

Functional stuff: GADT, Existential types, Rank-N-Types, ...

Rafał Łasocha

Wrocław, 25th March 2015

- A **phantom type** is a parametrised type whose parameters do not all appear on the right-hand side of its definition

```
data FormData a = FormData String
```

```
data FormData a = FormData String
```

```
data Validated
```

```
data Unvalidated
```

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data FormData a = FormData String
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```
formData :: String -> FormData Unvalidated
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formData str = FormData str
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```
useData :: FormData Validated -> IO ()
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```
useData (FormData str) = ...
```

```
liftStringFn :: (String -> String) -> FormData a -> FormData a
```

```
liftStringFn fn (FormData str) = FormData (fn str)
```



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```
liftStringFn :: (String -> String) -> FormData a -> FormData a
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```
liftStringFn fn (FormData str) = FormData (fn str)
```

```
dataToUpper :: FormData a -> FormData a
```

```
dataToUpper = liftStringFn (map toUpper)
```

- **Generalised Algebraic Datatypes** (*GADTs*) are datatypes for which a constructor has a non standard type.

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data List x y where
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safeHead :: List x NonEmpty -> x
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silly 1 = Cons 1 Nil
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```

```
silly 0 = Nil
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- it can't infer proper type!

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data Worker x =  
  forall b. Buffer b => Worker {buffer :: b, input :: x}
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data MemoryBuffer = MemoryBuffer  
memoryWorker = Worker MemoryBuffer (1 :: Int)
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data Worker x =  
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memoryWorker :: Worker Int
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- it's impossible for function to **demand** specific Buffer
- you're more limited in what you can do with Worker like that

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data Worker x =
  forall b. Buffer b ==> Worker {buffer :: b, input :: x}

data MemoryBuffer = MemoryBuffer
memoryWorker = Worker MemoryBuffer (1 :: Int)
memoryWorker :: Worker Int

data NetBuffer = NetBuffer
netWorker = Worker NetBuffer (2 :: Int)
netWorker :: Worker Int
```

```
data Worker b x = Worker {buffer :: b, input :: x}
data Worker x =
  forall b. Buffer b ==> Worker {buffer :: b, input :: x}

data MemoryBuffer = MemoryBuffer
memoryWorker = Worker MemoryBuffer (1 :: Int)
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data NetBuffer = NetBuffer
netWorker = Worker NetBuffer (2 :: Int)
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workers = [netWorker, memoryWorker]
workers :: [Worker Int]
```

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```
ghci> let putInList x = [x]
ghci> liftTup putInList (5, "Blah") # We want to achieve this!
([5], ["Blah"])
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```
ghci> liftTup (\x -> [x]) (5, "Blah") # We want to achieve this!  
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-- *First try:*

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ghci> let liftTup liftFunc (a, b) = (liftFunc a, liftFunc b)
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ghci> liftTup (\x -> [x]) (5, "Blah") # We want to achieve this!  
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ghci> let liftTup liftFunc (a, b) = (liftFunc a, liftFunc b)  
ghci> liftTup (\x -> [x]) (5, "Hello")  
No instance for (Num [Char]) arising from literal '5'  
...
```

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ghci> liftTup (\x -> [x]) (5, "Blah") # We want to achieve this!  
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ghci> :t liftTup  
liftTup :: (t -> t1) -> (t, t) -> (t1, t1)
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— *test.hs:*

```
liftTup :: (x -> f x) -> (a, b) -> (f a, f b)  
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liftTup :: (x -> f x) -> (a, b) -> (f a, f b)  
liftTup liftFunc (t, v) = (liftFunc t, liftFunc v)
```

```
ghci> :l test.hs
```

Couldnt match expected type 'x' against inferred type 'a'

...

```
ghci> liftTup (\x -> [x]) (5, "Blah") # We want to achieve this!  
([5], ["Blah"])
```

-- *Second try*

```
liftTup :: (x -> f x) -> (a, b) -> (f a, f b)  
...
```

-- *Third try*

```
liftTup :: (forall x. x -> f x) -> (a, b) -> (f a, f b)
```

```
ghci> liftTup putInList (5, "Hello")  
([5], ["Hello"])
```

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- It adds abstract layer above IO
- It describes how our program want to use IO
- ...but it doesn't show to the user internals of the communication
- It's pure!

```
class Monad m => MonadPrompt p m where  
  prompt :: p a -> m a
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```
data Prompt p r  
instance MonadPrompt p (Prompt p)  
instance MonadPrompt p (PromptT p m)
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```
runPrompt :: (forall a. p a -> a) -> Prompt p r -> r
runPromptM :: Monad m => (forall a. p a -> m a) -> Prompt p r ->
```

data Request a where

Echo :: **String** -> Request ()

GetLine :: Request (**Maybe String**)

GetTime :: Request UTCTime

```
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handleIO :: Request a -> IO a
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```
data Request a where
  Echo :: String -> Request ()
  GetLine :: Request (Maybe String)
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handleIO :: Request a -> IO a
handleIO (Echo s) = putStrLn s
handleIO GetLine = catchJust
  (guard . isEOFError)
  (Just <$> getLine)
  (const (return Nothing))
handleIO GetTime = getCurrentTime
```

```
handleIO :: Request a -> IO a
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```

```
cat :: Prompt Request ()
```

```
cat = do
```

```
  line <- prompt GetLine
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  maybe (return ()) (\x -> prompt (Echo x) >> cat) line
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  m r
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```
runCat :: IO ()
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```
runCat = runPromptM handleIO cat
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- Why runPromptM needs to be Rank2Type function?

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data Request a where
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- What we achieved?

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- Why is that cool?

RWS monad

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- Writer: Something, to which we can write data (for example, logging to file)
 - `tell` function is writing to the Writer monad
- State we already know.
 - `get` gets the state
 - `put` sets the state

```
type Input = [String]  
type Output = [String]
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```

```
handleRWS :: Request a -> RWS r Output Input a
handleRWS (Echo s) = tell (return s)
handleRWS GetLine = do
  lines <- get
  if null lines
  then return Nothing
  else do
    put (tail lines)
    return (Just (head lines))
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rwsCat :: RWS r Output Input ()
rwsCat = runPromptM handleRWS cat
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```

```
simulateCat :: Input -> Output
simulateCat input = snd $ evalRWS rwsCat undefined input
```

- Red-black trees properties:
 - every black node has two black children
 - every path from the root to an empty tree passes through the same number of black nodes

```
type Tr t a b = (t a b, a, t a b)
data Red t a b = C (t a b) | R (Tr t a b)
data Black a b = E | B(Tr (Red Black) a [b])
```

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- Red is really MaybeRed
- C (...) constructor means that there's Black node inside
- When there's Red node inside of Red node, we need to do something. Thus, type Red (Red Black) a b means dangerous situation.

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- Red is really MaybeRed
- C (...) constructor means that there's Black node inside
- When there's Red node inside of Red node, we need to do something. Thus, type Red (Red Black) a b means dangerous situation.
- Note that Black nodes are always increasing their depth, by passing it down increased by one (look at [b] at Black constructor)

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```

type Tr t a b = (t a b, a, t a b)
data Red t a b = C (t a b) | R (Tr t a b)
data Black a b = E | B(Tr (Red Black) a [b])

balanceL :: Red (Red Black) a [b] -> a -> Red Black a [b] -> Red Bl
  
```

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data Red t a b = C (t a b) | R (Tr t a b)
data Black a b = E | B(Tr (Red Black) a [b])

balanceL :: Red (Red Black) a [b] -> a -> Red Black a [b] -> Red Black a [b]
balanceL (R(R(a,x,b),y,c)) z d = R(B(C a,x,C b),y,B(c,z,d))
    
```

```

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balanceL (R(R(a,x,b),y,c)) z d = R(B(C a,x,C b),y,B(c,z,d))
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balanceL (R(C a,x,C b)) z d = C(B(R(a,x,b),z,d))
balanceL (C a) x b = C(B(a,x,b))
```

```
insB :: Ord a => a -> Black a b -> Red Black a b
insB x E = R(E,x,E)
insB x t@(B(a,y,b))
  | x<y = balanceL (insR x a) y b
  | x>y = balanceR a y (insR x b)
  | otherwise = C t
```

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insB :: Ord a => a -> Black a b -> Red Black a b
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  | otherwise = C t
```

```
insR :: Ord a => a -> Red Black a b -> RR a b
insR x (C t) = C(insB x t)
insR x t@(R(a,y,b))
  | x<y = R(insB x a,y,C b)
  | x>y = R(C a,y,insB x b)
  | otherwise = C t
```

```
tickB :: Black a b -> Black a c
tickB E = E
tickB (B(a,x,b)) = B(tickR a,x,tickR b)
```



```
tickB :: Black a b -> Black a c
tickB E = E
tickB (B(a,x,b)) = B(tickR a,x,tickR b)

tickR :: Red Black a b -> Red Black a c
tickR (C t) = C(tickB t)
tickR (R(a,x,b)) = R(tickB a,x,tickB b)
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```
tickB :: Black a b -> Black a c
tickB E = E
tickB (B(a,x,b)) = B(tickR a,x,tickR b)
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```
tickR :: Red Black a b -> Red Black a c
tickR (C t) = C(tickB t)
tickR (R(a,x,b)) = R(tickB a,x,tickB b)
```

```
inc :: Black a b -> Black a [b]
inc = tickB
```

```
ghci> (E :: Black a [[[b]]])  
E  
it :: Black a [[[b]]]
```

```
ghci> (E :: Black a [[[b]]])
```

```
E
```

```
it :: Black a [[[b]]]
```

```
newtype Tree a = forall b . ENC (Black a b)
```

```
ghci> (E :: Black a [[[b]]])
```

```
E
```

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empty :: Tree a
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empty = ENC E
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```
insert :: Ord a => a -> Tree a -> Tree a
```

```
insert x (ENC t) = ENC(blacken (insB x t))
```

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ghci> (E :: Black a [[[b]]])
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```

```
insert x (ENC t) = ENC(blacken (insB x t))
```

```
blacken :: Red Black a b -> Black a b
```

```
blacken (C u) = u
```

```
blacken (R(a,x,b)) = B(C(inc a),x,C(inc b))
```

```
class Renderable a where
```

```
  boundingSphere :: a -> Sphere
```

```
  hit :: a -> [Fragment]  -- returns the "fragments" of all hits with ray
```



```
class Renderable a where
  boundingSphere :: a -> Sphere
  hit :: a -> [Fragment] -- returns the "fragments" of all hits with ray

hits :: Renderable a => [a] -> [Fragment]
hits xs = sortByDistance $ concatMap hit xs
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data AnyRenderable = forall a. Renderable a => AnyRenderable a
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data AnyRenderable = forall a. Renderable a => AnyRenderable a

instance Renderable AnyRenderable where
  boundingSphere (AnyRenderable a) = boundingSphere a
  hit (AnyRenderable a) = hit a
```

```
class Renderable a where
  boundingSphere :: a -> Sphere
  hit :: a -> [Fragment] -- returns the "fragments" of all hits with ray

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data AnyRenderable = forall a. Renderable a => AnyRenderable a

instance Renderable AnyRenderable where
  boundingSphere (AnyRenderable a) = boundingSphere a
  hit (AnyRenderable a) = hit a

[ AnyRenderable x
, AnyRenderable y
, AnyRenderable z ]
```

References

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