

COMPUTER VISION AND
PHOTOGRAMMETRY

TWO VIEW GEOMETRY

CONTENTS TODAY

- ▶ Quick review, Camera Geometry and Homographies
- ▶ Epipolar Geometry (review)
- ▶ Fundamental Matrix
- ▶ Projective Invariance and Projective Ambiguity
- ▶ Essential Matrix
- ▶ Computing F and E
- ▶ Canonical Cameras from F
- ▶ Triangulation

LET'S RECAP . . .

LAB 1
QUESTIONS?
PROBLEMS?

CAMERA PARAMETERS - NOTATION AND RELATIONSHIPS

- ▶ P
- ▶ K
- ▶ R
- ▶ T
- ▶ Q
- ▶ C
- ▶ $k_1, k_2, k\dots$

CAMERA PARAMETERS - NOTATION AND RELATIONSHIPS

- ▶ P = Projection matrix (pinhole camera model)
- ▶ takes a 3D point in world homogeneous coordinates
- ▶ gives a 2D point in image homogeneous coordinates
- ▶ includes intrinsic and extrinsic parameters
- ▶ 11 degrees of freedom $K[R \mid T]$
- ▶ doesn't include lens distortion

CAMERA PARAMETERS - NOTATION AND RELATIONSHIPS

- ▶ K = intrinsic matrix is 3x3 upper diagonal (calibration)
- ▶ focal length (in pixels)
 - ▶ how to calculate? Sensor size = 20mm X 20mm / focal length $f = 10\text{mm}$
 - ▶ image size 1000 by 1000, then 1pixel = $20/1000 \text{ mm}$,
 - ▶ $f = 10\text{mm} = 10 / (20/1000) = 500 \text{ pixels}$
 - ▶ pixel aspect ratio (non-squared pixels) (different x, and y, f)
 - ▶ not related to the image aspect ratio
 - ▶ skew: non-rectangular pixels
 - ▶ U_0 and V_0 center of the image (in pixels) near $w/2$ $h/2$

CAMERA PARAMETERS - NOTATION AND RELATIONSHIPS

- ▶ R
- ▶ rotation matrix, 3×3
- ▶ orthogonal matrix: change points in 3D world to points in camera world orientation

CAMERA PARAMETERS - NOTATION AND RELATIONSHIPS

- ▶ T and C
- ▶ T is Translation matrix 3x1
- ▶ is the (0,0,0) in the world coordinates in camera coordinates
- ▶ C is the center of the camera in world coordinates
- ▶ $T = -RC \rightarrow C = -R^{-1}T$
- ▶ RT transforms a point in the world coordinates to camera coordinates and projected into an ideal camera with $K = \text{diag}(1,1,1)$ *normalized coordinates*
- ▶ $RT X = x\text{-vector}$ is the direction of that point towards X
- ▶ $Kx\text{-vector} = x$ in image coordinates $\rightarrow K^{-1}x = x\text{-vector}$

CAMERA PARAMETERS - NOTATION AND RELATIONSHIPS

- ▶ $P - K, R, T, C$
- ▶ Q is the left 3×3 matrix of P
- ▶ Lets call P_4 the last column of P
- ▶ $C = -Q^{-1}P_4$
- ▶ K, R = rq-factorization of (Q) upper diagonal and orthogonal
- ▶ $T = K^{-1}P_4$
- ▶ $C = -R^{-1}T$

CAMERA PARAMETERS - NOTATION AND RELATIONSHIPS

- ▶ k_1, k_2 are the first two parameters for lens distortion
- ▶ usually enough
- ▶ $x\text{-distorted} = x + (x-U_o)(k_1r^2 + k_2r^4 + \dots)$
- ▶ r^2 and r^4 are calculated in *normalized coordinates* (x -vector)

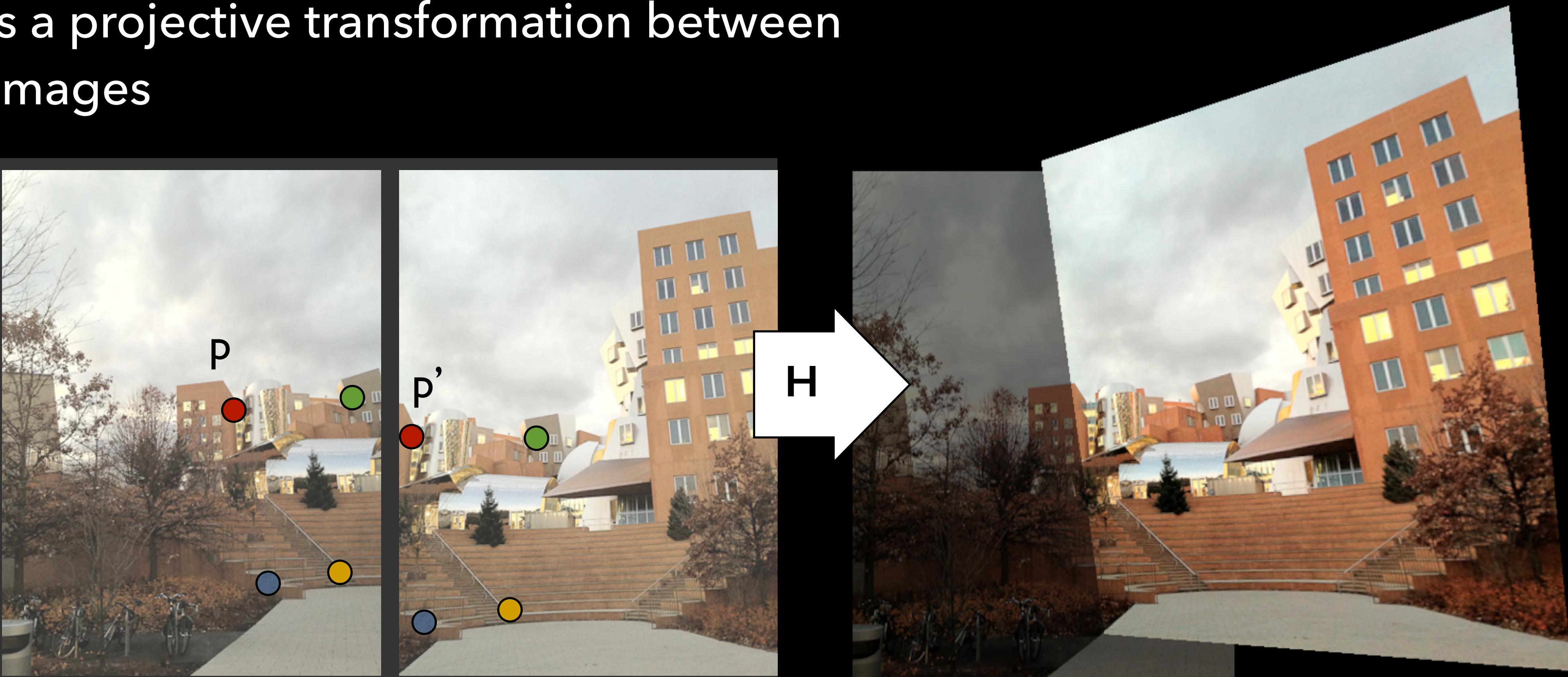
CAMERA PARAMETERS - NOTATION AND RELATIONSHIPS

- ▶ P can be calculated from 3D to 2D correspondences
- ▶ K , R , and T , can be recovered from P using RQ Factorization.
- ▶ k_1 , k_2 can be calculated from disparity between P and projections
- ▶ Non-linear iterative methods allow for adding known constraints and include Lens distortion in the model.

HOMOGRAPHIES

HOMOGRAPHIES

- ▶ 3x3 matrix
- ▶ Gives a projective transformation between two images



TEXT

CORRECT PERSPECTIVE

- ▶ We will use it for stereo rectification



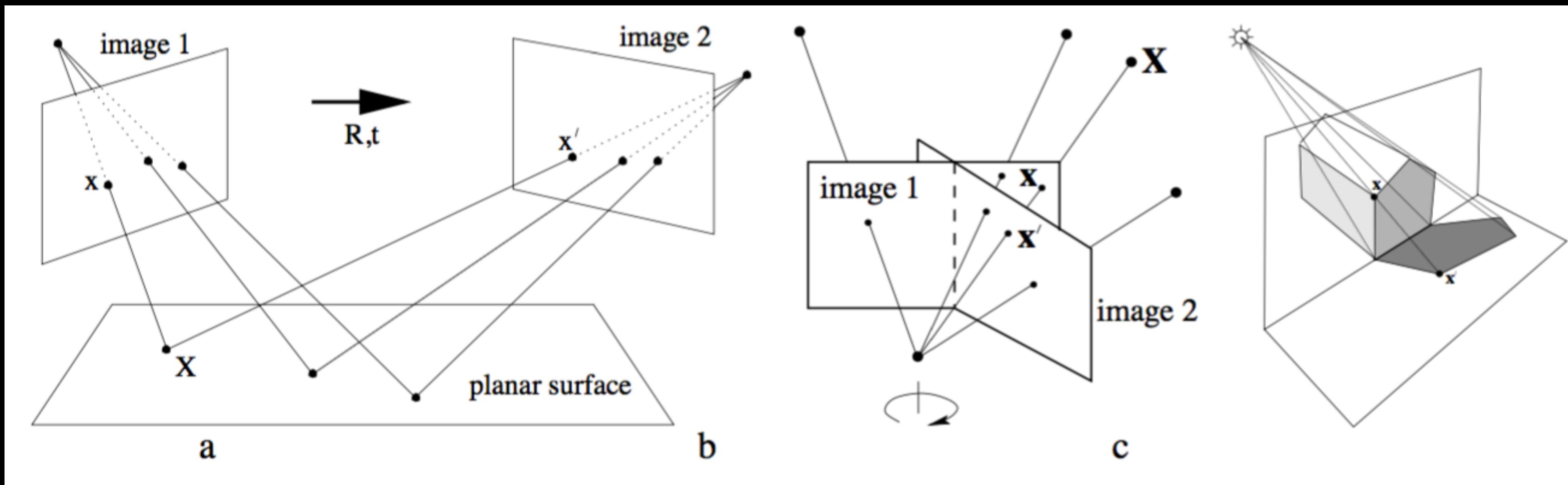
SCALE

- ▶ Homographies are defined up to a scale
- ▶ H and kH represent the same 2D transformation
- ▶ because (wx', wy', w') and $(kw'x', kw'y', kw')$ represent the same point

$$\begin{bmatrix} wx' \\ wy' \\ w \\ p' \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \\ H \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \\ p \end{bmatrix}$$

EXAMPLES OF HOMOGRAPHIES

- ▶ Rotation and Translation in 3D
- ▶ Rotation on camera center
- ▶ Shadows from a point light-source



CALCULATING AN HOMOGRAPHY

- ▶ Given correspondences
- ▶ Find homography matrix H that maps the p_i to p'_i



HOMOGRAPHY EQUATION

- ▶ We are given pairs of corresponding points
 - ▶ x, y, x', y' are known
- ▶ Unknowns: matrix coefficients and w'
- ▶ (the order in Y and X is only for convenience when

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} y \\ x \\ 1 \end{pmatrix} = \begin{pmatrix} y'w' \\ x'w' \\ w' \end{pmatrix}$$

LINEAR SYSTEM

$$ay + bx + c = y'(gy + hx + i)$$

$$dy + ex + f = x'(gy + hx + i)$$

$$\left(\begin{array}{ccccccccc} a & b & c & d & e & f & g & h & i \\ \dots & & & & & & & & \\ y & x & | & 0 & 0 & 0 & -yy' & -xy' & -y' \\ \dots & & & \dots & & & & & \\ \dots & & & & & & & & \end{array} \right) \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{pmatrix} = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$$

SOLVE THE SYSTEM USING LEAST SQUARES

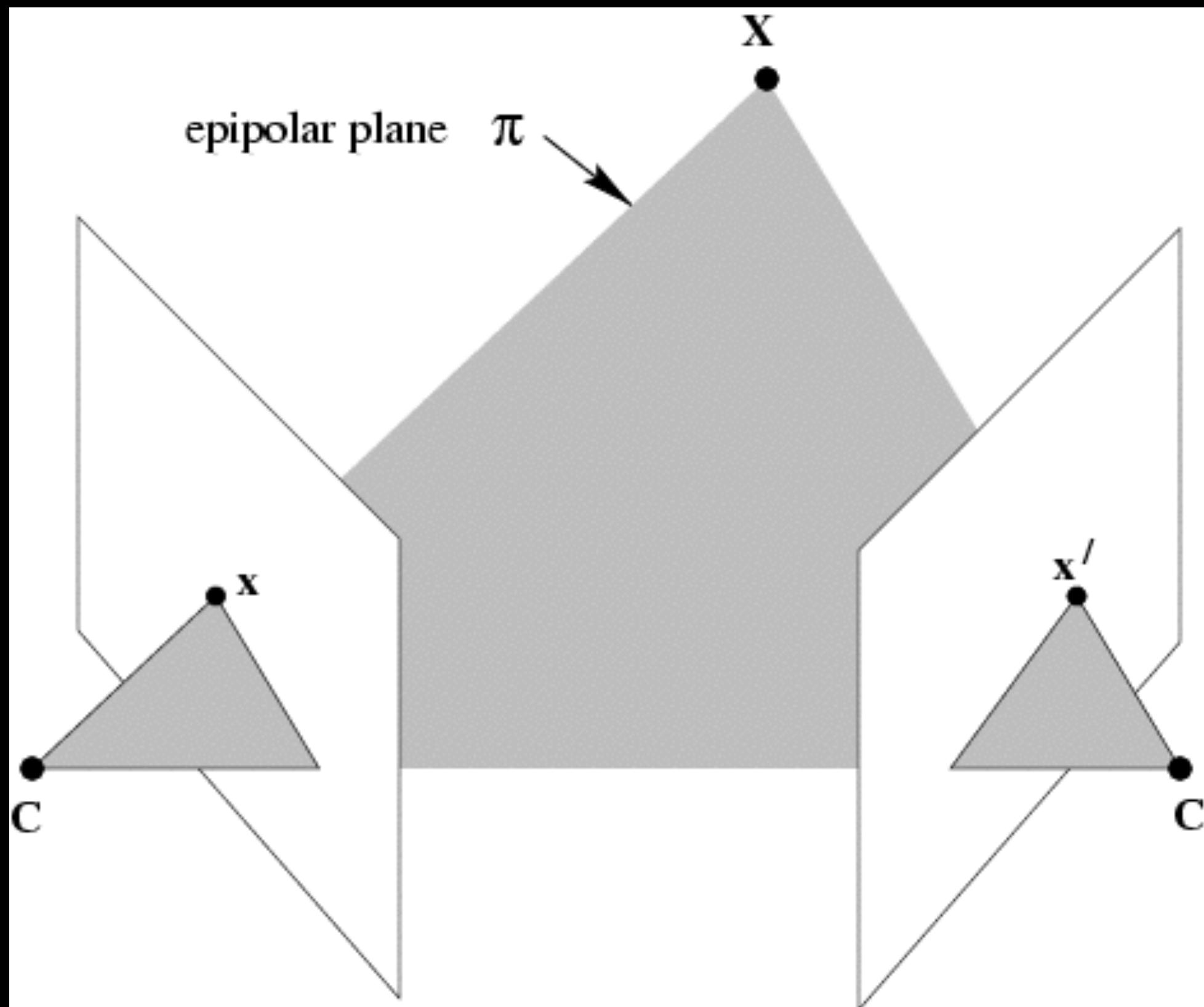
- ▶ 4 points - 8 equations will suffice
- ▶ Use SVD -> UsV or UsV^T
- ▶ The singular vector with singular value 0 is a solution
- ▶ This is a good solving method for homogeneous systems

?

EPIPOLAR GEOMETRY

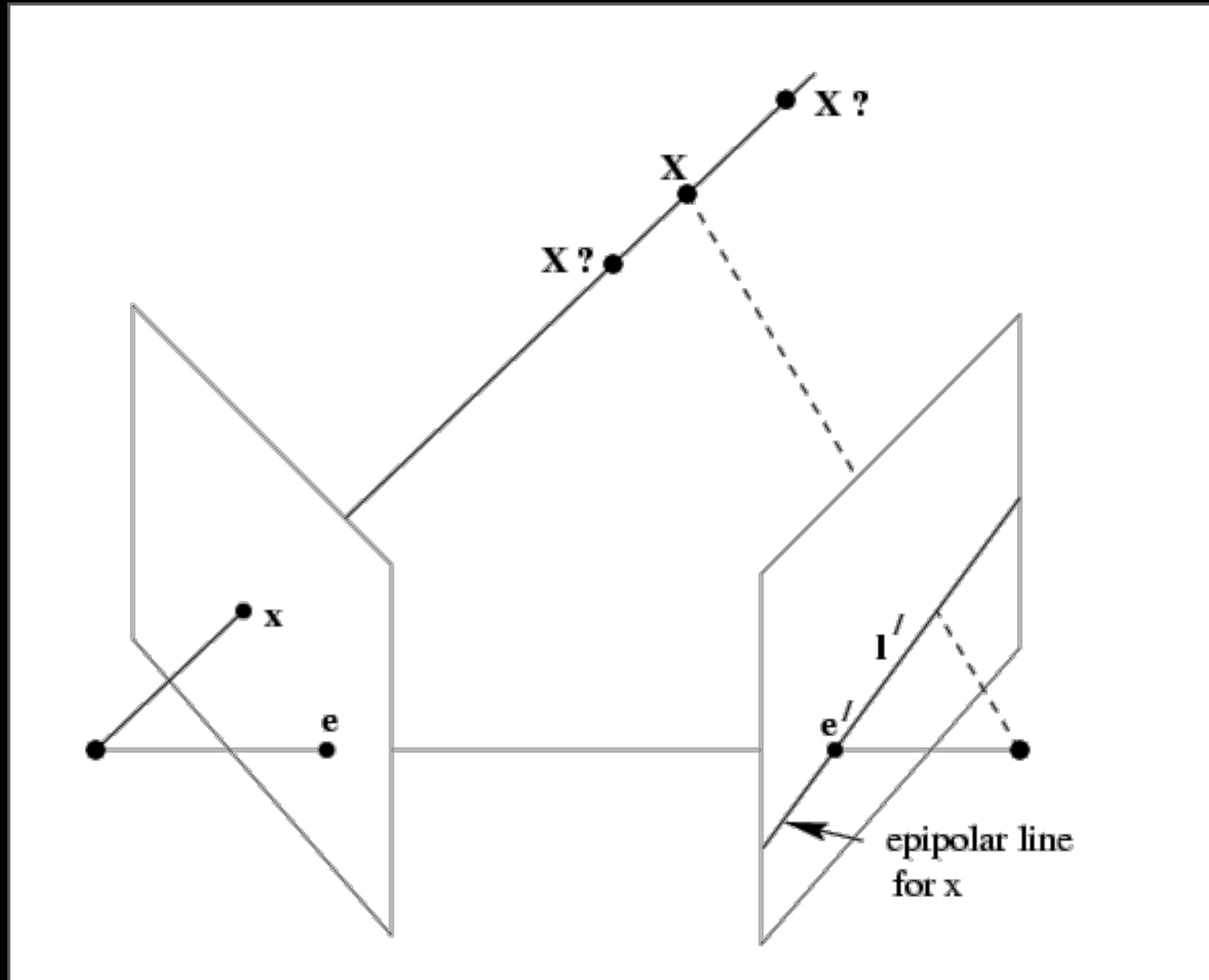
TEXT

THE EPIPOLAR GEOMETRY



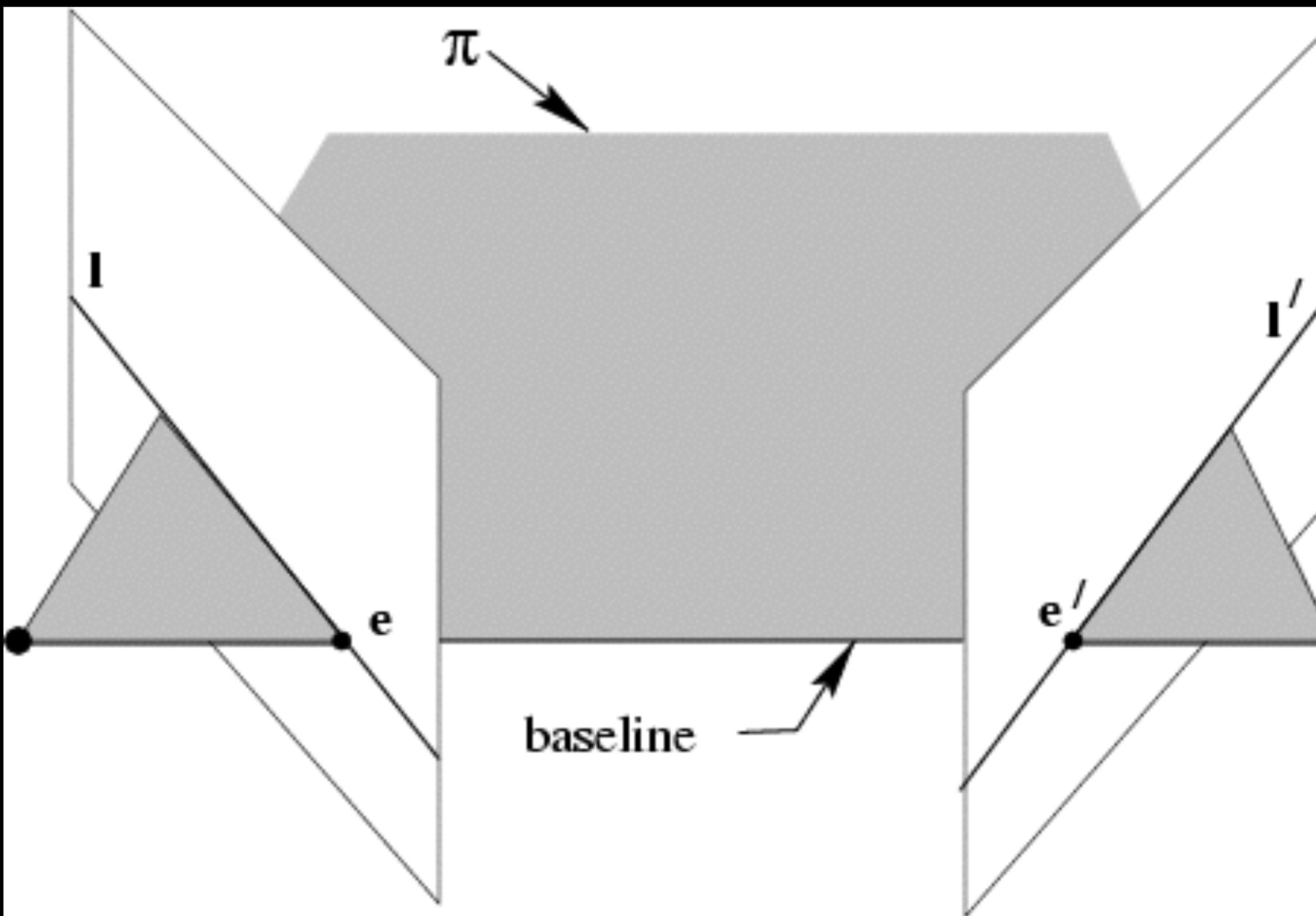
C,C',X,X' AND X ARE COPLANAR

THE EPIPOLAR GEOMETRY



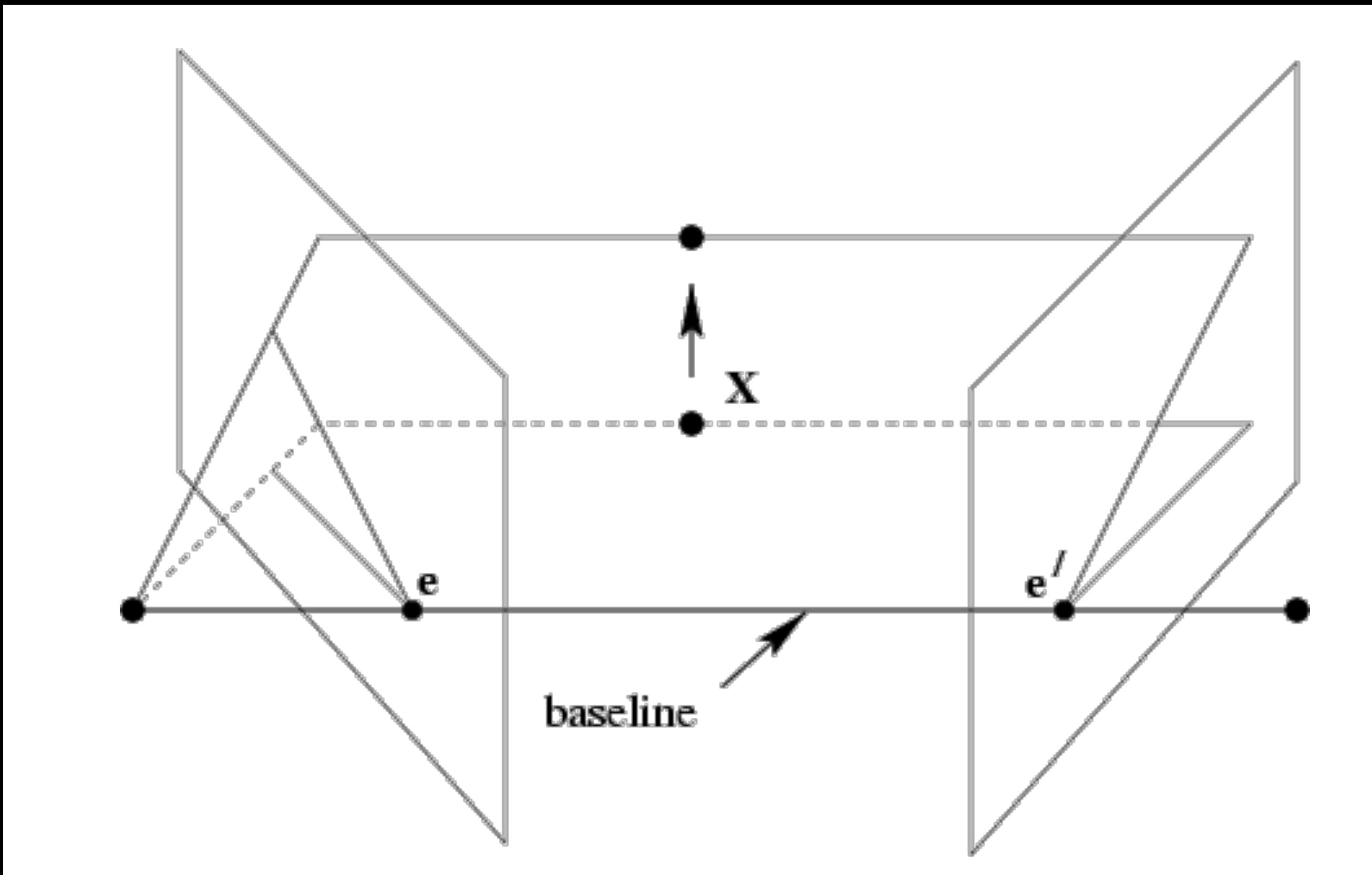
WHAT IF ONLY C, C', X ARE KNOWN?

THE EPIPOLAR GEOMETRY



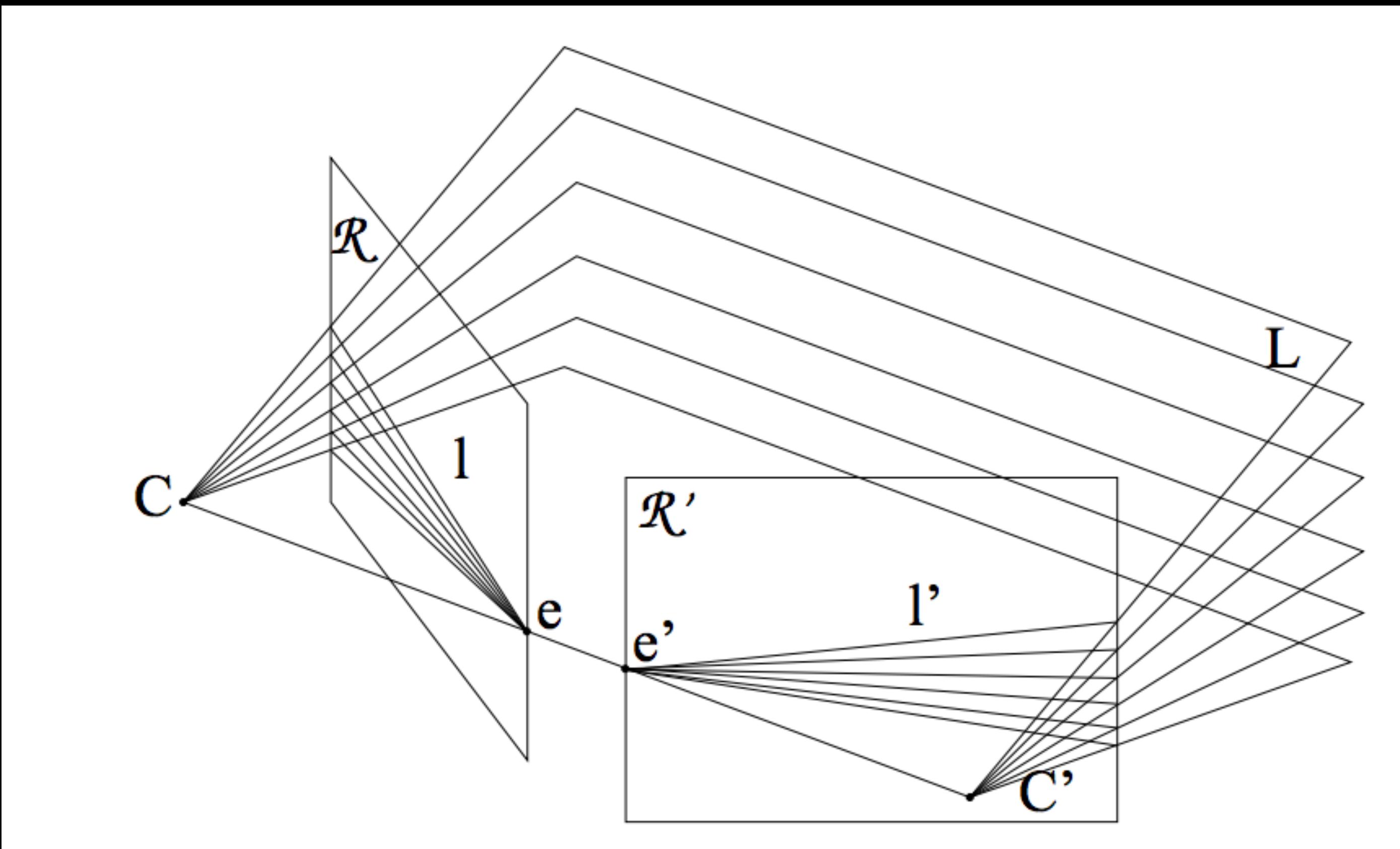
ALL POINTS ON π PROJECT ON l AND l'

THE EPIPOLAR GEOMETRY



FAMILY OF PLANES Π AND LINES L AND L'
INTERSECTION IN e AND e'

EPIPOLAR GEOMETRY: NOTATION

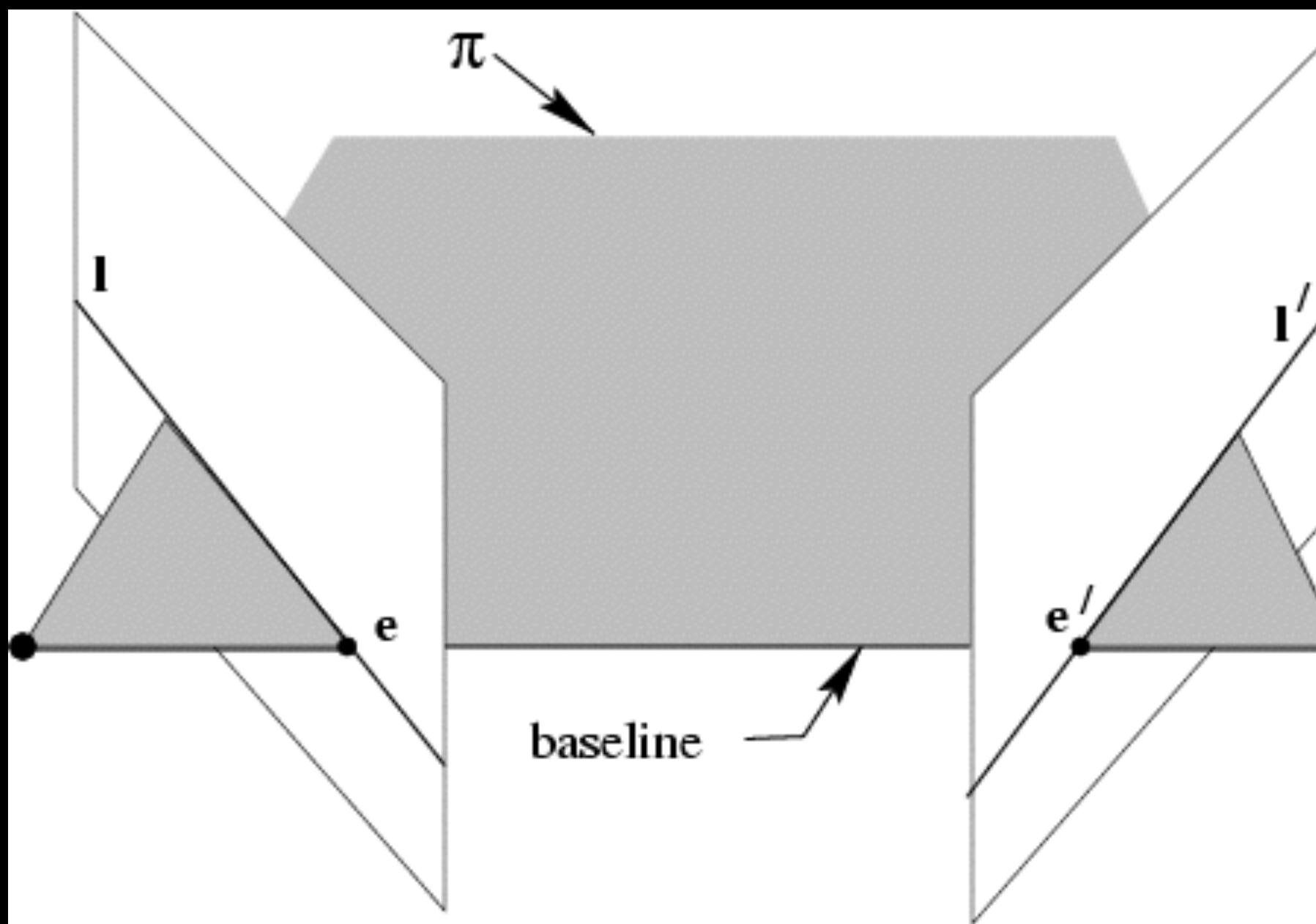


PENCIL OF PLANES OF THE BASELINE DEFINE EPIPOLAR LINES AND EPIPOLES

TEXT

EPIPOLES e, e'

- = INTERSECTION OF BASELINE WITH IMAGE PLANE
- = PROJECTION OF PROJECTION CENTER IN OTHER IMAGE
- = VANISHING POINT OF CAMERA MOTION DIRECTION

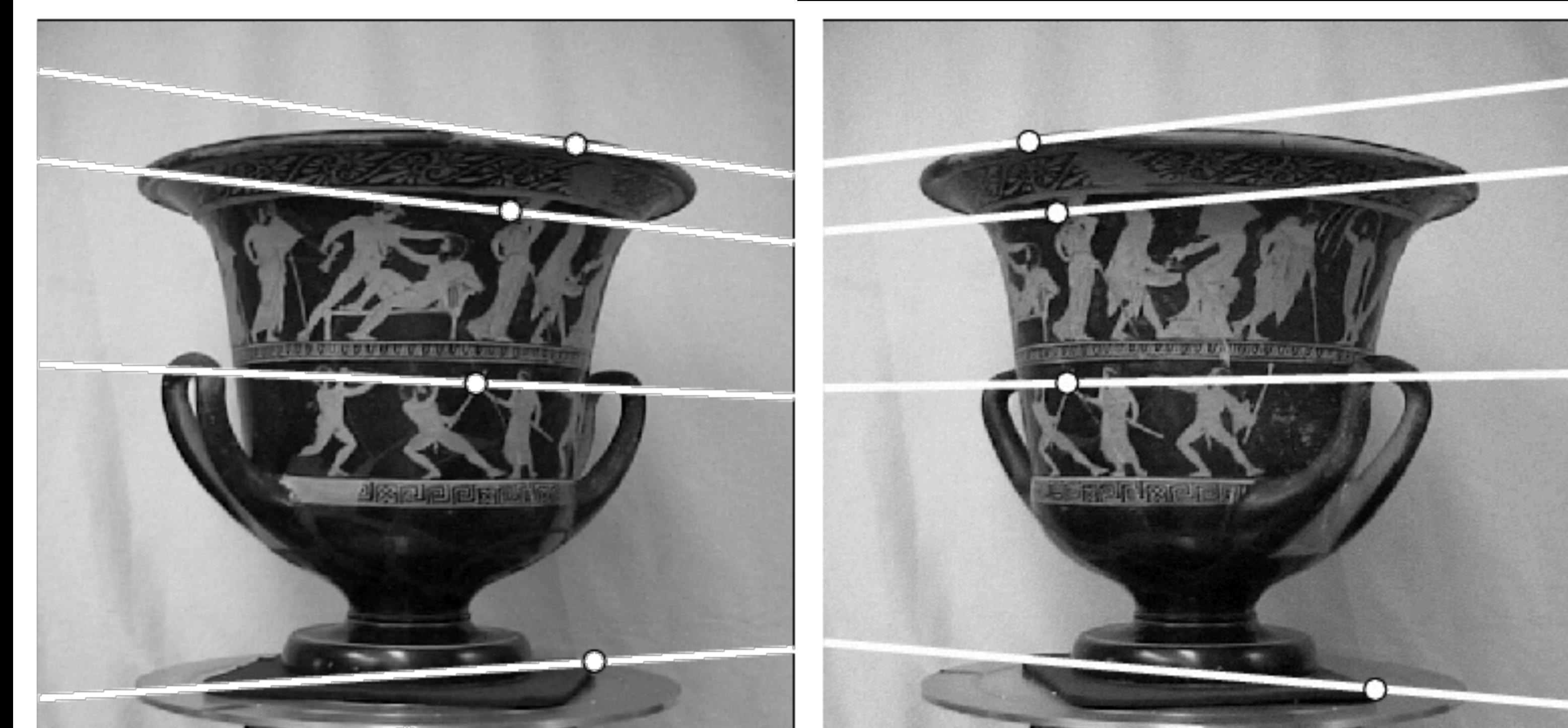
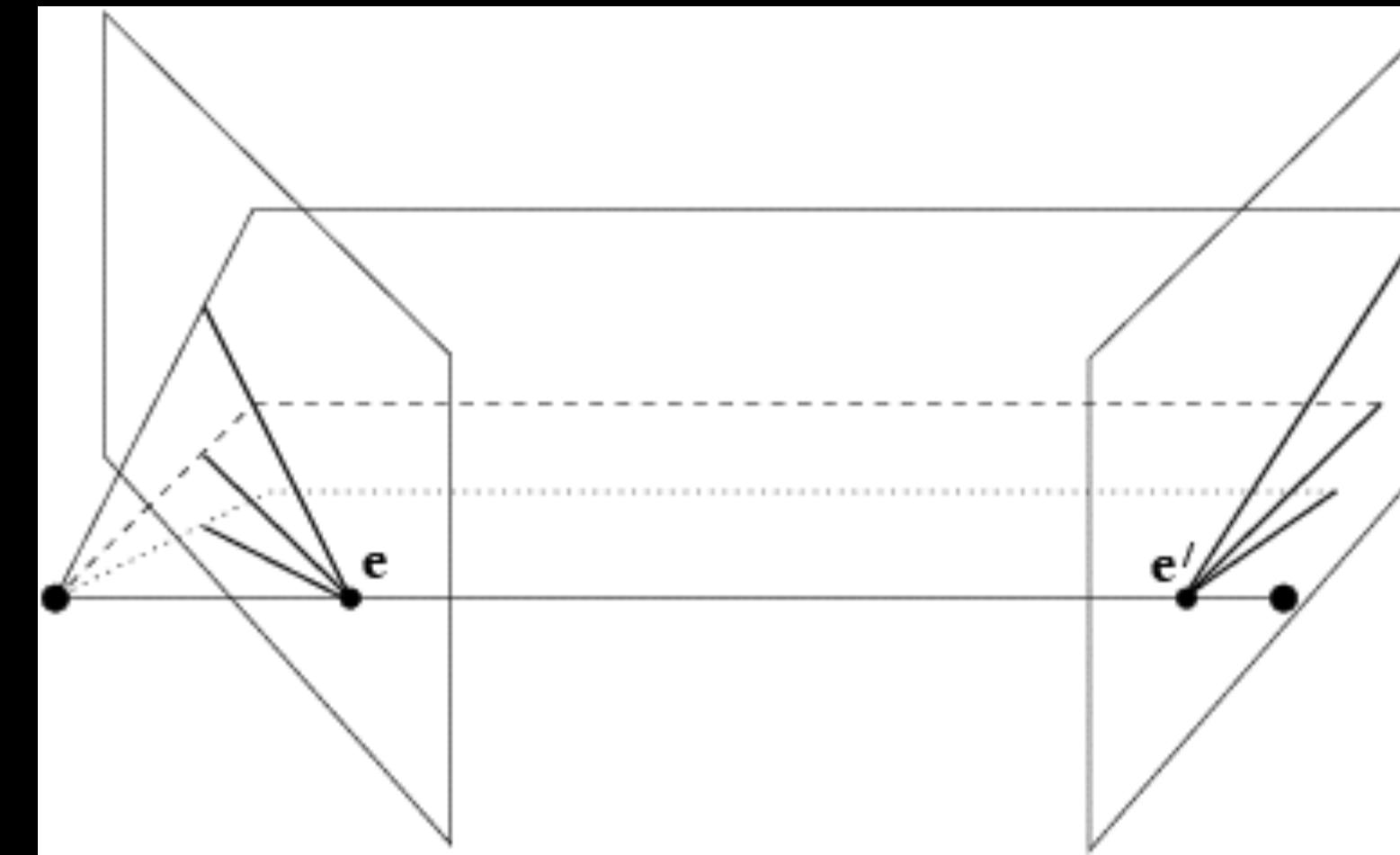


AN EPIPOLAR PLANE = PLANE CONTAINING BASELINE (1-D FAMILY)

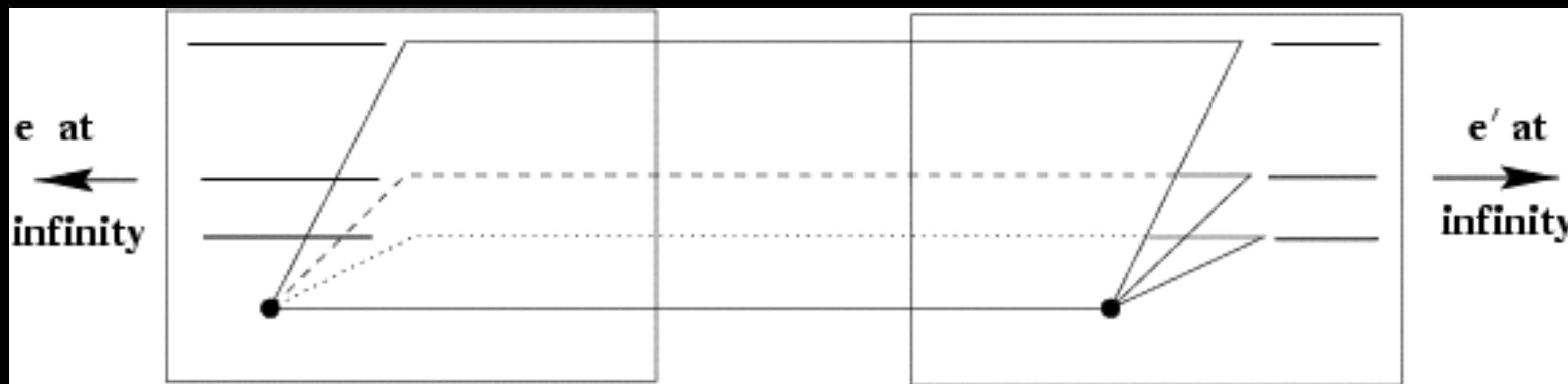
AN EPIPOLAR LINE = INTERSECTION OF EPIPOLAR PLANE WITH IMAGE
(ALWAYS COME IN CORRESPONDING PAIRS)

TEXT

EXAMPLE: CONVERGING CAMERAS

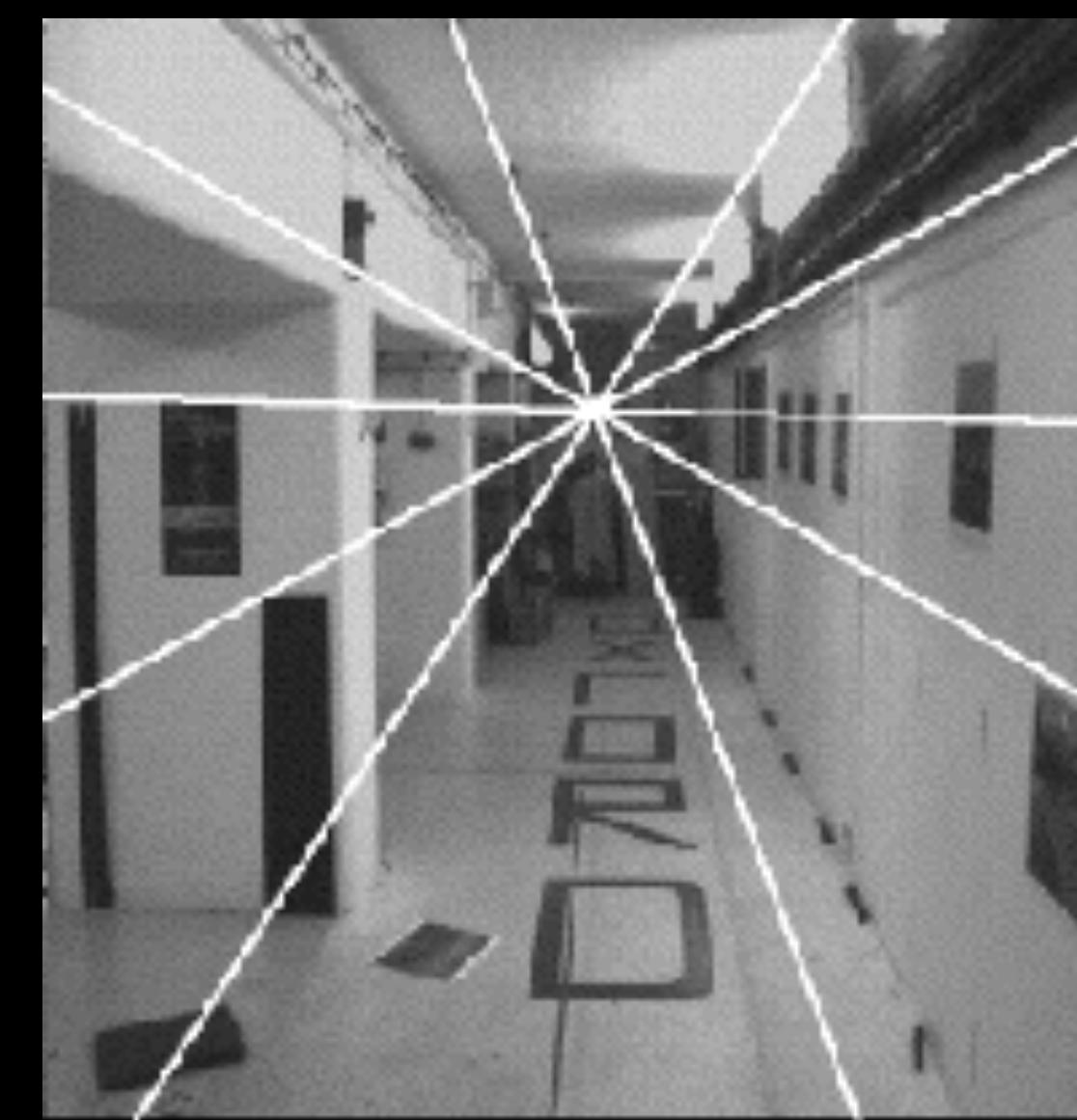
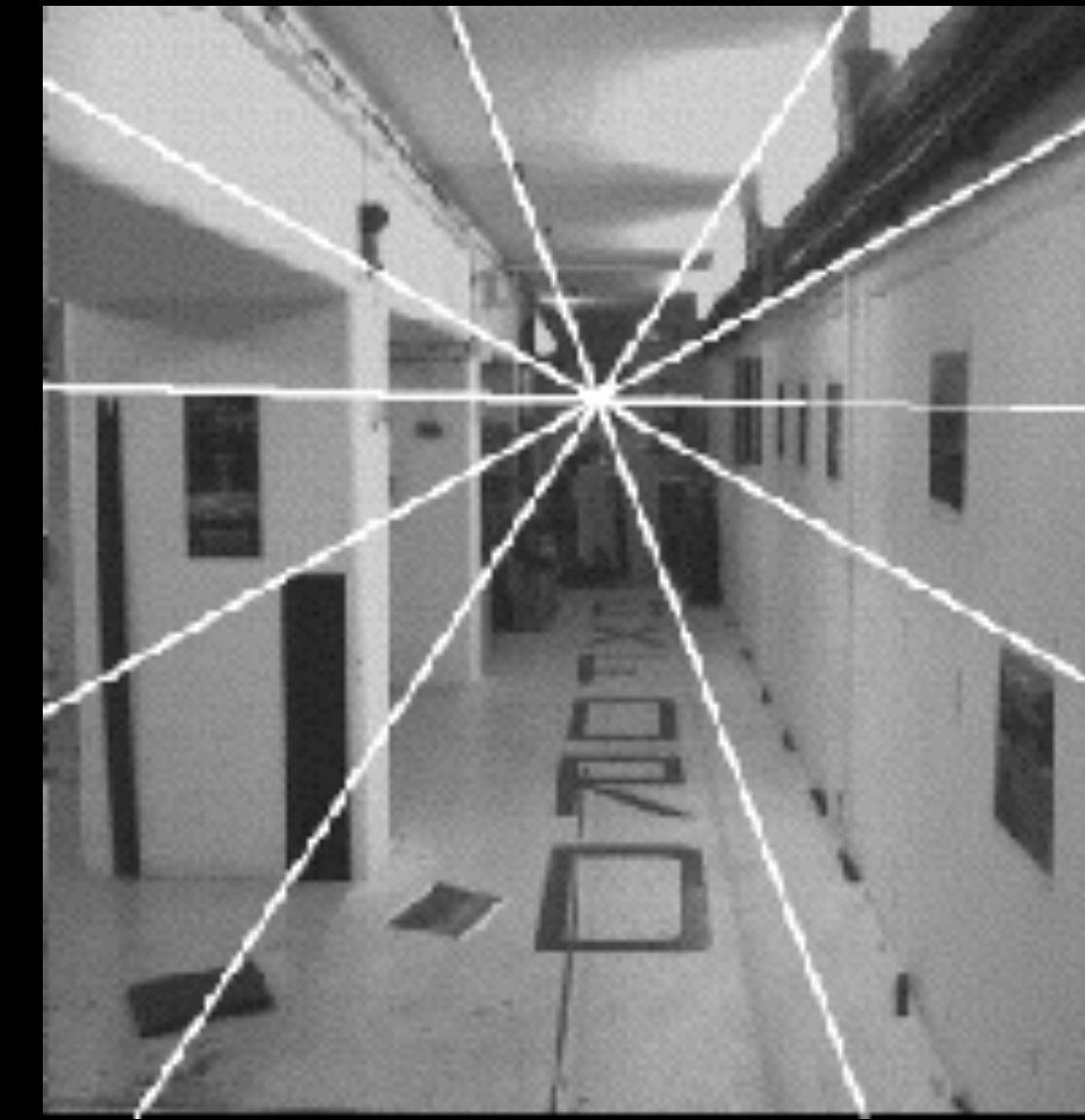
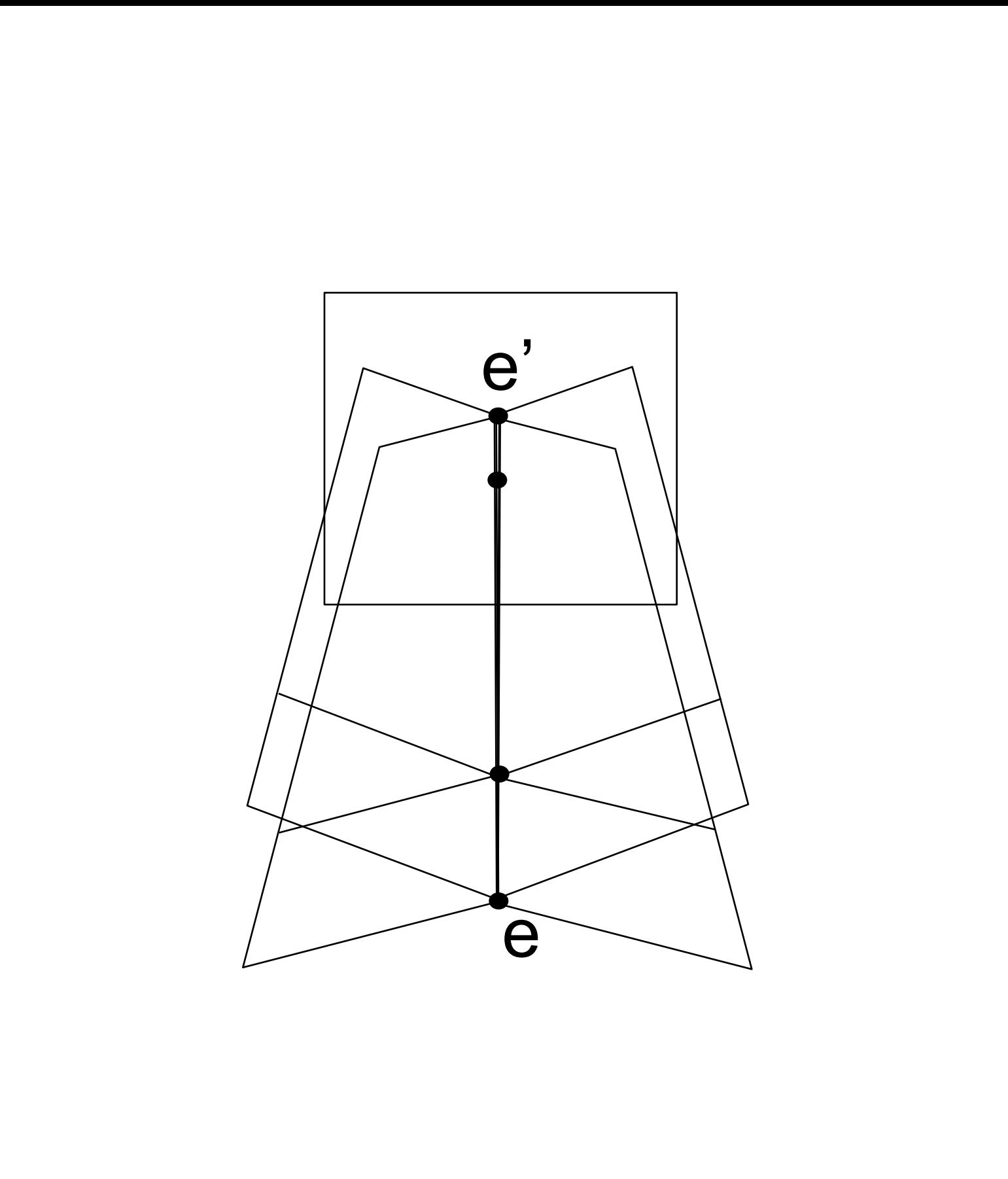


TEXT



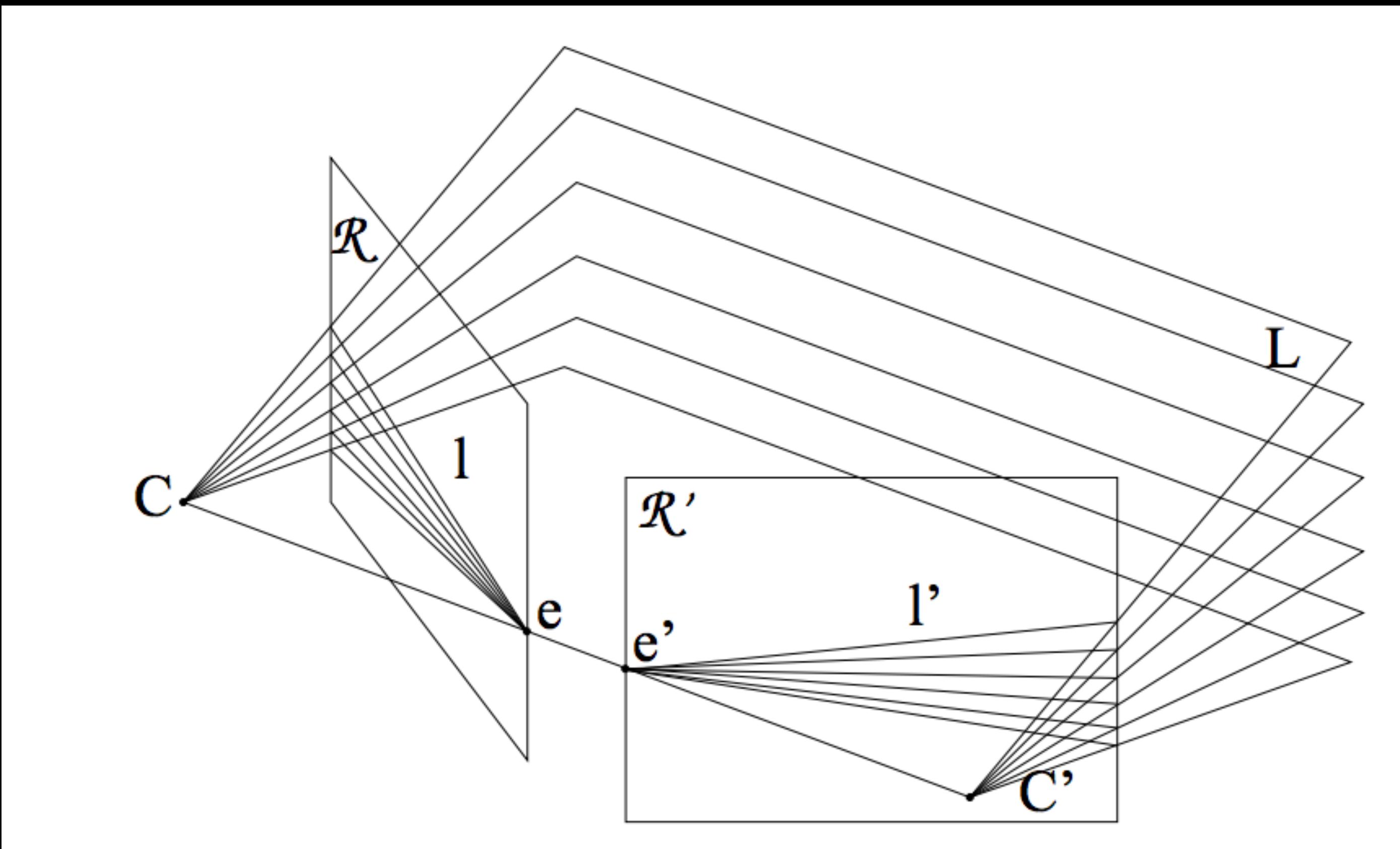
TEXT

EXAMPLE: FORWARD MOTION



TEXT

EPIPOLAR LINES CROSS AT THE EPIPOLE



BY DEFINITION, ALL EPIPOLAR LINES CROSS AT THE EPIPOLE

?

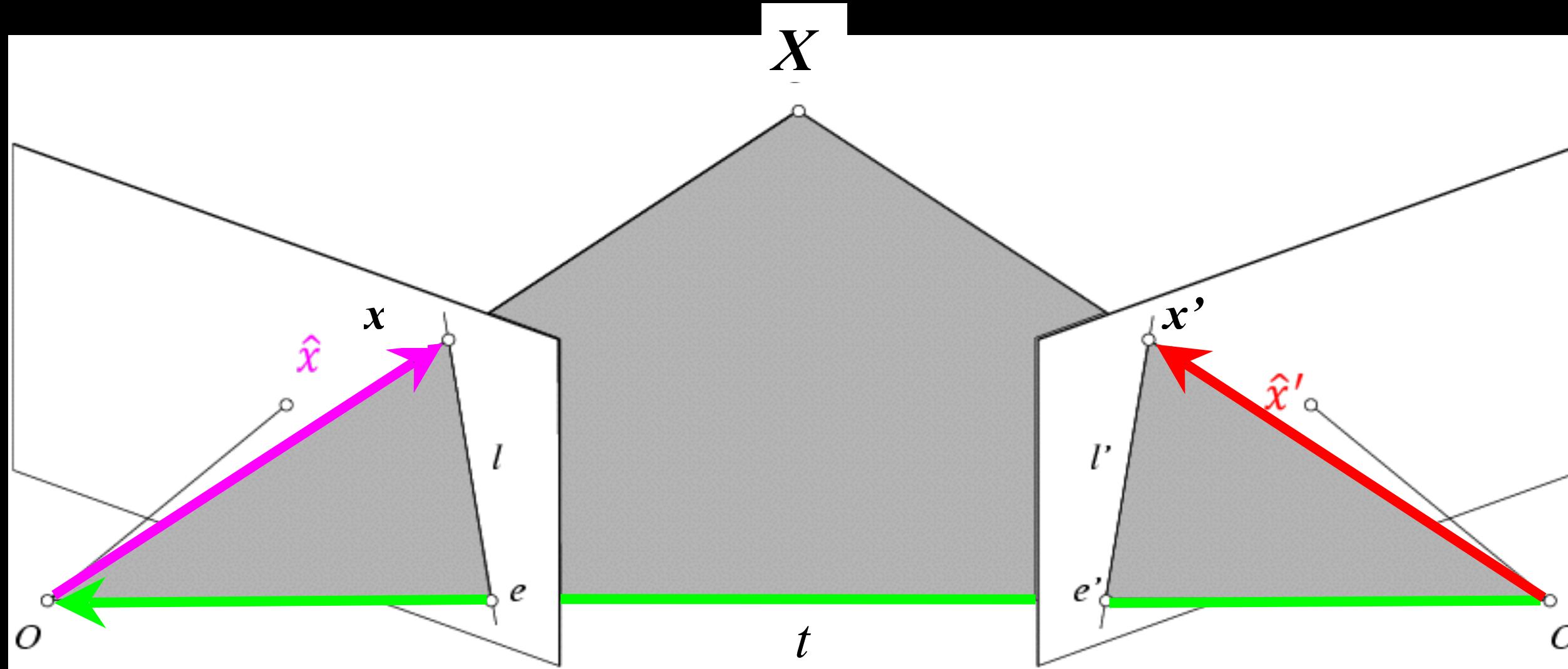
THE FUNDAMENTAL MATRIX F

ALGEBRAIC REPRESENTATION OF EPIPOLAR GEOMETRY

$$x \quad ? \quad l'$$

WE WILL SEE THAT MAPPING IS (SINGULAR) CORRELATION (I.E. PROJECTIVE MAPPING FROM POINTS TO LINES) REPRESENTED BY THE FUNDAMENTAL MATRIX F

EPIPOLAR CONSTRAINT: CALIBRATED CASE



$$\hat{x} = K^{-1}x = X$$

$$\hat{x} = R\hat{x}' + t \quad \rightarrow \quad \hat{x} \cdot [t \times (R\hat{x}')] = 0$$

(because \hat{x} , $R\hat{x}'$, and t are co-planar)

$$\hat{x}' = K'^{-1}x' = X'$$

$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_k R$$

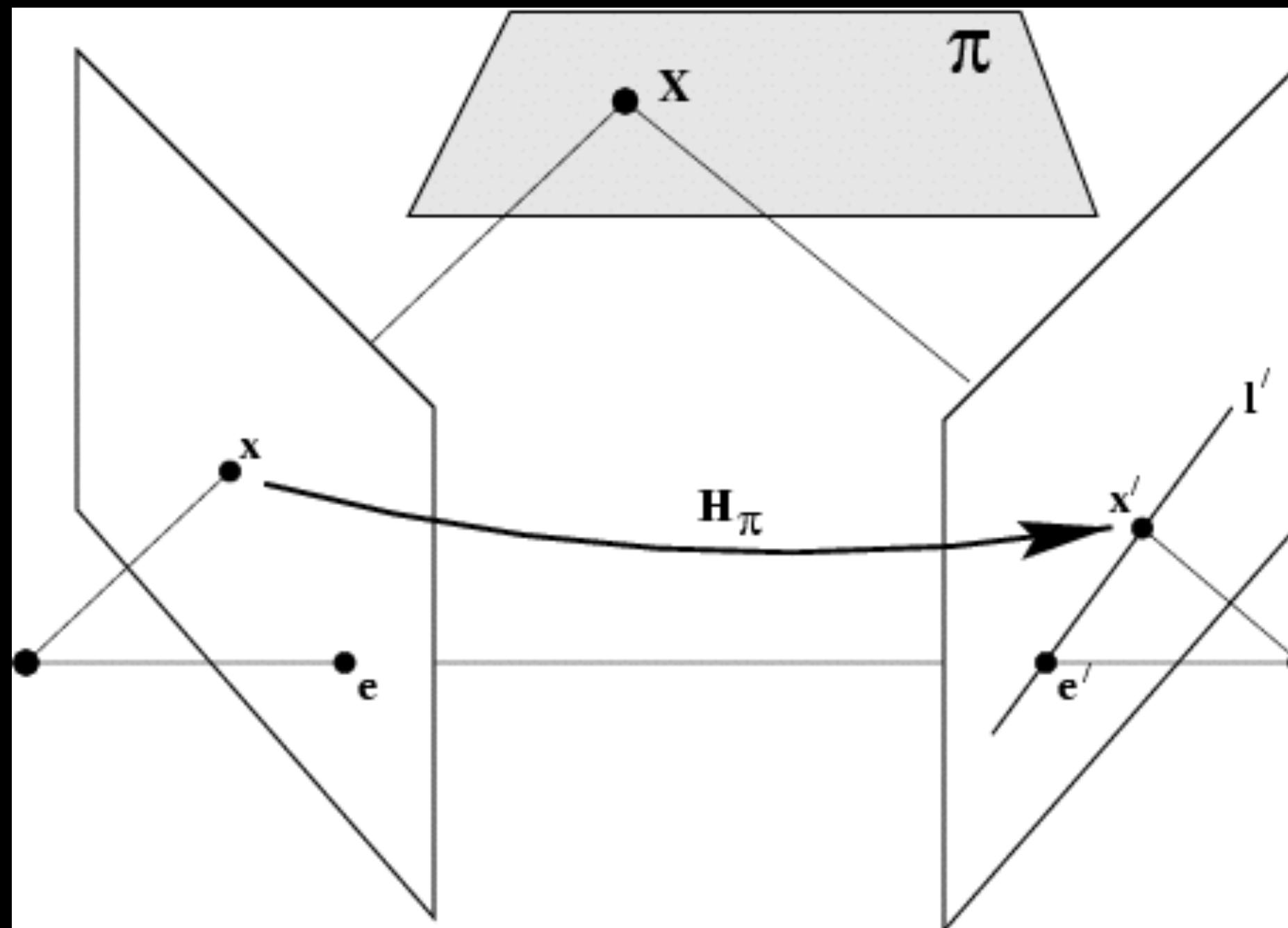
SKEW SYMMETRIC NOTATION

$$[\mathbf{a}]_{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_{\times} \mathbf{b} = (\mathbf{a}^T [\mathbf{b}]_{\times})^T.$$

THE FUNDAMENTAL MATRIX F

GEOMETRIC DERIVATION



$$x' = H_\pi x$$

$$l' = e' \times x' = [e'] H_\pi x = Fx$$

MAPPING FROM 2-D TO 1-D FAMILY (RANK 2)

THE FUNDAMENTAL MATRIX F

CORRESPONDENCE CONDITION

The fundamental matrix satisfies the condition that for any pair of corresponding points $x \leftrightarrow x'$ in the two images

$$x'^T F x = 0 \quad (x'^T l' = 0)$$

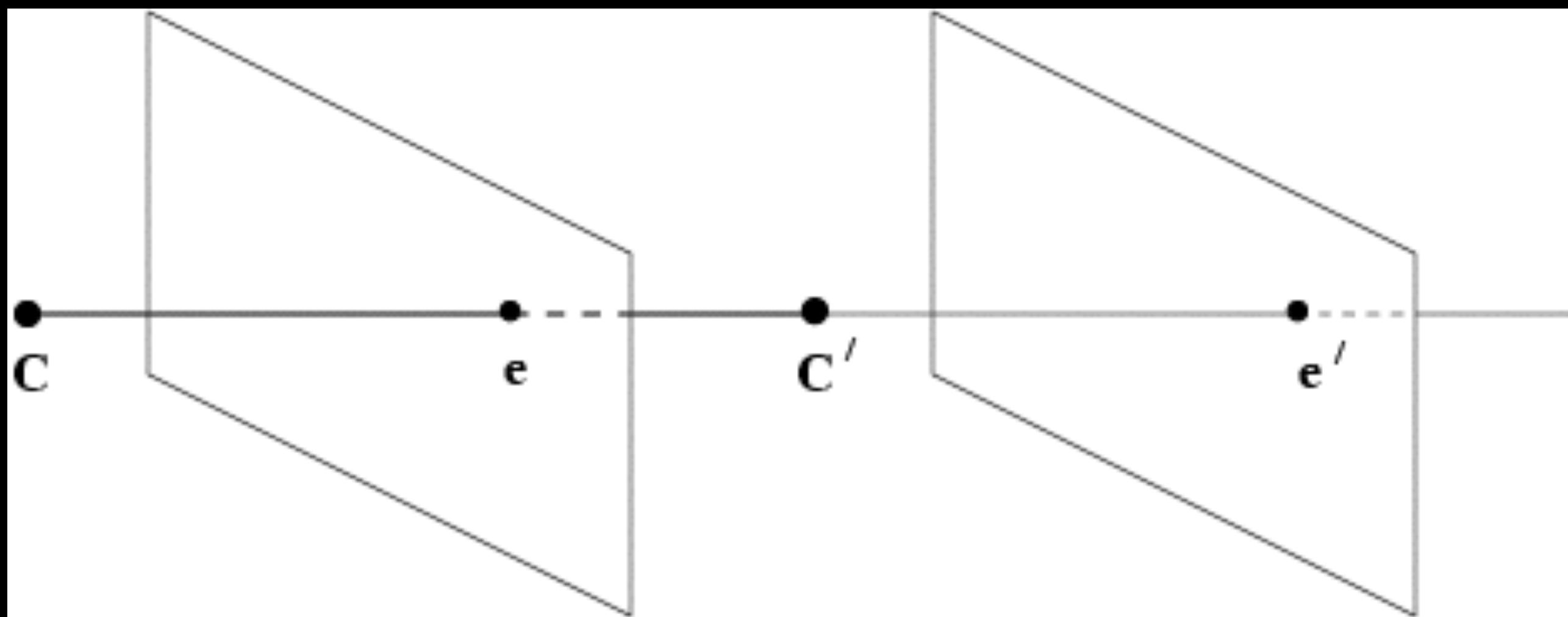
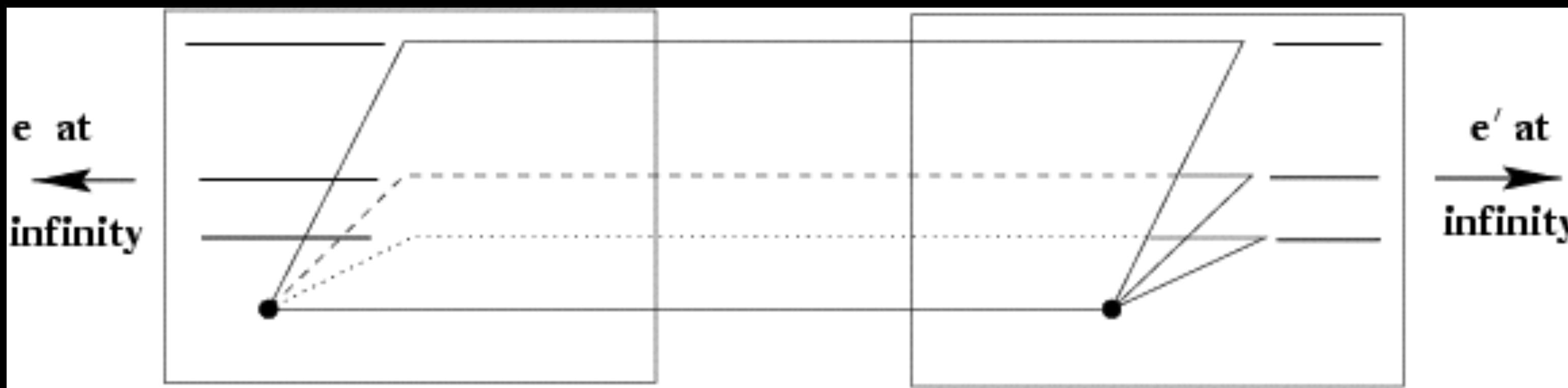
THE FUNDAMENTAL MATRIX F

F is the unique 3×3 rank 2 matrix that satisfies $x'^T F x = 0$ for all $x \leftrightarrow x'$

- (i) **Transpose:** if F is fundamental matrix for (P, P') , then F^T is fundamental matrix for (P', P)
- (ii) **Epipolar lines:** $l' = Fx$ & $l = F^T x'$
- (iii) **Eipoles:** on all epipolar lines, thus $e'^T F x = 0, \forall x \Rightarrow e'^T F = 0$, similarly $Fe = 0$
- (iv) F has 7 d.o.f. , i.e. $3 \times 3 - 1(\text{homogeneous}) - 1(\text{rank}2)$

TEXT

FUNDAMENTAL MATRIX FOR PURE TRANSLATION



FUNDAMENTAL MATRIX FOR PURE TRANSLATION

example:

$$e' = (1, 0, 0)^T \quad F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}_x$$

$$x'^T F x = 0 \Leftrightarrow y = y'$$

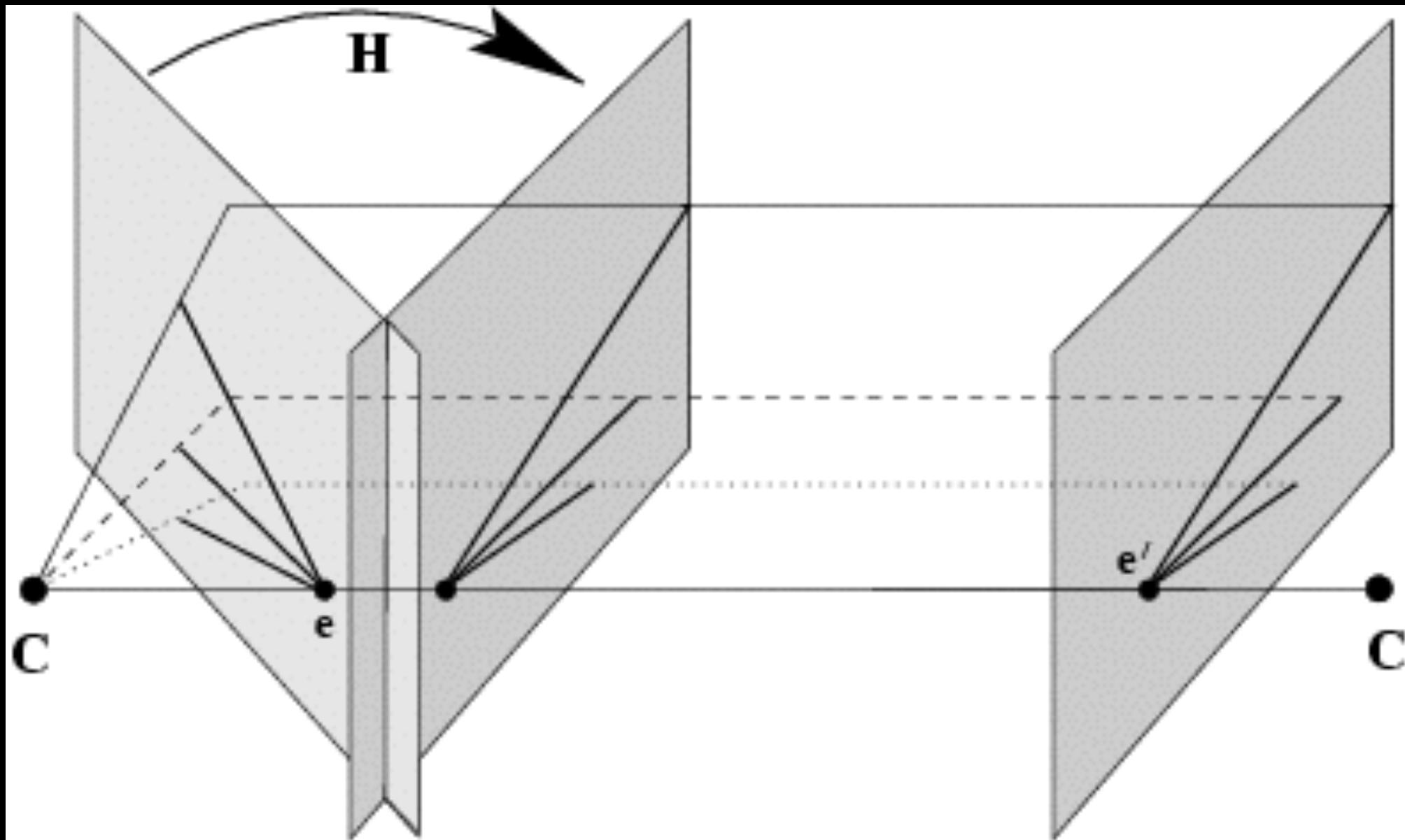
$$x = P X = K[I | 0]X \quad (X, Y, Z)^T = K^{-1}x/Z$$

$$x' = P' X = K[I | t] \begin{bmatrix} K^{-1}x \\ Z \end{bmatrix} \quad x' = x + Kt/Z$$

motion starts at x and moves towards e , faster depending on Z

pure translation: F only 2 d.o.f., $x^T [e]_x x = 0 \Rightarrow$ auto-epipolar

GENERAL MOTION



$$\mathbf{x} = \mathbf{P}\mathbf{X} = \mathbf{K}[\mathbf{I} \mid \mathbf{0}]\mathbf{X} \quad (\mathbf{X}, \mathbf{Y}, \mathbf{Z})^\top = \mathbf{K}^{-1}\mathbf{x}/Z$$

$$\mathbf{x}' = \mathbf{P}'\mathbf{X} = \mathbf{K}'[\mathbf{R}' \mid \mathbf{t}]\mathbf{X}$$

$$\mathbf{x}' = \mathbf{K}'\mathbf{R}\mathbf{K}^{-1}\mathbf{x} + \mathbf{K}'\mathbf{t}/Z$$

- ▶ To this point we have examined the properties of F and of image relations for a point correspondence $x \leftrightarrow x'$. We now turn to one of the most significant properties of F , that the matrix may be used to determine the camera matrices of the two views.

WE CAN GET PROJECTION MATRICES P AND P' UP TO A PROJECTIVE AMBIGUITY

$$\mathbf{P} = [\mathbf{I} \mid \mathbf{0}] \quad \mathbf{P}' = \left[\begin{matrix} \mathbf{e}' \\ \mathbf{F} \end{matrix} \right] \quad \mathbf{e}'^T \mathbf{F} = 0$$

\downarrow \nwarrow

K'*rotation K'*translation

SEE HZ P. 255-256

- ▶ Code:
- ▶

```
function P = vgg_P_from_F(F)
```
- ▶

```
[U,S,V] = svd(F);
```
- ▶

```
e = U(:,3);
```
- ▶

```
P = [-vgg_contreps(e)*F e];
```

PROJECTIVE TRANSFORMATION AND INVARIANCE

F invariant to transformations of projective 3-space

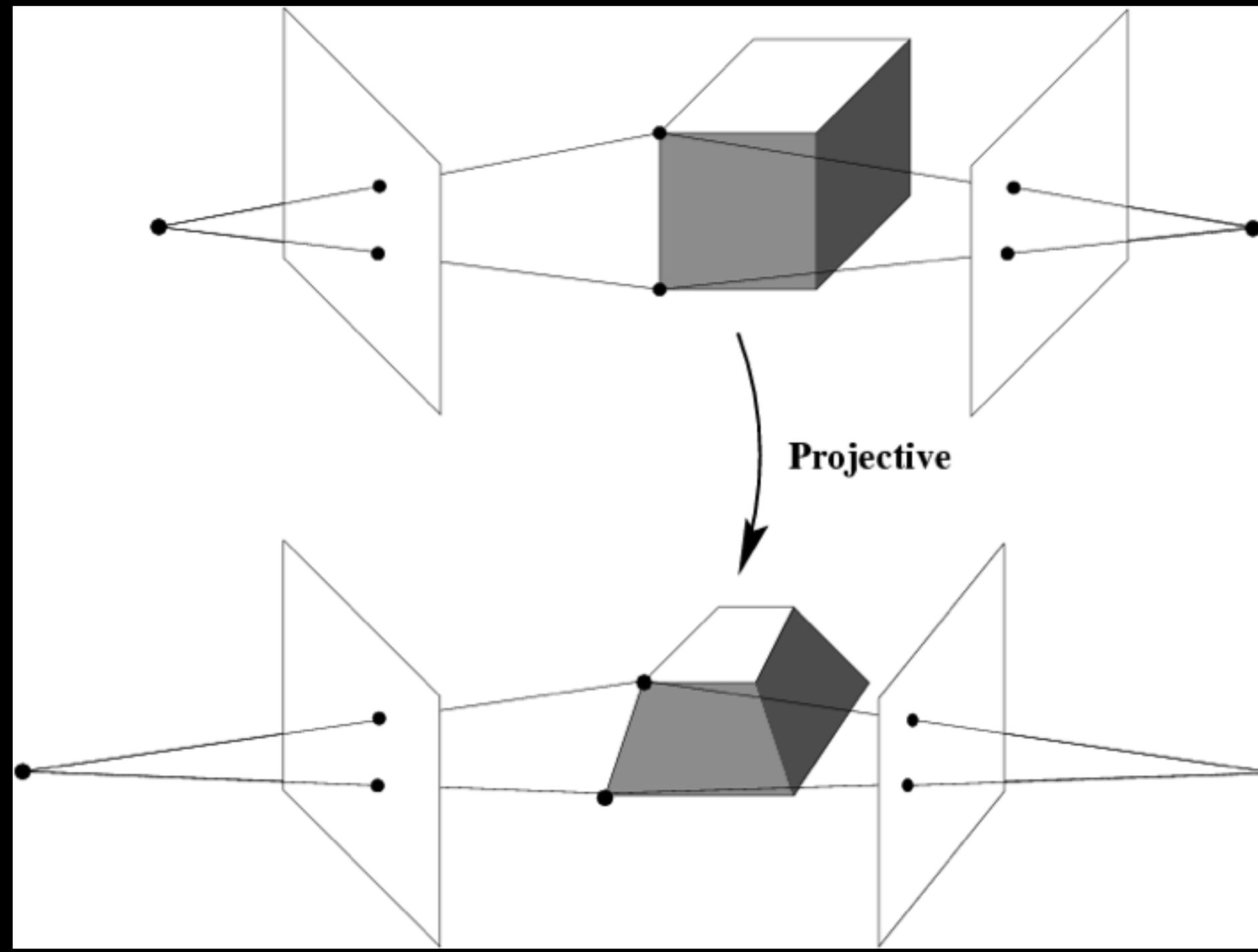
$$x = Px = (Ph)(h^{-1}x) = \hat{P}\hat{x}$$

$$x' = P'x = (P'h)(h^{-1}x) = \hat{P}'\hat{x}$$

(P, P') $\blacksquare F$ unique

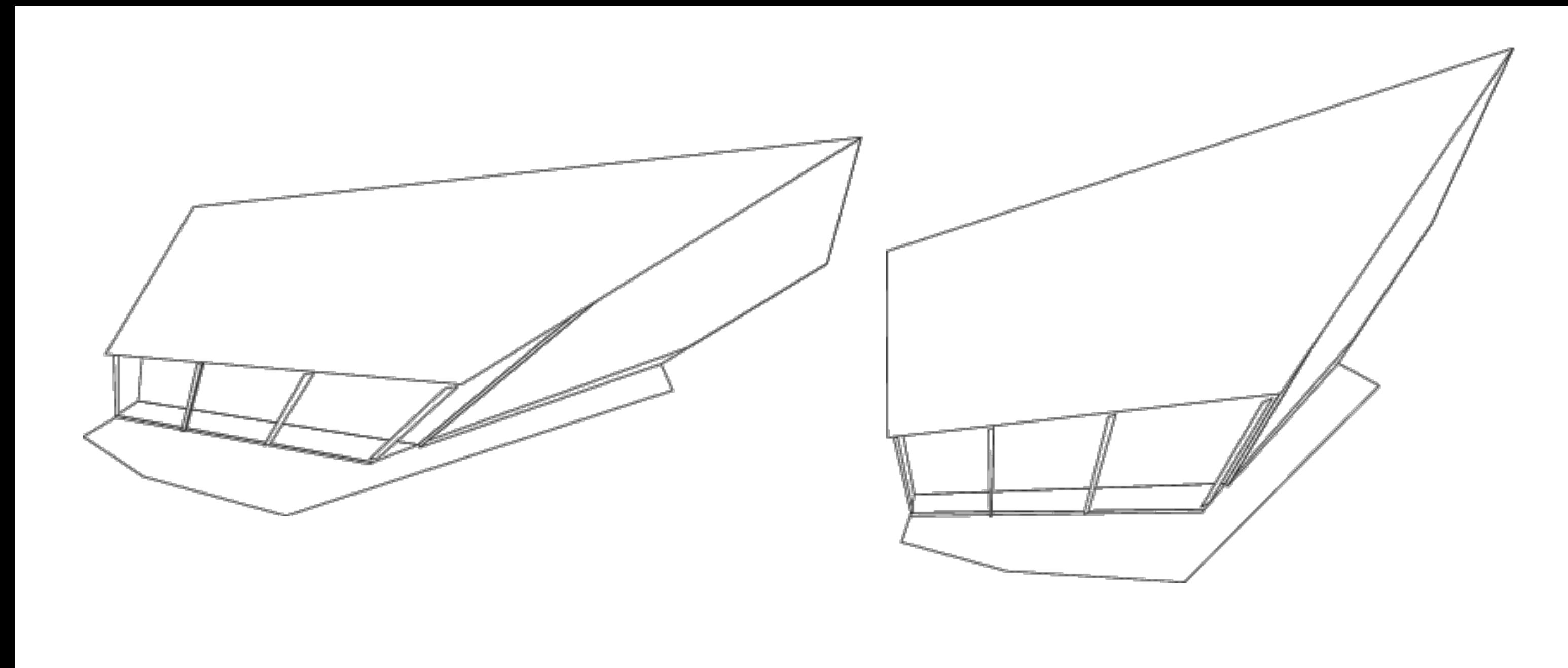
$F \blacksquare (P, P')$ not unique

RECONSTRUCTION AMBIGUITY: PROJECTIVE

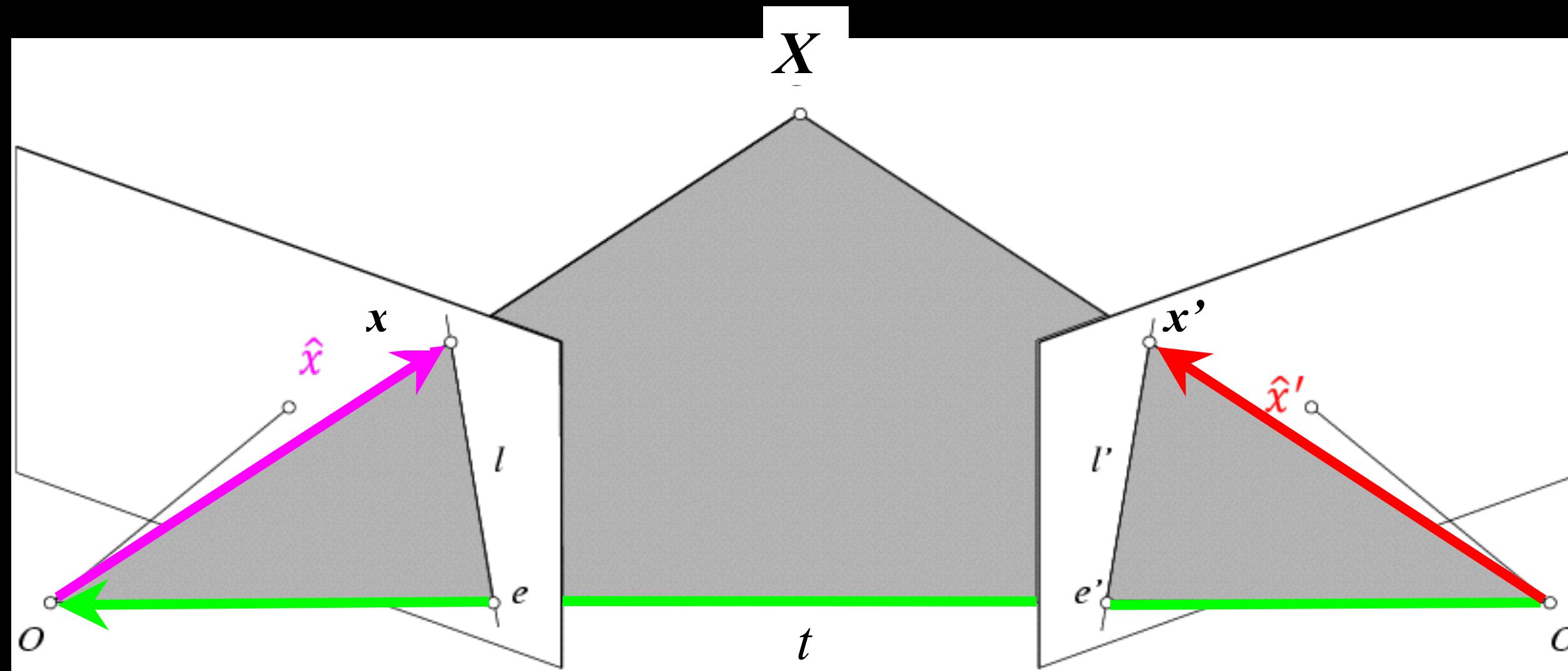


$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i = (\mathbf{P}\mathbf{H}_P^{-1})(\mathbf{H}_P \mathbf{X}_i)$$

TEXT



ESSENTIAL MATRIX



$$\hat{x} = K^{-1}x = X$$

$$\hat{x} = R\hat{x}' + t \quad \rightarrow \quad \hat{x} \cdot [t \times (R\hat{x}')] = 0$$

(because \hat{x} , $R\hat{x}'$, and t are co-planar)

$$\hat{x}' = K'^{-1}x' = X'$$

$$\hat{x} \cdot [t \times (R\hat{x}')] = 0 \quad \rightarrow \quad \hat{x}^T E \hat{x}' = 0 \quad \text{with} \quad E = [t]_R$$

THE ESSENTIAL MATRIX

~fundamental matrix for calibrated cameras (remove K)

$$E = [t]_x R = R[R^T t]_x$$

$$\hat{x}'^T E \hat{x} = 0 \quad (\hat{x} = K^{-1}x; \hat{x}' = K^{-1}x')$$

$$E = K'^T F K$$

5 d.o.f. (3 for R; 2 for t up to scale)

E is essential matrix if and only if
two singularvalues are equal (and third=0)

$$E = U \text{diag}(1,1,0) V^T$$

THE ESSENTIAL MATRIX

$$W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad Z = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

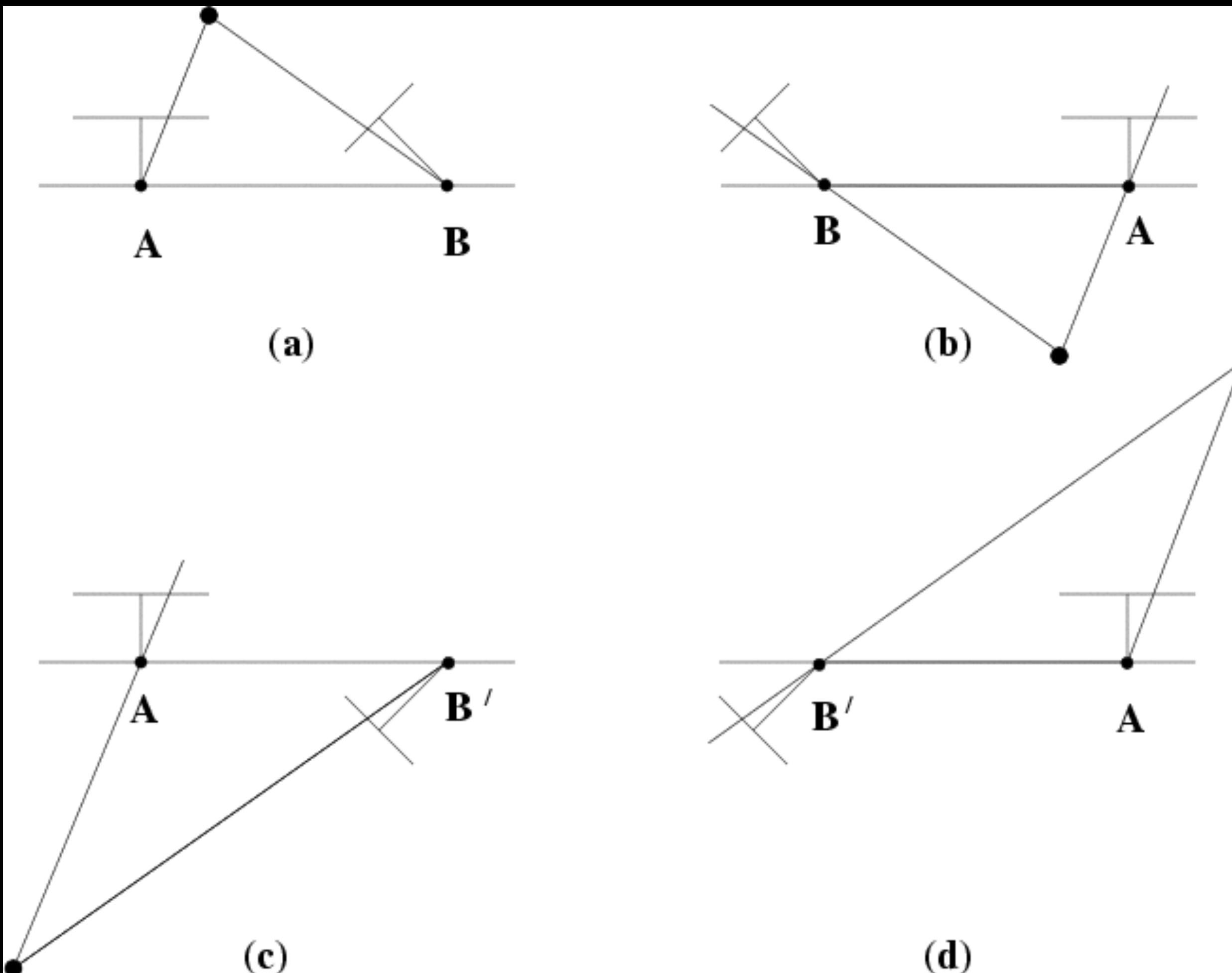
Result 9.18. Suppose that the SVD of E is $U \text{diag}(1, 1, 0) V^T$. Using the notation of (9.13), there are (ignoring signs) two possible factorizations $E = SR$ as follows:

$$S = UZU^T \quad R = UWV^T \quad \text{or} \quad UW^TV^T. \quad (9.14)$$

Result 9.19. For a given essential matrix $E = U \text{diag}(1, 1, 0) V^T$, and first camera matrix $P = [I \mid 0]$, there are four possible choices for the second camera matrix P' , namely

$$P' = [UWV^T \mid +\mathbf{u}_3] \quad \text{or} \quad [UWV^T \mid -\mathbf{u}_3] \quad \text{or} \quad [UW^TV^T \mid +\mathbf{u}_3] \quad \text{or} \quad [UW^TV^T \mid -\mathbf{u}_3].$$

FOUR POSSIBLE RECONSTRUCTIONS FROM E



(ONLY ONE SOLUTION WHERE POINTS IS IN FRONT OF BOTH CAMERAS)

COMPUTING F

EQUATIONS

- ▶ Solve a system of homogeneous linear equations
- ▶ Write down the system of equations

$$\mathbf{x}^T F \mathbf{x}' = 0$$

$$uu'f_{11} + uv'f_{12} + uf_{13} + vu'f_{21} + vv'f_{22} + vf_{23} + u'f_{31} + v'f_{32} + f_{33} = 0$$

$$A\mathbf{f} = \begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ \vdots & \vdots \\ u_nu_n' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{33} \end{bmatrix} = \mathbf{0}$$

ESTIMATING THE FUNDAMENTAL MATRIX

- ▶ 8-point algorithm
- ▶ Least squares solution using SVD on equations from 8 pairs of correspondences
- ▶ Enforce $\det(F)=0$ constraint using SVD on F
- ▶ Minimize reprojection error
- ▶ Non-linear least squares

Note: estimation of F (or E) is degenerate for a planar scene.

8-POINT ALGORITHM

- ▶ Solve a system of homogeneous linear equations
- ▶ Write down the system of equations
- ▶ Solve f from $Af=0$ using SVD

Matlab:

```
[U, S, V] = svd(A);  
f = V(:, end);  
F = reshape(f, [3 3])';
```

THE SINGULARITY CONSTRAINT

$$\mathbf{e}'^T \mathbf{F} = 0 \quad \mathbf{F}\mathbf{e} = 0 \quad \det \mathbf{F} = 0 \quad \text{rank } \mathbf{F} = 2$$

SVD from linearly computed F matrix (rank 3)

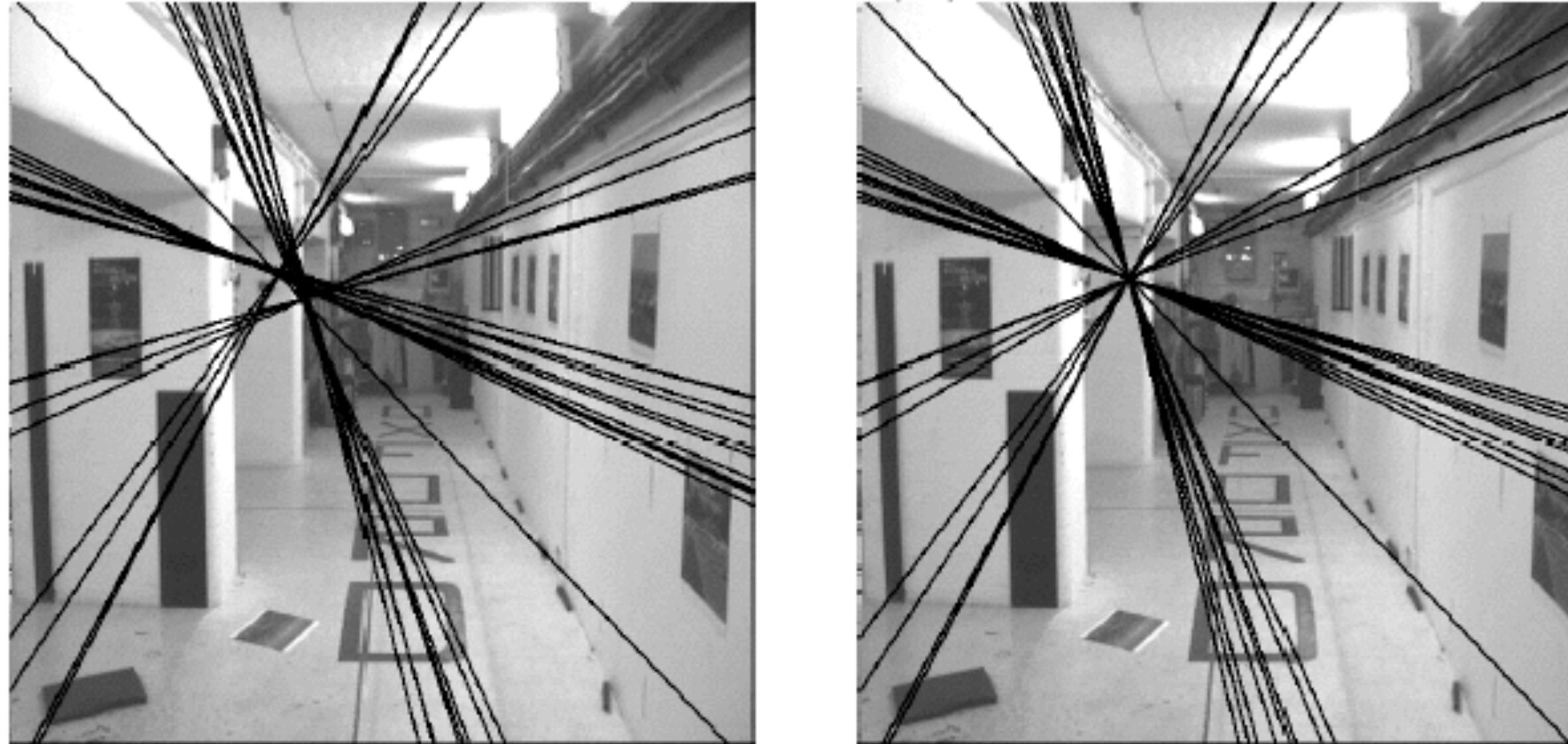
$$\mathbf{F} = \mathbf{U} \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} \mathbf{V}^T = \mathbf{U}_1 \sigma_1 \mathbf{V}_1^T + \mathbf{U}_2 \sigma_2 \mathbf{V}_2^T + \mathbf{U}_3 \sigma_3 \mathbf{V}_3^T$$

Compute closest rank-2 approximation $\min \|\mathbf{F} - \mathbf{F}'\|_F$

$$\mathbf{F}' = \mathbf{U} \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & 0 \end{bmatrix} \mathbf{V}^T = \mathbf{U}_1 \sigma_1 \mathbf{V}_1^T + \mathbf{U}_2 \sigma_2 \mathbf{V}_2^T$$

NEED TO ENFORCE SINGULARITY CONSTRAINT

Fundamental matrix has rank 2 : $\det(F) = 0$.



Left : Uncorrected F – epipolar lines are not coincident.

Right : Epipolar lines from corrected F .

8-POINT ALGORITHM

- ▶ Solve a system of homogeneous linear equations

- ▶ Write down the system of equations
- ▶ Solve f from $Af=0$ using SVD

Matlab:
`[U, S, V] = svd(A);
f = V(:, end);
F = reshape(f, [3 3])';`

- ▶ Resolve $\det(F) = 0$ constraint using SVD

Matlab:
`[U, S, V] = svd(F);
S(3,3) = 0;
F = U*S*V';`

8-POINT ALGORITHM

- ▶ Solve a system of homogeneous linear equations
 - ▶ Write down the system of equations
 - ▶ Solve f from $Af=0$ using SVD
- ▶ Resolve $\det(F) = 0$ constraint by SVD
- ▶ Notes:
- ▶ Use RANSAC to deal with outliers (sample 8 points)
 - ▶ How to test for outliers?

PROBLEM WITH EIGHT-POINT ALGORITHM

$$\begin{bmatrix} x_1x_1' & y_1x_1' & x_1' & x_1y_1' & y_1y_1' & y_1' & x_1 & y_1 & 1 \\ x_2x_2' & y_2x_2' & x_2' & x_2y_2' & y_2y_2' & y_2' & x_2 & y_2 & 1 \\ \boxed{\text{?}} & \boxed{\text{?}} \\ x_nx_n' & y_nx_n' & x_n' & x_ny_n' & y_ny_n' & y_n' & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

~10000 ~10000 ~100 ~10000 ~10000 ~100 ~100 ~100 1



Orders of magnitude difference
Between column of data matrix
→ least-squares yields poor results

THE NORMALIZED EIGHT-POINT ALGORITHM

(Hartley, 1995)

- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels
- Use the eight-point algorithm to compute \mathbf{F} from the normalized points
- Enforce the rank-2 constraint (for example, take SVD of \mathbf{F} and throw out the smallest singular value)
- Transform fundamental matrix back to original units: if \mathbf{T} and \mathbf{T}' are the normalizing transformations in the two images, than the fundamental matrix in original coordinates is $\mathbf{T}'^T \mathbf{F} \mathbf{T}$

LET'S RECAP...

- ▶ Fundamental matrix song

TRIANGULATION

EQUATIONS

$$x_j = \frac{p_{00}^{(j)}X + p_{01}^{(j)}Y + p_{02}^{(j)}Z + p_{03}^{(j)}W}{p_{20}^{(j)}X + p_{21}^{(j)}Y + p_{22}^{(j)}Z + p_{23}^{(j)}W}$$

$$y_j = \frac{p_{10}^{(j)}X + p_{11}^{(j)}Y + p_{12}^{(j)}Z + p_{13}^{(j)}W}{p_{20}^{(j)}X + p_{21}^{(j)}Y + p_{22}^{(j)}Z + p_{23}^{(j)}W},$$

Solve the system with the same methods as before

$$x(\mathbf{p}^{3\top}\mathbf{X}) - (\mathbf{p}^{1\top}\mathbf{X}) = 0$$

$$y(\mathbf{p}^{3\top}\mathbf{X}) - (\mathbf{p}^{2\top}\mathbf{X}) = 0$$

$$\mathbf{A} = \begin{bmatrix} x\mathbf{p}^{3\top} - \mathbf{p}^{1\top} \\ y\mathbf{p}^{3\top} - \mathbf{p}^{2\top} \\ x'\mathbf{p}'^{3\top} - \mathbf{p}'^{1\top} \\ y'\mathbf{p}'^{3\top} - \mathbf{p}'^{2\top} \end{bmatrix}$$

MORE ACCURATE

- ▶ Geometric error cost function
- ▶ Given a measured point correspondence $x \leftrightarrow x'$, and a fundamental matrix F , compute the corrected correspondences $\hat{x} \leftrightarrow \hat{x}'$ that minimize the geometric error $(C(x, x'))$ subject to the epipolar constraint $\hat{x}'^T F \hat{x} = 0$. ($d()$ is the Euclidian distance)

$$\mathcal{C}(x, x') = d(x, \hat{x})^2 + d(x', \hat{x}')^2 \quad \text{subject to } \hat{x}'^T F \hat{x} = 0$$

AN OPTIMAL SOLUTION

$$\min_{\hat{\mathbf{x}}} \mathcal{C} = d(\mathbf{x}, \hat{\mathbf{x}})^2 + d(\mathbf{x}', \hat{\mathbf{x}}')^2 = \min_t \mathcal{C} = d(\mathbf{x}, l(t))^2 + d(\mathbf{x}', l'(t))^2.$$

- ▶ Parametrize the pencil of epipolar lines in the first image by a parameter t . Thus an epipolar line in the first image may be written as $l(t)$.
- ▶ Using the fundamental matrix F , compute the corresponding epipolar line $l'(t)$ in the second image.
- ▶ Express the distance function $d(x, l(t))^2 + d(x', l'(t))^2$ explicitly as a function of t .
- ▶ Find the value of t that minimizes this function
- ▶ Finding the real roots of a polynomial of degree 6

OUTLINE OF RECONSTRUCTION

- (i) Compute F from correspondences
- (ii) Compute camera matrices from F
- (iii) Compute 3D point for each pair of corresponding points

computation of F

8+ (least-squares) (more on this next class)

computation of camera matrices

use

$$P = [I \mid 0] \quad P' = [[e']]_x F + e' v^T \mid \lambda e'$$

triangulation

compute intersection of two backprojected rays

FROM EPIPOLAR GEOMETRY TO CAMERA CALIBRATION

- ▶ Estimating the fundamental matrix is known as “weak calibration”
- ▶ We know the calibration matrices of the two cameras, we can estimate the essential matrix: $E = K^T F K'$
- ▶ The essential matrix gives us the relative rotation and translation between the cameras, or their extrinsic parameters

LAB 2

COMPUTE THE FUNDAMENTAL MATRIX

- ▶ 8-point algorithm
- ▶ Normalized 8-point algorithm
- ▶ Draw epipolar lines
- ▶ Compute Essential Matrix
- ▶ Compute Canonical P1 and P2
- ▶ Triangulate Points
- ▶ Reprojection error

REFERENCES AND FURTHER READING

- ▶ Computer Vision: Algorithms and Applications. Richard Szeliski
- ▶ Multiple View Geometry in Computer Vision. Hartley and Zisserman.