Motivation: What are the dynamics of vortex knots in a superfluid? Addressing this question in a satisfactory manner touches on a few important open problems in condensed matter theory. For example, understanding how the dynamics of two topologically identical knots are related requires an understanding of what dynamical properties of a superfluid are topologically invariant, a question highlighted by this year's Nobel Physics prize [3]. Attempts like [4] have been made to address this question and investigate the dynamics of quantum vortex knots. This study thoroughly researched the shape preserving properties and dynamics of topological structures in Bose Einstein condensates (BECs), as modeled by the Gross-Pitaevskii equation (GPE).

Despite [4], the production of quantum vortices and how they interact with other vortices in the superfluid, or the flow of the superfluid itself, is still not well understood [2]. In order to make progress it would be helpful to make an analogy to a simpler case, perhaps to vortices in incompressible inviscid fluid (IIF). To probe this the Irvine Lab at the University of Chicago experimentally produced knotted vortices in water. The trefoil knot dynamics in water [2] were very similar to those found in the simulated superfluid [4]. This led to suspicion that in some cases the superfluid can be treated as an incompressible inviscid fluid (IIF) with 'sound waves'. However, the correspondence is not ideal since [4] generated vortices numerically using a given vortex profile, whereas [2] generated vortices experimentally by dragging an airfoil. One way to remove this discrepancy and to ensure a closer corresponence is to simulate dragging the same airfoil through the superfluid, then compare the resultant vortex dynamics to the experimental case in [2]. Removing this discrepancy is one of many ways to simplify and discover the dynamics of vortex knots in a superfluid.

Research Plan: The problem oulined above personally interests me for two reasons. First, I have a longstanding interest in fluids and nonlinear dynamics (see statement) and my Goldwater winning experience with IIFs in Riemann ellipsoids make the analogy between IIFs and superfluids especially engaging for me. Second, the possibility that this analogy will reveal properties of vortex dynamics conserved by topology speaks directly to my enjoyment of simplifying complex problems through symmetries and pictures. Due to my interest I have spent the last summer working with Professor William Irvine at the University of Chicago in order to build a simulation of the motion of an airfoil through a 2D superfluid. The airfoil chosen was from a class of airfoils that have well understood IIF fluid flow around them. This allows direct comparison of our results for superfluid flow to the analytical expressions for IIF flow around the foil. The simulation I built used the GPE as in [4] in order to simulate low temperature BECs. To build the simulation I used, among other things, split-step Fourier methods to numerically solve the GPE as a nonlinear Scroödinger equation. I have also been optimizing the simulation, as well as building a suite of analytical tools.

I plan to continue researching vortex nucleation and dynamics for my Ph.D. I hope to attend the University of Chicago to study condensed matter theory, where I will continue my research in the Irvine lab. Specifically I will extend the analogy between IIF and superfluid, and probe topologically conserved dynamical quantities. The first step is to extend my simulation of the GPE to three dimensions. I could then simulate dragging the same airfoil used in water in [2] through superfluid. Generating vortices in this way would allow a more robust comparison between [4] and [2] and be free from questions about the viability of creating vortex rings from scratch. The simulation would also allow further study of the dynamics of an enourmous variety of vortex knots without needing to generate the knots analytically. Further modification could be made to obtain more robust results. For example, the simulation could be modified to dynamically change the geometric, but not topological, properties of the foil as it generated vortices. Doing this would give a key look into how much topology matters in dynamics, even as the vortex knots nucleate.

Additionally, I will study the flow around the airfoil that nucleates the quantum vortices. Any interesting analogs I find in my current research between IIF flow and superfluid flow around the 2D airfoil could be investigated during my graduate research in the 3D case. If the analogs hold there, then such a confirmed duality between IIFs with 'sound waves' and superfluids could lead to using the IIF to predict vortex dynamics. These predictions could then be tested against the vortex generator I will build, giving even greater confidence in the analogy between IIF and superfluid. Finally, I will probe the same questions theoretically. I would be very interested in trying to find elegant theoretical explanations for dynamical properties conserved by topology as well as the reason for any dualism between IIF with 'sound waves' and superfluids.

Potential to Accomplish: My current research in the Irvine lab has more than prepared me to tackle this research plan. For example, I recently used a suite of analytical tools I built independently to deduce that the circulation around the foil moving through a superfluid seems to be identical to the circulation around the same foil in an IIF; a deduction accomplished by vortex counting in the superfluid. This finding suggests further similarities between IIFs and superfluids besides those shown in [4] and [2]. I am currently buidling other analysis tools to extend the analogy even further. Work like this on the 2D foil highlights the fact that I am uniquely prepared to extend to the case of a 3D airfoil and vortex knot dynamics there. It also shows I am dedicated enough to see my research plan through to the end. Additionally the Irvine lab, being the first to experimentally produce a vortex knot [2], has members with plenty of experience with whom I could collaborate. Finally, receiving the NSF GRF would allow me to initially focus wholly on the large overhead required to build optimized simulations. Once I built the simulations the grant would allow me to efficiently pursue the research plan I proposed above, monetarily and through resources like XSEDE's supercomputing time.

Broader Impacts: My research plan has the potential to have broader impacts on several levels. On the level of condensed matter physics, it will identify superfluid flow with IIF flow, potentially simplifying descriptions of turbulence. It will also uncover topologically conserved dynamical quantities of vortex knots in a superfluid. Next, it will have an effect on broader fields of science. For example, Type-II superconductors possess vortices, similar to those in superfluids, that introduce resistance into the superconductors. By analogy with superfluids my graduate research project could suggest ways to control these topological defects, making for more efficient superconductors by lowering resistance [1]. Superconductors are used for technologies ranging from improved cell phone performance to MRI machines, so more efficient superconductors could have a positive impact on society at large. These examples make it clear that my project could have a broader impact on condensed matter physics, building more efficient superconductors, and society at large.

References: [**1**] R. Córdoba, *Magnetic field-induced...*, Nature Communications **4** (2013). [**2**] D. Kleckner and W. Irvine, *Creation and dynamics of knotted vortices*, Nature Physics **9** (2013) 253-258. [**3**] *The 2016 Nobel Prize in Physics - Press Release*, Nobelprize.org. Nobel Media AB 2014. Web. [**4**] D. Proment, et al, *Torus quantum vortex knots...*, JPCS **544** (2014) 012022.