

COMPUTER VISION AND  
PHOTOGRAMMETRY

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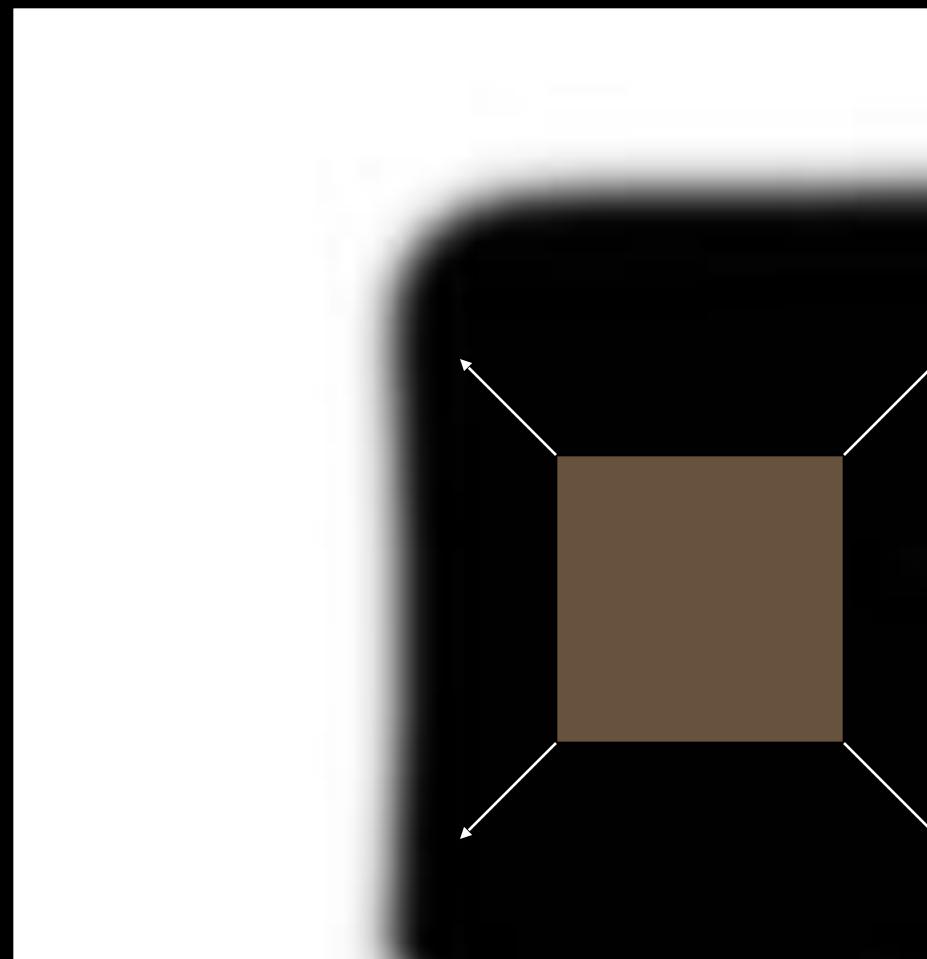
FEATURE DESCRIPTORS AND  
MATCHING

# LAST WEEK

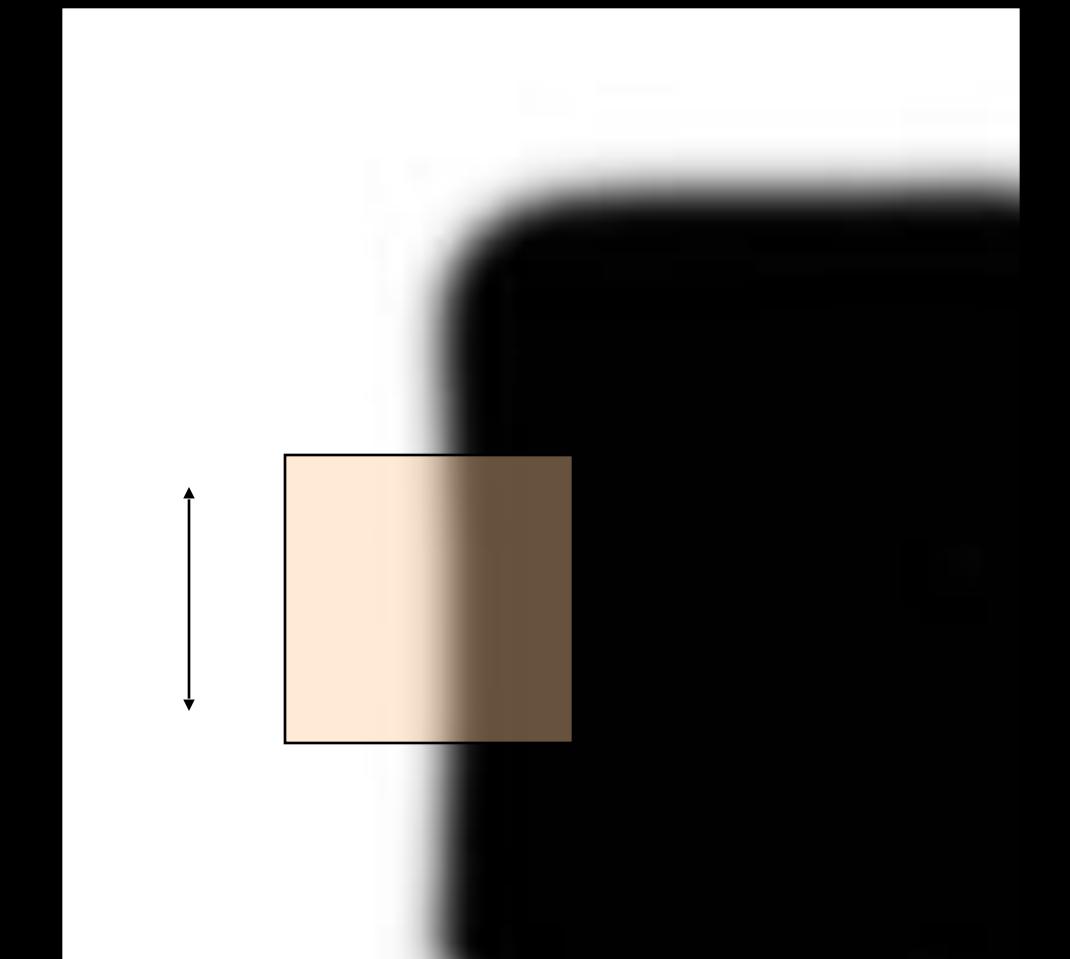
- ▶ General
- ▶ Keypoint detection
- ▶ Corners (Harris)
- ▶ Scale invariance: LoG and DoG, Harris-Laplace

## CORNER DETECTION: BASIC IDEA

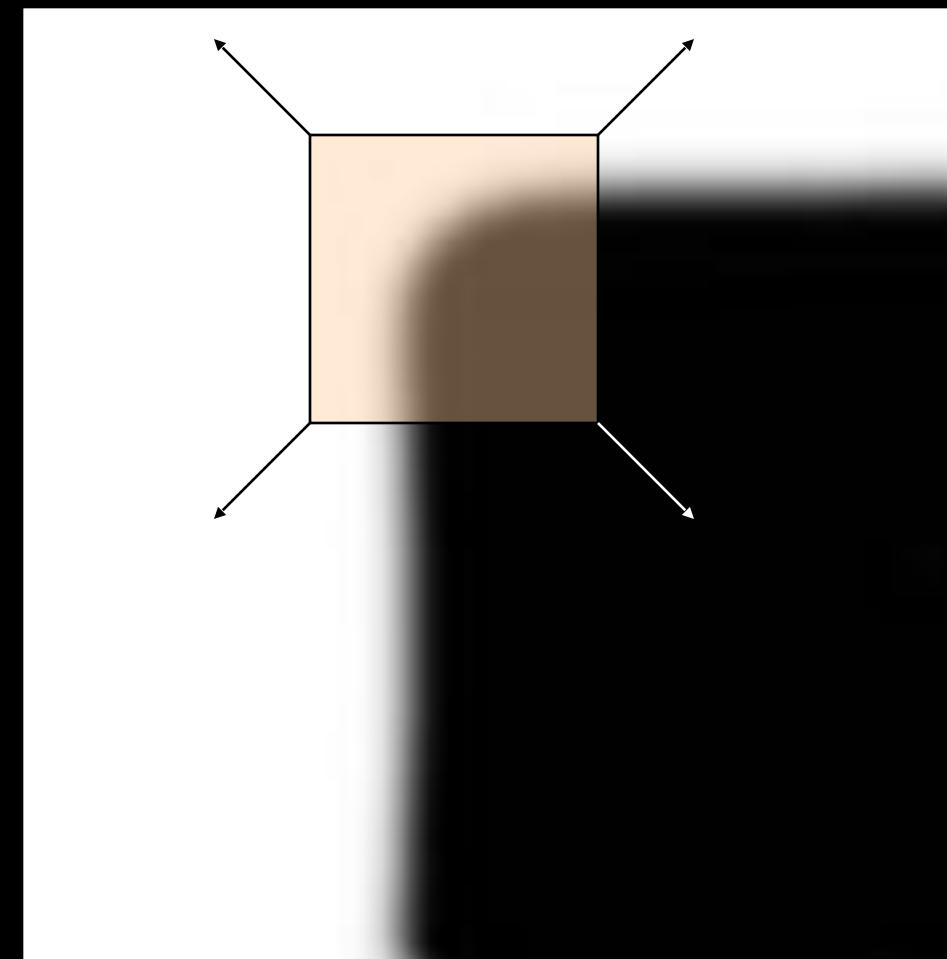
- ▶ We should easily recognize the point by looking through a small window
- ▶ Shifting a window in *any direction* should give *a large change* in intensity



“FLAT” REGION:  
NO CHANGE IN ALL  
DIRECTIONS



“EDGE”:  
NO CHANGE ALONG THE EDGE  
DIRECTION



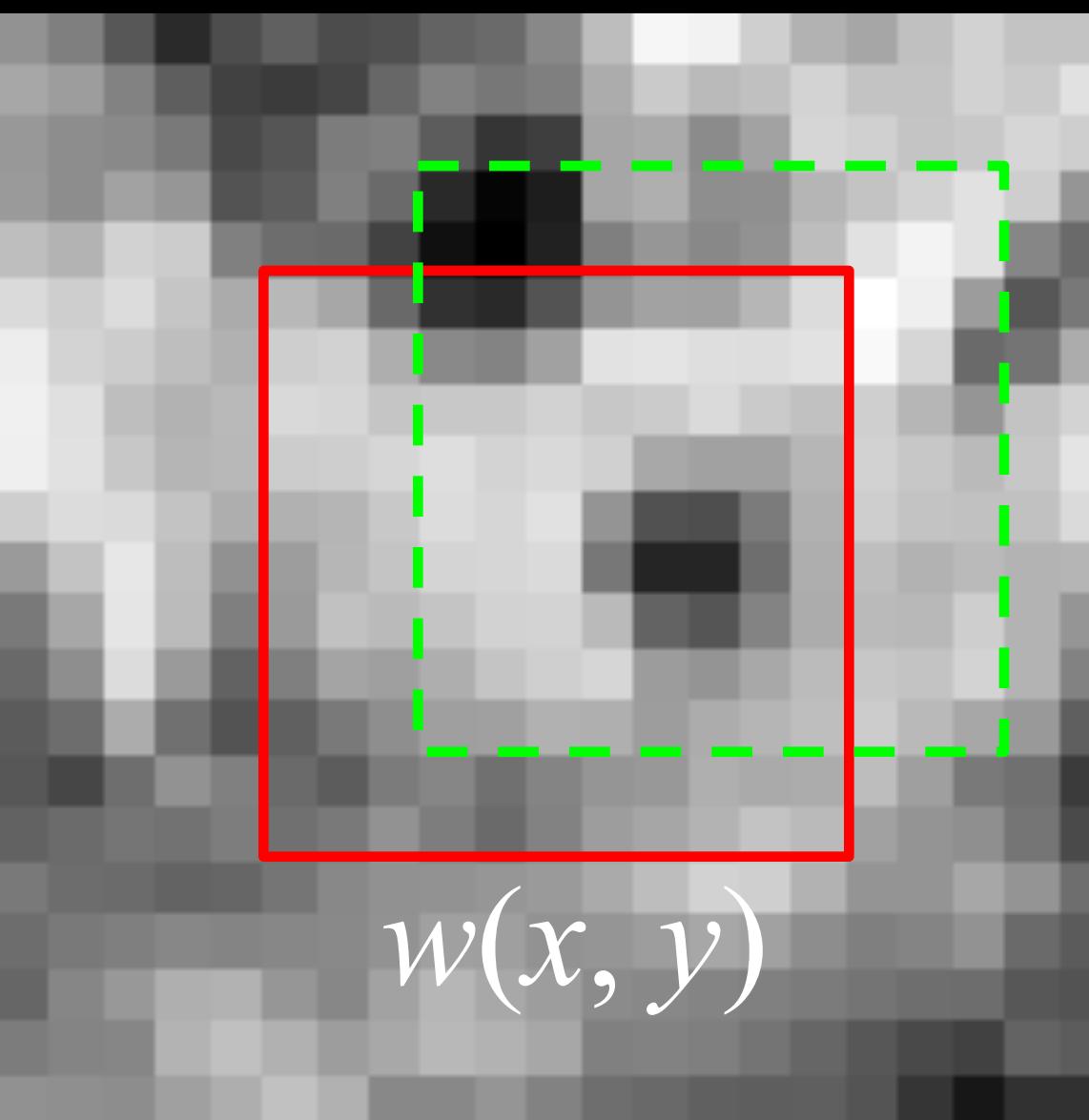
“CORNER”:  
SIGNIFICANT CHANGE IN ALL  
DIRECTIONS

## CORNER DETECTION: MATHEMATICS

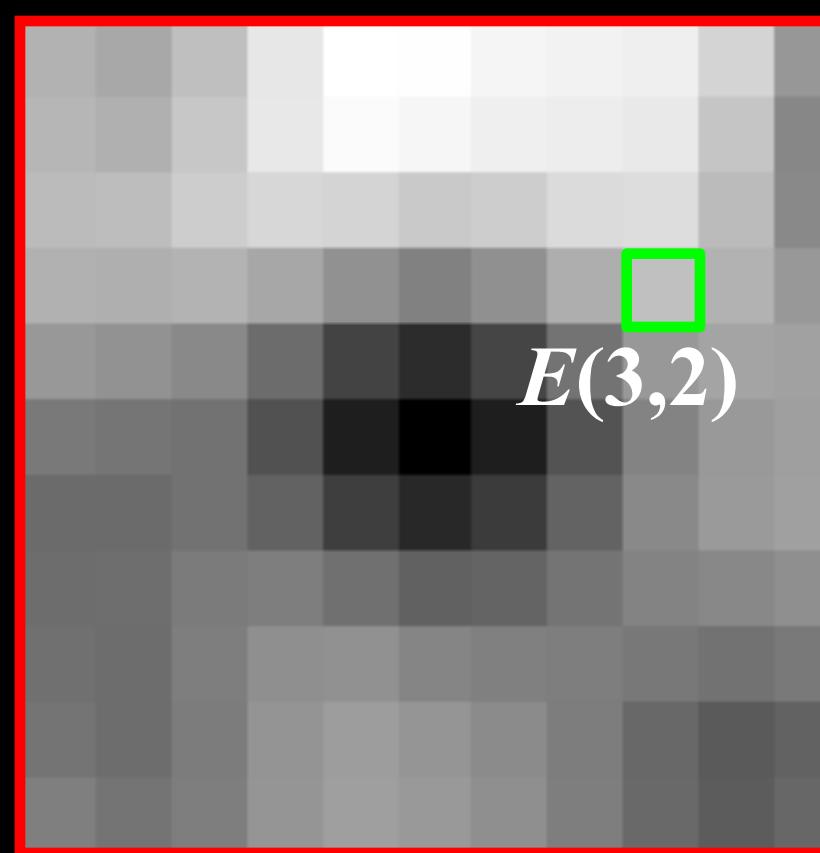
- ▶ Change in appearance of window  $w(x,y)$  for the shift  $[u,v]$ :

$$E(u, v) = \sum_{x,y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

$I(x, y)$



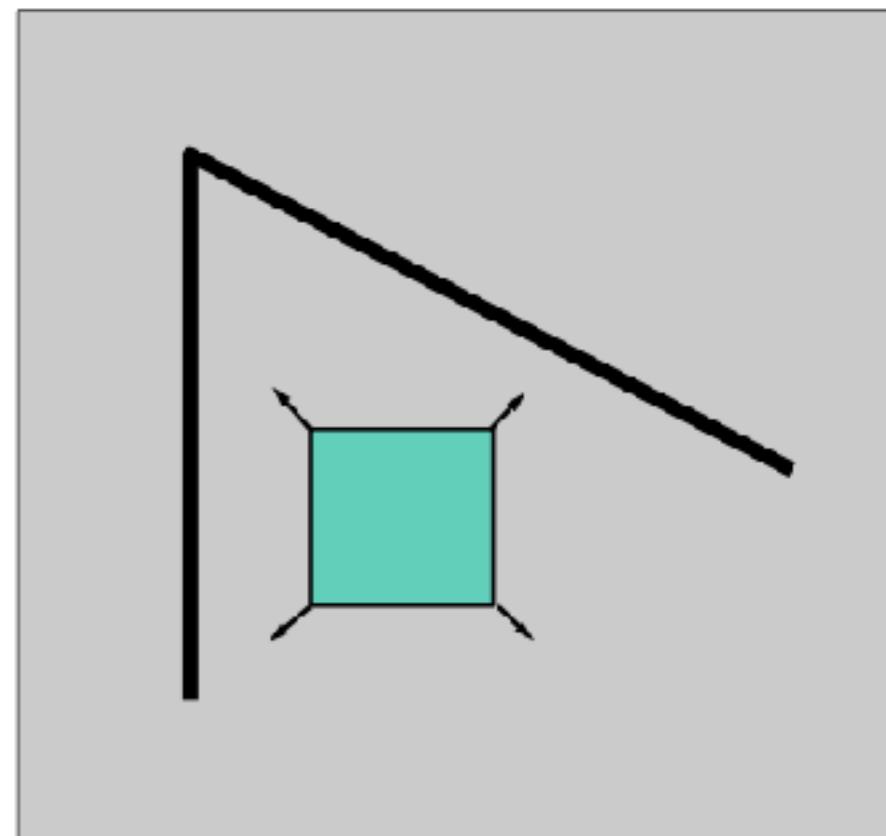
$E(u, v)$



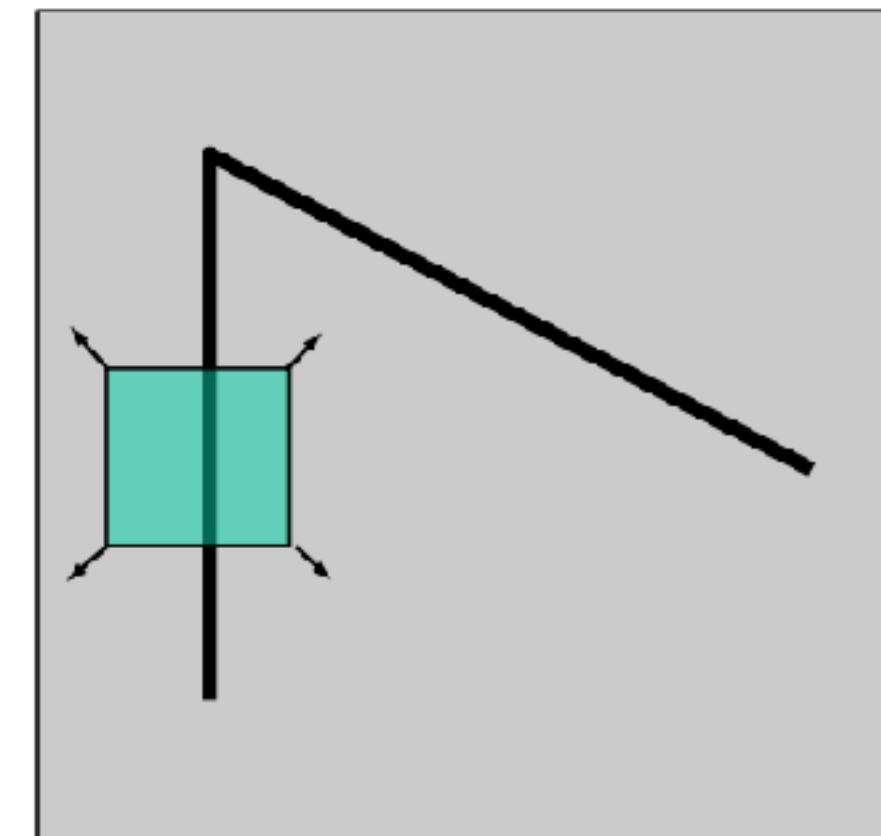
## HARRIS CORNER MATH

Local measure of feature uniqueness

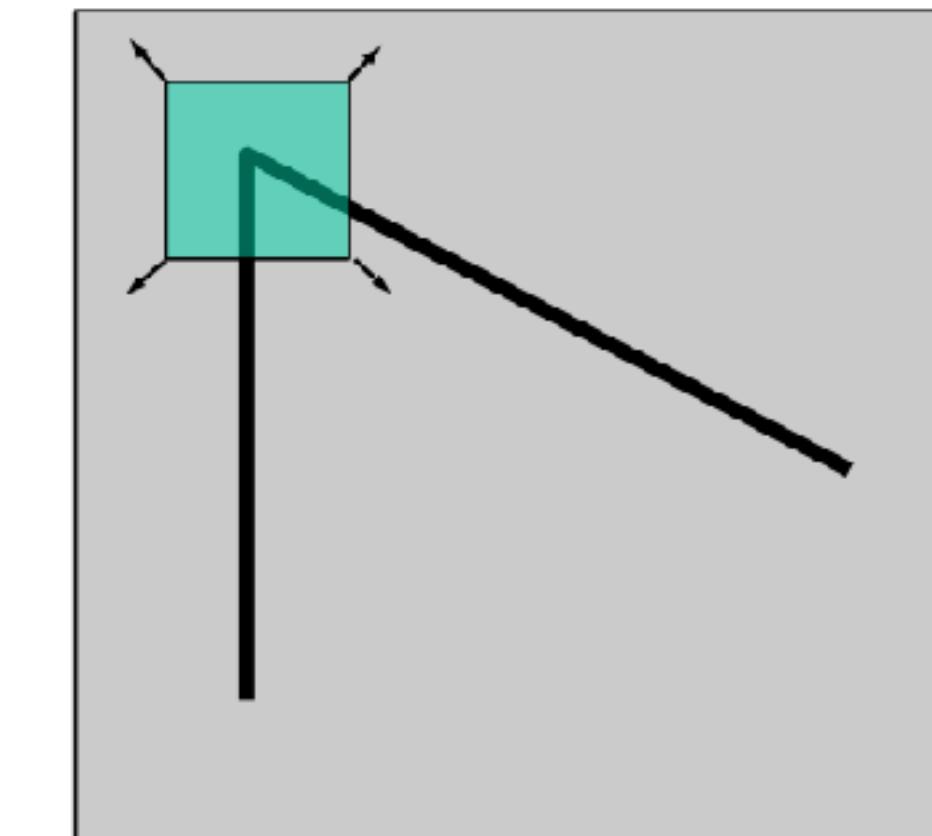
- $E(u,v)$  = amount of change when you shift the window by  $(u,v)$



$E(u,v)$  is small  
for **all** shifts



$E(u,v)$  is small  
for **some** shifts



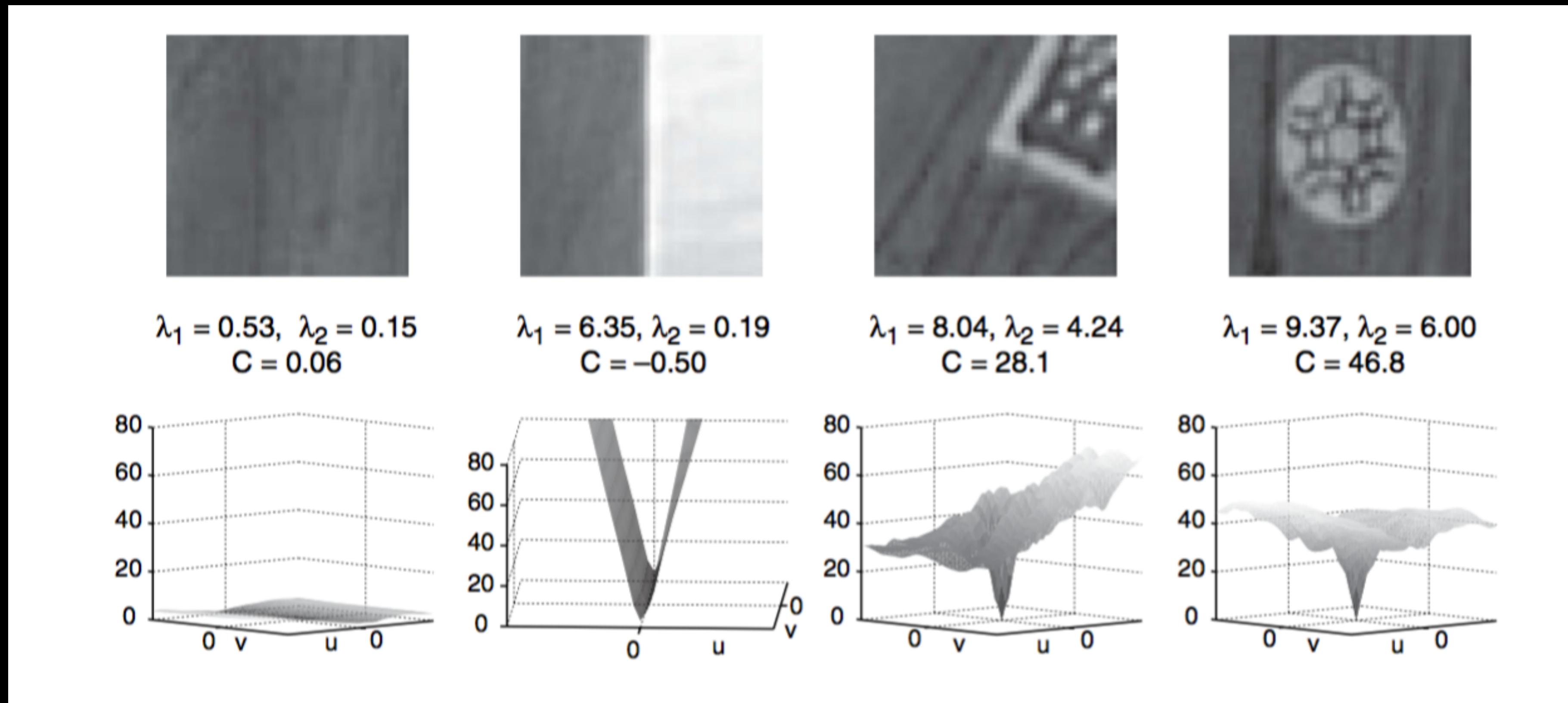
$E(u,v)$  is small  
for **no** shifts

We want  $\min_{(u,v)} E(u,v)$  to be large

TEXT

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## HARRIS CORNER MATH

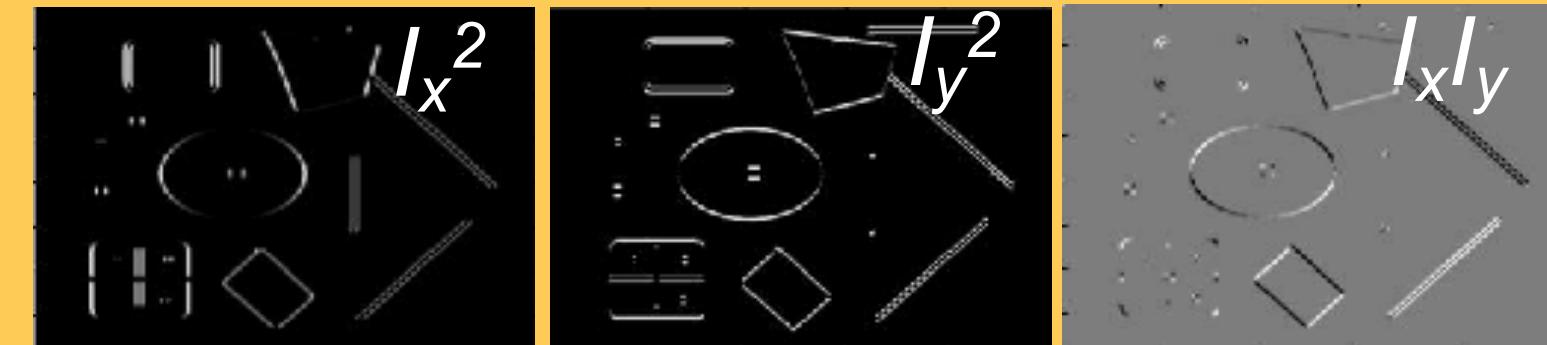
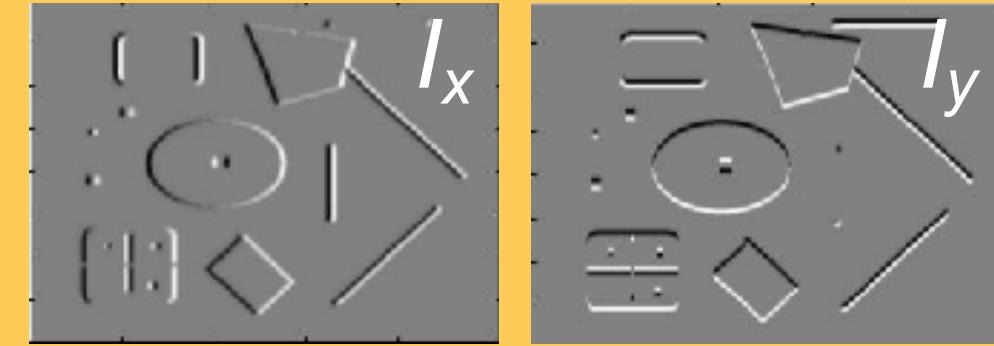
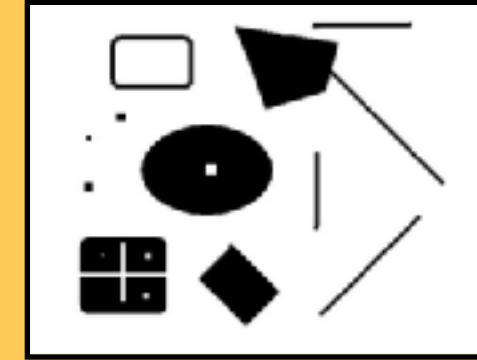


# Harris Detector [Harris88]

- Second moment matrix

$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives  
(optionally, blur first)



2. Square of derivatives

3. Gaussian filter  $g(\sigma_I)$



$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

4. Cornerness function – both eigenvalues are strong

$$\begin{aligned} har &= \det[\mu(\sigma_I, \sigma_D)] - \alpha [\text{trace}(\mu(\sigma_I, \sigma_D))^2] = \\ &= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2 \end{aligned}$$

5. Non-maxima suppression

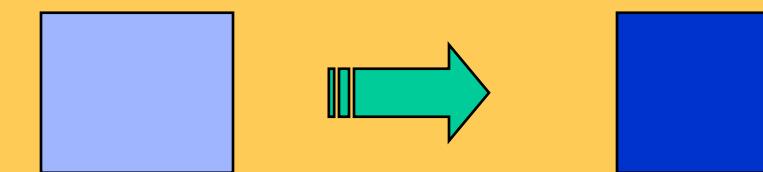


## INvariance AND COVARIANCE

- ▶ We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
- ▶ **Invariance:** image is transformed and corner locations do not change
- ▶ **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

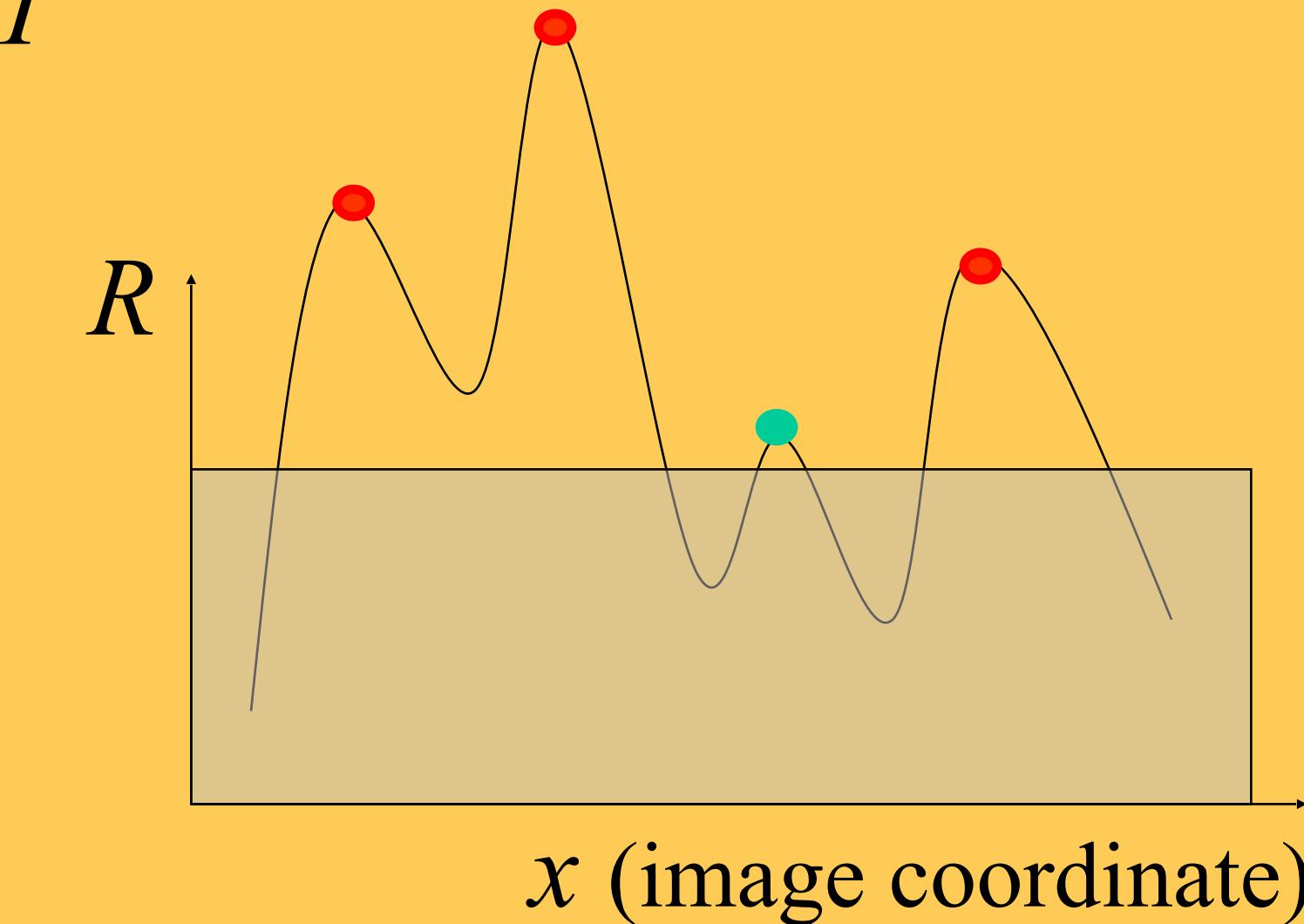
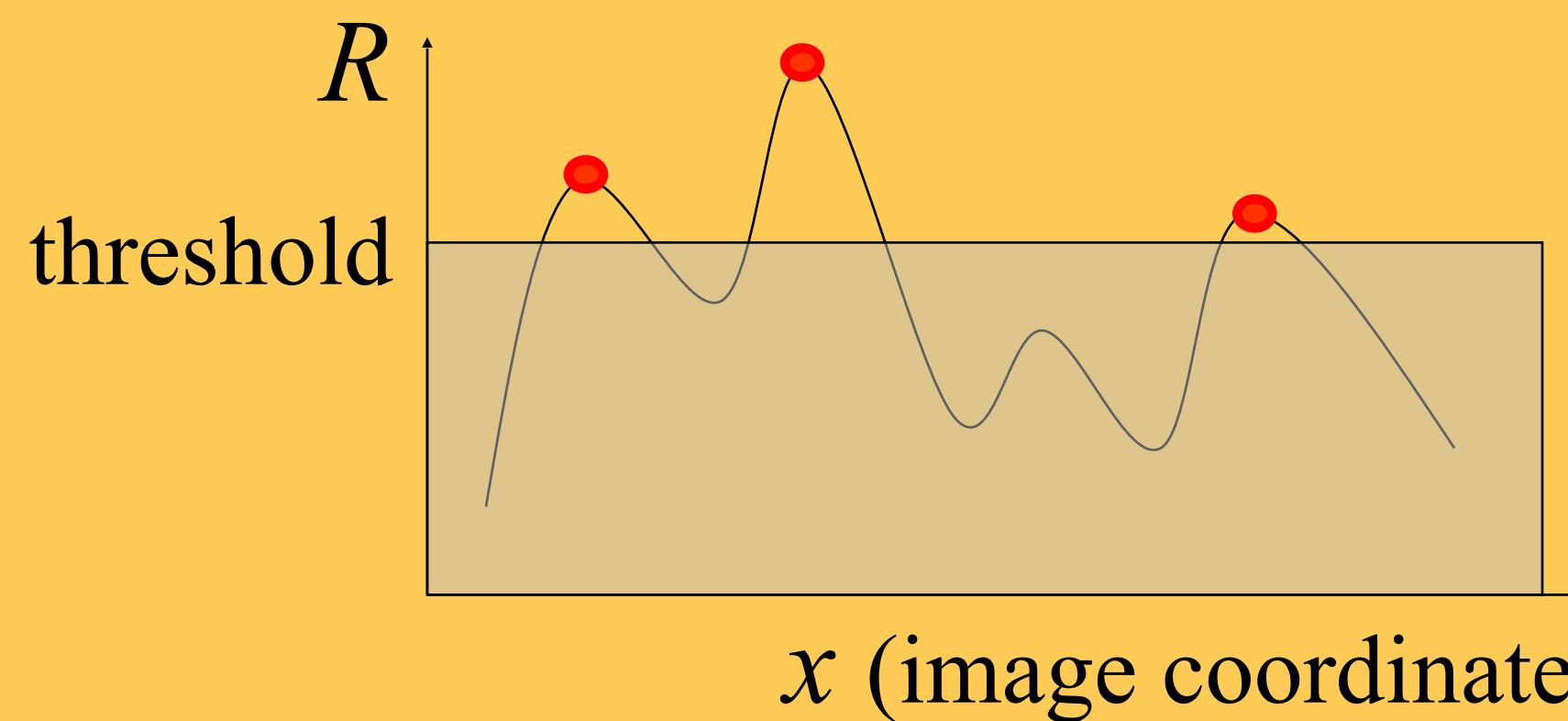


# AFFINE INTENSITY CHANGE



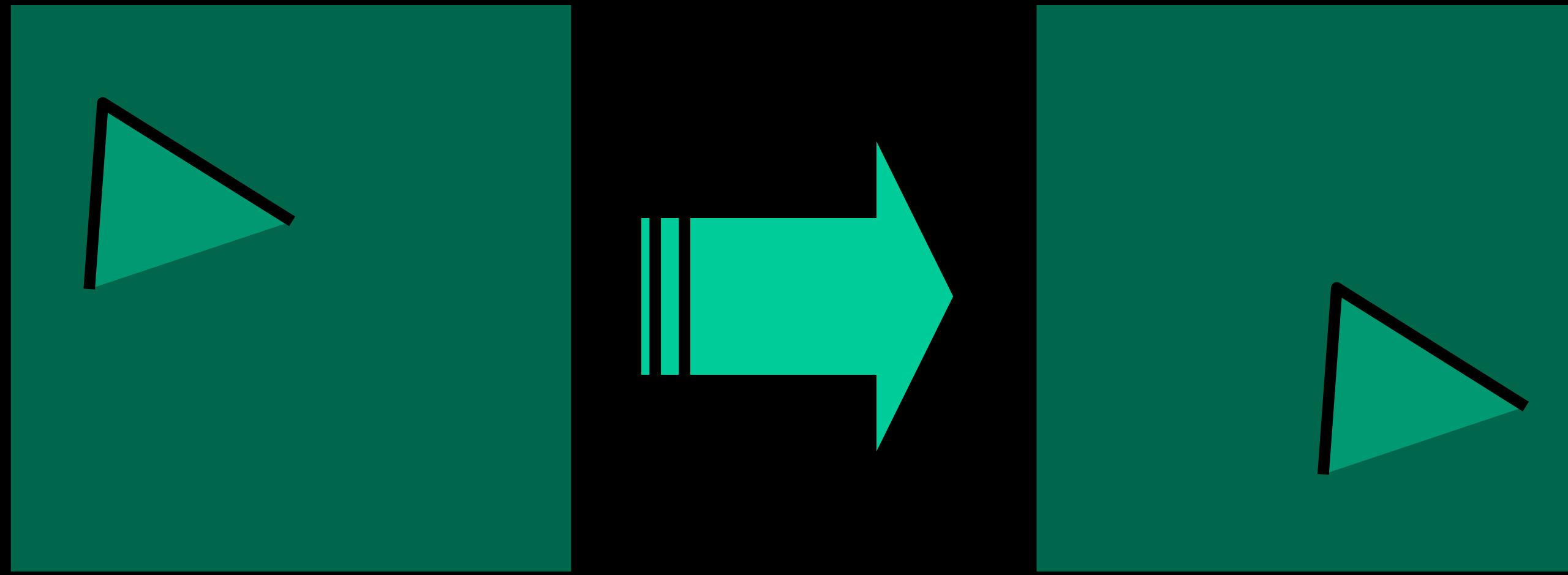
$$I \rightarrow a I + b$$

- Only derivatives are used => invariance to intensity shift  $I \rightarrow I + b$
- Intensity scaling:  $I \rightarrow a I$



*Partially invariant to affine intensity change*

## IMAGE TRANSLATION



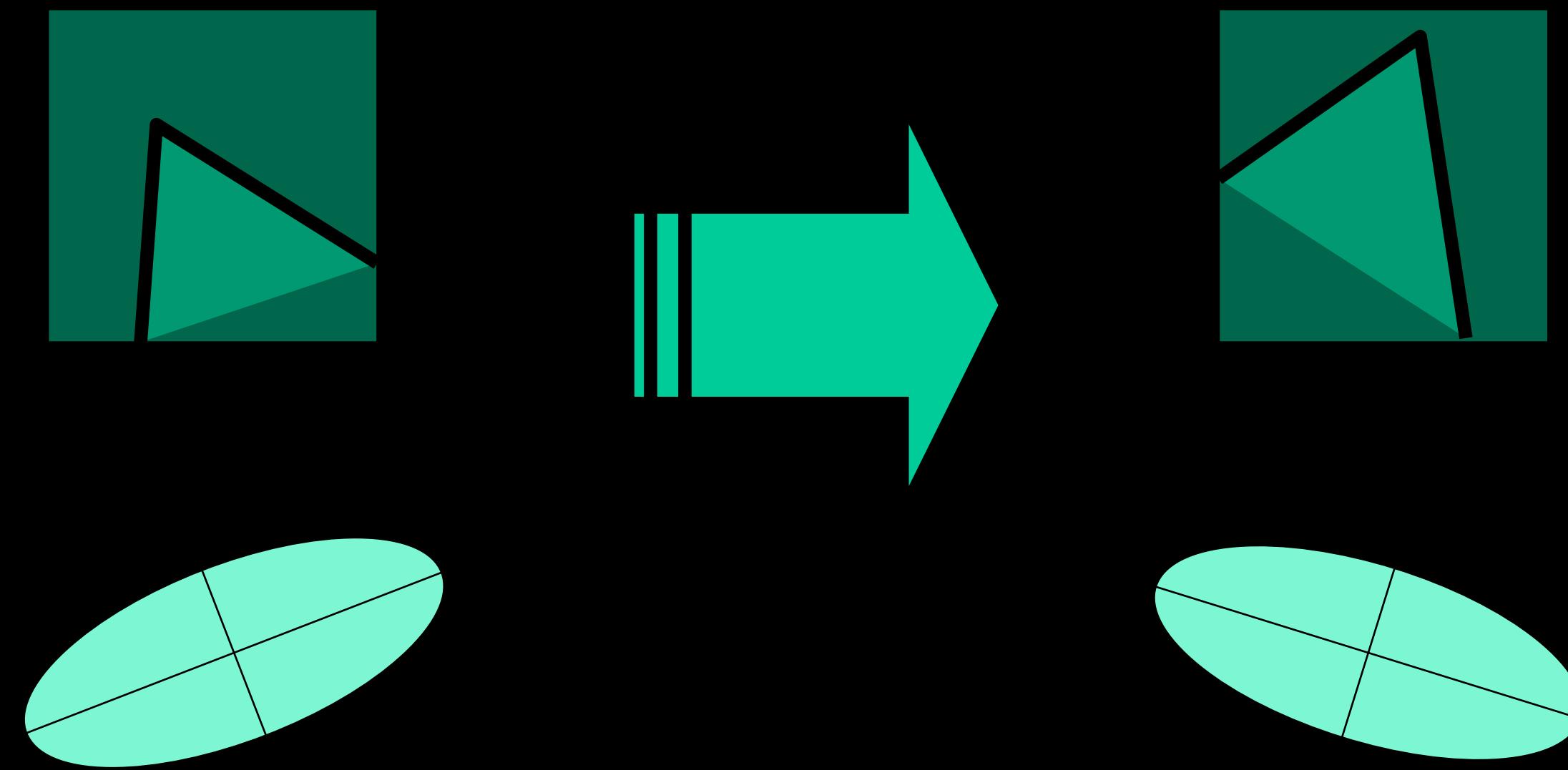
- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

TEXT

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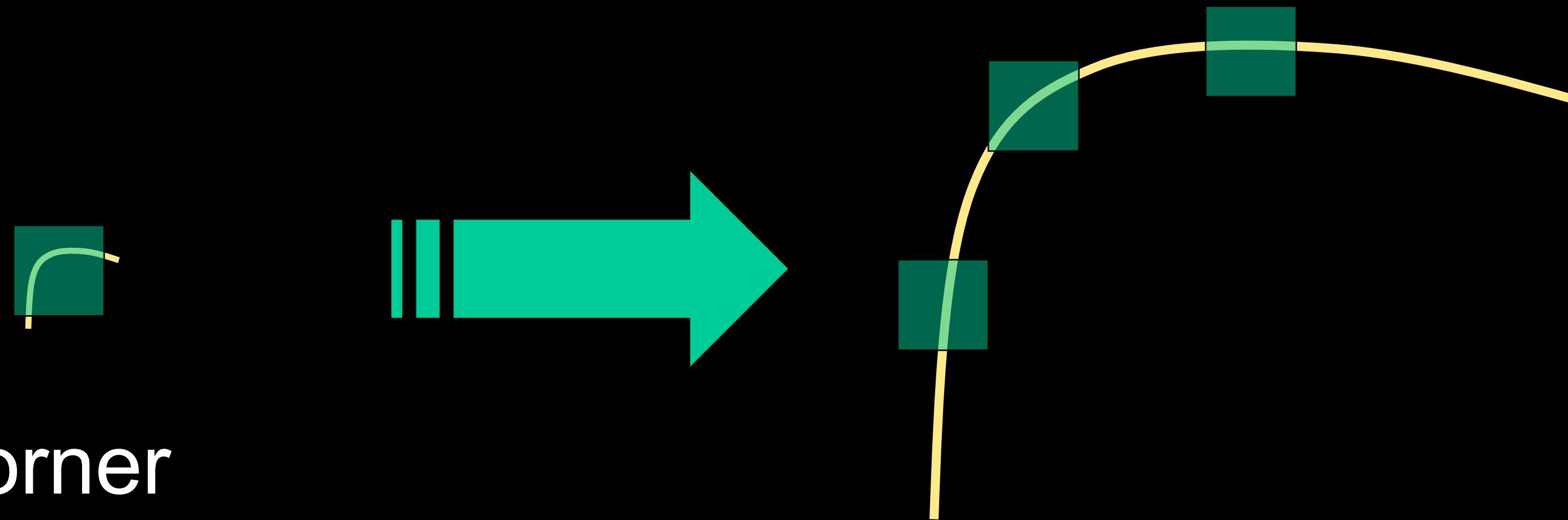
## IMAGE ROTATION



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

## SCALING

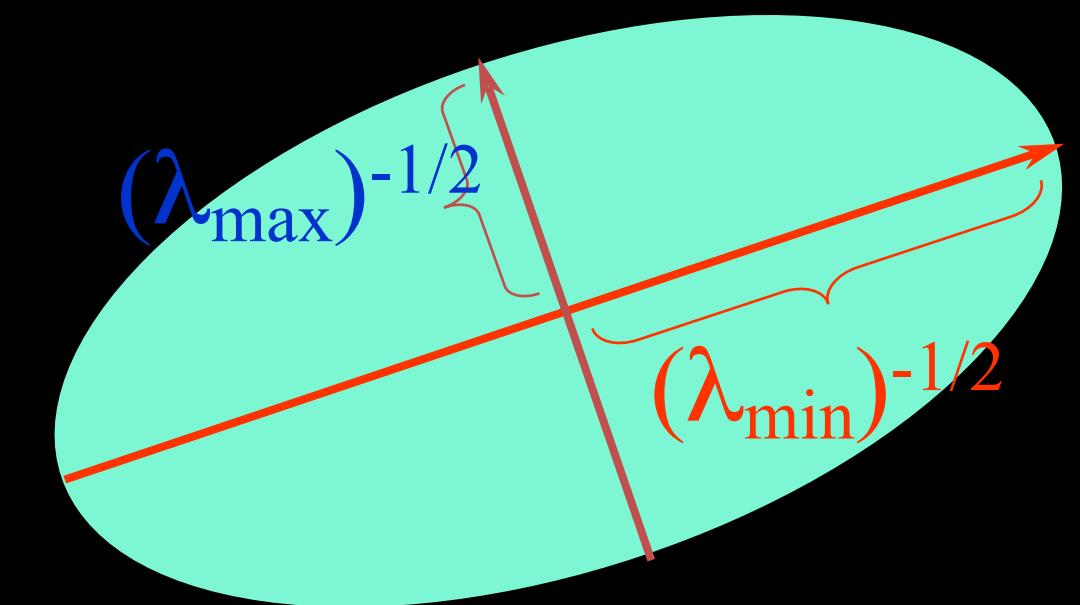


All points will  
be classified as  
edges

Corner location is not covariant to scaling!

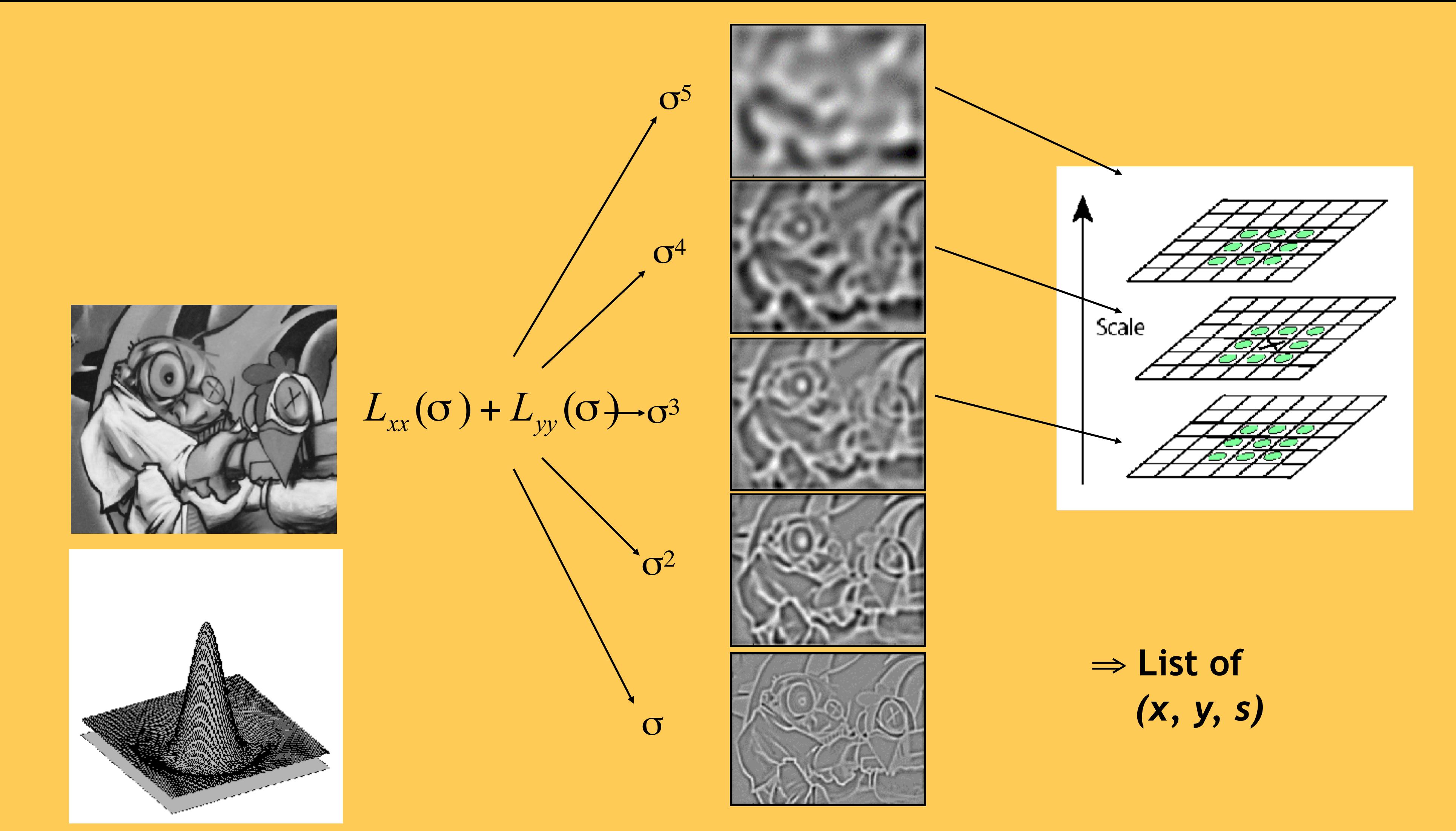
## REVIEW: HARRIS CORNER DETECTOR

- ▶ Approximate distinctiveness by local auto-correlation.
- ▶ Approximate local auto-correlation by second moment matrix
- ▶ Quantify distinctiveness (or cornerness) as function of the eigenvalues of the second moment matrix.
- ▶ But we don't actually need to compute the eigenvalues by using the determinant and trace of the second moment matrix.



TEXT

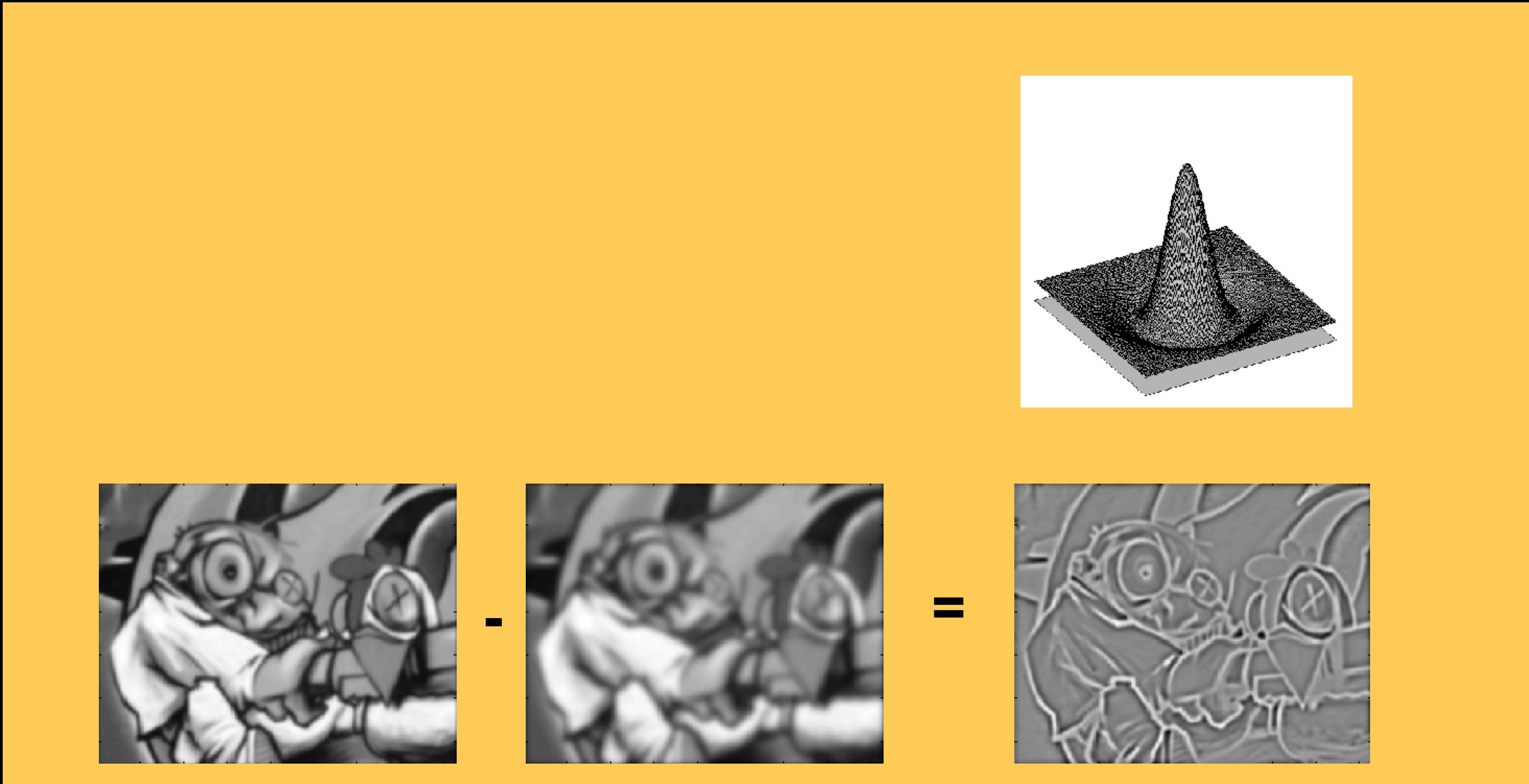
## FIND LOCAL MAXIMA IN POSITION-SCALE SPACE



TEXT

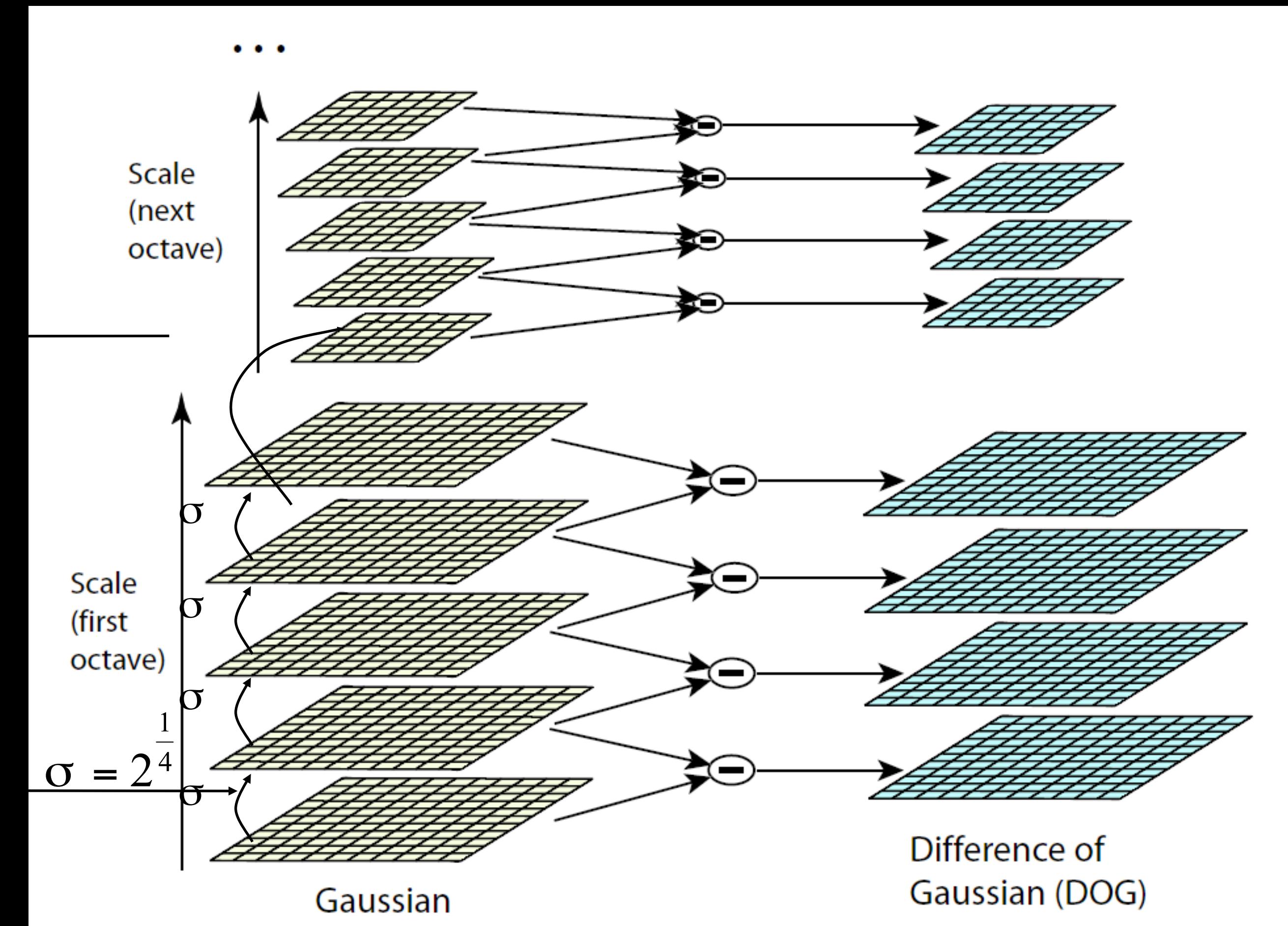
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## DIFFERENCE-OF-GAUSSIAN (DOG)



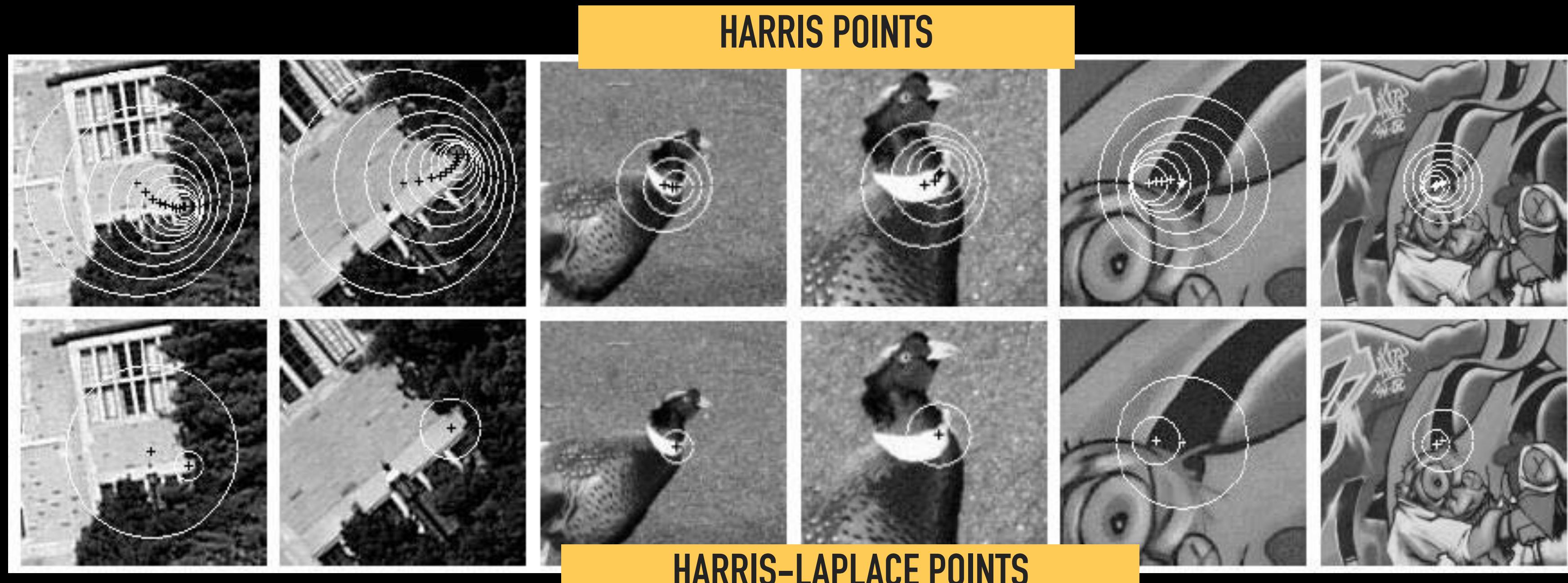
# DOG – EFFICIENT COMPUTATION

▶ Computation in Gaussian scale pyramid



## HARRIS-LAPLACE [MIKOŁAJCZYK '01]

1. Initialization: Multiscale Harris corner detection
2. Scale selection based on Laplacian  
(same procedure with Hessian  $\Rightarrow$  Hessian-Laplace)



# TODAY: FEATURE DESCRIPTORS

- ▶ Orientation estimation
- ▶ Feature descriptors
  - ▶ SIFT and variations
  - ▶ PCA-SIFT
  - ▶ GLOH
  - ▶ SURF
- ▶ Matching Strategy
- ▶ RANSAC

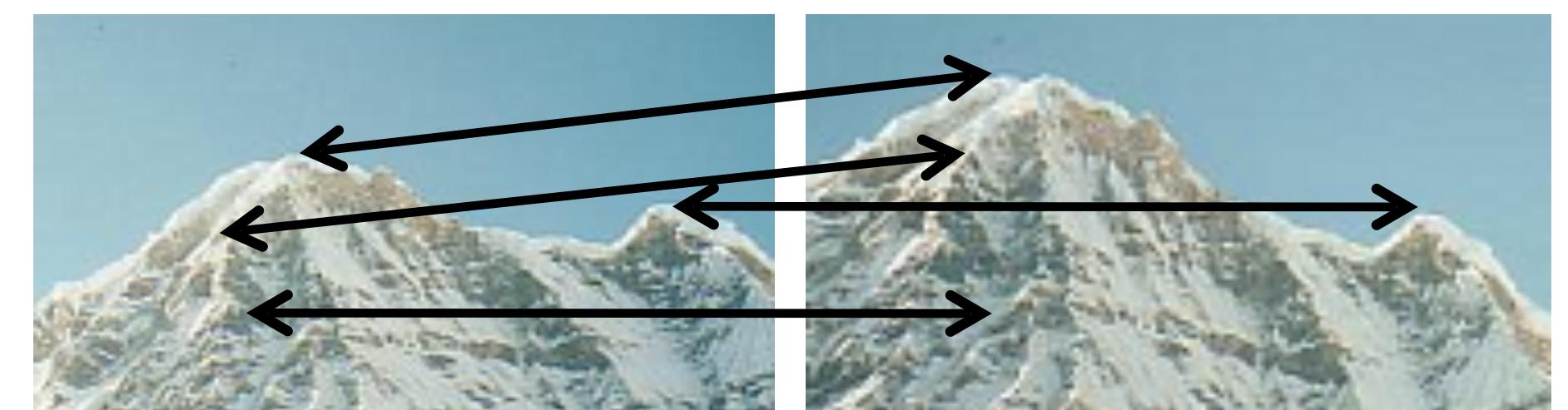
## LOCAL FEATURES: MAIN COMPONENTS

- ▶ Detection: Identify the interest points
- ▶ Description: Extract vector feature descriptor surrounding each interest point.
- ▶ Matching: Determine correspondence between descriptors in two views



$$\mathbf{x}_1 = [x_1^{(1)}, \boxed{?}, x_d^{(1)}]$$

The diagram illustrates the extraction of feature descriptors  $\mathbf{x}_1$  and  $\mathbf{x}_2$  from two views of a mountain peak. The first view on the left shows a cluster of interest points with red arrows pointing to specific points in the second view on the right. The second view also shows a cluster of interest points. The descriptors are represented as vectors:  $\mathbf{x}_1 = [x_1^{(1)}, \boxed{?}, x_d^{(1)}]$  and  $\mathbf{x}_2 = [x_1^{(2)}, \boxed{?}, x_d^{(2)}]$ , where the question marks indicate the descriptor vectors.



## DESCRIPTORS

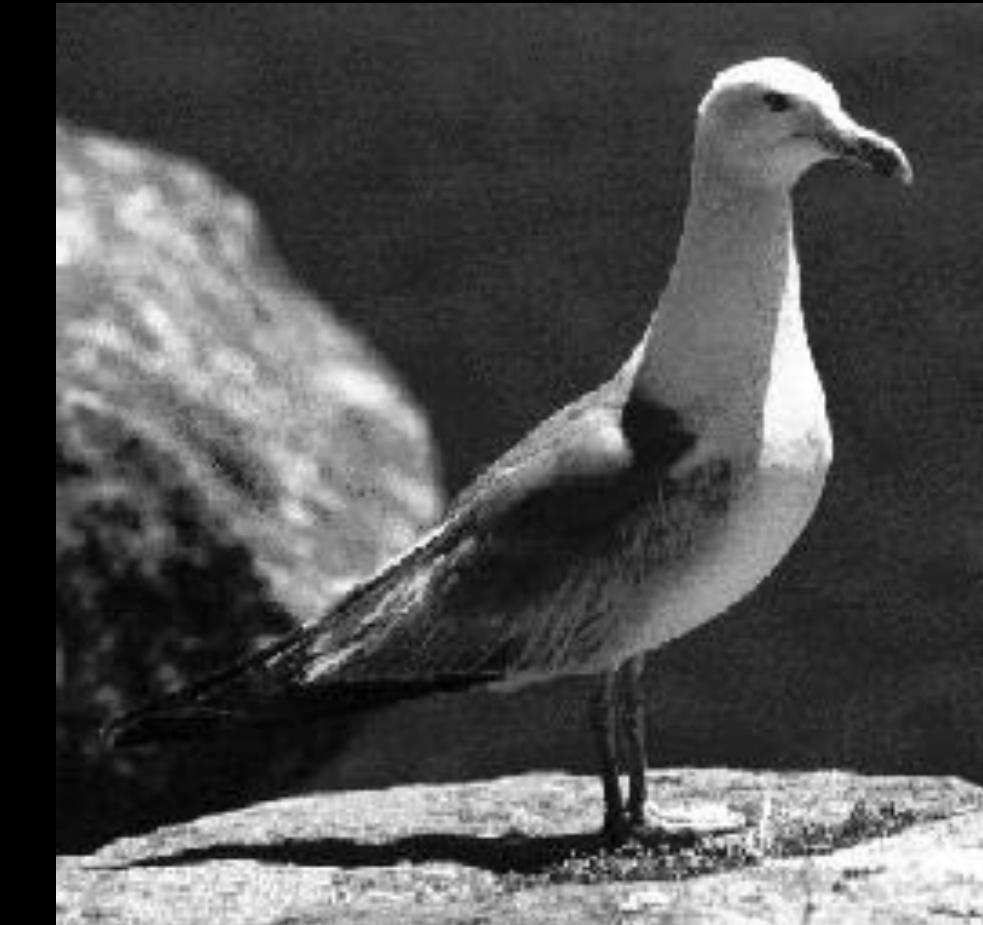
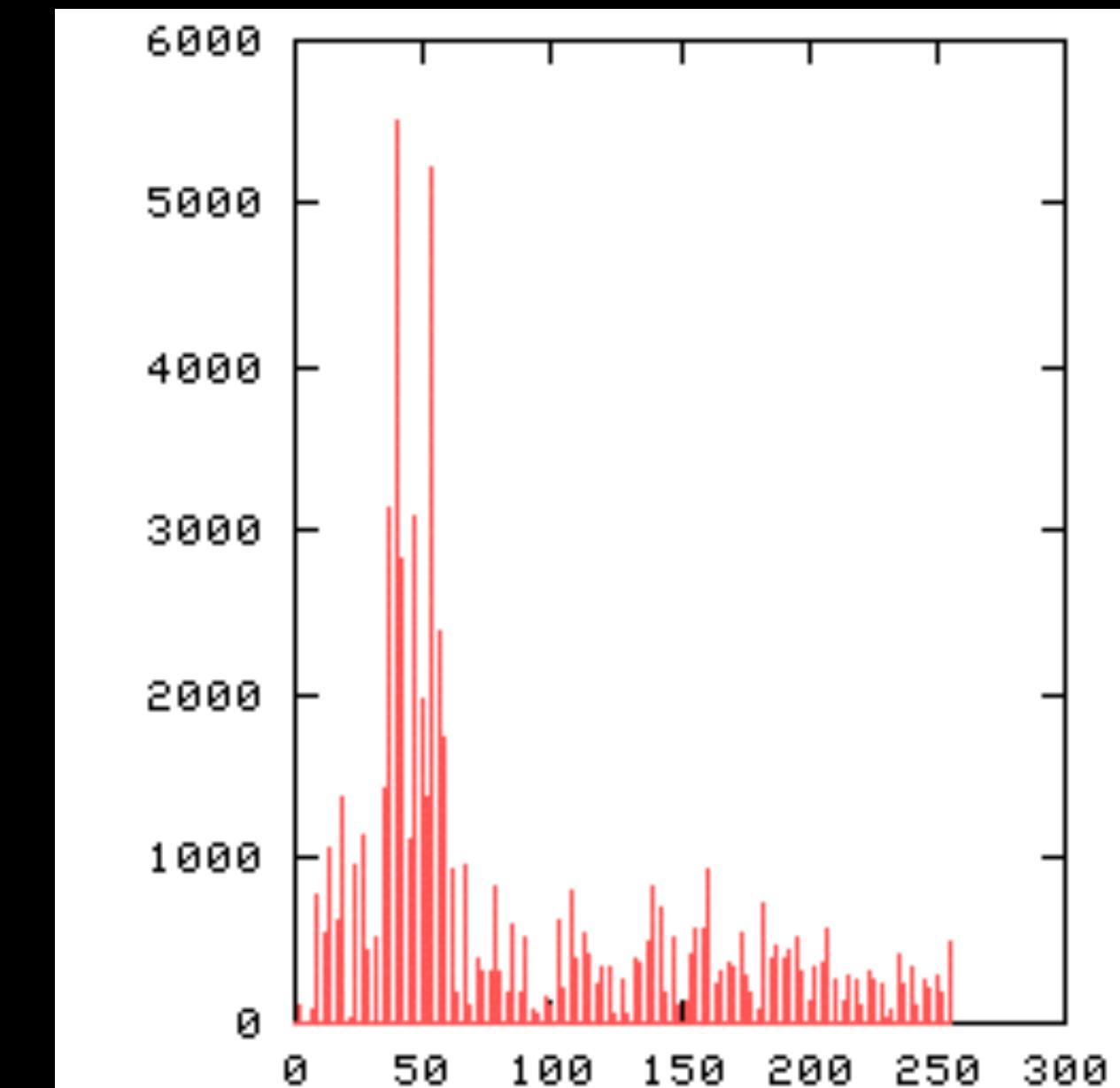
- ▶ Stereo: Patches around keypoints
  - ▶ After rectification, it's only translations, so it's fine
- ▶ What about changes in orientation, scale, affine transformations?
  - ▶ Rectification for those things... patches will still vary significantly.
- ▶ Descriptors ***invariant*** but keeping ***discriminability***

## IMAGE REPRESENTATIONS

- ▶ Templates
  - ▶ Intensity, gradients, etc.
  - ▶ SDD, NCC
- ▶ Histograms
  - ▶ Color, texture, SIFT descriptors, etc.
  - ▶ Orientation, scale, affine transformations

## IMAGE REPRESENTATIONS: HISTOGRAMS

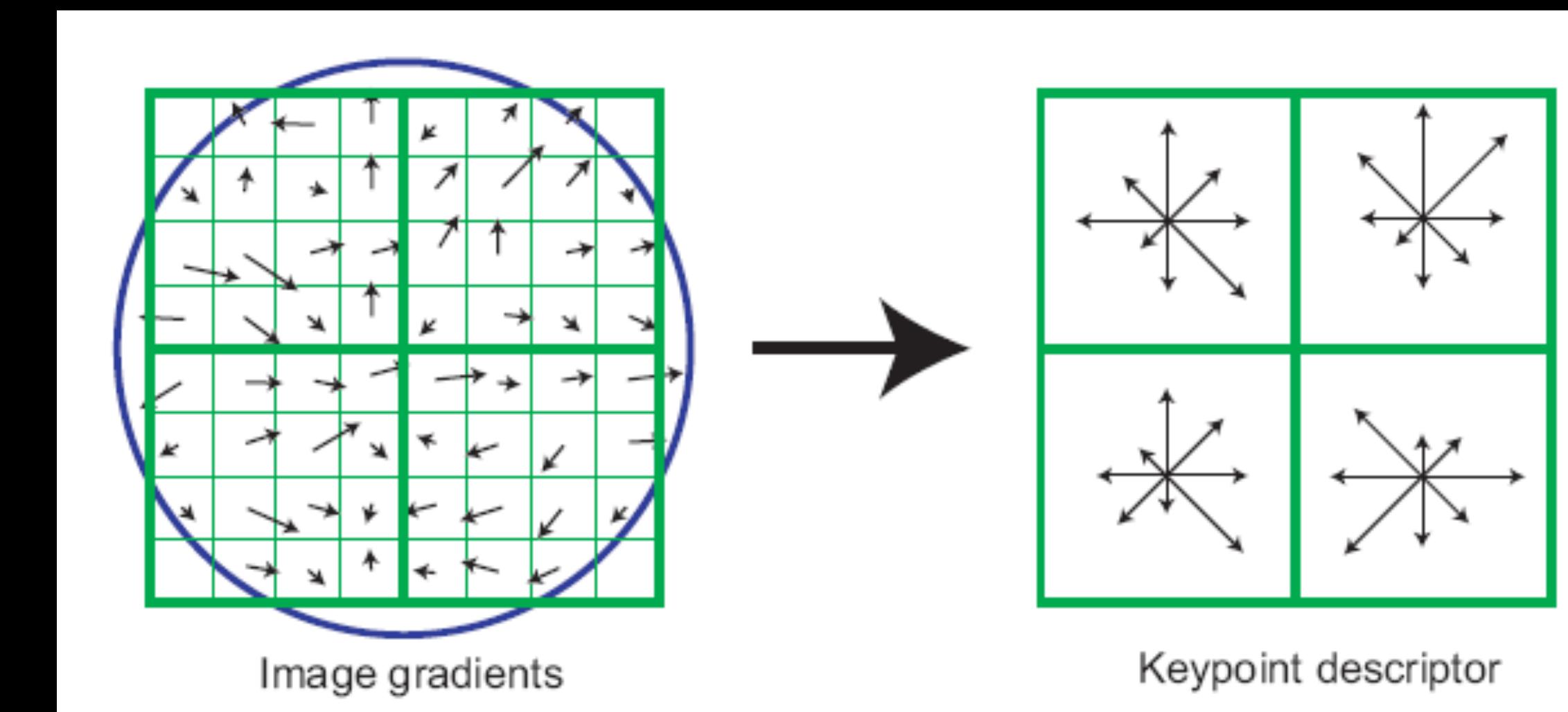
- ▶ Global histogram
- ▶ Represent distribution of features
- ▶ Color, texture, depth, ...



# SIFT

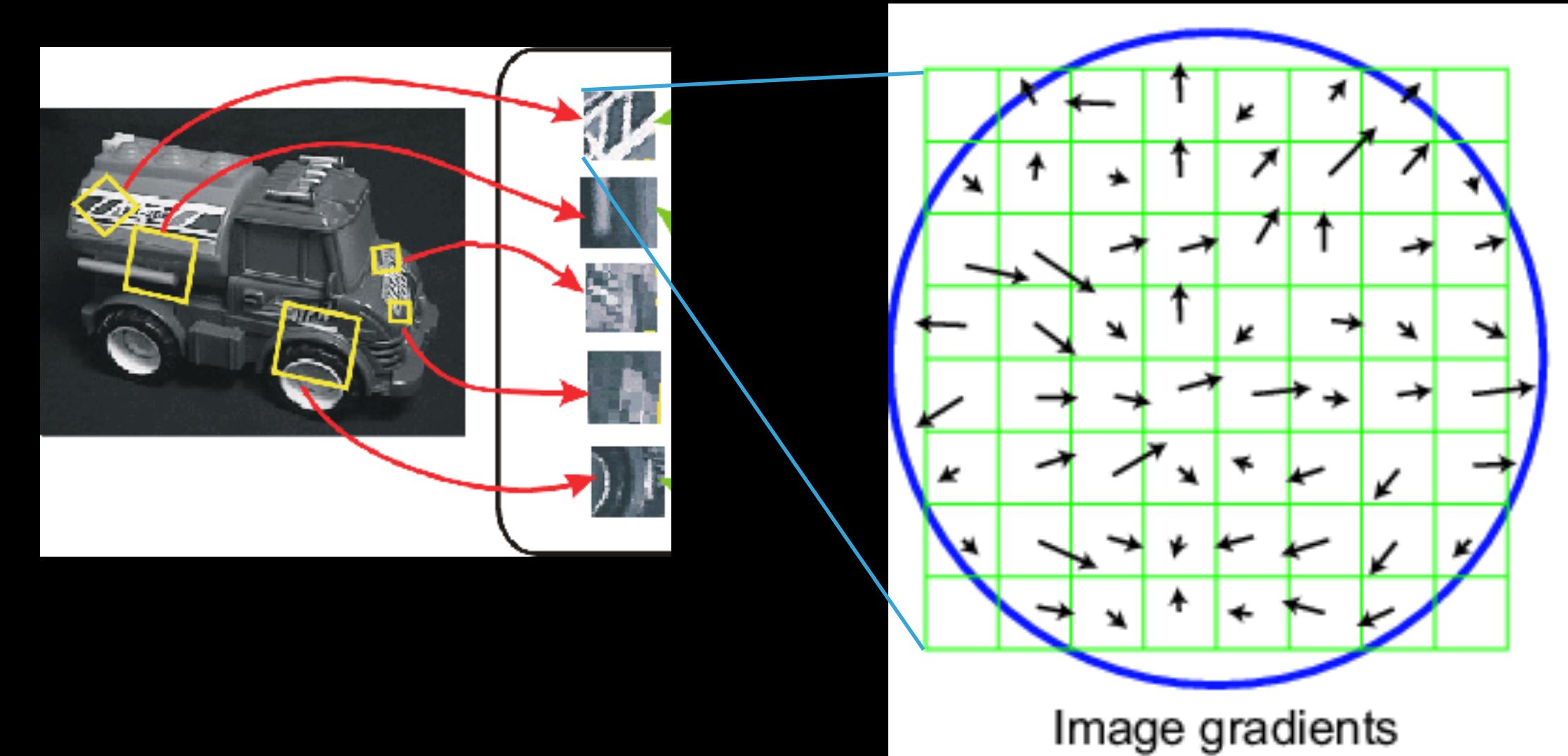
# SIFT

- ▶ Histograms of oriented gradients



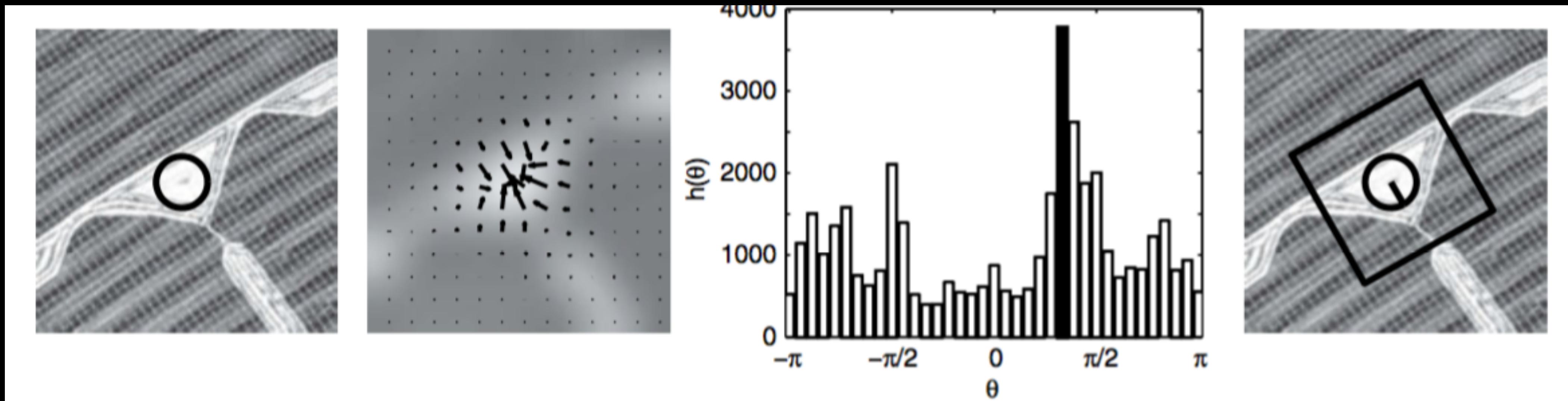
## SIFT VECTOR FORMATION

- ▶ Computed on rotated and scaled version of window according to computed orientation & scale
- ▶ resample the window
- ▶ Based on gradients weighted by a Gaussian of variance half the window (for smooth falloff)



## SIFT VECTOR FORMATION

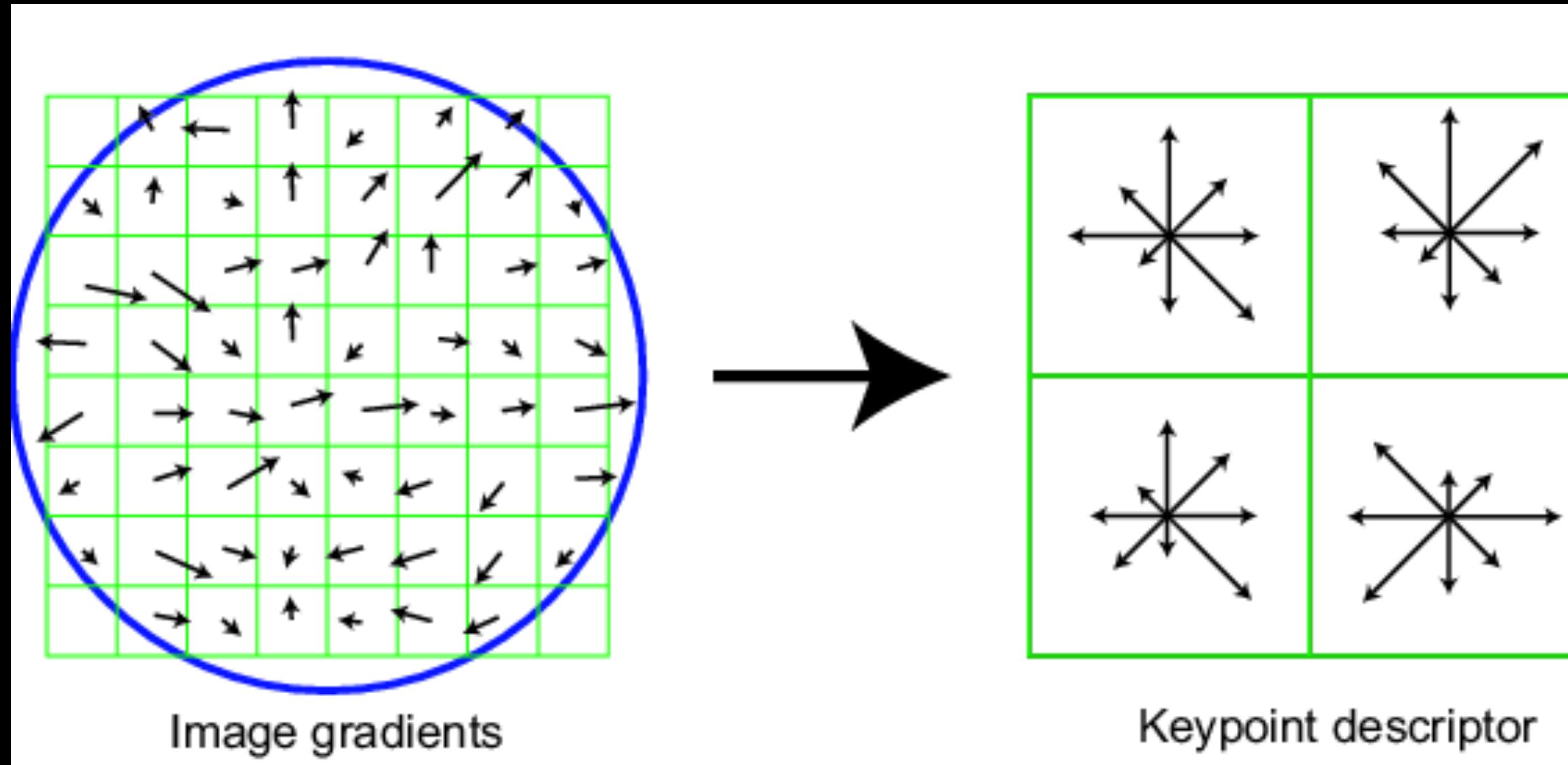
- ▶ Computed on rotated and scaled version of window according to computed orientation & scale



- ▶ 36 bins. Take maximum. If more than one peak, two keypoints are set with different orientations.

## SIFT VECTOR FORMATION

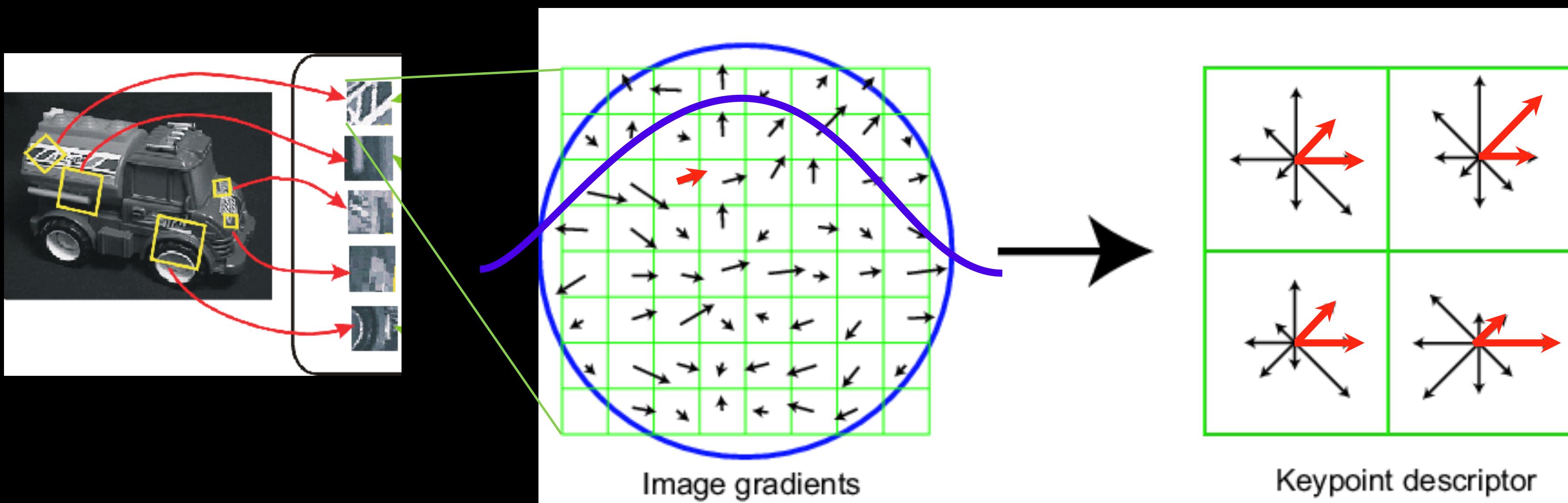
- ▶ 16 by 16 window
- ▶ 4x4 array of gradient orientation histogram weighted by magnitude
- ▶ 8 orientations x 4x4 array = 128 dimensions
- ▶ Motivation: some sensitivity to spatial layout, but not too much.



showing only 2x2 here but is 4x4

## ENSURE SMOOTHNESS

- ▶ Gaussian weight
- ▶ Trilinear interpolation
- ▶ a given gradient contributes to 8 bins: 4 in space times 2 in orientation



## REDUCE EFFECT OF ILLUMINATION

- ▶ 128-dim vector normalized to 1
- ▶ Threshold gradient magnitudes to avoid excessive influence of high gradients
  - ▶ after normalization, clamp gradients  $>0.2$
  - ▶ renormalize

## SUMMARY SIFT

- ▶ Use DoG to get keypoints and scale
- ▶ Take a region around the point and compute dominant orientation, (mode of histogram of directions)
- ▶ Sample 16x16 pixels at the detected scale and orientation
- ▶ Compute 4x4 grid of histograms of gradient directions using 8 bins = 128 dimensional feature descriptor
  - ▶ Sum of magnitudes.
  - ▶ Scaled by a gaussian
- ▶ Normalize, clip, re-normalize

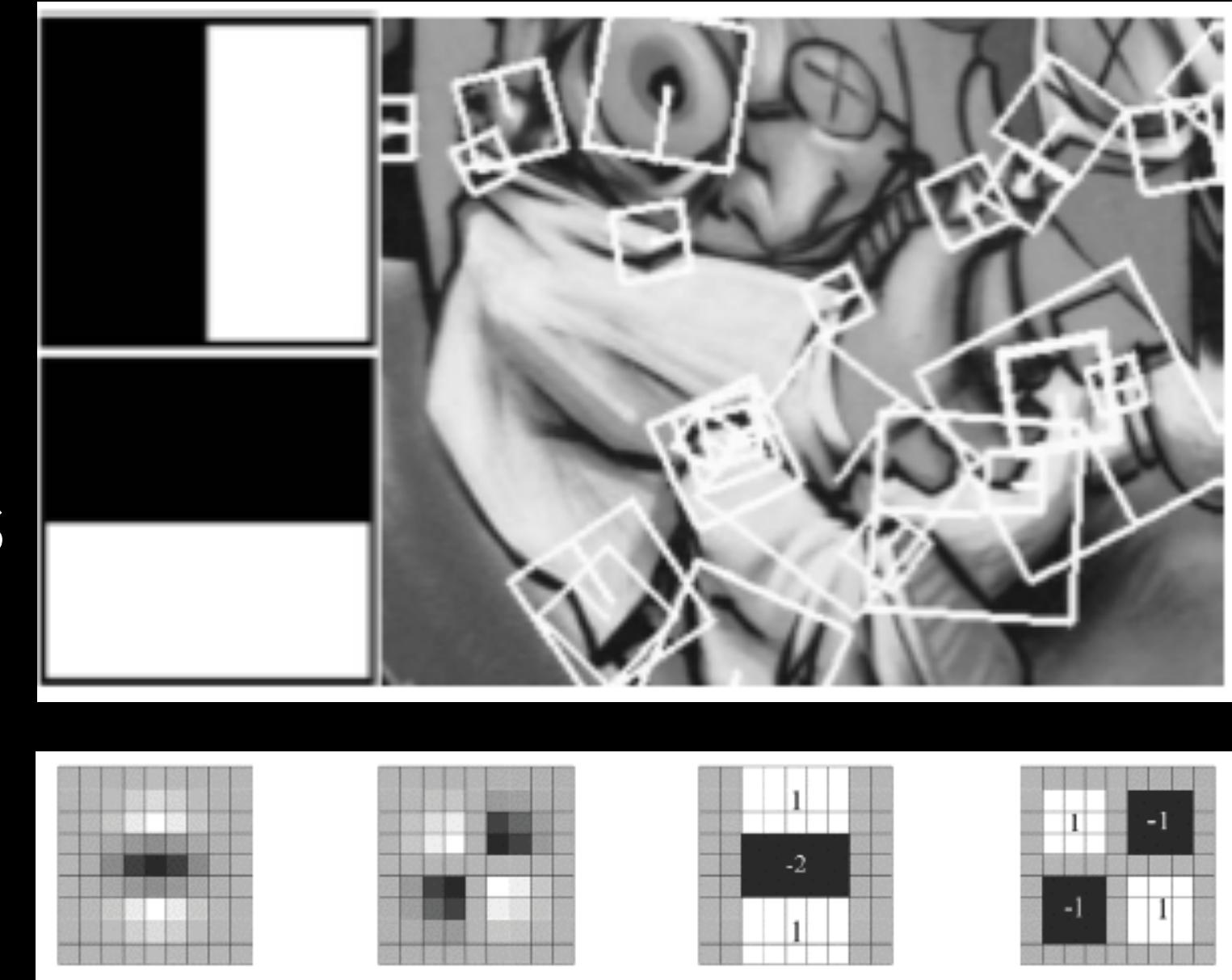
# VARIATIONS OF SIFT

## PCA-SIFT

- ▶ Takes 39x39 patch of gradients around the keypoint.
- ▶ Uses PCA learned from a large databased to reduce the 3042-dimensions to 36

## LOCAL DESCRIPTORS: SURF

- ▶ Fast approximation of SIFT idea
- ▶ Efficient computation by 2D box filters & integral images  
⇒ 6 times faster than SIFT
- ▶ Equivalent quality for object identification



[Bay, ECCV'06], [Cornelis, CVGPU'08]

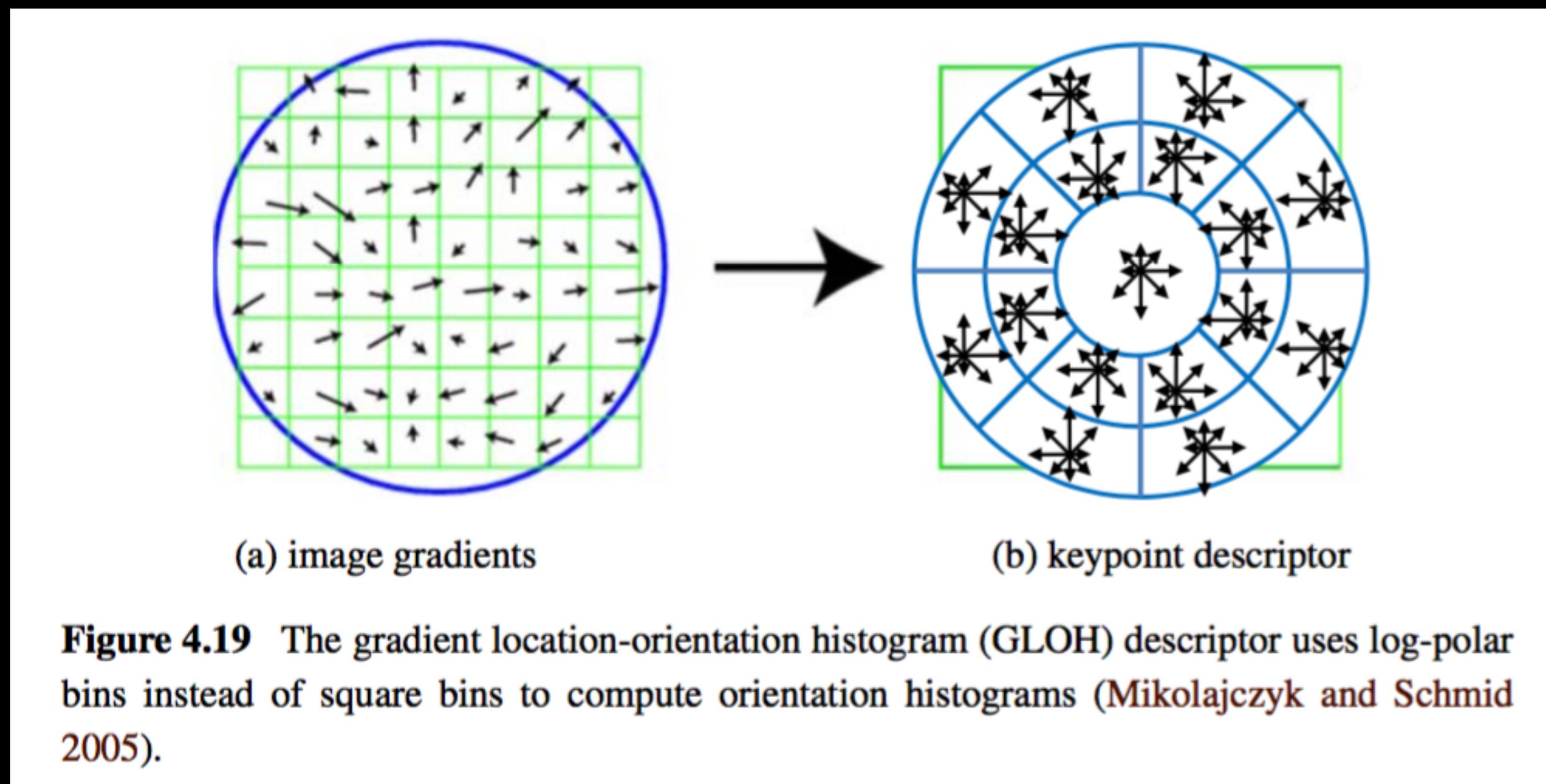
GPU implementation available

Feature extraction @ 200Hz  
(detector + descriptor, 640×480 img)

<http://www.vision.ee.ethz.ch/~surf>

## GLOH

- ▶ 3 circles, 8 regions per circle, 16 direction bins = 272
- ▶ reduced to 128 using PCA from a large descriptor database



# OTHER DESCRIPTORS

## LOCAL DESCRIPTORS

- ▶ Most features can be thought of as templates, histograms (counts), or combinations
- ▶ The ideal descriptor should be
  - ▶ Robust
  - ▶ Distinctive
  - ▶ Compact
  - ▶ Efficient
- ▶ Most available descriptors focus on edge/gradient information
  - ▶ Capture texture information
  - ▶ Color rarely used

## LOCAL FEATURES: MAIN COMPONENTS

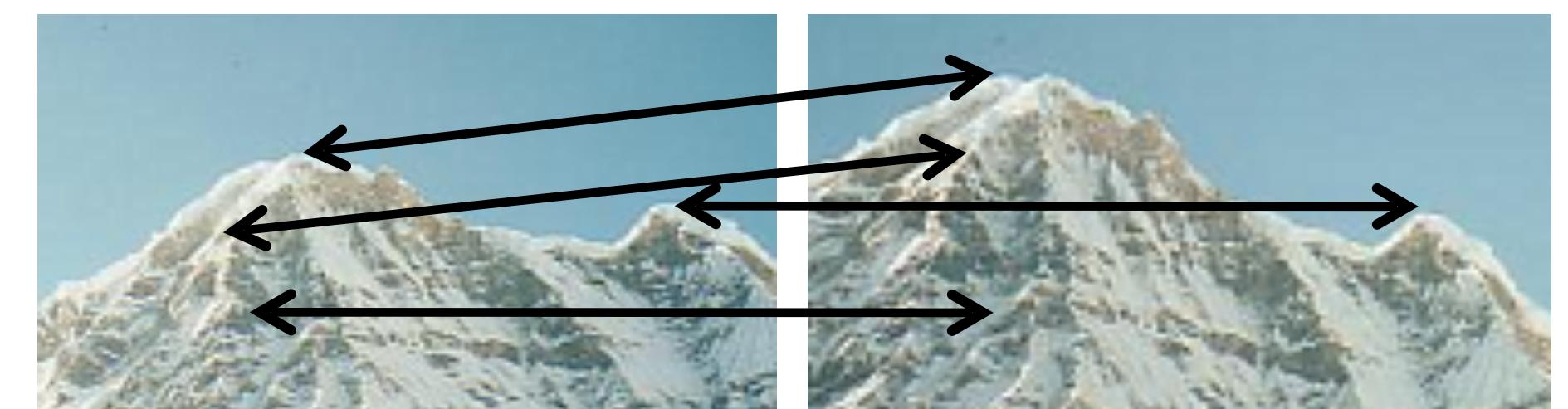
- ▶ Detection: Identify the interest points
- ▶ Description: Extract vector feature descriptor surrounding each interest point.
- ▶ Matching: Determine correspondence between descriptors in two views



$$\mathbf{x}_1 = [x_1^{(1)}, \boxed{?}, x_d^{(1)}]$$

A diagram illustrating the extraction of a local feature descriptor. A red box highlights a cluster of points around an interest point. Arrows point from these points to the corresponding feature vectors  $x_1^{(1)}$ ,  $\boxed{?}$ , and  $x_d^{(1)}$  in the equation above. Below this, another red box highlights a similar cluster of points in a second view, with arrows pointing to the corresponding feature vectors  $x_1^{(2)}$ ,  $\boxed{?}$ , and  $x_d^{(2)}$  in the equation below.

$$\mathbf{x}_2 = [x_1^{(2)}, \boxed{?}, x_d^{(2)}]$$



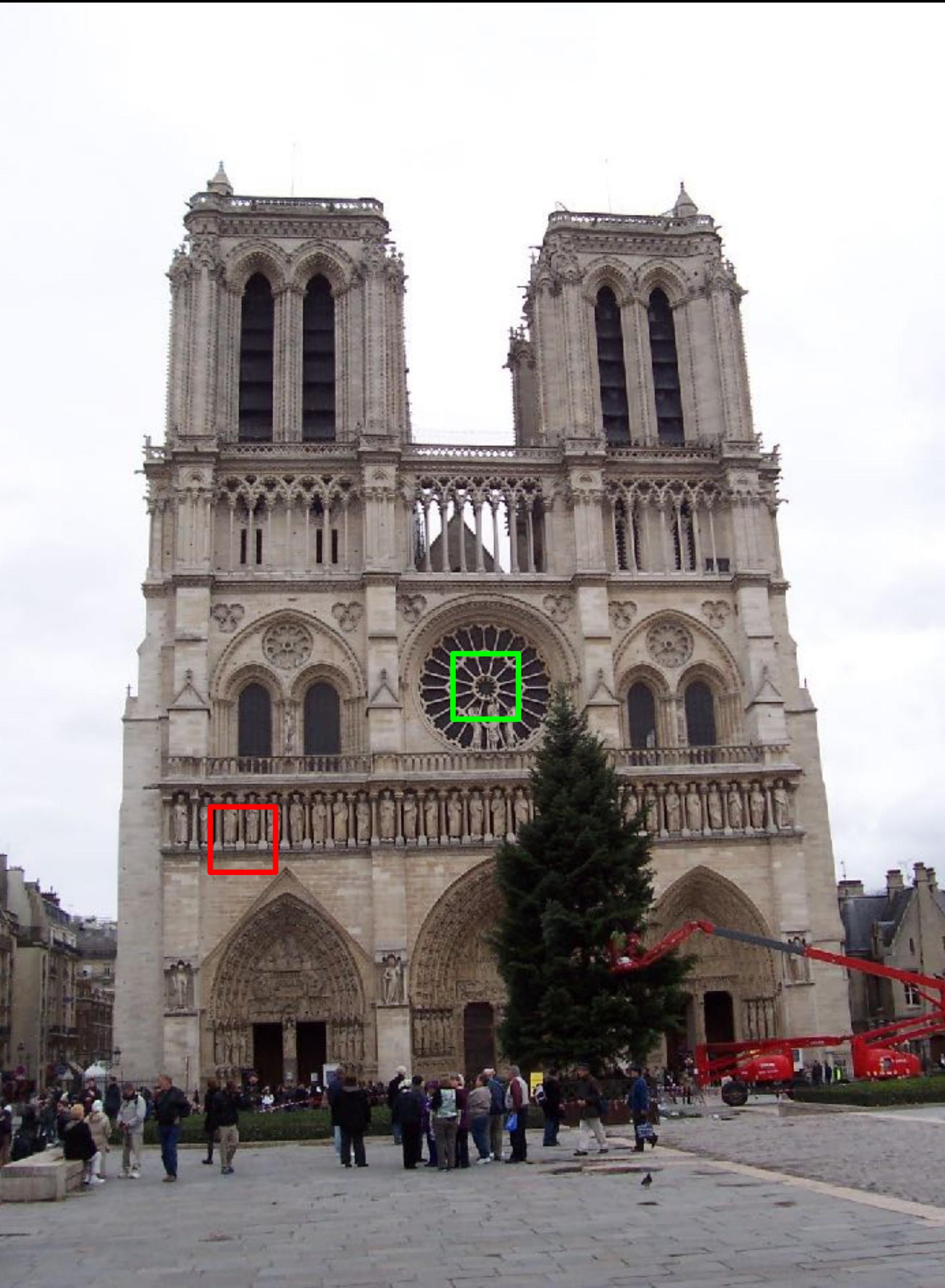
# MATCHING

## FEATURE MATCHING

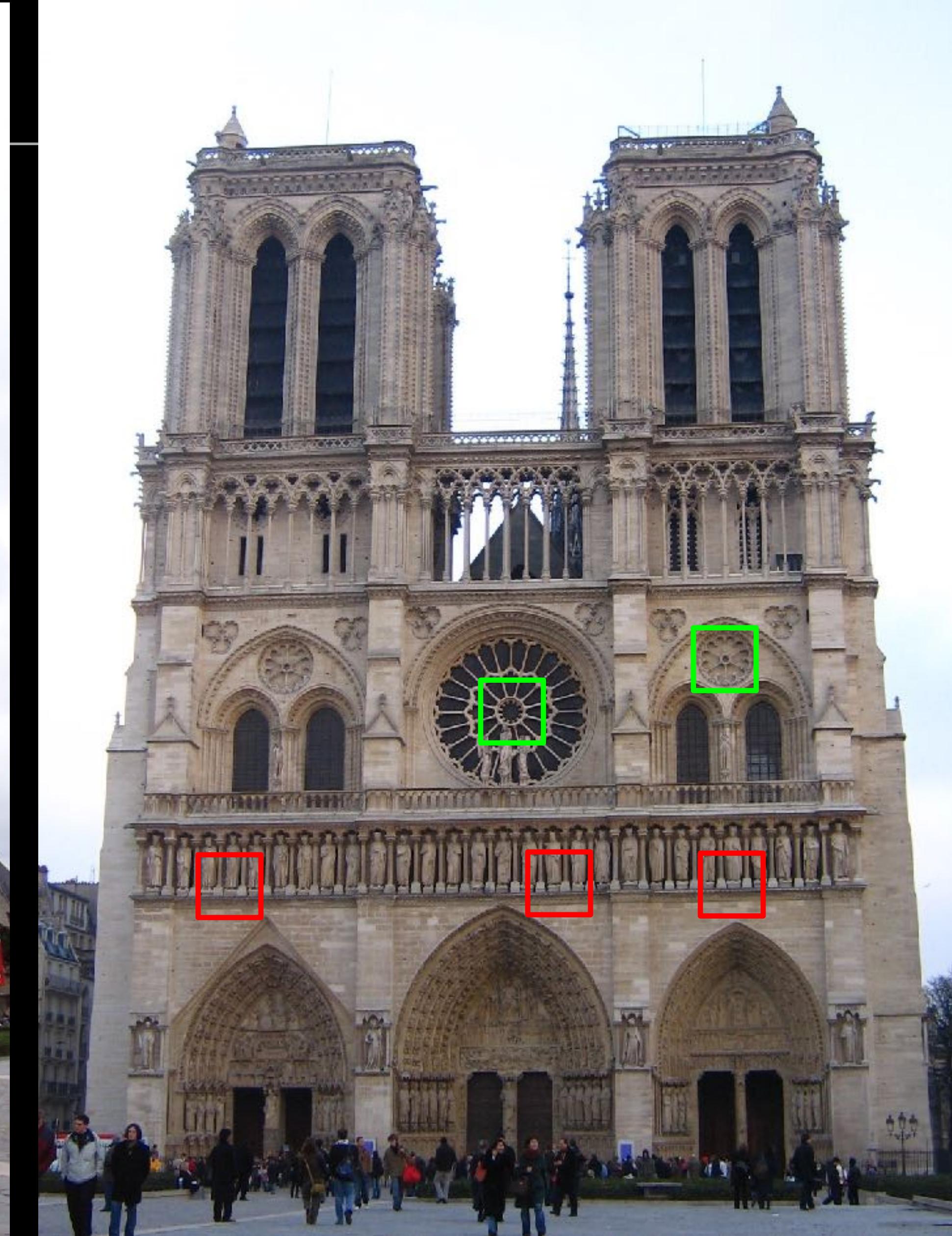
- ▶ For a feature in Image1 how to find the best match in Image2?
- ▶ Define distance function that compares two descriptors
- ▶ Test all the features in Image2, find the one with min distance

## FEATURE MATCHING

- ▶ distances in feature space can be directly used for ranking
- ▶ Euclidean distance should be meaningful
- ▶ If some features have more importance, they should be scaled first



Distance: 0.34, 0.30, 0.40



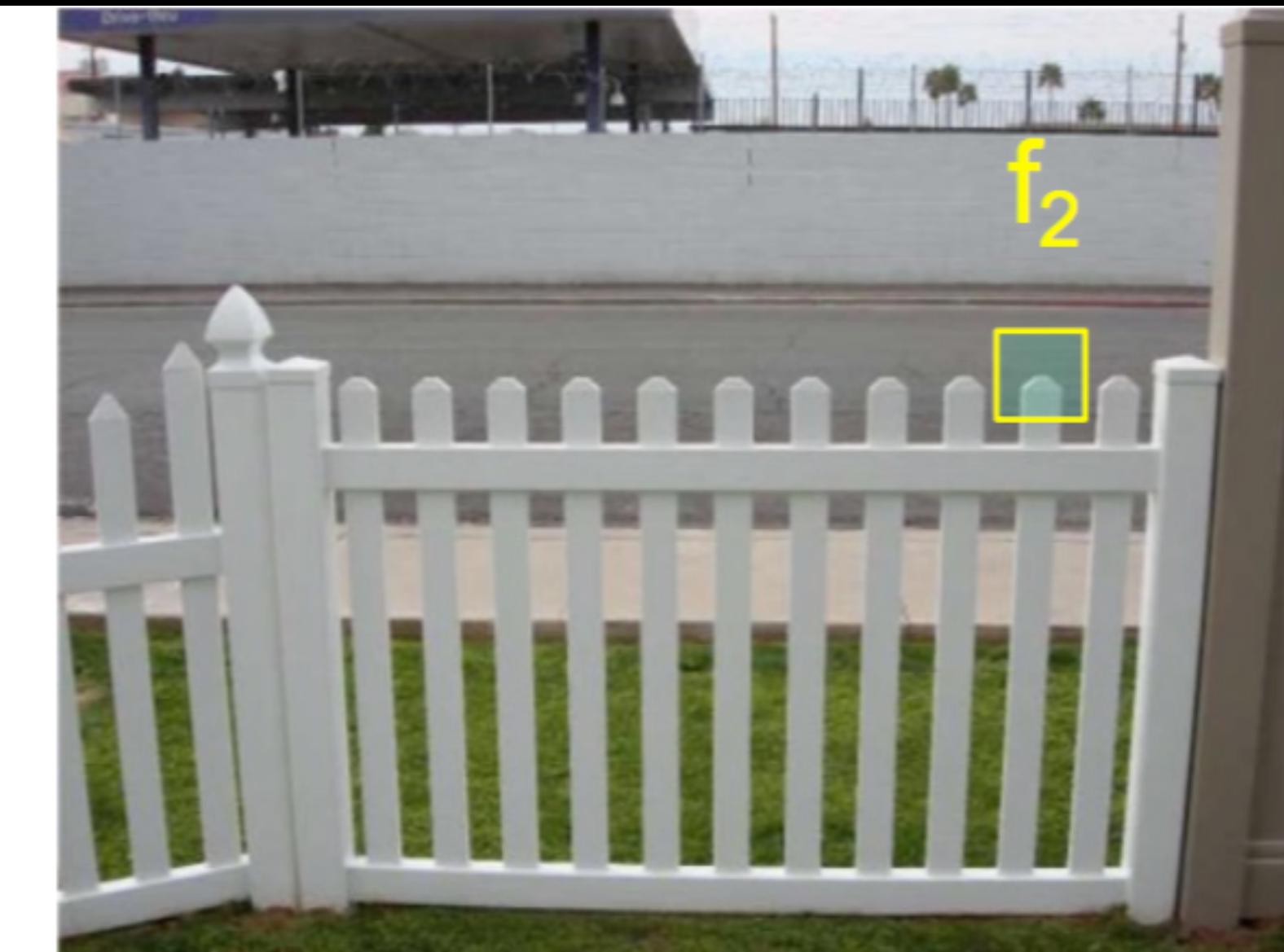
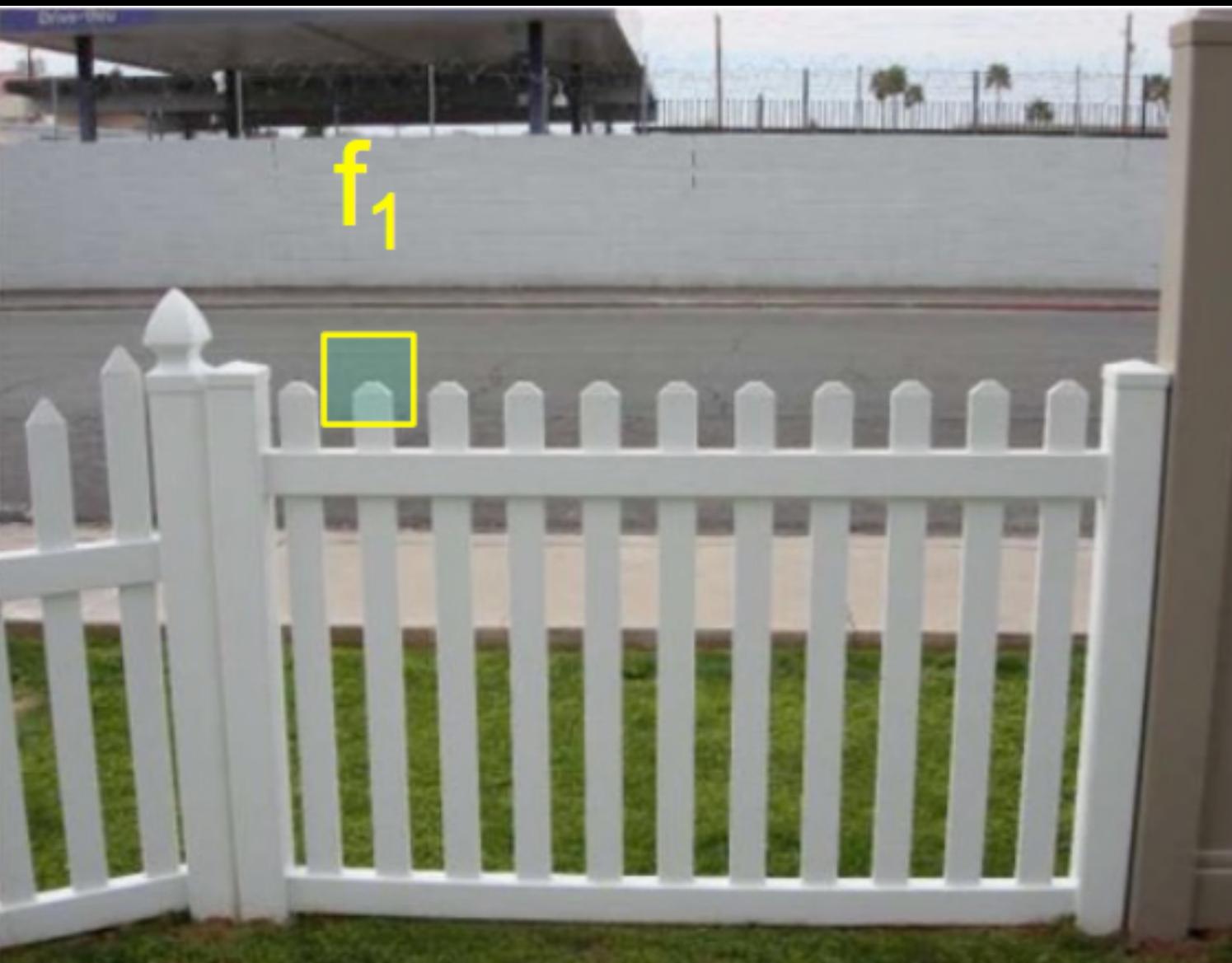
Distance: 0.61  
Distance: 1.22

TEXT

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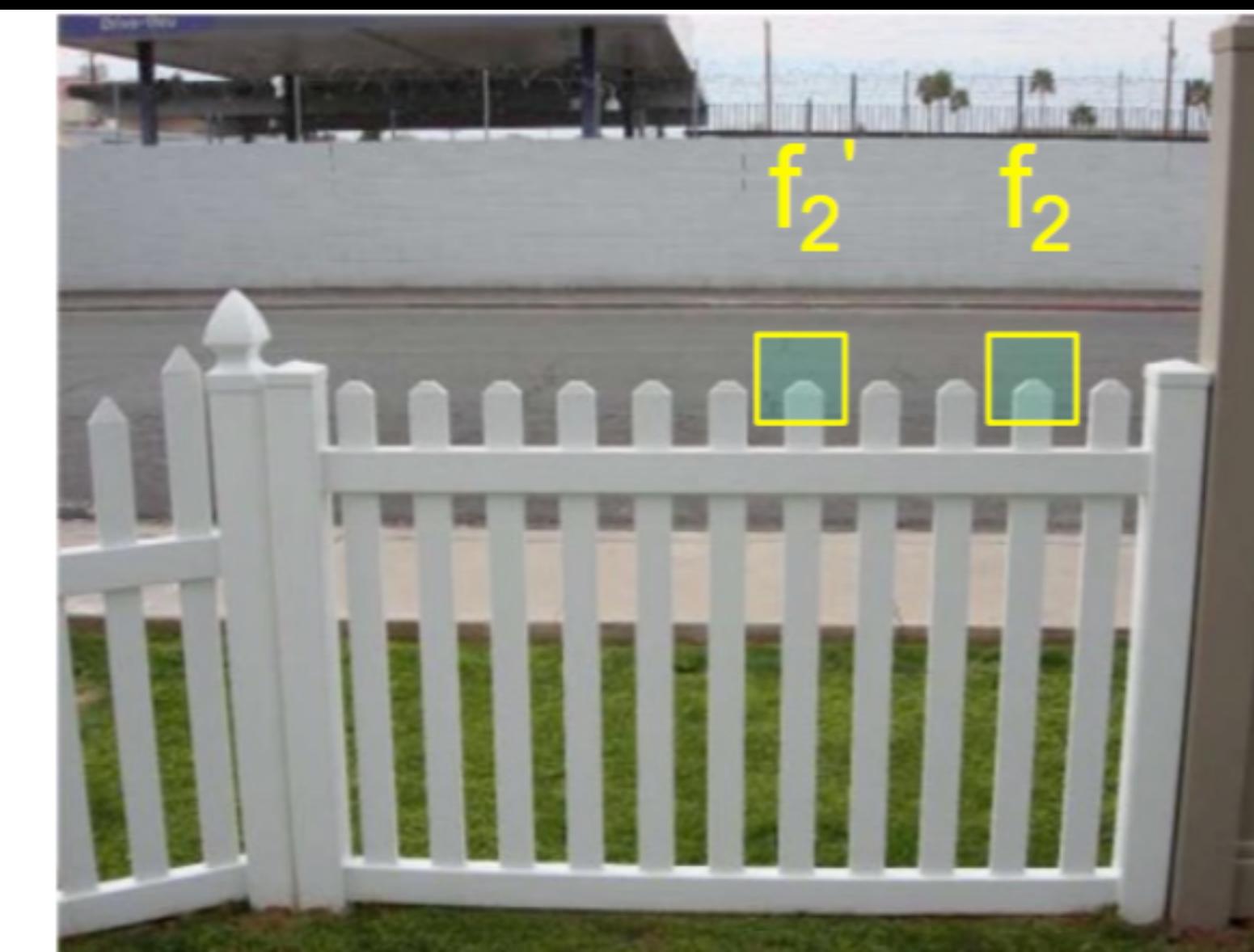
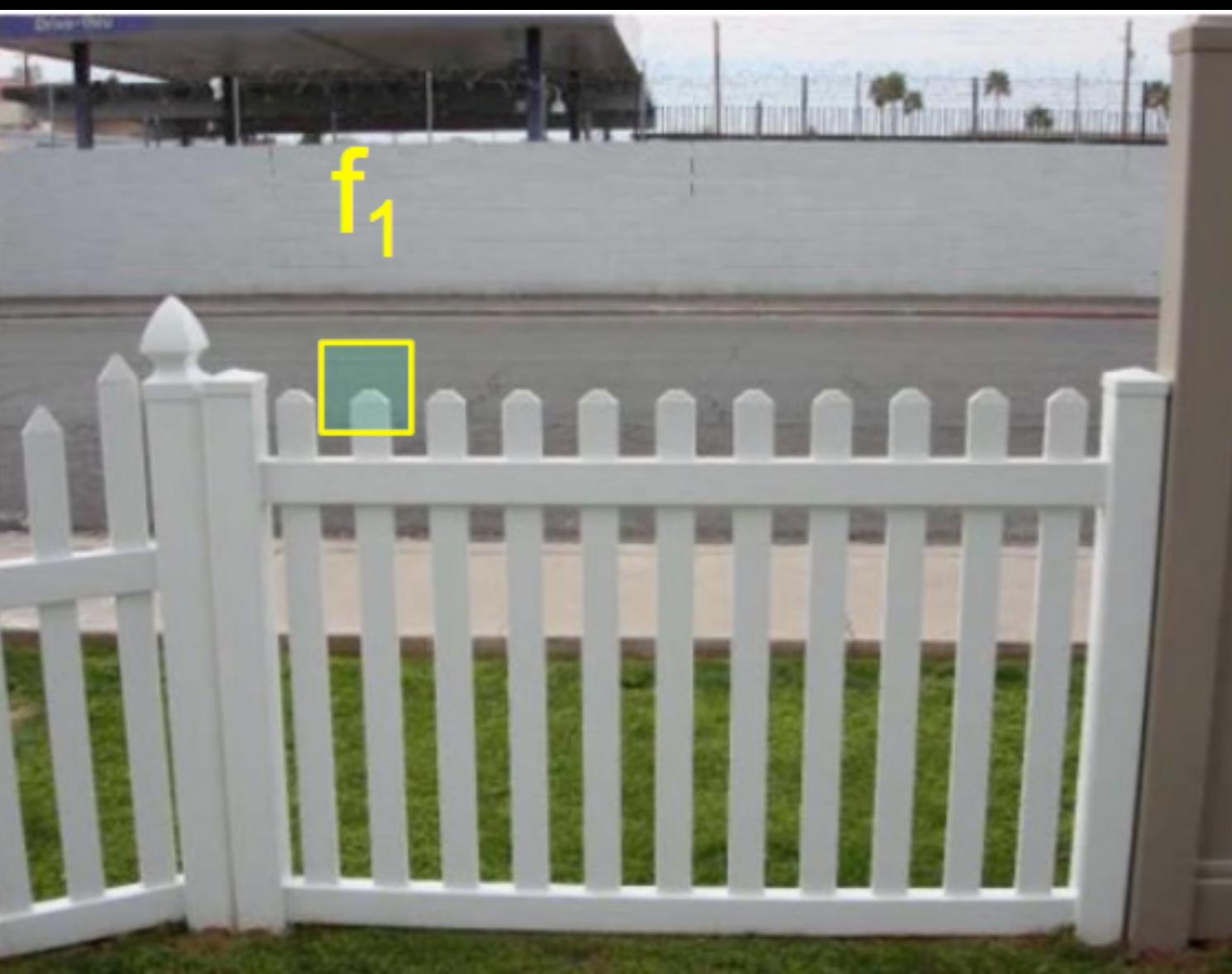
## FEATURE MATCHING

- ▶ Let's assumes SDD =  $|f_1 - f_2|^2$



## FEATURE MATCHING

- ▶ Fails in ambiguous cases
- ▶ ideas?



## FEATURE MATCHING

- ▶ Ratio distance =  $\text{SSD}(f_1, f_2) / \text{SSD}(f_1, f_2')$
- ▶  $f_2$  is best SSD match to  $f_1$  in Image2
- ▶  $f_2'$  is 2nd best SSD match to  $f_1$  in Image2



The value is in the range [0, 1.0]

## NEAREST NEIGHBOR DISTANCE RATIO

- ▶ NN1/NN2
- ▶ where NN1 is the distance to the first nearest neighbor and NN2 is the distance to the second nearest neighbor
- ▶ Sorting by this ration puts matches in order of confidence

## MATCHING

- ▶ Simplest approach: Pick the nearest neighbor. Threshold on absolute distance
- ▶ Problem: Lots of self similarity in many photos
- ▶ Use ratio for confidence

## EVALUATING MATCHING PERFORMANCE

- ▶ Throw out features with distance > threshold
- ▶ The threshold affects performance
- ▶ True positives = # of detected matches that are correct
  - ▶ Suppose we want to maximize these–how to choose threshold?
- ▶ False positives = # of detected matches that are incorrect
  - ▶ Suppose we want to minimize these–how to choose threshold?

## EVALUATING MATCHING PERFORMANCE

	We find a match	We did not find a match
Correct match	True positive (TP)	False negative (FN)
Incorrect match	False positive (FP)	True negative (TN)

- ▶ what is TP + FN?
- ▶ what is FP + TN?

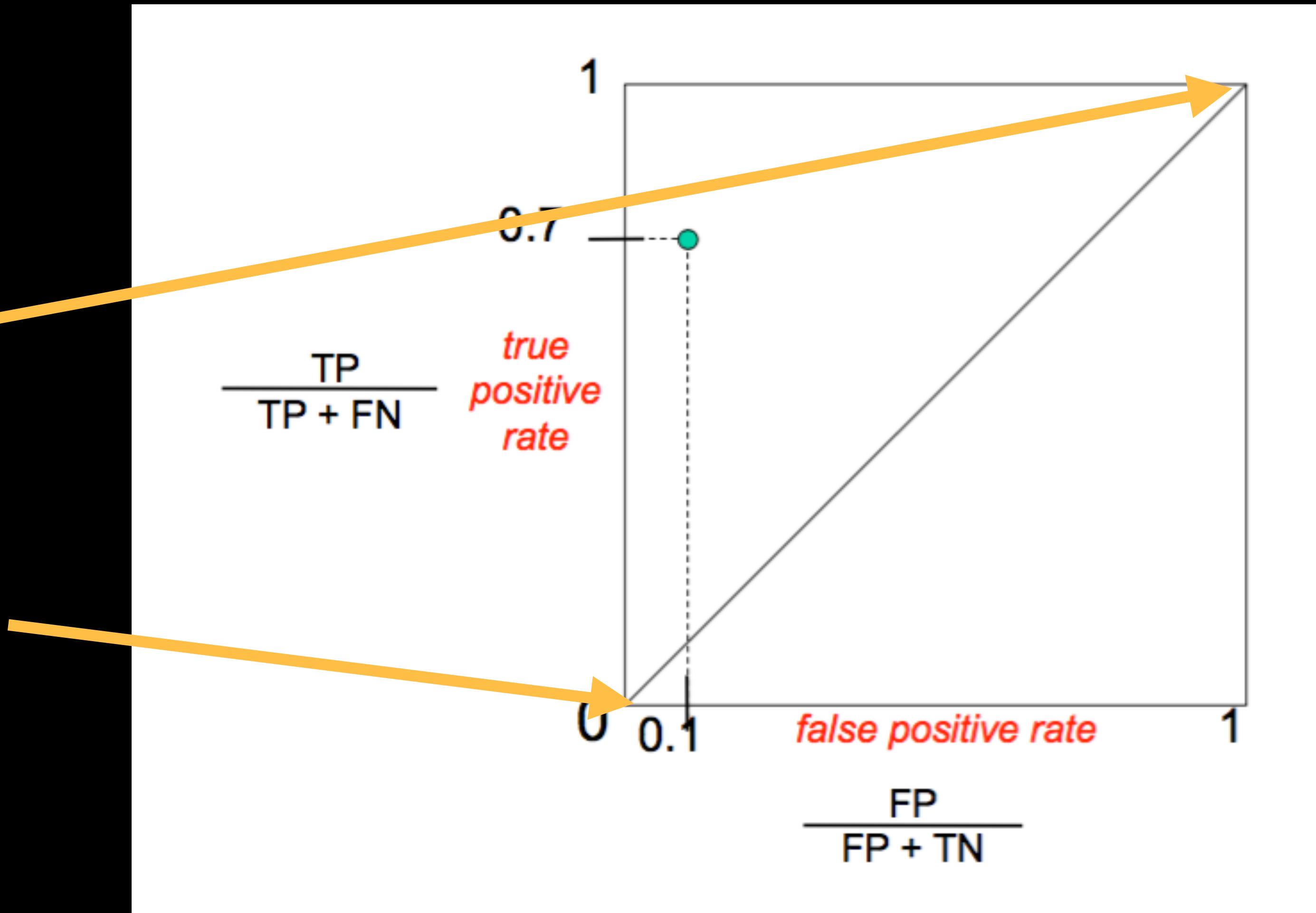
## EVALUATING MATCHING PERFORMANCE

- ▶ True positive rate: RECALL
  - ▶  $TPR = TP / (TP + FN) = TP / P$
- ▶ False positive rate
  - ▶  $FPR = FP / (FP + TN) = TP / N$
- ▶ positive predictive value: PRECISION
  - ▶  $PPV = TP / (TP + FP) = TP / P'$
- ▶ Accuracy
  - ▶  $TRP = TP + TN / (P+N)$

# EVALUATING MATCHING PERFORMANCE

Threshold = 1: Accept Everything

Threshold = 0: Reject Everything



# EVALUATING MATCHING PERFORMANCE

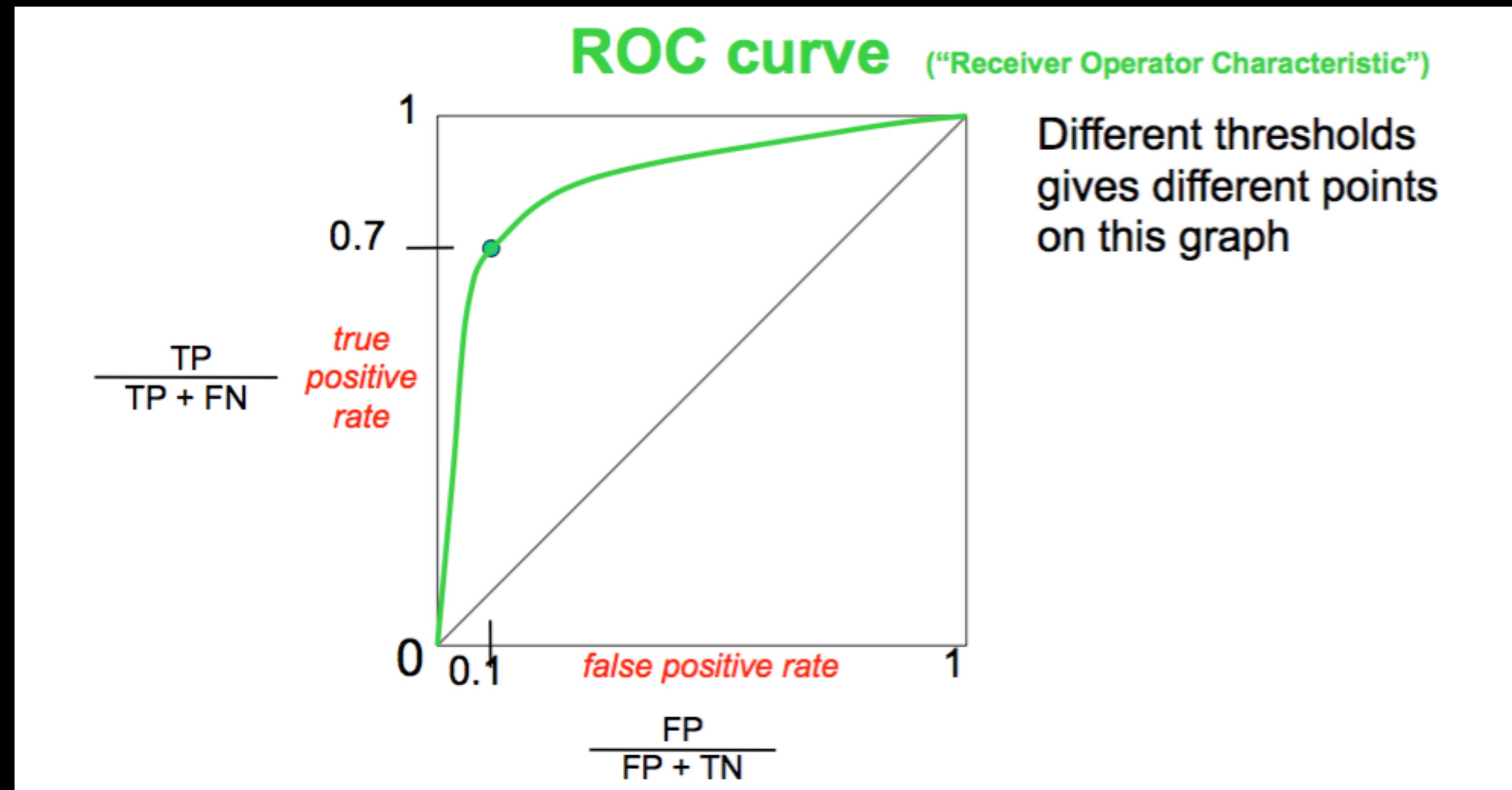
ROC Curves

Want to maximize area under the curve  
(AUC)

Useful for comparing different feature  
matching methods

For more info: [http://en.wikipedia.org/  
wiki/Receiver\\_operating\\_characteristic](http://en.wikipedia.org/wiki/Receiver_operating_characteristic)

Recall close to 1



## FEATURE MATCHING

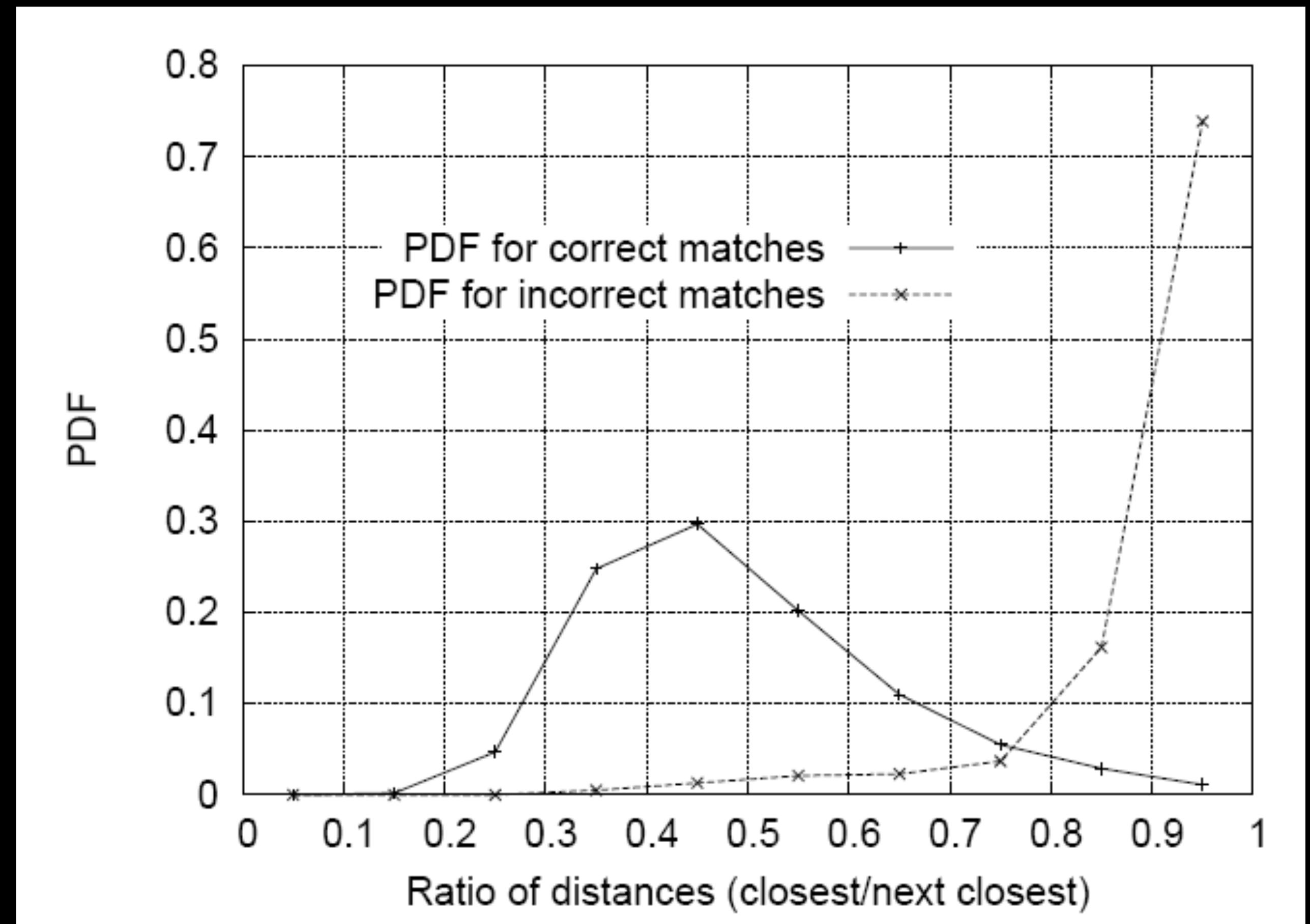
- ▶ Simple criteria: One feature matches to another if those features are nearest neighbours and their distance is below some threshold.
- ▶ Problems:
  - ▶ Ideally, threshold adapts to different regions of the feature space using training data
  - ▶ If not possible, the ratio with the second NN is a good heuristic

## FEATURE MATCHING

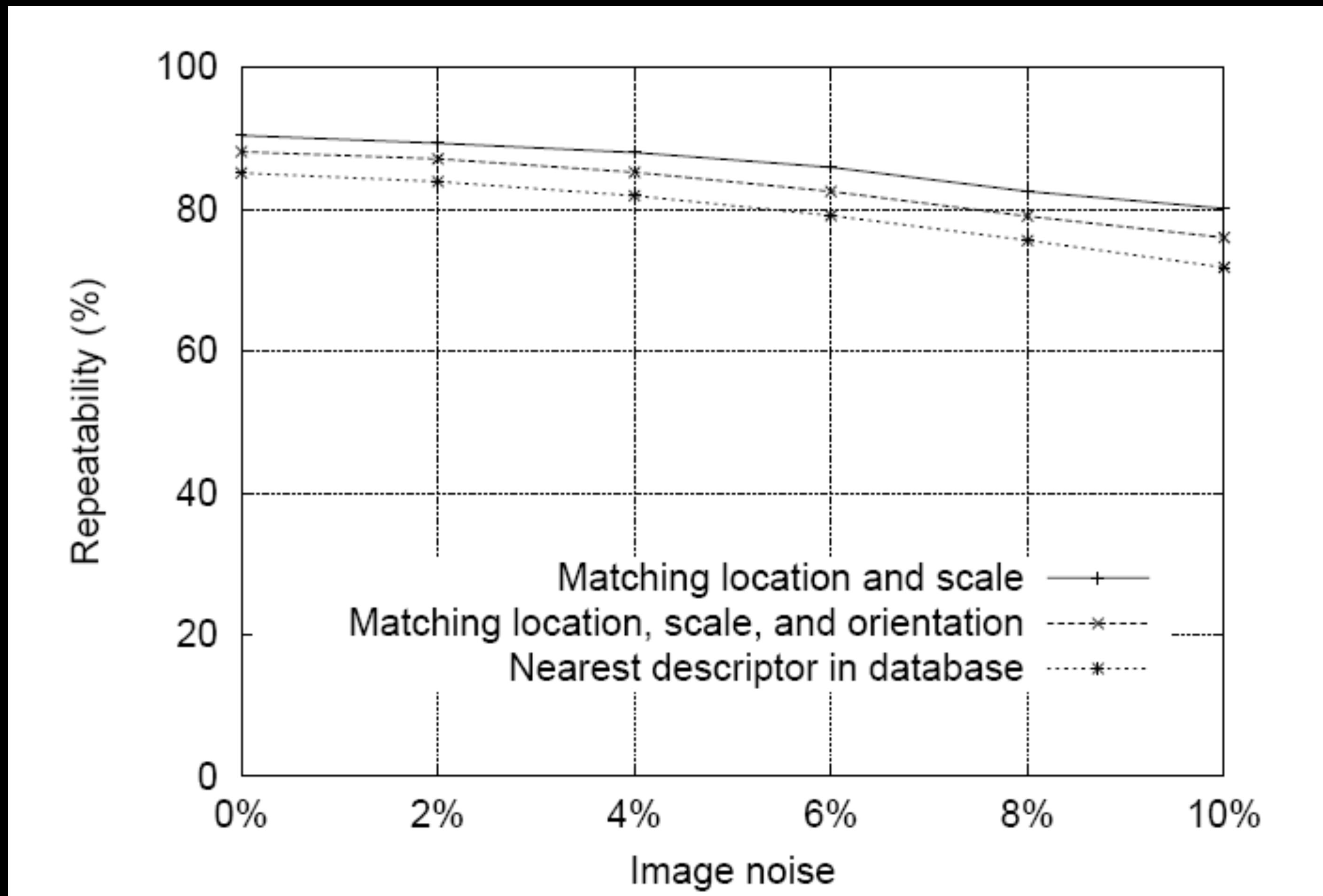
- ▶ Matching Strategy
  - ▶ determines which correspondences are passed to next stage
  - ▶ threshold
  - ▶ nearest neighbor
  - ▶ NN distance ratio
- ▶ Data Structures and Algorithms
  - ▶ perform the matching as quickly as possible

## MATCHING LOCAL FEATURES

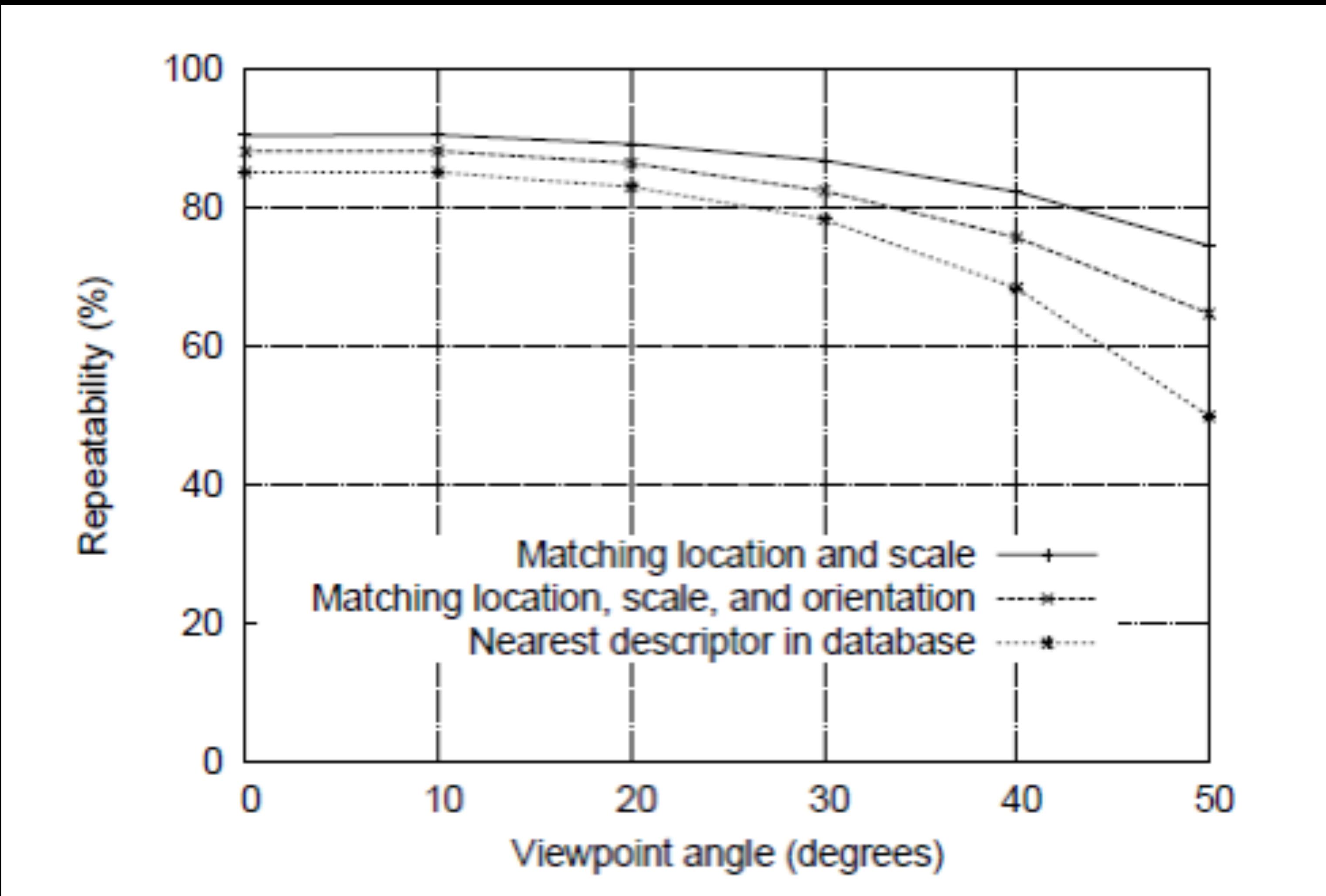
- ▶ Nearest neighbor  
(Euclidean distance)
- ▶ Threshold ratio of nearest  
to 2<sup>nd</sup> nearest descriptor
- ▶ 0.8 rejects 90% of false  
matches



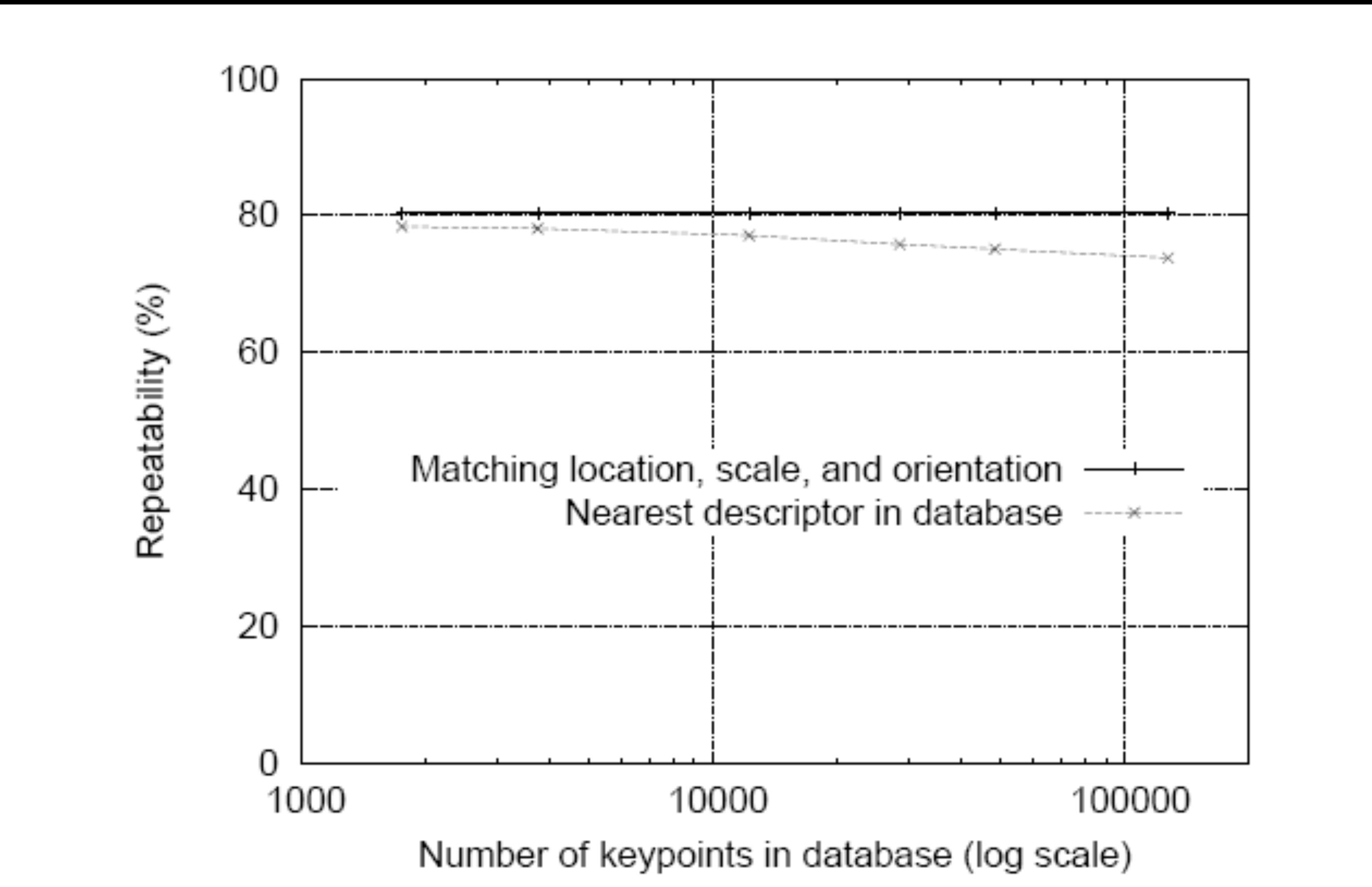
# SIFT REPEATABILITY



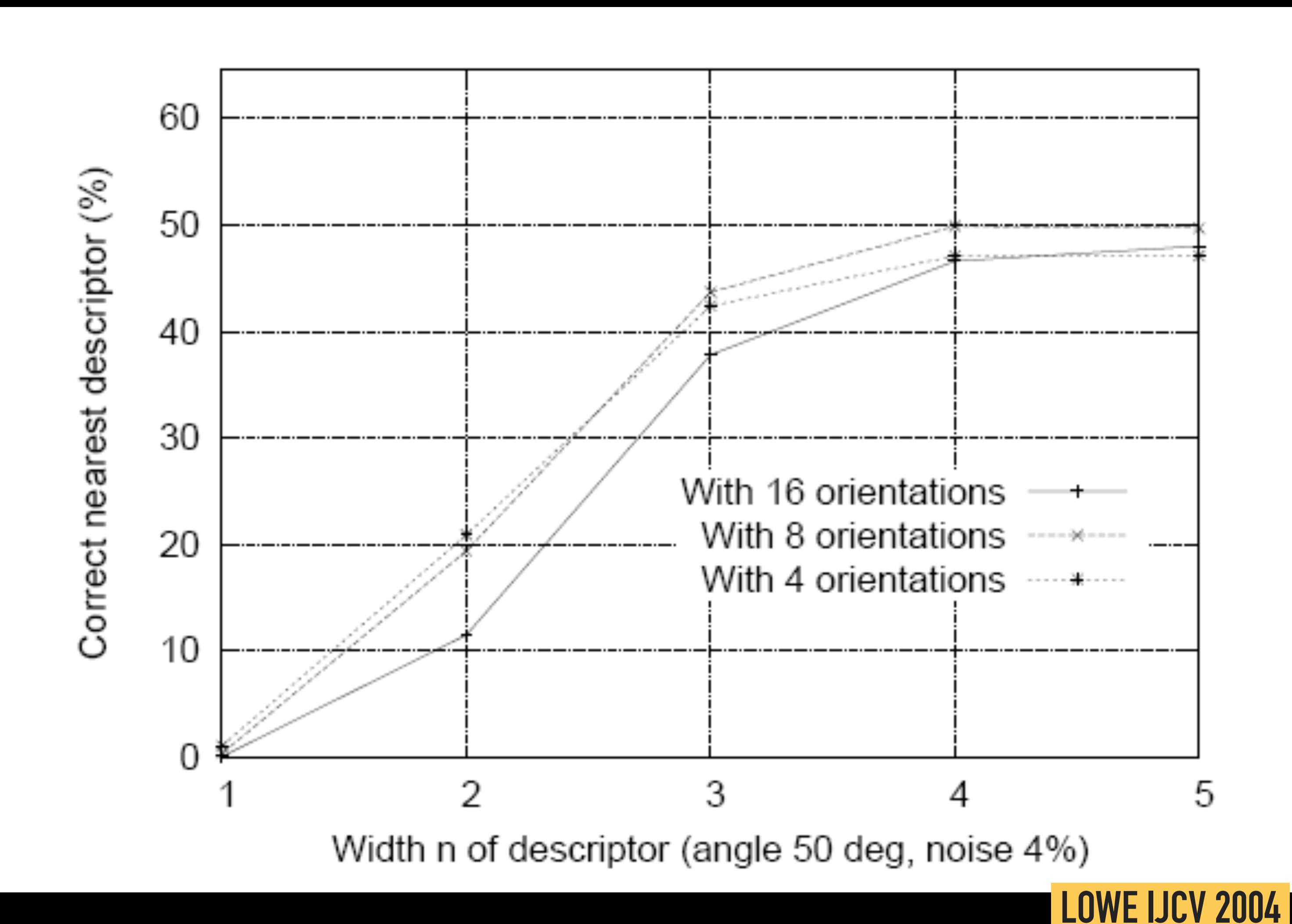
## SIFT REPEATABILITY



## SIFT REPEATABILITY



## SIFT REPEATABILITY



## CHOOSING A DETECTOR

- ▶ What do you want it for?
  - ▶ Precise localization in x-y: Harris
  - ▶ Good localization in scale: Difference of Gaussian
  - ▶ Flexible region shape: MSER
- ▶ Best choice often application dependent
  - ▶ Harris-/Hessian-Laplace/DoG work well for many natural categories
  - ▶ MSER works well for buildings and printed things
- ▶ Why choose?
  - ▶ Get more points with more detectors
- ▶ There have been extensive evaluations/comparisons
  - ▶ [Mikolajczyk et al., IJCV'05, PAMI'05]
  - ▶ All detectors/descriptors shown here work well

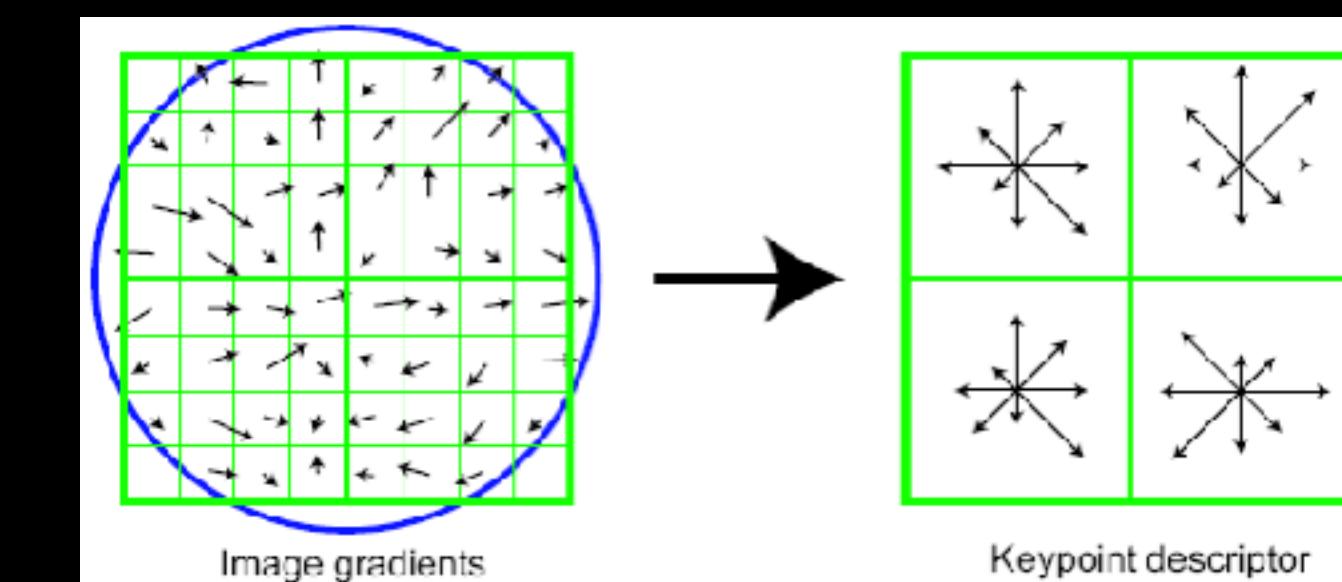
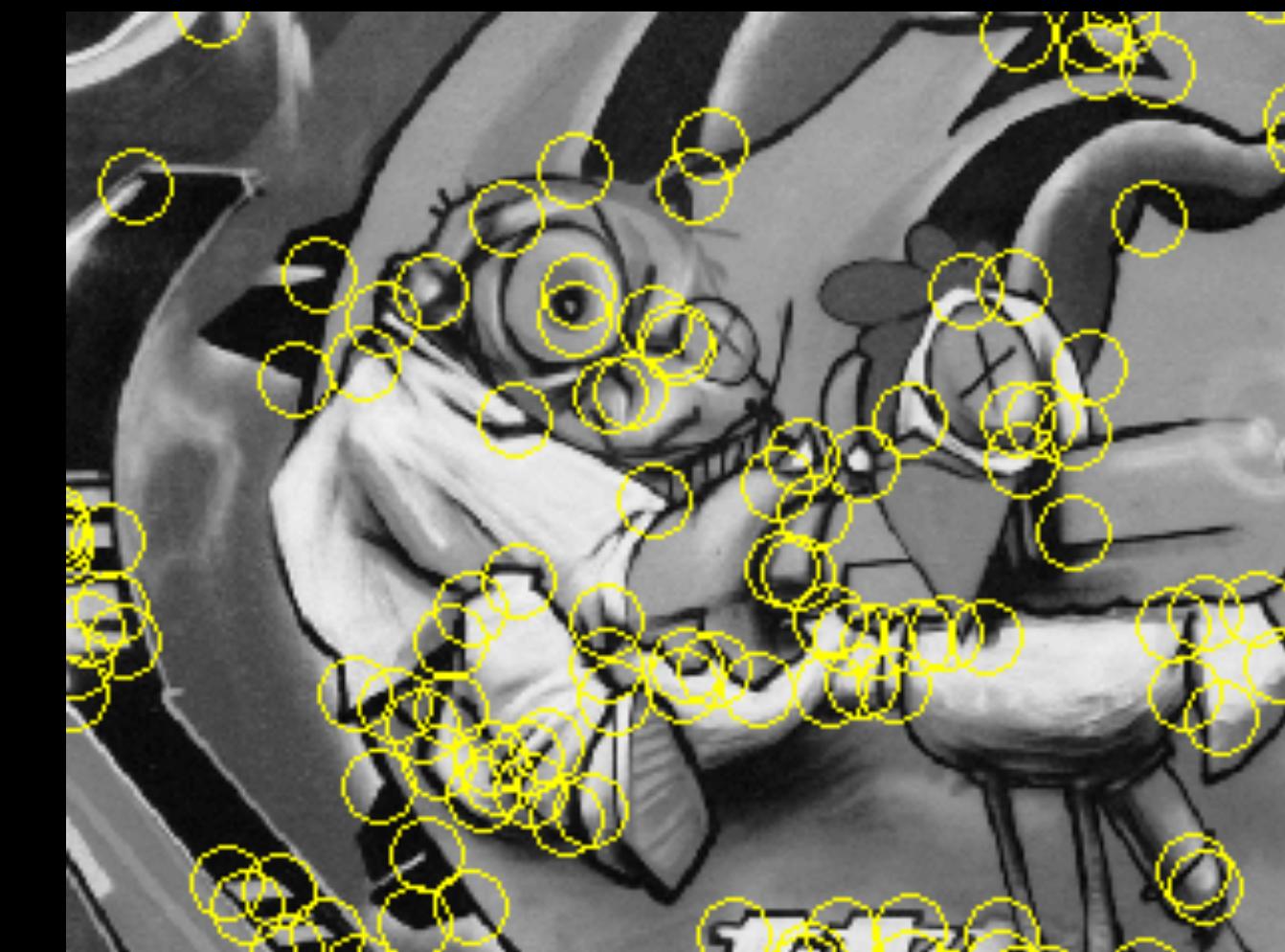
# COMPARISON OF KEYPOINT DETECTORS

Table 7.1 Overview of feature detectors.

Feature Detector	Corner	Blob	Region	Rotation	Scale	Affine	Localization			
				invariant	invariant	invariant	Repeatability	accuracy	Robustness	Efficiency
Harris	✓			✓			+++	+++	+++	++
Hessian		✓		✓			++	++	++	+
SUSAN	✓			✓			++	++	++	+++
Harris-Laplace	✓	(✓)		✓	✓		+++	+++	++	+
Hessian-Laplace	(✓)	✓		✓	✓		+++	+++	+++	+
DoG	(✓)	✓		✓	✓		++	++	++	++
SURF	(✓)	✓		✓	✓		++	++	++	+++
Harris-Affine	✓	(✓)		✓	✓	✓	+++	+++	++	++
Hessian-Affine	(✓)	✓		✓	✓	✓	+++	+++	+++	++
Salient Regions	(✓)	✓		✓	✓	(✓)	+	+	++	+
Edge-based	✓			✓	✓	✓	+++	+++	+	+
MSER		✓		✓	✓	✓	+++	+++	++	+++
Intensity-based		✓		✓	✓	✓	++	++	++	++
Superpixels		✓		✓	(✓)	(✓)	+	+	+	+

## THINGS TO REMEMBER

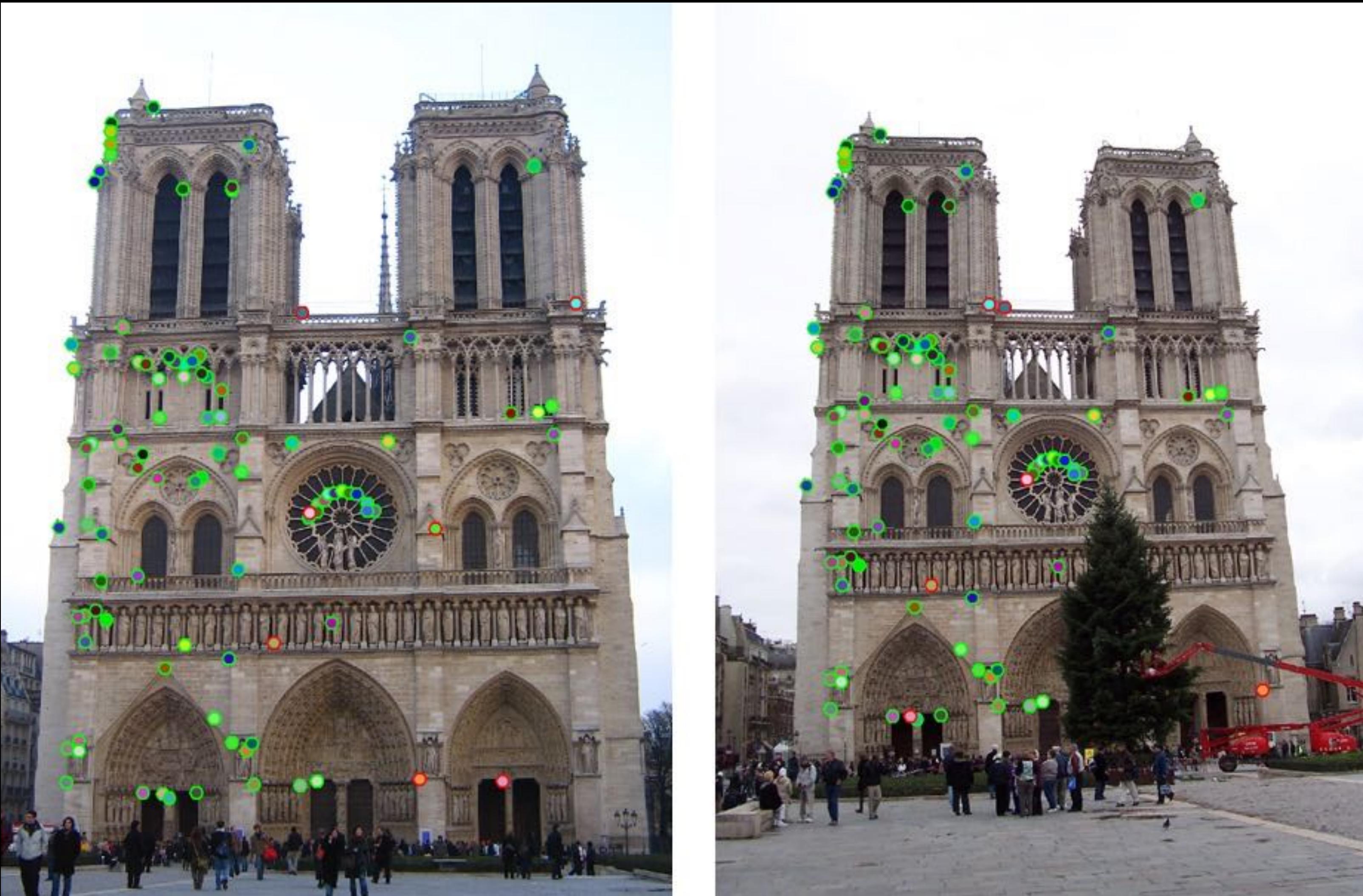
- ▶ Keypoint detection: repeatable and distinctive
  - ▶ Corners, blobs, stable regions
  - ▶ Harris, DoG
- ▶ Descriptors: robust and selective
  - ▶ spatial histograms of orientation
- ▶ SIFT



TEXT

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## HOW DO WE DECIDE WHICH FEATURES MATCH?



## HOW DO WE DECIDE WHICH FEATURES MATCH?

- ▶ Fitting: find the parameters of a model that best fit the data
- ▶ Alignment: find the parameters of the transformation that best align matched points

## FITTING AND ALIGNMENT

- ▶ Design challenges
- ▶ Design a suitable goodness of fit measure
- ▶ Similarity should reflect application goals
- ▶ Encode robustness to outliers and noise
- ▶ Design an optimization method
- ▶ Avoid local optima
- ▶ Find best parameters quickly

## FITTING AND ALIGNMENT: METHODS

- ▶ Global optimization / Search for parameters
  - ▶ Least squares fit
  - ▶ Robust least squares
  - ▶ Iterative closest point (ICP)
- ▶ Hypothesize and test
  - ▶ Generalized Hough transform
  - ▶ RANSAC

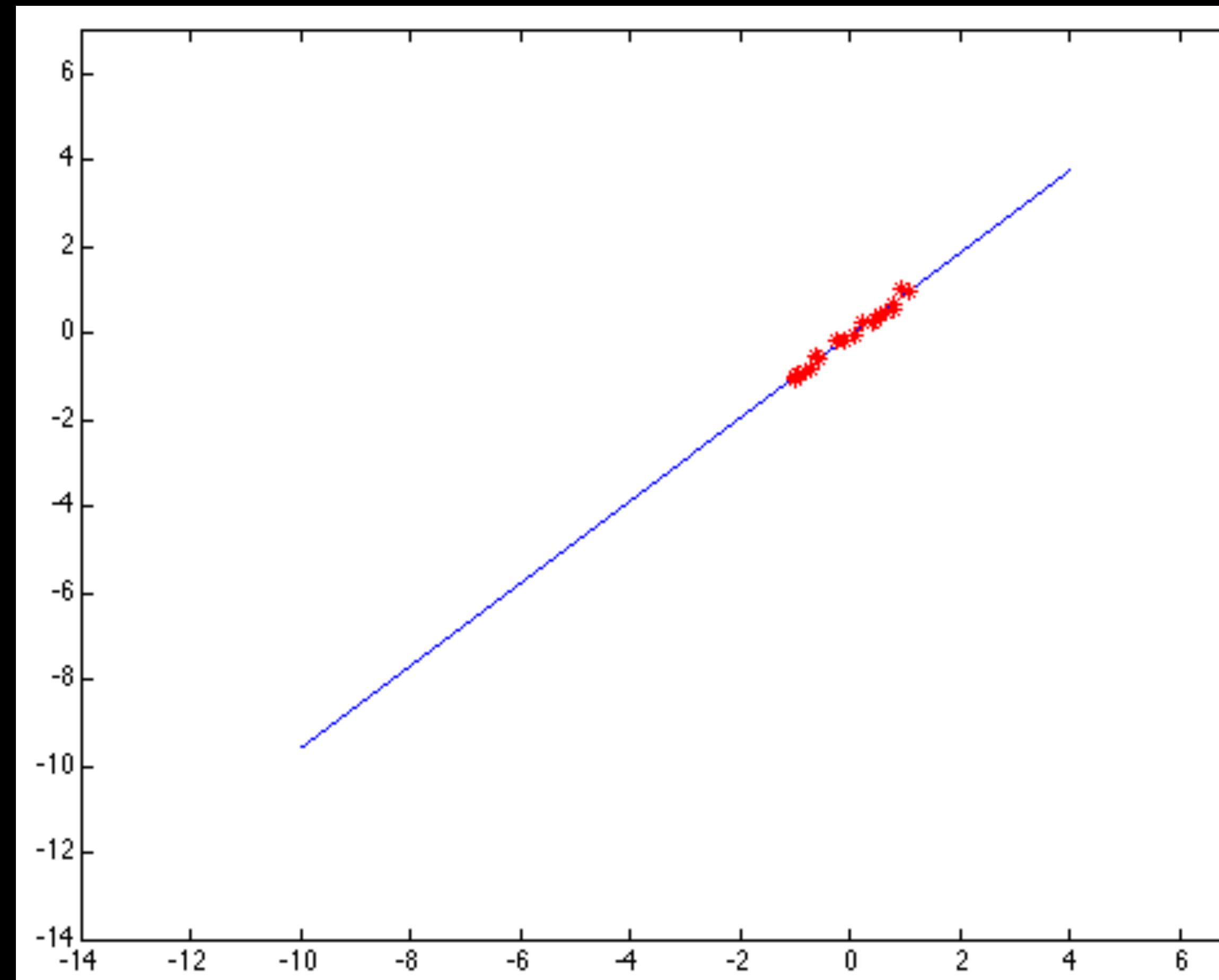
# **SIMPLE EXAMPLE: FITTING A LINE**

## LEAST SQUARES (GLOBAL) OPTIMIZATION

- ▶ Good
  - ▶ Clearly specified objective
  - ▶ Optimization is easy
- ▶ Bad
  - ▶ May not be what you want to optimize
  - ▶ Sensitive to outliers
  - ▶ Bad matches, extra points
  - ▶ Doesn't allow you to get multiple good fits
  - ▶ Detecting multiple objects, lines, etc.

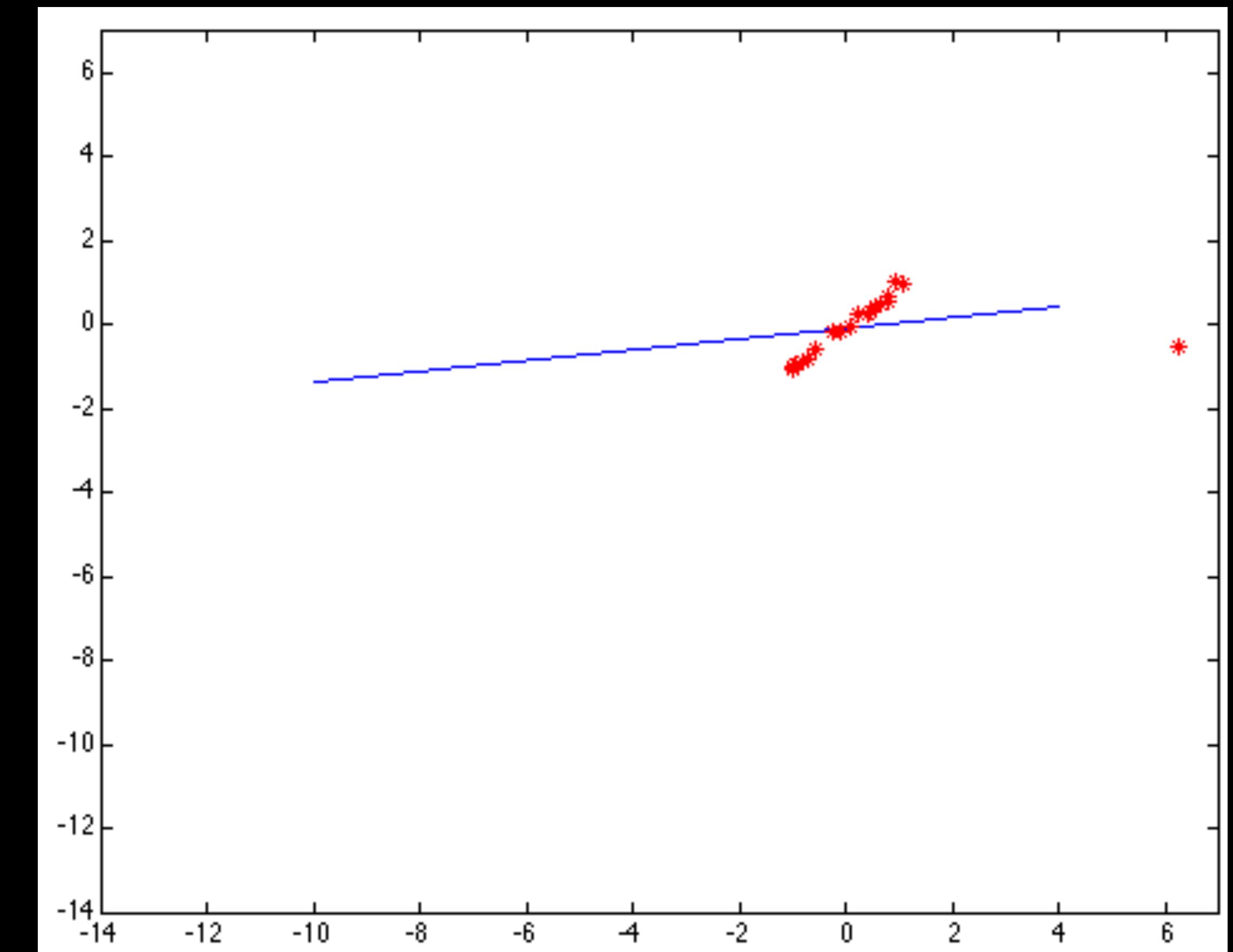
## LEAST SQUARES: ROBUSTNESS TO NOISE

- ▶ Least squares fit to the red points:



## LEAST SQUARES: ROBUSTNESS TO NOISE

- ▶ Least squares fit with an outlier:
- ▶ Problem: squared error heavily penalizes outliers



# Robust least squares (to deal with outliers)

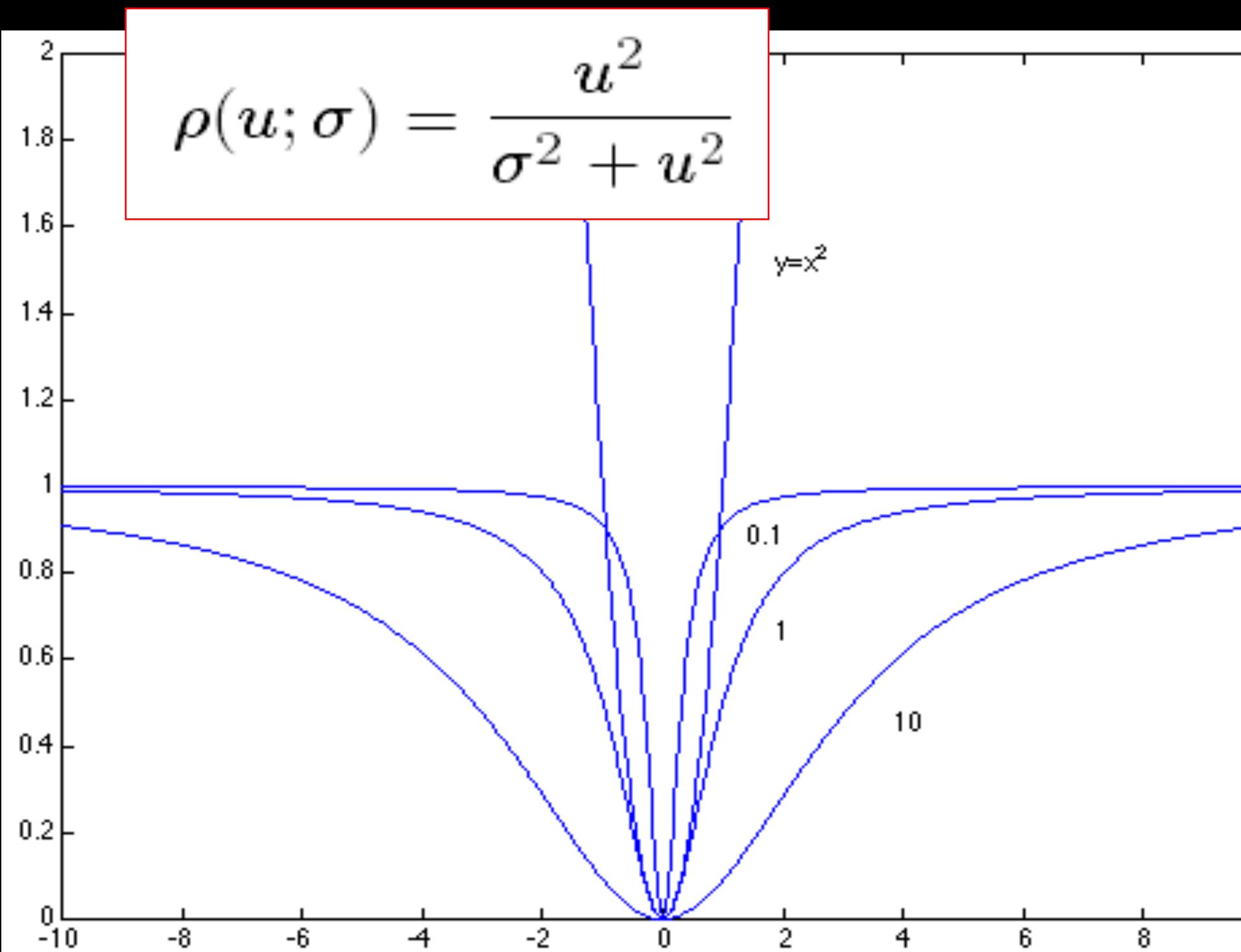
General approach:

minimize

$$\sum_i \rho(u_i(x_i, \theta), \sigma) \quad u^2 = \sum_{i=1}^n (y_i - mx_i - b)^2$$

$u_i(x_i, \theta)$  - residual of  $i^{th}$  point w.r.t. model parameters  $\theta$

$\rho$  - robust function with scale parameter  $\sigma$



The robust function  $\rho$

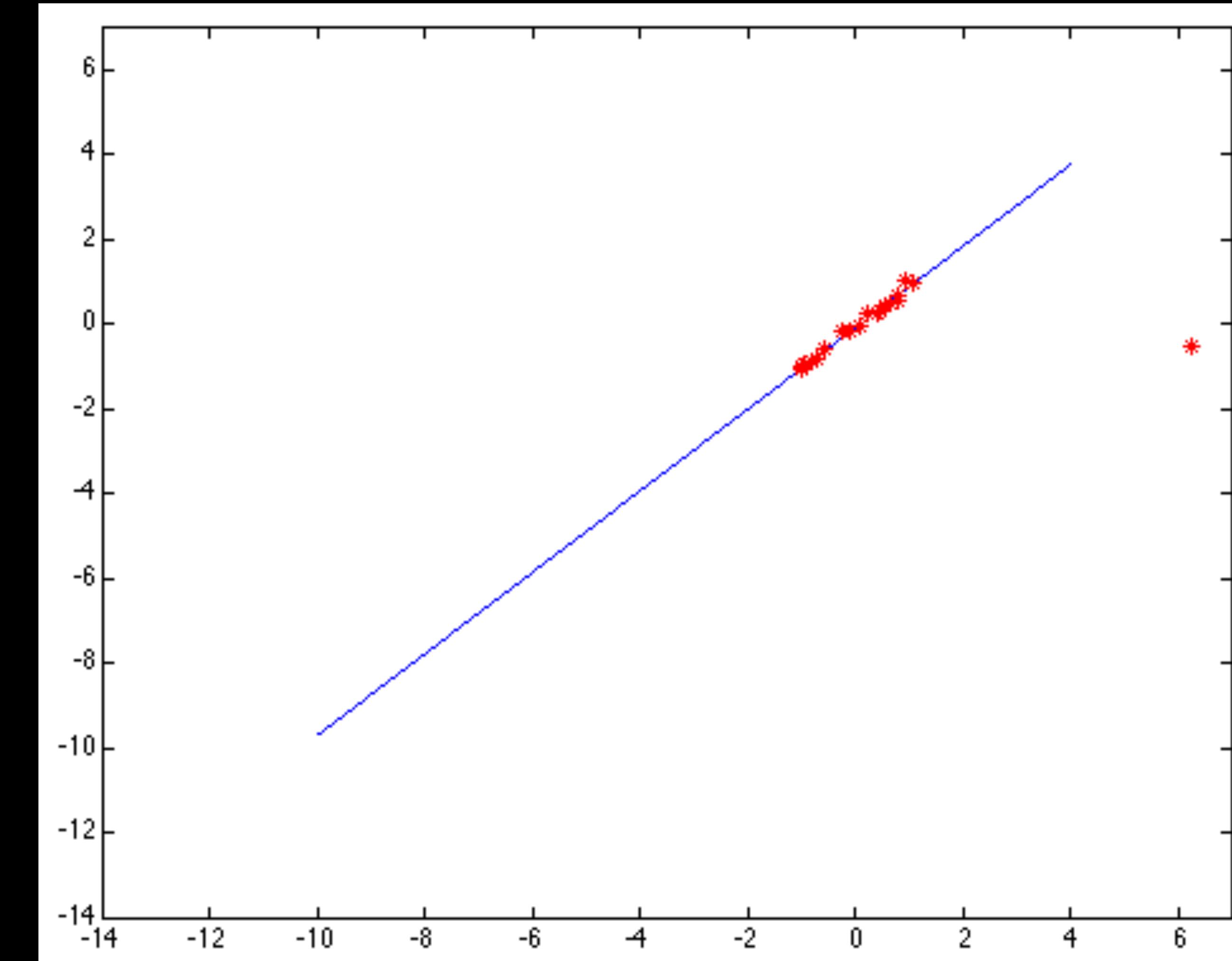
- Favors a configuration with small residuals
- Constant penalty for large residuals

TEXT

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## CHOOSING THE SCALE: JUST RIGHT

THE EFFECT OF THE OUTLIER IS MINIMIZED

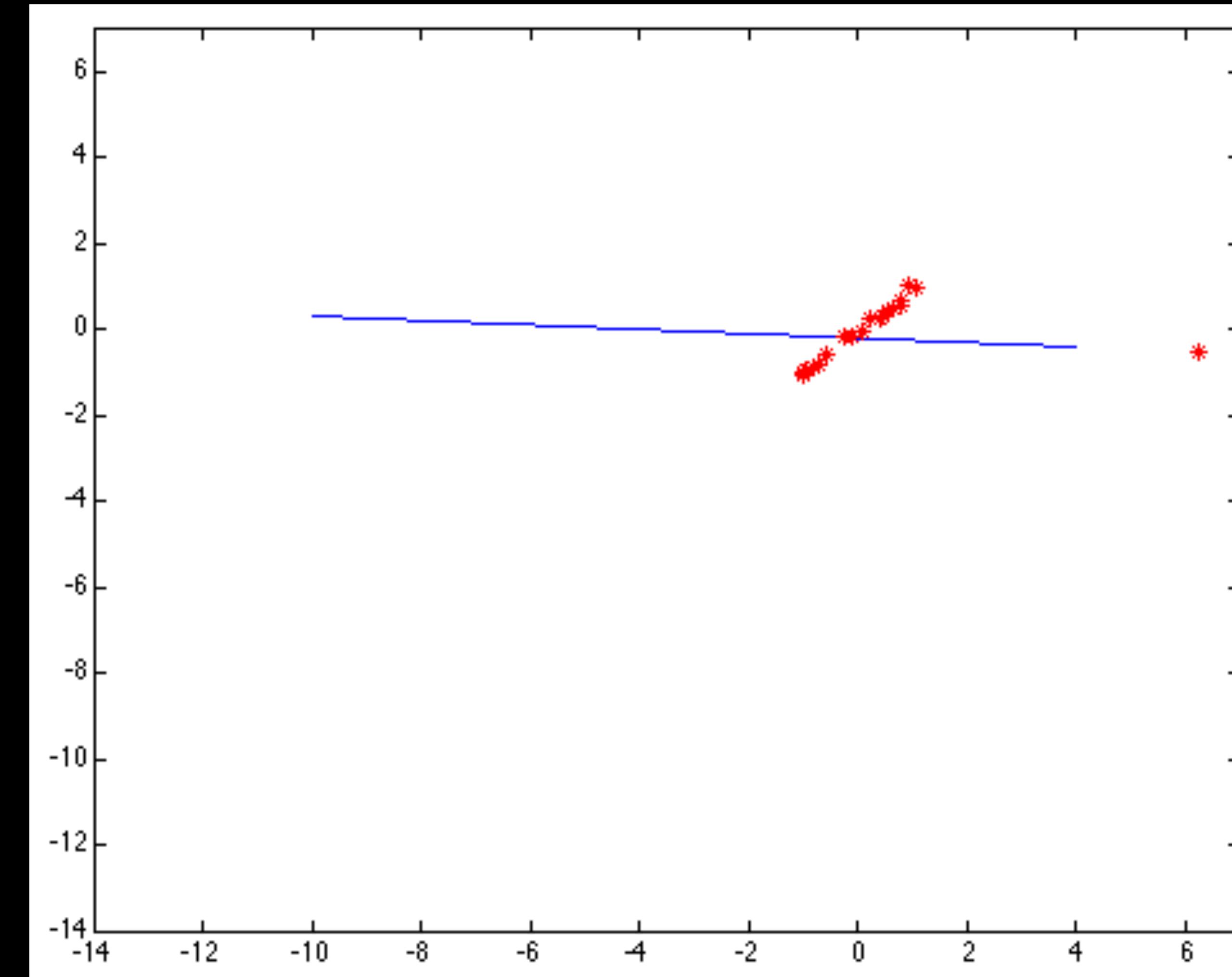


TEXT

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## CHOOSING THE SCALE: TOO SMALL

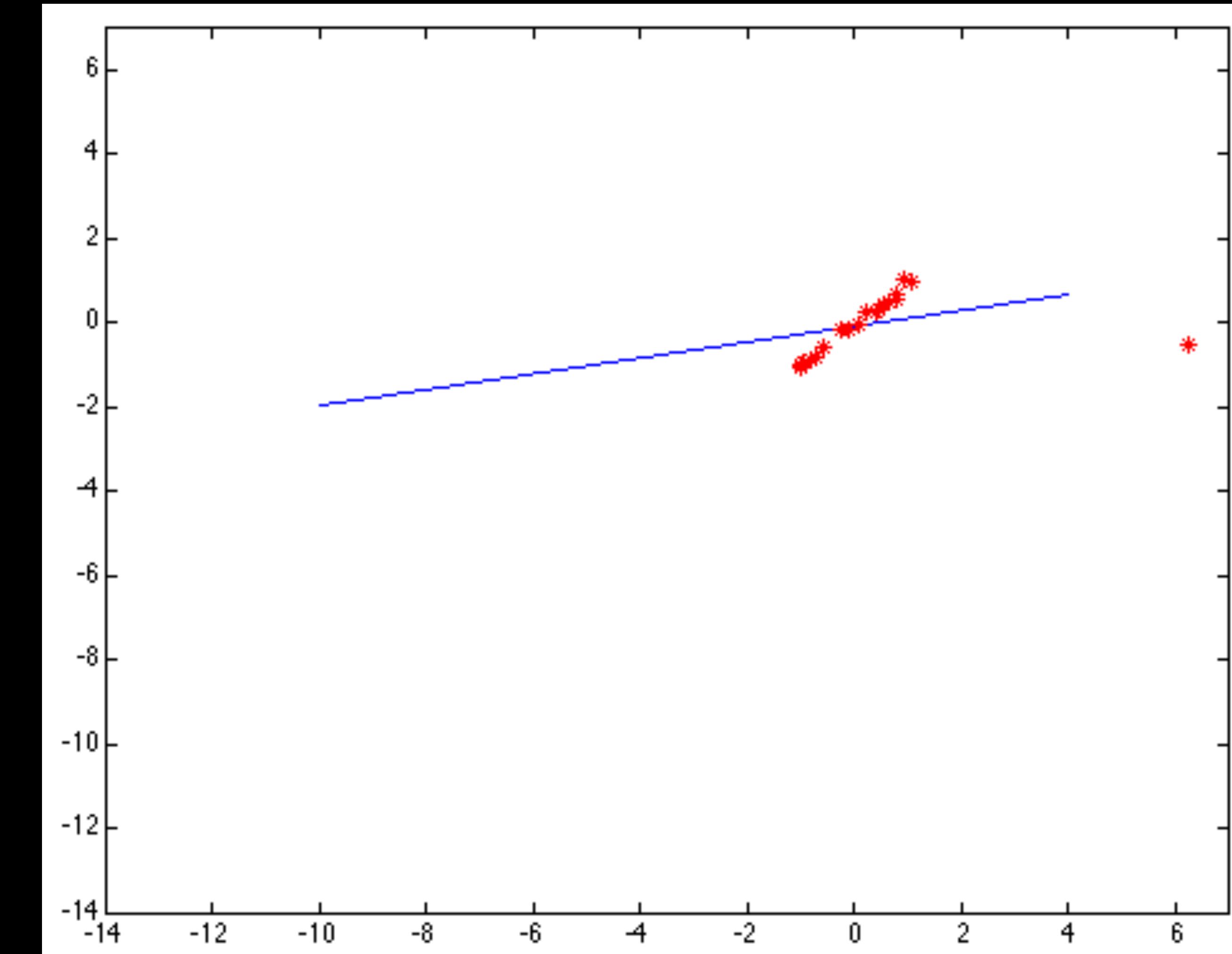
THE ERROR VALUE IS ALMOST THE SAME FOR EVERY POINT AND THE FIT IS VERY POOR



TEXT

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## CHOOSING THE SCALE: TOO LARGE



BEHAVES MUCH THE SAME AS LEAST SQUARES

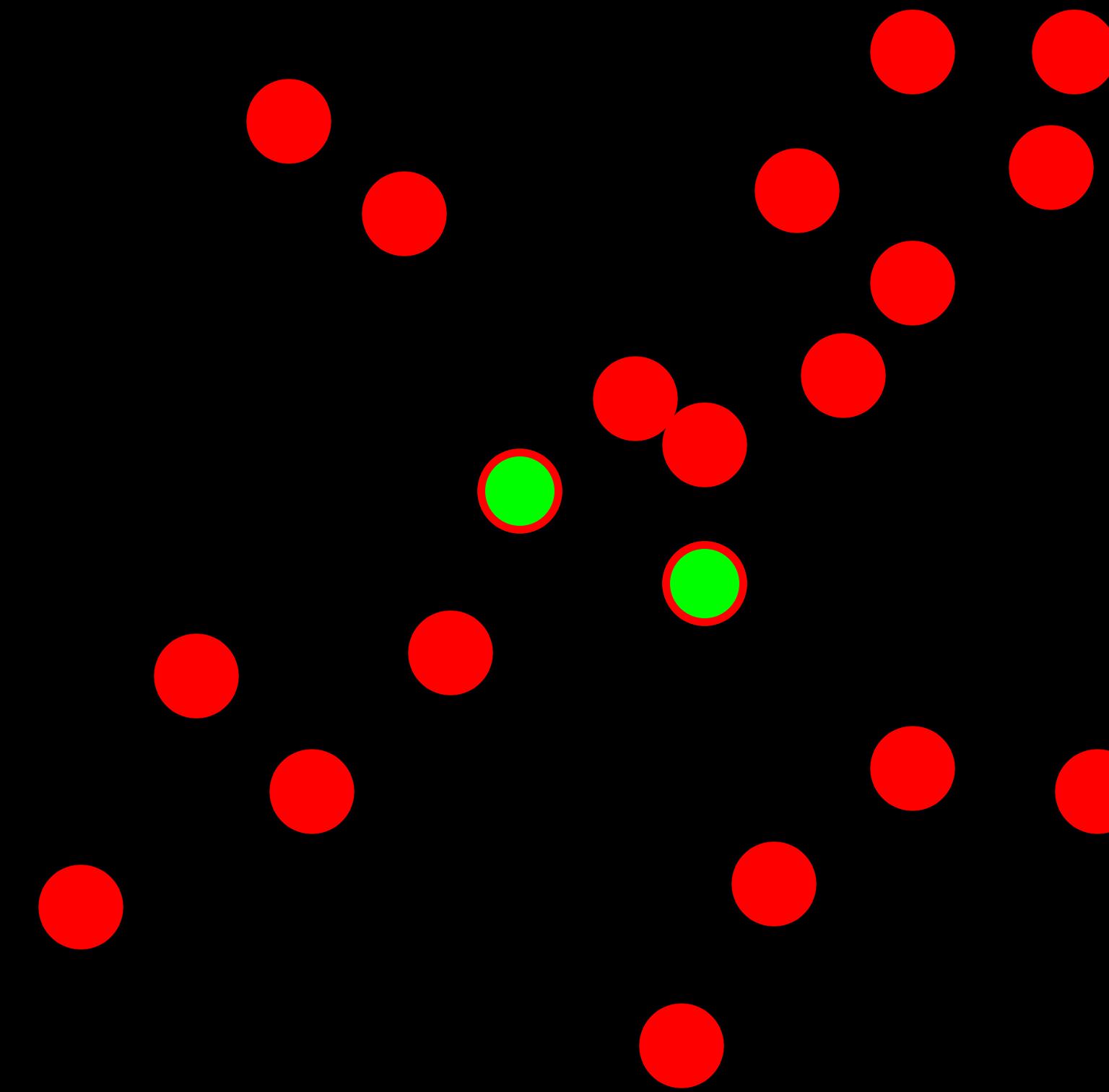
# RANSAC

## RANSAC (RANDOM SAMPLE CONSENSUS) :

- ▶ Algorithm:
- ▶ 1. Sample (randomly) the number of points required to fit the model
- ▶ 2. Solve for model parameters using samples
- ▶ 3. Score by the fraction of inliers within a preset threshold of the model
- ▶ Repeat 1-3 until the best model is found with high confidence

# RANSAC

## LINE FITTING EXAMPLE



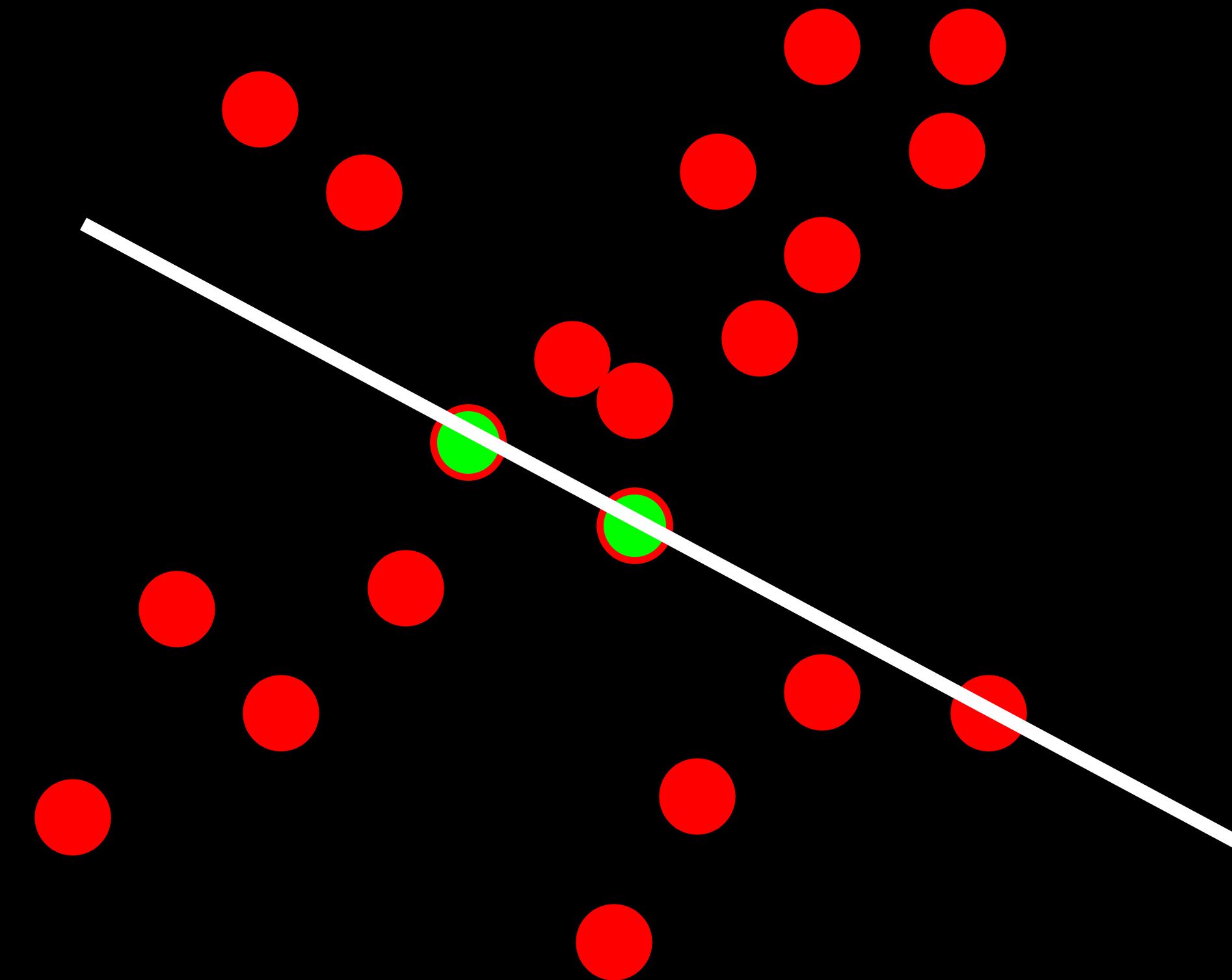
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

# RANSAC

## LINE FITTING EXAMPLE



Algorithm:

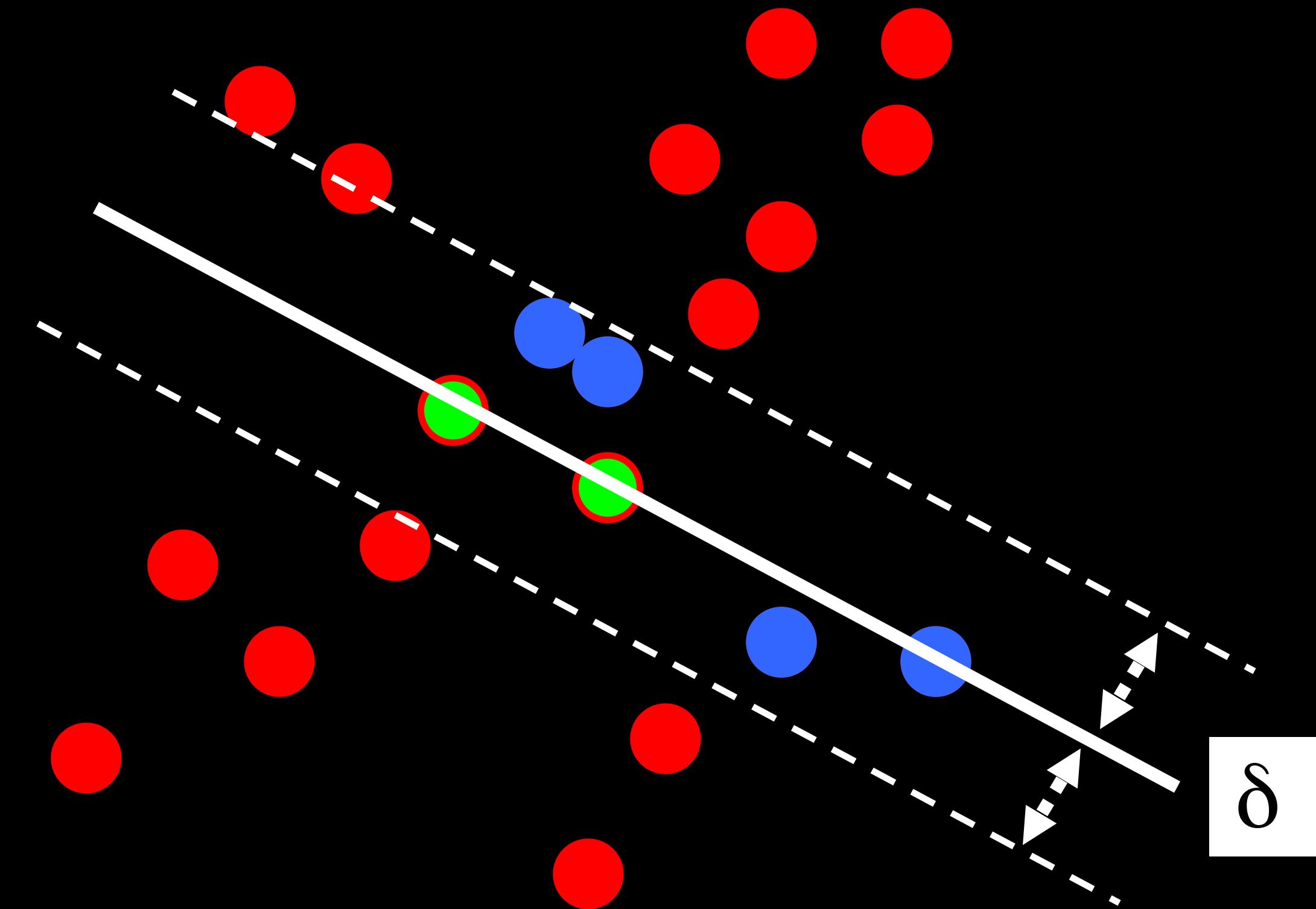
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

# RANSAC

## LINE FITTING EXAMPLE

$$N_I = 6$$



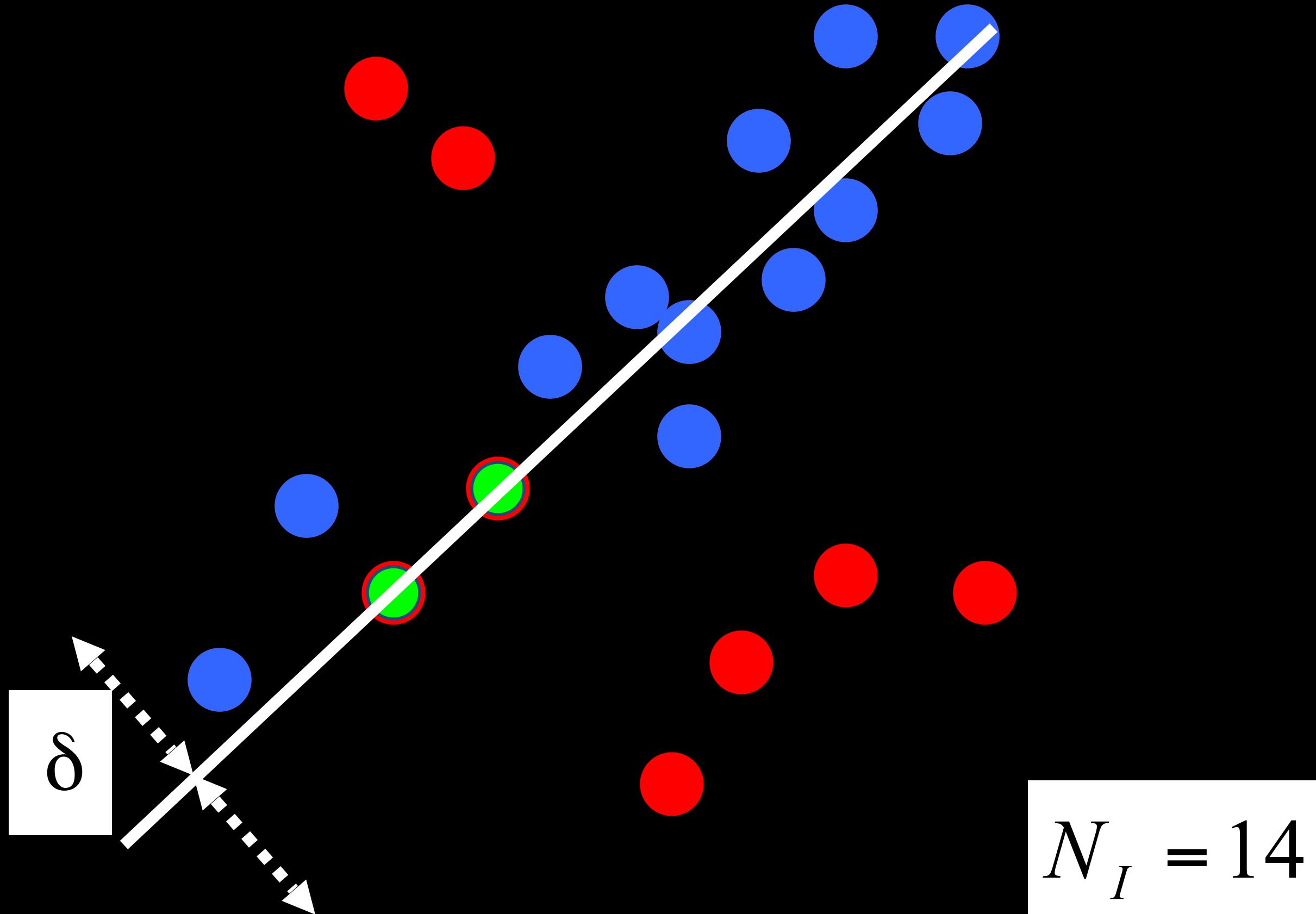
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

# RANSAC

## LINE FITTING EXAMPLE



Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

**Repeat** 1-3 until the best model is found with high confidence

# HOW TO CHOOSE PARAMETERS?

- Number of samples  $N$ 
  - Choose  $N$  so that, with probability  $p$ , at least one random sample is free from outliers (e.g.  $p=0.99$ ) (outlier ratio:  $e$  )
- Number of sampled points  $s$ 
  - Minimum number needed to fit the model
- Distance threshold  $\delta$ 
  - Choose  $\delta$  so that a good point with noise is likely (e.g., prob=0.95) within threshold
  - Zero-mean Gaussian noise with std. dev.  $\sigma$ :  $t^2=3.84\sigma^2$

$$N = \log(1 - p) / \log(1 - (1 - e)^s)$$

$s$	proportion of outliers $e$							
	5%	10%	20%	25%	30%	40%	50%	
2	2	3	5	6	7	11	17	
3	3	4	7	9	11	19	35	
4	3	5	9	13	17	34	72	
5	4	6	12	17	26	57	146	
6	4	7	16	24	37	97	293	
7	4	8	20	33	54	163	588	
8	5	9	26	44	78	272	1177	

## RANSAC CONCLUSIONS

Good

- ▶ Robust to outliers

Bad

- ▶ Computational time grows quickly with fraction of outliers and number of parameters

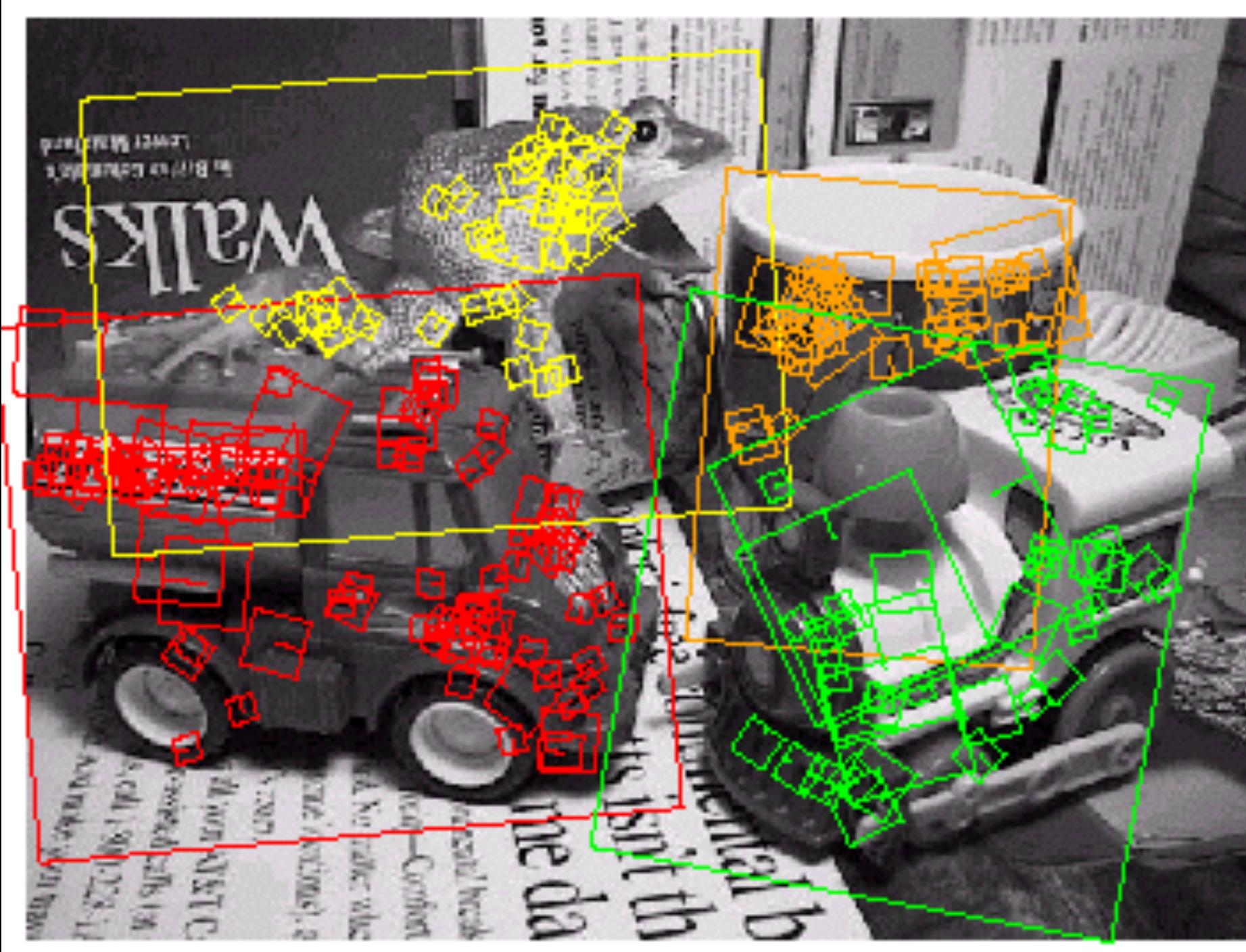
Common applications

- ▶ Estimating fundamental matrix (relating two views)
- ▶ Computing a homography (e.g., image stitching)

TEXT

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## EXAMPLES OF RECOGNIZED OBJECTS



- ▶ Most slides taken from James Hays
- ▶ <http://www.cc.gatech.edu/~hays/>