COMPUTER VISION AND PHOTOGRAMMETRY

RECONSTRUCTION FROM SEQUENCES

SUMMARY

3D MODELLING FROM IMAGES PIPELINE

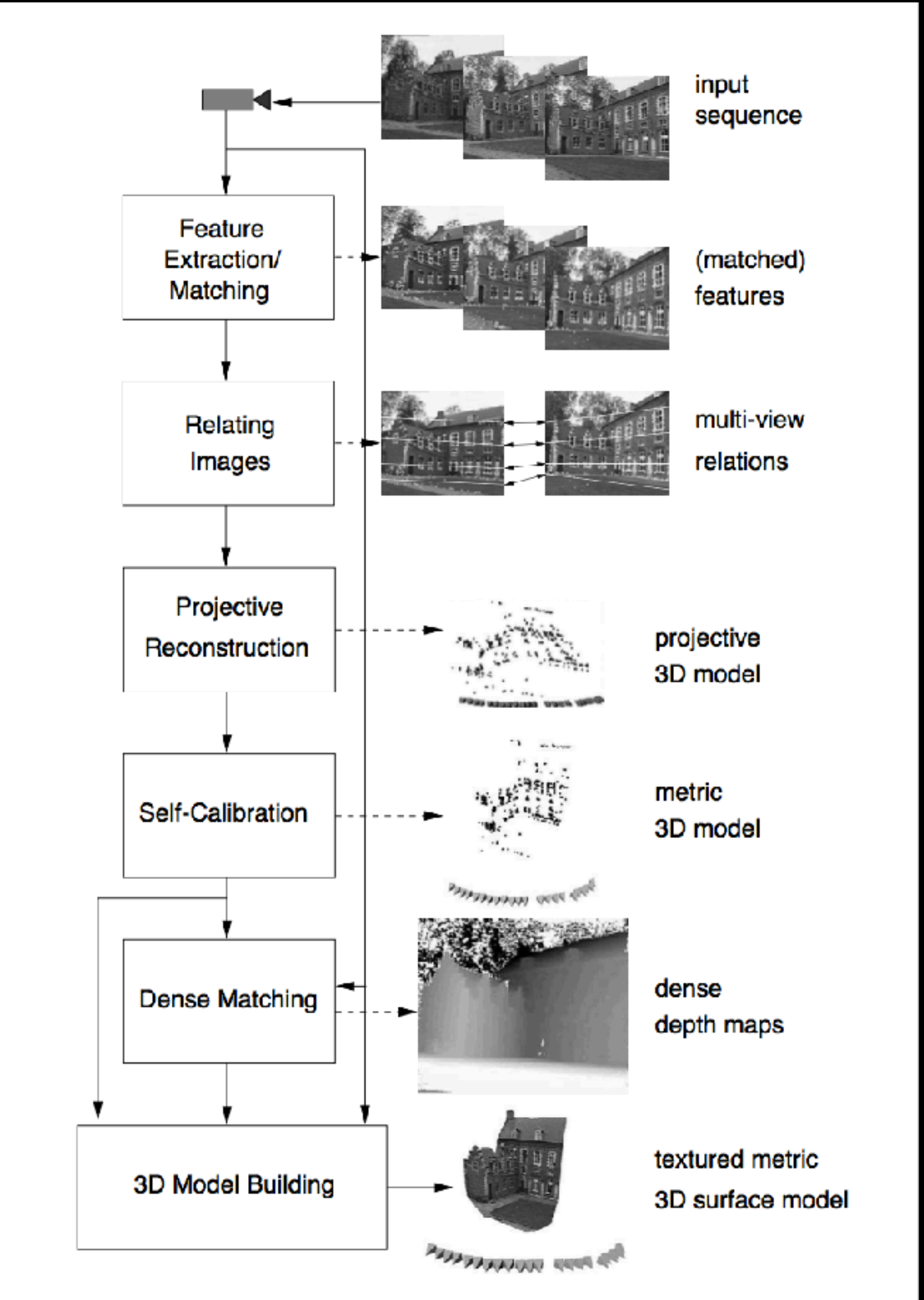


Figure 1.7: Overview of the presented approach for 3D modeling from images

PROJECTIVE GEOMETRY AND SELF CALIBRATION

HOMOGENEOUS COORDINATES

Homogeneous representation of lines

$ax + by + c = 0 \qquad (a,b,c)^T$

$(ka)x + (kb)y + kc = 0, \forall k \neq 0 \qquad (a,b,c)^T \sim k(a,b,c)^T$

equivalence class of vectors, any vector is representative

Set of all equivalence classes in $\mathbf{R}^3-(0,0,0)^T$ forms $\mathbf{P}^2$

Homogeneous representation of points

$\mathrm{x} = (x, y)^T$ on $\mathrm{l} = (a,b,c)^T$ if and only if $ax + by + c = 0$

$(x,y,1)(a,b,c)^T = (x,y,1)\mathrm{l} = 0 \qquad (x, y,1)^T \sim k(x, y,1)^T, \forall k \neq 0$

The point x lies on the line l if and only if $\mathrm{x}^T\mathrm{l}=\mathrm{l}^T\mathrm{x}=0$

Homogeneous coordinates $(x_1, x_2, x_3)^T$ but only 2DOF

Inhomogeneous coordinates $(x, y)^T$

POINTS FROM LINES AND VICE-VERSA

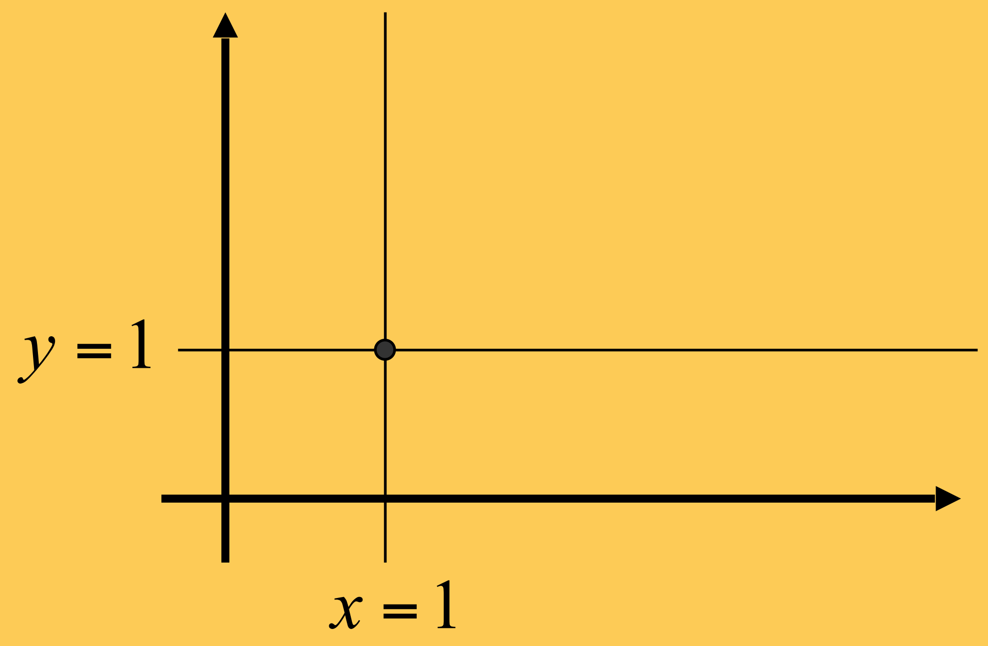
Intersections of lines

The intersection of two lines $\mathbf{l}$ and $\mathbf{l}'$ is $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$

Line joining two points

The line through two points $\mathbf{x}$ and $\mathbf{x}'$ is $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$

Example

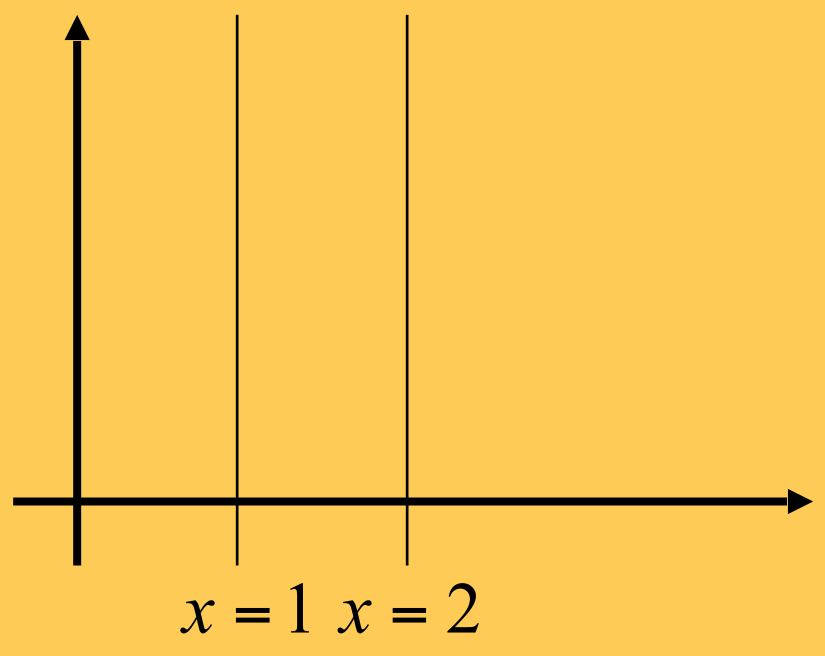


IDEAL POINTS AND THE LINE AT INFINITY

Intersections of parallel lines

$$\mathbf{l} = (a, b, c)^\mathsf{T} \text{ and } \mathbf{l}' = (a, b, c')^\mathsf{T} \qquad \mathbf{l} \times \mathbf{l}' = (b, -a, 0)^\mathsf{T}$$

Example



$(b, -a)$ tangent vector

(a, b) normal direction

Ideal points $(x_1, x_2, 0)^\mathsf{T}$

Line at infinity $\mathbf{l}_\infty = (0, 0, 1)^\mathsf{T}$

$$\mathbf{P}^2 = \mathbf{R}^2 \cup \mathbf{l}_\infty$$

Note that in $\mathbf{P}^2$ there is no distinction between ideal points and others

DUALITY

$$\mathbf{x} \longleftrightarrow \mathbf{l}$$

$$\mathbf{x}^{\mathsf{T}}\mathbf{l} = 0 \longleftrightarrow \mathbf{l}^{\mathsf{T}}\mathbf{x} = 0$$

$$\mathbf{x} = \mathbf{l} \times \mathbf{l}' \longleftrightarrow \mathbf{l} = \mathbf{x} \times \mathbf{x}'$$

Duality principle:

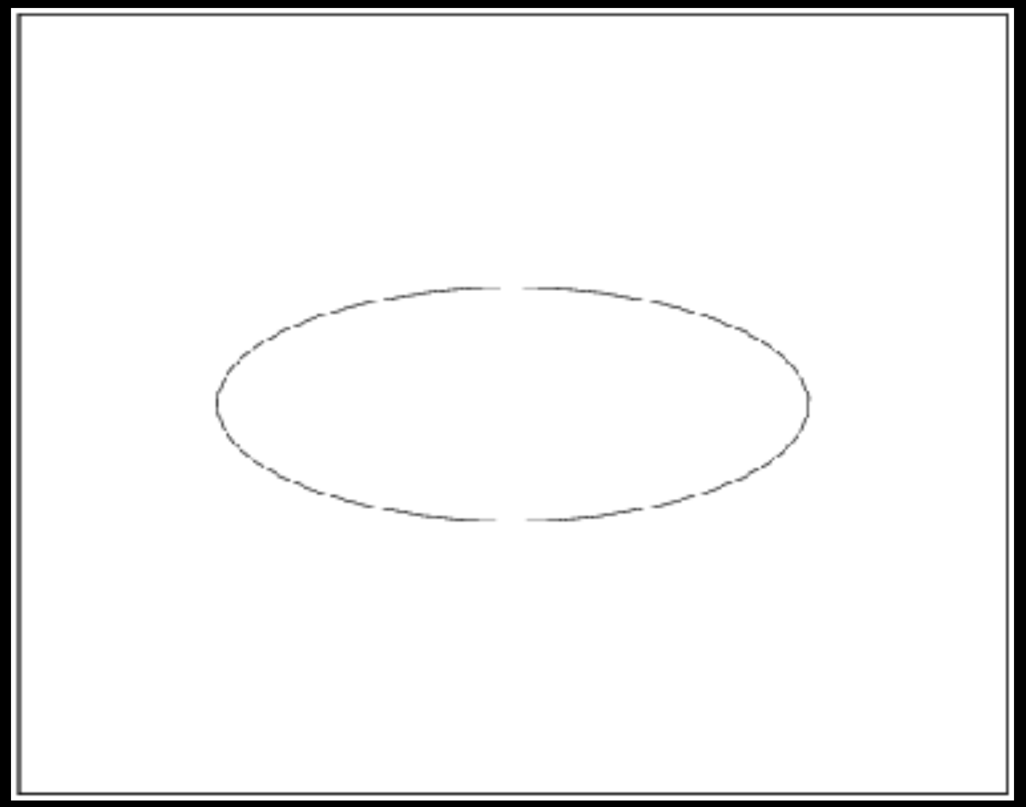
To any theorem of 2-dimensional projective geometry there corresponds a dual theorem, which may be derived by interchanging the role of points and lines in the original theorem

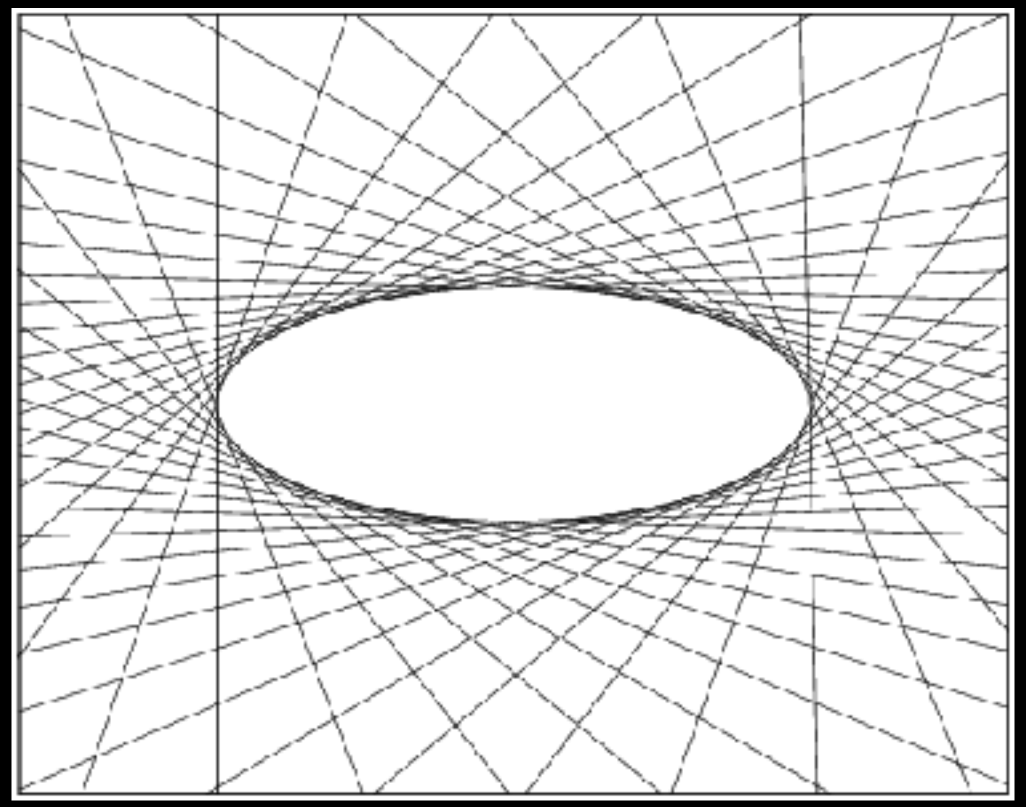
DUAL CONICS

A line tangent to the conic $\mathbf{C}$ satisfies $\mathbf{l}^{\mathsf{T}} \mathbf{C}^{*} \mathbf{l} = 0$

In general ($\mathbf{C}$ full rank): $\mathbf{C}^{*} = \mathbf{C}^{-1}$

Dual conics = line conics = conic envelopes





PLANES

3D plane

$$\pi_1 X + \pi_2 Y + \pi_3 Z + \pi_4 = 0$$

$$\pi_1 X_1 + \pi_2 X_2 + \pi_3 X_3 + \pi_4 X_4 = 0$$

$$\pi^{\mathsf{T}} \mathrm{X} = 0$$

Transformation

$$\mathrm{X}' = \mathbf{H}\,\mathrm{X}$$

$$\pi' = \mathbf{H}^{-\mathsf{T}}\,\pi$$

Dual: points ↔ planes

QUADRICS AND DUAL QUADRICS

- Quadric

$$X^{T}QX = 0$$ (Q : 4x4 symmetric matrix)

$$Q = \begin{bmatrix} \bullet & \bullet & \bullet & \bullet \\ ? & \bullet & \bullet & \bullet \\ ? & ? & \bullet & \bullet \\ ? & ? & ? & \bullet \end{bmatrix}$$

1. 9 d.o.f.
2. (plane ∩ quadric)=conic $C = M^{T}QM$
3. transformation $Q' = H^{-T}QH^{-1}$

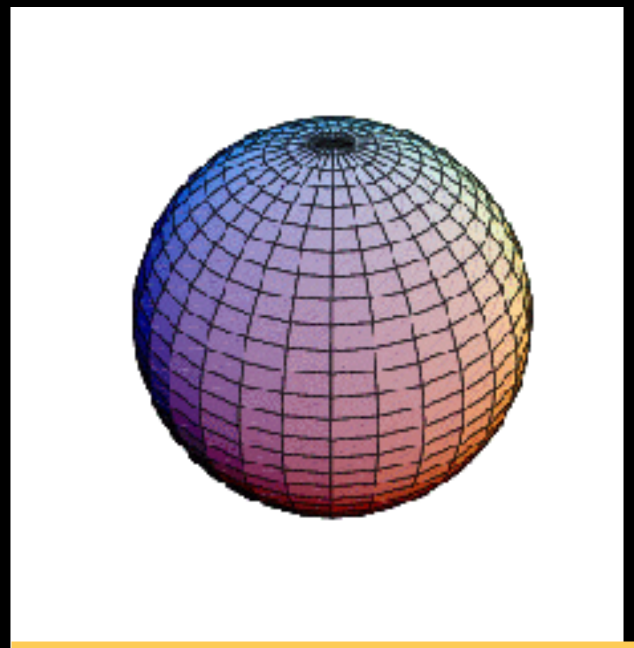
- Dual Quadric

$$\pi^{T}Q^{*}\pi = 0$$

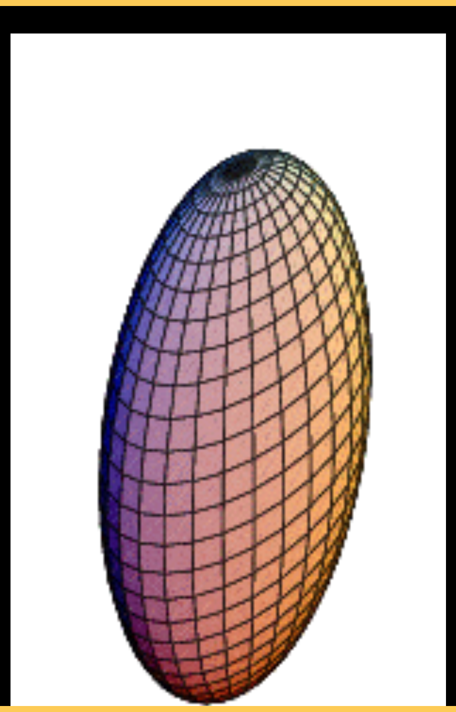
1. relation to quadric $Q^{*} = Q^{-1}$ (non-degenerate)
2. transformation $Q'^{*} = HQ^{*}H^{T}$

QUADRIC CLASSIFICATION

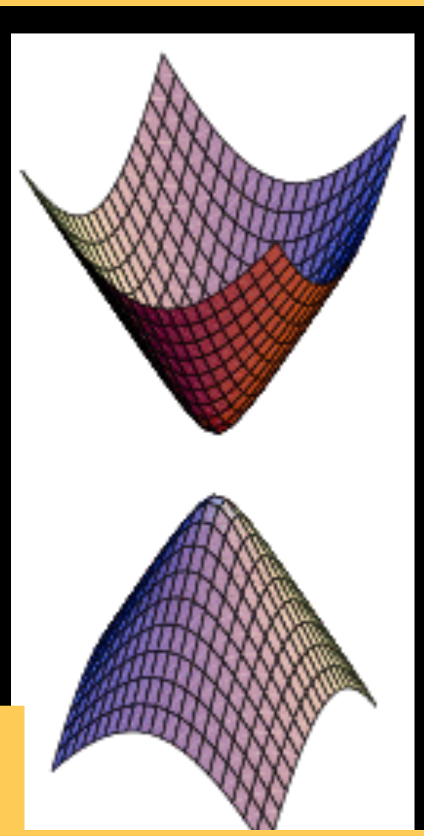
PROJECTIVELY EQUIVALENT TO *SPHERE:*



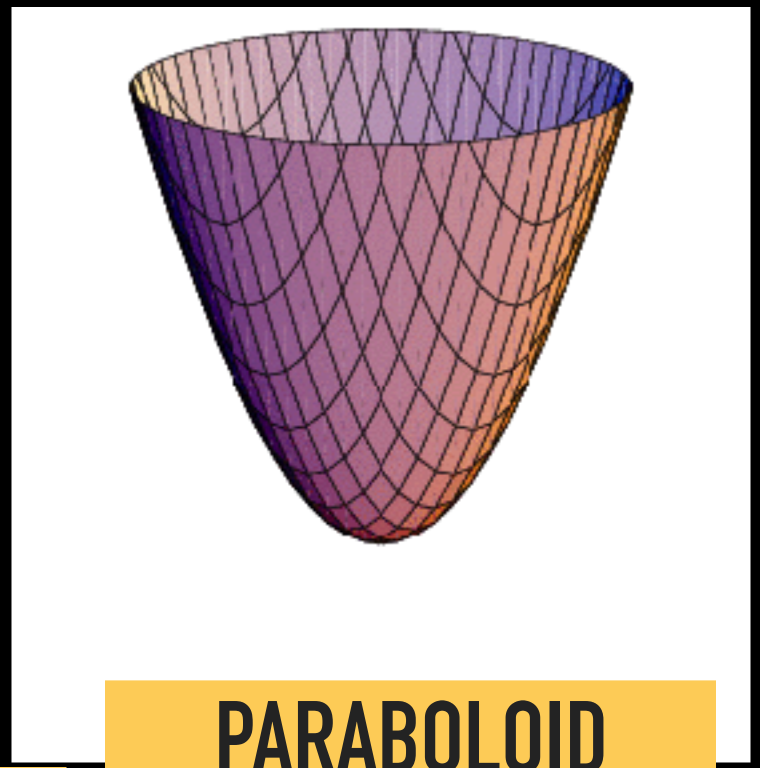
SPHERE



ELLIPSOID

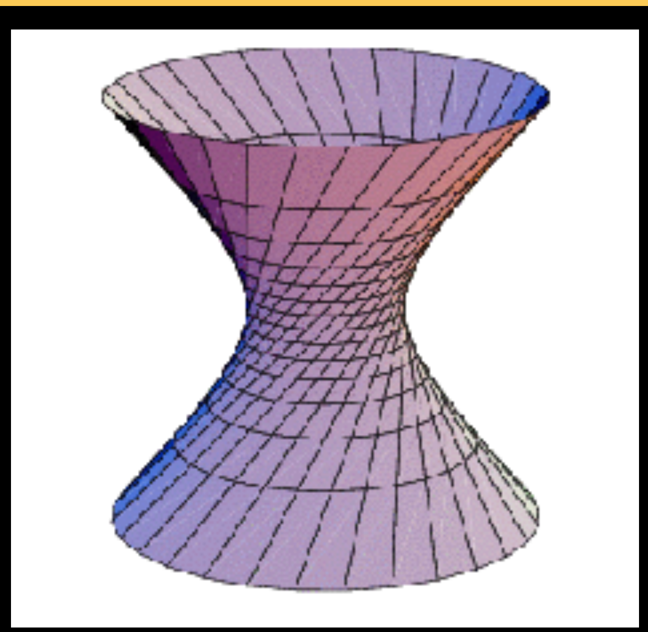


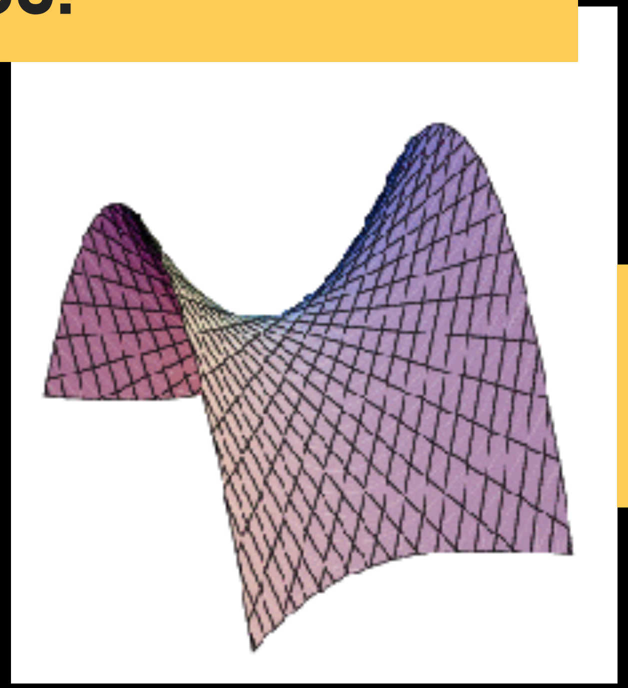
HYPERBOLOID OF TWO SHEETS



PARABOLOID

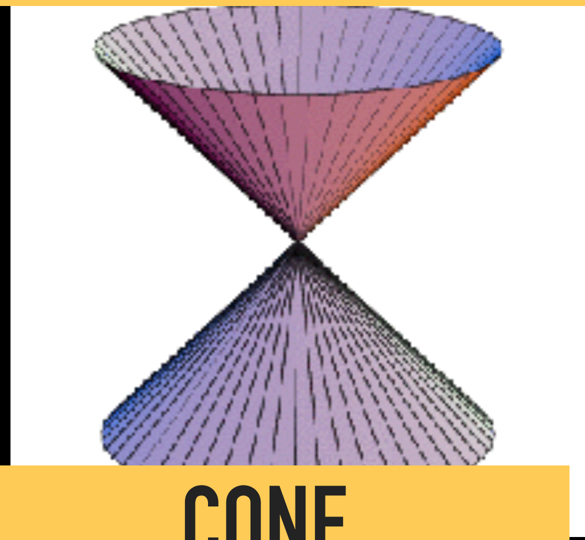
RULED QUADRICS:



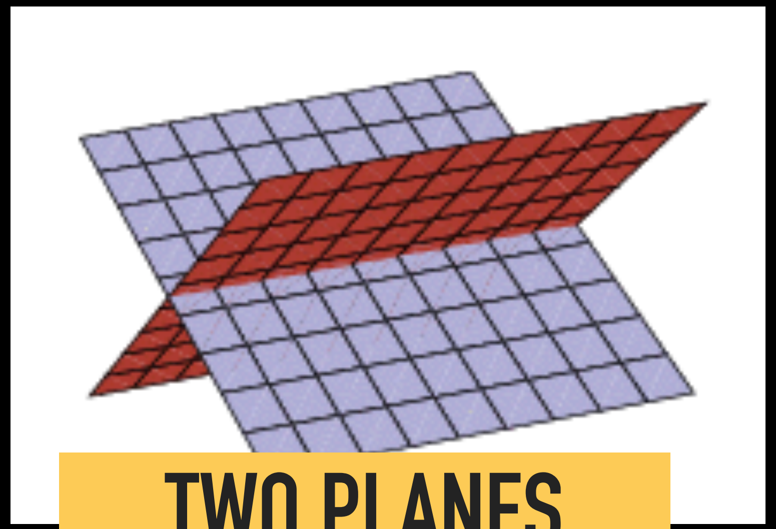


HYPERBOLOIDS OF ONE SHEET

DEGENERATE RULED QUADRICS:

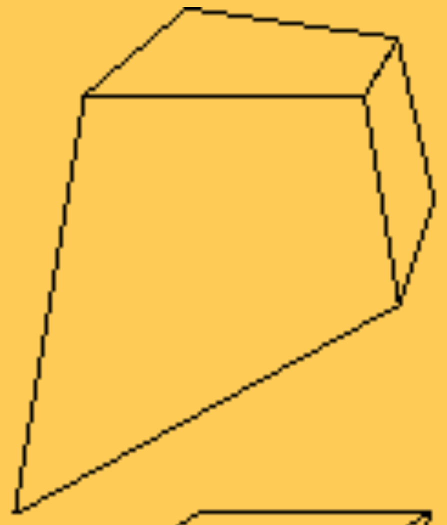
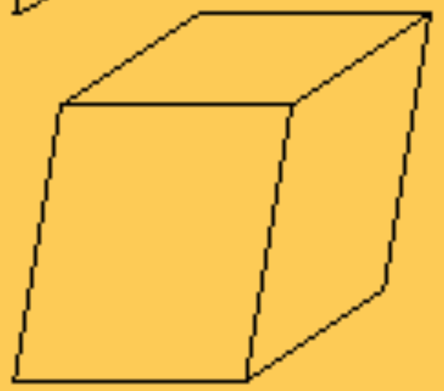
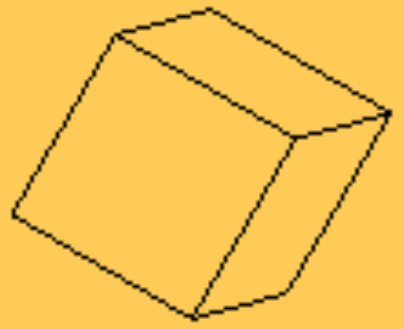
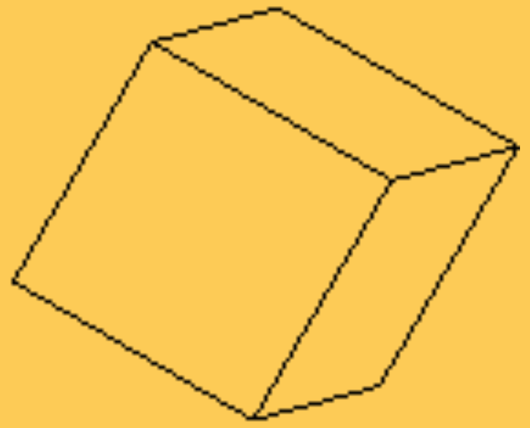


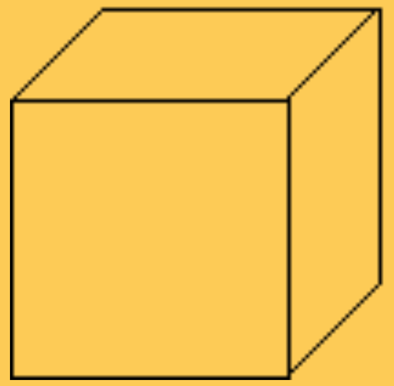
CONE



TWO PLANES

HIERARCHY OF TRANSFORMATIONS

Projective 15dof	$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$		Intersection and tangency
Affine 12dof	$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$		Parallellism of planes, Volume ratios, centroids, **The plane at infinity π_∞**
Similarity 7dof	$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$		**The absolute conic Ω_∞**
Euclidean 6dof	$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$		Volume



THE PLANE AT INFINITY

$$\pi'_\infty = \mathbf{H}_A^{-\mathsf{T}}\pi_\infty = \begin{bmatrix} \mathbf{A}^{-\mathsf{T}} & 0 \\ -\mathbf{A}\,\mathrm{t} & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \pi_\infty$$

The plane at infinity π_∞ is a fixed plane under a projective transformation H if H is an affinity

1. canical position $\pi_\infty = (0,0,0,1)^\mathsf{T}$
2. contains directions $\mathrm{D} = (X_1, X_2, X_3, 0)^\mathsf{T}$

THE ABSOLUTE CONIC

The absolute conic Ω_∞ is a (point) conic on π_∞.

In a metric frame:

$$\left.\begin{array}{c} X_1^2 + X_2^2 + X_3^2 \\ X_4 \end{array}\right\} = 0$$

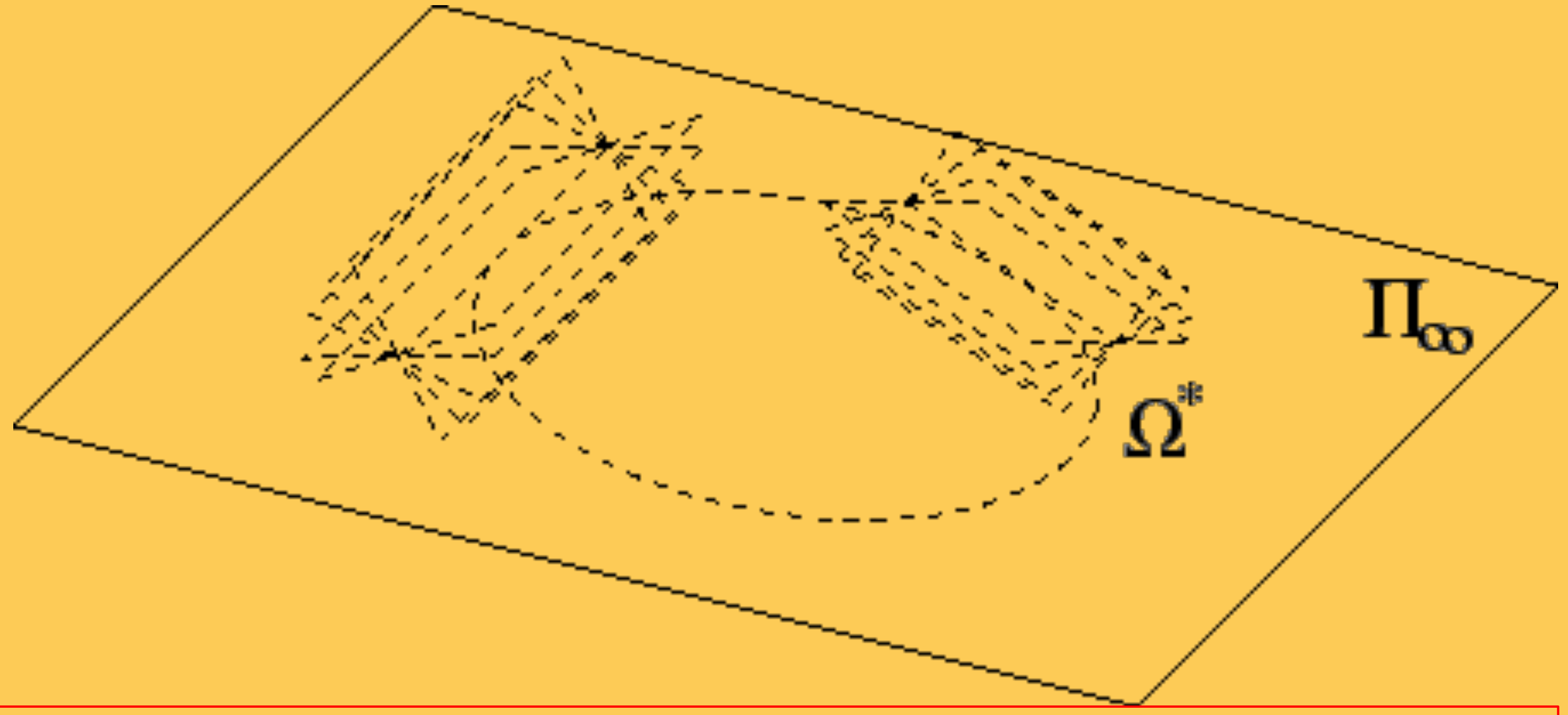
or conic for directions: $(X_1, X_2, X_3)\mathrm{I}(X_1, X_2, X_3)^\mathsf{T}$
(with no real points)

The absolute conic Ω_∞ is a fixed conic under the projective transformation $\mathbf{H}$ if $\mathbf{H}$ is a similarity

1. Ω_∞ is only fixed as a set
2. Circle intersect Ω_∞ in two points
3. Spheres intersect π_∞ in Ω_∞

THE DUAL ABSOLUTE QUADRIC

$$\Omega^*_\infty = \begin{bmatrix} \mathrm{I} & 0 \\ 0^\mathsf{T} & 0 \end{bmatrix}$$



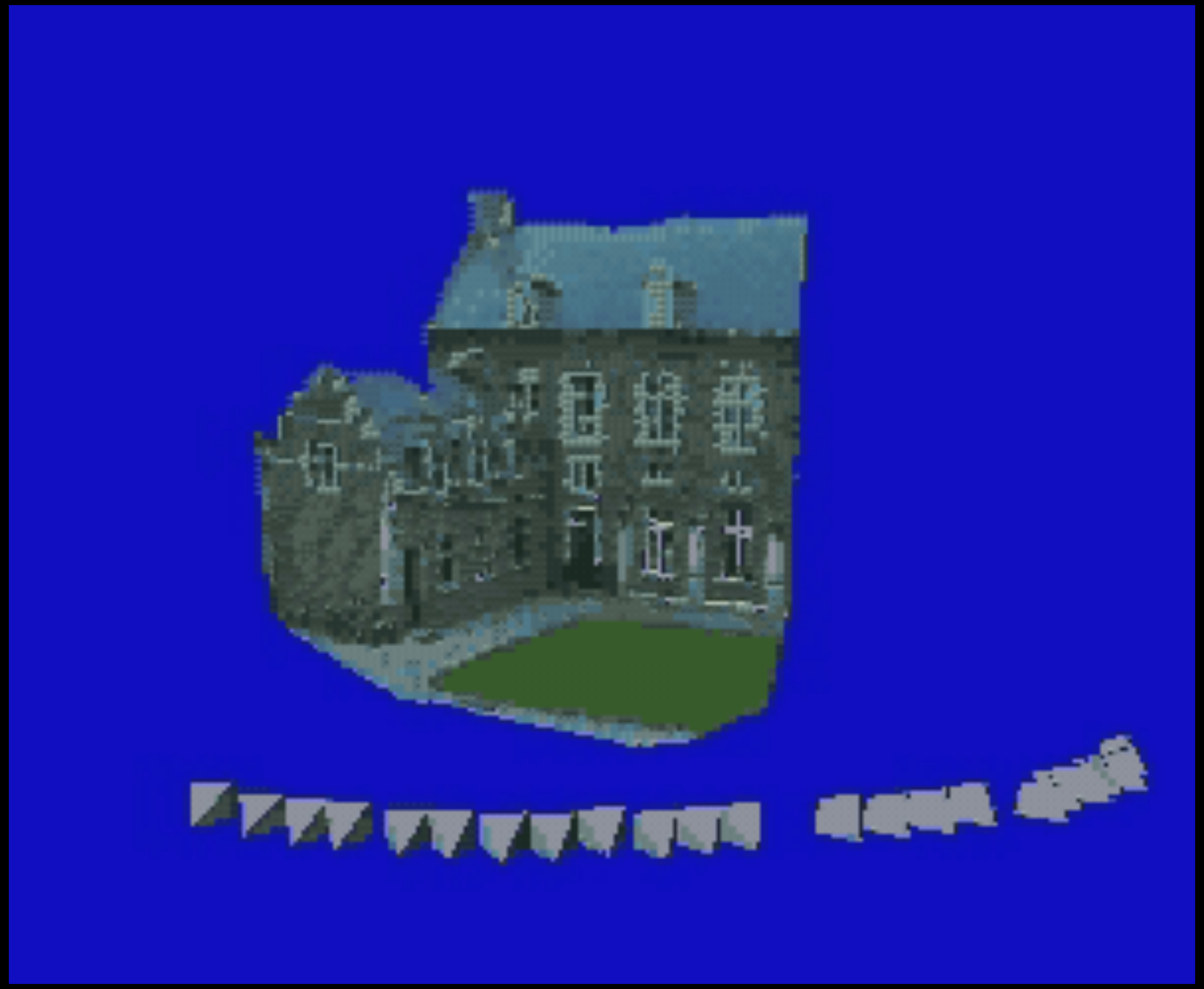
The absolute conic Ω^*_∞ is a fixed conic under the projective transformation $\mathbf{H}$ if $\mathbf{H}$ is a similarity

1. 8 dof
2. plane at infinity π_∞ is the nullvector of Ω_∞

PROJECTIVE AMBIGUITY

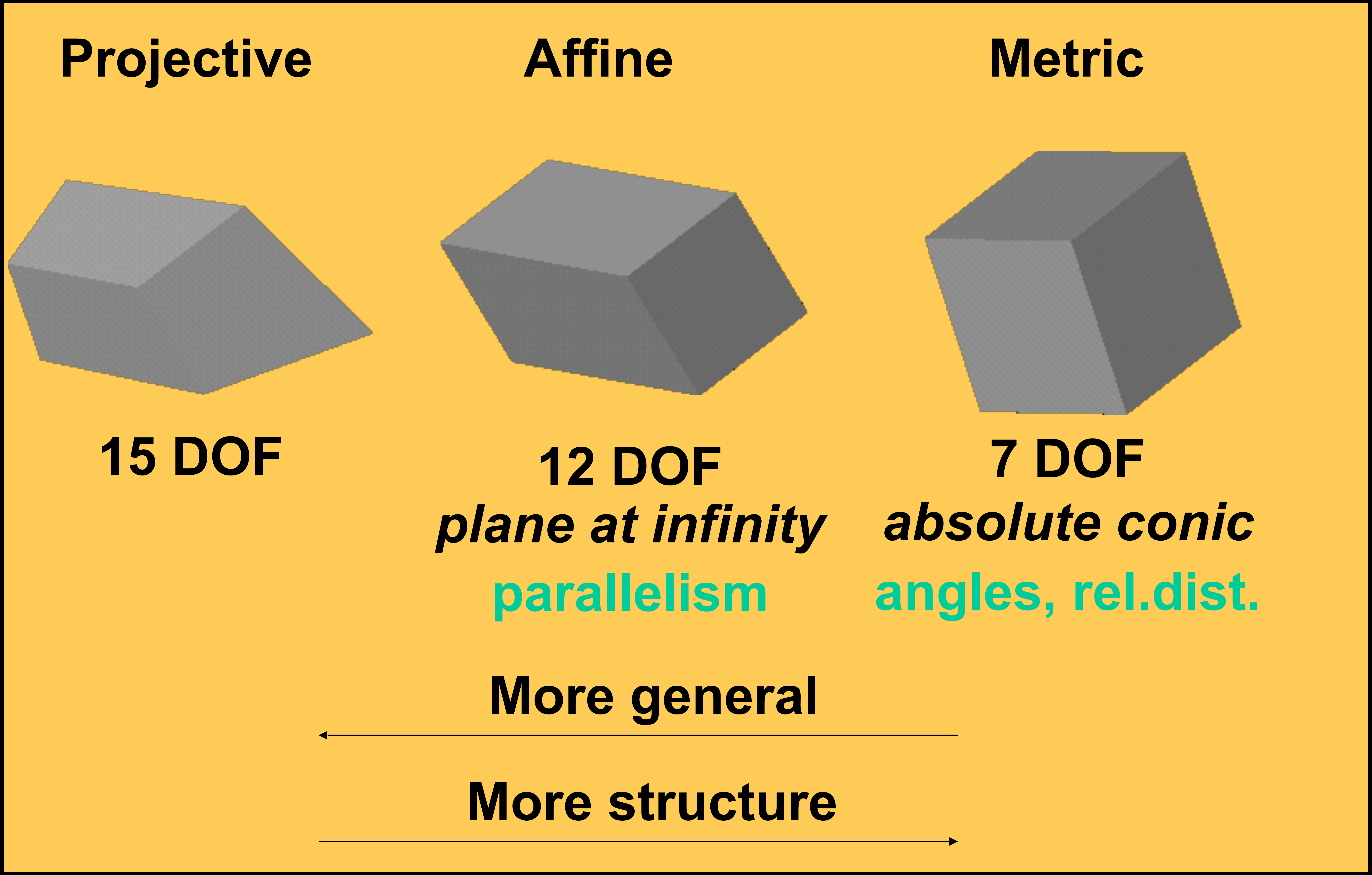
Reconstruction from uncalibrated images

$\Rightarrow$ projective ambiguity on reconstruction



$$\mathrm{m} = \mathbf{P}\mathrm{M} = (\mathbf{P}\mathbf{T}^{-1})(\mathbf{T}\mathrm{M}) = \mathbf{P}'\mathrm{M}'$$

STRATIFICATION OF GEOMETRY



CONSTRAINTS ?

- Scene constraints
  - Parallellism, vanishing points, horizon, ...
  - Distances, positions, angles, ...

  Unknown scene → no constraints

- Camera extrinsics constraints
  – Pose, orientation, ...

  Unknown camera motion → no constraints

- Camera intrinsics constraints
  – Focal length, principal point, aspect ratio & skew

  Perspective camera model too general
  → some constraints

ABSOLUTE DUAL QUADRIC AND SELF-CALIBRATION

Eliminate extrinsics from equation

$$\mathbf{K}\left[\mathbf{R}^{\mathsf{T}} \quad \cancel{-\mathbf{R}^{\mathsf{T}}\mathbf{t}}\right] \longrightarrow \mathbf{K}\cancel{\mathbf{R}^{\mathsf{T}}\mathbf{R}}\mathbf{K}^{\mathsf{T}} \longrightarrow \mathbf{K}\mathbf{K}^{\mathsf{T}}$$

Equivalent to projection of dual quadric

$$\mathbf{P}\Omega_{\infty}^{*}\mathbf{P}^{\mathsf{T}} \propto \mathbf{K}\mathbf{K}^{\mathsf{T}} \qquad \Omega_{\infty}^{*} = \mathrm{diag}(1110)$$

Abs.Dual Quadric also exists in projective world

$$\mathbf{K}\mathbf{K}^{\mathsf{T}} \propto \mathbf{P}\Omega_{\infty}^{*}\mathbf{P}^{\mathsf{T}} \propto (\mathbf{P}\mathbf{T}^{-1})(\mathbf{T}\Omega_{\infty}^{*}\mathbf{T}^{\mathsf{T}})(\mathbf{T}^{-\mathsf{T}}\mathbf{P}^{\mathsf{T}})$$

$$\propto \mathbf{P}'\Omega_{\infty}'^{*}\mathbf{P}'^{\mathsf{T}}$$

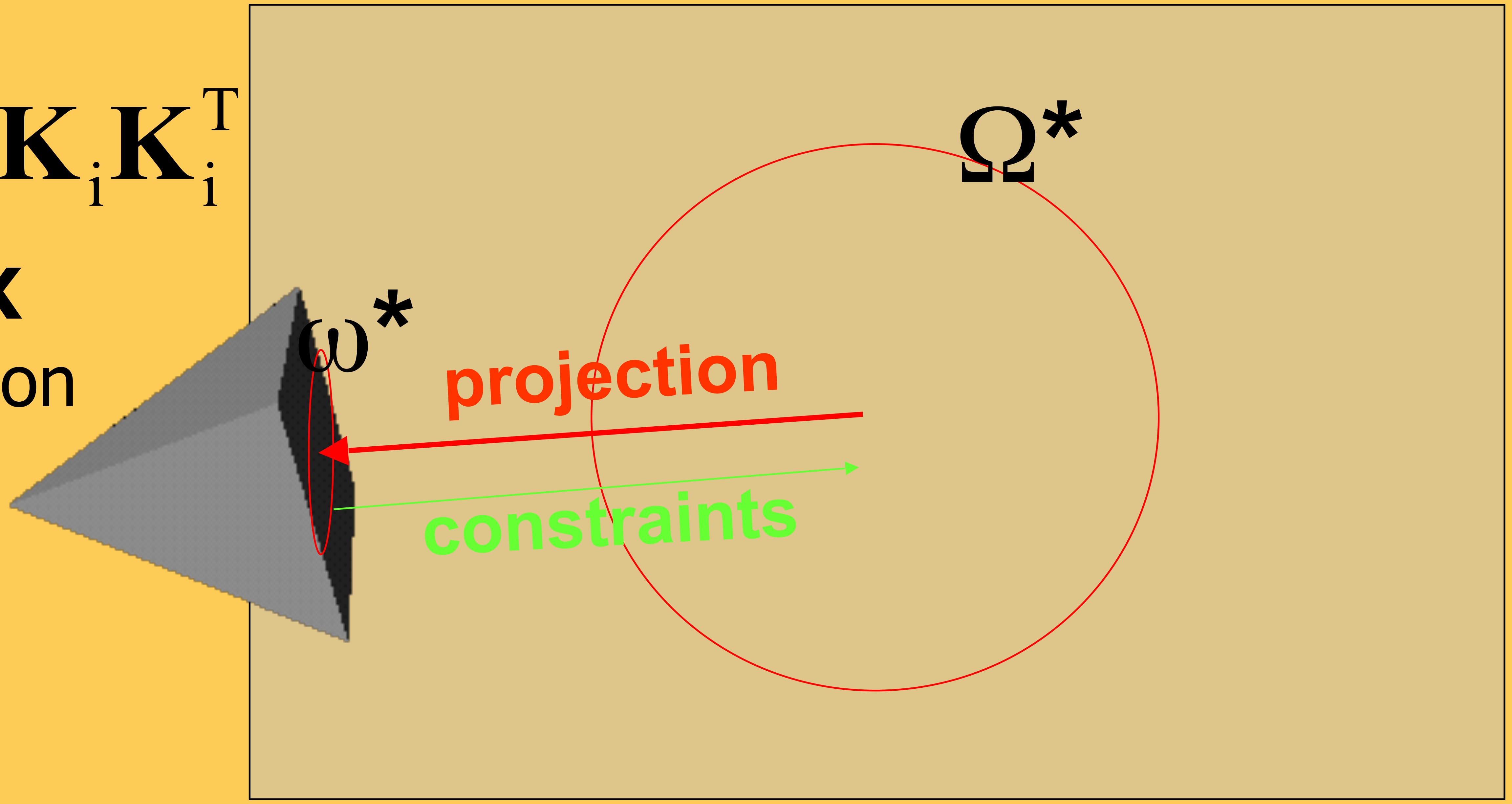
Transforming world so that $\Omega_{\infty}'^{*} \rightarrow \Omega_{\infty}^{*}$
reduces ambiguity to metric

IMAGE OF THE ABSOLUTE CONIC

Projection equation:

$$\omega_i^* \propto P_i \Omega^* P_i^T \propto K_i K_i^T$$

Translate constraints on **K** through projection equation to constraints on Ω^*



Absolute conic = calibration object which is always present but can only be observed through constraints on the intrinsics

CONSTRAINTS ON Ω^*_∞

$$\omega^*_\infty = \begin{bmatrix} f_x^2 + s^2 + c_x^2 & sf_y + c_x c_y & c_x \\ sf_y + c_x c_y & f_y^2 + c_y^2 & c_y \\ c_x & c_y & 1 \end{bmatrix}$$

condition	constraint	type	#constraints
Zero skew	$\omega^*_{12}\omega^*_{33} = \omega^*_{13}\omega^*_{23}$	quadratic	m
Principal point	$\omega^*_{13} = \omega^*_{23} = 0$	linear	$2m$
Zero skew (& p.p.)	$\omega^*_{12} = 0$	linear	m
Fixed aspect ratio (& p.p.& Skew)	$\omega^*_{11}\omega'^*_{22} = \omega^*_{22}\omega'^*_{11}$	quadratic	m-1
Known aspect ratio (& p.p.& Skew)	$\omega^*_{11} = \omega^*_{22}$	linear	m
Focal length (& p.p. & Skew)	$\omega^*_{33} = \omega^*_{11}$	linear	m

LINEAR ALGORITHM

(Pollefeys et al.,ICCV´98/IJCV´99)

Assume everything known, except focal length

$$\omega^* \approx \begin{bmatrix} \hat{f}^2 & 0 & 0 \\ 0 & \hat{f}^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \propto \mathbf{P}\Omega^*\mathbf{P}^\top$$

$$\begin{aligned} \left(\mathbf{P}\Omega^*\mathbf{P}^\top\right)_{11} - \left(\mathbf{P}\Omega^*\mathbf{P}^\top\right)_{22} &= 0 \\ \left(\mathbf{P}\Omega^*\mathbf{P}^\top\right)_{12} &= 0 \\ \left(\mathbf{P}\Omega^*\mathbf{P}^\top\right)_{13} &= 0 \\ \left(\mathbf{P}\Omega^*\mathbf{P}^\top\right)_{23} &= 0 \end{aligned}$$

Yields 4 constraint per image

Note that rank-3 constraint is not enforced

PROJECTIVE TO METRIC

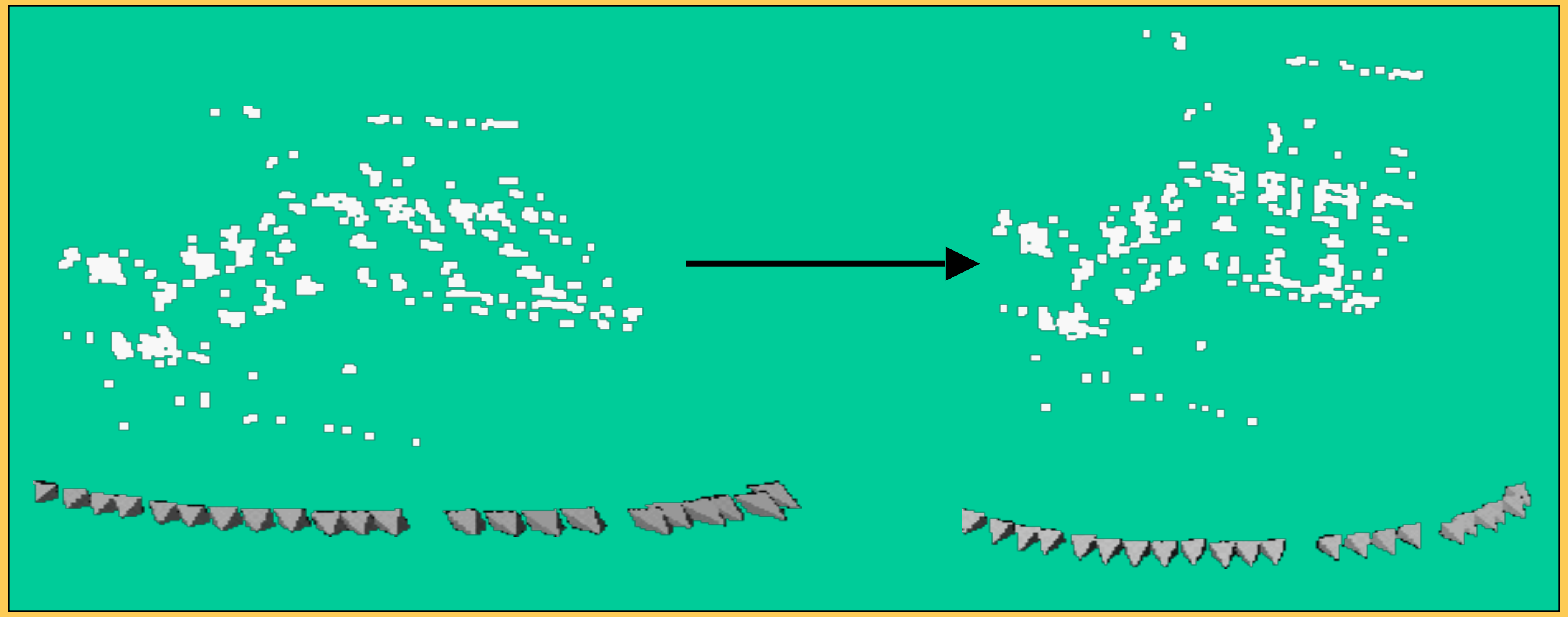
Compute $\mathbf{T}$ from

$$\tilde{\mathbf{I}} = \mathbf{T}\Omega^*_\infty\mathbf{T}^\top \text{ or } \mathbf{T}^{-1}\tilde{\mathbf{I}}\mathbf{T}^{-\top} = \Omega^*_\infty \text{ with } \tilde{\mathbf{I}} = \begin{bmatrix} \mathbf{I} & 0 \\ 0^\top & 0 \end{bmatrix}$$

using eigenvalue decomposition of Ω^*_∞

and then obtain metric reconstruction as

$$\mathbf{PT}^{-1} \text{ and } \mathbf{TM}$$

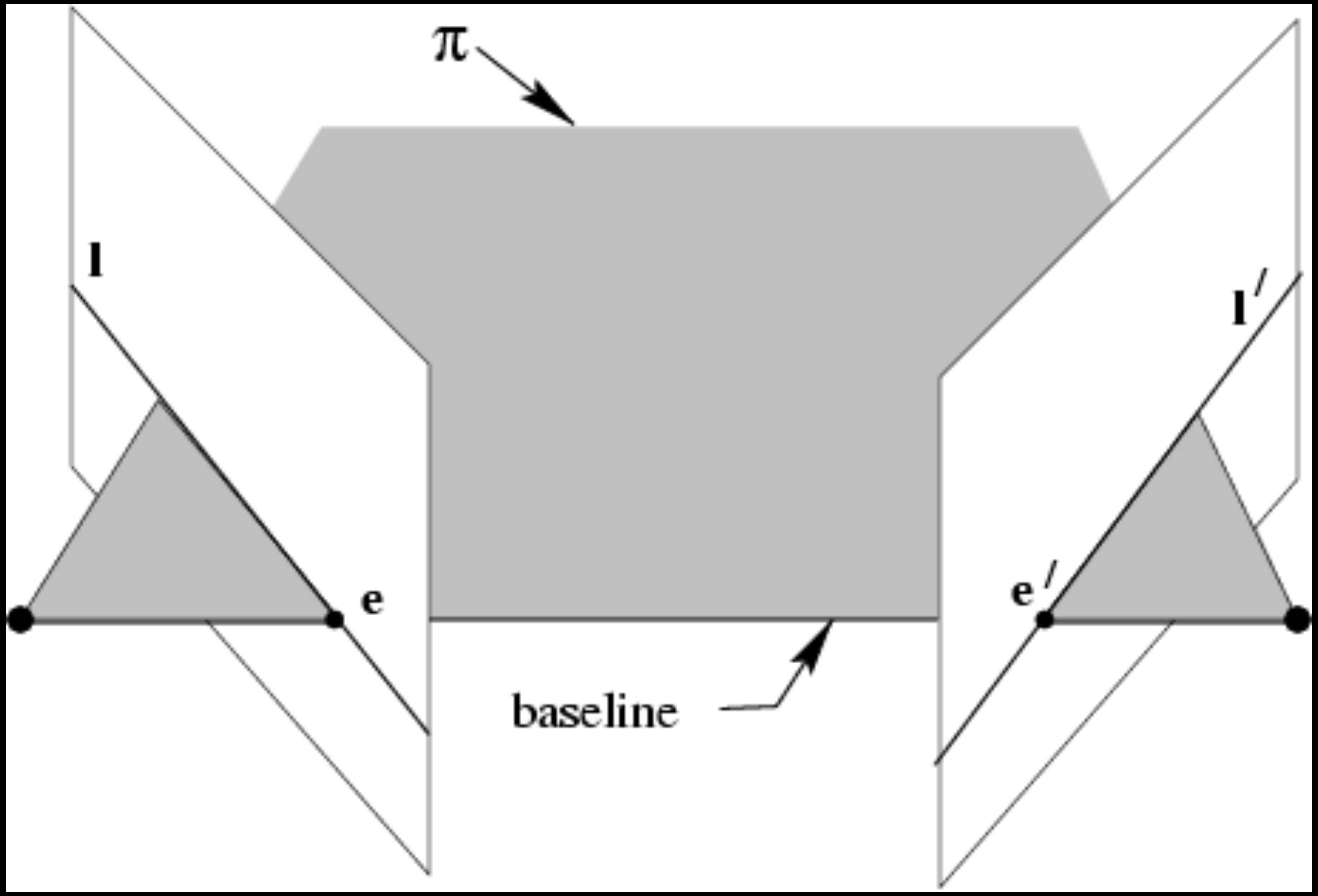


RELATING IMAGES

2 VIEWS

- Epipolar Geometry
- Fundamental Matrix
- Projection Matrix from F

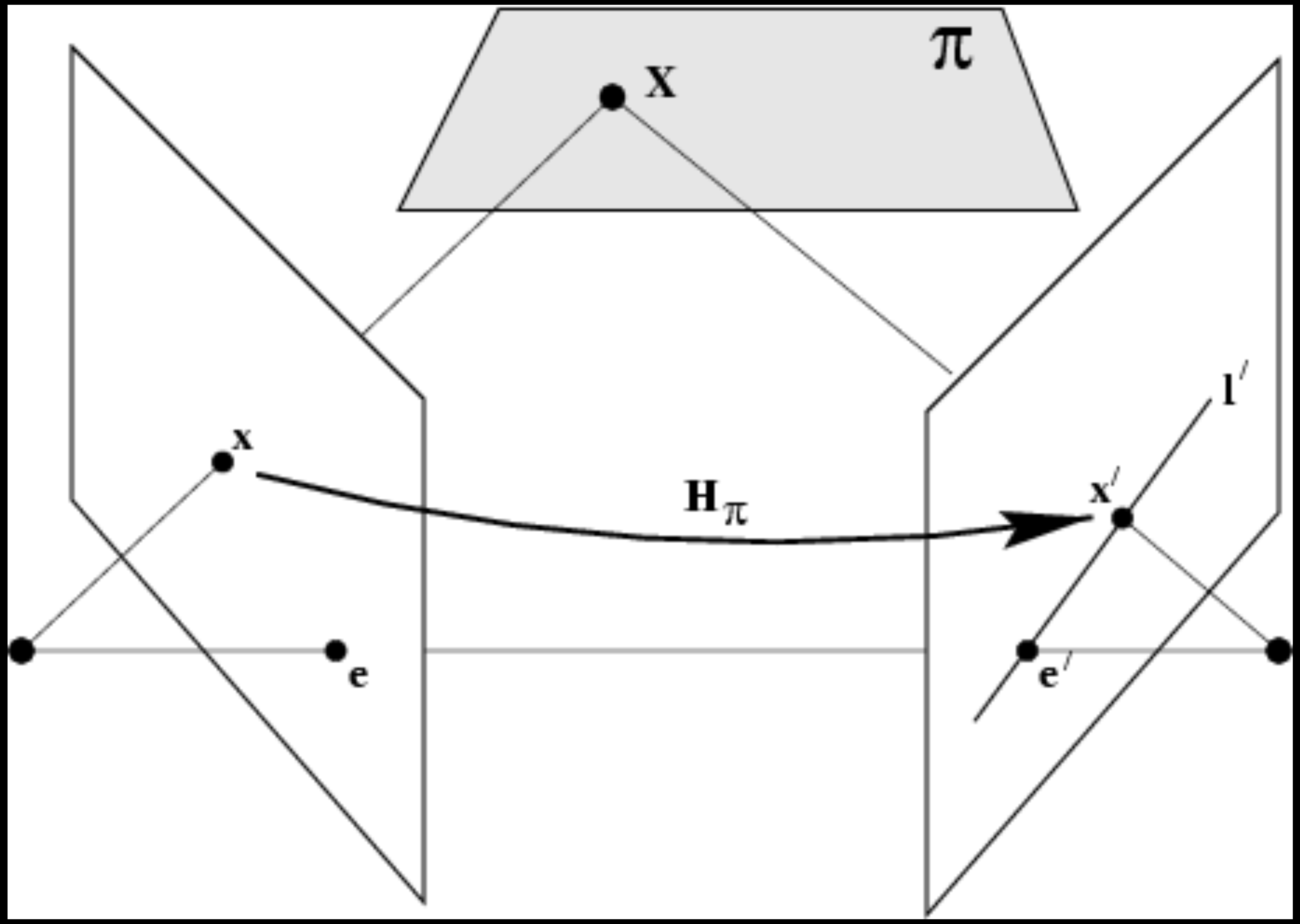
THE EPIPOLAR GEOMETRY



ALL POINTS ON Π PROJECT ON L AND L'

THE FUNDAMENTAL MATRIX F

GEOMETRIC DERIVATION



$$x' = H_\pi x$$

$$l' = e' \times x' = [e']_\times H_\pi x = Fx$$

MAPPING FROM 2-D TO 1-D FAMILY (RANK 2)

HOMOGRAPHY FROM A PLANE TO THE IMAGE

$$\Pi \sim [\pi^\top \, 1]^\top \qquad \mathbf{M}_\Pi \sim [\mathbf{m}_\Pi^\top \, 1]^\top$$

$$0 = \Pi^\top \mathbf{M}_\Pi = \pi^\top \mathbf{m}_\Pi + 1 \qquad \mathbf{M}_\Pi \sim \begin{bmatrix} \mathbf{m}_\Pi \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{m}_\Pi \\ -\pi^\top \mathbf{m}_\Pi \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{3\times3} \\ -\pi^\top \end{bmatrix} \mathbf{m}_\Pi \, .$$

$$\mathbf{P}_i = [\mathbf{A}_i | \mathbf{a}_i]$$

$$\mathbf{m}_{\Pi i} \sim \mathbf{P}_i \mathbf{M}_\Pi = [\mathbf{A}_i | \mathbf{a}_i] \begin{bmatrix} \mathbf{I}_{3\times3} \\ -\pi^\top \end{bmatrix} \mathbf{m}_\Pi = [\mathbf{A}_i - \mathbf{a}_i \pi^\top] \mathbf{m}_\Pi \, .$$

$$\boxed{\mathbf{H}_{\Pi i} \sim \mathbf{A}_i - \mathbf{a}_i \pi^\top}$$

HOMOGRAPHY FROM A PLANE TO THE IMAGE

$$\boxed{\mathbf{H}_{\Pi i} \sim \mathbf{A}_i - \mathbf{a}_i \pi^\top}$$

$$\mathbf{P}_i = [\mathbf{A}_i | \mathbf{a}_i]$$

$$\mathbf{P}_1 = [\mathbf{I}_{3\times3} | 0_3]$$

$$\mathbf{H}^{\Pi}_{ij} = \mathbf{H}_{\Pi j} \mathbf{H}^{-1}_{\Pi i}$$

$$\mathbf{H}_{\Pi 1} \sim \mathbf{I}_{3\times3}$$

$$Pj = \left[\mathbf{H}^{\Pi}_{ij} + \mathbf{a}_i \pi^\top \;|\; a_i \right]$$

HOMOGRAPHY FROM A PLANE TO THE IMAGE

$$P_j = [\mathbf{H}_{ij}^{\Pi} + \mathbf{a}_i \pi^\top \mid a_i]$$

$\mathbf{l}_j$ $\quad \Pi \quad$ $\mathbf{m}_{\Pi i}$ $\quad$ $\mathbf{F}_{ij}\mathbf{m}_{\Pi i}$ $\quad$ $\mathbf{l}_j \times \mathbf{F}_{ij}\mathbf{m}_{\Pi i}$

$\mathbf{H}_{ij}^{\Pi} = [\mathbf{l}_j]_\times \mathbf{F}_{ij}$ $\quad$ $\mathbf{l}_j \sim \mathbf{e}_{ij}$ $\quad$ $\mathbf{e}_{ij}^\top \mathbf{e}_{ij} \neq 0$

CHOOSE A LINE THAT IS NOT COINCIDANT WITH THE EPIPOLAR LINE

$$\begin{aligned} \mathbf{P}_1 &= [\mathbf{I}_{3\times 3} \mid \mathbf{0}_3] \\ \mathbf{P}_2 &= [[\mathbf{e}_{12}]_\times \mathbf{F}_{12} + \mathbf{e}_{12}\pi^\top \mid \mathbf{e}_{12}] \end{aligned}$$

P FROM F

- Has 4 more degrees of freedom

$$
\begin{aligned}
\mathbf{P}_1 &= [\quad \mathbf{I}_{3\times3} \quad | \quad 0_3 \quad] \\
\mathbf{P}_2 &= [\quad [\mathbf{e}_{12}]_\times \mathbf{F}_{12} + \mathbf{e}_{12}\mathbf{a}^\top \quad | \quad \sigma \mathbf{e}_{12} \quad]
\end{aligned}
$$

$$\mathbf{a} = [0\,0\,0]^\top$$

WE CAN GET PROJECTION MATRICES P AND P' UP TO A PROJECTIVE AMBIGUITY

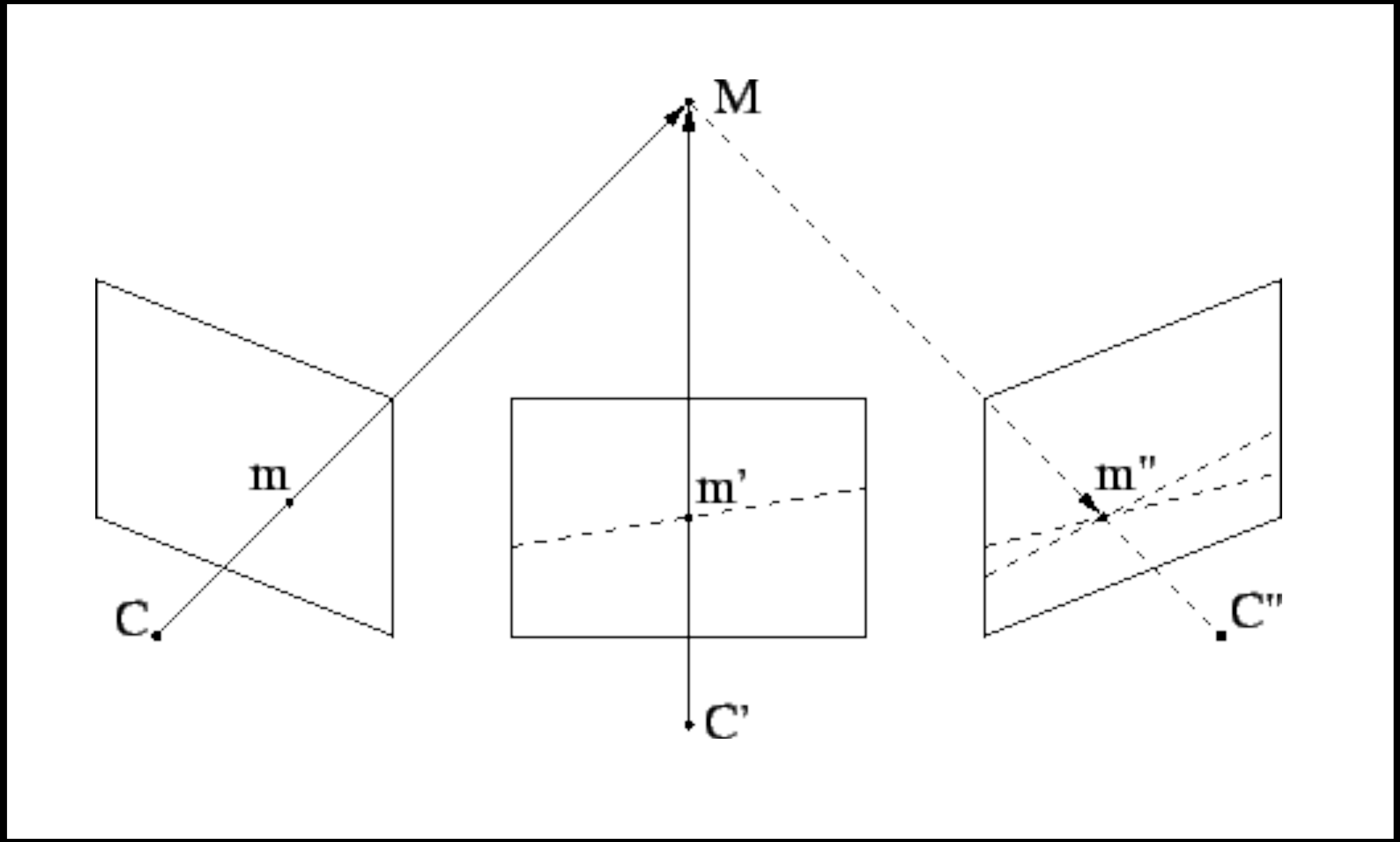
$$\mathbf{P} = [\mathbf{I} \mid \mathbf{0}] \qquad \mathbf{P}' = [\underbrace{[\mathbf{e}']_\times \mathbf{F}}_{\text{K'*rotation}} \mid \underbrace{\mathbf{e}'}_{\text{K'*translation}}] \qquad \mathbf{e}'^T \mathbf{F} = 0$$

SEE HZ P. 255-256

- Code:
- function P = vgg_P_from_F(F)
- [U,S,V] = svd(F);
- e = U(:,3);
- P = [-vgg_contreps(e)*F e];

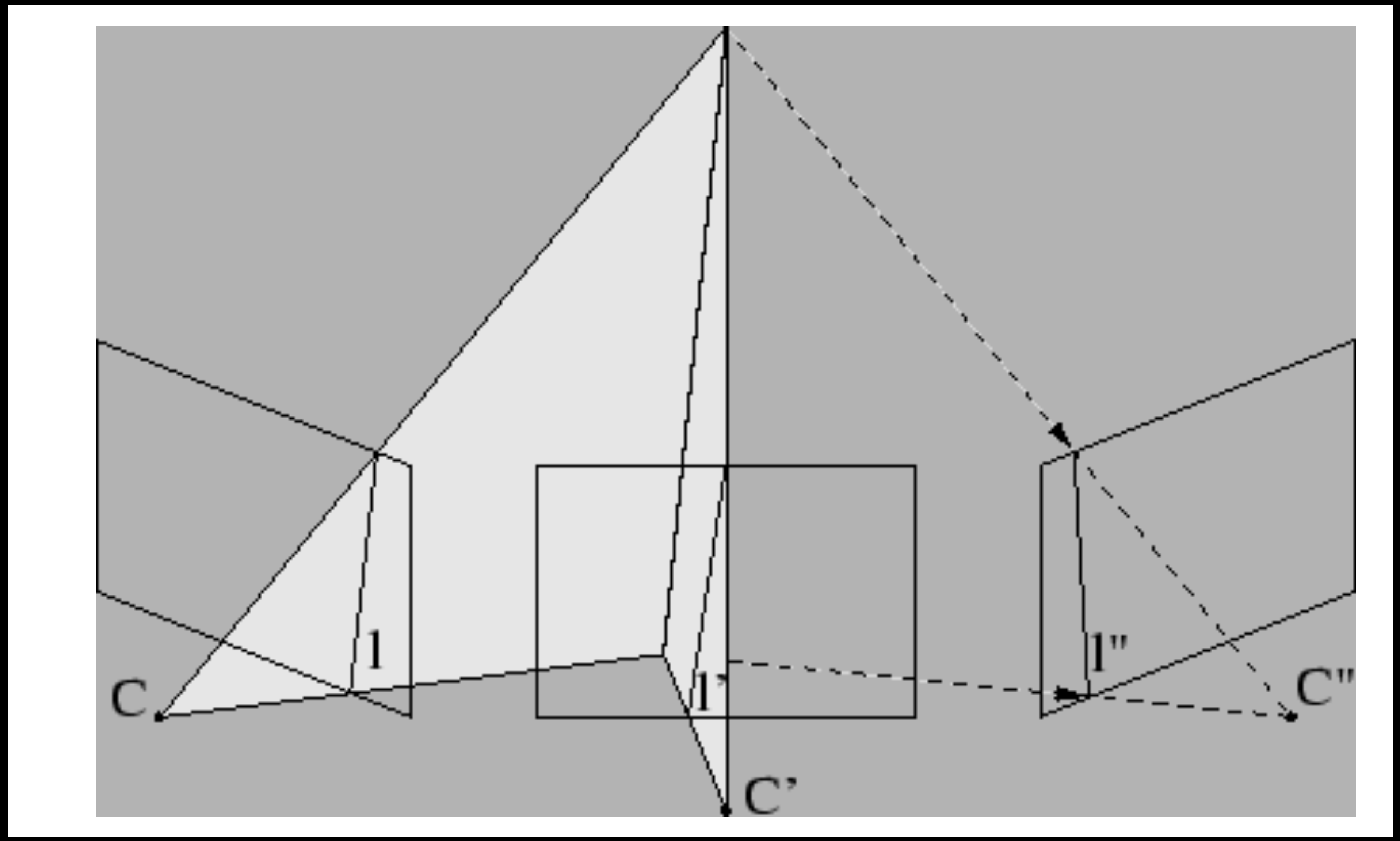
THREE VIEW GEOMETRY

- In the three view case not all the information is described by the epipolar geometry. These additional relationships are described by the trifocal tensor.



THREE VIEW GEOMETRY

- Relation between the image of a line in three images. While in the two view case no constraints are available for lines, in the three view case it is also possible to predict the position of a line in a third image from its projection in the other two. This transfer is also described by the trifocal tensor.



THREE VIEW GEOMETRY

- The trifocal tensor T is a 3 X 3 X 3 tensor. It contains 27 parameters, only 18 of which are independent due to additional nonlinear constraints.

THREE VIEW GEOMETRY

(i) Line–line–line correspondence

$$\mathbf{l}'^{\top}[\mathtt{T}_1, \mathtt{T}_2, \mathtt{T}_3]\mathbf{l}'' = \mathbf{l}^{\top} \quad \text{or} \quad \left(\mathbf{l}'^{\top}[\mathtt{T}_1, \mathtt{T}_2, \mathtt{T}_3]\mathbf{l}''\right)[\mathbf{l}]_{\times} = \mathbf{0}^{\top}$$

(ii) Point–line–line correspondence

$$\mathbf{l}'^{\top}\left(\sum_i x^i \mathtt{T}_i\right)\mathbf{l}'' = 0 \;\; \text{for a correspondence } \mathbf{x} \leftrightarrow \mathbf{l}' \leftrightarrow \mathbf{l}''$$

(iii) Point–line–point correspondence

$$\mathbf{l}'^{\top}\left(\sum_i x^i \mathtt{T}_i\right)[\mathbf{x}'']_{\times} = \mathbf{0}^{\top} \;\; \text{for a correspondence } \mathbf{x} \leftrightarrow \mathbf{l}' \leftrightarrow \mathbf{x}''$$

(iv) Point–point–line correspondence

$$[\mathbf{x}']_{\times}\left(\sum_i x^i \mathtt{T}_i\right)\mathbf{l}'' = \mathbf{0} \;\; \text{for a correspondence } \mathbf{x} \leftrightarrow \mathbf{x}' \leftrightarrow \mathbf{l}''$$

(v) Point–point–point correspondence

$$[\mathbf{x}']_{\times}\left(\sum_i x^i \mathtt{T}_i\right)[\mathbf{x}'']_{\times} = 0_{3\times 3}$$

RECONSTRUCTION FROM A SEQUENCE

SEQUENTIAL STRUCTURE AND MOTION RECOVERY

Initialize structure and motion from 2 views

For each additional view

- Determine pose
- Refine and extend structure

INITIAL STRUCTURE AND MOTION

Epipolar geometry ↔ Projective calibration

$$\mathbf{m}_2^T \mathbf{F} \mathbf{m}_1 = 0$$

$$\mathbf{P}_1 = [\mathbf{I} \quad \mathbf{0}]$$

$$\mathbf{P}_2 = [[\mathbf{e}]_\times \mathbf{F} + \mathbf{e}\mathbf{a}^T \quad \mathbf{e}]$$

compatible with F

Yields correct projective camera setup

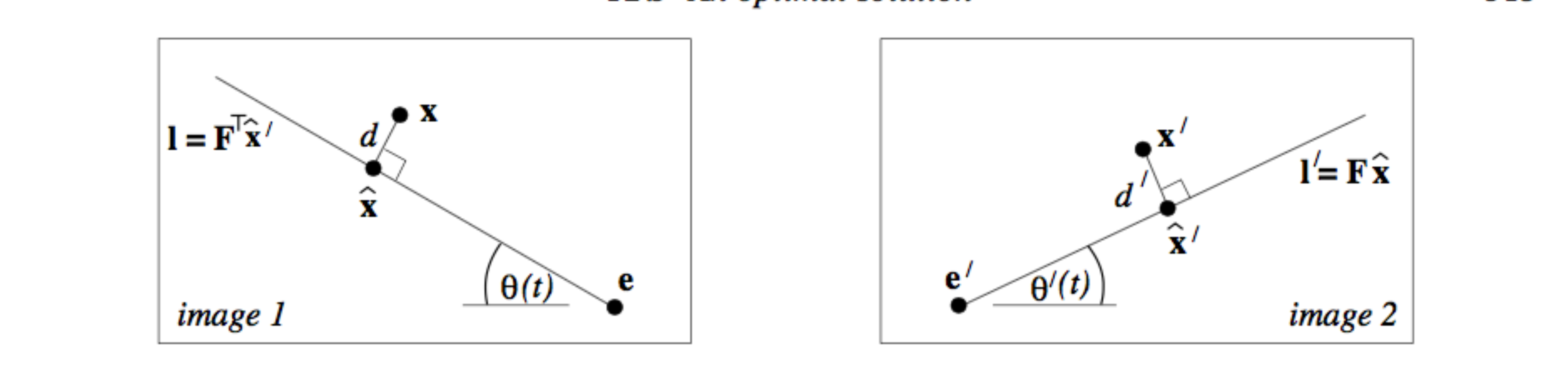
(Faugeras´92,Hartley´92)

INITIAL STRUCTURE AND MOTION

- The first step consists of selecting two views that are suited for initializing the sequential structure and motion computation.
- On the one hand it is important that sufficient features are matched between these views, on the other hand the views should not be too close to each other so that the initial structure is well-conditioned.
- In practice the selection of the initial frame can be done by maximizing the product of the number of matches and a image-based distance based on the planar tomography between images

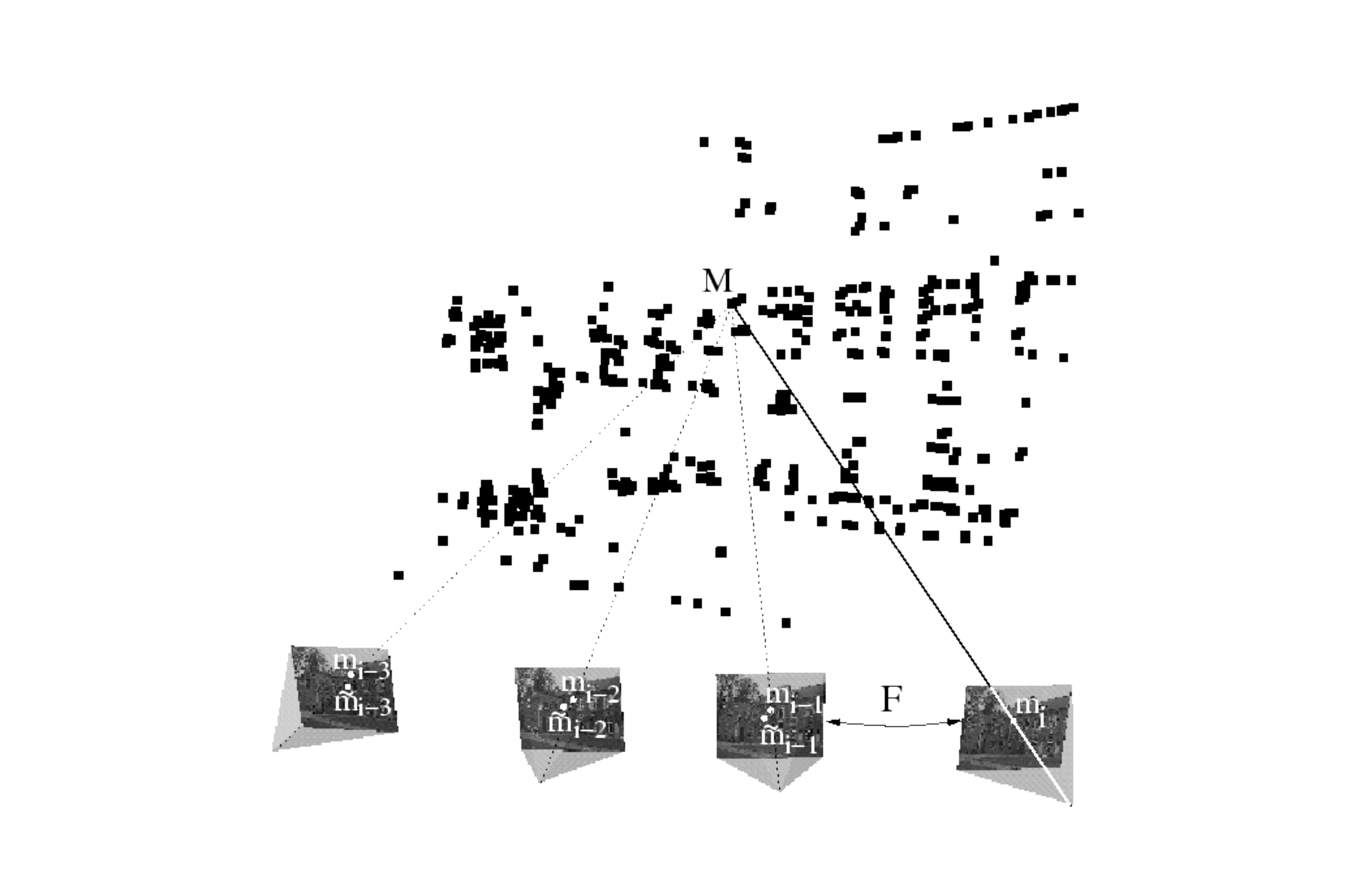
INITIAL STRUCTURE AND MOTION

- Homogeneous method (seen earlier)
- One can pre-adjust points so they satisfy the epipolar constraint.



$$d(\mathbf{x}, \mathbf{l})^2 + d(\mathbf{x}', \mathbf{l}')^2$$

DETERMINE POSE TOWARDS EXISTING STRUCTURE



Compute $\mathbf{P}_{i+1}$ using robust approach (6-point RANSAC)
Extend and refine reconstruction

COMPUTE P WITH 6-POINT RANSAC

- Generate hypothesis using 6 points

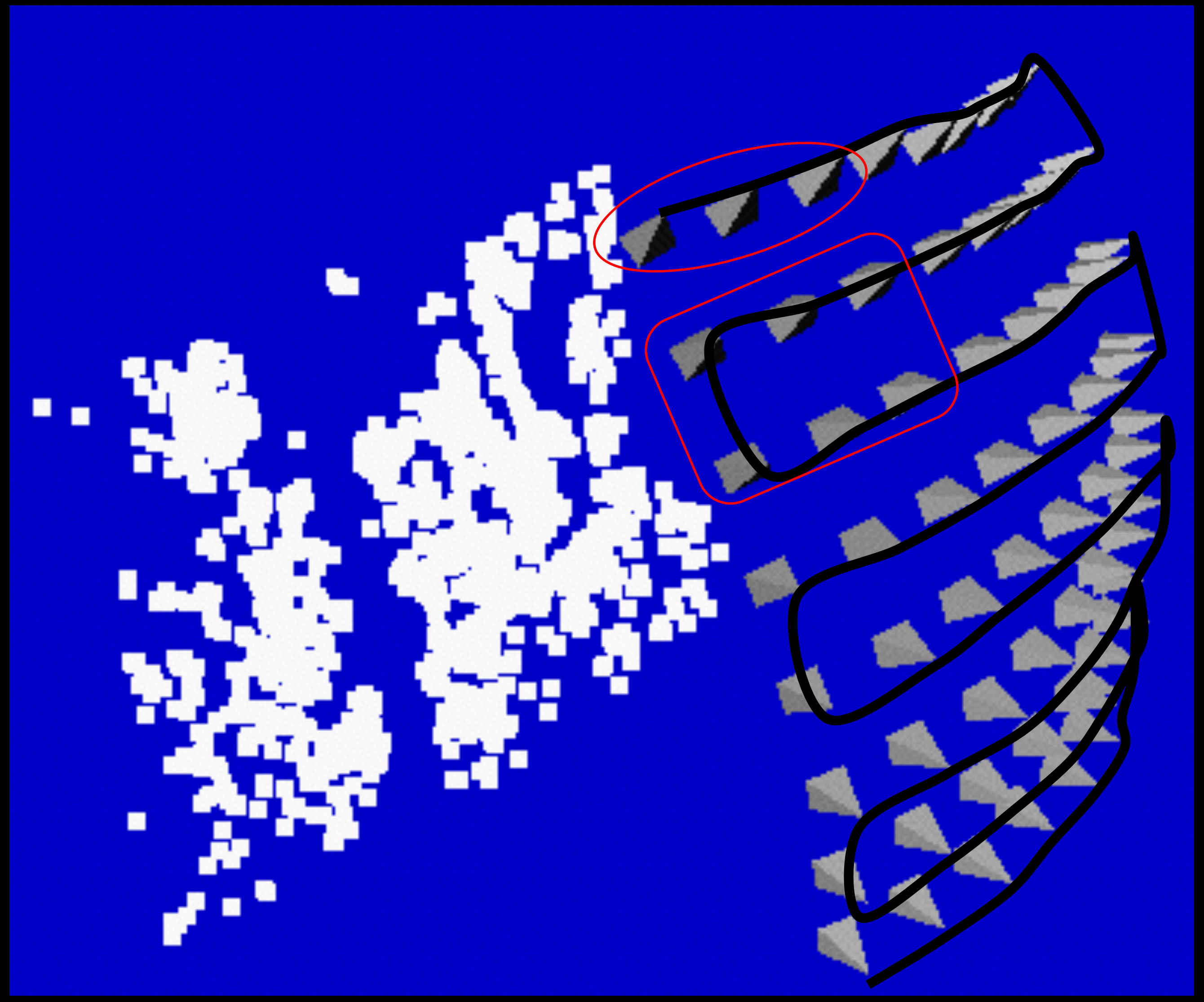
$$\begin{bmatrix} \mathbf{0}^\top & -w_i \mathbf{X}_i{}^\top & y_i \mathbf{X}_i{}^\top \\ w_i \mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i \mathbf{X}_i{}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = 0$$

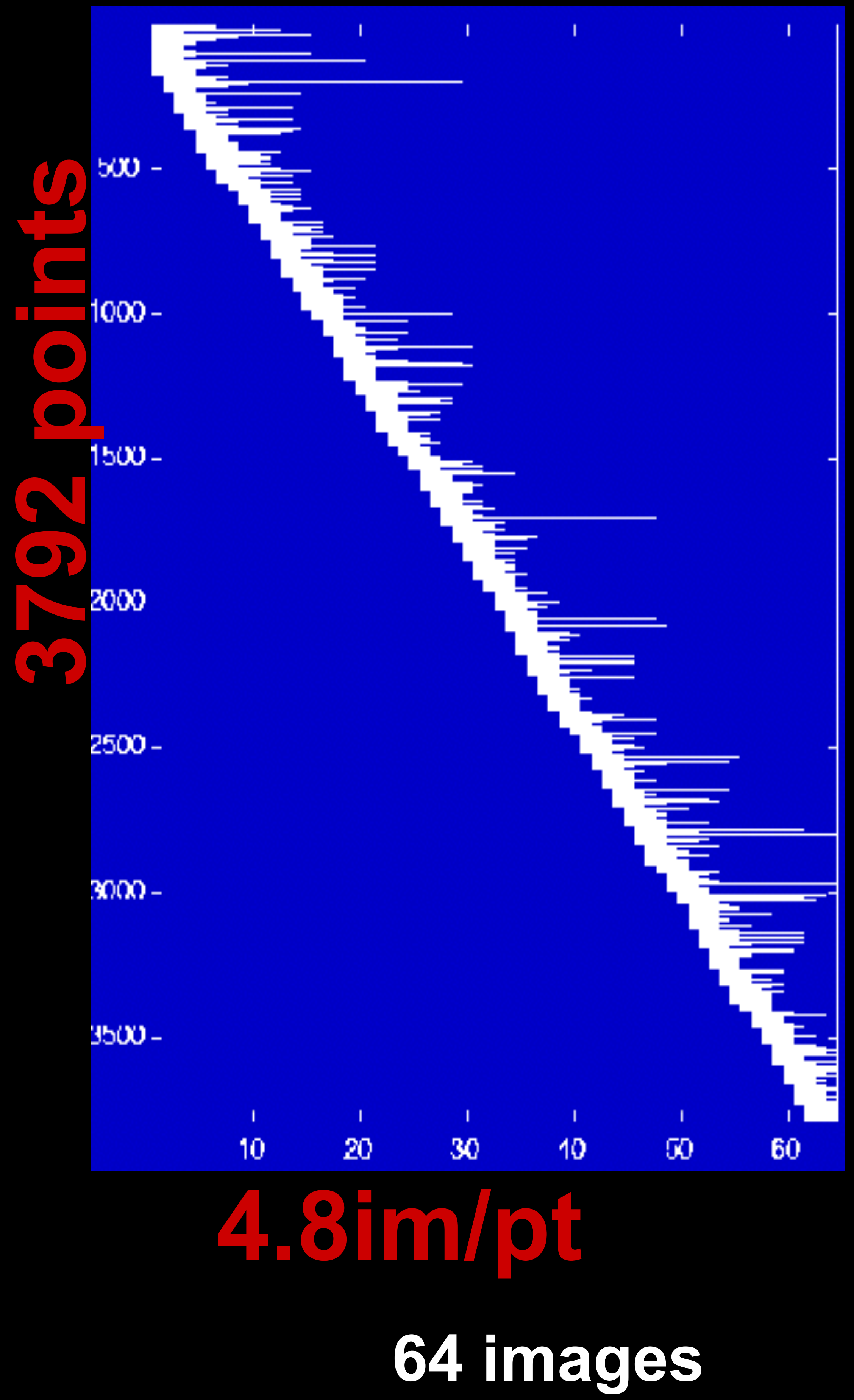
- Count inliers
  - Re-projection error $d\left(\mathrm{P}_i \mathrm{X}\left(\mathrm{x}_1, \ldots, \mathrm{x}_{i-1}, \mathrm{x}_i\right), \mathrm{x}_i\right) < t$

- Expensive testing? Abort early if not promising
  - Also verify at random, abort if e.g. P(wrong)>0.95

(Chum and Matas, BMVC'02)

NON-SEQUENTIAL IMAGE COLLECTIONS





Problem:
Features are lost and reinitialized as new features

Solution:
Match with other *close* views

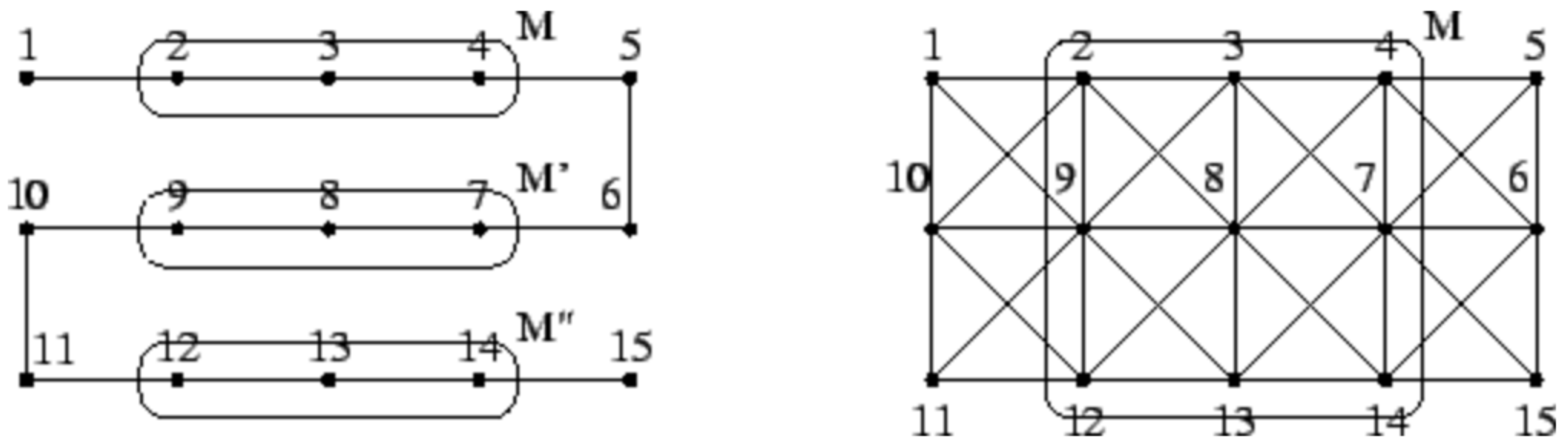


Figure 5.2: *Sequential approach (left) and extended approach (right). In the traditional scheme view 8 would be matched with view 7 and 9 only. A point* `M` *which would be visible in views 2,3,4,7,8,9,12,13 and 14 would therefore result in 3 independently reconstructed points. With the extended approach only one point will be instantiated. It is clear that this results in a higher accuracy for the reconstructed point while it also dramatically reduces the accumulation of calibration errors.*

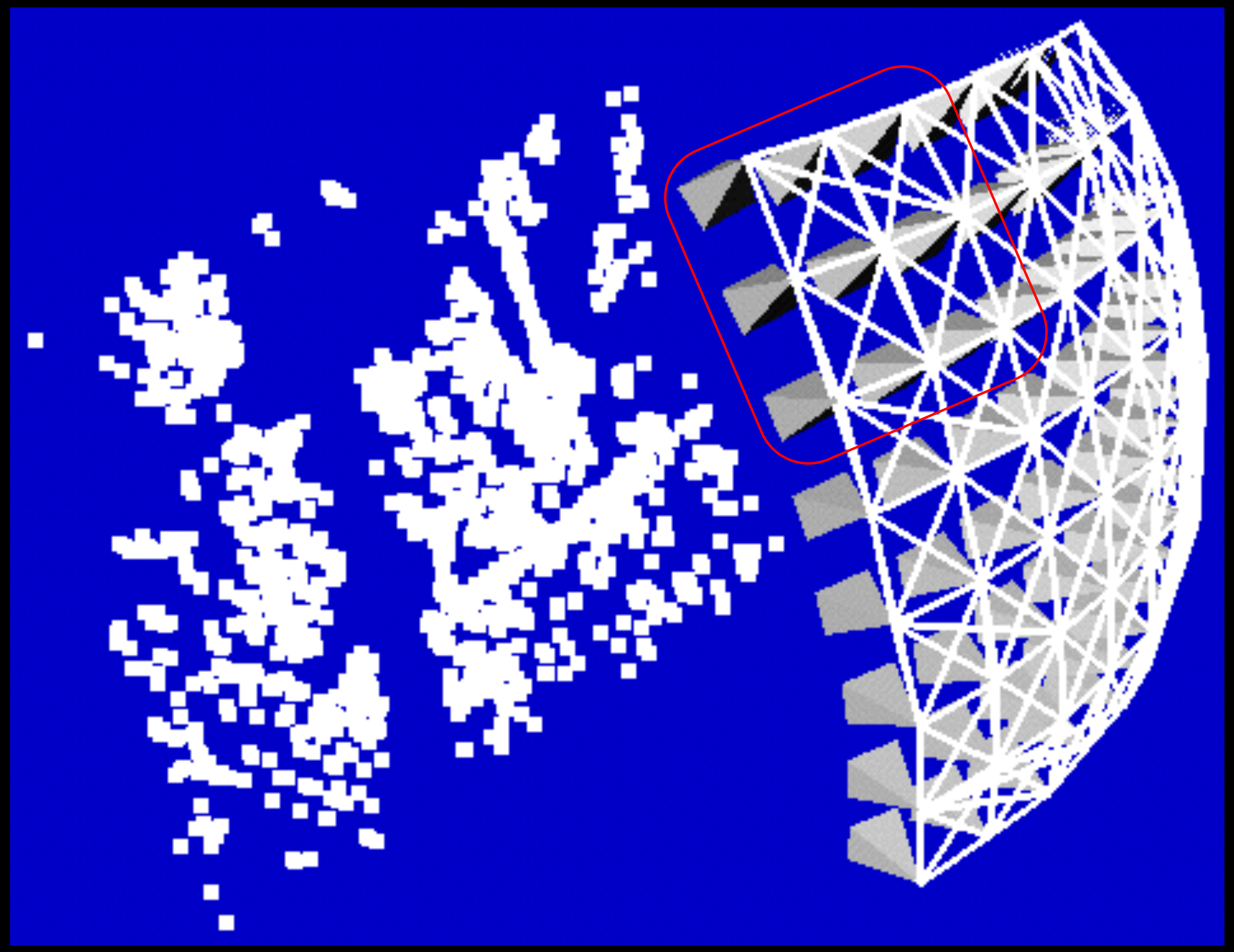
DETERMINING CLOSE VIEWS

- If viewpoints are close then most image changes can be modelled through a planar homography
- Qualitative distance measure is obtained by looking at the residual error on the best possible planar homography

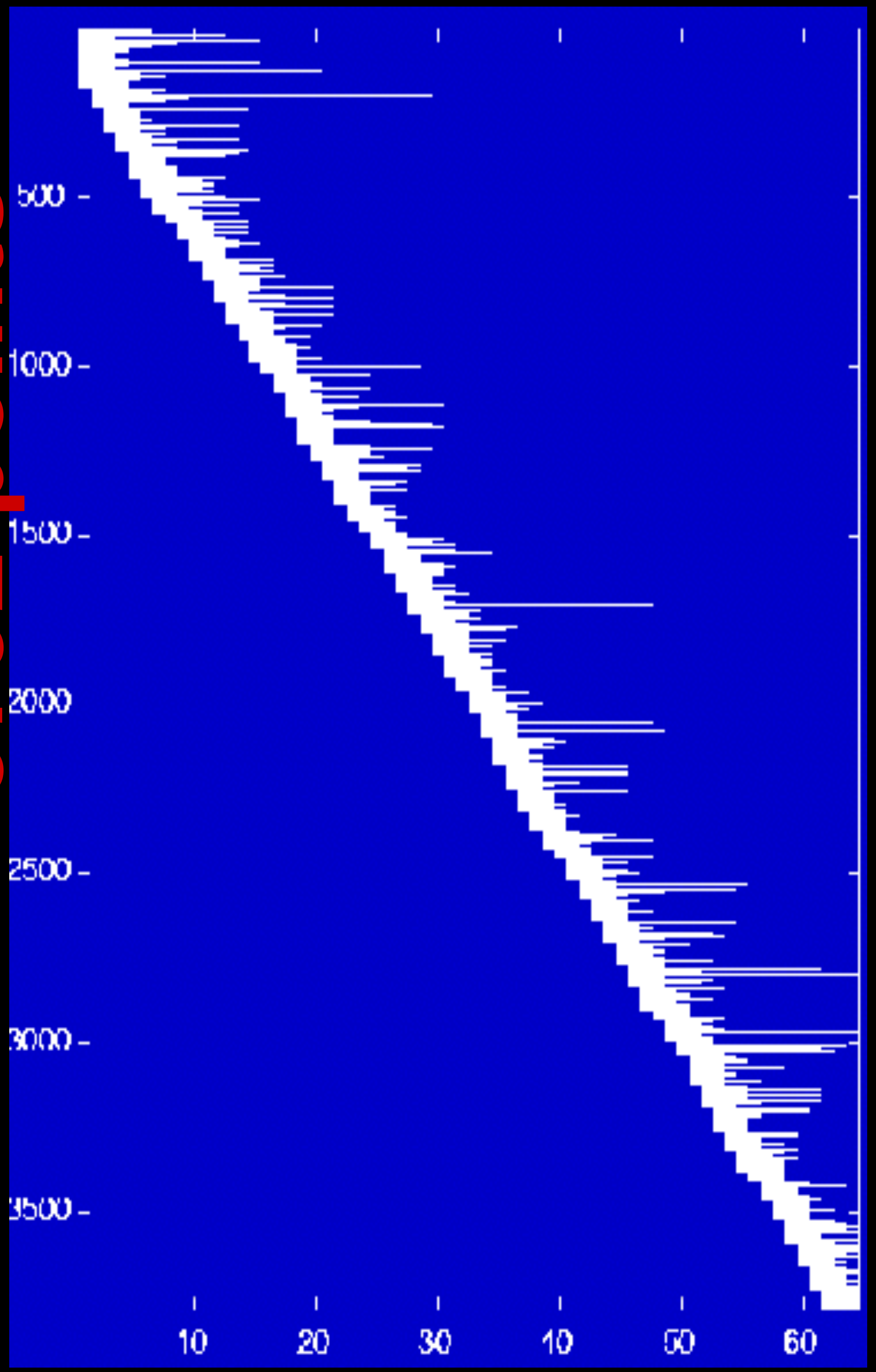
Distance = $\min \operatorname{median} D\left(\mathbf{H}\mathrm{m}, \mathrm{m}'\right)$

$$\mathbf{H} = [\mathbf{e}]_\times \mathbf{F} + \mathbf{e}\mathbf{a}_{min}^\top \text{ with } \mathbf{a}_{min} = \operatorname{argmin}_{\mathbf{a}} \sum_i D(([\mathbf{e}]_\times \mathbf{F} + \mathbf{e}\mathbf{a}^\top)\mathbf{m}_i, \mathbf{m}'_i)^2$$

NON-SEQUENTIAL IMAGE COLLECTIONS (2)



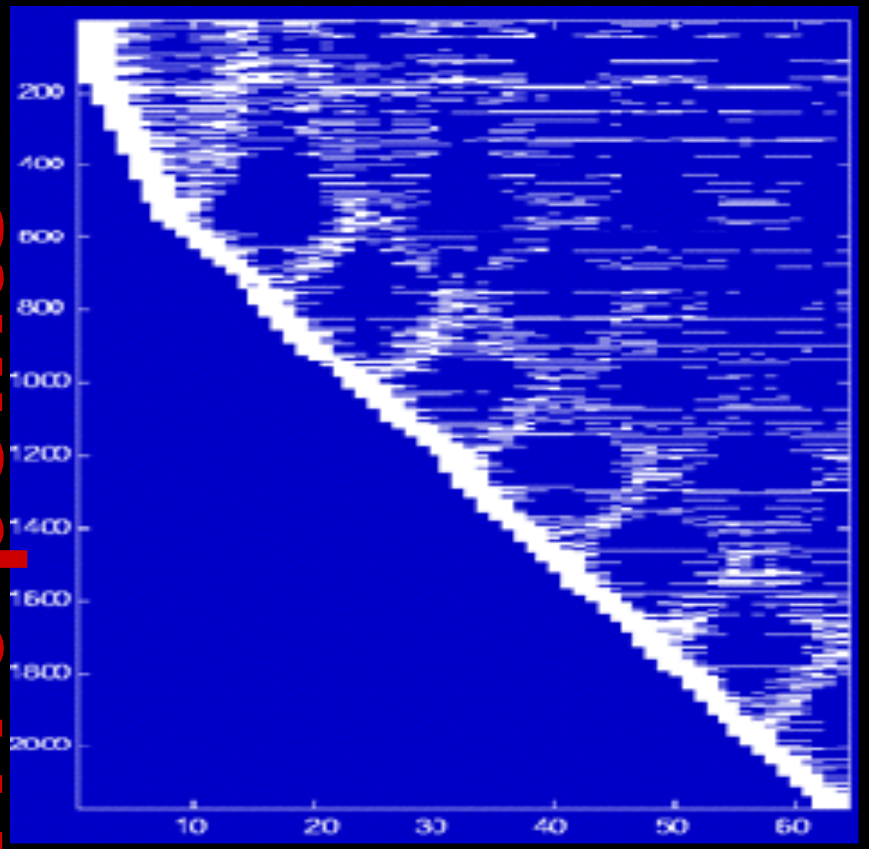
3792 points



4.8im/pt

64 images

2170 points



9.8im/pt

64 images

REFINING GEOMETRY

- By solving this system of equations through SVD a normalized homogeneous point is automatically obtained. If a 3D point is not observed the position is not updated

$$\frac{1}{\mathrm{P}_3\tilde{\mathrm{M}}}\begin{bmatrix} \mathrm{P}_3 x - \mathrm{P}_1 \\ \mathrm{P}_3 y - \mathrm{P}_2 \end{bmatrix}\mathrm{M} = 0$$

REFINING GEOMETRY

- One can check if the point was seen in a sufficient number of views to be kept in the final reconstruction.
- This minimum number of views can for example be put to three. This avoids to have an important number of outliers due to spurious matches.
- Of course in an image sequence some new features will appear in every new image. If point matches are available that were not related to an existing point in the structure, then a new point can be initialized

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Step 1. Match or track points over the whole image sequence.

Step 2. Initialize the structure and motion recovery

- step 2.1. Select two views that are suited for initialization.
- step 2.2. Relate these views by computing the two view geometry.
- step 2.3. Set up the initial frame.
- step 2.4. Reconstruct the initial structure.

Step 3. For every additional view

- step 3.1. Infer matches to the structure and compute the camera pose using a robust algorithm.
- step 3.2. Refine the existing structure.
- step 3.3. (optional) For already computed views which are "close"
  - 3.4.1. Relate this view with the current view by finding feature matches and computing the two view geometry.
  - 3.4.2. Infer new matches to the structure based on the computed matches and add these to the list used in step 3.1.

  Refine the pose from all the matches using a robust algorithm.
- step 3.5. Initialize new structure points.

Step 4. Refine the structure and motion through bundle adjustment.