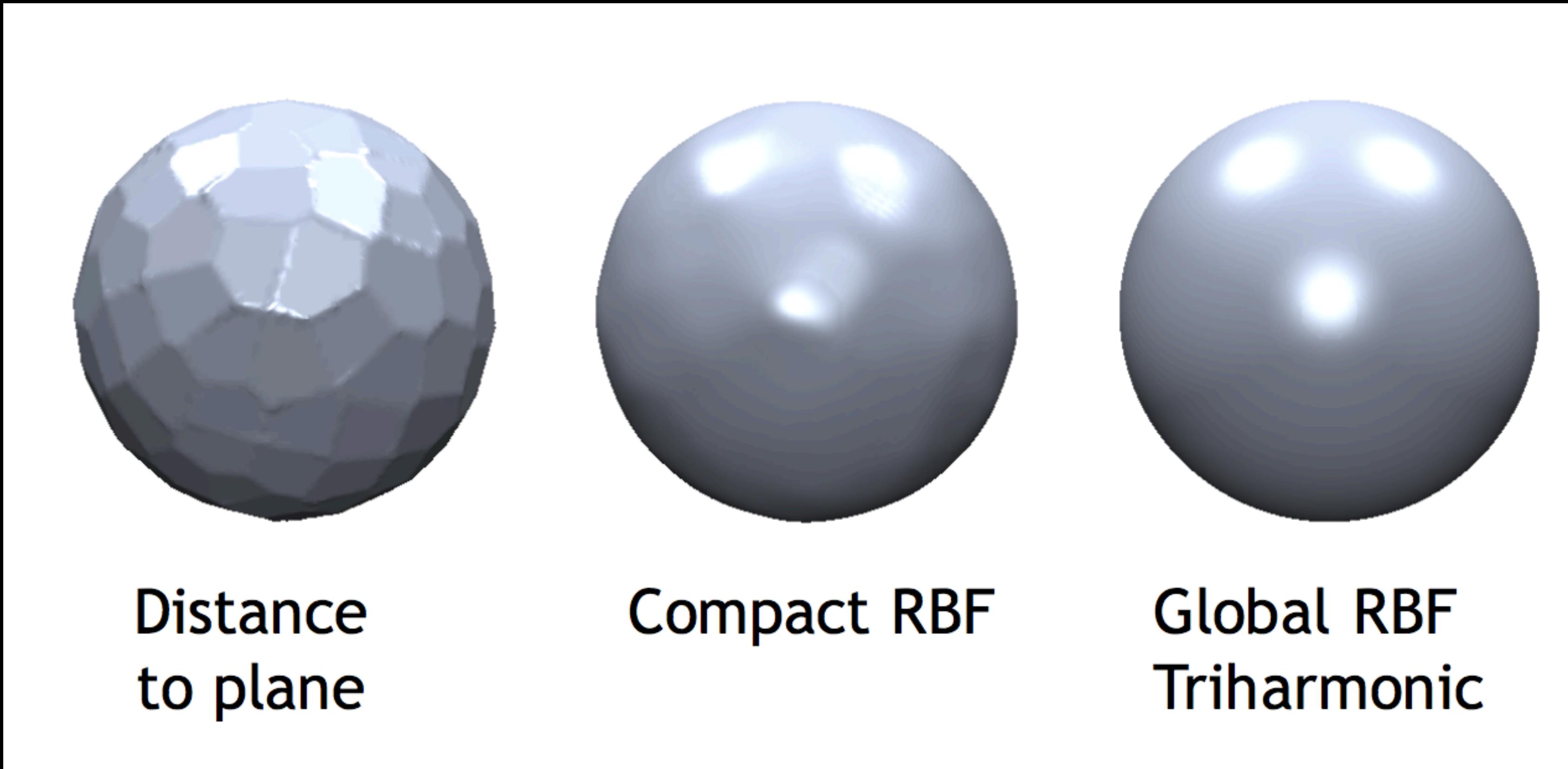
TEXT

RECONSTRUCTION RESULTS WITH RBF



TEXT

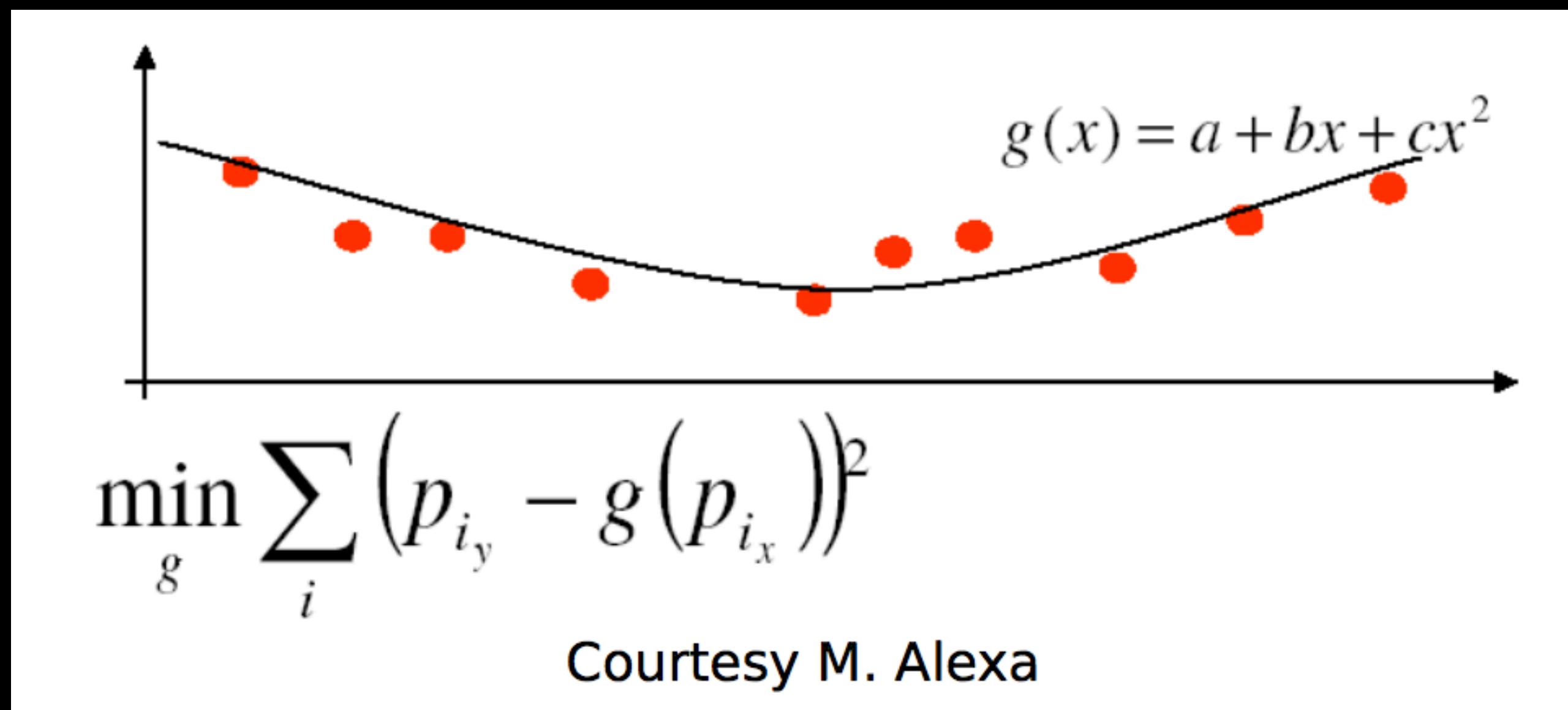
RBF KERNELS



MOVING LEAST SQUARES

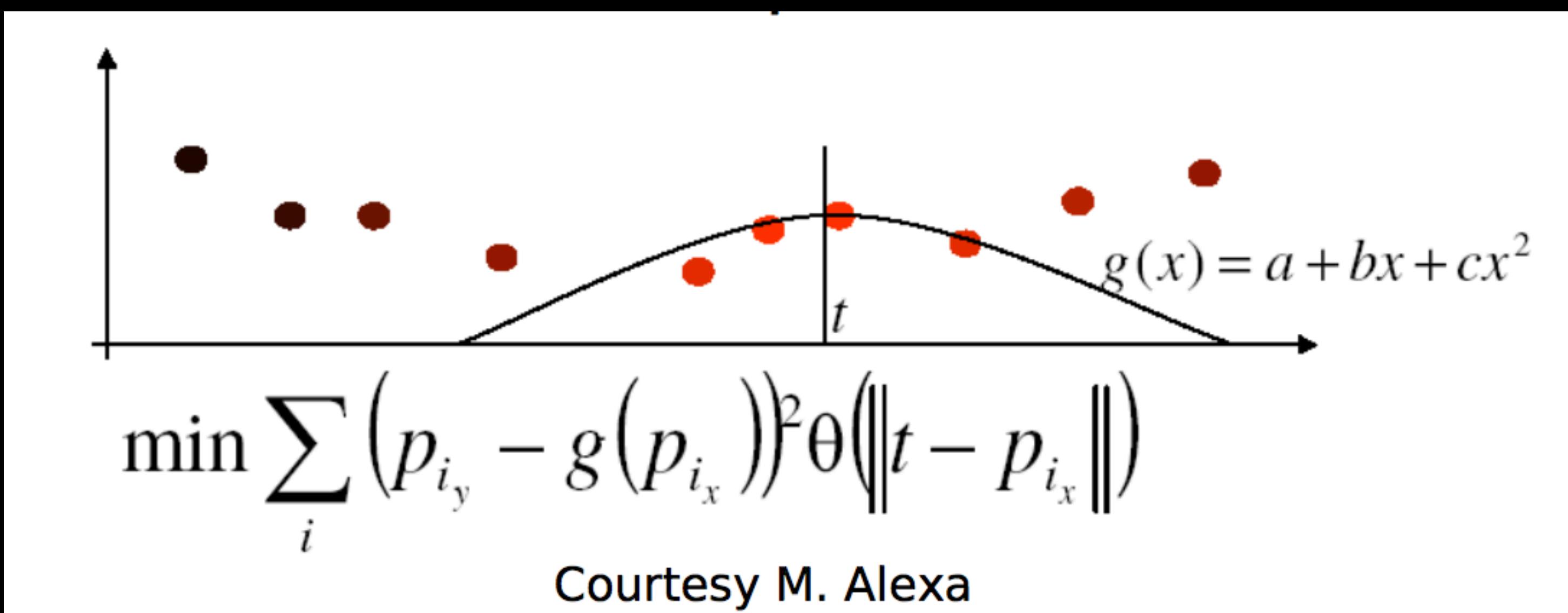
LEAST SQUARES

- ▶ Goal: fit a primitive (e.g. Polynomial function) to scattered data
- ▶ Idea: minimize square distance between the point's values and the primitive



MOVING LEAST SQUARES

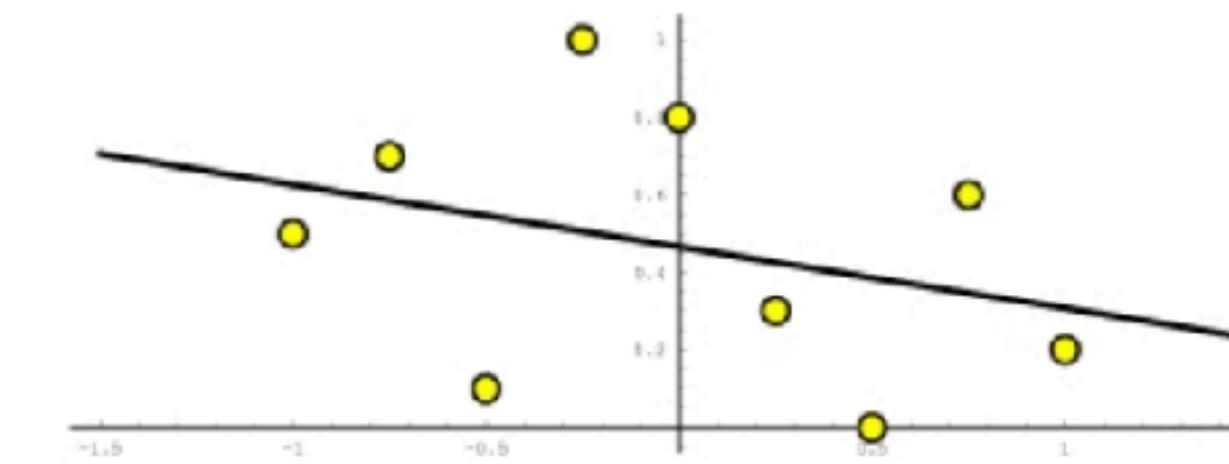
- ▶ Compute a local LS approximation at t
- ▶ Weight points based on distance to t
- ▶ Decrease when going far from t



MOVING LEAST SQUARES

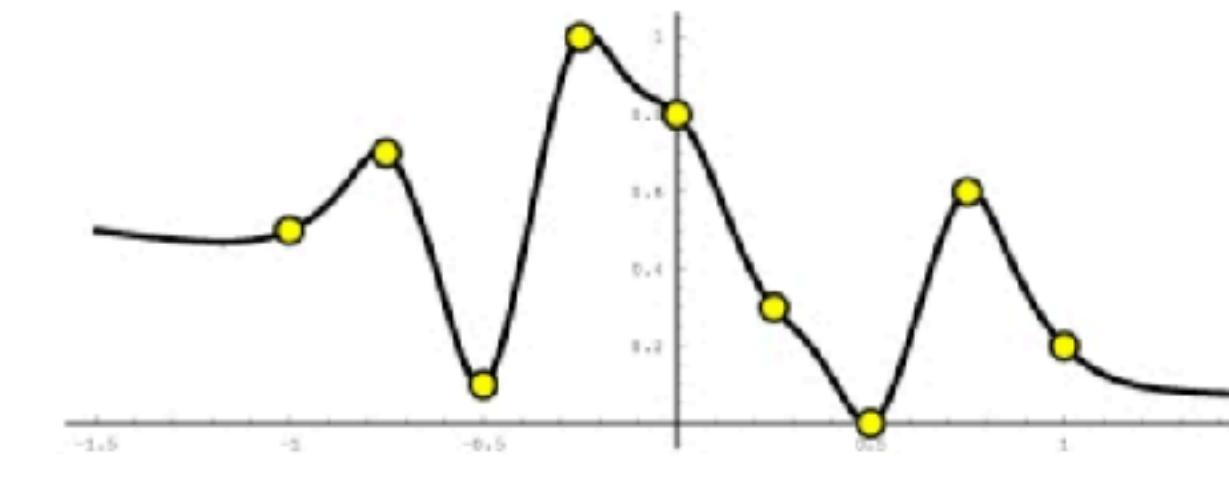
- Weighted LS where the weights depend on position

- Global least squares with linear basis



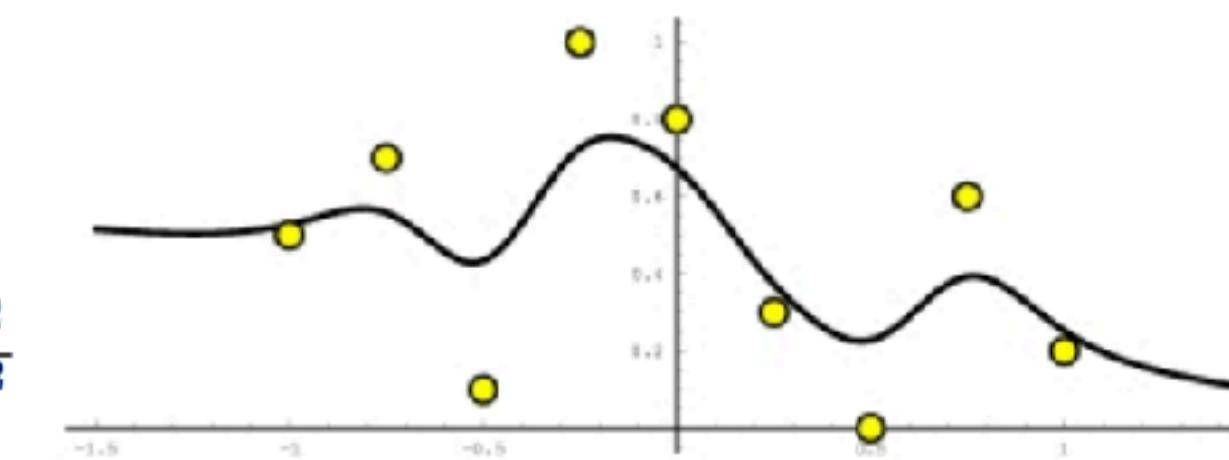
- MLS with (nearly) singular weight function

$$\theta(r) = \frac{1}{r^2 + \varepsilon^2}$$



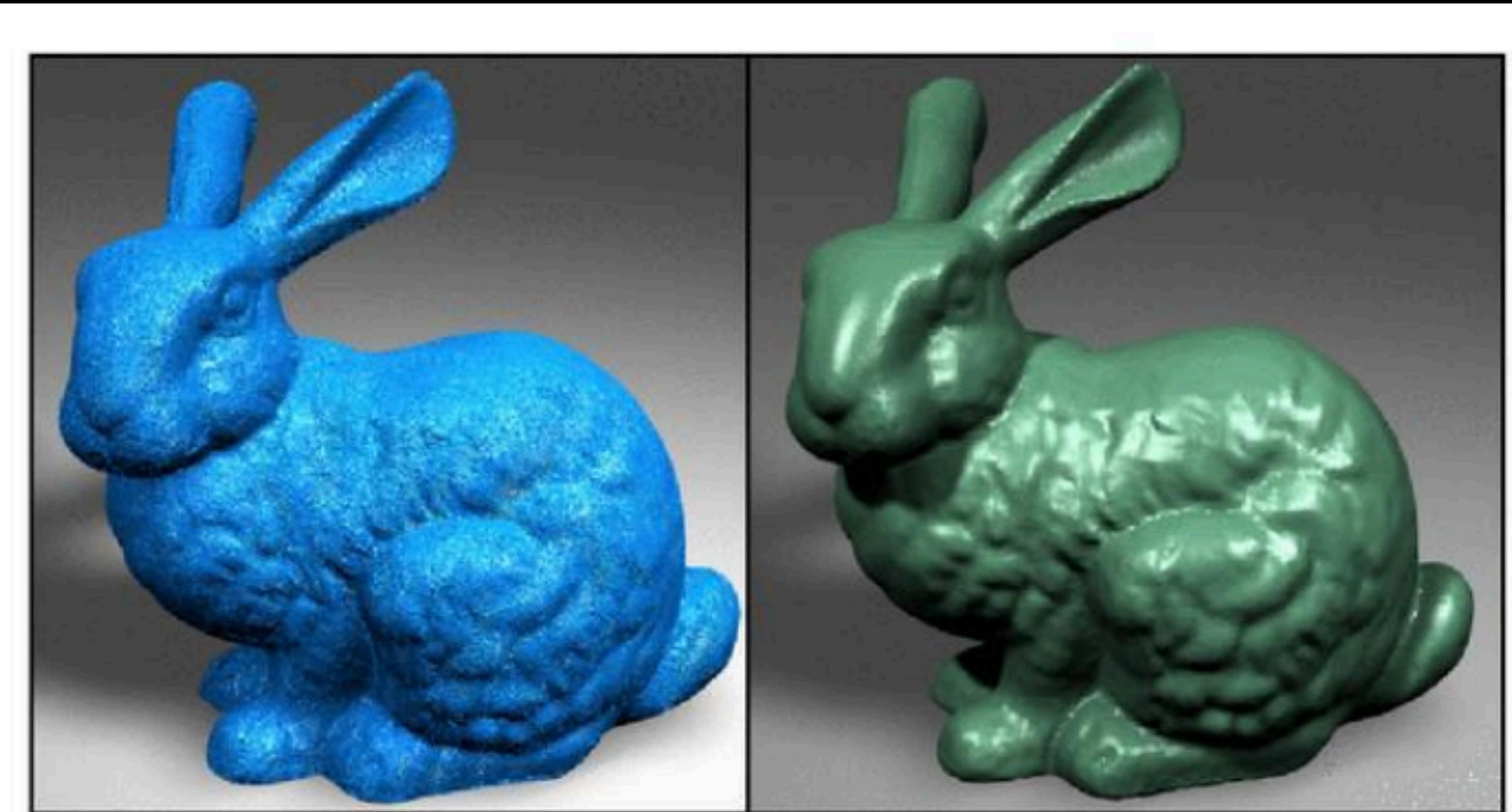
- MLS with approximating weight function $\theta(r) = e^{-\frac{r^2}{h^2}}$

$$\theta(r) = e^{-\frac{r^2}{h^2}}$$



TEXT

MOVING LEAST SQUARES



MLS

POISSON RECONSTRUCTION

THE INDICATOR FUNCTION

- We reconstruct the surface of the model by solving for the indicator function of the shape.

$$\chi_M(p) = \begin{cases} 1 & \text{if } p \in M \\ 0 & \text{if } p \notin M \end{cases}$$

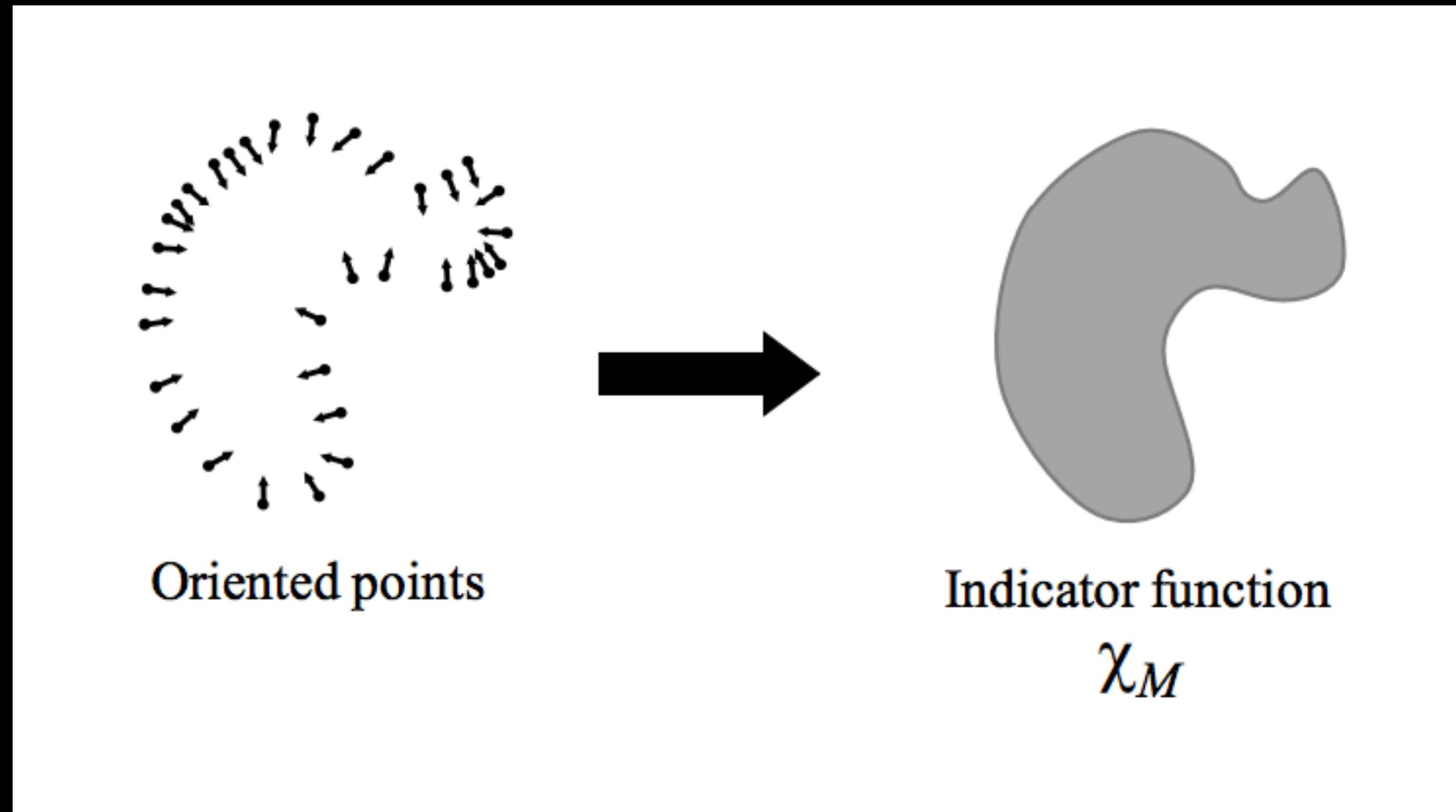


Indicator function

χ_M

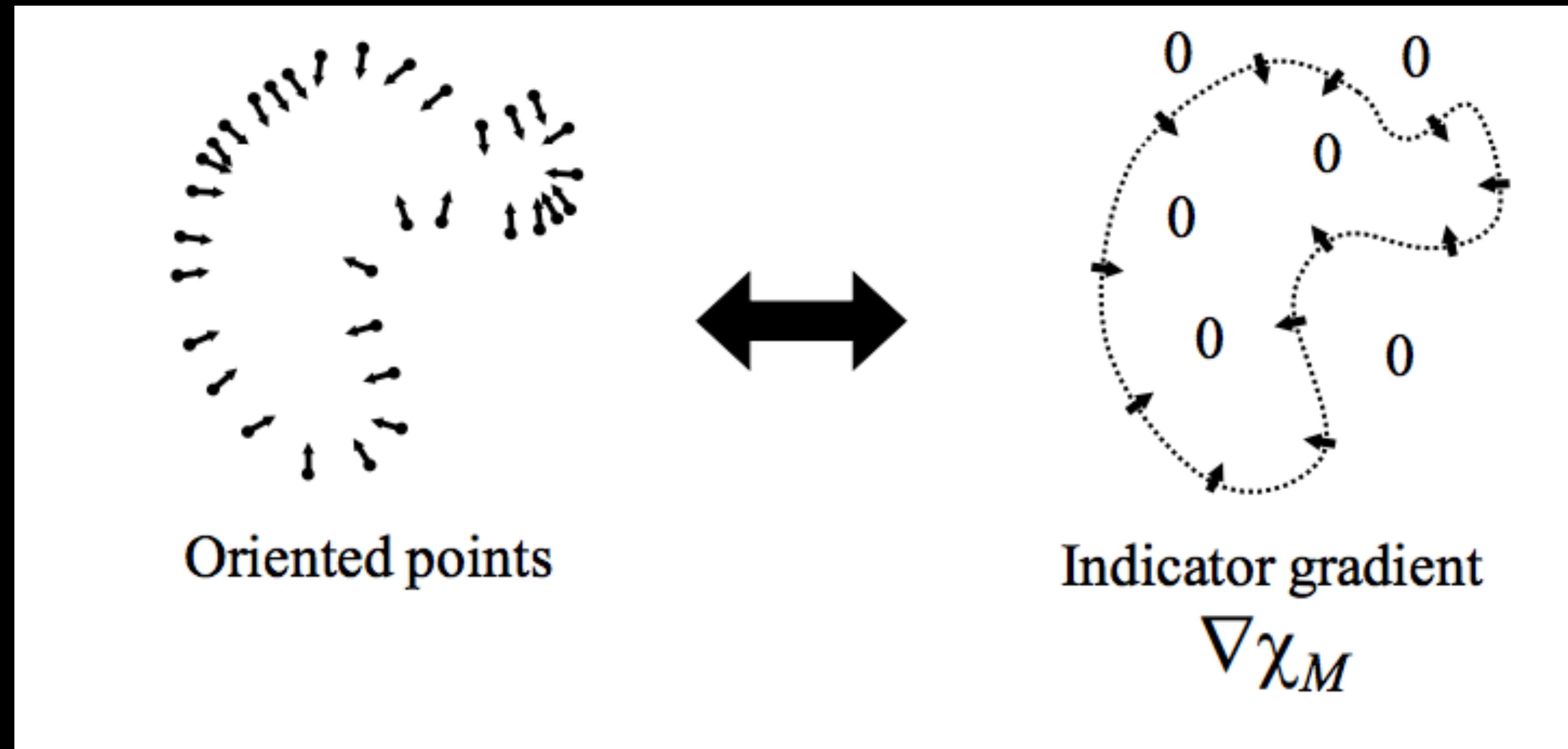
CHALLENGE

- ▶ How to construct the indicator function?



GRADIENT RELATIONSHIP

- ▶ There is a relationship between the normal field and gradient of indicator function



INTEGRATION

- ▶ Represent the points by a vector field V
- ▶ Find the function X whose gradient best approximates :

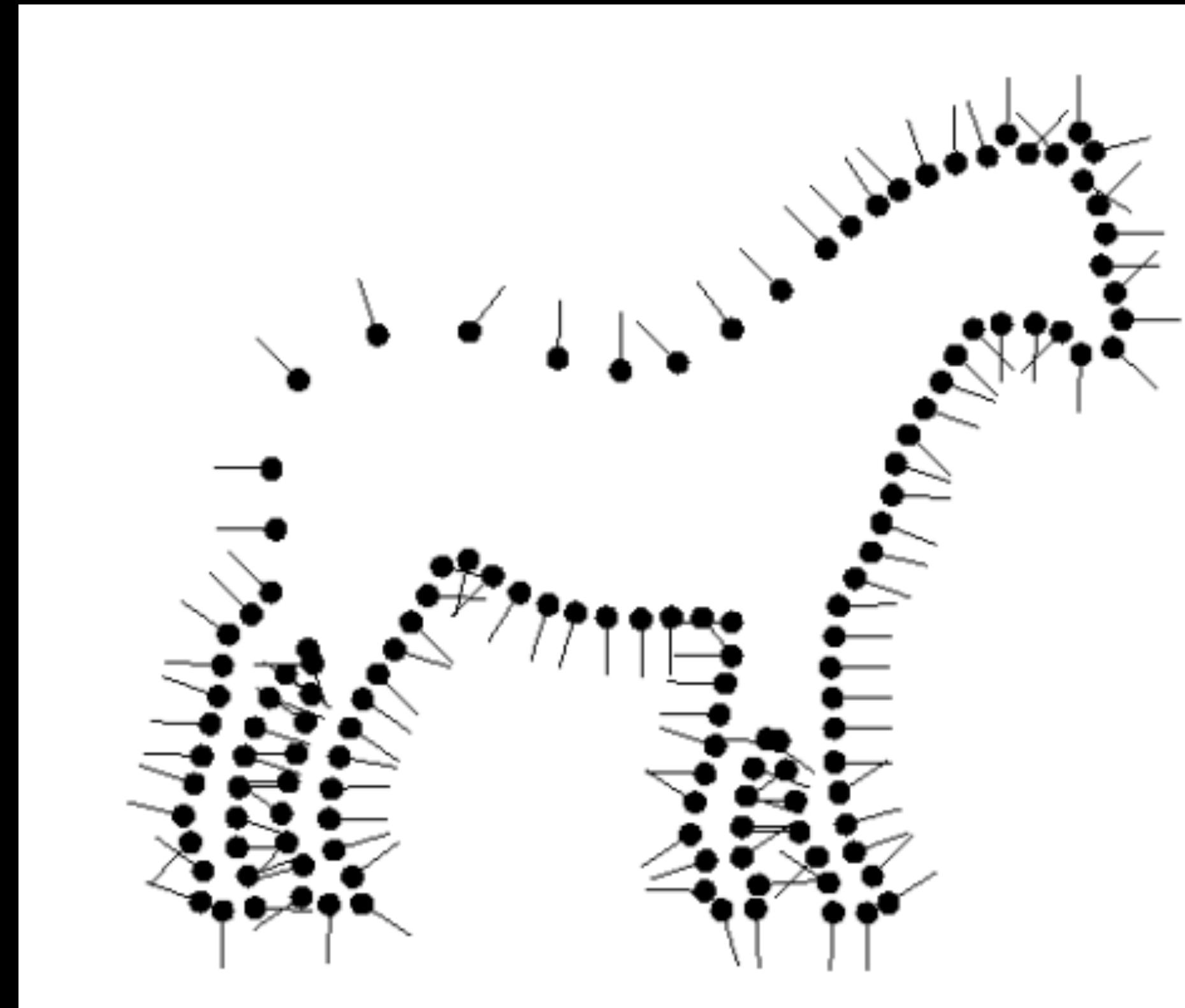
$$\min_{\chi} \|\nabla \chi - \vec{V}\|$$

- ▶ Applying the divergence operator, we can transform this into a Poisson problem:

$$\nabla \cdot (\nabla \chi) = \nabla \cdot \vec{V} \quad \Leftrightarrow \quad \Delta \chi = \nabla \cdot \vec{V}$$

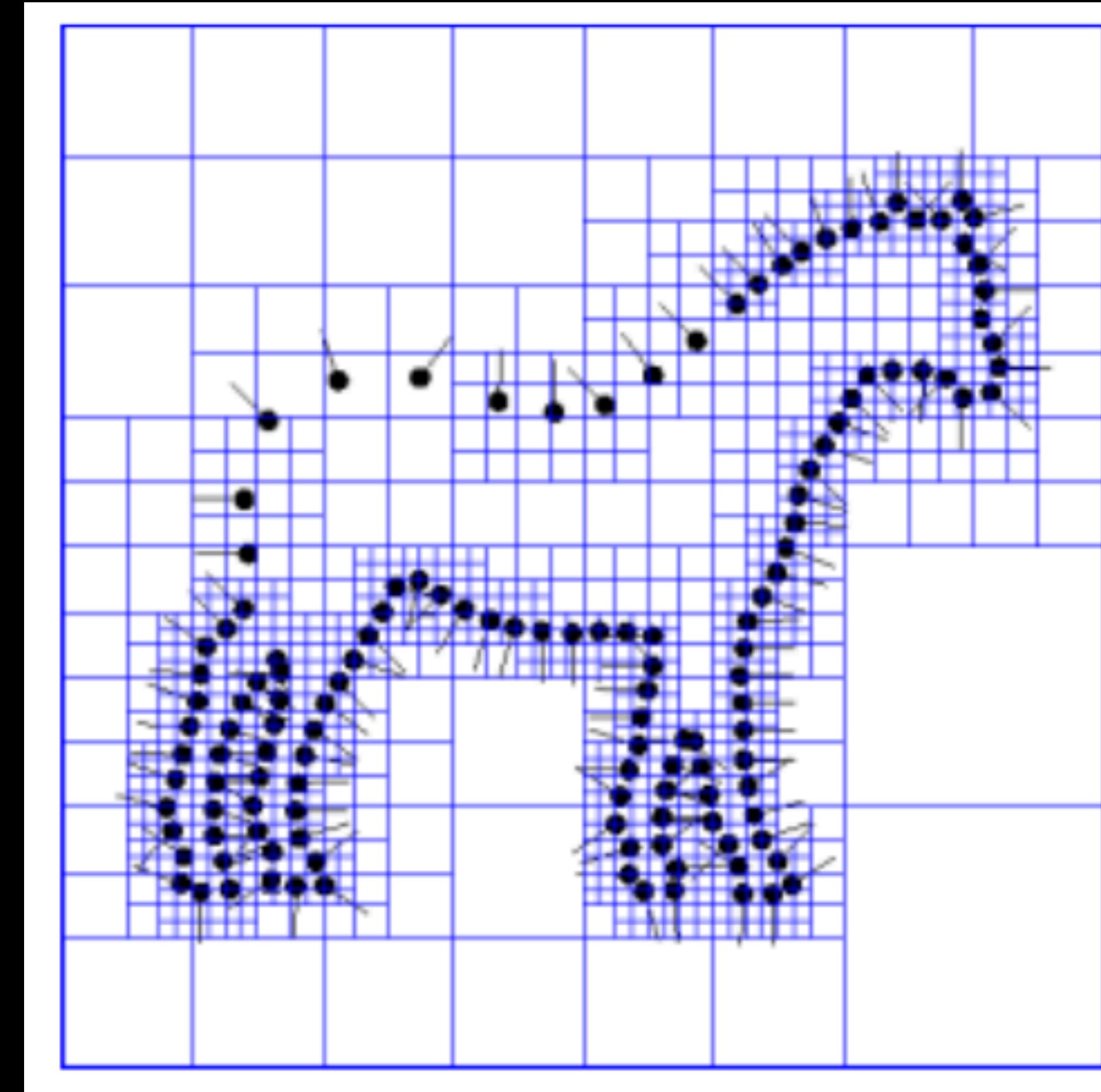
INTEGRATION

- ▶ Given the Points:
 - ▶ Set octree
 - ▶ Compute vector field
 - ▶ Compute indicator function
 - ▶ Extract iso-surface



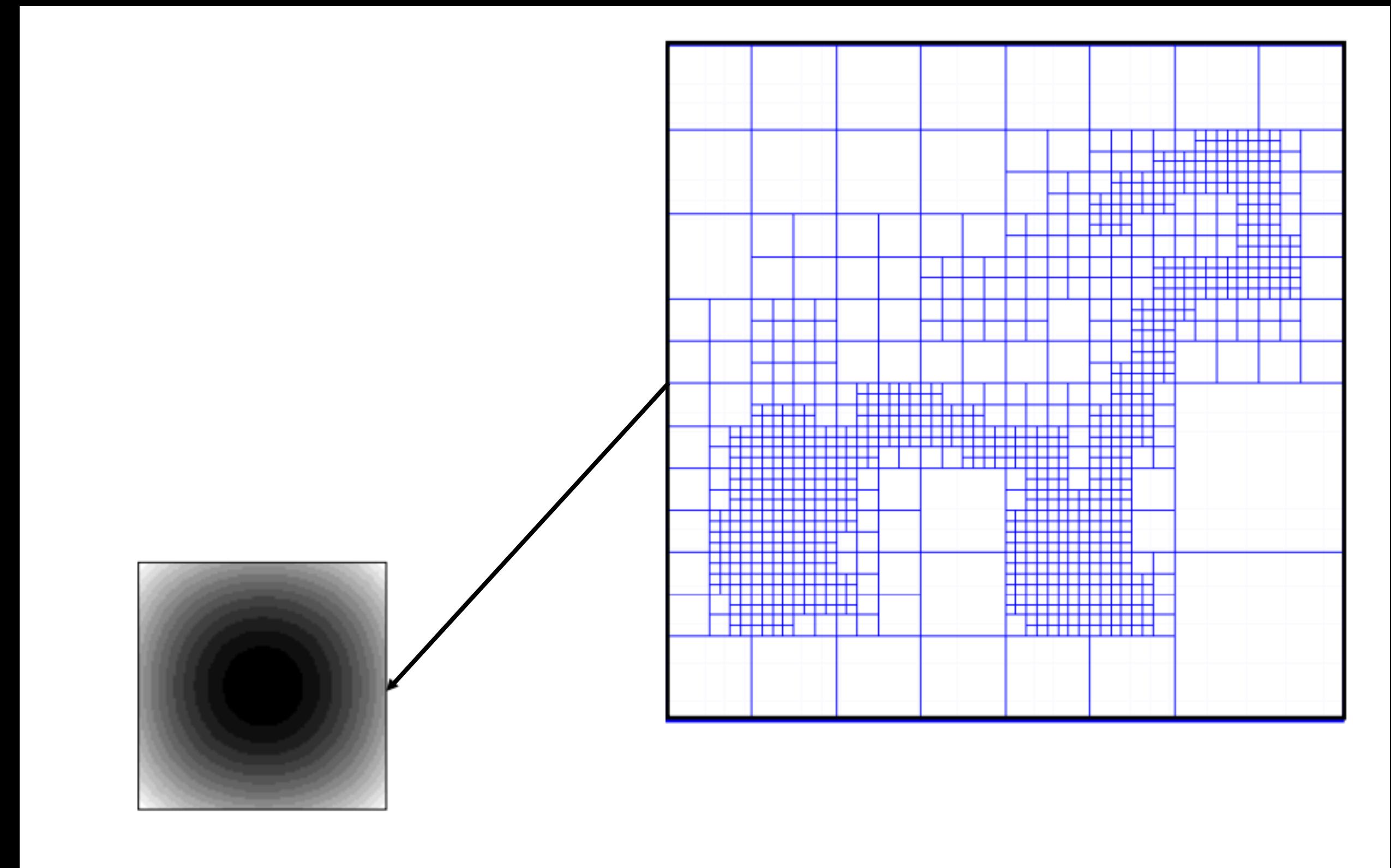
INTEGRATION

- ▶ Given the Points:
- ▶ Set octree



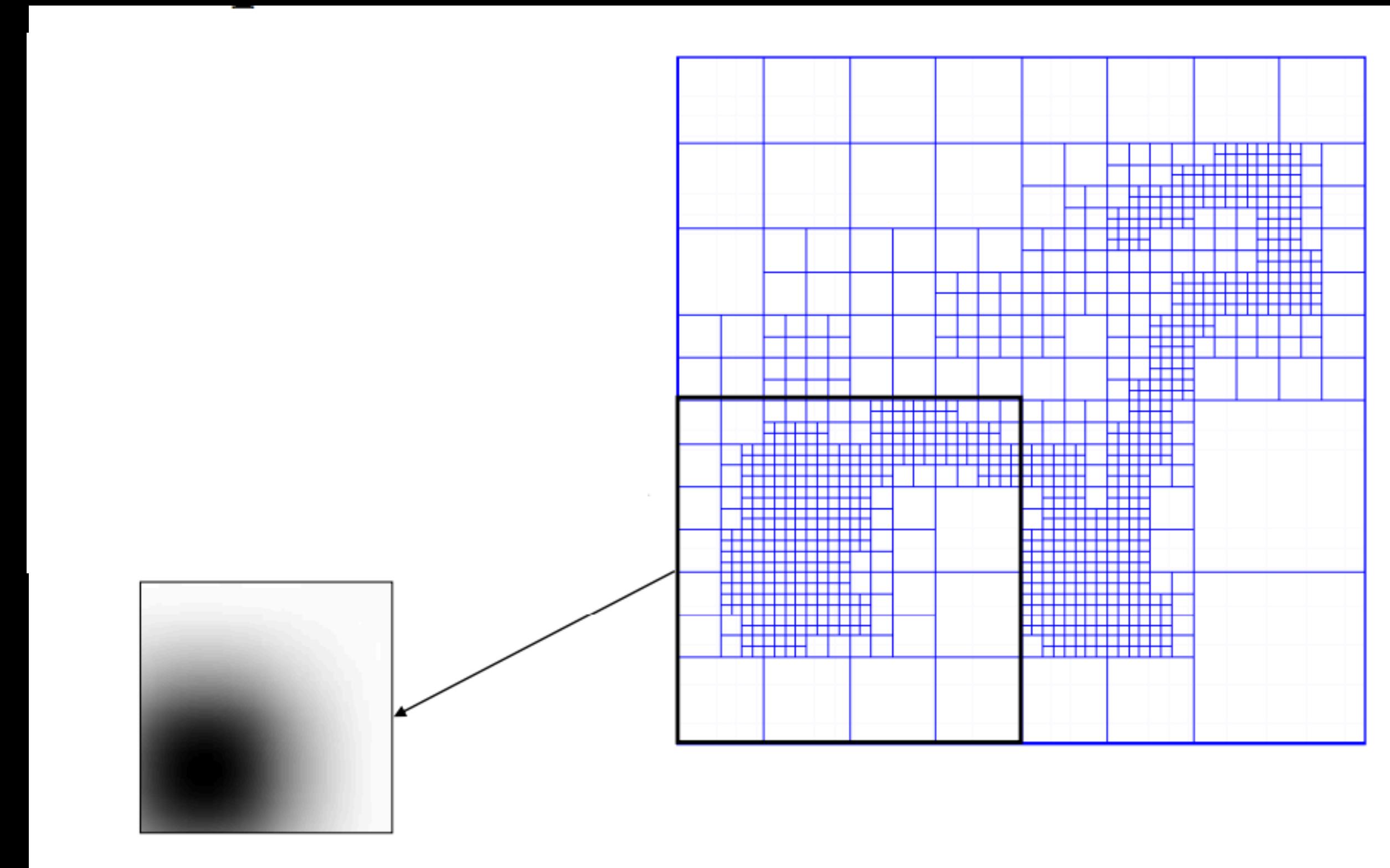
INTEGRATION

- ▶ Given the Points:
 - ▶ Set octree
 - ▶ Compute vector field
 - ▶ define function space



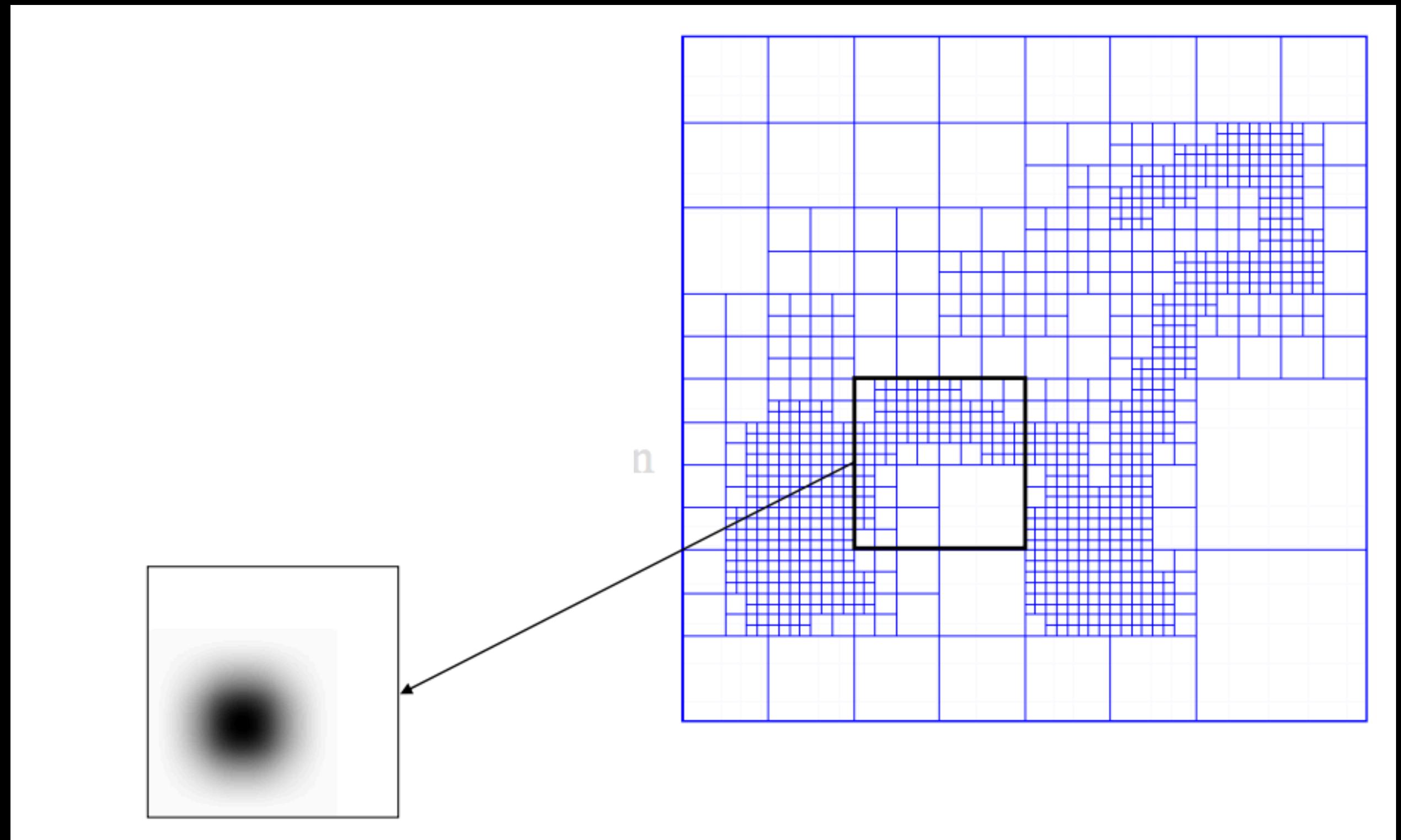
INTEGRATION

- ▶ Given the Points:
 - ▶ Set octree
 - ▶ Compute vector field
 - ▶ define function space



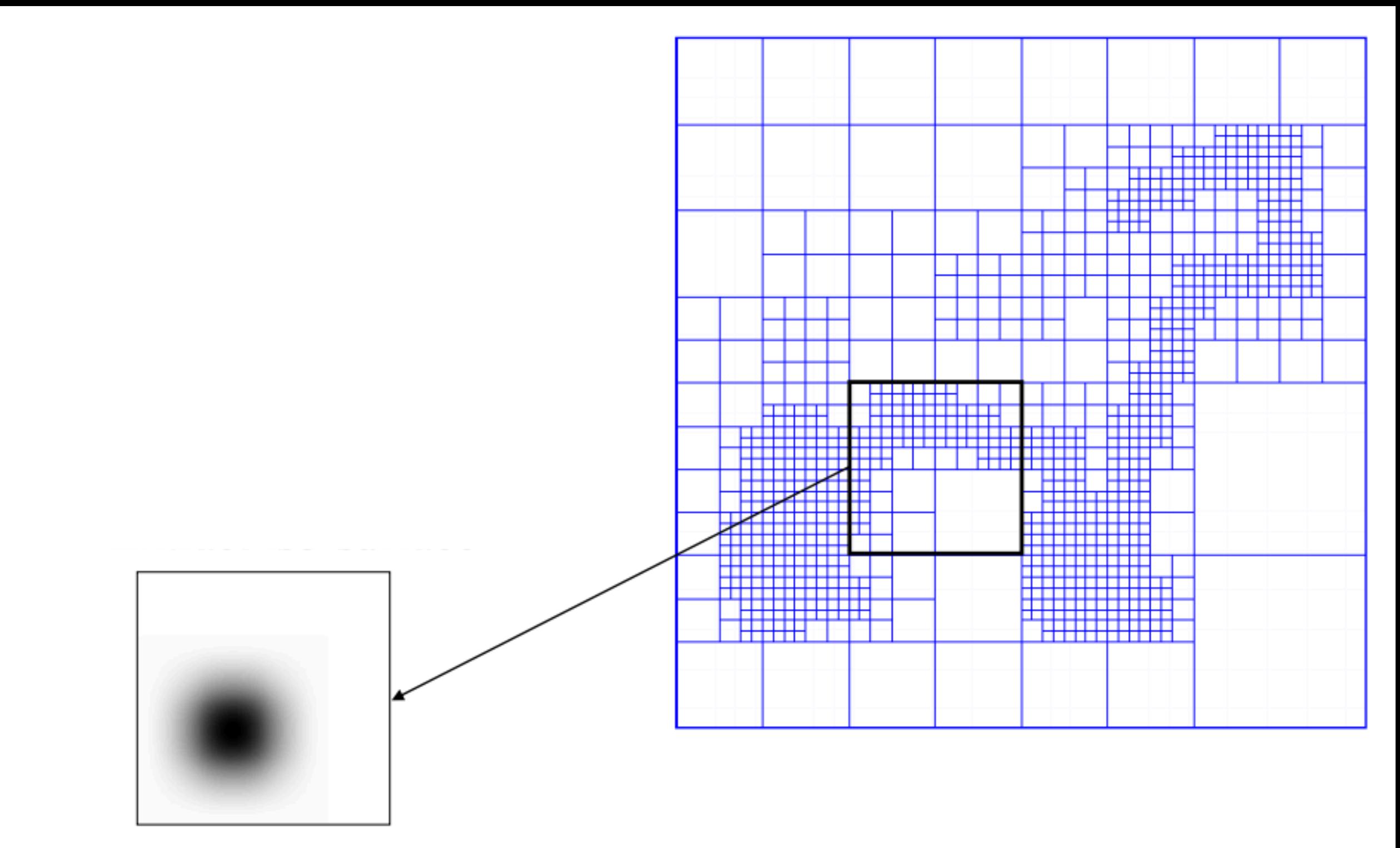
INTEGRATION

- ▶ Given the Points:
 - ▶ Set octree
 - ▶ Compute vector field
 - ▶ define function space



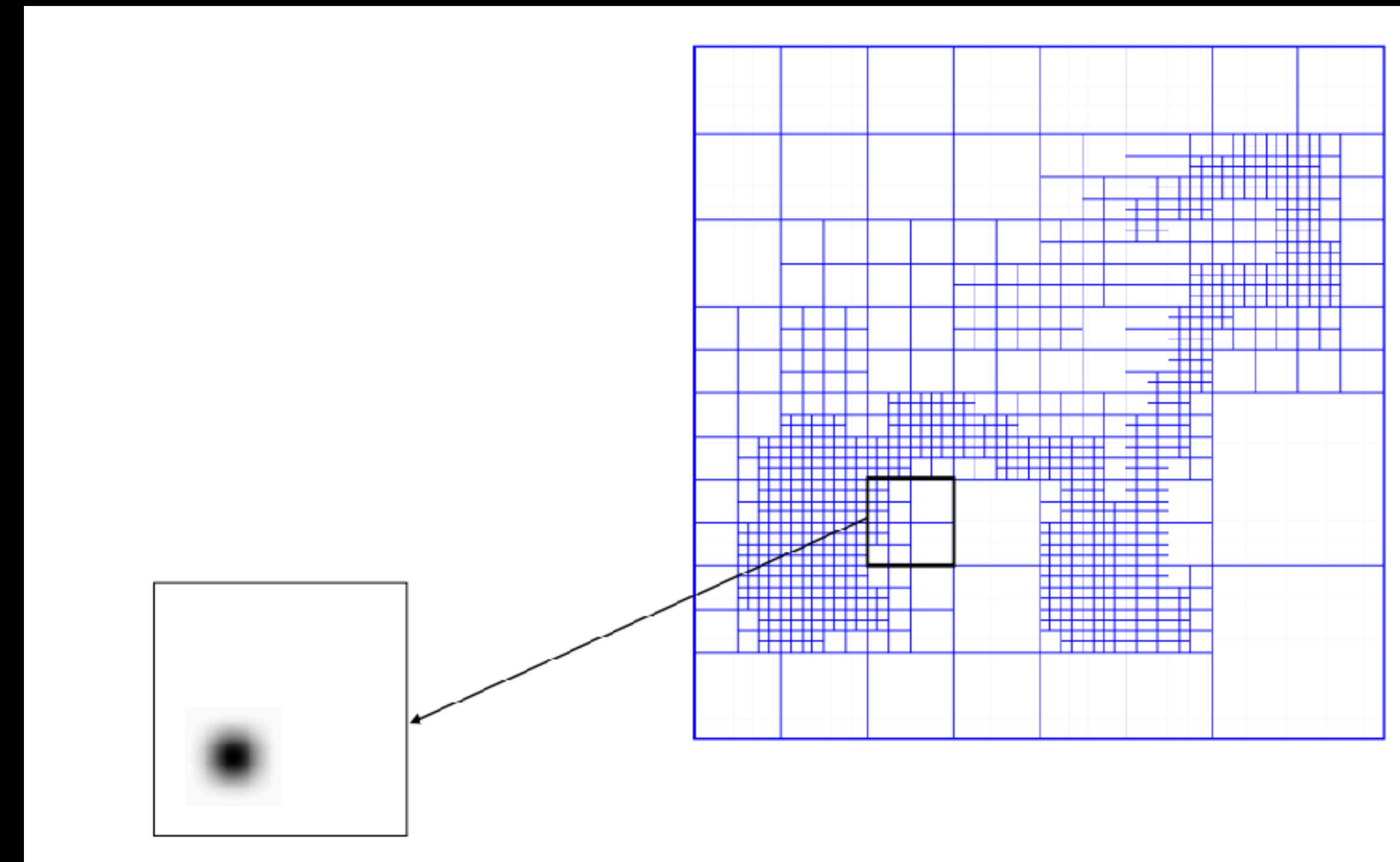
INTEGRATION

- ▶ Given the Points:
 - ▶ Set octree
 - ▶ Compute vector field
 - ▶ define function space



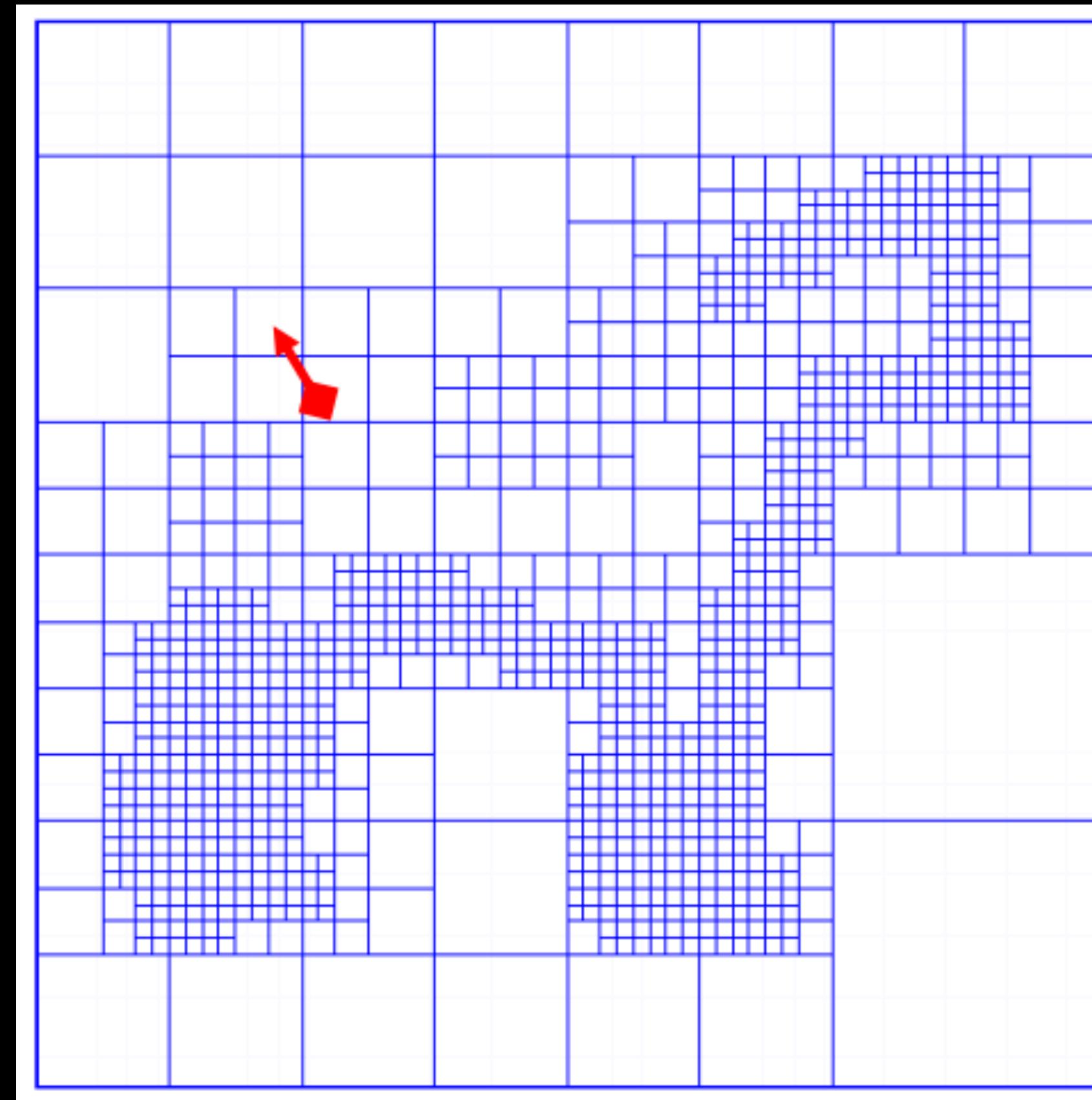
INTEGRATION

- ▶ Given the Points:
- ▶ Set octree
- ▶ Compute vector field
- ▶ define function space



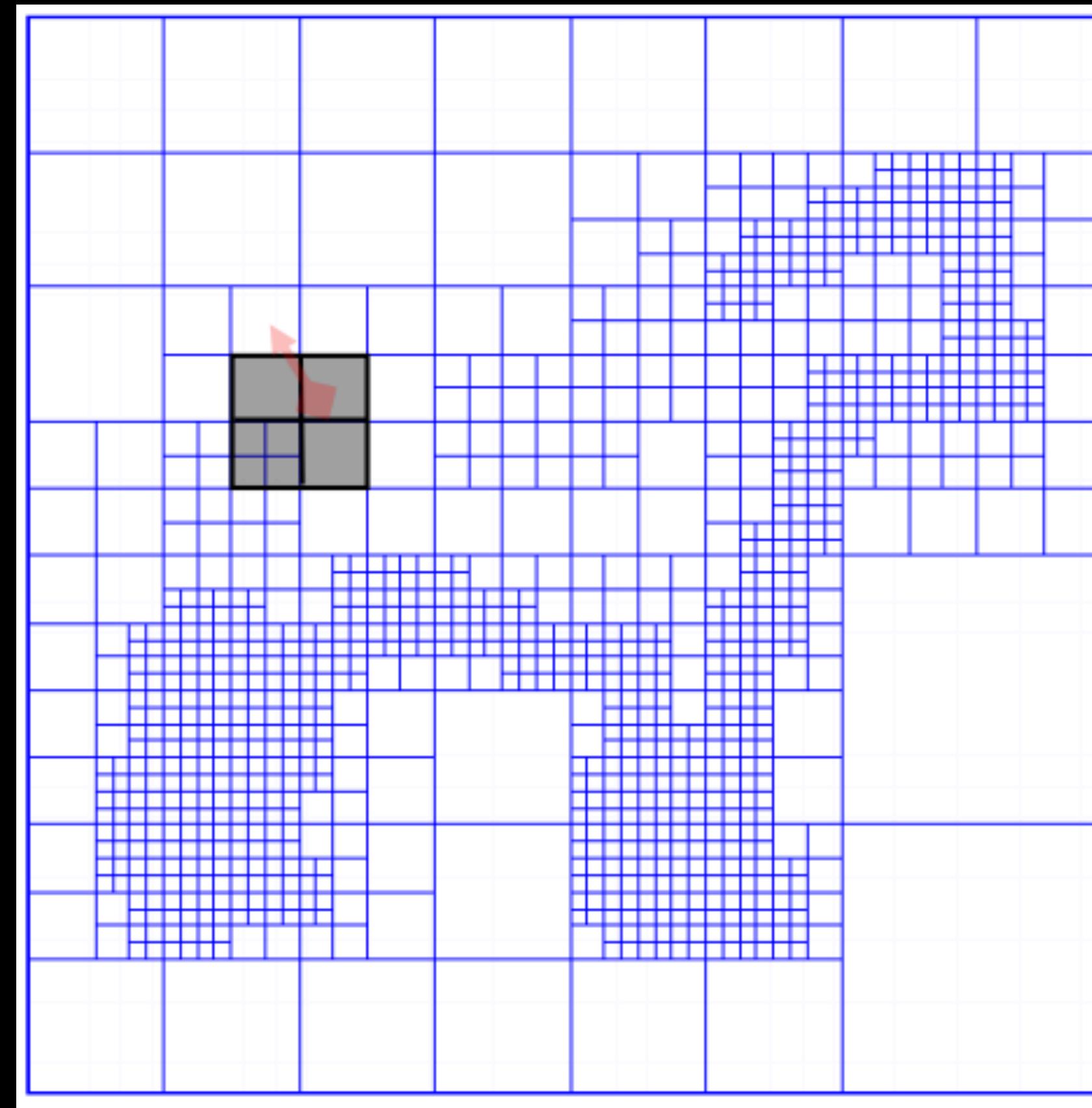
INTEGRATION

- ▶ Given the Points:
- ▶ Set octree
- ▶ Compute vector field
- ▶ define function space
- ▶ splat the samples



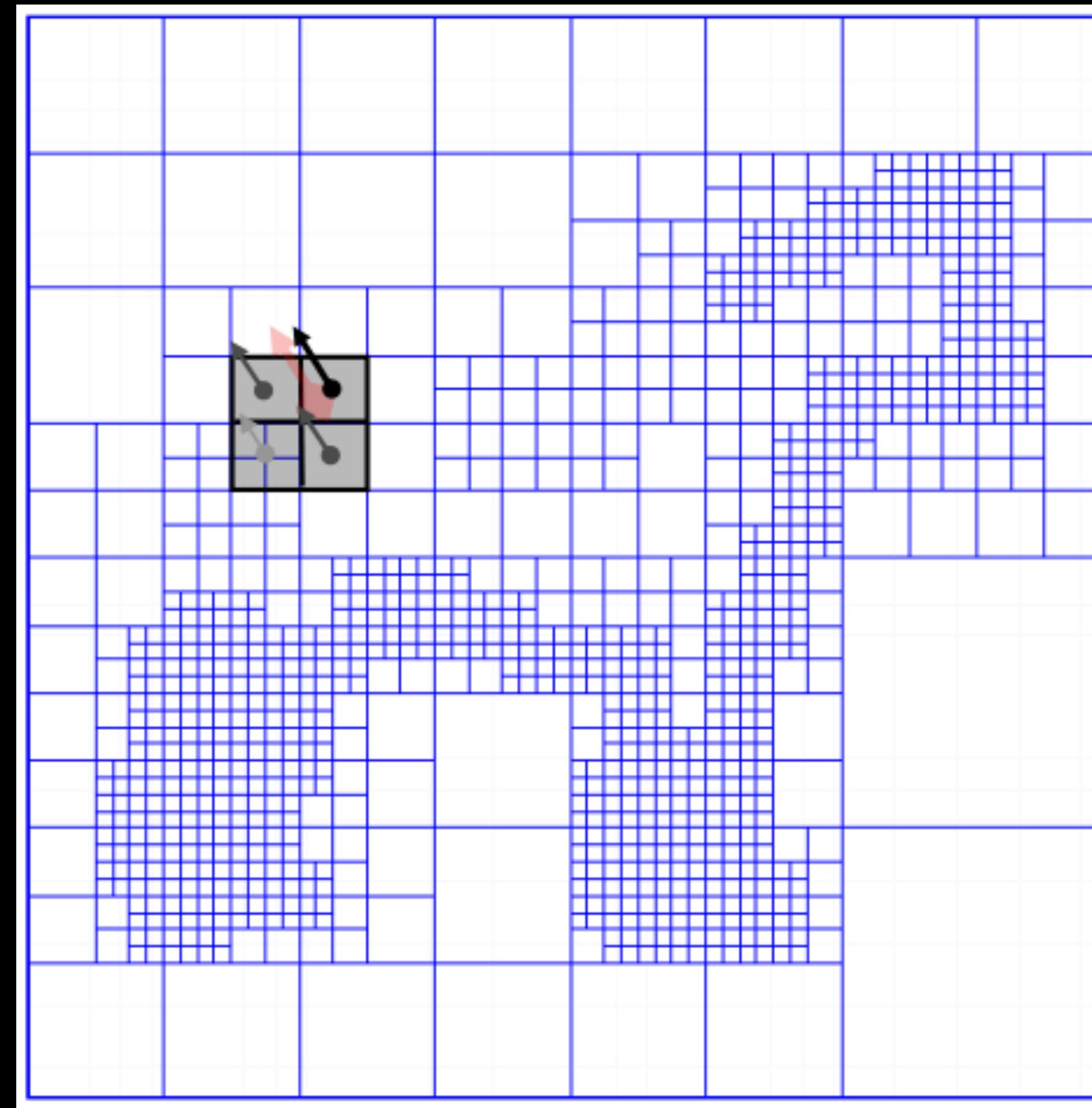
INTEGRATION

- ▶ Given the Points:
- ▶ Set octree
- ▶ Compute vector field
- ▶ define function space
- ▶ splat the samples



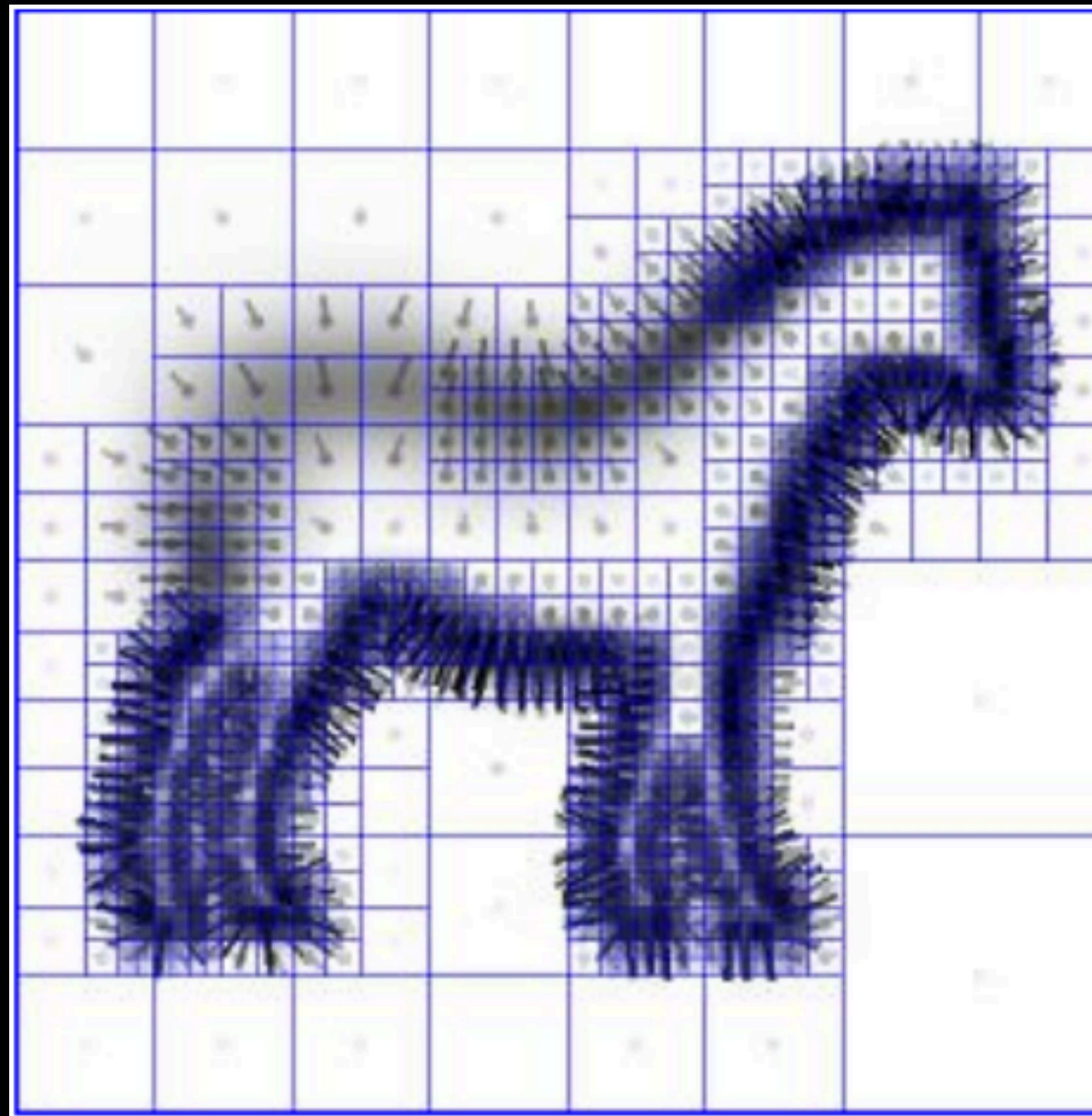
INTEGRATION

- ▶ Given the Points:
- ▶ Set octree
- ▶ Compute vector field
- ▶ define function space
- ▶ splat the samples



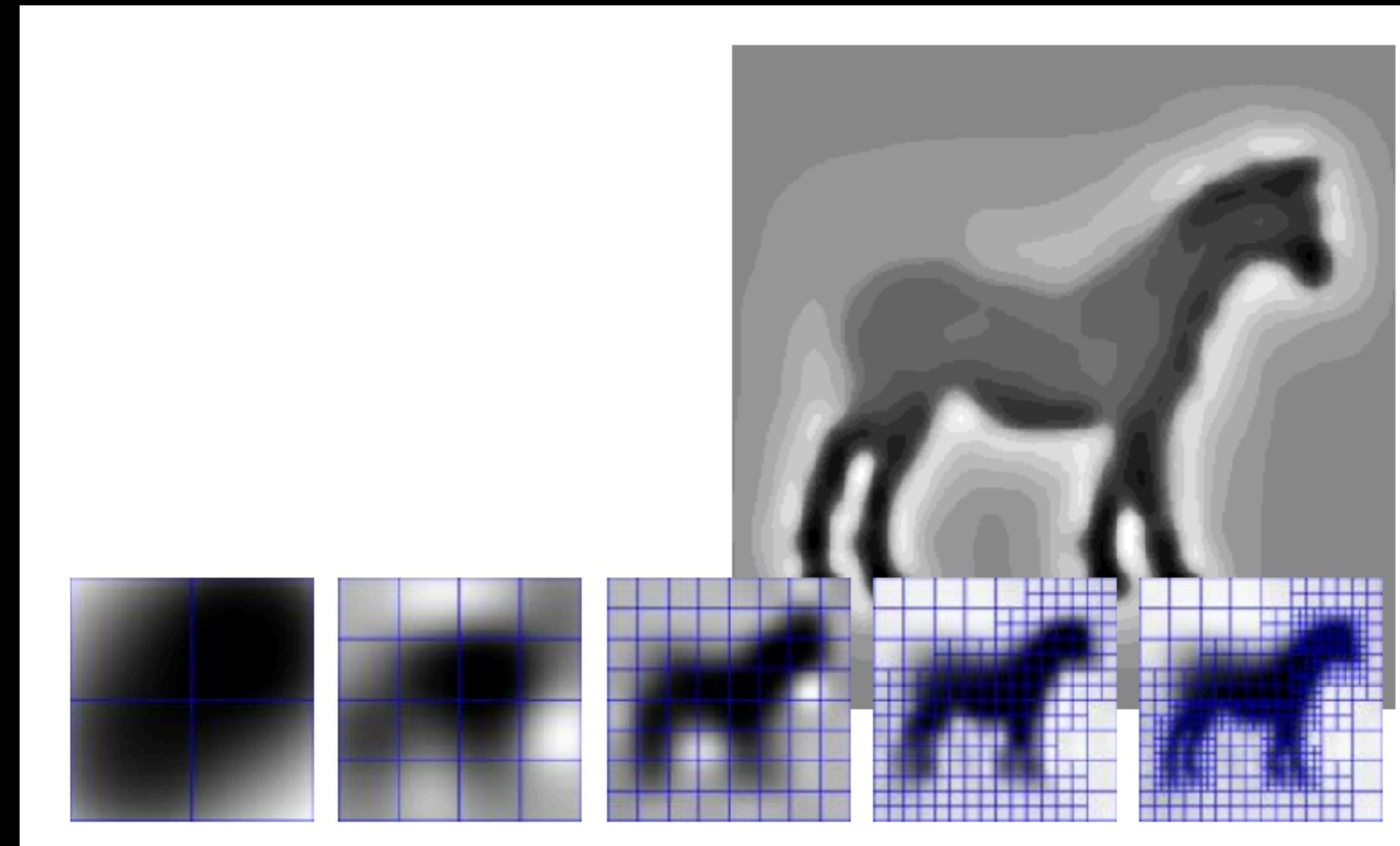
INTEGRATION

- ▶ Given the Points:
- ▶ Set octree
- ▶ Compute vector field
- ▶ define function space
- ▶ splat the samples



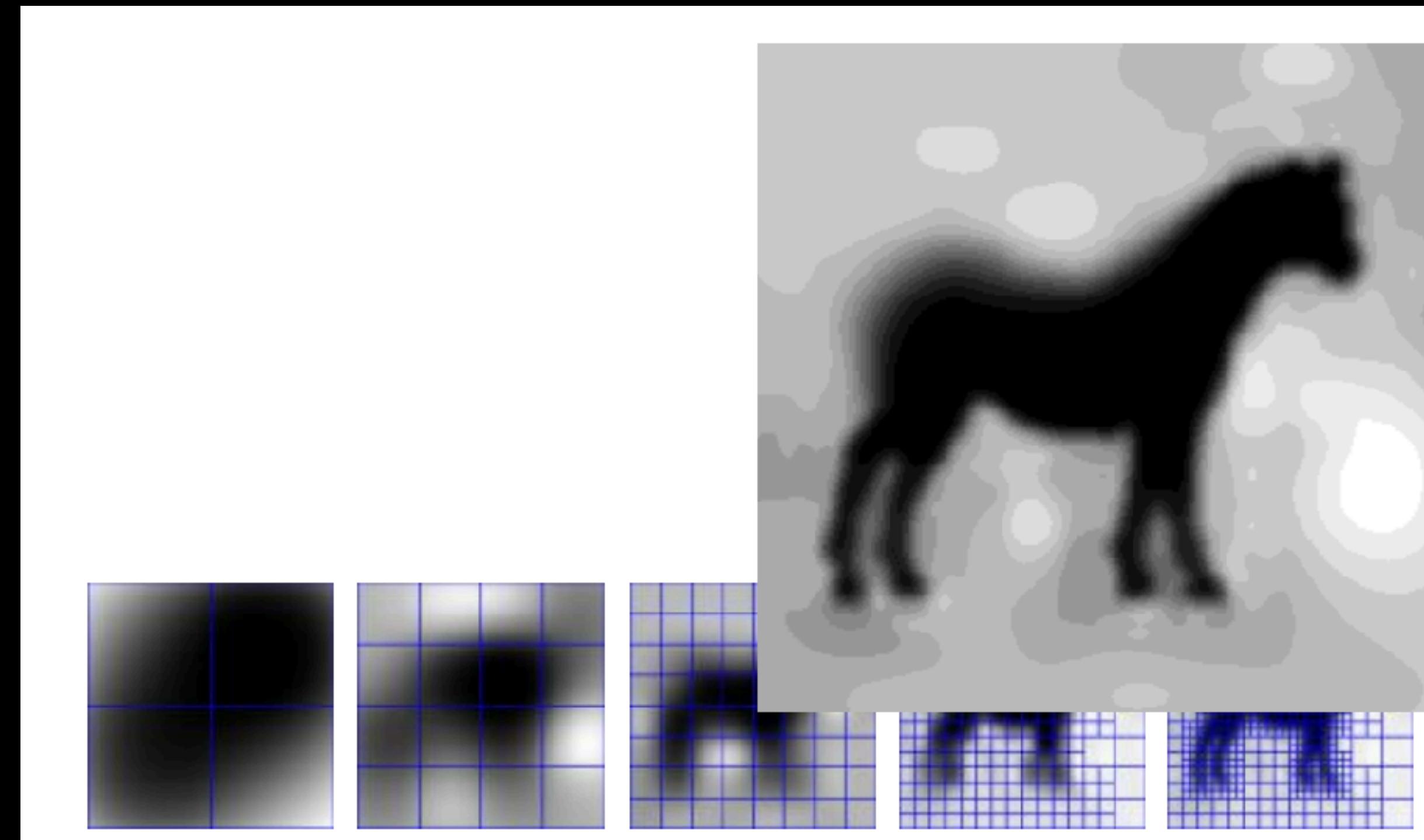
INTEGRATION

- ▶ Given the Points:
 - ▶ Set octree
 - ▶ Compute vector field
 - ▶ Compute indicator function
 - ▶ Solve Poisson Equation



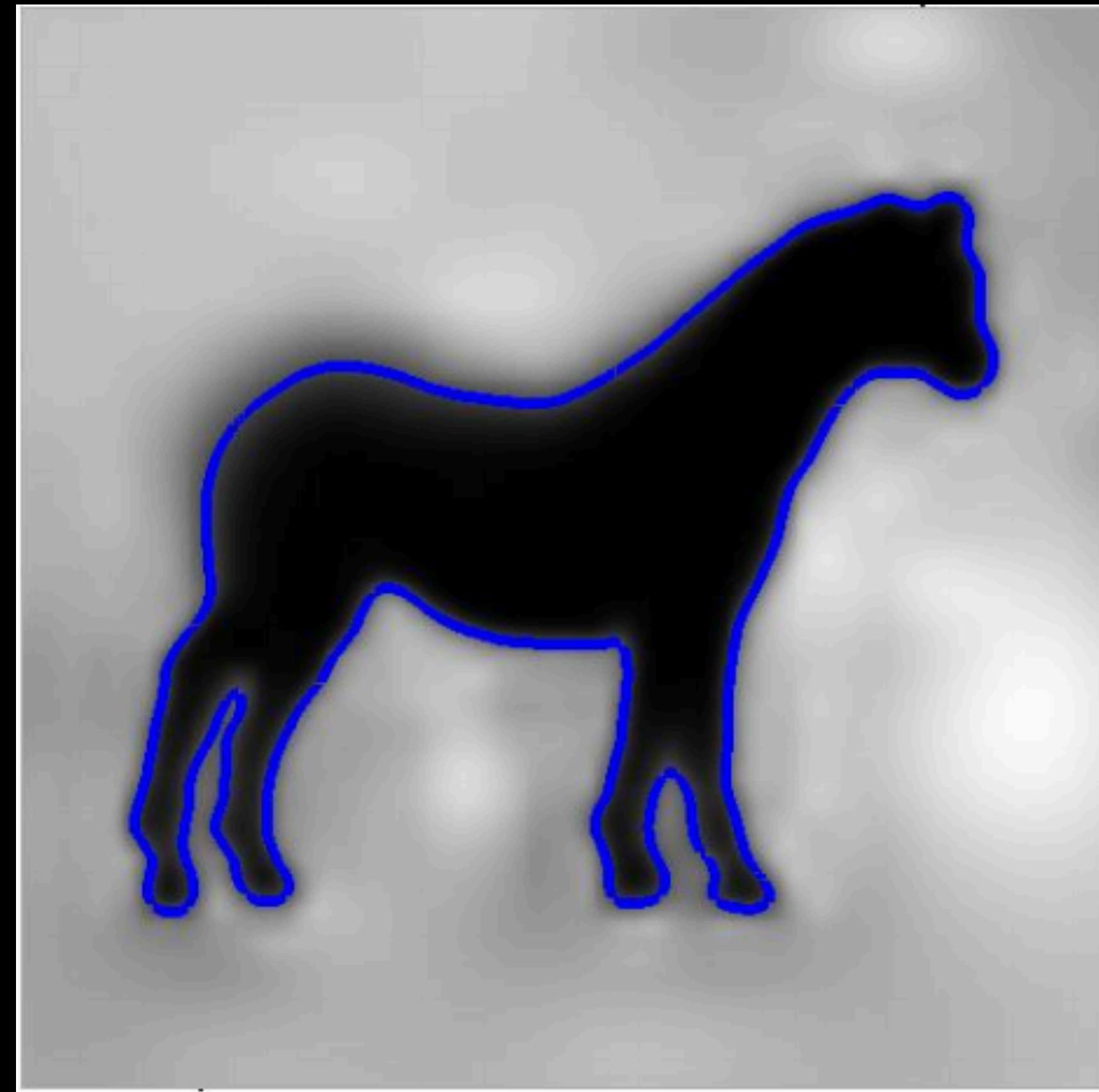
INTEGRATION

- ▶ Given the Points:
- ▶ Set octree
- ▶ Compute vector field
- ▶ Compute indicator function
- ▶ Compute divergence
- ▶ Solve Poisson Equation



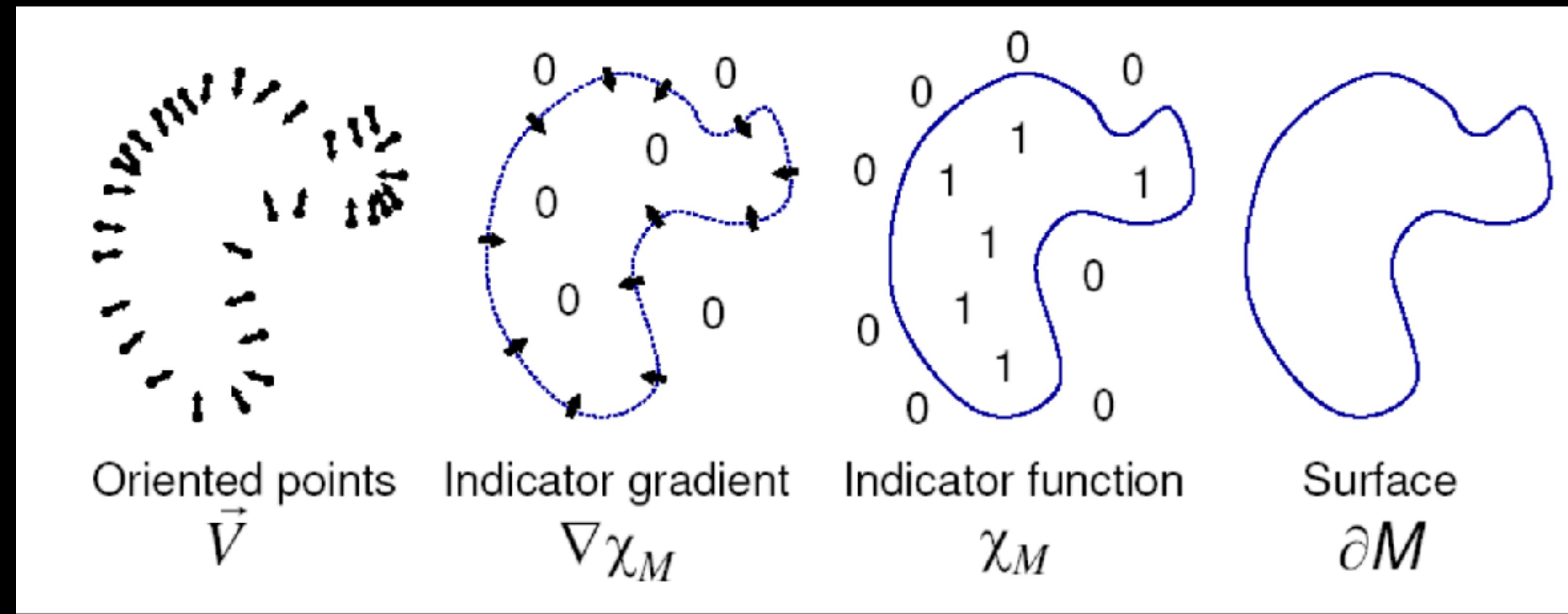
INTEGRATION

- ▶ Given the Points:
- ▶ Set octree
- ▶ Compute vector field
- ▶ Compute indicator function
- ▶ Extract iso-surface



PROBLEM MODELING

- ▶ Input: points with associated normals
- ▶ • Indicator function: 1 inside, 0 outside
- ▶ => gradient = 0 everywhere except near surface



POISSON RECONSTRUCTION

- ▶ Problem: find the indicator function starting
- ▶ from the gradient
- ▶ - $\min_{\chi} \|\nabla \chi - \vec{V}\|$
- ▶ - \vec{V} gradient field defined by the points
- ▶ Transforms to a Poisson equation:

$$\Delta \chi \equiv \nabla \cdot \nabla \chi = \nabla \cdot \vec{V}$$

POISSON RECONSTRUCTION

- ▶ Basis functions with local support (\neq RBF)
- ▶ \Rightarrow sparse system, fast to solve
- ▶ Implicit function constrained everywhere, not only near input points
- ▶ Good result even for noisy data
- ▶ Main drawback: consistent normal orientation

KAZHDAN ET AL. SGP 2006

- ▶ “Poisson surface reconstruction”
- ▶ Discretization of space: not a uniform grid (Hoppe et al.), but an adaptive octree (Ohtake et al.)
- ▶ Time and memory complexity for a given octree depth = $O(n)$
- ▶ Octree depth $\pm 1 \Rightarrow$ time and memory complexity + number of output triangles \sim multiplied by 4

TEXT

KAZHDAN ET AL. SGP 2006



MICHELANGELO'S DAVID



- ▶ 215 million data points from 1000 scans
- ▶ 22 million triangle reconstruction
- ▶ Maximum tree depth of 11
- ▶ Compute Time: 2.1 hours
- ▶ Peak Memory: 6600MB

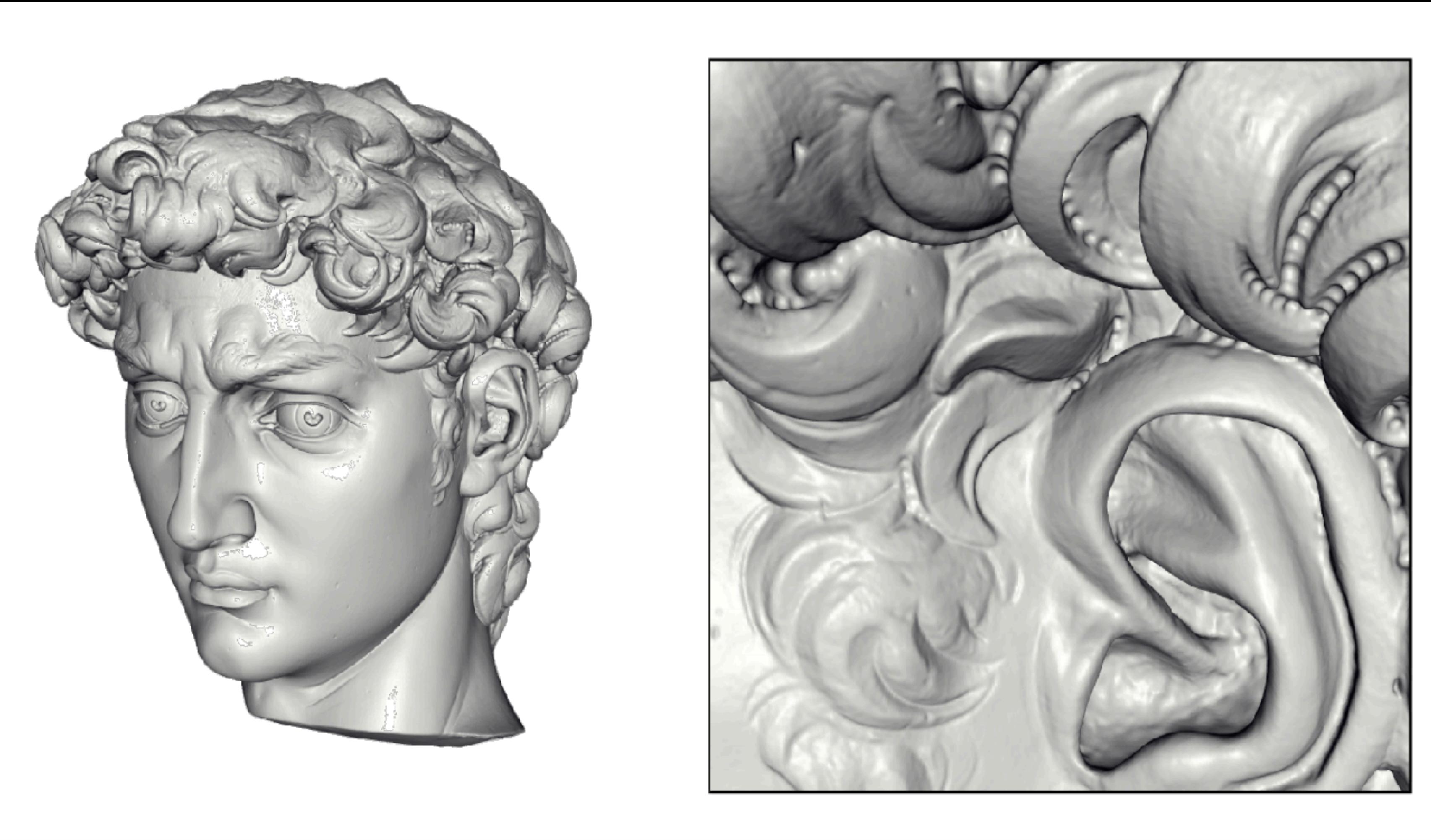
TEXT

MICHELANGELO'S DAVID - CHISEL MARKS



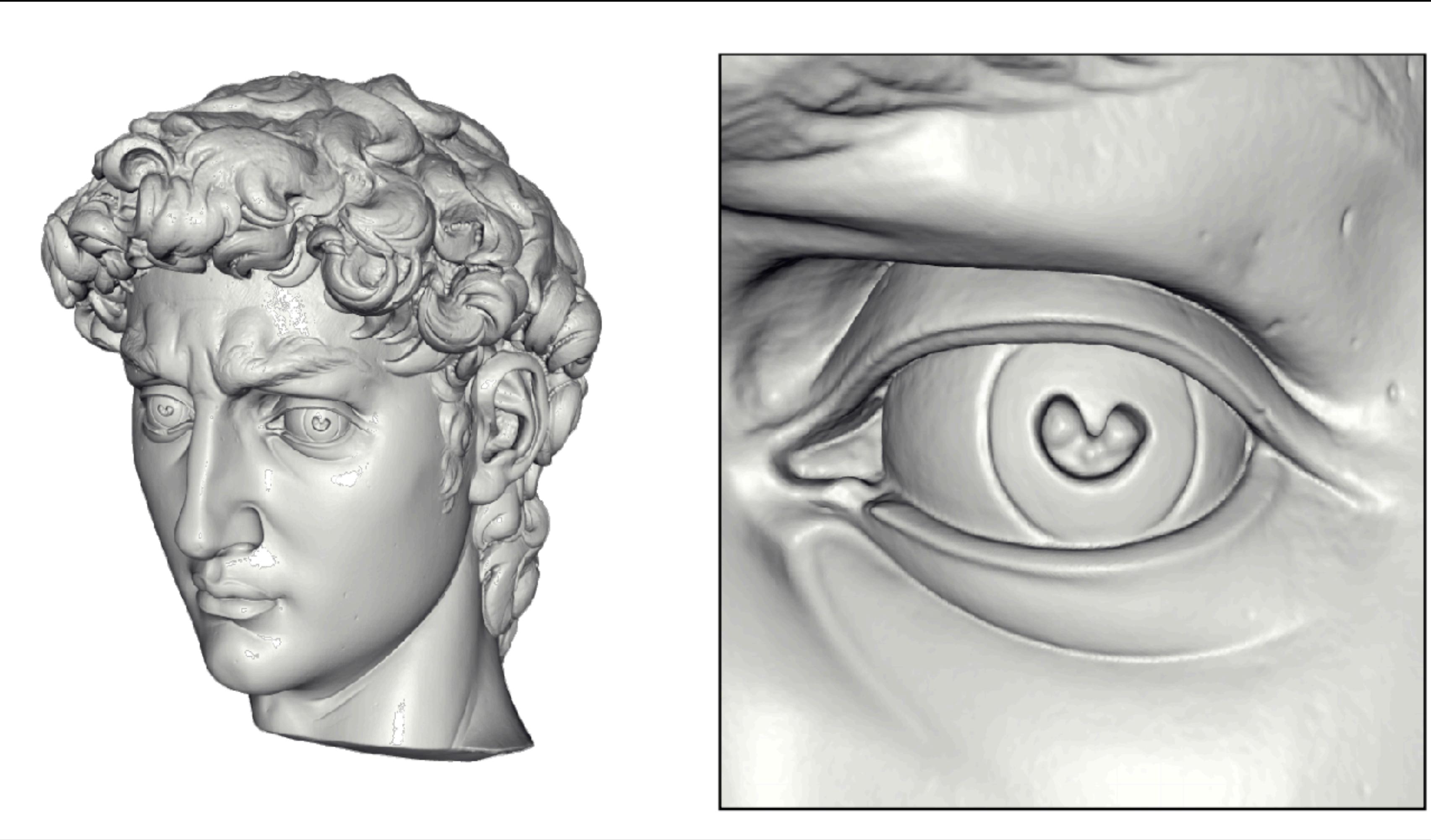
TEXT

MICHELANGELO'S DAVID - DRILL MARKS



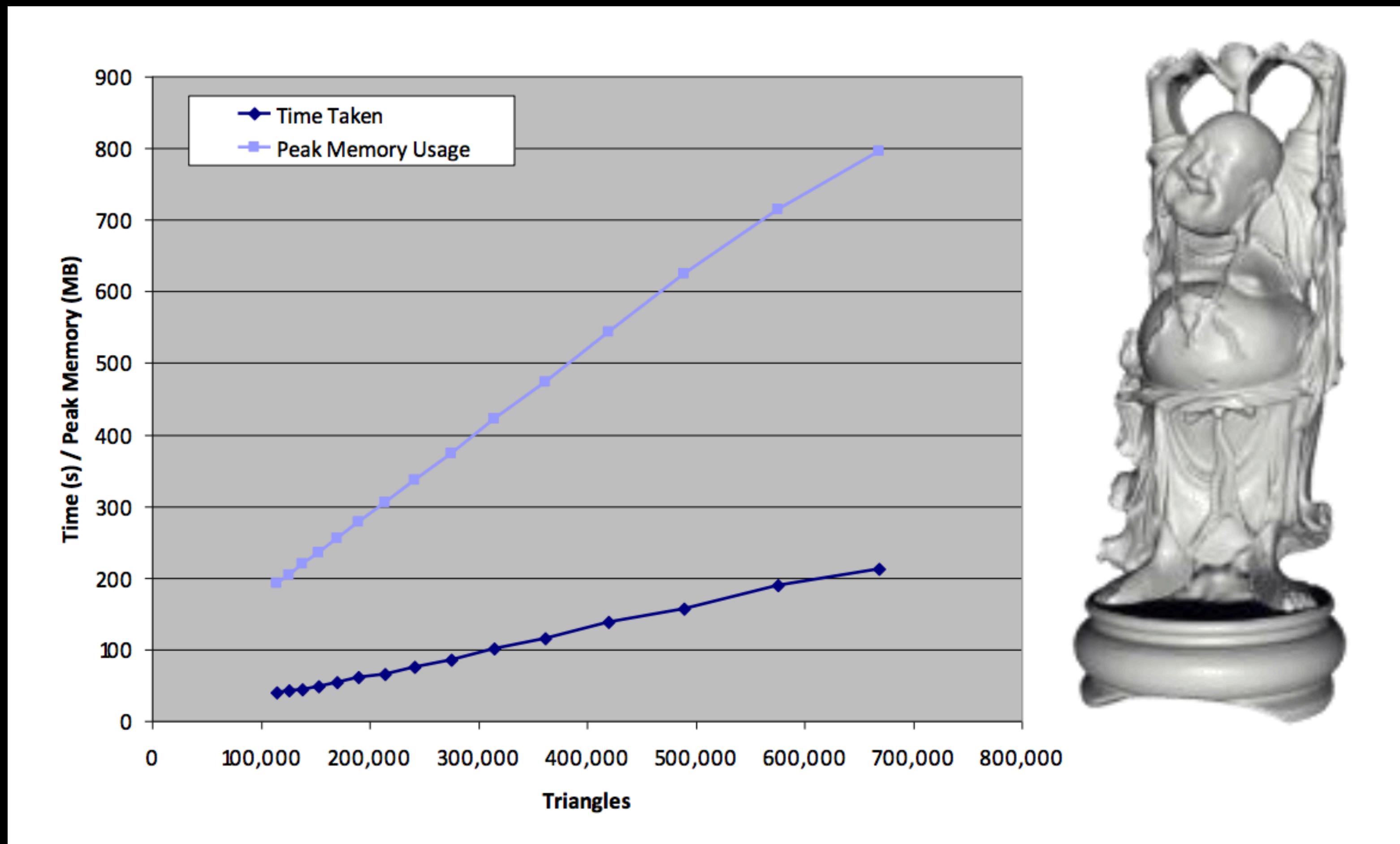
TEXT

MICHELANGELO'S DAVID - EYE



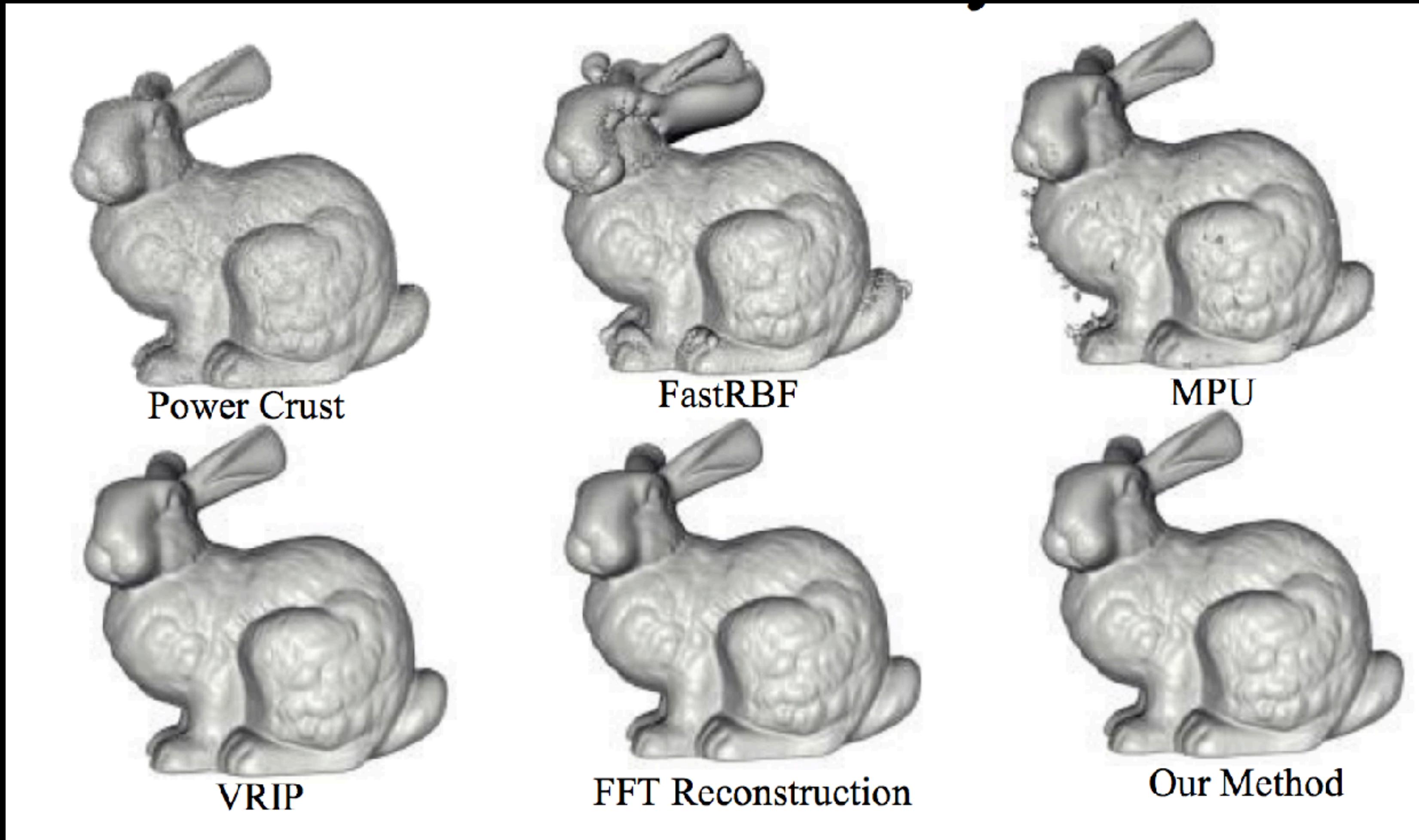
TEXT

SCALABILITY - BUDDHA MODEL



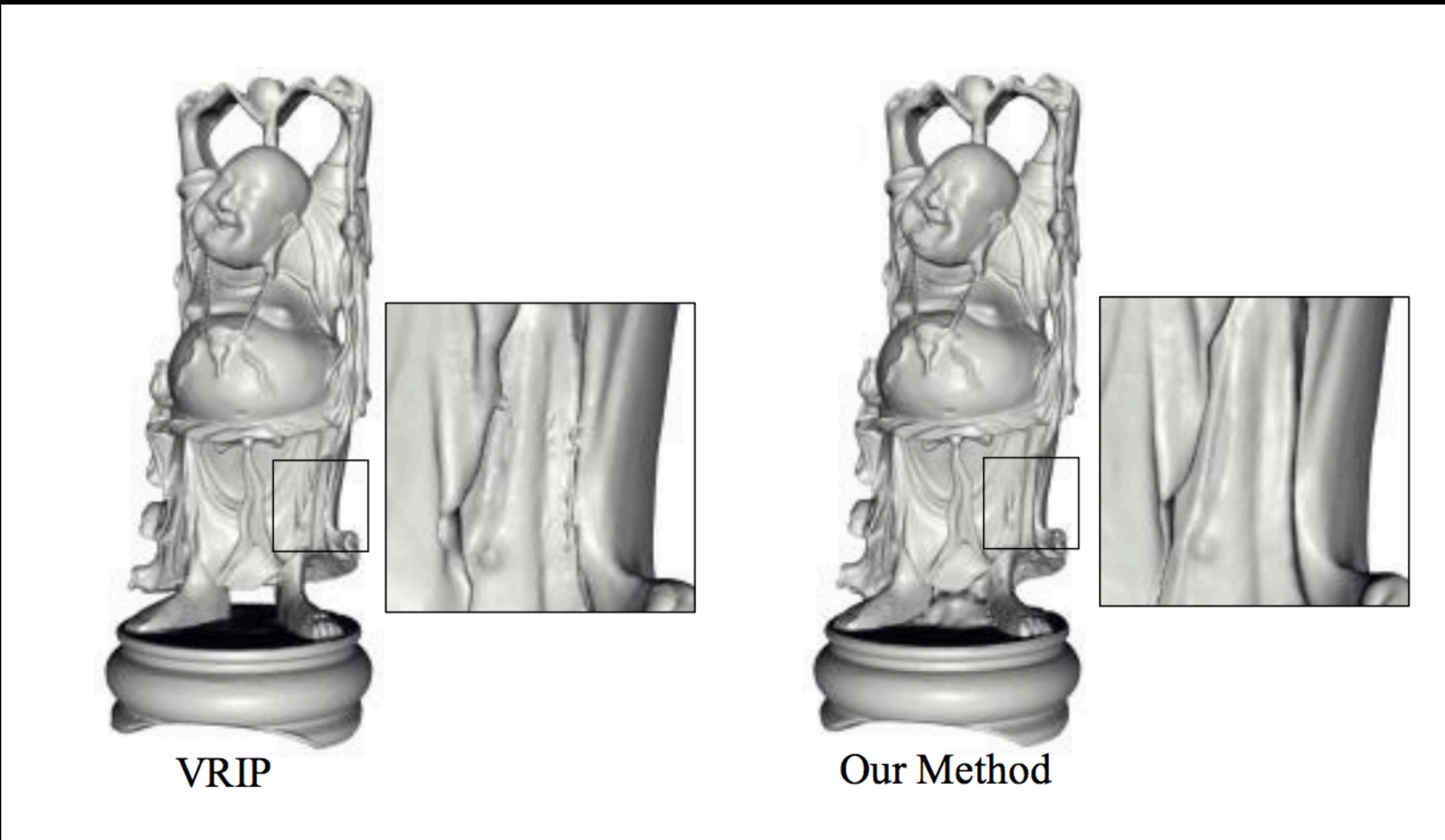
TEXT

STANFORD BUNNY MODEL



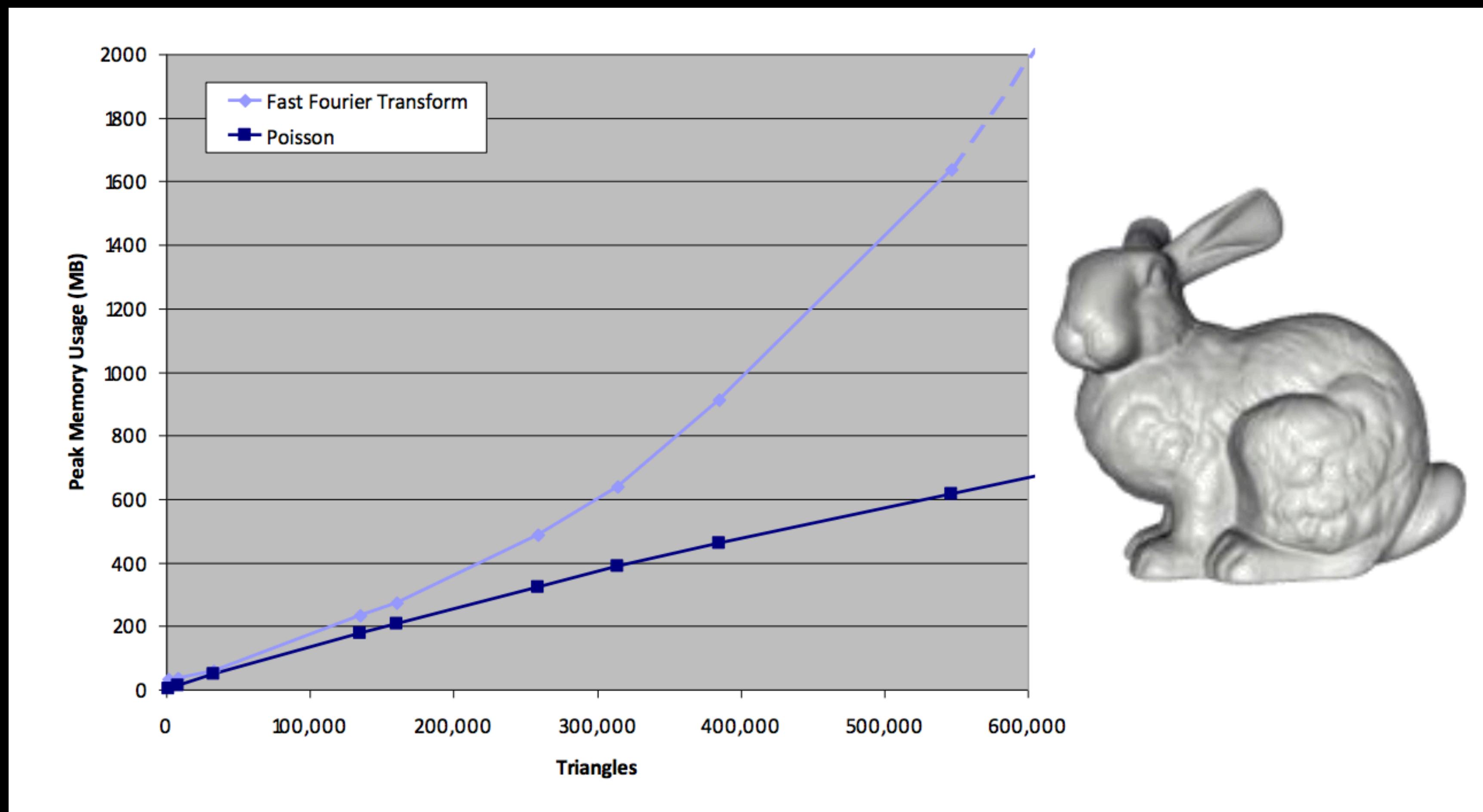
TEXT

VRIP COMPARISON



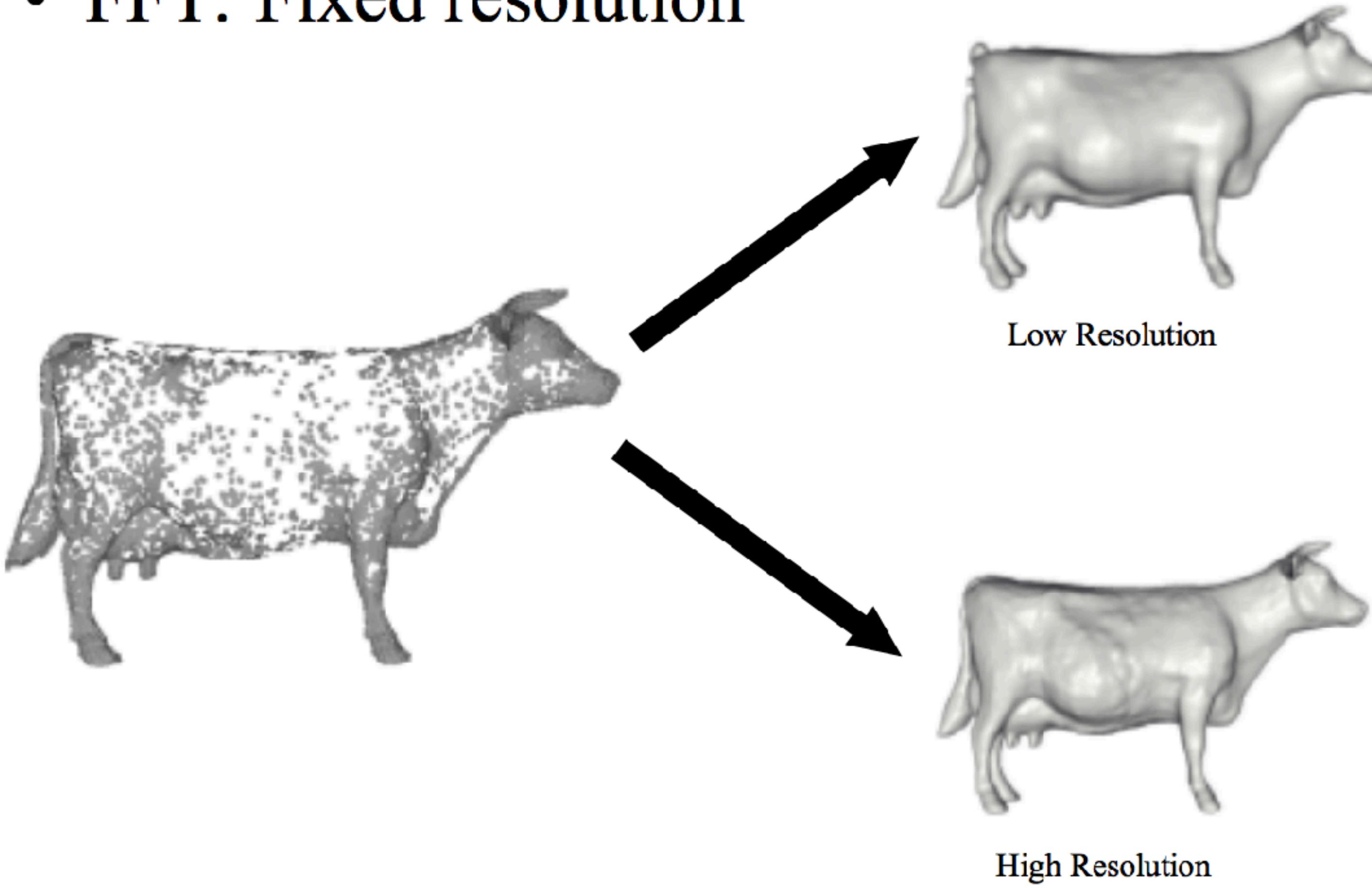
TEXT

FFT RECONSTRUCTION COMPARISON



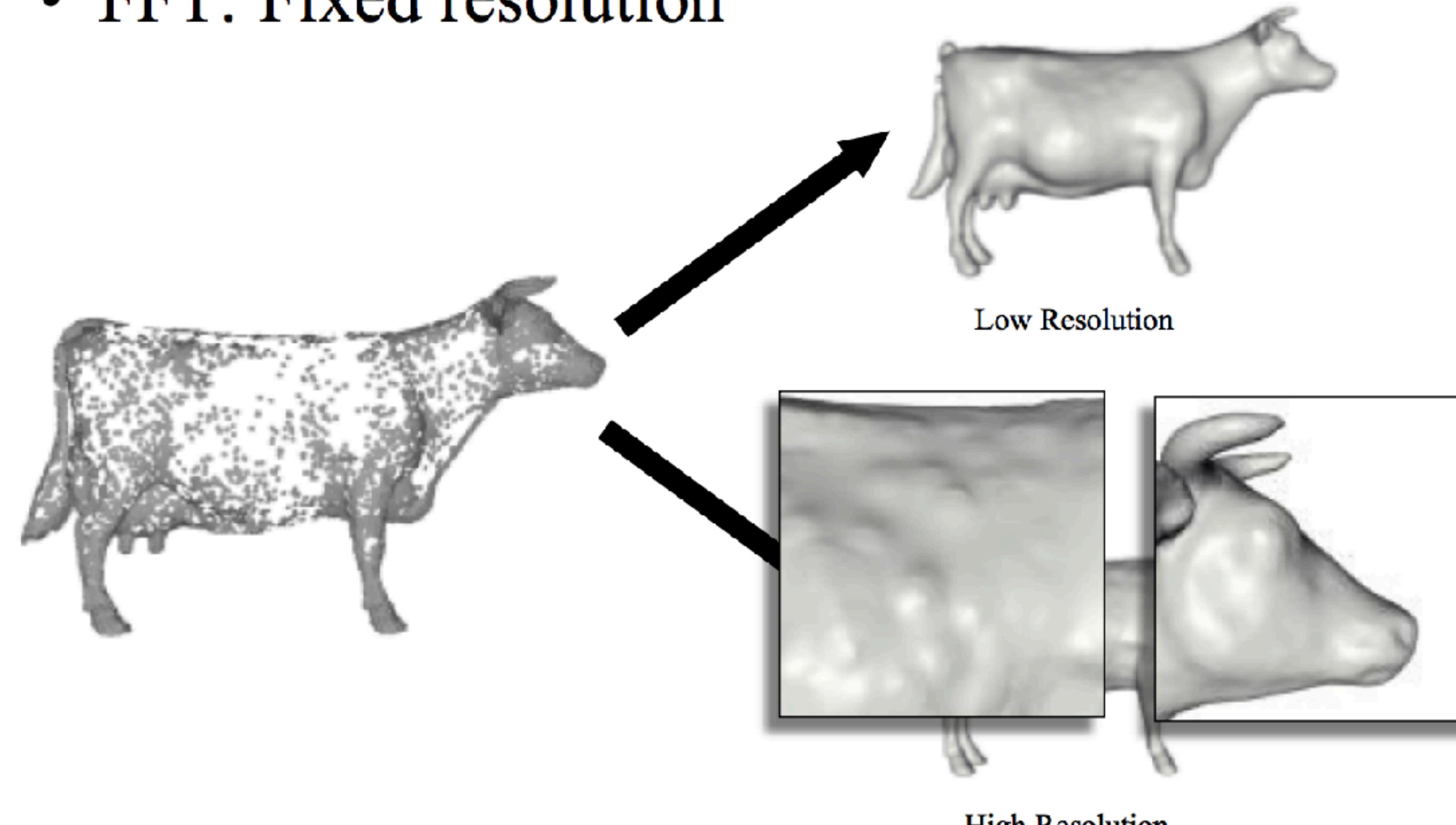
FFT RECONSTRUCTION COMPARISON

- FFT: Fixed resolution



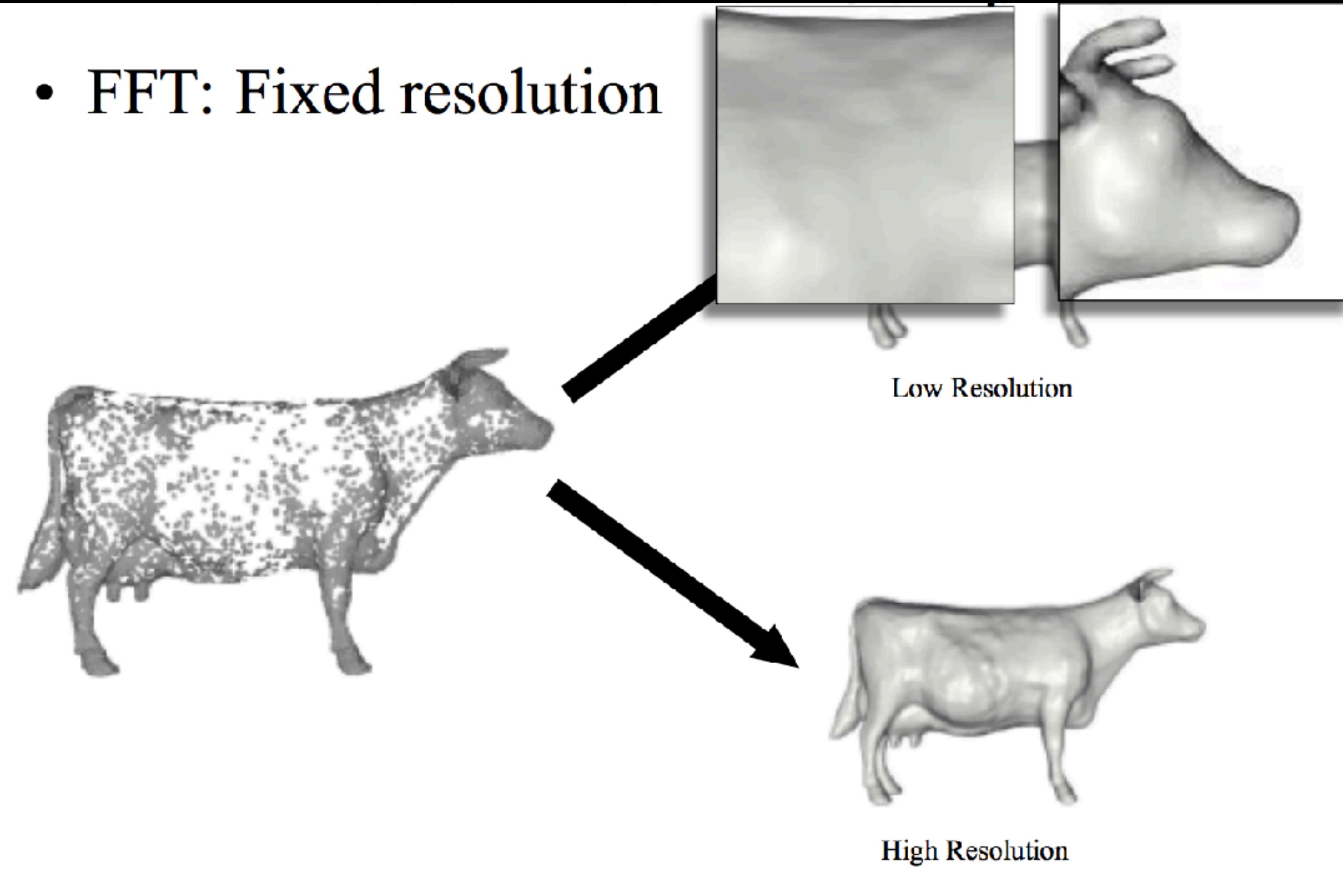
FFT RECONSTRUCTION COMPARISON

- FFT: Fixed resolution



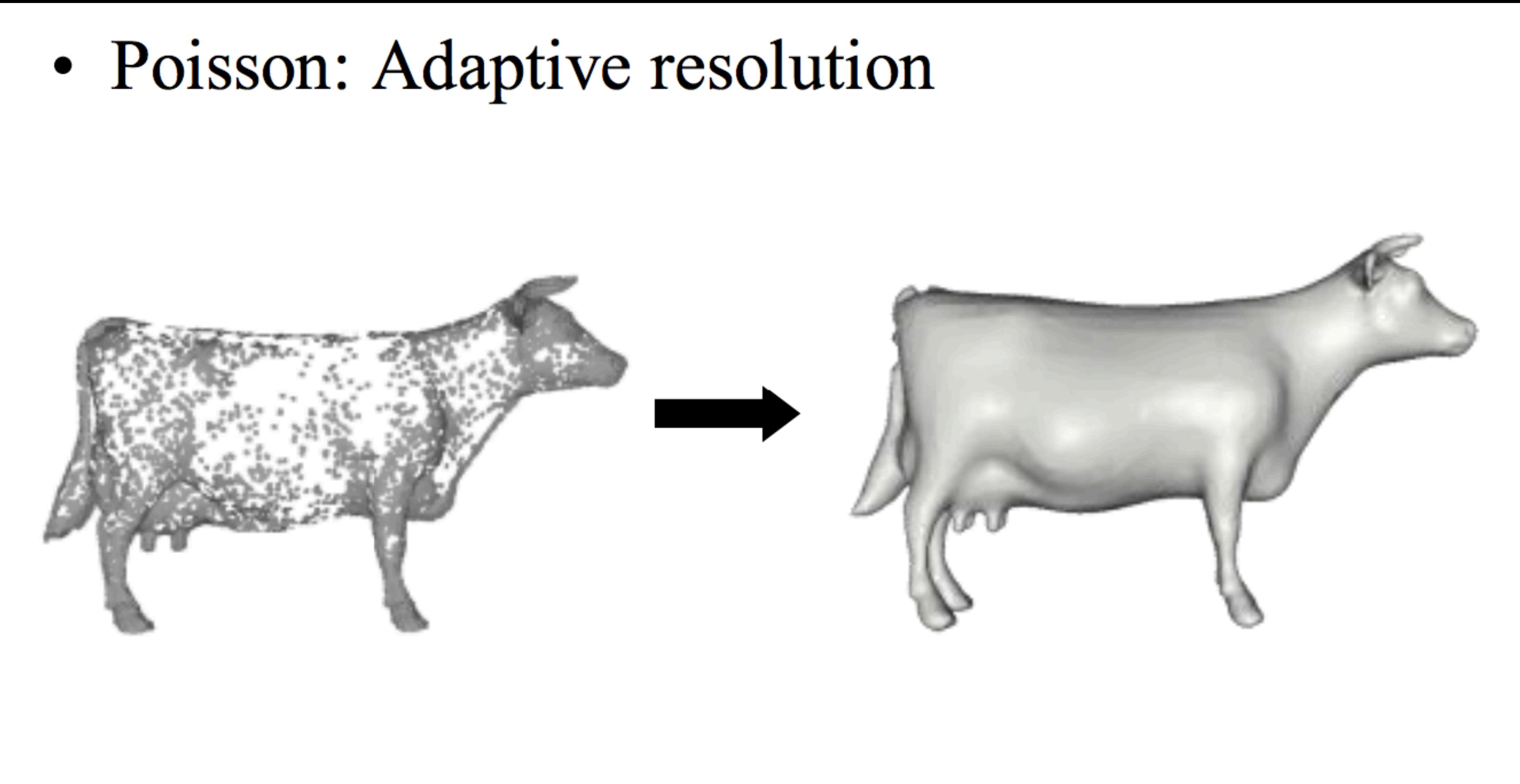
FFT RECONSTRUCTION COMPARISON

- FFT: Fixed resolution



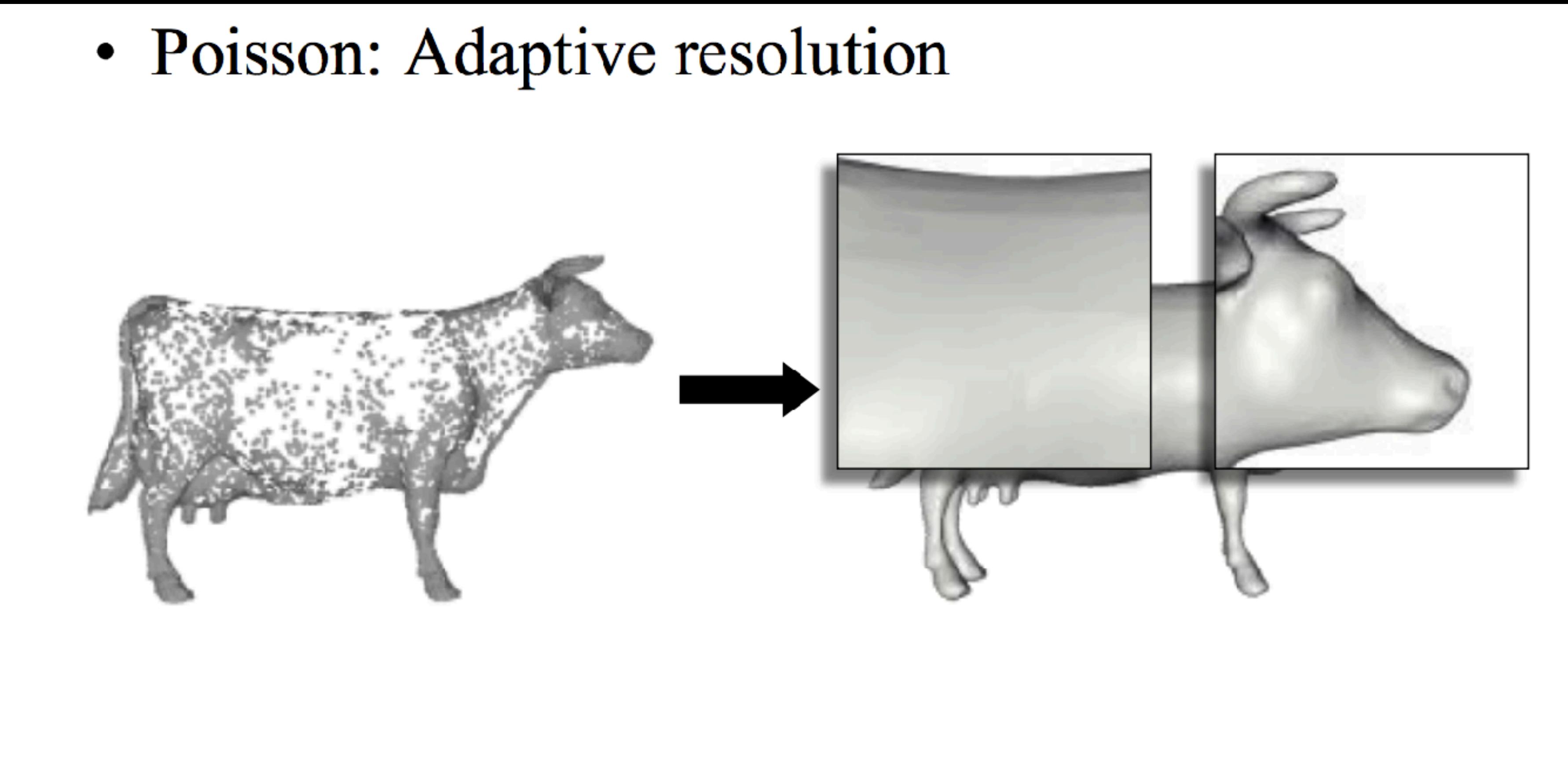
FFT RECONSTRUCTION COMPARISON

- Poisson: Adaptive resolution



FFT RECONSTRUCTION COMPARISON

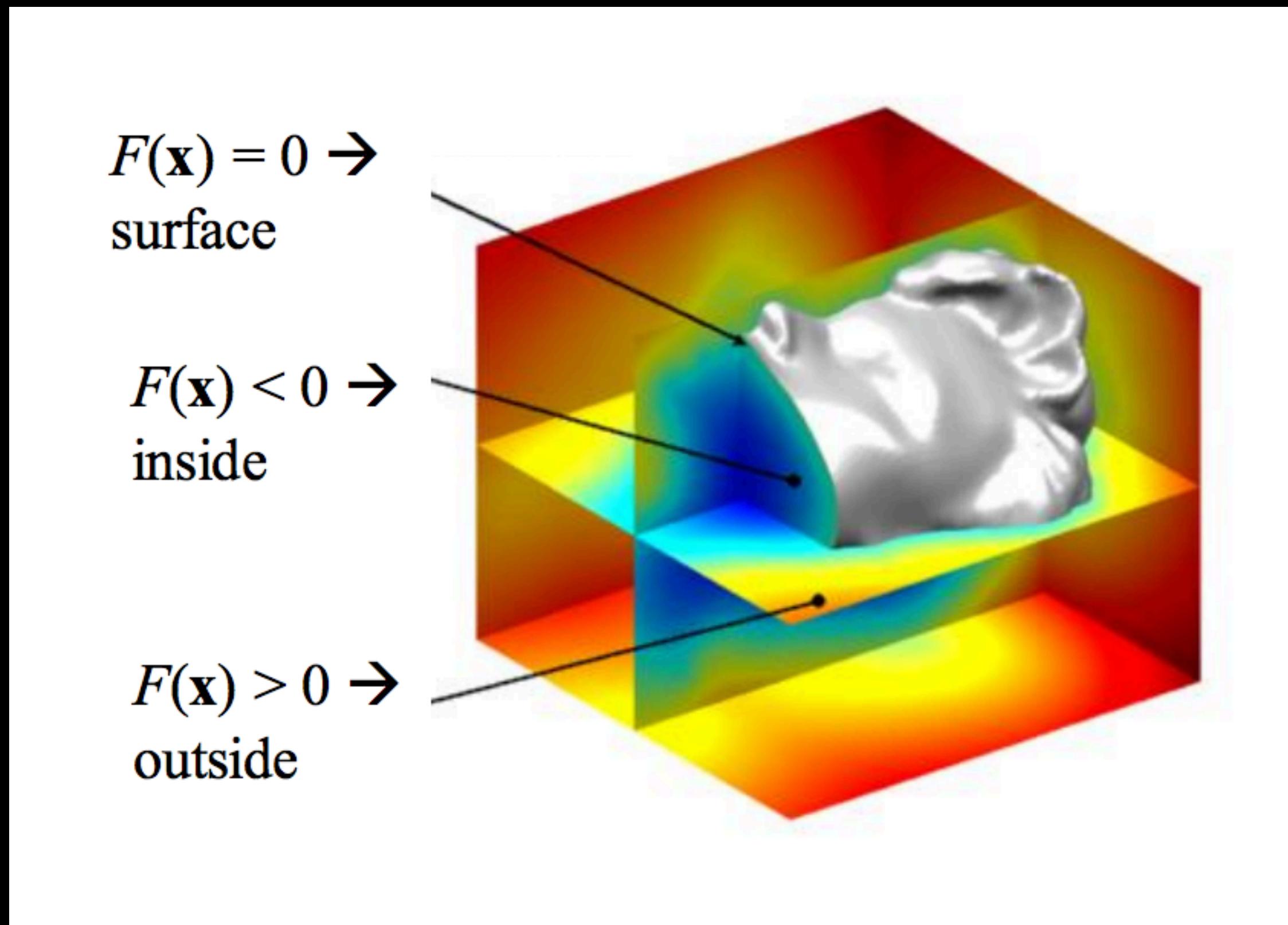
- Poisson: Adaptive resolution



EXTRACTING THE SURFACE

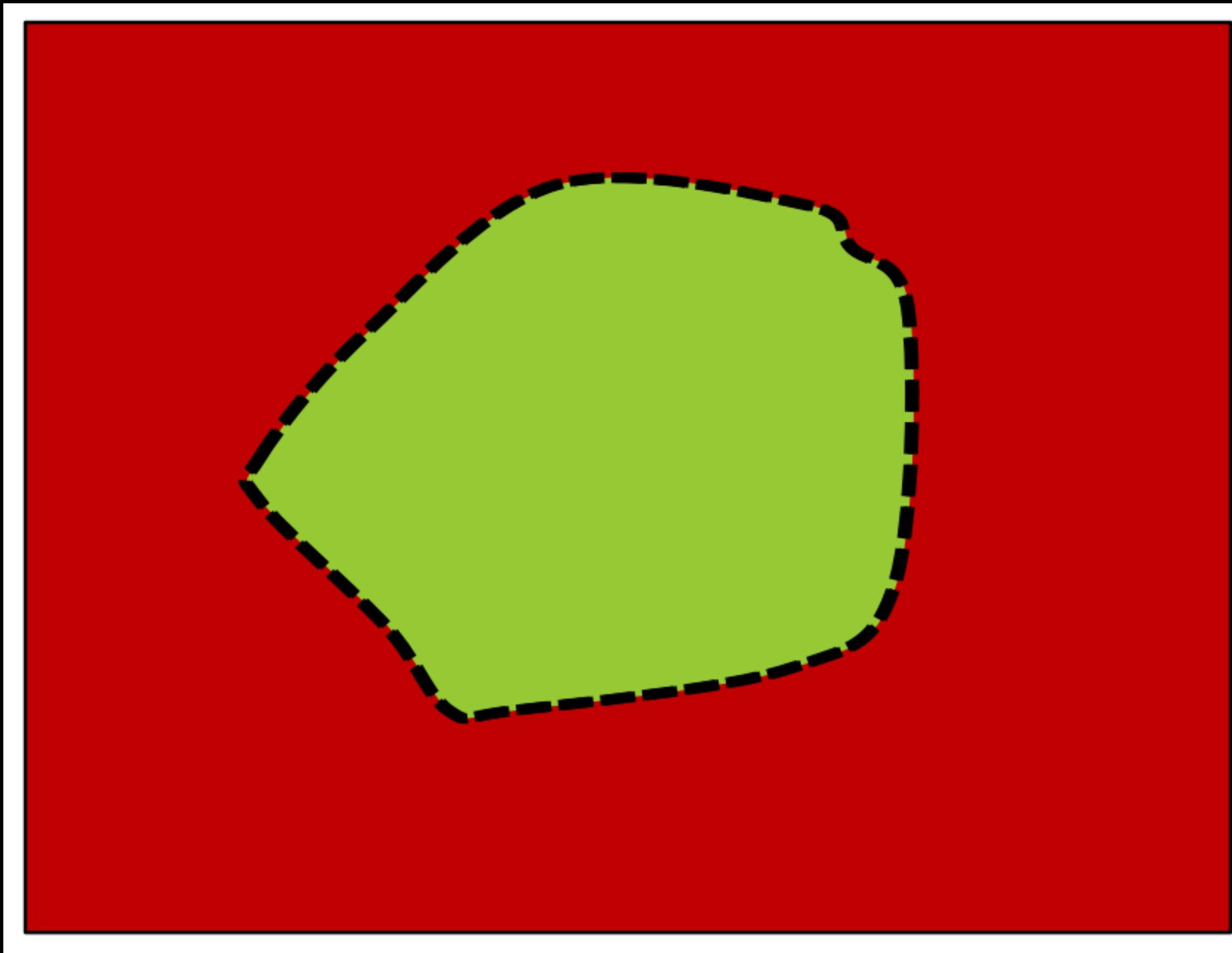
EXTRACTING THE SURFACE

- ▶ How do we make a manifold mesh of the SDF?



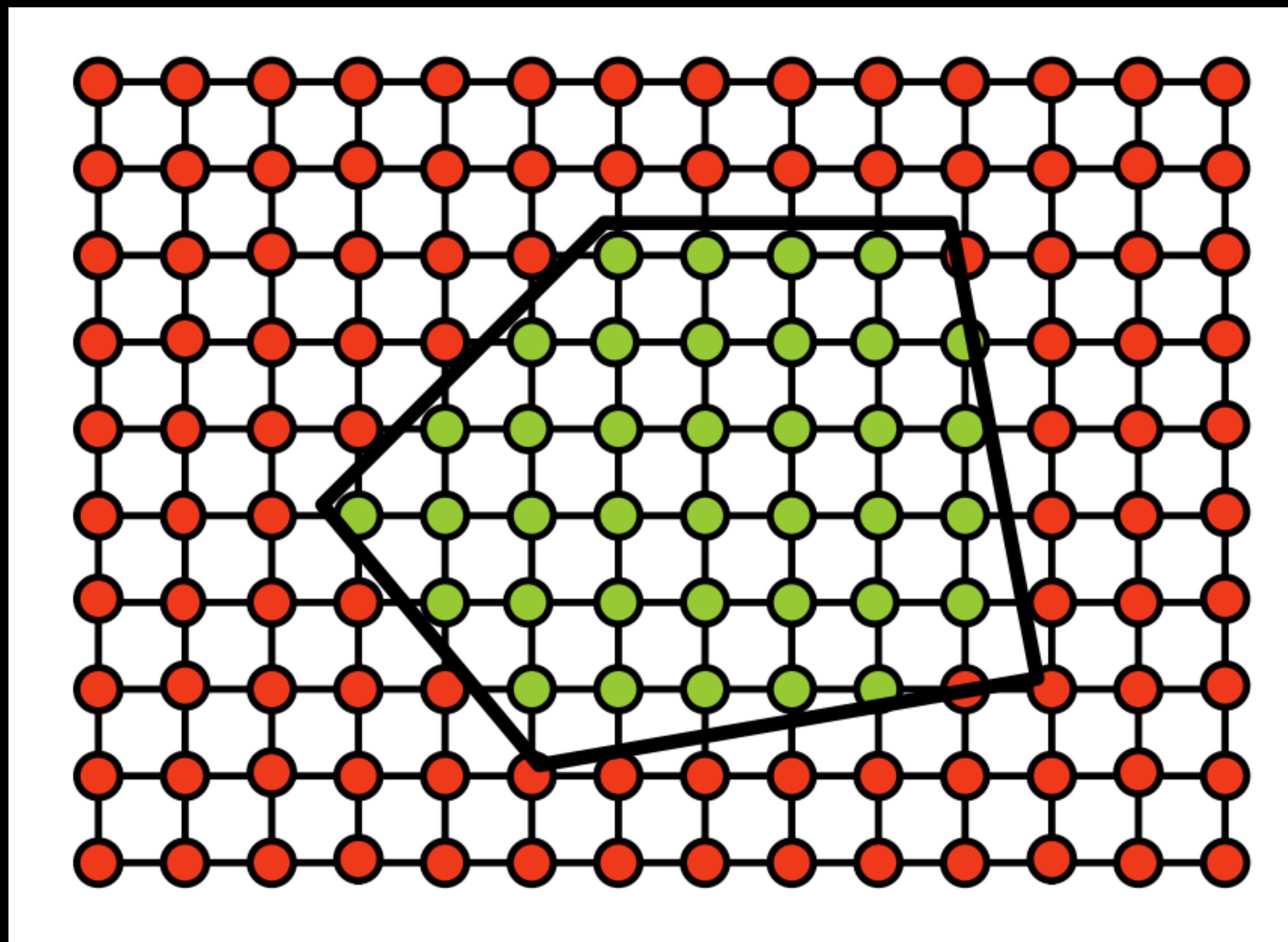
TEXT

EXTRACTING THE SURFACE



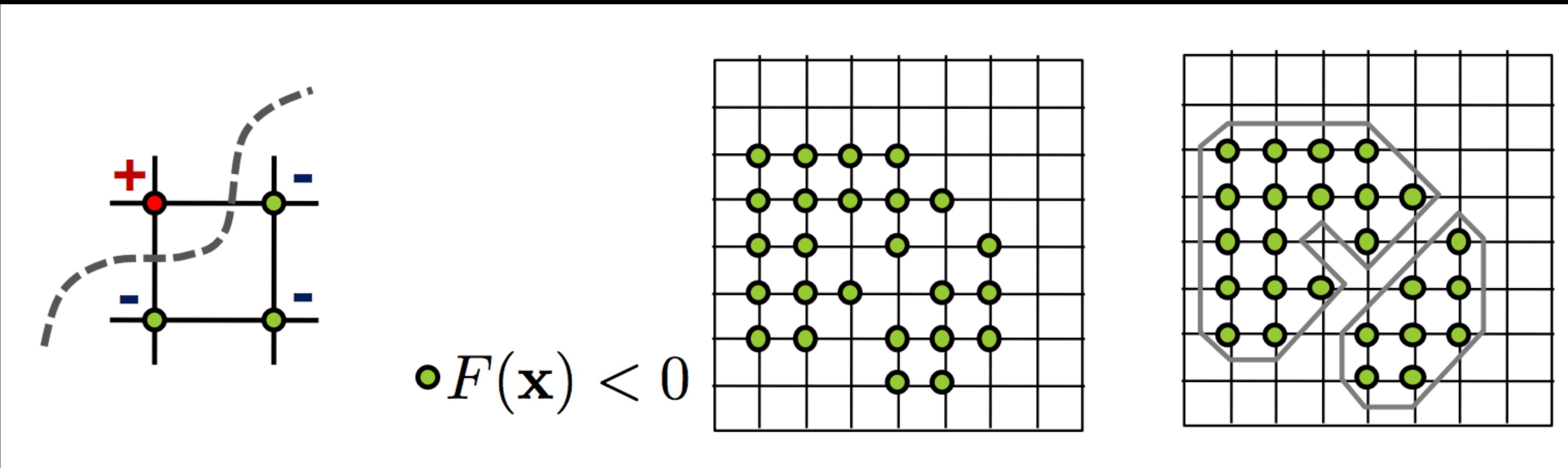
TEXT

EXTRACTING THE SURFACE



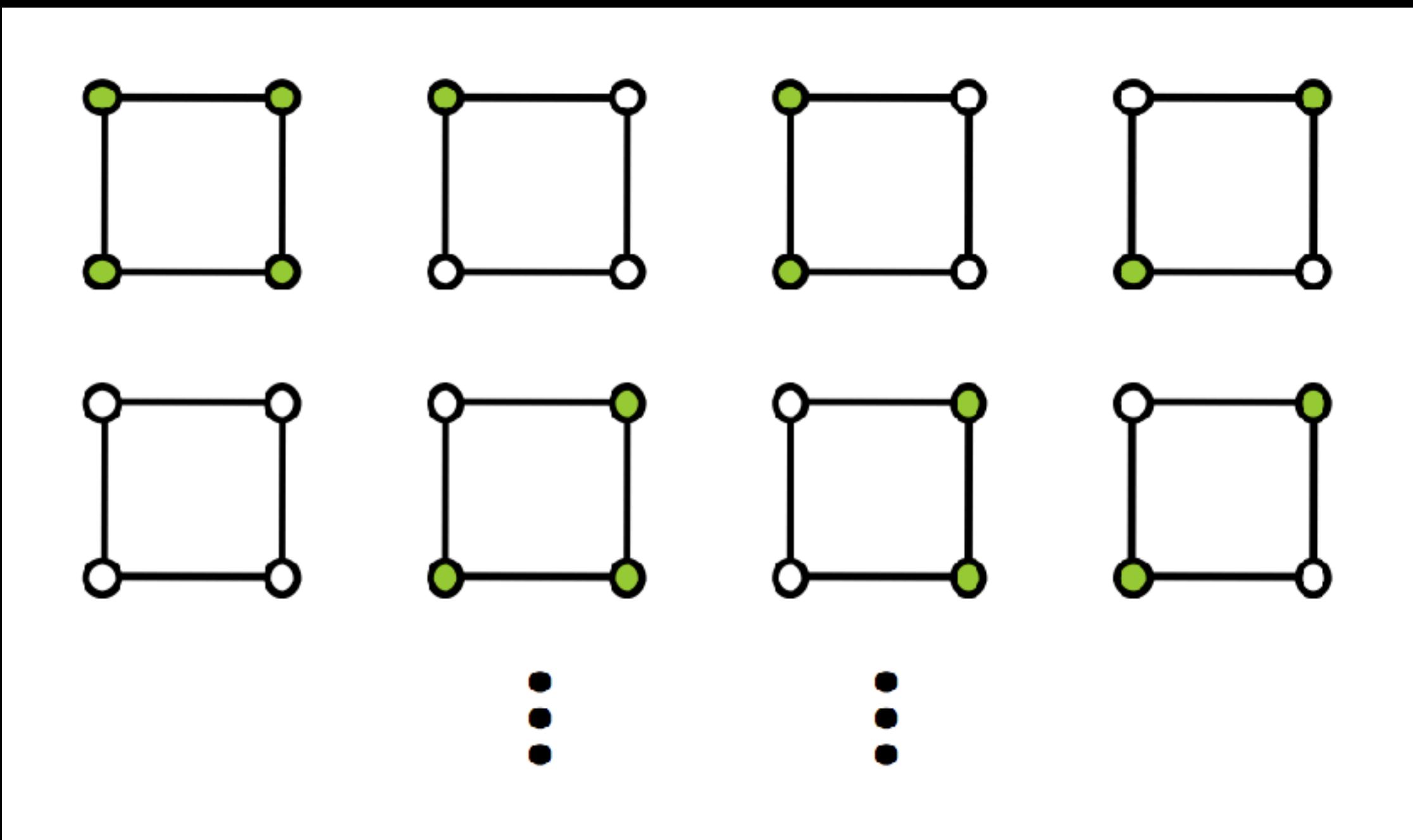
TESSELLATION

- ▶ Idea: sample and assume function is linear within the sampled grid.
- ▶ Zero set passes only where different signs exist in a grid cell.



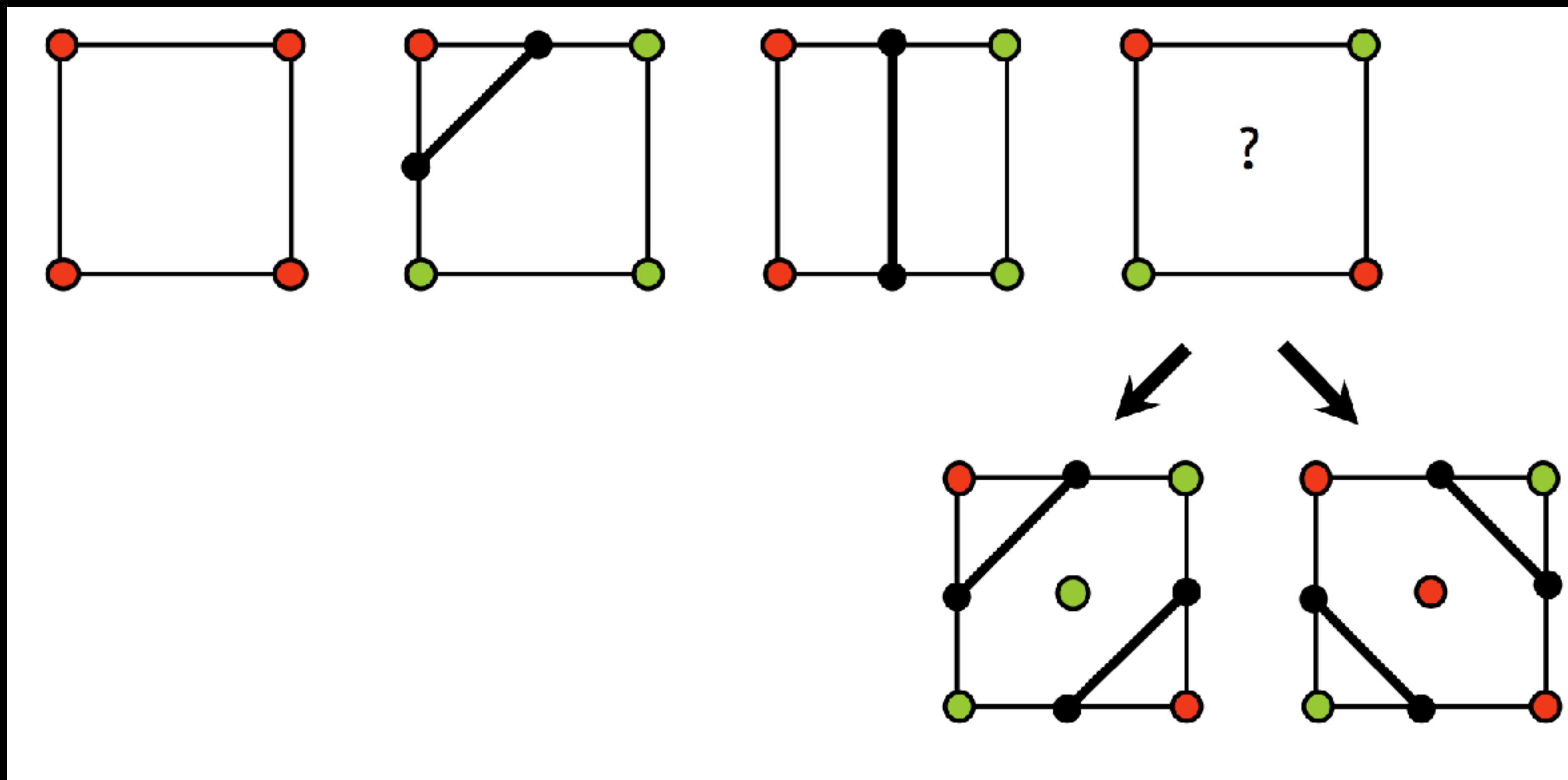
2D MARCHING SQUARES

- ▶ 16 different configurations in 2D.
- ▶ 4 equivalence classes (up to rotational and reflection symmetry + complement).



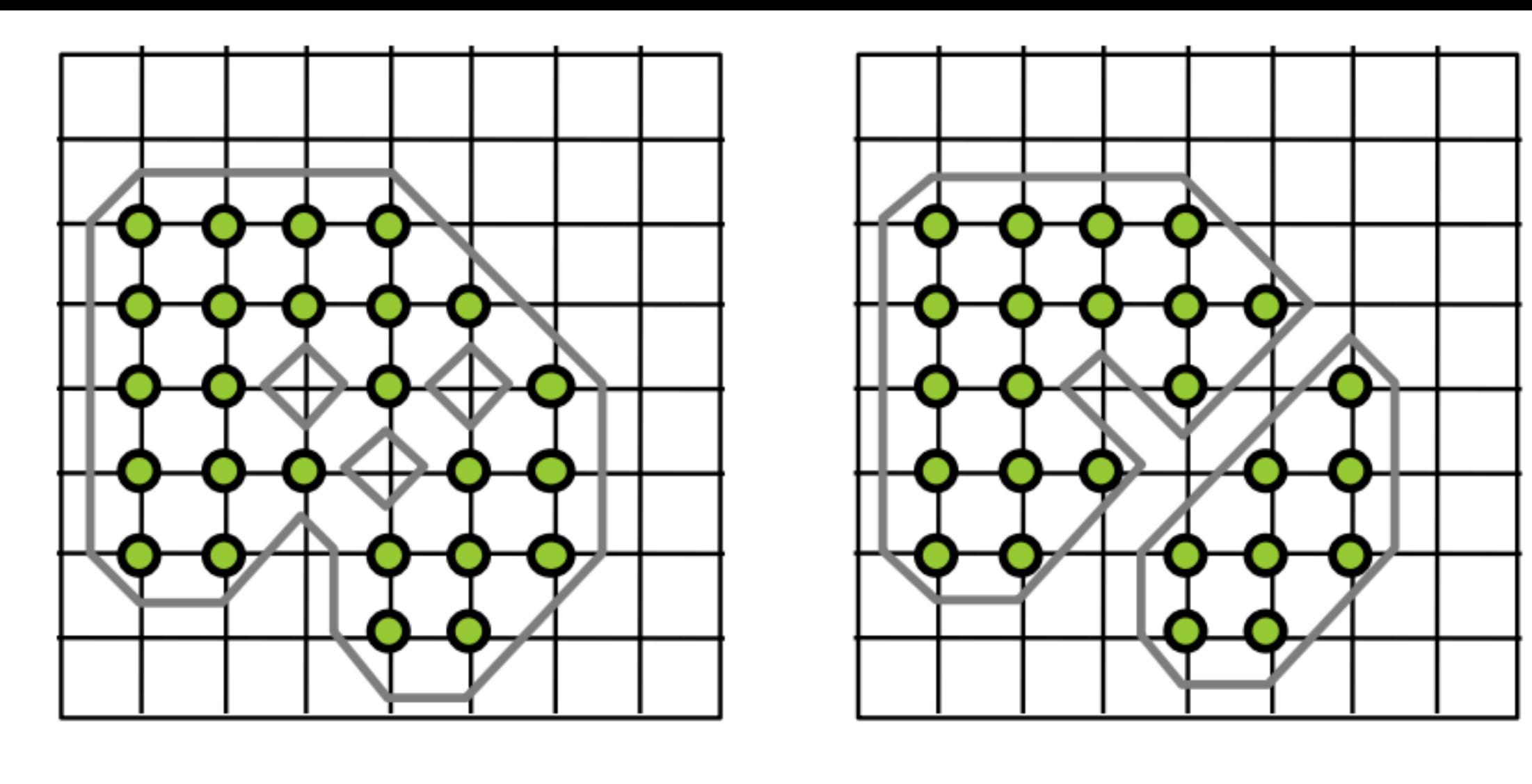
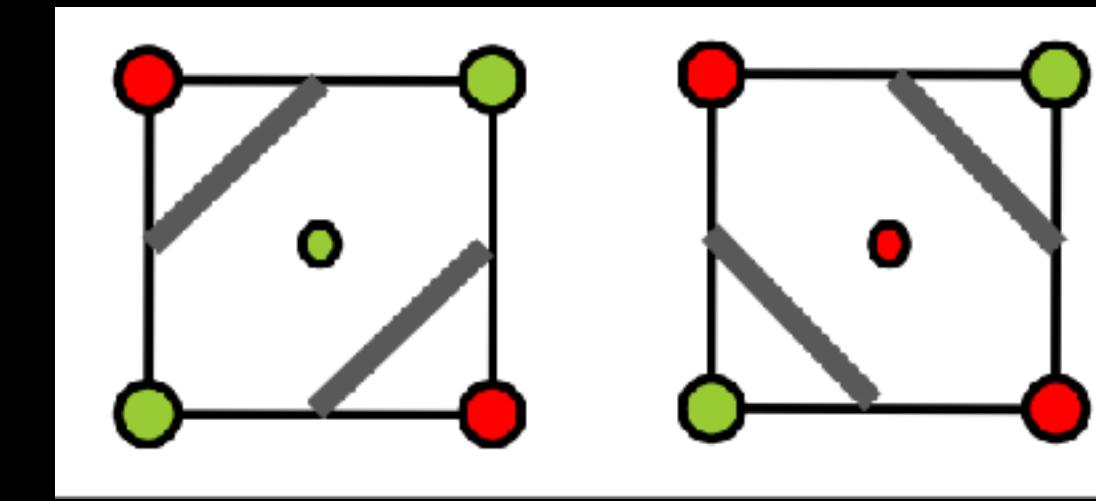
TESSELLATION IN 2D

- ▶ 4 equivalence classes (up to rotational and reflection symmetry + complement)



TESSELLATION IN 2D

- ▶ Case 4 is ambiguous:
- ▶ Choose arbitrarily; but consistently.
- ▶ To avoid problems with the resulting mesh.

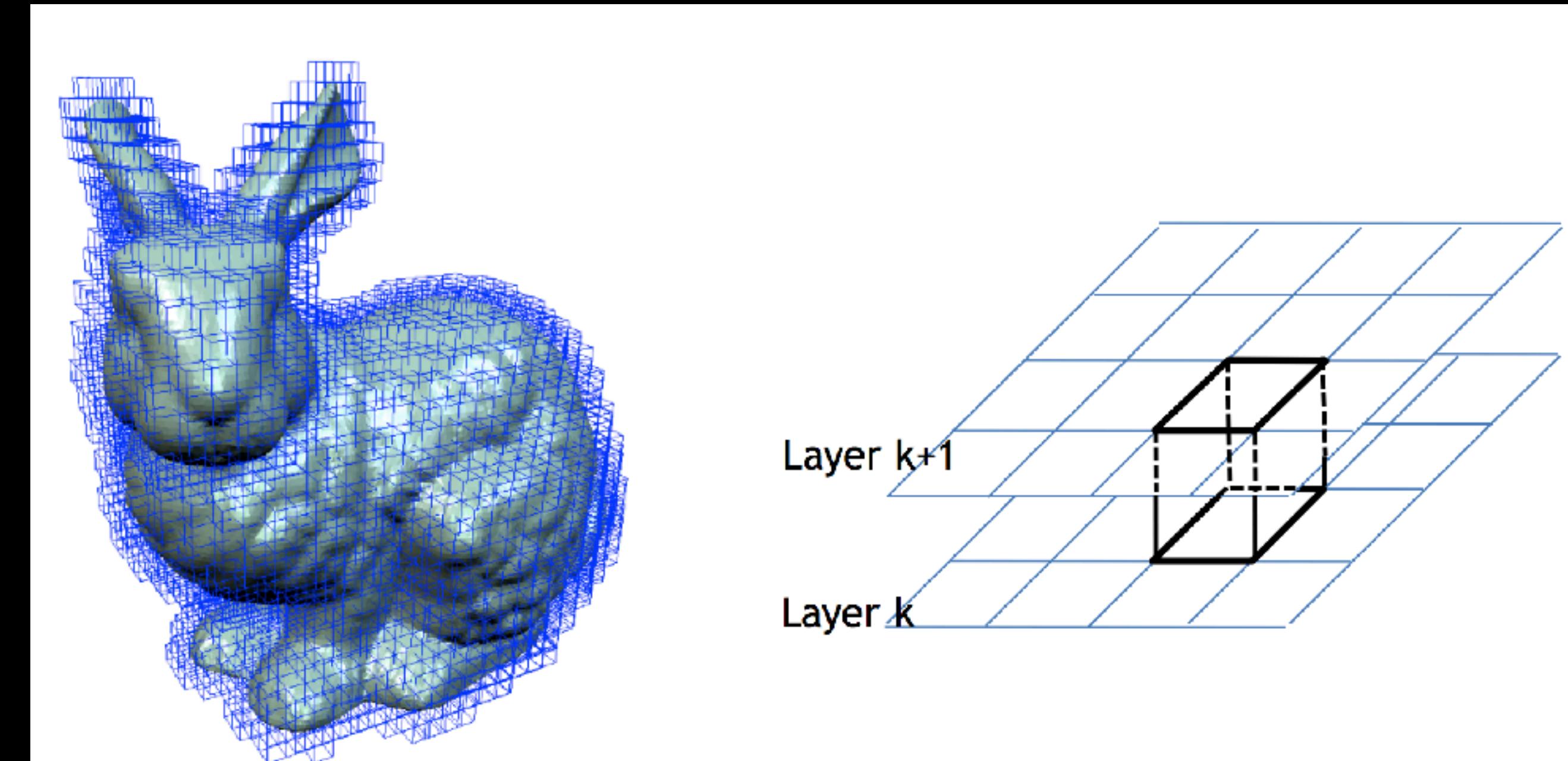


3D: MARCHING CUBES

- ▶ Classify grid nodes as inside/outside inside/outside
- ▶ Classify cell: 2^8 configurations
- ▶ Linear interpolation along edges
- ▶ Look-up table for patch configuration
- ▶ Disambiguation more complicated

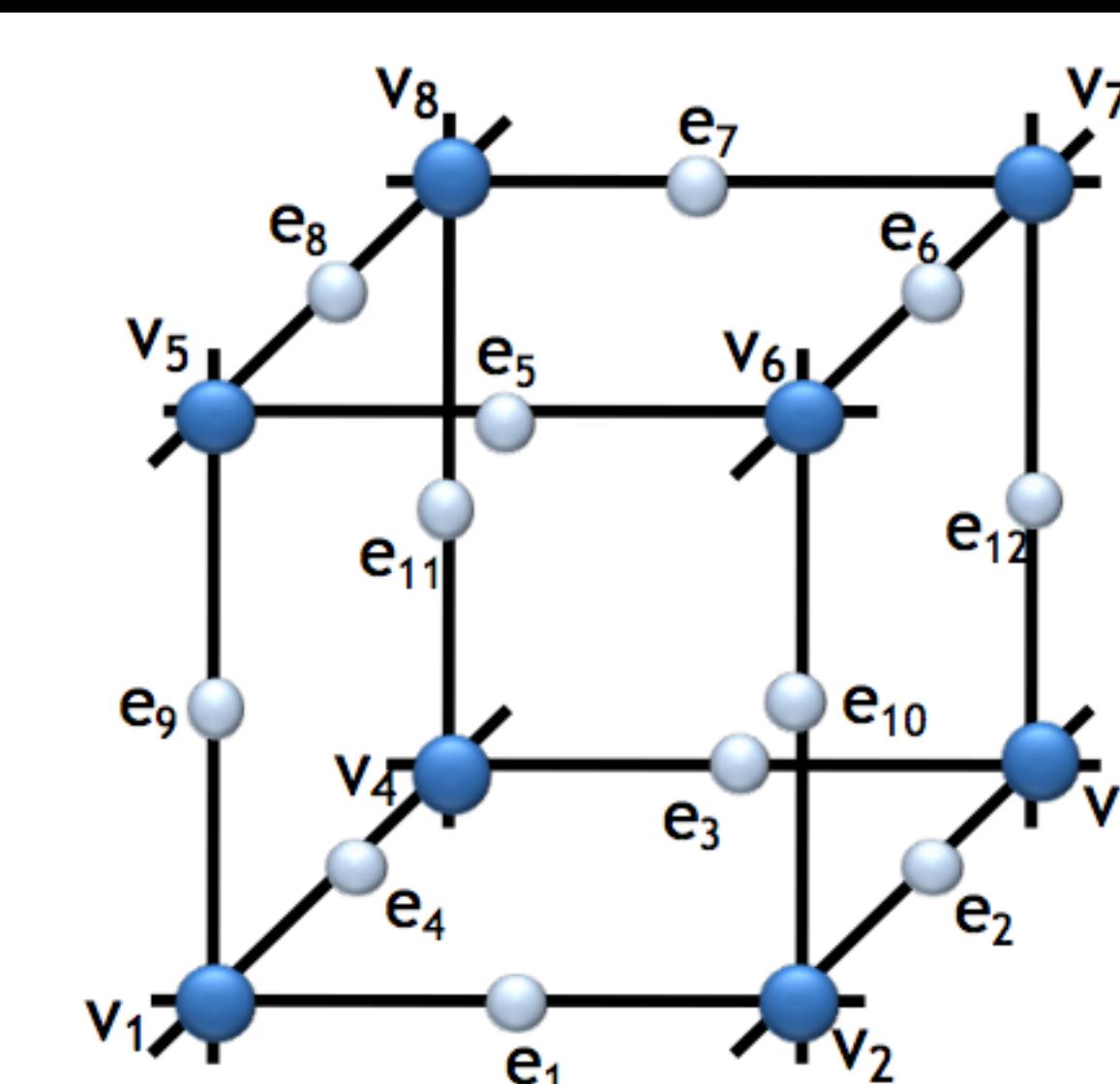
3D: MARCHING CUBES

- ▶ Cell classification:
 - ▶ Inside
 - ▶ Outside
 - ▶ Intersecting



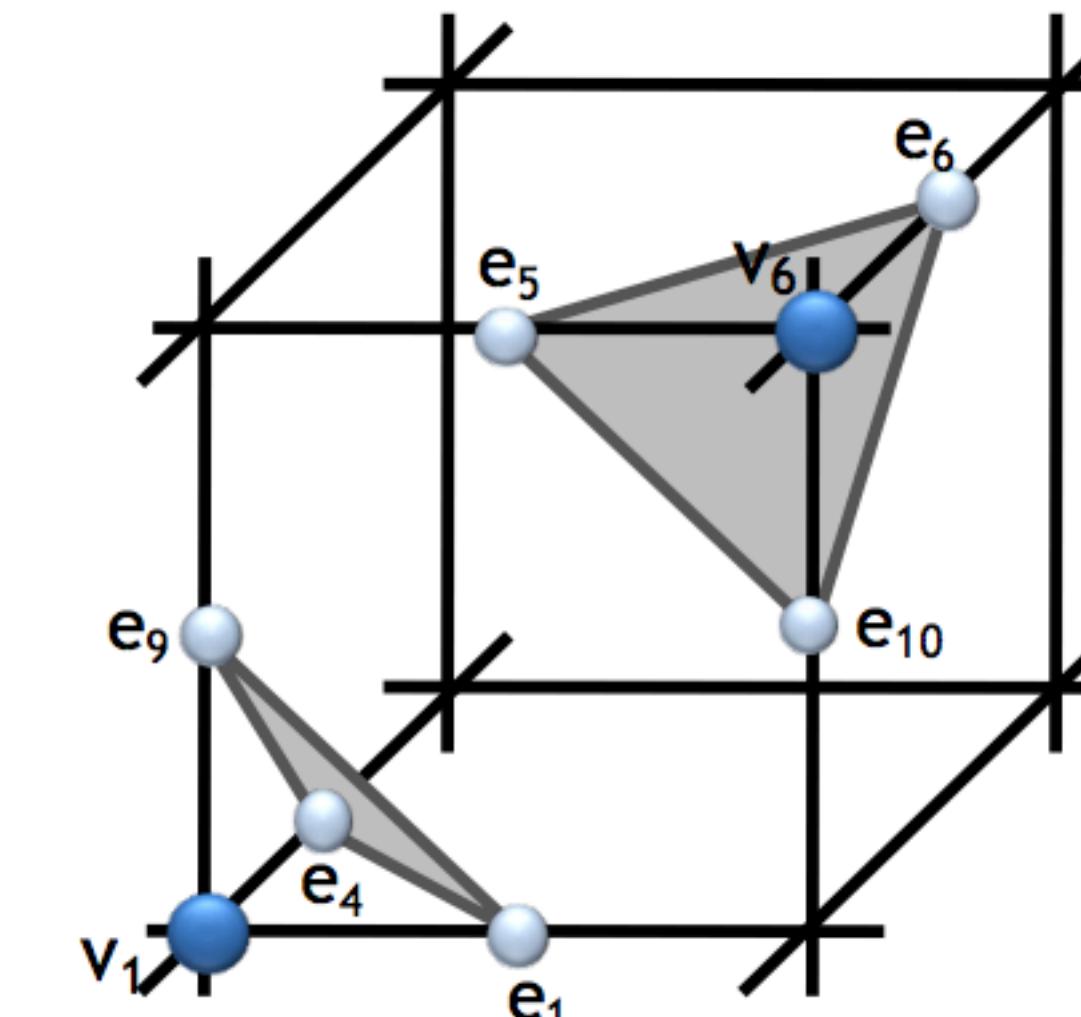
MARCHING CUBES

- ▶ Compute case index. We have $2^8 = 256$ cases (0/1 for each of the eight vertices) - can store as 8 bit (1 byte) index.



index =

v ₁	v ₂	v ₃	v ₄	v ₅	v ₆	v ₇	v ₈
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------



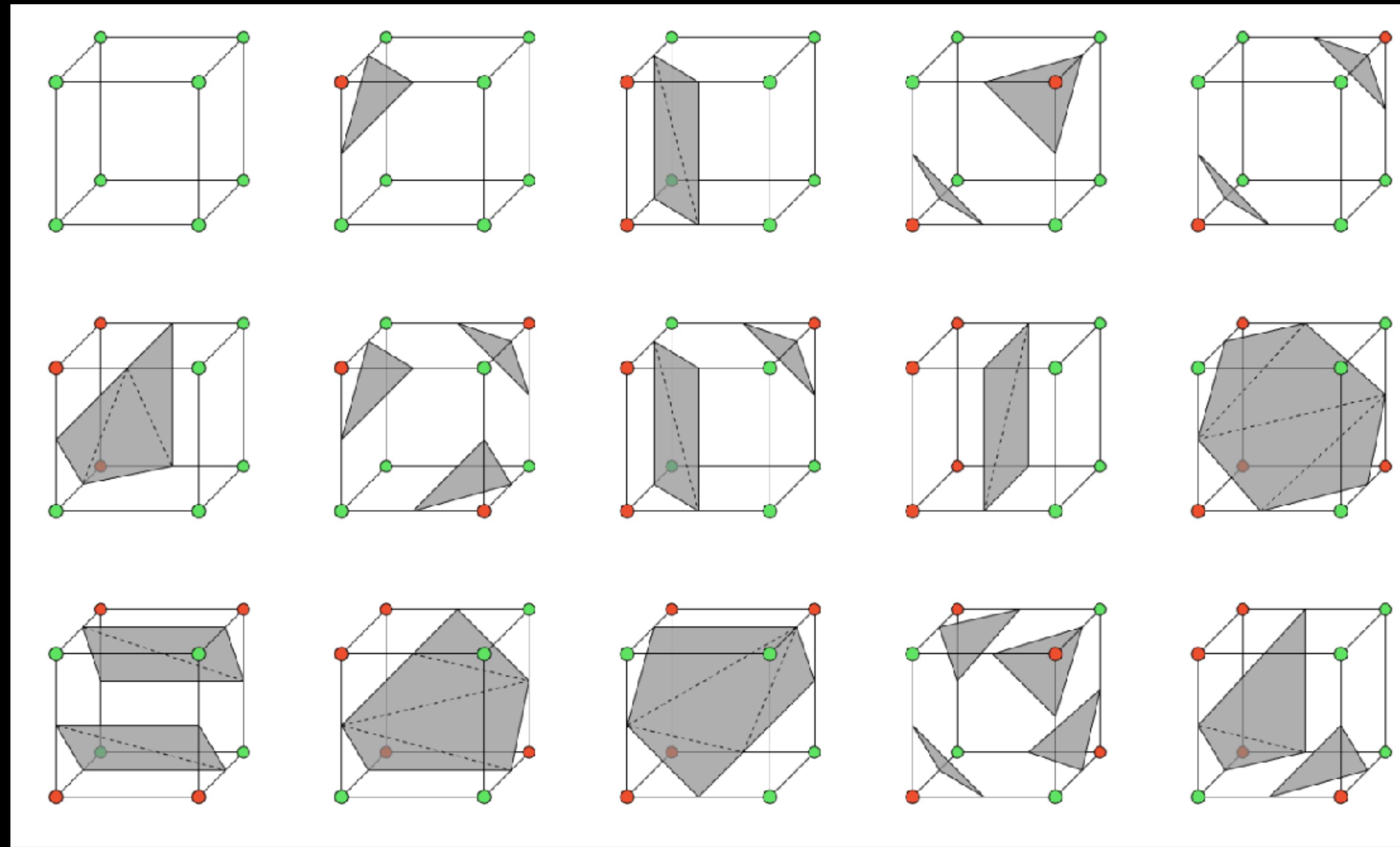
index =

0	0	1	0	0	0	0	1
---	---	---	---	---	---	---	---

 = 33

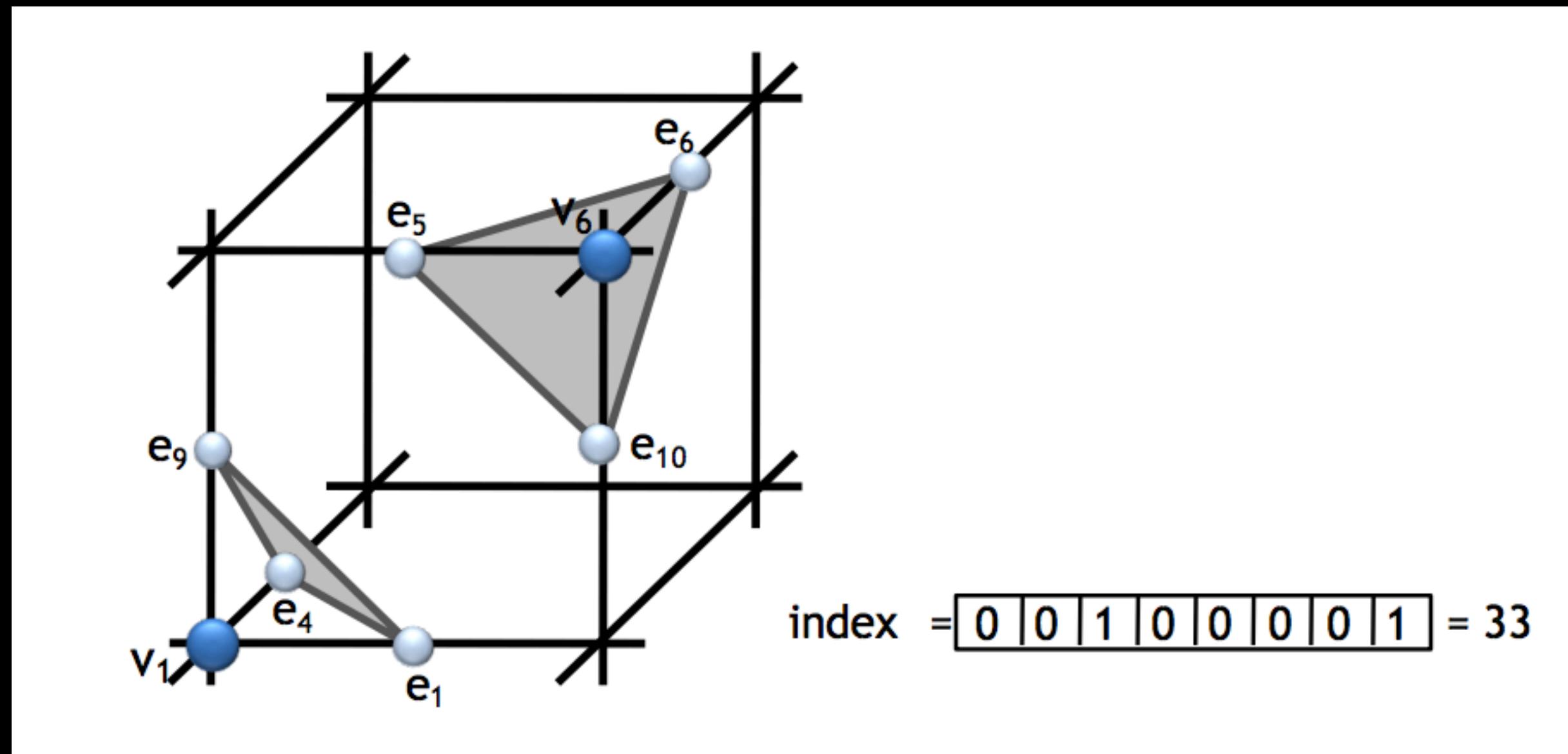
MARCHING CUBES

- ▶ Unique cases (by rotation, reflection and complement)



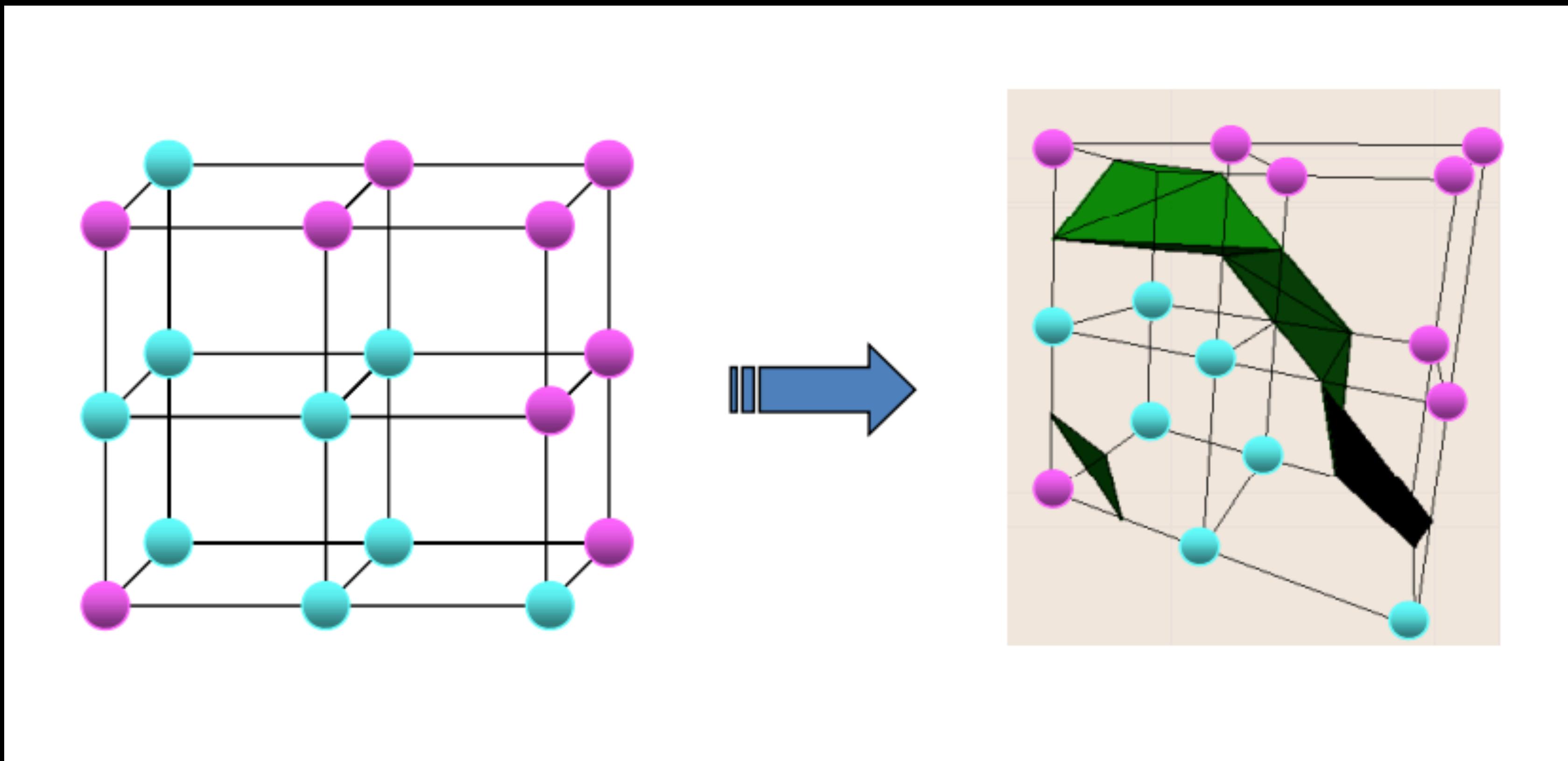
TESSELLATION

- ▶ Using the case index, retrieve the connectivity in the look-up table.
- ▶ Example: the entry for index 33 indicates:
 - ▶ Cut edges are e_1 ; e_4 ; e_5 ; e_6 ; e_9 and e_{10} ;
 - ▶ Output triangles are $(e_1; e_9; e_4)$ and $(e_5; e_{10}; e_6)$.



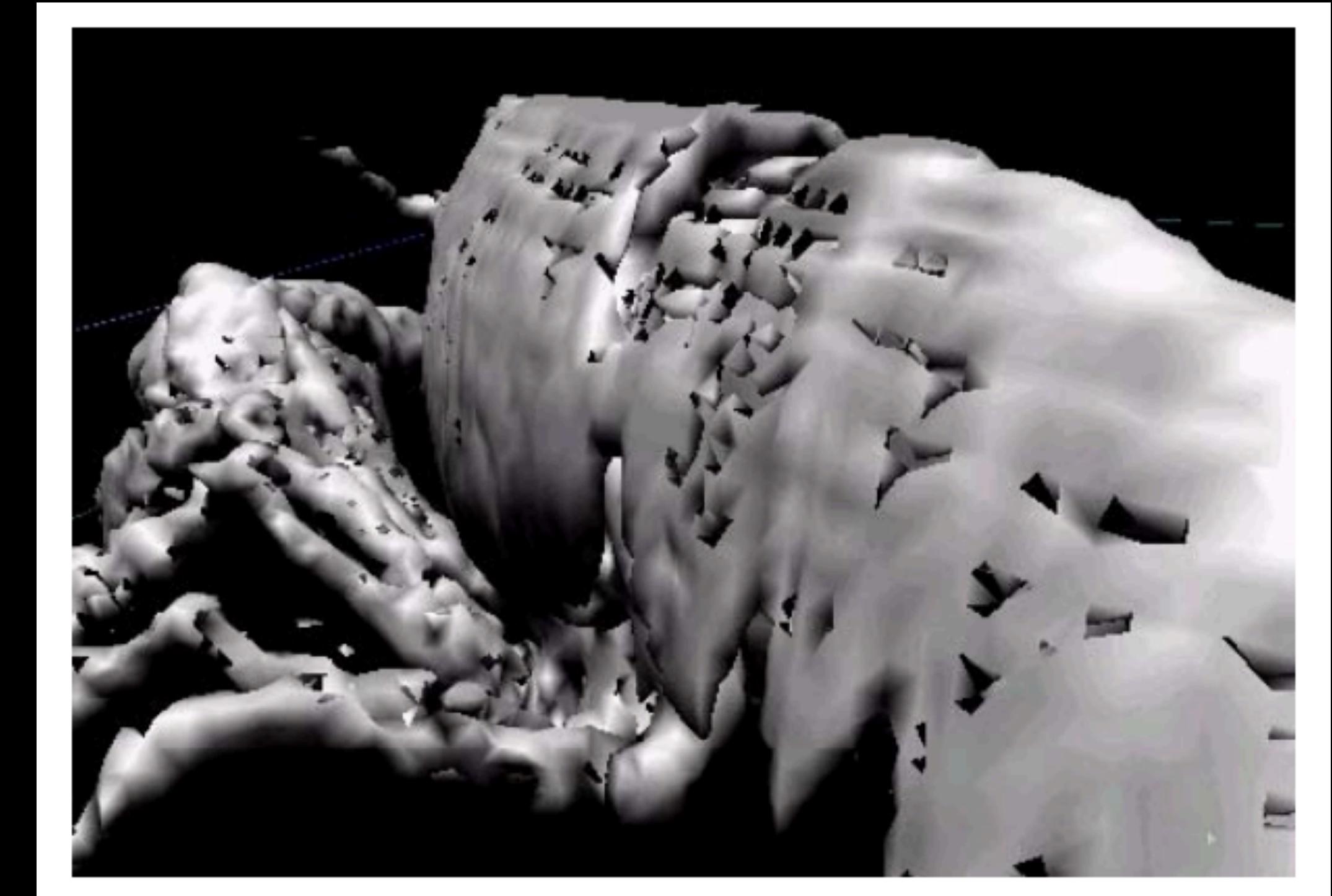
TEXT

MARCHING CUBES



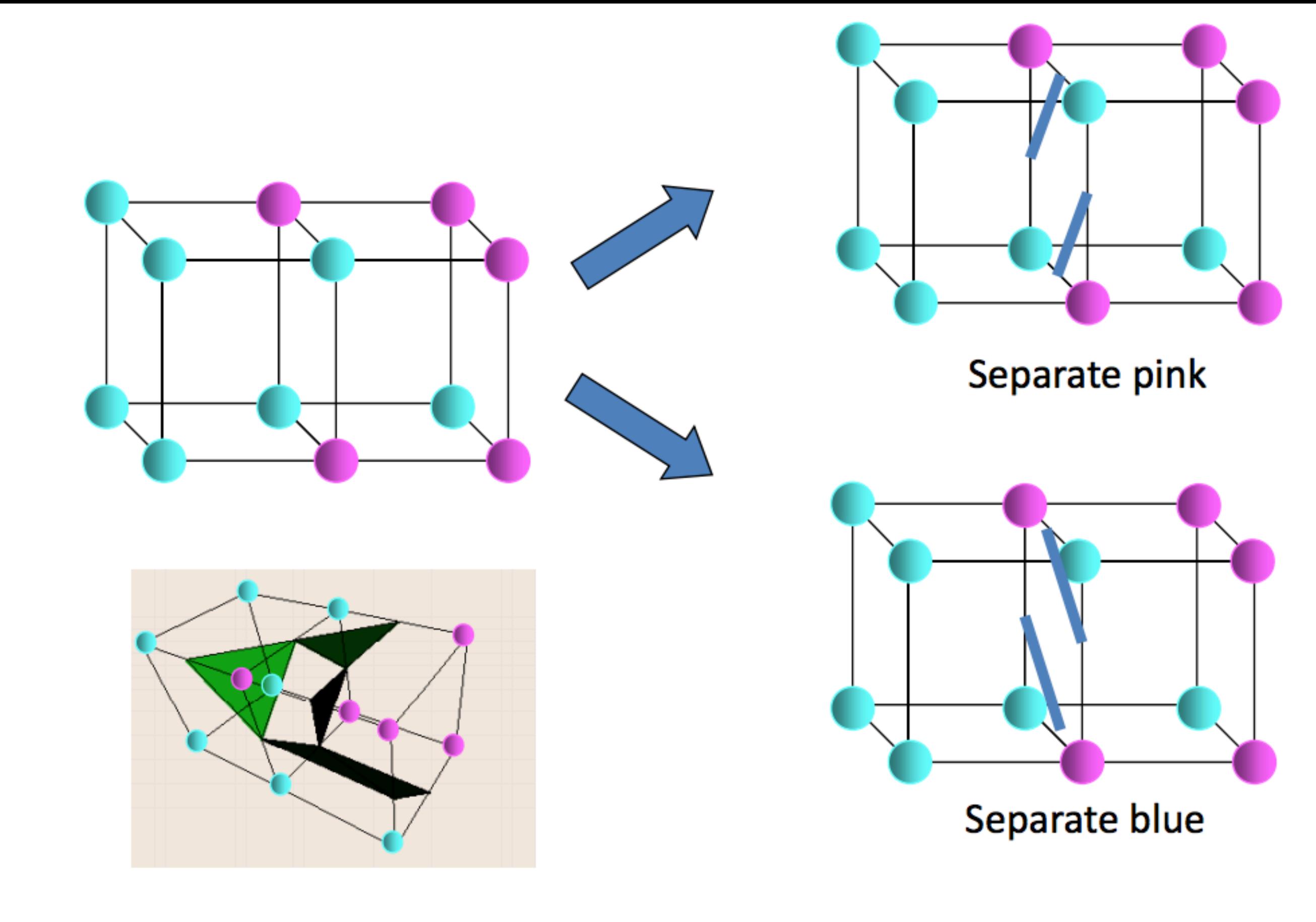
MARCHING CUBES PROBLEMS

- ▶ Ambiguity
- ▶ Holes
- ▶ Generates **HUGE** meshes
- ▶ Millions of polygons



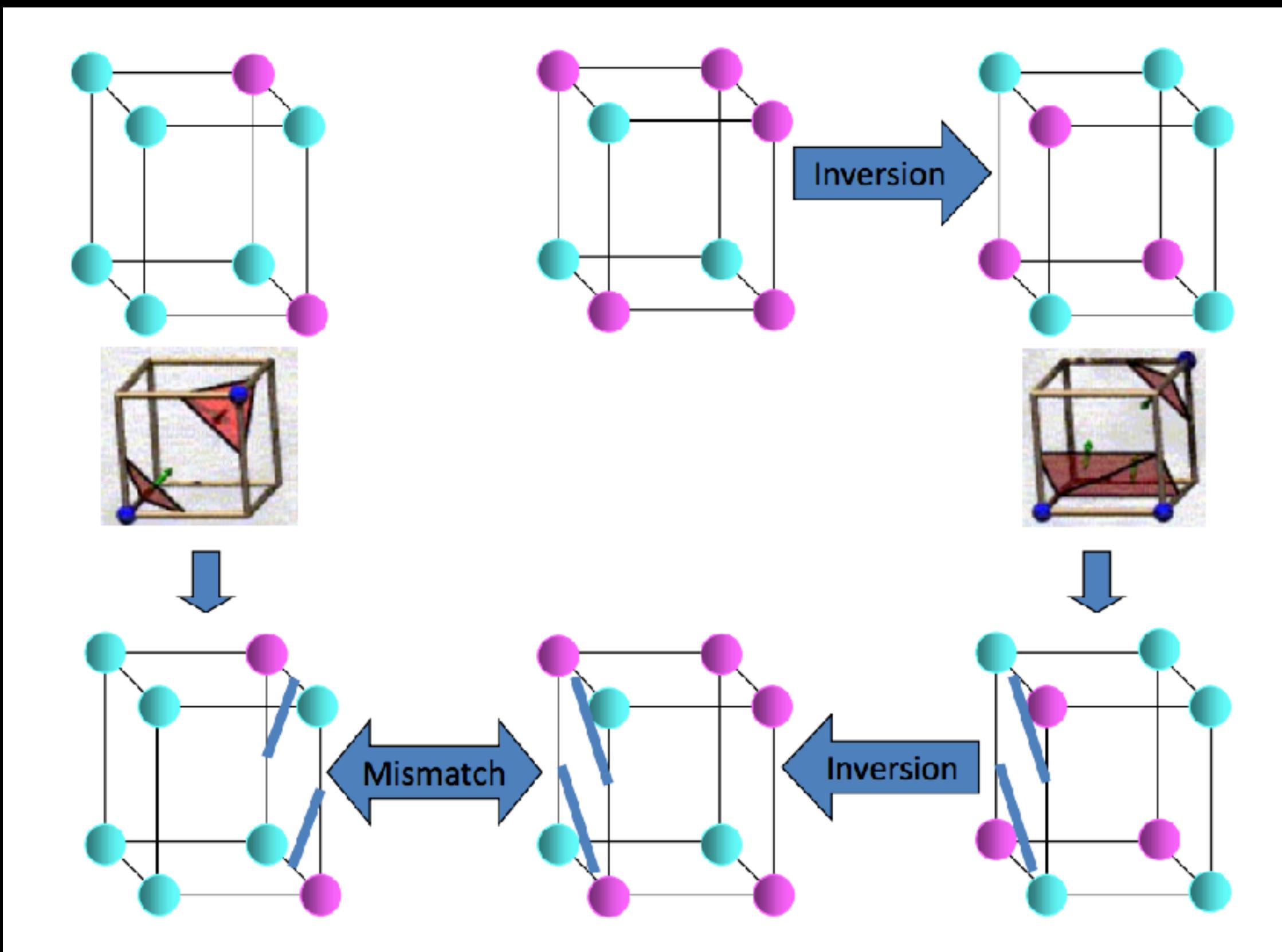
TEXT

AMBIGUITY



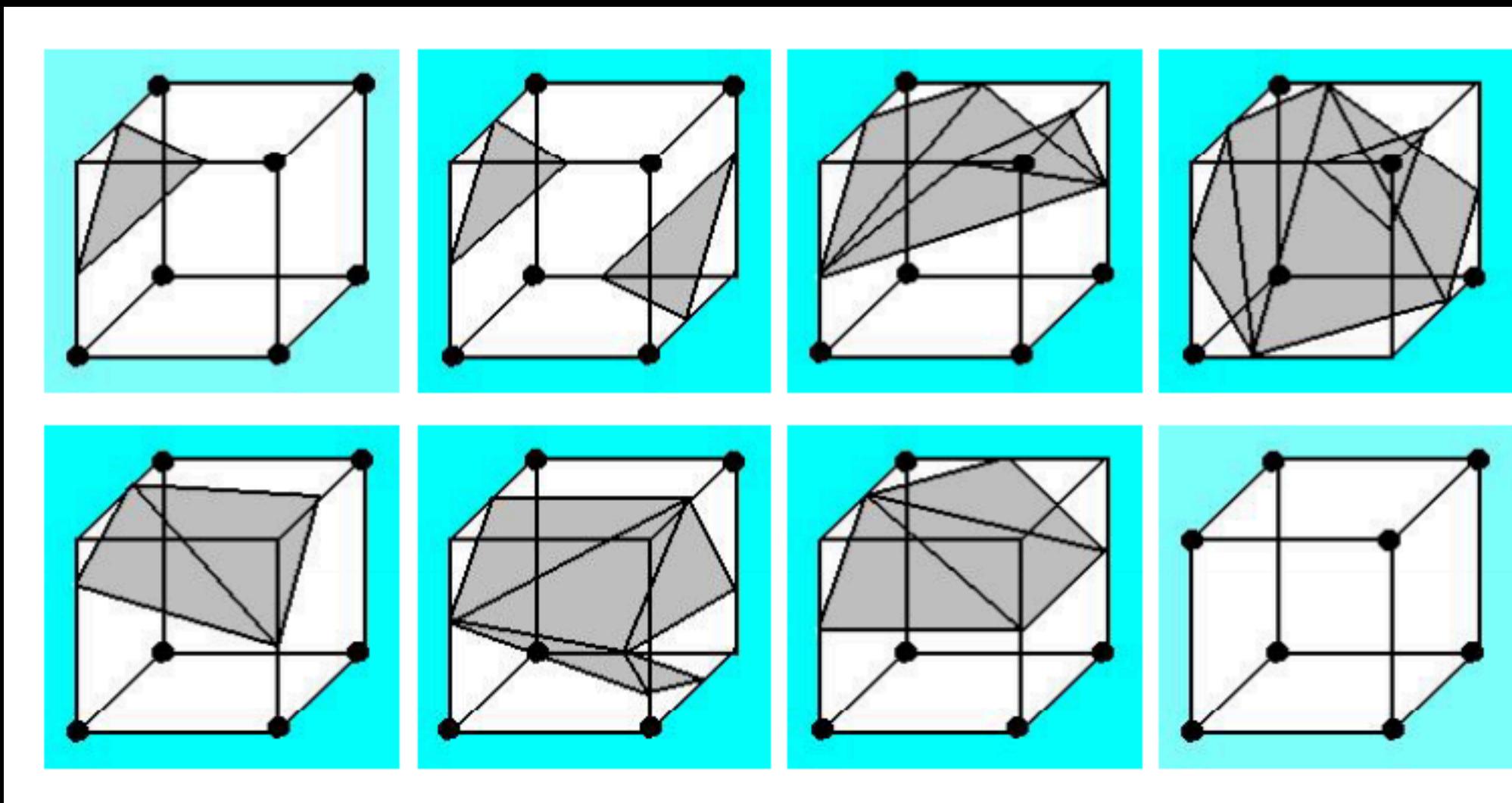
THE INVERSION PROBLEM

- ▶ Reduction from 256 to 15 cases includes inversion



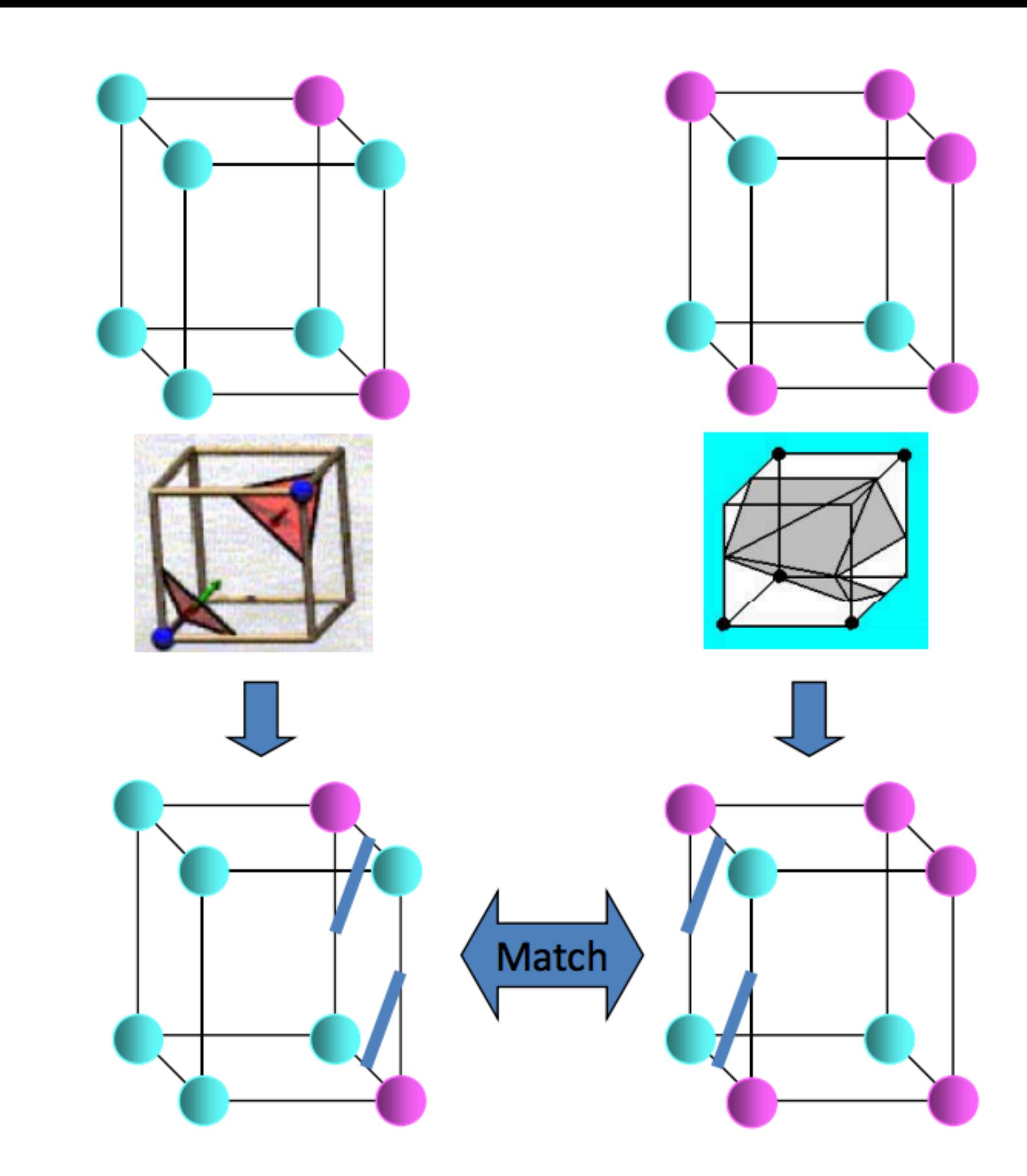
AMBIGUITY SOLUTION

- ▶ 256 cases -> 23 cases
 - ▶ Rotation only
 - ▶ Always separate same “color”
 - ▶ Ambiguous faces triangulated consistently
- ▶ 8 new cases



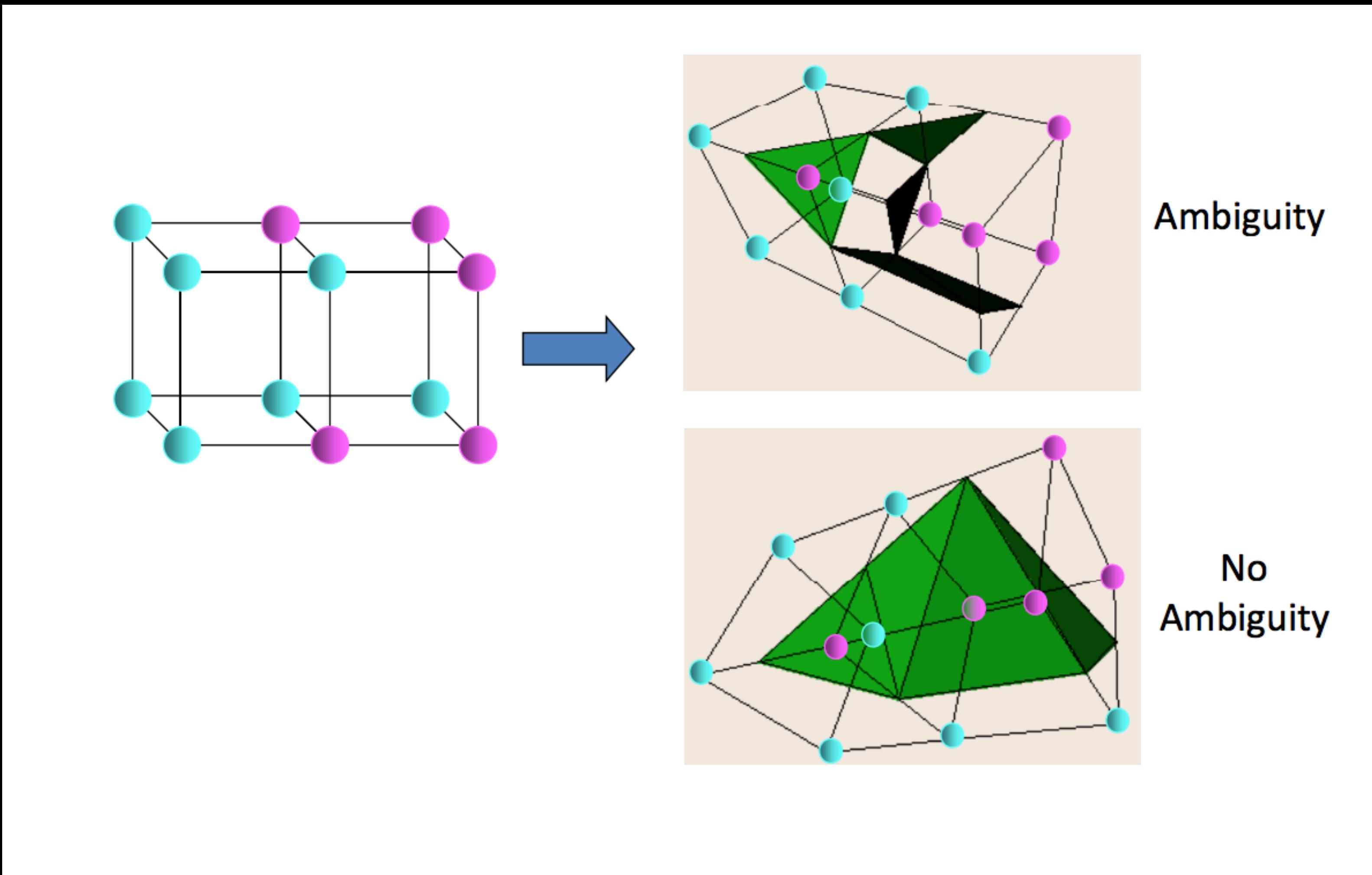
TEXT

WITHOUT INVERSION

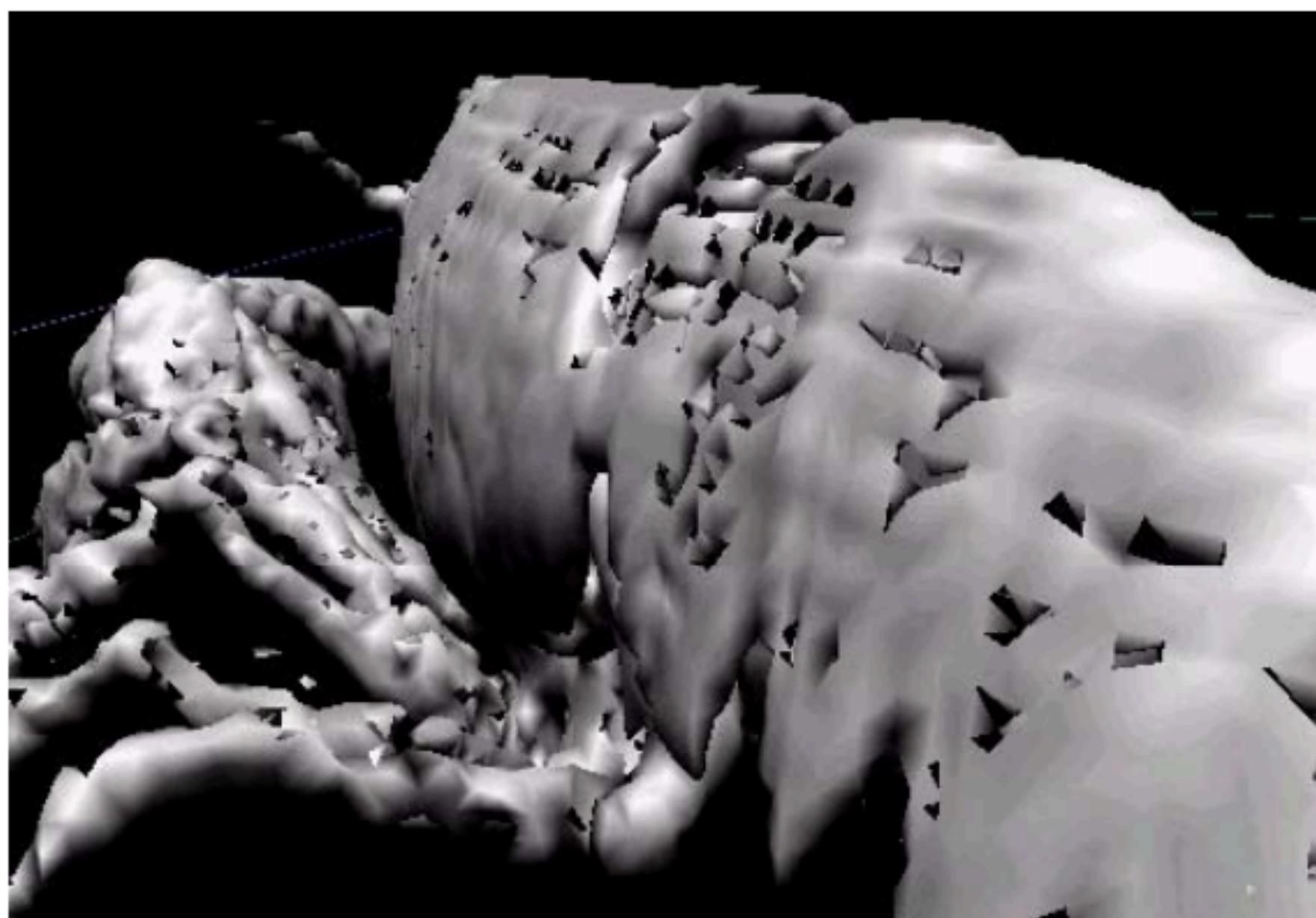


TEXT

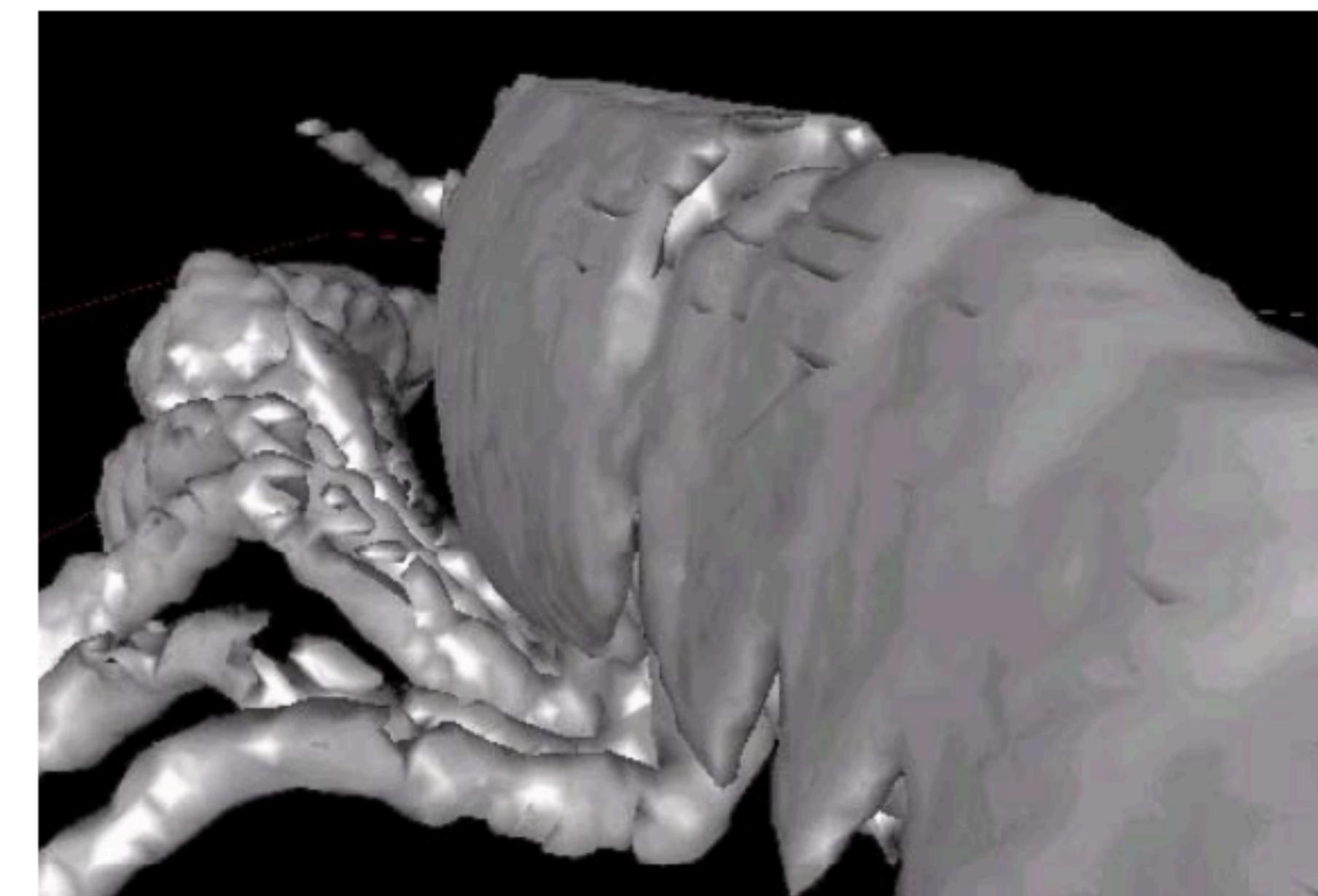
WITHOUT INVERSION



TEXT



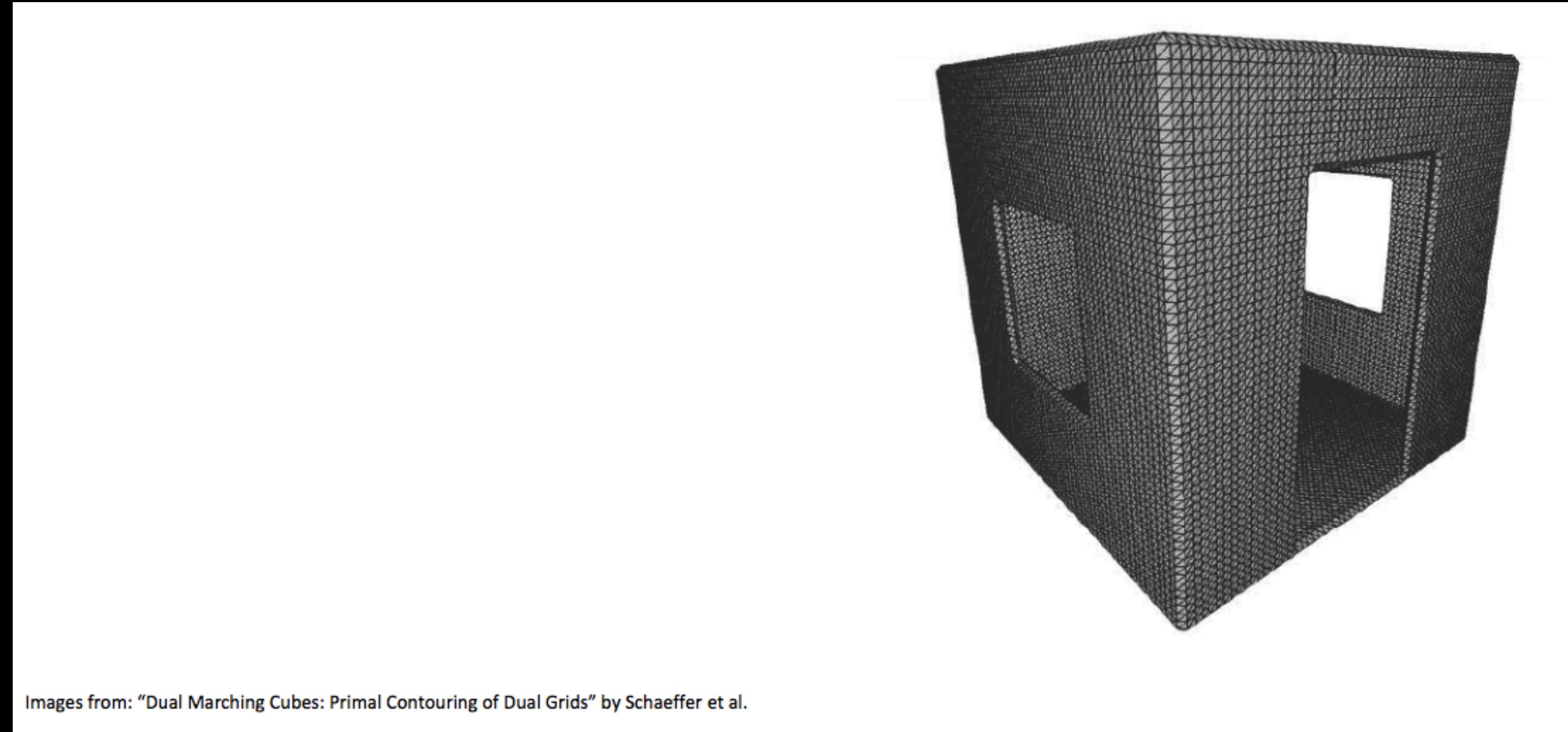
Ambiguity



No Ambiguity

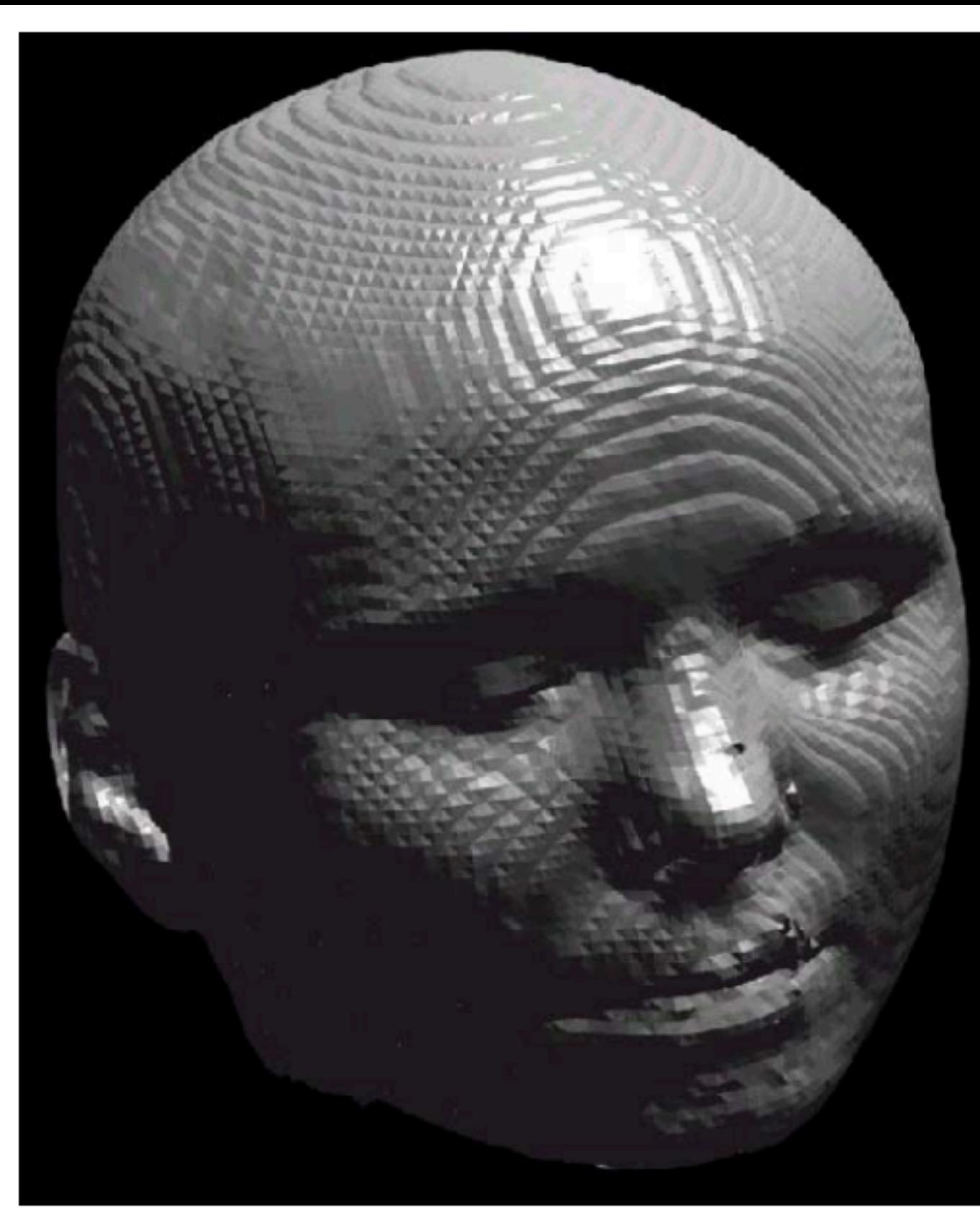
MARCHING CUBES ISSUES

- ▶ Grid not adaptive
- ▶ • Many polygons are used regardless of the geometric feature size.



Images from: "Dual Marching Cubes: Primal Contouring of Dual Grids" by Schaeffer et al.

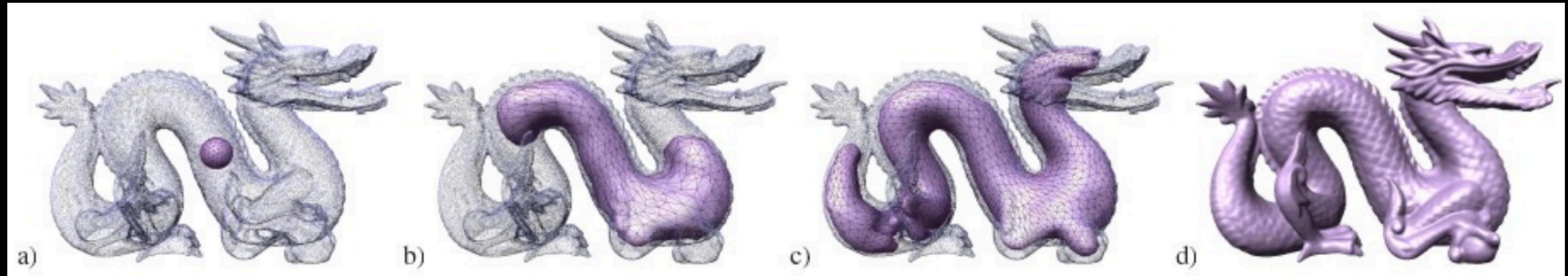
TEXT



DEFORMABLE MODELS

DEFORMABLE MODELS

- ▶ Define a close surface that will deform to fit the input points
- ▶ Suppose the surface to be watertight
- ▶ Can be combined with previous approaches

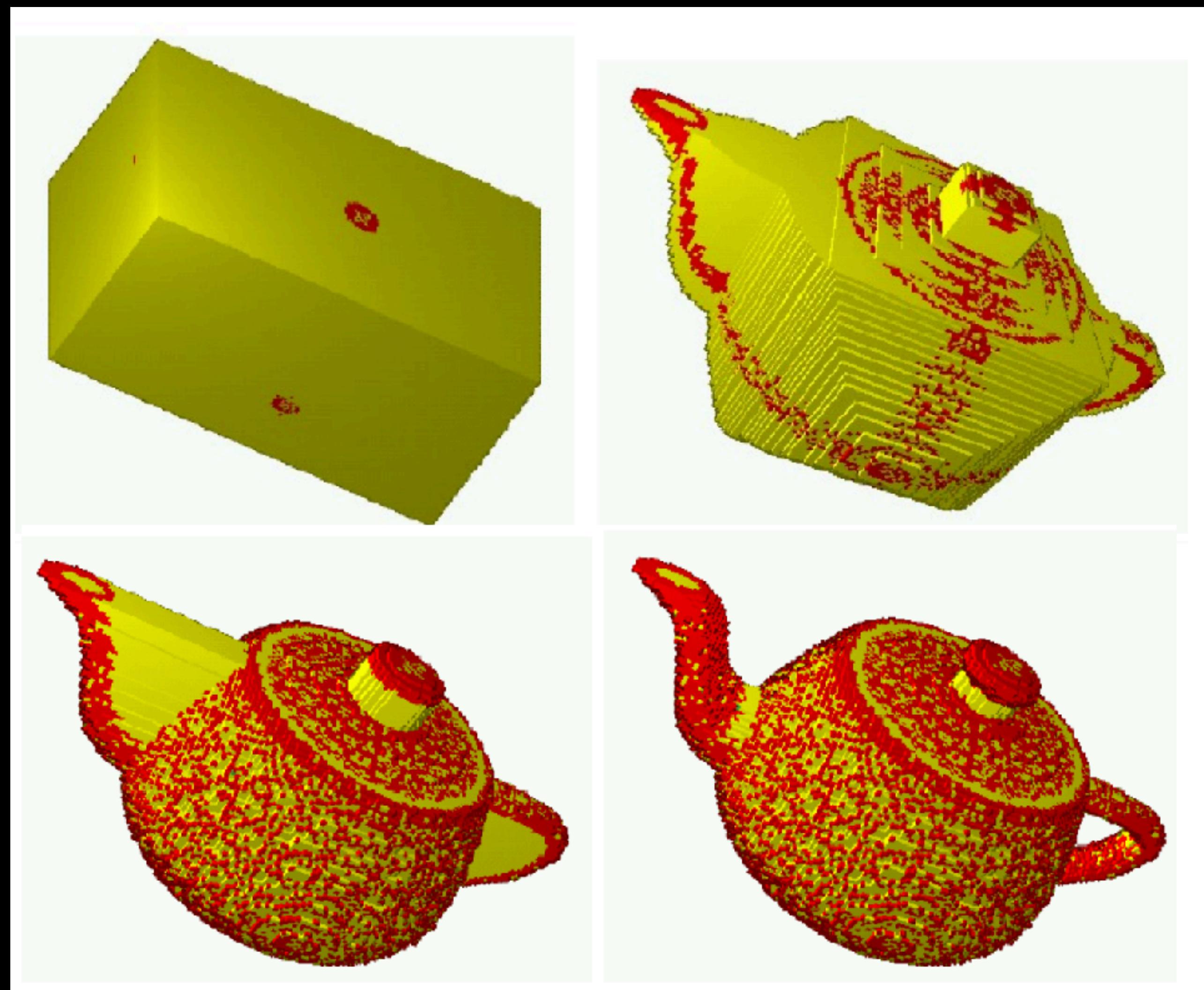


ESTEVE ET AL. 2005

- ▶ Discretization into a regular grid
- ▶ Discrete membrane = close connected set of voxels
- ▶ At the beginning: boundary voxels of the grid
- ▶ Then shrunk until it contains input points
- ▶ Operations: contraction, undo contraction, freeze
- ▶ No use of normal information
- ▶ OK for non-uniform sampling

TEXT

ESTEVE ET AL. 2005

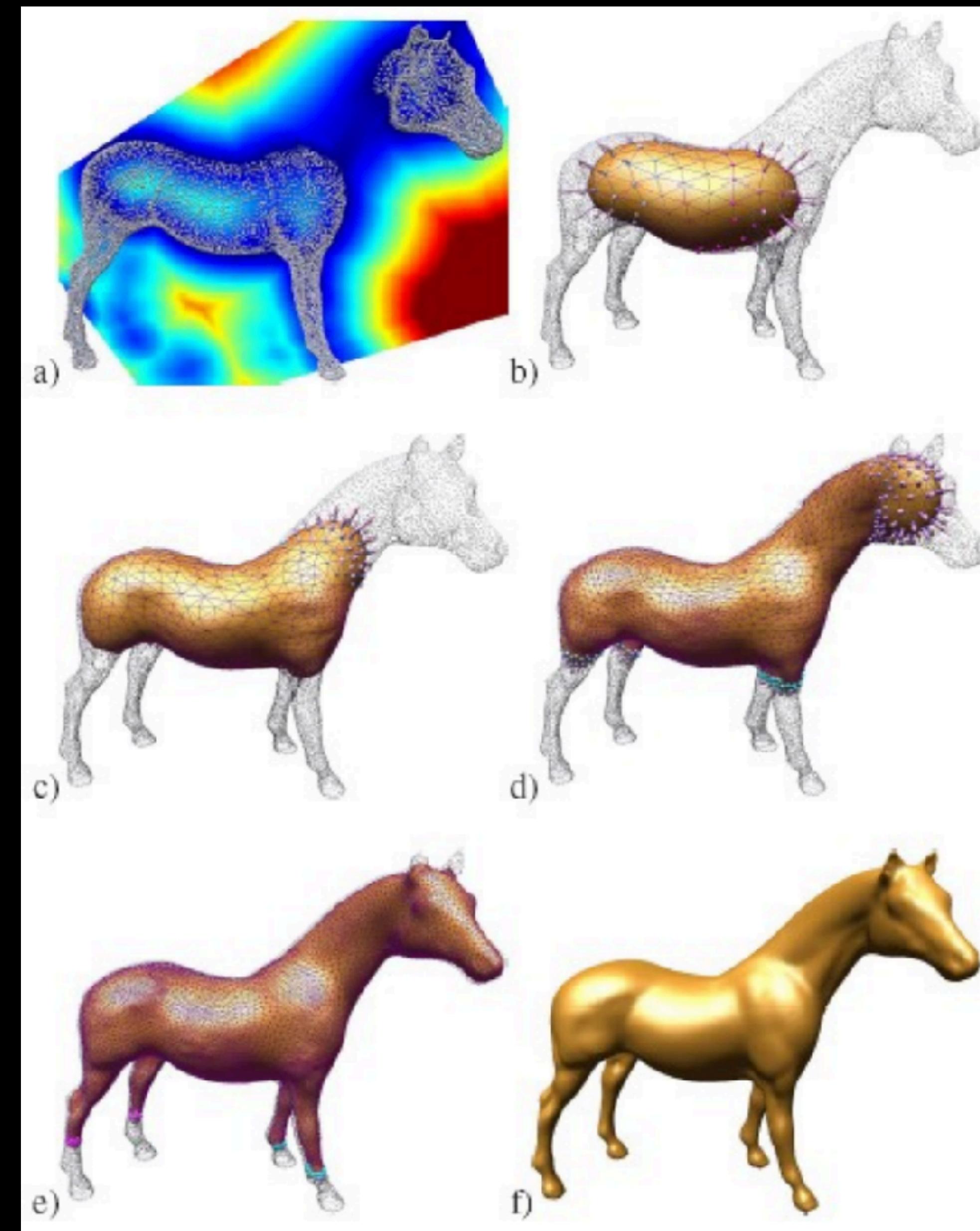


SHARF ET AL. 2006

- ▶ Start from a small sphere mesh inside the object
- ▶ Move its vertices in outward normal direction
- ▶ Using a volumetric distance map
- ▶ Adjust to local curvature and features (subdivision)
- ▶ Heuristics to handle topology changes

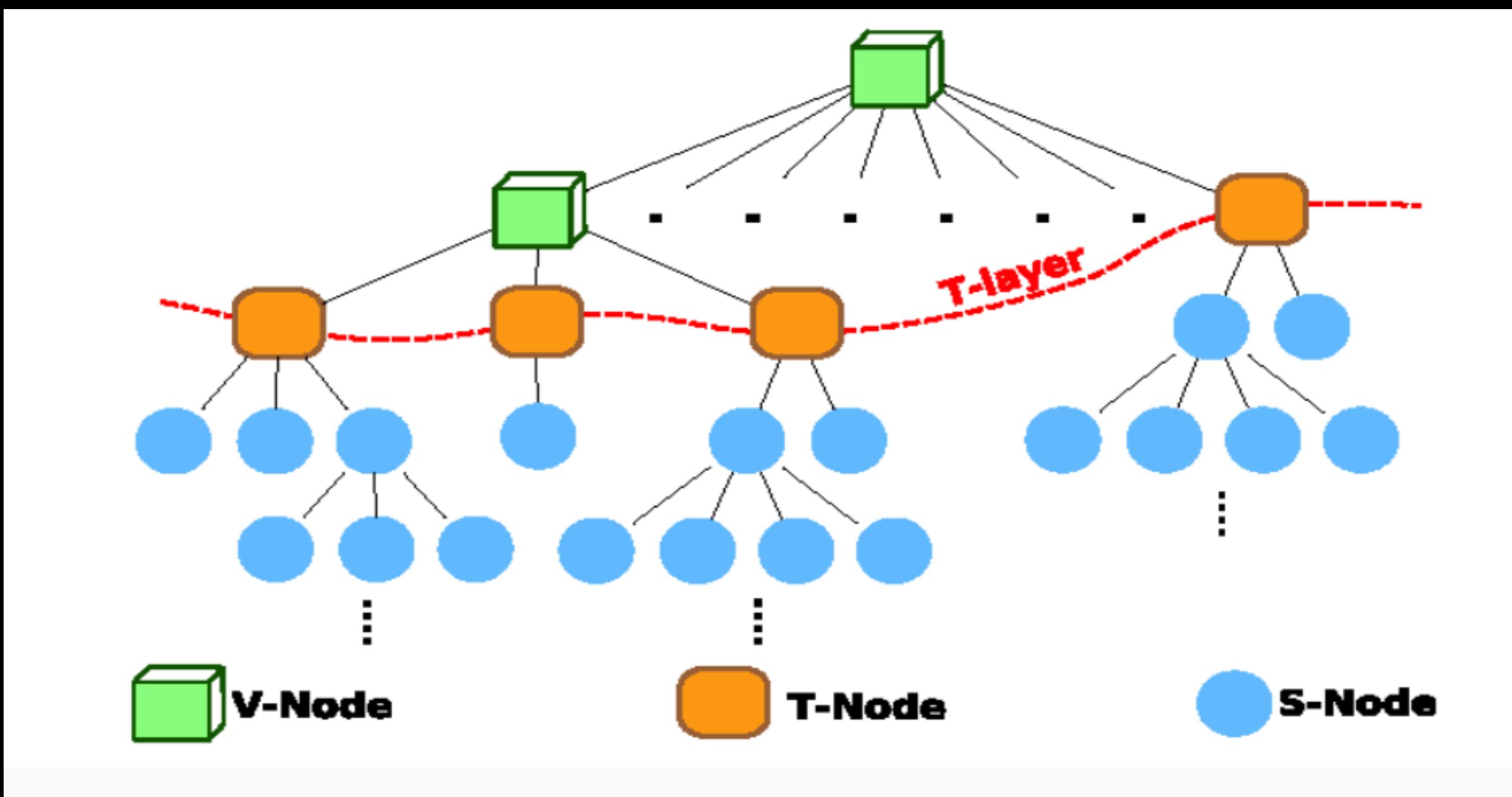
TEXT

SHARF ET AL. 2006



BOUBEKEUR ET AL. 2006

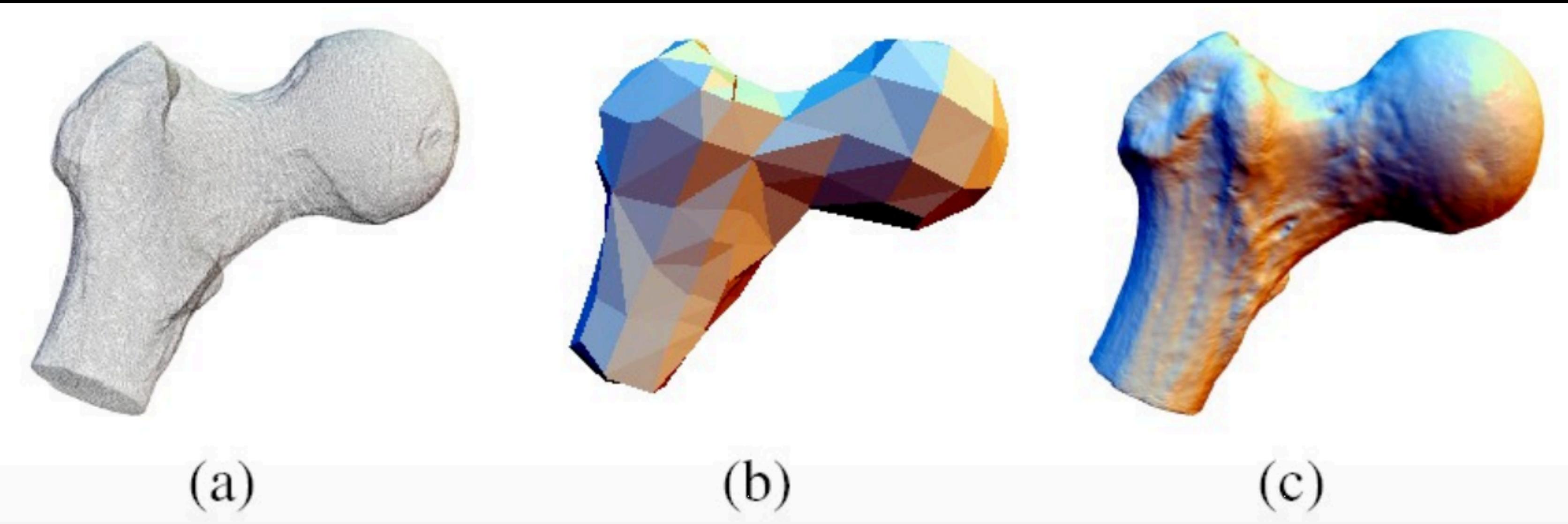
- ▶ Part of a more general paper presenting a new hierarchical space subdivision tool: VStrees
- ▶ ~ octree with surface leaves, forming a mesh



BOUBEKEUR ET AL. 2006

- ▶ Reconstruction process:

1. VS-tree construction
2. Coarse mesh constructed using the T-layer and MPU implicit reconstruction
3. Several refinement tricks



SUMMARY

- ▶ Implicit and Explicit Meshing
 - ▶ Implicit is more adequate for us (noisy and irregular data)
- ▶ Need normals
 - ▶ Estimated with [Hoppe92], PCA on KNN
 - ▶ Check orientation with camera
- ▶ Moving Least Squares, Radial Basis Functions, Poisson.
- ▶ Marching Cubes to Extract the Surface from the Function

SOURCES

- ▶ <https://www.cs.princeton.edu/courses/archive/fall12/cos526/lectures/11-reconstruction.pdf>
- ▶ <https://graphics.ethz.ch/publications/tutorials/points/powerpoint/Representation.Alexa.pdf>
- ▶ <http://www-evasion.imag.fr/people/Franck.Hetroy/Teaching/Geo3D/seance06-1.pdf>
- ▶ <http://www.hao-li.com/cs599-ss2015/slides/Lecture06.2.pdf>
- ▶ <http://www.cs.uu.nl/docs/vakken/ddm/2016-2017/Lecture%203%20-%20Reconstruction.pdf>
- ▶ http://www.cs.technion.ac.il/~cs236329/lectures/poisson_reconstruction.pdf
- ▶ <http://www.cs.uu.nl/docs/vakken/ga/slides9alt.pdf>
- ▶ http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/04_Surface_Reconstruction.pdf

RESOURCES

- ▶ <http://hhoppe.com/poissonrecon.pdf>
- ▶ [http://www.cs.jhu.edu/~misha/Code/PoissonRecon/
Version9.01/](http://www.cs.jhu.edu/~misha/Code/PoissonRecon/Version9.01/)
- ▶ <http://hhoppe.com/proj/recon/>
- ▶ [http://www-lb.cs.umd.edu/class/spring2005/cmsc828v/
papers/siggraph01.pdf](http://www-lb.cs.umd.edu/class/spring2005/cmsc828v/papers/siggraph01.pdf)