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An Activity-Based Ontology for Dates

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Abstract

The representation of dates and their relationship to time and duration has long been recognized as an important problem in commonsense reasoning. However, existing date ontologies, such as OWL-Time and Date-Time Foundation Vocabulary from the Object Modeling Group, take either over-simplistic or convoluted approaches to defining the key semantics for dates. We show that such approaches are inadequate and provide an improved solution: a first-order Date Ontology that is an extension of the Process Specification Language and an existing duration ontology. Rather than treat dates as a class of timepoints, we axiomatize dates as a class of complex activities which have multiple periodic occurrences. We consider two modules of the Date Ontology, and characterize the models of the Date Ontology up to elementary equivalence.

Introduction

Reasoning about durations in the context of a calendar composed of dates is a fundamental capability required for many applications, from supply-chain management and e-commerce to narrative analysis. For ontologies to support these sorts of applications they need to be able to infer, for example, that an order is late if we know that today is Friday and the order was sent on Monday, but the order should take only two days to arrive.

Earlier approaches to ontologies for durations, dates, and calendars lack complete axiomatizations – the intended semantics is specified in documentation but there are insufficient axioms to guarantee that all models are intended, or else arithmetic is used as the intended model (that is, reasoning about timedurations reduces to reasoning about real numbers). We want to find a first-order theory with the minimal set of assumptions that axiomatizes the class of intended models, rather than use a specific intended model.

In this paper we extend earlier work with durations (Grüninger 2010) and introduce a first-order ontology that axiomatizes intuitions about dates and calendars. The fundamental ontological commitment of this paper is based on the observation that since dates have multiple repeatable occurrences at different timepoints, we can define them as a class of activities. Calendars are complex activities composed of dates, and the axioms that specify how such complex activities occur are also introduced. Specific calendars, such as the Gregorian and lunar calendars, are domain theories that can be defined as extensions of the Date Ontology.

The Date Ontology introduced in this paper consists of two modules (referred to as $T_{date\_periodic}$ and $T_{date\_compose}$) that formalize fundamental intuitions about dates. The remainder of the paper will explore the axiomatizations in these modules more closely; we will also characterize the models of the Date Ontology.

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Dates in Practice

Although it is tempting and even intuitive to consider a date to simply be a way of naming and referring to particular time points, it is easy to see that they are much more than that. One way to understand this is to look at how we think about and use the concept in practice. Dates contain much more information than simply their relative order on a time line. Consider all of the knowledge that is implicitly captured in the date: Monday, October 20. We are confronted with an intuition of repeatability – we know that there was a Monday before this Monday, and that there will be another Monday a week after it. We know that not only did this particular Monday take place in October, but so did the previous (and so will the following) Monday. Further, we know that the October we are referring to in this date was part of a year. Our ability to make such inferences is based on two key properties of dates: *periodicity* and *composition*. In order to represent and reason about dates correctly, taking full advantage of this sort of knowledge, these properties need to be addressed.

One additional characteristic stems from an important distinction which must be made between the *date concepts* that are the focus of this ontology, such as "Monday", "week", "year" and the dates that we encounter in day-to-day conversation, such as "Monday, October 20". Whereas the properties of periodicity and composition are more or less observable in both, there is a stronger semantics that might easily be missed if our understanding of the distinction between these concepts is not clear. There is, in fact, *a meaningful relationship between the concepts of date and time* that must be captured to accurately represent the semantics of dates. Consider for example, a particular day like today. Today has some date, which we know will correspond to a particular day of the week. Further, we know that this date must correspond to a *unique* day of the week. We also know that dates have specific start and end times – today, whichever day it is, started at midnight and will end at 11:59 P.M. We can construct similar examples to illustrate this semantics for other calendar constructs (Gregorian or otherwise), such as months.

Other Approaches to Dates

A well-recognized approach to representing dates is found in the XML schema standard (Biron & Malhotra 2004) with datatypes such as date, dateTime, gDay (i.e. "Gregorian day"), etcetera. This standard is based on ISO 8601 (ISO 1988), the standard for Representations of dates and times. Both standards are restricted to the Gregorian calendar, and while they take into consideration the properties of aggregation and repeatability in the definition of their parts, they are essentially syntactic restrictions on the format and thus offer little toward the goal of defining the semantics of dates.

An unfortunately common, overly simplistic approach to include the concept of dates in an ontology builds on these standards by leveraging the date datatypes as timestamp-valued labels for temporal concepts via a sort of 'has-date' property (we refer to this as the "datatype approach"). Not only are we unable to perform the type of reasoning described in the previous section with such a formalism, it also leads to an inaccurate representation of the key semantics of aggregation. Further, this type of approach fails to capture any of the semantics of the relationship between dates and time; rather the relationship is simply defined as a sort of label without any of the underlying meaning.

In ontologies in which dates are conceptualized as time-identifiers in this way, it is possible to have situations where a time point represents the same date as an interval (e.g. January 22 and the interval represented as starting at the beginning of January 22 12:00 A.M. and ending at 11:59 P.M.). Similarly, it would be consistent for a time point (e.g. January 22) to contain one or many intervals (e.g. beginning at January 22 09:00 A.M. and ending at January 22 10:00 P.M.). Our understanding of time precludes us from including the aggregation of dates in this approach. When they are defined as specific time points it only makes sense to discuss one level of granularity at a time. These observations point to a need for an ontology of dates to formally characterise these properties such that they can be implemented in practice for more accurate reasoning and representation. This need has in fact been recognized previously in three attempts to formally axiomatize the semantics of the concept of dates: the Date-Time Vocabulary (DTV) (Linehan, Barkmeyer, & Hendryx 2012), OWL-Time (Hobbs & Pan 2004; Pan & Hobbs 2005), and Pat Hayes' Calendars in (Hayes 1996).

The DTV has a concept of dates somewhat in line with the datatype approach, where a date is defined as a "time coordinate" (a name given to a point in time). However additional axioms account for the properties of aggregation and repeatability by reasoning about sequences and sets of sequences. OWL-Time represents aggregation and repeatability by identifying different types of relationships between temporal sequences of different types, opting for the use of second-order formulations in their axiomatization. The Calendars defined in (Hayes 1996) also quantify over sequences, where repeatability and aggregation are described with a "rhythm" that defines a sequence of durations that makes up a particular calendar unit of measure.

No existing date ontologies accurately capture the semantics of the relationship between dates and time. This is likely a result of a focus only on modelling the *behaviour* of dates that is, attempting to imitate the way in which dates unfold with respect to time, as opposed to considering the *semantics* of the concept. We speculate that this focus is also the cause of the awkward and unnatural form of the axiomatizations. With such axiomatizations it would appear that implementing any of the reasoning tasks that served as motivation for an ontology of dates would be cumbersome and unlikely to scale well. Further, none of the existing axiomatizations have been proven to be consistent, nor have their models been characterized. On the other hand, the Date Ontology presented in this paper is a straightforward first-order theory, and we are able to characterize its models up to isomorphism.

Periodicity in the Date Ontology

The Date Ontology introduced in this paper consists of two modules that formalize fundamental intuitions about dates. We first consider the module $T_{date\_periodic}$, which axiomatizes the two intuitions about the periodicity of dates:

- Every date has multiple occurrences, each of which is associated with a pair of timepoints that are the beginning and end of the occurrence of the date (for example, each Monday begins at midnight and ends twenty-four hours later). Furthermore, all occurrences of the date have the same duration.
- Dates occur periodically, at fixed intervals along the timeline (for example, there is an occurrence of Monday in every week).

Dates as Activities

Following the first intuition, dates appear to have the same properties as activities in the PSL Ontology (Gruninger 2009). We therefore axiomatize dates as classes of activities in an extension of the PSL Ontology.

Within the theory $T_{pslcore}$[1] in the PSL Ontology, an *activity* is a repeatable pattern of behaviour, while an *activity occurrence* corresponds to a concrete instantiation of this pattern. The relationship between activities and activity occurrences is represented by the $occurrence\_of(o, a)$ relation. Activities may have multiple occurrences, or there may exist activities which never occur. Any activity occurrence corresponds to a unique activity.

Underlying the intuition that activity occurrences are the instantiations of activities is the notion that each activity occurrence is associated with unique timepoints that mark the begin and end of the occurrence. The PSL Ontology introduces two functions $beginof$ and $endof$ for this purpose.

[1] http://colore.oor.net/psl_core/

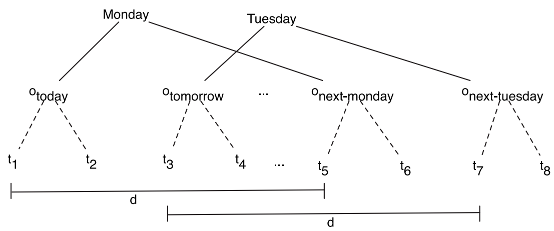

Figure 1: Structures for date periodicity.

Figure 1 depicts part of a structure which we want to axiomatize. We can see two dates, $\mathbf{Monday}$ and $\mathbf{Tuesday}$, each of which has two occurrences; $\mathbf{o_{today}}$ is the specific day which is an occurrence of $\mathbf{Monday}$, which would make the next specific day $\mathbf{o_{tomorrow}}$ to be an occurrence of $\mathbf{Tuesday}$. There is another occurrence $\mathbf{o_{next\_Monday}}$ of $\mathbf{Monday}$ seven days later. Each day which is an occurrence of $\mathbf{Monday}$ or $\mathbf{Tuesday}$ has a timepoint at which the day begins and a timepoint at which the day ends. Notice also that a day is not an instance of a date, since $\mathbf{Monday}$ or $\mathbf{Tuesday}$ are both elements of the domain, rather than classes.

Structures for Date Periodicity

The second basic intuition about dates is that they occur at discrete periods along the timeline. For example, in Figure 1, there are occurrences of the dates $\mathbf{Monday}$ and $\mathbf{Tuesday}$ every 7 days. In order to formalize this intuition, we first need to reuse an ontology for durations.

subsubsectionDuration Ontology The notion of duration plays a key role in the formalization of the second intuition about periodicity for dates. As with dates, there are several existing ontologies for durations that have been developed ((Barber 1993), (Hayes 1996), (Hobbs & Pan 2004)); these ontologies use a particular algebraic field (such as the rational or real numbers) to reason about duration. On the other hand, in the ontology for duration $T_{duration}$[2] proposed in (Grüninger 2010) timedurations do not form a field, since we do not multiply timedurations, although we do want to multiply timedurations by a scalar (i.e. element of a field). Instead, timedurations form an ordered vector space $\mathcal{D}$.

The ontology $T_{duration}$ also axiomatizes a duration function, which maps pairs of timepoints to a unique timeduration, and hence can be considered to be a vector-valued function. Within differential topology, a vector-valued function on the space $\mathbb{R}^n$ is known as a vector field (Auslander & MacKenzie 1977), in which a unique vector is associated with each element of $\mathbb{R}^n$. However, since we want a mapping δ from $\mathcal{T} \times \mathcal{T}$ to the vector space $\mathcal{D}$, we need to generalize the notion of vector field.

There are, of course, many possible functions from $\mathcal{T} \times \mathcal{T}$ to the vector space $\mathcal{D}$. The class of vector-valued functions for duration is defined by using the automorphisms of the structure, that is, mappings from the structure to itself that preserve values of the duration function δ:

$$\delta(\mathbf{t_1}, \mathbf{t_2}) = \delta(\mathbf{t_3}, \mathbf{t_4}) \Leftrightarrow \varphi(\delta(\mathbf{t_1}, \mathbf{t_2})) = \varphi(\delta(\mathbf{t_3}, \mathbf{t_4}))$$

Intuitively, automorphisms of the timeline (which form a group denoted by $Aut(\mathcal{T})$ should preserve the duration function – if we shift timepoints along the timeline while preserving their ordering, the values of the duration function should not change. This insight leads to the following class of structures:

Definition 1. *Let $\mathcal{T}$ be a timeline and let $\mathcal{D}$ be an ordered vector space.*

The structure $\mathbb{V} = \langle \delta, \mathcal{T} \times \mathcal{T}, \mathcal{D} \rangle$ is a vector map iff $\delta : \mathcal{T} \times \mathcal{T} \rightarrow \mathcal{D}$ and $Aut(\mathbb{V}) \cong (Aut(\mathcal{T}) \times Aut(\mathcal{T}))$

The following result from (Grüninger 2010) characterizes the models of the Duration Ontology[3] and http://colore.oor.net/algebra/vectorspace.clif which we use in this paper::

Theorem 1. *$\mathcal{M} \in Mod(T_{duration})$ iff $\mathcal{M} = \langle T \cup D, \mathbf{timepoint}, \mathbf{before}, \mathbf{timeduration}, \mathbf{duration}, \mathbf{add}, \mathbf{mult}, \mathbf{zero}, \mathbf{one}, \mathbf{lesser} \rangle$ such that:*

1. *$\mathcal{T} = \langle T, \mathbf{timepoint}, \mathbf{before} \rangle \in Mod(T_{lp\_ordering})$;*
2. *$\mathcal{D} = \langle D, \mathbf{timeduration}, \mathbf{duration}, \mathbf{add}, \mathbf{mult}, \mathbf{zero}, \mathbf{one}, \mathbf{lesser} \rangle \in Mod(T_{ordered\_vector\_space})$;*
3. *$\mathbb{V} = \langle \mathbf{duration}, \mathcal{T} \times \mathcal{T}, \mathcal{D} \rangle$ is a vector map.*

The vector space for timedurations and the vector map for the $duration$ function in models of $T_{duration}$ will play a key role in the axiomatization of periodicity of dates.

Discrete Vector Maps $T_{duration}$ allows us to say that two occurrences of a date correspond to a specific timeduration (e.g. there are 7 days between one Monday and the next). However, we still need to axiomatize the property that all occurrences of a date occur at discrete periods along the timeline. We therefore introduce the substructures of the timeduration vector space that are definable by occurrences of dates.

Definition 2. *$\mathcal{D}'$ is semilattice in the vector space $\mathcal{D}$ iff it is a discrete additive subsemigroup of $\mathcal{D}$.*

The intuition of "regular intervals" arises from the following property of semilattices:

Lemma 1. *If $\mathcal{D}'$ is a semilattice in the vector space $\mathcal{D}$, then there exists a unique $\mathbf{d} \in D$ such that for any $\mathbf{d_1}, \mathbf{d_2} \in D'$, there exists $\mathbf{x}$ such that*

$$\mathbf{d_2} = \mathbf{add}(\mathbf{d_1}, \mathbf{mult}(\mathbf{x}, \mathbf{d})) \Leftrightarrow \langle \mathbf{d_1}, \mathbf{d_2} \rangle \in \mathbf{lesser}$$

We will refer to the element $\mathbf{d} \in D$ as the period of the semilattice $\mathcal{D}'$. In Figure 1, the period is the duration 7 days.

Although semilattices allow us to define sets of timepoints along the timeline, there are two sets of timepoints in which we are especially interested, namely, the timepoints at which occurrences of a date begin and end. We can use these timepoints to specify two special classes of substructures the vector map.

[2] http://colore.oor.net/duration/

[3] The CLIF axiomatization of the theories referred to in the Theorem can be found at http://colore.oor.net/ordering/lp_ordering.clif

Definition 3. *Suppose* $\mathbb{V} = \langle \delta, \mathcal{T} \times \mathcal{T}, \mathcal{D} \rangle$ *is a vector map.*
$\mathbb{V}^b(\mathbf{a}) = \langle \delta, \mathcal{T}(\mathbf{a}) \times \mathcal{T}(\mathbf{a}), \mathcal{D}(\mathbf{a}) \rangle$ *is the substructure of* $\mathbb{V}$ *generated by the subset of timepoints*
$T^b(\mathbf{a}) = \{\mathbf{t} : \langle \mathbf{o}, \mathbf{a} \rangle \in \mathbf{occurrence\_of}, \mathbf{t} = \mathbf{beginof}(\mathbf{o})\}$ *and the subset of timedurations* $D^b(\mathbf{a}) = \{\mathbf{d} : \mathbf{d} = \delta(\mathbf{t_i}, \mathbf{t_j}),\ \mathbf{t_i}, \mathbf{t_j} \in T(\mathbf{a})\}$
$\mathbb{V}^e(\mathbf{a}) = \langle \delta, \mathcal{T}(\mathbf{a}) \times \mathcal{T}(\mathbf{a}), \mathcal{D}(\mathbf{a}) \rangle$ *is the substructure of* $\mathbb{V}$ *generated by the subset of timepoints*
$T^e(\mathbf{a}) = \{\mathbf{t} : \langle \mathbf{o}, \mathbf{a} \rangle \in \mathbf{occurrence\_of}, \mathbf{t} = \mathbf{endof}(\mathbf{o})\}$ *and the subset of timedurations* $D^e(\mathbf{a}) = \{\mathbf{d} : \mathbf{d} = \delta(\mathbf{t_i}, \mathbf{t_j}),\ \mathbf{t_i}, \mathbf{t_j} \in T(\mathbf{a})\}$

In general, $\mathcal{D}^b(\mathbf{a})$ and $\mathcal{D}^e(\mathbf{a})$ will not be semilattices in the vector space $\mathcal{D}$, since it is not the case that all activities have periodic occurrences. Of course, this is precisely the class of activities which we are interested in for dates.

Definition 4. *Suppose* $\mathbb{V} = \langle \delta, \mathcal{T} \times \mathcal{T}, \mathcal{D} \rangle$ *is a vector map.*
The vector map $\mathbb{V}(\mathbf{a}) = \langle \delta, \mathcal{T} \times \mathcal{T}, \mathcal{D}(\mathbf{a}) \rangle$ *is a discrete vector map iff* $\mathcal{D}(\mathbf{a})$ *is a semilattice in* $\mathcal{D}$.

Using the notion of discrete vector maps, we can specify the required structures for the periodicity in the Date Ontology:

Definition 5. *Let* $\mathfrak{M}^{date\_periodic}$ *be the following class of structures:*
$\mathcal{M} \in \mathfrak{M}^{date\_periodic}$ *iff* $\mathcal{M}$ *is the amalgamation of the structures* $\mathbb{V}$ *and* $\mathcal{N}$ *such that*

1. $\mathbb{V}$ *is a vector map;*
2. $\mathcal{N} \in Mod(T_{pslcore})$*;*
3. *if* $\langle \mathbf{a} \rangle \in \mathbf{date}$ *then* $\mathbb{V}^b(\mathbf{a})$ *and* $\mathbb{V}^e(\mathbf{a})$ *are discrete vector maps;*
4. $\langle \mathbf{a}, \mathbf{d} \rangle \in \mathbf{freq}$ *iff* $\mathbb{V}^b(\mathbf{a})$ *and* $\mathbb{V}^e(\mathbf{a})$ *are discrete vector maps, and* $\mathbf{d}$ *is the period of the semilattices* $\mathcal{D}^b(\mathbf{a})$ *and* $\mathcal{D}^e(\mathbf{a})$.

Now that we have specified the required structures for this part of the Date Ontology, we turn to the problem of axiomatizing this class of structures.

Axiomatization of Date Periodicity

The Date Ontology imports $T_{pslcore}$ (to axiomatize the distinction between dates and their occurrences) and $T_{duration}$ (to axiomatize timedurations and the $duration$ function). The remaining axioms of the $T_{date\_periodic}$ module within the Date Ontology can be found in Figure 2. Dates form a class of activities, each of which has a unique period (captured by the $freq$ relation). Axiom 3 ensures that the set of occurrences of a date forms a discrete ordering. The period of an activity is the timeduration that is the value of the duration between successive occurrences of the date. By Axiom 4, all occurrences of a date have the same duration; together with the definition of frequency, this axiom guarantees that the set of **beginof** timepoints for occurrences of the date generate a semilattice with the same period as the the semilattice generated by the set of **endof** timepoints for occurrences of the date. Note that the axiomatization of $T_{date\_periodic}$ depends on $T_{duration}$ to guarantee the existence of the timepoint at which the next occurrence of a date begins. We can prove the representation theorem for models of $T_{date\_periodic}$:

$$(\forall a)\ date(a) \supset (\exists d)\ freq(a, d) \tag{1}$$

$$(\forall a, d_1, d_2)\ freq(a, d_1) \wedge freq(a, d_2) \supset (d_1 = d_2) \tag{2}$$

$$\begin{gathered}(\forall a, d, t, o_1, o_2)\ freq(a, d) \wedge occurrence\_of(o_1, a) \wedge \\ occurrence\_of(o_2, a) \wedge (d = duration(beginof(o_1), t)) \\ \supset \neg between(beginof(o_1), t, beginof(o_2))\end{gathered} \tag{3}$$

$$\begin{gathered}(\forall a, d, t, o_1, o_2)\ date(a) \wedge \\ occurrence\_of(o_1, a) \wedge occurrence\_of(o_2, a) \supset \\ (duration(beginof(o_1), endof(o_1)) = \\ duration(beginof(o_2), endof(o_2)))\end{gathered} \tag{4}$$

$$(\forall a, t)\ occurs(a, t) \equiv \tag{5}$$

$$(\forall o) between(beginof(o), t, endof(o)) \supset occurrence\_of(o, a)$$

$$(\forall a, d)\ freq(a, d) \equiv (activity(a) \wedge timeduration(d) \tag{6}$$

$$\begin{gathered}\wedge((\forall o)\ occurrence\_of(o, a) \supset \\ (\exists t)\ (d = duration(beginof(o), t)) \wedge occurs(a, t)))\end{gathered}$$

Figure 2: $T_{date\_periodic}$: Axiomatization of dates as periodic activities. Additional axioms are imported from $T_{pslcore}$ and $T_{duration}$.

Theorem 2. *The mapping*
$\mu : Mod(T_{date\_periodic} \cup T_{duration} \cup T_{pslcore}) \rightarrow \mathfrak{M}^{date\_periodic}$ *is a bijection.*

Composition in the Date Ontology

The third intuition about dates is that they can be composed into more complex dates that correspond to calendars and temporal aggregates. For example, each year is composed of months, each month is composed of days, and each week is composed of days. This also corresponds to the intuitions about temporal aggregates in (Pan & Hobbs 2005), e.g. "five business days", "every third Monday in 2014", "every morning for the past four years", "four consecutive Sundays", "three weekdays after January 10". In this section, we present the module $T_{date\_compose}$ of the Date Ontology, which axiomatize dates as a particular class of complex activities in the PSL Ontology. Since we are treating dates as activities, this means that we will need to consider a process ontology which axiomatizes complex activities.

Complex Activities in the PSL Ontology

A useful feature of process formalisms is the ability to compose simpler activities to form new complex activities (or conversely, to decompose any complex activity into a set of subactivities). The PSL Ontology incorporates this idea while making several distinctions between different kinds of composition that arise from the relationship between composition of activities and composition of activity occurrences.

The PSL Ontology uses the $subactivity$ relation to capture the basic intuitions for the composition of activities. A model of the subtheory $T_{subactivity}$ is known as a subactivity ordered set, and it is equivalent to discrete partial ordering in which primitive activities are the minimal elements. This theory alone does not specify any relationship between the occurrence of an activity and occurrences of its subactivities.

Corresponding to the composition relation over activities, $subactivity\_occurrence$ is the composition relation over activity occurrences. A model of the subtheory T_{actocc} is known as a subactivity occurrence ordering, and it is a discrete partial orderings which is homomorphic to the subactivity ordered set. Occurrences of atomic activities are the minimal elements in this composition ordering – they do not have any nontrivial subactivity occurrences. One can consider the subactivity occurrence to be a temporal part of the complex activity occurrence. The axioms of T_{actocc} guarantee that any subactivity occurrence is "during" an occurrence of the complex activity.

The basic structure that characterizes occurrences of complex activities within models of the subtheory $T_{complex}$ is the activity tree, which consists of all possible sequences of atomic subactivity occurrences. The relation $root(s, a)$ denotes that the subactivity occurrence s is the root of an activity tree for a and the relation $leaf(s, a)$ denotes that the subactivity occurrence s is a leaf of an activity tree for a. Elements of the tree are ordered by the $min\_precedes$ relation; each branch of an activity tree is a linearly ordered set of occurrences of subactivities of the complex activity. The relation $mono(s_1, s_2, a)$ indicates that s_1 and s_2 are occurrences of the same subactivity on different branches of the activity tree for a. In addition, there is a one-to-one correspondence between occurrences of complex activities and branches of the associated activity trees. The set of activity trees in a model of $T_{complex}$ is known as a complex activity structure.

Structures for Date Composition

Since we will define dates to be a class of complex activities in the PSL Ontology, we first introduce the classes of substructures that will be used to characterize the subactivities and activity trees for dates, as well as the substructures that characterize the subactivity occurrences of occurrences of a date.

Definition 6. *$\mathcal{A}^{\mathbf{a}}$ is the substructure of the subactivity ordered set $\mathcal{A}$ that is generated by the activity* $\mathbf{a}$.

$\mathcal{C}^{\mathbf{a}}$ is the set of activity trees for the activity $\mathbf{a}$ *in the complex activity structure $\mathcal{C}$.*

$\mathbb{C}^{\mathbf{a}}$ is the substructure of the complex activity occurrence ordering $\mathbb{C}$ generated by occurrences of the activity $\mathbf{a}$.

Figure 3(a) is a depiction of $\mathcal{A}^{\mathbf{gregorian\_week}}$, which is the substructure of $\mathcal{A}$ consisting of subactivities of the date $\mathbf{gregorian\_week}$. Figure 3(b) is a depiction of $\mathbb{C}^{\mathbf{o_{gregorian\_week}}}$ for a specific occurrence of $\mathbf{gregorian\_week}$.

The axiomatization of the ontology will be guided by more detailed intuitions for the composition of dates. First, we observe that the days in a week, and the hours in day,

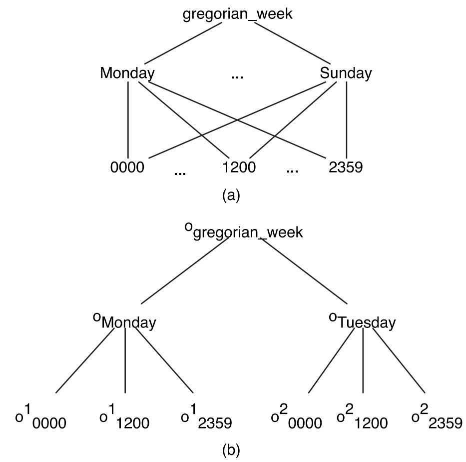

Figure 3: Structures for date composition of subactivities and subactivity occurrences.

are nested within each other. Further, the days in a week do not overlap each other, and similarly with the hours in a day. Thus, the occurrences of subactivities of a date correspond to intervals that are either disjoint or nested, and the resulting ordering is known as a series-parallel poset (Branstaedt, Le, & Spinrad 1999).

The second intuition about the composition of dates is that every subactivity of a date has a unique occurrence whenever the date occurs. For example, each occurrence of a week contains unique occurrences of the dates Sunday through Saturday. In terms of the structures within the models of the PSL Ontology, there is a bijection between the set of subactivities covered by a date in the subactivity ordered set $\mathcal{A}$ and the set of subactivity occurrences covered by an occurrence of the date in $\mathbb{C}$. We make this intuition precise with the following notion:

Definition 7. *A mapping $\varphi : \mathcal{M} \to \mathcal{N}$ is a partial isomorphism iff there exist substructures $\mathcal{M}' \subseteq \mathcal{M}$ and $\mathcal{N}' \subseteq \mathcal{N}$ such that $\varphi' : \mathcal{M}' \subseteq \mathcal{M} \to \mathcal{N}'$ is an isomorphism, where φ' is the restriction of the mapping to the domain of $\mathcal{M}'$.*

Combining these ideas leads us to the specification of the required structures for date composition in the Date Ontology:

Definition 8. *Let $\mathfrak{M}^{date\_compose}$ be the following class of structures: $\mathcal{M} \in \mathfrak{M}^{date\_compose}$ iff $\mathcal{M}$ is an expansion of some $\mathcal{N} \in \mathfrak{M}^{psl}$ such that for every $\langle \mathbf{a} \rangle \in \mathbf{date}$ we have*

1. *$\omega : \mathbb{C}^{\mathbf{a}} \to \mathcal{A}^{\mathbf{a}}$ is a partial isomorphism, where $\langle \mathbf{o}, \omega(\mathbf{o}) \rangle \in \mathbf{occurrence}$;*
2. *$\mathcal{C}^{\mathbf{a}} \cong \mathcal{P} \times \mathbb{A}$, where $\mathcal{P}$ is a series-parallel poset and $\mathbb{A}$ is an infinite antichain.*

Condition (2) captures the additional intuition that all occurrences of a date have the same structure, that is, they are all occurrences of the same subactivities, and these subactivities always occur in the same linear ordering.

$$(\forall a_1, a_2)\ date(a_1) \wedge subactivity(a_2, a_1) \supset date(a_2) \tag{7}$$

$$(\forall a_1, a_2, o_2)\ date(a_1) \wedge subactivity(a_2, a_1)$$
$$\wedge occurrence\_of(o_1, a_1) \supset (\exists o_2)\ occurrence\_of(o_2, a_2)$$
$$\wedge subactivity\_occurrence(o_2, o_1) \tag{8}$$

$$(\forall a, s_1, s_2, s_3, s_4)\ date(a) \wedge min\_precedes(s_1, s_2, a) \tag{9}$$
$$\wedge mono(s_1, s_3, a) \wedge mono(s_2, s_4, a) \supset min\_precedes(s_3, s_4, a)$$

$$(\forall a_1, a_2, o_1, o_2, o_3)\ date(a_1) \wedge occurrence(o_1, a_1) \tag{10}$$
$$\wedge subactivity\_occurrence(o_2, o_1) \wedge subactivity\_occurrence(o_3, o_1)$$
$$\wedge occurrence(o_2, a_2) \wedge occurrence(o_3, a_2) \wedge (o_2 \neq o_3)$$
$$\supset (\exists a_3)\ subactivity(a_3, a_1) \wedge subactivity(a_2, a_3)$$
$$\wedge (a_3 \neq a_1) \wedge (a_3 \neq a_2)$$

$$(\forall a, a_1, a_2, s, s_1, s_2, s_3, s_4)\ date(a)$$
$$\wedge subactivity(a_1, a) \wedge subactivity(a_2, a)$$
$$\wedge root(s_1, a_1) \wedge leaf(s_3, a_1) \wedge root(s_2, a_2) \wedge root(s_4, a_2)$$
$$\wedge min\_precedes(s_1, s_2, a) \wedge min\_precedes(s_2, s_3, a)$$
$$\supset min\_precedes(s_4, s_3, a) \tag{11}$$

Figure 4: $T_{date\_compose}$: Axiomatization of date composition. Additional axioms are imported from T_{actocc}, $T_{complex}$ and $T_{subactivity}$.

Axiomatization of Date Composition

The axioms of $T_{date\_compose}$ can be found in Figure 4. By Axiom 7, a date is a complex activity that is composed only of dates as subactivities, and by Axiom 11, the $subactivity$ relation is isomorphic to a tree when restricted to dates. Axioms 8 and 9 guarantee that all occurrences of a calendar will have isomorphic activity trees, each of which consists of a unique branch. Axiom 10 corresponds to the property that the $subactivity\_occurrence$ relation is isomorphic to a tree when restricted to occurrences of dates.

We can prove that the models of $T_{date\_compose}$ are equivalent to the required structures in $\mathfrak{M}^{date\_compose}$:

Theorem 3. *The mapping*
$\mu : Mod(T_{date\_compose} \cup T_{psl}) \rightarrow \mathfrak{M}^{date\_compose}$
is a bijection.

Using the Date Ontology

The axiomatization we have provided defines the concepts of dates independent of any given calendar. In this way, the Date Ontology can be used to represent and reason about any particular calendar by adding a *domain theory* to define the calendar's constructs. In the following example, we illustrate a portion of the axiomatization of such a domain theory for the Gregorian calendar.

The composition of days into weeks – Monday is part of a week, as is Tuesday:

$$subactivity(Monday, Gregorian\_week)$$
$$subactivity(Tuesday, Gregorian\_week)$$

The duration aspect of periodicity – Mondays and Tuesdays are always a day long, and they occur at weekly intervals:

$$(\forall o)\ occurrence\_of(o, Monday) \supset$$
$$duration((beginof(o), endof(o)) = Day$$
$$(\forall o)\ occurrence\_of(o, Tuesday) \supset$$
$$duration((beginof(o), endof(o)) = Day$$
$$freq(Monday, Week)$$
$$freq(Tuesday, Week)$$

The ordering aspect of periodicity – Monday comes before Tuesday in every week:

$$(\forall o)\ occurrence\_of(o, Gregorian\_week) \supset$$
$$(\exists s_1, s_2)\ leaf(s_1, Monday) \wedge root(s_2, Tuesday)$$
$$\wedge min\_precedes(s_1, s_2, Gregorian\_week)$$

Note that the same approach can be taken regardless of the calendar. In fact, multiple calendars can be used concurrently with the Date Ontology, simply by defining multiple such domain theories.

Summary

Although the representation of dates has long been addressed by the community, earlier ontologies have utilised complex, unnatural axiomatizations that failed to fully capture all of the intuitions about dates. In this paper, we have introduced a first-order axiomatization for dates. We have specified the classes of intended structures for these ontologies, proven characterization theorems for these classes of structures, and proven representation theorems for the models of the axioms. The resulting ontology captures all three key intuitions in an elegant framework. Rather than becoming entangled in the more direct approach of attempting to model the semantics and behaviour of dates from-scratch, we recognized a similarity between the intuitions of dates and those of activities. This allowed us to approach the problem of a representation of dates through the reuse of existing theories, resulting in an effective, clean axiomatization with only eleven additional axioms. This concise extension of existing ontologies is not only able to capture all of our intuitions about dates, it does so without committing to a particular calendar type and sacrificing adaptability.

Future work will look at using the Date Ontology for scheduling. When scheduling activities at particular dates we are in fact specifying occurrences of concurrent activities, since dates themselves are activities in the ontology. We expect that this use of the Date Ontology will result in a relatively concise and novel formalization of scheduling applications.

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