Carbon taxation and precautionary savings*

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Abstract

This paper asks how precautionary savings affect the level of the optimal carbon tax. I augment a heterogeneous-agent incomplete-markets model with a climate sector and estimate its structural parameters with indirect inference. As households in the model engage in precautionary saving behavior, it replicates a stylized fact from the data that the marginal propensity to consume pollution-intensive goods decreases with income. Therefore, the carbon tax and the redistribution of its revenue have distributional consequences. When recycling the revenue lump-sum, the optimal carbon tax carbon tax also functions as a means of redistribution and serves as an insurance device for the uninsurable idiosyncratic productivity shocks in two ways. First, through transfers and second, by increasing wages and interest rates due to lower climate damages on production and thus higher factor productivity. As a consequence, the optimal tax is increasing in the level of idiosyncratic risk and precautionary savings, and higher than what is required to internalize the negative climate externality. This result depends on the availability of tax instruments of the social planner.

Keywords: heterogenous agents, precautionary savings, carbon taxation
JEL codes: C23, D12, D31, Q50

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1 Introduction

A carbon tax on polluting economic activity is seen as the most efficient way to tackle anthropogenic global warming[1], "the greatest market failure the world has ever seen" (Stern, 2007). Economic insights about the optimal *level* of the carbon tax stem predominantly from models where households can perfectly insure shocks to their labor income. Hence, realistic market features such as borrowing constraints and uninsurable idiosyncratic risk play no role in these models. Yet, from the quantitative macroeconomics literature we know that these features affect household demand in non-trivial ways, as agents accumulate precautionary savings.[2] As a result, the optimal carbon tax - in addition to the climate externality - might also take into account distributional and insurance concerns. The goal of the present paper is thus to investigate how and to what extent the optimal level of the carbon tax changes when precautionary saving behavior is taken into account.

To do so, I augment a heterogeneous-agent incomplete-markets model with a climate sector, and estimate its structural parameters with indirect inference. In particular, the demand side of the model, where households consume a clean and a pollution-intensive (dirty) good, captures two salient stylized facts. Both have important implications for the optimal carbon tax. First, the marginal propensity to consume dirty goods, which I henceforth term *marginal propensity to pollute* (MPPs), is decreasing in income (Weber and Matthews, 2008; Levinson and O'Brien, 2019; Sager, 2019). This heterogeneity in MPPs, so far ignored in the quantitative literature, arises naturally from my model with incomplete markets. The key is that agents engage in precautionary saving against future idiosyncratic risk that renders their consumption function concave in current income and wealth (Carroll and Kimball, 1996; Huggett and Ospina, 2001). Indeed, as my main theoretical result, I show that Engel curves - which describe the relation between *income* and consumption - are concave under quasi-homothetic utility over both goods and uninsurable productivity shocks.

This is important because, ceteris paribus, decreasing MPPs (or concave Engel curves) give rise to a so-called "equity-pollution dilemma" (Sager, 2019) such that progressive redistribution of income increases aggregate pollution. This feedback might call for a lower carbon tax. However, the revenues from carbon taxes can be used to provide insurance against the idiosyncratic risk and thus might call for a higher carbon tax.

To get an intuitive understanding for the concavity result, note that the dynamic problem of the household can be divided into two steps. First, she decides how much to save for the next period and how much to spend on consumption. Second, given the amount of total expenditure, the household decides how much to spend on the dirty and the clean good, respectively. This latter problem is void of any dynamic decisions and hence, static in nature. Finally, since in the static problem quasi-homothetic preferences are used, the decision rule from total expenditure to clean or dirty consumption is linear. However, total expenditure is concave in current assets and

[1] The "Economists' Statement on Carbon Dividends" has been signed by over 3500 economists, including 45 nobel laureates, and, among other things, states that "A carbon tax offers the most cost-effective lever to reduce carbon emissions at the scale and speed that is necessary." (www.econstatement.org)

[2] In recent papers, for instance, Holm (2018) and Carroll, Holm and Kimball (2021) provide analytical insights about the interaction between precautionary saving and borrowing constraints.

income and hence, consumption in clean or dirty goods is also concave.

Turning to the second stylized fact, poorer households spend a larger fraction of their income on dirty goods. I follow the literature and model this by introducing a subsistence level of dirty goods consumption in household preferences. Again, this implies a trade-off. On the one hand, taxing the dirty consumption good is very efficient as the subsistence level lowers the elasticity of substitution between the clean and the dirty good. On the other hand, the incidence of the carbon tax is now regressive.[3]

The supply side of the model follows Barrage (2020) and is specified as follows. A final goods firm uses capital, labor, and energy as inputs to production, while energy in turn is produced by a second firm using capital and labor only. Importantly, energy production is pollution intensive and entails a climate externality: it increases the stock of carbon in the atmosphere which in turn decreases economic productivity due to a damage function (Nordhaus, 1993; Golosov, Hassler, Krusell and Tsyvinski, 2014).

Finally, a government levies labor income, capital income, and carbon taxes to finance interest payment on bonds and lump-sum transfers. Throughout, the government keeps labor income and capital income taxes fixed and chooses the lump-sum transfer to balance its budget constraint.

The main computational exercise then proceeds in two steps. First, I calibrate and estimate a subset of the model's structural parameters using simulation methods and U.S. data. In my estimation, I use a comprehensive U.S. household panel data set - the Panel Study of Income Dynamics (PSID) - which includes information on demographics, income, wealth, and consumption. Second, I compute the the level of the carbon tax which yields the highest (utilitarian) welfare in the steady-state of the economy when carbon revenue is recycled lump-sum. Lastly, I study how this welfare-maximizing tax changes in economic environments without borrowing constraints and idiosyncratic risk.

In the first step, I externally calibrate parameters, including labor income and capital income taxes, to match features of the U.S economy. Moreover, using the PSID data, I estimate the stochastic productivity process on (imputed) wage data from the PSID using minimum distance estimation. Lastly, I estimate the remaining structural parameters of the model using indirect inference as in Guvenen and Smith, Jr. (2014) and more recently in Stoltenberg and Uhlendorff (2023). In particular, I use the household decision rule in the static framework as my auxiliary model. The parameters are precisely estimated and I provide numerical evidence for global identification.

In the second step, I use my estimated model as a laboratory and optimize over the carbon tax in a general equilibrium steady state under an utilitarian welfare criterion. Due to concavity of the utility function, the planner (i.e. the government) has an implicit preference for redistribution. The optimal carbon tax in this economy is about 141 $\$/CO_2$ and emissions are reduced by almost half. To understand the effect of idiosyncratic risk and precautionary savings on the level of the carbon tax, I then repeat this optimization when I scale up and down the labor productivity risk that agents face in the economy.

[3] See Hummel and Ziesemer (2021) for a similar reasoning in the context of food subsidies.

As my main result, I find that the carbon tax is increasing in the level of idiosyncratic risk and precautionary savings in the economy. This results holds for a comparative statics exercise, where model parameters are kept fixed, and for a case in which I recalibrate the model prior to optimization to match average hours worked and the capital-to-output ratio from the benchmark. Intuitively, the carbon tax is higher for two reasons. First, carbon tax revenue is redistributed lump-sum back to households, thus improving insurance and equality in the economy. Indeed, under optimal carbon taxes both consumption and net income inequality decrease. Second, as polluting is now more expensive, emissions and economic damages decrease, total factor productivity increases and thus also interest rates and wages. Especially higher wages are valued more by poorer households in an more uncertain environment.

In order to better understand what components gives rise to a non-zero carbon tax in this economy and to the positive gradient with respect to the degree labor productivity risk, I conduct several exercises that change various features of the model. First, I introduce a skill-specific skill-specific incidence rule for lump-sum transfers. Transfers are now proportional to ones labor productivity risk and are hence less redistributive. In response to this, the optimal carbon tax is reduced by half. Since carbon tax revenue now contributes less to redistribution and insurance, the planner reduces the tax as poorer households still pay relatively more for dirty goods.

Second, I optimize the carbon tax if I remove the subsistence level and damages, respectively. Eliminating the subsistence level increases the carbon tax only slightly. In this respect, the subsistence level does not seem to have a big quantitative impact. Eliminating carbon taxes, on the other hand, reduces the carbon tax to a measly 9 $\$/CO_2$. Naturally, this indicates that a carbon tax is an ineffective way to raise revenue for redistribution and insurance. In this way, it merely increases the price of the dirty good without bringing the benefit of increasing wages and interest rates.

Note that the utilitarian criterion used so far conflates redistribution and insurance motives of the planner. For this reason, I also use a criterion inspired by Bénabou (2002) that places no weight on interpersonal redistribution. The carbon tax under the efficient welfare criterion is about 6% lower than under the utilitarian welfare criterion. Furthermore, combining the efficient welfare criterion with no damages in the economy pushes the carbon tax to almost zero. These findings suggest that redistribution due to equality concerns of the planner is larger than for insurance concerns.

Finally, all of the above exercises keep other taxes in the economy, in particular labor income and capital income taxes, fixed. This exercise has major practical relevance, however, it is unsatisfactory from an economic theory perspective, as all inefficiencies and redistributive shortcomings of the economy are corrected by the carbon tax. As a result, I repeat the main exercises from above, but now also let the government adjust labor income taxes. I find that average labor income taxes are increasing in idiosyncratic risk and precautionary savings. This is intuitive, as the planner now engages in progressive redistribution by increasing labor taxes. At the same time, however, carbon taxes are now *decreasing*. Hence, if the government has additional tax instruments at its disposal, the relationship between precautionary savings and carbon taxes reverses.

Related literature and contribution My paper contributes to several strands of the literature overarching optimal fiscal policy, consumption dynamics, and environmental economics.

My key contribution to this literature is the *joint analysis* of optimal carbon taxation in an environment with idiosyncratic risk which generates precautionary savings. The quantitative model combines a heterogeneous-agent incomplete market economy in the spirit of Bewley (1986); Huggett (1993); Aiyagari (1994) with a climate sector, which yields an endogenous distribution over income and wealth, and heterogeneity in marginal propensities to consume. Hence, my model allows to study the interaction of climate policies and economic inequalities in a unified framework. Thereby, I connect two lines of research.

The first line is a rapidly growing literature which analyzes optimal carbon taxation in quantitative macroeconomic models. Building on the seminal work by Nordhaus (1992, 1993), who developed the first integrated assessment model (IAM) to analyze climate damages within a centralized economic framework, several papers moved to decentralized market structure. For instance, Golosov et al. (2014) derive a formula for the optimal carbon tax in a dynamic stochastic general-equilibrium model with an externality and resource scarcity. Building on their quantitative work, Barrage (2020) quantifies optimal carbon taxation in a model with tax distortions, and in turn, Douenne, Hummel and Pedroni (2023) quantify the additional impact of inequality.[4] None of these papers investigate settings with idiosyncratic risk, which is my main contribution compared to existing frameworks.[5]

The second line investigates the impact of idiosyncratic uncertainty and borrowing constraints on individual consumption demand. In particular, in the presence of idiosyncratic risk both prudence in preferences as well as borrowing constraints give rise to a precautionary saving motive which renders the consumption function concave in current income and wealth (Leland, 1968; Sandmo, 1970; Zeldes, 1989*b*,*a*; Kimball, 1990*a*,*b*; Carroll and Kimball, 1996; Huggett and Ospina, 2001; Carroll et al., 2021).[6]

Second, my exercise builds on the theoretical literature on optimal carbon taxation. In particular, Jacobs and van der Ploeg (2019) show that the optimal carbon tax should be equal to the marginal external damage of pollution if Engel curves are linear and the social planner has access to a non-individual lump-sum transfer and linear income taxes.[7] In other words, the optimal car-

[4] Other quantitative examples study the optimal environmental policy in response to business cycles (Heutel, 2012) or nominal frictions and uncertainty (Annicchiarico and Di Dio, 2015), or in an overlapping generations framework (Kotlikoff, Kubler, Polbin and Scheidegger, 2021*a*; Kotlikoff, Kubler, Polbin, Sachs and Scheidegger, 2021*b*).

[5] An exception is Benmir and Roman (2022) who study the 2050 net zero emissions target for the U.S. in a HANK model. The main difference to the present paper is that I focus on the optimal carbon tax and model household consumption with quasi-homothetic preferences and two goods.

[6] Lugilde, Bande and Riveiro (2019) survey the empirical literature on precautionary savings. They conclude that papers which "test the effect of uncertainty about future income on consumption/saving decisions, especially [those] using micro data, tend to provide robust and convincing results as regards the existence of a precautionary motive for saving" (p.507). Examples of micro-panel studies in different countries include Carroll and Samwick (1997, 1998); Guariglia and Rossi (2002); Guariglia (2003); Lugilde, Bande and Riveiro (2018).

[7] This result is reminiscent of earlier studies by Angus Deaton (Deaton, 1979, 1981) in which he demonstrates that uniform commodity taxation is desirable under linear Engel curves and separability in consumption and leisure.

bon tax follows the Pigouvian rule (Pigou, 1920).[8] Intuitively, any demand change induced by the carbon tax can be undone by changing the lump-sum transfer and the income tax. The main difference in this paper is that I consider a quantitative model with idiosyncratic risk, CRRA utility, and borrowing constraints in which, as explained above, non-linear Engel curves are microfounded.[9]

Recent studies further extend these theoretical analyses under deterministic environments with tax distortions (Barrage, 2020) and inequality (Douenne et al., 2023). Compared to this theoretical literature I do not have analytical results concerning the optimal carbon tax, because a closed-form solution is not obtainable within the class of models I consider. Instead, I conduct counterfactual analyses to disentangle the main forces behind my results, as is common in this literature (see e.g. Conesa, Kitao and Krueger, 2009; Dyrda and Pedroni, 2023).

In addition, my paper relates to the literature on subsistence consumption of carbon-intensive goods and the incidence of taxation (Klenert and Mattauch, 2016; Klenert, Schwerhoff, Edenhofer and Mattauch, 2018). I contribute to this literature by proposing a novel strategy to estimate the structural parameters - including the subsistence level of dirty goods consumption - of the model via indirect inference (Guvenen and Smith, 2014; Stoltenberg and Uhlendorff, 2023).[10]

Lastly, my paper builds on the literature which studies how to optimally recycle carbon tax revenue (Fried, Novan and Peterman, 2018, 2021; Goulder, Hafstead, Kim and Long, 2019). This paper, on the other hand, examines the optimal level of the carbon tax, and the method of revenue recycling is, for now, set to lump-sum transfers.

The paper is organized as follows. Section 2 describes the quantitative model. Section 3 presents the data and outlines the estimation strategy. Section 4 shows the estimation results and discusses identification. Section 5 briefly discusses the main computational exercise. Section 6 presents the results.

2 An Economy with Idiosyncratic Risk, Two Goods, and a Climate Externality

This section presents an Aiyagari (1994) economy with a clean and a dirty good, and endogenous labor supply. The structure of production and the climate sector largely follows Barrage (2020) and Golosov et al. (2014), respectively. In this setting, the first-best is not attainable, as the government does not have access to individualized lump-sum transfers.

A key notion will be that the household problem in this more elaborate setting can be broken into two steps. First, the household chooses how much to save and how much to spend. Second, once total spending has been determined, the household decides in a static subproblem how much to spend on the clean and how much to spend on the dirty good.

[8] This refers to Proposition 2 in Jacobs and van der Ploeg (2019).

[9] Jacobs and van der Ploeg (2019) is a specific application of a more general result that the optimal carbon tax equals the Pigouvian rate adjusted by the marginal cost of public funds (Sandmo, 1975; Bovenberg and de Mooij, 1994), which equals one under the optimal tax system (Jacobs and de Mooij, 2015; Jacobs, 2018).

[10] The references in the text refer to recent applications of indirect inference to dynamic macroeconomic models. A theoretical treatment can be found in Gourieroux, Monfort and Renault (1993) and Smith, Jr. (1993).

2.1 Setup

Households Time is discrete, $t \in \{0, 1, \dots\}$, and there is no aggregate risk. The time period in the model is five years. The economy is populated by a continuum of infinitely-lived households of measure one. Households' preferences are represented by the utility function

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_{it}, d_{it}, n_{it}) \tag{1}$$

where c_{it} denotes the consumption flow of the clean good, d_{it} denotes the consumption flow of the dirty good, and n_{it} denotes labor supply of household i at time t. The future is discounted with factor β.

Households are subject to idiosyncratic productivity risk captured by a first-order Markov chain $\theta_t \in \Theta$ with $|\Theta| = S < \infty$ and transition matrix $\underset{S\times S}{\Gamma}$. An agents' pre-tax income is then determined by her productivity, the equilibrium wage per unit of productivity, w_t, and the amount of labor supply: $y_t^{pre} = w_t \theta_t n_t$. Pre-tax income is transformed into net (or after-tax) income using a net income function $\mathcal{T}(y) = y - T^y(y)$, where the tax function $T^y(\cdot)$ is to be specified below. Moreover, households have access to a one-period risk-free bond, a, as consumption insurance instrument. Capital income is taxed at rate τ^k and borrowing is restricted by an ad-hoc constraint $\underline{a}$. Lastly, share $(1-\mu)$ of energy produced is dirty and hence potentially subject to a carbon tax τ_d, which the energy producer passes-through at rate ϖ. The government pays lump-sum transfers g to the household.

Hence, the household budget constraint is

$$c_t + (p_d + (1-\mu)\varpi\tau_d)d_t + a_{t+1} = \mathcal{T}(y^{pre}) + (1 + r(1-\tau^k))a_t + g,$$

where p_d denotes the price of the dirty good, respectively, r is the equilibrium interest rate. In the following, I define $\tilde{p} \equiv p_d + (1-\mu)\varpi\tau_d$.

Production I model two production sectors (Barrage, 2020; Douenne et al., 2023).

Final good sector In the final goods sector, indexed by 1, a final good Y is produced using a neoclassical aggregate production function

$$Y = (1 - \mathfrak{D}(S))\tilde{X}\tilde{F}_1(K_1, L_1, E^p) = X(S)\tilde{F}_1(K_1, L_1, E^p) = F_1(K_1, L_1, E^p; \tilde{X}, S) \tag{2}$$

with K_1 units of capital, L_1 efficiency units of labor, E^p units of energy as inputs, and total factor productivity $\tilde{X}$. The final good can either be consumed or invested. $\mathfrak{D}(S)$ represents climate damages to output as a function of the stock of atmospheric carbon S with $\mathfrak{D}'(S) > 0$. This modelling approach of climate damages follows the seminal work by Nordhaus (1991) and the more recent environmental macroeconomic literature.

Energy sector In the energy sector, indexed by 2, energy E is produced using a neoclassical aggregate production function

$$E = F_2(K_2, L_2) \tag{3}$$

with K_2 units of capital and L_2 efficiency units of labor. Energy is either consumed by households (dirty good) or used in production of the final good such that $E = E^p + D$. Following Barrage (2020), producers can provide a share μ from clean energy production, such that only $E^m = (1-\mu)E$ contributes to the stock of emissions. This clean technology is available at a cost of $\Psi(\mu)$ per unit of energy.

Lastly, capital and labor are fully mobile across sectors such that market clearing implies:

$$K = K_1 + K_2 \tag{4}$$

$$L = L_1 + L_2 \tag{5}$$

Government The government levies labor taxes on pre-tax income y^{pre} using the possibly non-linear labor tax function $T^y(y^{pre})$, a linear capital income tax τ^k as well as a carbon tax on dirty goods consumption τ_d. Moreover, it issues government debt B, and chooses lump-sum transfers g to balance its budget:

$$B_{t+1} + g_t = (1+r)B_t + \mathfrak{T}_t, \tag{6}$$

where $\mathfrak{T}_t$ denotes total tax revenue from labor, capital, and carbon taxes.

Climate sector

Carbon cycle The current level of atmospheric carbon concentration, S_t, depends on current and past emissions. In my case, emissions are related to energy produced net of the abated share:

$$S_t = \sum_{\tau=0}^{\infty}(1-\Phi_\tau)\left[(1-\mu_{t-\tau})E_{t-\tau}\right] = \sum_{\tau=0}^{\infty}(1-\Phi_\tau)E^m_{t-\tau}$$

where $1-\Phi_\tau = \varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^\tau$ with the following interpretation: φ_L is the share of carbon emitted which stays in the atmosphere forever; a share of $1-\varphi_0$ of the remaining $1-\varphi_L$ exits the atmosphere immediately; and a remaining share $(1-\varphi_L)\varphi_0$ that decays at geometric rate φ. To write it recursively, following Känzig (2023), I set and $\varphi_L = 0$ and write

$$S_t = (1-\varphi)S_{t-1} + \varphi_0 E^m_t \tag{7}$$

Recursive problem An agent is characterized by the by the pair $(a_{it} = a, \theta_{it} = \theta)$, the household state, and solves the following optimization problem

$$
\begin{aligned}
V(a,\theta) = \max_{c,d,n,a'} & \; u\big(c,d,n\big) + \beta \mathbb{E}_\theta V\big(a',\theta'\big) \\
& \text{subject to} \\
& c + \tilde{p}d + a' \leq (1 + r(1-\tau^k))a + \underbrace{w\theta n - T^y(w\theta n)}_{\mathcal{T}(w\theta n)} + g \\
& a' \geq \underline{a}
\end{aligned} \tag{8}
$$

2.2 Equilibrium

Let $A \equiv [\underline{a}, \overline{a}]$ be the set of possible values for a_{it}. Define the state space by $\mathbb{S} \equiv A \times \Theta$ and let the σ-algebra $\Sigma_{\mathbb{S}}$ be defined as $B_A \otimes P(\Theta)$, where B_A is the Borel σ-algebra on A and $P(\Theta)$ is the power set of Θ. Finally, let $\mathcal{S} = (\mathcal{A} \times \Theta)$ denote a typical subset of $\Sigma_{\mathbb{S}}$. I define a steady-state equilibrium as follows.

Definition 1 (Steady-state equilibrium)**.** A steady-state equilibrium is a government policy $\{\tau_d, g\}$, a vector of aggregate quantities $\{Y, K_1, K_2, L_1, L_2, \mu, E, S\}$, a probability measure Λ defined over the measurable space $(\mathbb{S}, \Sigma_{\mathbb{S}})$, a set of policy functions $\{\boldsymbol{c}(a,\theta), \boldsymbol{d}(a,\theta), \boldsymbol{n}(a,\theta),$
$\boldsymbol{a}'(a,\theta)\}$, a set of prices $\{r, w, p_d\}$, and a set of policies $\{g, \tau_0, \tau_1, \tau_2, \tau^d, \tau_d\}$ such that: (i) given policies and prices, the decision rules solve the optimization problem Equation (8), (ii) the final goods firm chooses capital K_1, labor in efficiency units L_1, and energy E^p to maximize profits, (iii) the energy producer chooses capital K_2, labor in efficiency units L_2, and abatement μ to maximize profits, (iv) the government budget constraint

$$g + rB = \int_{(A \times \Theta)} T^y(w\theta \boldsymbol{n}(a,\theta)) d\Lambda + \tau^k r A + \tau_d (1-\mu) E$$

holds, (v) the asset market clears

$$A \equiv \int_{(A \times \Theta)} \boldsymbol{a}'(a,\theta) d\Lambda = B + K$$

(vi) the goods market clears[11]

$$\int_{(A \times \Theta)} \boldsymbol{c}(a,\theta) d\Lambda + \delta K + \Psi(\mu) E = Y,$$

(vii) Λ is an invariant probability measure and satisfies for all $\mathcal{S} \in \Sigma_{\mathbb{S}}$

$$\Lambda(\mathcal{S}) = \int_{(A \times \Theta)} Q((a,\theta), \mathcal{S}) d\Lambda,$$

where Q is the associated Markov transition function induced by Γ and $\boldsymbol{a}'$, and (viii) the stock of

[11] This market clearing condition is actually redundant by Walras's law, but is nevertheless a useful check whether all equilibrium conditions are properly computed (Appendix A.3).

emissions stays constant at $S = \frac{\varphi_0}{\varphi}(1-\mu)E$.

3 Quasi-homothetic preferences

This section brings forward the functional form assumption regarding the flow utility $u(\cdot)$ in Equation (1). In particular, the two goods setup is cast in an environmental context where one good plays the role of a *clean* good, and the other the role of the *dirty* good. The latter good is subject to a subsistence requirement and in the literature often interpreted as food, fuel, or electricity consumption.

I assume that the utility function is separable in the consumption composite $\tilde{c} = c^{\eta}(d-\underline{d})^{1-\eta}$ and labor. I normalize the time endowment to 1 to specify

$$u(c,d,n) = \frac{\left(c^{\eta}(d-\underline{d})^{1-\eta}\right)^{1-\gamma}}{1-\gamma} + \chi\frac{(1-n)^{1-\epsilon}}{1-\epsilon}. \tag{9}$$

The first part of Equation (9) nests Stone-Geary utility in a CRRA specification. In particular, γ denotes the coefficient of relative risk aversion and $\underline{d}$ is the subsistence level for the dirty consumption goods. It is important to note that the elasticity of substitution between the clean and the dirty good is decreasing in the subsistence level (Baumgärtner, Drupp and Quaas, 2017).[12] η and $(1-\eta)$ are expenditure shares based on total income net subsistence consumption, as will become clear below. Regarding the second part, χ denotes the disutility of labor supply, and ϵ is related to the Frisch elasticity of labor supply,$\frac{1}{\epsilon}\frac{1-n}{n}$.

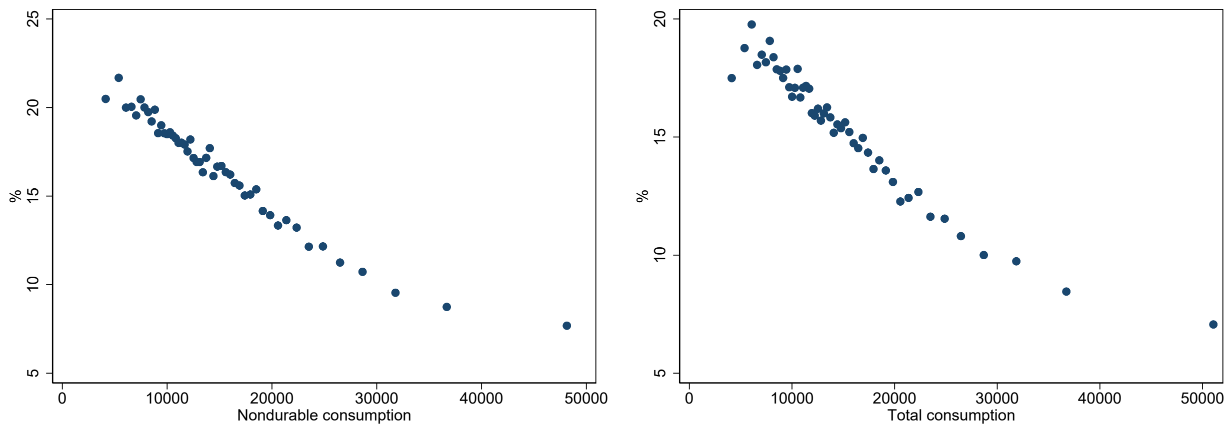
The reason for choosing Stone-Geary type preferences is that empirical evidence shows a declining expenditure share of carbon-intensive goods (dirty goods), such as energy, with respect to expenditures or income. Indeed, this relationship also holds for my data from the PSID. As Figure 1 shows, the expenditure share of US households on energy - defined as the sum of home fuel, heating, and electricity expenditure as a share of two different consumption measures in the PSID - decreases from 20% at the lower end of the expenditure distribution to around 8% at the upper end. Under Cobb-Dogulas utility, the expenditure share would be constant and independent of the expenditure level. The introduction of a subsistence level, $\underline{d}$, generates this pattern as households first have to cover the subsistence level before equating the (price-weighted) marginal utilities of the two goods (Equation (10)).

Static subproblem To further understand the role of the subsistence level, and to facilitate the discussion below, it is instructive to separate the household problem into a dynamic and a static one. In the dynamic problem, the household chooses how much to save for the next period, a_{it+1}, and how much to spend on consumption. Denote this latter total expenditure by e_{it}. In the static problem, the household allocates total expenditure between the clean and the dirty good, respectively.

[12] Under no subsistence consumption this elasticity is one (usual Cobb-Douglas case).

Figure 1: Energy expenditure relative to total expenditure

(a) Nondurable consumption expenditure (b) Total consumption expenditure



Note. This figure shows energy expenditure relative to consumption expenditure with and without durables for households in consumption expenditure 100 bins. Consumption includes expenses for food, gasoline, rent, utilities, communication, transportation, education, childcare, medical needs, vacations, clothing, and recreational activities. Durable components are car repair expenses, down-, loan-, and lease-payments for vehicle loans as well as other expenditure regarding vehicles. All variables have been adjusted using the OECD equivalence scale and are expressed in 2010-$.

Formally, the household solves the following simple problem, in which e_{it} is predetermined:

$$u(e_{it}) = \max_{c_{it},d_{it}} c_{it}^{\eta}(d_{it} - \underline{d})^{1-\eta}$$

subject to:

$$c_{it} + \tilde{p}d_{it} = e_{it}$$

$$c_{it} \geq 0, \quad d_{it} \geq \underline{d}$$

The intra-temporal first-order condition of this problem is

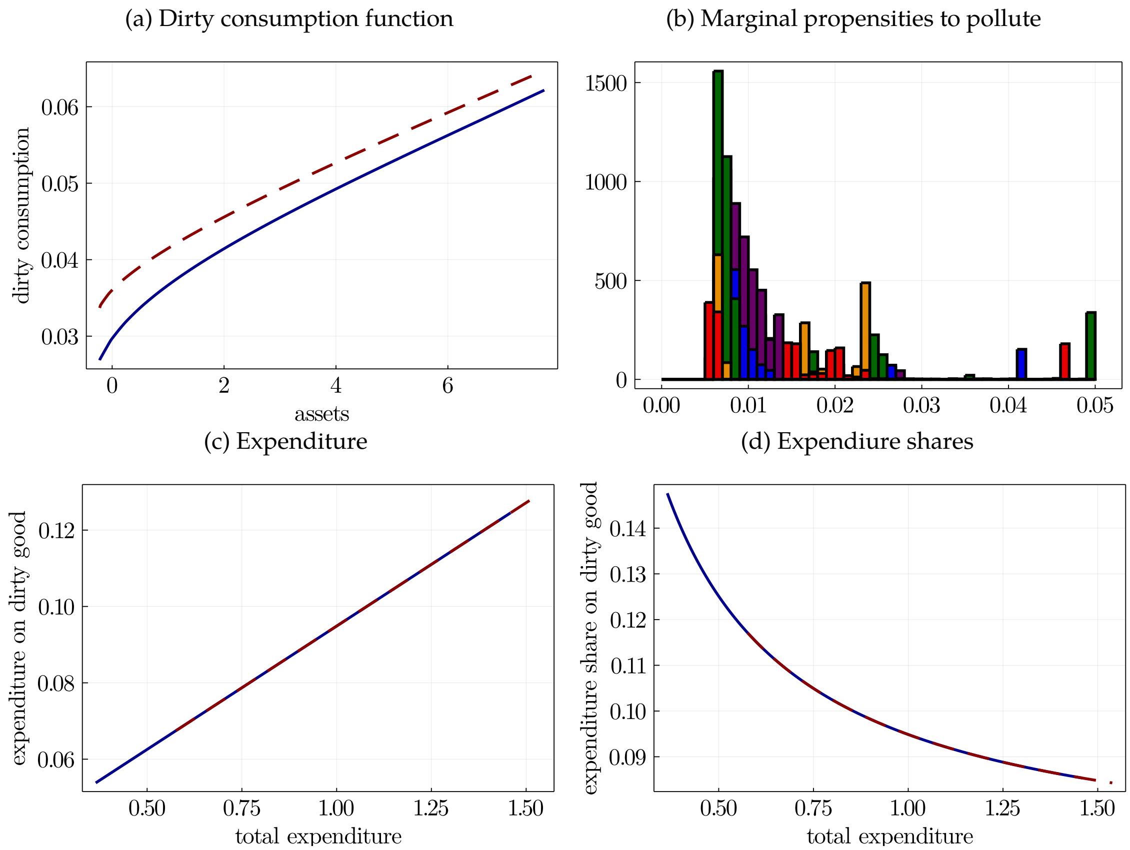
$$u_d(c_{it}, d_{it}) = \tilde{p}u_c(c_{it}, d_{it}) \quad \Rightarrow \quad (1-\eta)c_{it} = \eta\tilde{p}(d_{it} - \underline{d}). \tag{10}$$

The solution to this problem is

$$\begin{aligned} c_{it} &= \eta\Big(e_{it} - \tilde{p}\underline{d}\Big), \\ d_{it} &= (1-\eta)\frac{e_{it}}{\tilde{p}} + \eta\underline{d}. \end{aligned} \tag{11}$$

Hence, in this simple setting, decision rules for clean and dirty consumption are linear in total expenditure. The subsistence level is merely a shifter of the expenditure expansion paths. This is an important feature of these particular preferences. In fact, the system of demand equations implied by them are referred to as the Linear Expenditure System (Stone, 1954).

Figure 2: Decision rules, expenditure, and marginal propensities to pollute



Note.

3.1 Concavity of the consumption function(s)

Recall again Equation (11) where consumption on either good is linear in expenditure e_{it}. Moreover, Jacobs and van der Ploeg (2019) show, in a static setting where expenditure equals income, that under linear Engel curves, externality correcting taxes should be set at the Pigouvian rate.

It is worth noting, however, that Engel curves are *not* linear under the model described in Section 2.1, even with (CRRA-nested) Stone-Geary preferences. The key to this observation lies in the concavity of the consumption function in heterogeneous-agent incomplete-markets models (Zeldes, 1989*b*; Carroll and Kimball, 1996). Due to uncertain future income or productivity, households accumulate precautionary savings and especially so when asset and/or income levels are low. Hence, poorer households with relatively more precautionary savings have low consumption and hence, higher marginal propensities to consume. In other words, the Engel curve is non-linear.[13]

Figure 2 illustrates these points. Panel 2a shows consumption functions for two productivity types as a function of assets; both are clearly concave and more so for lower levels of assets. Panel 2c shows expenditure on the dirty good as a function of total expenditure. We see that this relation

[13] Carroll and Kimball (1996) discuss two cases under which the consumption function is linear. First, under isoelastic utility and interest rate uncertainty but no idiosyncratic uncertainty. Second, under CARA utility and labor income risk only.

is linear, relating to the static subproblem of the household (Equation (11)). Panel 2b shows the marginal propensity to consume the dirty good, what I term the *marginal propensity to consume (MPP)*, out of a windfall income gain of 1% of average income. We see that there is a distribution of MPPs, with higher marginal propensities for the lower productivity type. This heterogeneity is a clear indication of non-linear Engel curves.

The following proposition formalizes this discussion:

Proposition 1 (Non-linear Engel curves)**.** *Under (quasi-)homothetic preferences, inelastic labor supply, and for any labor-productivity Markov chain which induces non-negative consumption decisions, both the clean and dirty consumption good exhibits concave Engel curves w.r.t. to income and wealth:*

$$\boldsymbol{c}_{aa}(a,\theta) < 0,\ \boldsymbol{c}_{\theta\theta}(a,\theta) < 0 \quad \textit{and} \quad \boldsymbol{d}_{aa}(a,\theta) < 0,\ \boldsymbol{d}_{\theta\theta}(a,\theta) < 0$$

Proof. The proof of this proposition is a straightforward application of Theorem 1 in Carroll and Kimball (1996) and the fact that the composition of a linear (decision rule in the static problem) and a concave function (decision rule in the dynamic problem) yields a concave function. □

The key takeaway of Proposition 1 and the preceding discussion is that the optimal carbon tax does take into account distributional concerns in quantitative heterogeneous-agent incomplete market models with precautionary savings.

4 Bringing the model to the data

The last section suggests that the presence of idiosyncratic risk and precautionary savings matters - in theory - qualitatively for optimal carbon taxation. In light of this, the rest of this paper asks whether these features are also quantitatively important? Hence, I calibrate the model from Section 2.1 and have to choose functional forms and parameter values. The latter are chosen in two steps. First, I set values according to the literature or to match aggregate data targets. Second, I estimate the remaining set of structural parameters, which only belong to the household problem, using indirect inference in partial equilibrium (PE). PE requires to take a stance on the determination of prices, since I do not impose market clearing, which I will discuss below. Table 1 summarizes the parameter values.

4.1 Calibration

Preferences I choose a standard value for relative risk aversion, $\gamma = 2$, and set ϵ to target an average Frisch elasticity of labor supply of one. Equation (9) shows the functional form.

Labor productivity process I model the idiosyncratic productivity process as the sum of a persistent and a transitory shock (plus measurement error):

$$\begin{aligned} \log(\theta_{it}) &= \kappa_{it} + \psi_{it} + \nu_{it} \\ \kappa_{it} &= \rho\kappa_{it-1} + \varepsilon^{\kappa}_{it}. \end{aligned}$$

In particular, the persistence process κ is modelled as an AR(1) with persistence ρ and variance of its innovation of $\sigma^2_{\varepsilon^\kappa}$; the transitory shocks ψ are independently and identically distributed with zero mean and variance σ^2_ψ; ν denotes (classical) measurement error with σ^2_ν.

I determine the (annual) variances of the parameters using pre-tax wage residuals estimated from PSID data between 2000 and 2006, following the strategy by Flodén and Lindé (2001), and translate them into the 5-year period unit of the model. Moreover, I follow Heathcote, Storesletten and Violante (2010) and Straub (2019) and set $\sigma^2_\nu = 0.02$ as estimated in French (2004) to identify the (annual) transitory shock. In Appendix B.2, I describe the estimation procedure in detail.

Final goods production The technology $\tilde{F}_1$ is assumed to be of the constant elasticity of substitution (CES) form

$$\left[(1-s)(K_1^\alpha L_1^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E^p)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \tag{12}$$

with λ as the elasticity of substitution between the capital-labor bundle and energy, and s a share parameter.[14] In equilibrium, the factors of production are rented at rates $r+\delta$, w, and $\tilde{p}$, such that by Euler's theorem: $Y = (r+\delta)K_1 + wL_1 + \tilde{p}E^p$, where δ denotes capital depreciation.

I fix the gross capital share in production α at 0.36 based on standard estimates from the literature (Rognlie, 2016) and the elasticity of substitution between the capital-labor composite and energy λ at 0.547 as found in van der Werf (2008). I follow Straub (2019) and set δ to match a capital-to-output ratio of 3.05. The implied wealth to output ratio is 3.8, close to the most recent estimate of 4 in Piketty and Zucman (2014, Figure IV) for the US. The share parameter s is set to match an energy share of production of five percent.

I normalize output to unity using the technology parameter X. Moreover, recall that X is a product of net of climate damages and damages: $X = \tilde{X}(1-\mathfrak{D}(S))$. Hence, during estimation, I ignore the stock of carbon in the atmosphere in the economy, for I could always update $\tilde{X}$ to cancel out any resulting damages.

Energy production Energy is produced using a Cobb-Douglas technology in capital and labor:

$$E = K_2^{\alpha_E} L_2^{1-\alpha_E}. \tag{13}$$

I set $\alpha_E = 0.597$ following Barrage (2020). Moreover, the abatement cost function is

$$\Psi(\mu) = c_1 \mu^{c_2}. \tag{14}$$

I follow DICE and set c_2 to 2.6. Hence, the cost function is convex in μ, implying that marginal costs are increasing in abatement. To pin down a_1, I use initial steady-state values from Douenne et al. (2023), who largely follow DICE 2016 in their calibration. In particular, the backstop price describes the price of emissions at which there is full abatement, $\mu = 1$, which implies for marginal

[14] van der Werf (2008) writes that "the (KL)E nesting structure, that is a nesting structure in which capital and labour are combined first, fits the data best, but we generally cannot reject that the production function has all inputs in one CES function". Another recent example where this particular nesting structure is used is Hassler, Krusell and Olovsson (2021).

abatement costs: $c_1 c_2 \mu^{c_2-1} E = c_1 c_2 E = P^{backstop} E$. The parameter c_1 is then chosen such that the backstop-price implied energy costs to GDP ratio in initial steady state, $\frac{P^{backstop} E}{Y}$, is equal to 0.27 as in Douenne et al. (2023).

Government I use the three parameter functional form by Gouveia and Strauss (1994) to model the labor income tax function:

$$T(y^{pre}) = \tau_0 \left(y^{pre} - \left((y^{pre})^{-\tau_1} + \tau_2 \right)^{-1/\tau_1} \right). \tag{15}$$

Gouveia and Strauss (1994) report $\tau_0 = 0.258$ and $\tau_1 = 0.768$ for the year 1989 - their most recent estimate. τ_2 is determined in estimation and is adjusted such that the government budget constraint holds. I set the capital income tax τ^k to 0.36 as in Trabandt and Uhlig (2011). Lump-sum transfers g are set to 0.114 to match a transfer-to-GDP ratio of 11.4% (Dyrda and Pedroni, 2023).

Climate sector As discussed above, during estimation I ignore climate damages. For completeness, however, I also now describe how I model the climate sector of the economy, which - in the spirit of Nordhaus's DICE model (Nordhaus, 1992, 1993) - follows Golosov et al. (2014).

Carbon cycle To calibrate φ and φ_0 I follow Golosov et al. (2014).[15] φ is set to capture the fact that excess carbon has a mean-lifetime of about 300 years such that $(1-\varphi)^{60} = 0.5$, while the calibration for φ_0 captures that half of the CO_2 emissions into the atmosphere are removed after 30 years: $\varphi_0 = \frac{0.5}{(1-\varphi)^6}$

Damage function The functional form for the damage function is taken from Golosov et al. (2014):

$$1 - \mathfrak{D}(S) = e^{-\xi S_t}, \tag{16}$$

where ξ governs the strength of output damages of a marginal increase in atmospheric carbon.[16] Later, I set the parameter ξ such that in the initial steady state, without carbon taxes, damages imply a total loss of 5% of GDP.

Prices and quantities Prices and quantities are pinned down using a system of equations implied by the supply side of the economy. I fix $\{r, L, D\}$ and solve for $\{w, K, K_1, K_2, L_1, L_2, p_d, E, E^p, Y\}$ using i) five first-order conditions of the firms ii) two technology definitions iii) energy market clearing and iv) factor market clearing. The annual interest rate fixed at 3%, slightly below the estimate of Jordà, Knoll, Kuvshinov, Schularick and Taylor (2019) for the post-1980 period. I fix $L = N \sum_S \theta_s f(\theta_s)$, where $f(\cdot)$ denotes the invariant productivity type distribution induced by the Markov chain and where I set $N = 0.363$ since I target this number in estimation. Given arbitrary model units, there is no observable counterpart to D and it is difficult to derive a theoretical starting point, as aggregate dirty goods consumption depends on the distribution of agents in the economy.

[15] Golosov and co-authors, in turn, cite Archer (2005) and the 2007 technial summary of the IPCC report (IPCC, 2007)

[16] As Golosov et al. (2014) explain, Equation (16) is an approximation that conflates the *concave* relationship between CO_2 concentrations and temperature, and a *convex* relationship between temperature and damages. In particular, it implies constant marginal damages - measured as a share of GDP: $\frac{\partial Y / \partial S}{Y} = -\xi$

Table 1: Preset parameters for estimation

	Description	Value	Target/source
	Preferences		
γ	Risk aversion	2.0	literature
ϵ	Curvature of utility from leisure	4.06	Average Frisch elasticity of unity
	Productivities (annual)		
ρ	Productivty shock persistence	0.9327	PSID
$\sigma^2_{\varepsilon^\kappa}$	Variance of innovations to persistent shock	0.0426	PSID
σ^2_ψ	Variance of transitory shocks	0.0507	PSID
σ^2_ν	Variance of measurement error	0.02	French (2004, p.608, Table 5)
	Production		
	Final goods production		
λ	Substitution elasticity	0.547	van der Werf (2008, p.2972, Table 3)
α	Capital share	0.36	literature
δ	Depreciation (annual)	0.140	annual $K/Y = 3.05$ (FRED)
X	Net total factor productivity	2.75	Normalize output to unity
s	Share parameter	0.0054	Energy share in production of 5%
ω	Pass-through coefficient	0.25	
	Energy production		
α_2	Capital share	0.597	Barrage (2020)
	Abatement		
c_1	Scale abatement cost function	1.64	Backstop price to GDP (see text)
c_2	Exponent abatement cost function	2.6	DICE 2016
	Government		
τ_0	Average labor income tax	0.258	Gouveia and Strauss (1994, p.323, Table 1)
τ_1	Progressivity of labor income tax	0.768	Gouveia and Strauss (1994, p.323, Table 1)
τ_2	Scaling parameter	0.525	Government budget constraint
τ^k	Capital income tax	0.36	Trabandt and Uhlig (2011, p.311,Table 1)
B/Y	Public debt (annual) / GDP	0.73	FRED
g/Y	Transfers / GDP	0.114	Dyrda and Pedroni (2023)
	Climate sector		
	Damages		
ξ	Damage parameter	0.016	GDP loss of 5% under BAU
	Carbon cycle		
φ	Emissions decay parameter	$1 - \exp(\log(0.5)/60)$	Golosov et al. (2014)
φ_0	Emissions share parameter	$0.5/((1-\varphi)^6)$	Golosov et al. (2014)

Note. This table shows preset and calibrated parameters of the quantitative model which is used to estimate the remaining parameters via indirect infernce. FRED datasources can be found in Appendix B.1.

Hence, I fix a starting point of $D = 0.041$ from based on test estimation runs on a coarser grid. Moreover, I later verify post estimation that the implied aggregate dirty goods consumption is in line with this initial guess.

The remaining structural parameters are (i) the utility elasticity η, (ii) the subsistence level $\underline{d}$, (iii) the disutility of labor χ, (iv) the borrowing limit $\underline{a}$, and (v) the discount factor β. I will estimate these parameters using indirect inference. In the following two subsections, I will describe the micro data and targets I use. Thereafter, I will describe the estimation procedure in more detail.

4.2 Data

I use data from the Panel Study of Income Dynamics between 2005-2018 to compute the micro moments which I target in estimation. The PSID is a widely used longitudinal survey containing information on household demographics, income, and wealth. In the waves of 1999 and 2005, respectively, the PSID extended its collection of consumption expenditure data. It now captures over 70 percent of all consumption items available in the Consumer Expenditure Survey (CEX) and around 70 percent of aggregate consumption in the national income and product accounts (NIPA) (Blundell, Pistaferri and Saporta-Eksten, 2016). The PSID was attested to be a high quality dataset in terms of general low sample attrition rates and high response rates (Andreski, Li, Samancioglu and Schoeni, 2014).

Variables The following variables are all on the household level. For instance, income refers to income from both the head and the spouse in the household, if present. Moreover, all monetary variables in the analysis have been adjusted using the OECD equivalence scale and are expressed in 2010-$.

Income *Labor income* refers to all income from wages, salaries, commissions, bonuses, overtime and the labor part of business income *Total income* in addition includes transfers such as as well as social security income. Both income variables are net of taxes, which were computed using NBER's Taxsim program.

Consumption *Nondurable consumption* includes expenses for food, gasoline, rent, utilities, communication, transportation, education, childcare, medical needs, vacations, clothing, and recreational activities. *Total consumption* also includes durable components such as car repair expenses, down-, loan-, and lease-payments for vehicle loans as well as other expenditure regarding vehicles. Moreover, I define *energy expenditure* as expenditure on gasoline, electricity, and heating. All three categories are greenhouse gas intensive goods and are thus used as a data counterpart for the dirty consumption good in the model.

Wealth My wealth variable refers to financial wealth net of liabilities. In particular, I include the value of one's real estate assets net of remaining mortgages, checking and saving accounts, stocks, bonds, business assets, IRAs or other annuities, and cars. I subtract liabilities such as credit card debt, student debt, outstanding medical bills, legal debt, loans obtained from relatives, and business debt.

Sample My baseline sample includes all PSID waves from 2005-2019, and consists of households where the head is between 25 and 60 years. I exclude observations for which information on consumption, income, wealth, education, household size, and region is missing. Furthermore, I remove observations with labor income below half the state minimum wage as well as top and bottom 1% of the remaining observations on consumption, income, and wealth. This leaves me with a sample of 21,750 households, around 2700 observations per year.

Table D.1 in the appendix shows descriptive statistics about the data. The typical household head is 42 years old, male, and married with 3 family members in total.

4.3 Data targets in estimation

Using the PSID sample and variables as just described, I construct the following moments.

Hours worked The cross-sectional average of weekly working hours of household heads in my sample is 40.61. Given that a full week has 168 hours and assuming 8 hours per day for sleep and other personal care leaves 112 hours per week as time endowment (Guerrieri and Lorenzoni, 2017). Hence, average hours worked as a share of the total time endowment gives 36.3% which is targeted in estimation.

Wealth-to-income ratio To consider the distribution of endogenous variables in my model, I follow Stoltenberg and Uhlendorff (2023) and target two moments of the wealth-to-income distribution: the 10^{th} percentile as well as the median. A 10^{th} and 50^{th} quantile regression on a constant yields $\hat{\beta}_{10} = -0.253$ and $\hat{\beta}_{50} = 1.056$, respectively. Both values are precisely estimated.

Dirty good allocation rule In the data I only observe expenditures, that is, the product of price and quantity. Hence, the data counterpart to Equation (11) is

$$p_d d_{it} = \delta_0 + \delta_1 e_{it} + \mathbf{X}'_{it}\omega + \varepsilon_{it}, \tag{17}$$

where now e_{it} denotes *observed* total expenditure, $p_d d_{it}$ *observed* expenditure on dirty goods of household i at time t. $\mathbf{X}$ is a vector of controls including household-size dummies, household head's five-year age bracket, region, household and year dummies (Pedroni, Singh and Stoltenberg, 2022; Straub, 2019). Total expenditure will be instrumented by total income as in Blundell, Chen and Kristensen (2007). Lastly, to obtain a proper mapping between arbitrary units in the model and data variables in 2010-$ units, I scale all data variables with the average dirty goods expenditure, $D^{avg} = p_d \bar{d}$.[17]

Table 2 shows estimation results for the baseline specification (1) and various robustness exercises (2)-(4). Column (2) omits the region dummies in the control vector. Column (3) additionally

[17] When presenting this paper at a conference, Alkis Blanz brought to my attention that Blanz and Kalkuhl (2022); Blanz, Eydam, Heinemann, Kalkuhl and Moretti (2023) also calibrate their model starting from such a dirty allocation rule under Stone-Geary preferences.

Table 2: Dirty good regression

	dirty goods expenditure			
	(1)	(2)	(3)	(4)
Total expenditure	0.0646	0.0643	0.0623	0.0486
	(0.0152)	(0.0152)	(0.0156)	(0.0113)
Constant	0.3571	0.3517	0.3682	0.4169
	(0.1004)	(0.0743)	(0.1016)	(0.0896)
Observations	22033	22033	22033	22027
R-squared	0.3059	0.2996	0.3077	0.3100

Note. This table shows second stage (IV) coefficients δ_1 and δ_0 of Equation (17) for different specifications. Column (1) is the baseline case as specified in the text. Column (2) omits the region dummies in the control vector. Column (3) additionally controls for liquid assets. Column (4) uses an alternative measure for consumption as the (instrumented) regressor. Standard errors are corrected for heteroskedasticity and clustered at the household level.

Table 3: Targeted moments and estimated parameters

Moment	Estimated Parameters	Description
Wealth-to-income ratio - 10^{th} percentile	$\underline{a}$	Borrowing limit
Wealth-to-income ratio - Median	β	Discount factor
Expenditure regression (IV), δ_1	η	Clean good elasticity
Expenditure regression (IV), δ_0	$\underline{d}$	Subsistence level
Average hours worked	χ	Disutility of labor

Note. This table shows targeted moments an estimated parameters using indirect inference.

controls for liquid assets. Column (4) uses an alternative measure for consumption as the (instrumented) regressor. In all specifications, the first stage F-statistic is well above 10.[18]

To interpret the coefficients, let us look at column (1). The coefficient on total expenditure is equal to 0.0646. According to Equation (11), this coefficient identifies $1-\eta$, which suggests an η of 0.9354. Second, δ_0 identifies $\eta \frac{p_d d\eta}{p_d D^{avg}} = \eta \frac{d}{D^{avg}}$. This constant as well as δ_1 will be targeted in estimation.

4.4 Estimation with indirect inference

The remaining structural parameters $\{\eta, \underline{d}, \chi, \underline{a}, \beta\}$ of the model will be *jointly* estimated using indirect inference in a just identified system (same number of targets as unknown parameters) as shown in Table 3.

Objective function Denote the parameter vector by $\Theta = (\eta, \underline{d}, \chi, \underline{a}, \beta)$ and the vector of moments (as a function of the parameters) by $\mathcal{M} = \left(\delta_0(\Theta), \delta_1(\Theta), \frac{a}{y}\Big|_{50}(\Theta), \frac{a}{y}\Big|_{10}(\Theta), \overline{N}(\Theta)\right)$. Then define the vector of percentage deviations as

$$\mathcal{D} = \left(\mathcal{M} - \widehat{\mathcal{M}}\right) \oslash \widehat{\mathcal{M}}, \tag{18}$$

[18] This rule of thumb is valid as I only consider one endogenous regressor and one instrument (Stock and Yogo, 2005).

Table 4: Estimation results

	Parameter estimates				
	$\underline{d}$	η	χ	β	$\underline{a}$
	0.016	0.9354	0.4651	0.8614	-0.2233
	(0.0045)	(0.0165)	(0.0272)	(0.0008)	(0.0206)
	Auxiliary model and other moments				
	δ_0	δ_1	$\overline{N}$	$\frac{a}{y}\Big\vert_{50}$	$\frac{a}{y}\Big\vert_{10}$
Data	0.3571	0.0646	0.3627	1.0551	-0.2527
Model	0.3571	0.0646	0.3626	1.0562	-0.2530

Note. Asymptotic standard errors in parentheses are computed using a non-parametric panel bootstrap with 200 repetitions; see Appendix C for details.

where $\widehat{\mathcal{M}}$ denotes the data counterpart of the respective moment conditions, and $\oslash$ denotes element-wise division. The objective function is then

$$\Theta^* = \arg\min_{\Theta} \mathcal{D}' W \mathcal{D}, \tag{19}$$

where W is a positive definite weighting matrix. I use the identity matrix as weighting matrix.

Given a parameter vector, I solve the model in partial equilibrium, where the tax parameter τ_2 is adjusted such that the government budget constraint holds. With a PE solution in hand, I construct the model-implied moments and use them in the objective function.

The objective function is minimized using a multistart global optimization algorithm ("TikTak"). For local minimization routines in this algorithm, I use the derivative-free BOBYQA routine from starting points that are determined using the Sobol sequence in a pre-testing phase (Guvenen, 2011; Arnoud, Guvenen and Kleineberg, 2019). See Appendix C for details.

4.5 Estimation results

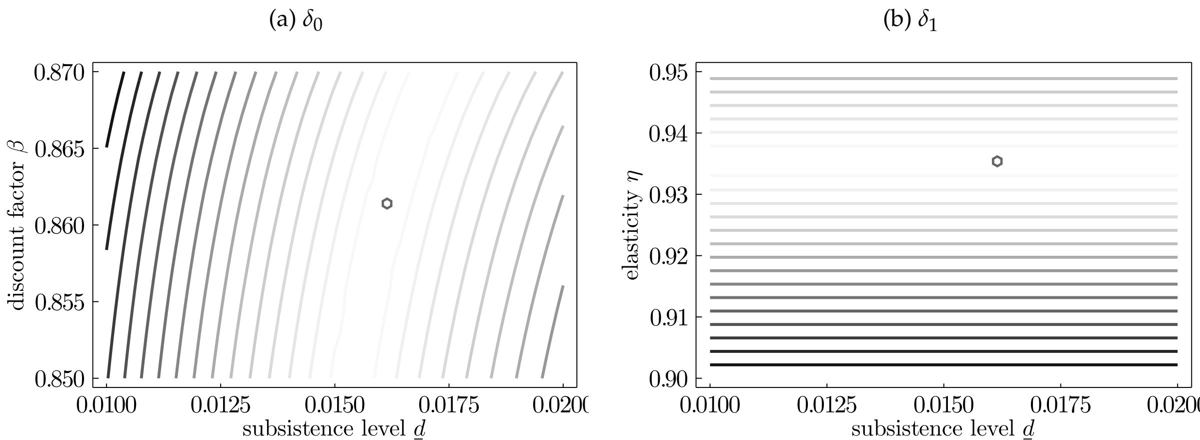
4.5.1 Parameter estimates

Table 4 shows the estimation results. I construct asymptotic standard errors using a non-parametric panel bootstrap with 200 repetitions (Appendix C).

Regarding the utility parameters, I estimate $\eta = 0.9354$ and $\underline{d} = 0.016$. Both parameters are precisely estimated and both coefficients of the auxiliary regression are close to their respective model counterpart. η is comparable with values in the literature, whereas $\underline{d}$ is relatively larger (Fried et al., 2018).[19]

[19] Different calibrations, of course, give rise to different subsistence levels of dirty goods consumption. Hence, when comparing $\underline{d}$ to the literature, I compute expenditure on subsistence consumption, $p_d\underline{d}$.

Figure 3: Numerical identification



Note. Absolute deviation of moments implied by the structural model from their data counterparts when parameters on the respective axis are varying. Other parameters are fixed at their estimated value. Darker regions indicate higher deviation.

Average hours worked, as a fraction of agents' time endowment, is pinned down well by the disutility of labor χ, which is estimated to equal 0.4651. Again, this parameter is estimated precisely.

Given an intertemporal substitution elasticity of $1/\gamma = 1/2$, the discount factor β is relatively high reflecting the 5-year time period of the model. The borrowing limit is $\underline{a} = -0.2233$, which amounts to 33% of average gross income that can be borrowed every period. The model matches the two targeted moments of the wealth-to-income distribution well.

4.5.2 Identification

As common in these type of models, I have no proof of global identification of my parameters. However, I want to mention two points that indicate proper identification of the five parameters. First, I assess how different model-implied moments are affected when I change two of the five parameters and fix the remaining three at their best fit value. Figure 3 shows the results of this exercise and plots of moments implied by the structural model from their data counterparts when varying the parameters on the axes; lighter areas depict smaller deviations.

Contour plots indicate that the utility parameters are identified in two steps, as indicated by the decision rule in the static model. The relative prefence parameter is identified by the model-implied estimate for δ_1. Conditional on this value, δ_0 identifies the subsistence parameter of consumption. Similar arguments hold for the identification of the discount factor and the borrowing limit.

Second, after searching a grid of potential values in my optimization algorithm I choose the best 10% and start a local search step from these points. This search procedure converges to the same best fit values for various starting points,[20] which suggests that the model is globally identified.

[20] By construction, the algorithm with every new local search puts more weight on the current local minimum when setting the new starting value. However, it also converges to the same values for early starting values.

5 Quantitative exercise

In this section, I use the estimated model as laboratory to answer the main question. To what extent do idiosyncratic risk and precautionary savings matter for the welfare-maximizing carbon tax? To this end, I first specifiy a social welfare function as the objective for the government (social planner). Next, I look for the welfare-maximizing carbon tax in general equilibrium, when the lump-sum transfer is adjusted to clear the government budget constraint, and other tax instruments are kept fixed. That is, the government collects carbon taxes and reimburses revenues via transfers. I then compare the level of this tax to various changes in the economic environment. Lastly, I repeat the main exercise if the government is also allowed to adjust the average labor income tax as well.

From now on, the economy is in general equilibrium (GE). In particular, I use the estimated and calibrated parameters, but let prices adjust to clear markets. Moreover, in the initial steady state without taxes, I back out the carbon cycle and damage function parameters as explained above. When optimizing over the carbon tax, the technology parameter $\tilde{X}$ is fixed, hence, any change in total factor productivity X is due to changes in climate damages.

Social welfare function I assume that the social planner is utilitarian and maximizes social welfare defined over households' value functions in stationary equilibrium:

$$SW = \int_{(A\times\Theta)} V_{(\tau_d)}(a,\theta)\, d\Lambda_{(\tau_d)}. \tag{20}$$

In this specification, every household gets the same welfare weight. However, due to concavity in the utility function, the government has an implicit preference for redistribution, as the marginal utility of poorer households is higher than that of the rich.

The planner chooses τ_d to maximize Equation (20) while setting g to balance the government budget. Hence, the subscript (τ_d) stresses that the value function and invariant distribution are associated with this particular carbon tax. I also allow the government in another exercise to adjust both the carbon tax and the average labor income tax τ_0.

5.1 Welfare-maxizing carbon taxes

Optimal carbon taxes The optimal carbon tax equals $\tau_d^* = 1.096$, about 42% of the energy price, and a clean energy share of $\mu = 0.357$. Taking into account the pass-through parameter ω, this translates into a carbon price of about 141\$ per ton of CO_2.[21]

Changes in aggregates and inequality Before discussing the impact of idiosyncratic risk and precautionary savings, it is instructive to first understand how the carbon tax affects aggregates and inequality. Thus, I compute and compare aggregate quantities and prices of the GE economy

[21] The first-order condition of the energy firm implies $\mu = ((\tau_d(1-\omega))/(c_1c_2))^{\frac{1}{c_2-1}}$. Using the definition of the backstop price and its implied value of 550\$ per ton of CO_2, I compute the implied \$/ton-CO_2 price as $(550/(1-\omega))\mu^{c_2-1}$.

Table 5: Changes in aggregate variables with optimal carbon tax

Variables	Percent change
Clean consumption, C	3.00
Dirty consumption, D	-16.82
Hours worked, N	-1.34
Assets, A	-2.12
Labor, L	-1.08
Capital, K	-2.57
Energy, E	-16.87
Output, Y	1.72
Wage, w	0.64
Interest rate, r	1.87
Transfers, g	41.54
Emissions, S	-46.54

Note. This table compares the changes in aggregate variables of the benchmark GE economy with and without the (optimized) carbon tax.

with no tax and the same economy with τ_d^*. Table 5 shows the results, where changes are displayed in percent between the two economies.

We see that households substitute away from dirty goods and increase their consumption in clean goods. Total dirty consumption decreases by 16.82% as the dirty good becomes more expensive. Moreover, average hours worked decrease. Note, however, that labor supply in efficiency units decreases by less, which suggests that labor supply shifts from less to more productive households. In fact, all three production inputs decrease. Nevertheless, overall output increases, because changes on the input side are dwarfed by reductions in emissions and thus environmental damages. Both interest rates and wages increase due to lower environmental damages which increase the marginal products of capital and labor, respectively.

Table 6 shows the results for household inequality statistics. In this case, I compare Gini coefficients for the different consumption goods and labor incomes before and after introducing the carbon tax. With respect to assets, which can attain negative values, I compare percentile differences.

We see four sets of results. First, irrespective of the carbon tax, the gini on dirty goods consumption is smaller than the one on clean goods consumption. Intuitively, the subsistence level compresses the dirty consumption distribution from below.

Second, the gini coefficient with the carbon tax is smaller for both the clean and dirty consumption than under no carbon tax, albeit for the clean good only slightly. A higher carbon tax increases the price of the dirty good, hence, households substitute away from dirty to clean. However, since the elasticity of substitution between the two goods is increasing in cash-on-hand due to the subsistence level (Baumgärtner et al., 2017), richer households decrease their dirty goods consumption by more and the dirty goods distribution gets compressed from above.

Third, gross labor income increases, whereas net labor income increases. Inequality in hours

Table 6: Inequality statistics

	Gini coefficient			
	Dirty consumption	Clean consumption	Gross labor income	Net labor income
No carbon tax	0.111	0.182	0.533	0.294
Optimal carbon tax	0.095	0.178	0.550	0.280
	Percentile difference in assets			
	P50-P5		P95-P50	
No carbon tax	0.685		2.767	
Optimal carbon tax	0.662		2.771	

Note. This table

worked decreases slightly, but higher wages increase inequality in gross labor income as work shifts to richer households. Net labor income, on the other hand, takes into account progressive taxation and lump-sum transfers. Redistributing carbon tax revenue as lump-sum transfers, then, decreases net labor income.

Fourth, asset inequality decreases. Instead of looking at gini coefficients I compute differences in the 5th and 50th percentiles that capture movements in the left part of the distribution as well as in the 50th and 95th percentiles that capture the right part of the distribution. Overall, the asset distribution shifts to the left with a higher share of agents at the borrowing constraint. The left part of the distribution moves by more than the right, as the P50-P5 distance decreases and the P95-P50 distance increases.

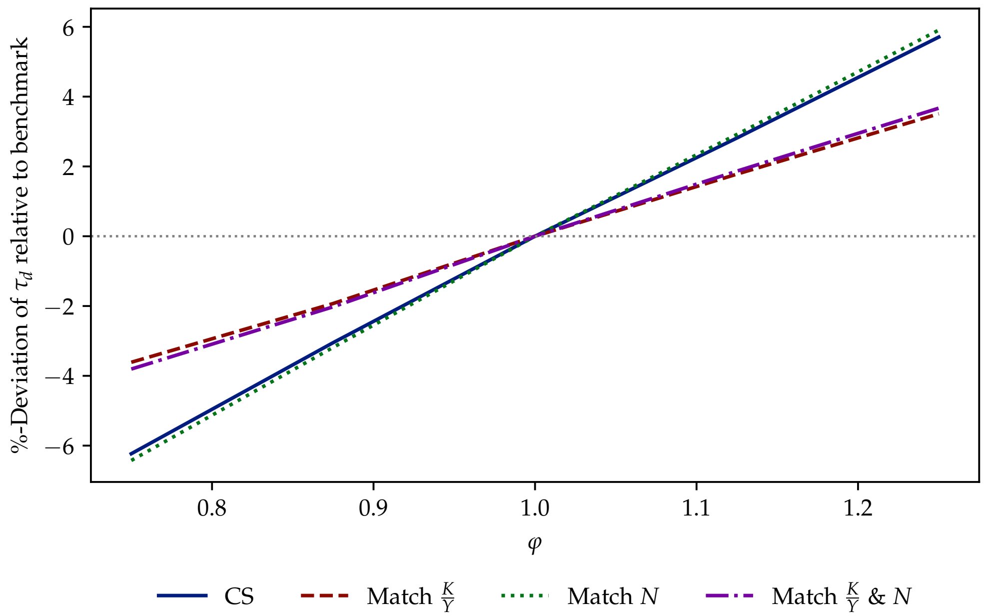
The role of precautionary savings To analyze the role of idiosyncratic risk and precautionary savings, I create a mean-preserving contraction/spread in the variance of the persistent part of the labor productivity of agents. In particular, I scale the variance of the persistent shock by a factor φ and re-normalize the mean productivity to one. Thus, $\varphi > 1$ represents an increase in the variance and so more risk. Moreover, I also compare different recalibrations: A comparative statics (CS) exercise in which all parameters are as in the benchmark, an exercise in which I adjust χ and β to match average hours worked and the capital-to-income ratio, respectively, as in the benchmark GE economy without the tax, and intermediate steps thereof.

The idea behind this exercise is that an increase in labor productivity risk increases the precautionary motive of households. Future labor productivity is more risky, hence, they respond by saving more or working more.

Figure 4 shows the results. I display the %-deviation to the benchmark carbon tax ($\varphi = 1$) under four different recalibrations and different levels of φ. Three things stand out. First, irrespective of recalibrating the model, an increase in idiosyncratic risk and precautionary savings increases the optimal carbon tax compared to the benchmark.

Second, the change in the carbon tax due to re-scaling labor productivity risk is fairly symmetric. For instance, the optimal carbon tax under CS decreases by approximately 6% when the variance

Figure 4: Optimal carbon tax for different levels of idiosyncratic productivity risk relative to benchmark



Note. This figure shows the percentage change of the optimal carbon tax relative to the benchmark for different levels of idiosyncratic risk, which is scaled with the factor φ. The different curves depict different choices regarding recalibration of the economy prior to optimizing the carbon tax: The blue solid curve is a compartive statics (CS) exercise that keeps all parameters as in benchmark and the purple dashed-dotted line matches both average hours worked, N, as well as the capital-to-output ratio, K/Y. Other curves show the intermediate cases.

of the persistent shock is reduced by 25% and increases by approximately 6% when the variance is raised by 25%.

Third, while matching average hours worked only slightly increases the change in the optimal carbon tax, compared to a CS exercise, matching the capital-to-output ratio decreases the impact of a change in labor productivity risk by about 1.2 percentage points. When matching both, an increase of the persistent shock variance by 25% raises the carbon tax by about 3.8%.

Contributors to the carbon tax In order to better understand what components gives rise to a non-zero carbon tax in this economy and to the positive gradient with respect to the degree labor productivity risk, I now conduct several exercises that change various features of the model. First, I introduce a skill-specific incidence rule, $\bar{g}(\theta)$, for lump-sum transfers. In the benchmark economy, every households gets the same level of transfers; g is non-individualized. With an incidence rule, I make the transfer proportional to a household's labor productivity type. In particular, the functional form for the rule is

$$\bar{g}(\theta) = \frac{\theta}{\sum_S \theta_s f(\theta_s)},$$

Table 7: Optimal carbon taxes under different model specifications

Benchmark: 1.096				
	Transfer incidence	No damages	No subsistence level	No damages and no subsistence level
Comparative statics	0.574	0.066	1.099	0.067
Match K/Y & N	0.518	0.067	1.164	0.092

Note. This table compares the optimal carbon tax under different model specifications. The first column implements the optimal carbon tax under a proportional skill-specific transfer rule instead of lump-sum transfers. The second column removes economic damages. The third column removes the subsistence level. The fourth column computes the optimal carbon tax under no subsistence level and no economic damages.

and the budget constraint is now specified as

$$c + \tilde{p}d + a' \leq (1 + r(1 - \tau^k))a + \mathcal{T}(w\theta n) + g\bar{g}(\theta).$$

A proportional incidence rule reduces the progressivity of the carbon tax-transfer combination. Moreover, and specific to the economy at hand, it also reduces the insurance and redistributive capabilities the government provides to households. When a negative labor productivity shock occurs for a household, the transfer received is also smaller than previous period.

The first column of Table 7 shows the optimal carbon taxes under this incidence rule. Compared to the benchmark economy tax is approximately halved. Forgoing a uniform distribution of transfers increases the carbon tax burden on poorer households, who now receive lower transfers while still spending relatively more for dirty goods.

The last three columns compute the optimal carbon tax under no damages, no subsistence level, and both, respectively. Under no damages, I set $\xi = 0$. The optimal carbon tax is then around 0.066, much lower than in the benchmark case. In this setup, a carbon tax merely increases the price of the dirty good without providing economic benefits by increasing total factor productivity. Especially, poorer households then profit from higher wages.

Under no subsistence level, I set $\underline{d} = 0$. In this case, the optimal carbon tax is slightly higher compared to the benchmark. The presence of subsistence needs makes the carbon tax, in and of itself, regressive. Hence, poorer households are affected relatively more by the tax. As such, removing the subsistence level gives room for the government to set higher carbon taxes. From a quantitative point of view, however, the change in carbon taxes is small. The reason for this is that even the poorest households' optimal dirty goods consumption is sufficiently far away from the subsistence level.

Equity vs efficiency The utilitarian welfare criterion used so far conflates redistributive and insurance concerns of the economy. Even in an economy without idiosyncratic labor productivity shocks, redistributing from rich to poor increases welfare due to the concavity of the utility function (Bénabou, 2002). Hence, to isolate equity and efficiency considerations, I follow Bénabou (2002)

Table 8: Optimal carbon taxes under the efficient welfare criterion

Utilitarian welfare	Efficient welfare	Efficient welfare & No damages
1.096	1.036	0.0047

Note. This table compares the optimal carbon tax under the utilitarian welfare criterion and an efficient welfare criterion inspired by Bénabou (2002) (middle column). The last column, in addition, removes economic damages.

and Bakış, Kaymak and Poschke (2015) and construct a different welfare measure which aggregates certainty-equivalent levels of consumption and hours worked, instead of utility levels. To be precise, the effiecient welfare is constructed as follows:

$$SW^{Eff} = \frac{1}{1-\gamma}\left(\int \bar{\tilde{c}}(a,\theta)d\Lambda\right)^{1-\gamma} + \frac{\chi}{1-\epsilon}\left(\int (1-\bar{n}(a,\theta))d\Lambda\right)^{1-\epsilon},$$

where $\bar{\tilde{c}}$ and $\bar{n}$ denote certainty-equivalent levels of the consumption-composite and hours-worked, respectively. The certainty-equivalent level of the consumption-composite solves the equation

$$\mathbb{E}\sum_t \beta^t \frac{\tilde{c}(a,\theta)^{1-\gamma}}{1-\gamma} = \frac{1}{1-\beta}\frac{\bar{\tilde{c}}(a,\theta)^{1-\gamma}}{1-\gamma};$$

similarly for hours worked.

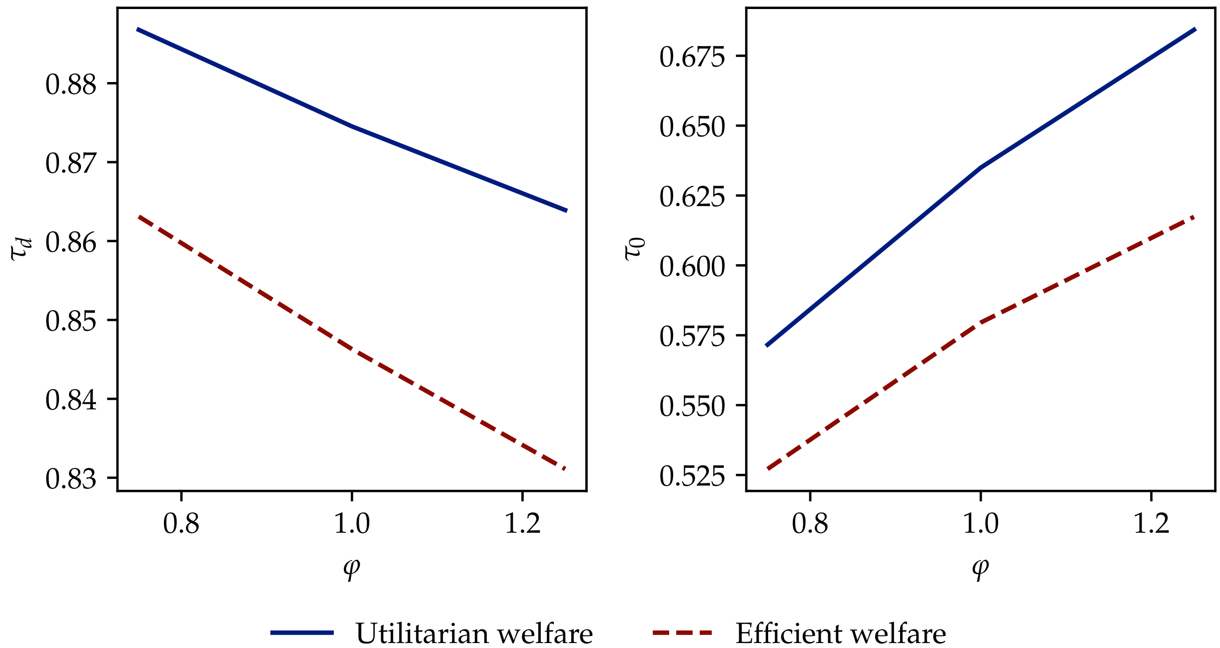
Table 8 shows the optimal carbon tax under this new welfare criterion and compares it to the utilitarian benchmark. The carbon tax under the efficient welfare criterion is about 6% lower than under the utilitarian welfare criterion. This suggests that equity concerns have a positive impact on the carbon tax. As discussed above, this is due to lump-sum redistribution of transfers, without which the carbon tax would be regressive.

To further understand the contribution of insurance considerations, the last column then shows the optimal carbon tax under the efficient welfare criterion when there are no damages ($\xi = 0$); in this case, the model has been recalibrated to match average hours worked and the capital-to-output ratio. The resulting optimal carbon tax very small with a optimized value of 0.005. Comparing this to the no-damage value from Table 7 suggests that redistribution due to equality concerns is much larger than for insurance concerns.

Adjusting labor income taxes The exercises conducted so far are essentially third-best policy exercises. I keep labor income and capital income taxes fixed and only adjust carbon taxes. While this is of major practical relevance, as governments do not always adjust other taxes when implementing corrective carbon taxation, it is unsatisfactory from a economic theory perspective. Changing the carbon tax in this economy means that it not only adjusts to internalize the climate externality, but it also raises revenue for redistribution and insurance. When the government has more instruments at its disposal, however, there are more suitable ways to achieve the latter.

In the following, I thus allow the government to adjust the average labor tax parameter τ_0, in addition to τ_d, to maximize welfare in the stationary equilibrium. Figure 5 shows the results. The

Figure 5: Optimal carbon taxes and optimal (average) labor income taxes



Note. This figure shows optimal carbon taxes and optimal average labor income taxes for different levels of idiosyncratic risk, scaled by the factor φ, if both taxes are allowed to be adjusted by the social planner. The solid line depicts results under utilitarian welfare. The dashed line depicts results under efficient welfare.

left panel shows the optimal carbon tax for different labor productivty risk for both utilitarian and efficient welfare. The right panel shows the corresponding average labor tax parameter.

The results suggest that the income tax does take over redistributive and insurance purposes that fell on the carbon tax before. First, the carbon tax is lower and the average labor income tax parameter is higher than under the benchmark steady state calibration. Second, the effect of idiosyncratic risk and the precautionary motive is now reversed! The carbon tax is decreasing in the scale parameter φ, whereas the average income tax parameter is increasing. These changes also hold when using the efficient welfare criterion.

6 Conclusion

In this paper, I studied the optimal carbon tax in a climate-economy model with idiosyncratic risk and borrowing constraints in general equilibrium. I first calibrated and estimated the model on U.S. household panel data. Thereby, I pay particular attention to the subsistence level of dirty pollution-intensive good consumption which I identify using indirect inference. In a next step, I used the model as a laboratory and optimize over the carbon tax in a general equilibrium steady state under an utilitarian welfare criterion.

To analyze the role of idiosyncratic risk and precautionary savings, I create a mean-preserving contraction/spread in the variance of the persistent part of the labor productivity of agents. I find that the optimal carbon tax is increasing in the desire for precautionary savings. When recycling the revenue lump-sum, the optimal carbon tax carbon tax also functions as a means of redistribution and serves as an insurance device for the uninsurable idiosyncratic productivity shocks through i) transfers and ii) increasing wages and interest rates due to lower climate damages on production and thus higher factor productivity. This result depends on the availability of tax instruments of the social planner.

Lastly, I find two more avenues of future research of interest. First, the novelty in this paper was the introduction of idiosyncratic risk. Another interesting direction would be to study the distributional consequences of *aggregate* climate uncertainty. This is in particular the case if there is heterogeneous incidence of pollution damages, for instance, due to different abilities in adaptation.

Second, political support for carbon taxation or other forms of corrective pricing has so far been weak.[22] This is also partly due to distributional concerns, as the yellow-vest movements in France or Canada demonstrate (Douenne and Fabre, 2022). Thus, an exploration of household heterogeneity in a political climate-economy would be worthy of future research.

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[22] As of 2020, only 18% of global CO_2 emissions are internalized (Ritchie and Rosado, 2022).

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Appendix

A Model - Details

A.1 Households

I repeat the recursive household problem for ease of exposition.

$$
\begin{aligned}
V(a,\theta) = \max_{c,d,n,a'} & \ u\big(c,d,n\big) + \beta \mathbb{E}_\theta V\big(a',\theta'\big) \\
& \text{subject to} \\
& c + (p_d + (1-\mu)\varpi\tau_d)d + a' \leq (1 + r(1-\tau^k))a + \underbrace{w\theta n - T^y(w\theta n)}_{\mathcal{T}(w\theta n)} + g \\
& a' \geq \underline{a} \quad n \geq 0
\end{aligned}
$$

Defining $r(1-\tau^k) \equiv \tilde{r}$ and $p_d + (1-\mu)\varpi\tau_d \equiv \tilde{p}_d$, the Bellman equation is

$$
\begin{aligned}
V(a,\theta) = \max_{c,d,n,a'} u\big(c,d,n\big) + \beta \mathbb{E}_\theta V\big(a',\theta'\big) - \pi^1 \Big(c + \tilde{p}_d d + a' - (1+\tilde{r})a - \mathcal{T}(w\theta n) - g\Big) \\
- \pi^2 \Big(\underline{a} - a'\Big) + \pi^3 n,
\end{aligned}
$$

where π^1, π^2, π^3 denote the Lagrange multipliers on the budget, borrowing, and non-negativity constraint, respectively.

In the following, I use the common notation that $\frac{\partial u(c,d,n)}{\partial c} \equiv u_c$. The first-order conditions of the household are

$$
\begin{aligned}
[c]: &\quad u_c = \pi^1 \\
[d]: &\quad u_d = \tilde{p}_d \pi^1 \\
[n]: &\quad u_n = -\pi^1 \mathcal{T}_n(w\theta n) w\theta - \pi^3 \\
[a']: &\quad \beta \mathbb{E}_\theta \left[V_a(a',\theta')\right] = \pi^1 - \pi^2
\end{aligned}
$$

Substituting out the multiplier π^1, assuming an interior solution for labor ($\pi^3 = 0$), and using the Envelope condition $V_a(a,\theta) = \pi^1(1+\tilde{r})$ we get

$$
u_c \tilde{p}_d = u_d \tag{A.21}
$$

$$
u_n = -u_c \mathcal{T}_n(w\theta n) w\theta \tag{A.22}
$$

$$
u_c \geq \beta(1+\tilde{r}) \mathbb{E}_\theta \left[u_{c'}\right] \tag{A.23}
$$

Equation (A.21) is the *intra-temporal* first-order condition between the two consumption goods. Re-arranging yields that in the optimum, the marginal rate of substitution (MRS) between the clean and the dirty good, u_c/u_d, equals the relative price of the clean good in terms of the dirty good,

$1/\tilde{p_d}$. Note that the marginal rate of transformation (MRT) between the two goods is $1/p_d$, hence, a positive carbon tax distorts the social optimal goods allocation.

Equation (A.22) is the *intra-temporal* first-order condition between clean consumption and labor. Similar arguments regarding the MRS and MRT as above apply. One could have stated the condition in terms of the dirty good. Again, a positive carbon tax (in addition to the labor tax) distorts the labor supply decision of the household, as it makes leisure cheaper relative to the dirty good.

Equation (A.23) is the *inter-temporal* first-order condition and is the familiar Euler equation. When the borrowing constraint is not binding, π^2 is zero and the equation holds with equality. Here, the capital income tax drives a wedge between MRS ($\frac{u_c}{\beta u_{c'}}$) and MRT $(1+r)$.

A.2 Firms

Energy producer The energy producer maximizes profits by choosing capital, K_2, labor L_2, and the fraction of abatement μ under perfect competition using a constant returns to scale technology. It takes prices (r, w, p_d), pass-through opportunities ϖ, as well as policy variables as given and obtains zero profits in equilibrium:

$$\max_{K_2,L_2,\mu} \quad \{p_d F_2(K_2,L_2) - (1-\mu)(1-\varpi)\tau_d F_2(K_2,L_2) - (r+\delta)K_2 - wL_2 - \Psi(\mu)F_2(K_2,L_2)\},$$

where I already substituted in the technology constraint $E = F_2(K_2, L_2)$.

The first-order conditions are

$$\begin{aligned}
[p_d - (1-\mu)(1-\varpi)\tau_d - \Psi(\mu)]F_{2,K_2} &= r+\delta \\
[p_d - (1-\mu)(1-\varpi)\tau_d - \Psi(\mu)]F_{2,L_2} &= w \\
\tau_d(1-\varpi) &= \Psi'(\mu)
\end{aligned}$$

Final good producer The final good firm maximizes its profits by choosing capital, K_1, labor L_1, and energy E^p under perfect competition using a constant returns to scale technology. Hence, it takes prices (r, w, p_d), policy variables as well as carbon-tax pass-through of the energy producer as given and obtains zero profits in equilibrium:

$$\max_{K,L,E} \quad \{F(K,E,L;X,S) - (r+\delta)K_1 - wL_1 - (p_d + (1-\mu)\varpi\tau_d)E^p\}$$

Moreover, the firm compensates households for depreciation, δ.

The first-order condition of the firm implied by profit maximization - substituting in the CES production function from the main text - are

$$
\begin{aligned}
p_d + (1-\mu)\varpi\tau_d = \frac{\partial F_1}{\partial E^p} &= X\left[(1-s)(K_1^\alpha L_1^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E^p)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{1}{\lambda-1}} s(E^p)^{-\frac{1}{\lambda}} \\
w = \frac{\partial F_1}{\partial L_1} &= X\left[(1-s)(K_1^\alpha L_1^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E^p)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{1}{\lambda-1}} (1-s)(K_1^\alpha L_1^{1-\alpha})^{\frac{-1}{\lambda}}(1-\alpha)K_1^\alpha L_1^{-\alpha} \\
r + \delta = \frac{\partial F_1}{\partial K_1} &= X\left[(1-s)(K_1^\alpha L_1^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E^p)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{1}{\lambda-1}} (1-s)(K_1^\alpha L_1^{1-\alpha})^{\frac{-1}{\lambda}}\alpha K_1^{\alpha-1} L_1^{1-\alpha}
\end{aligned}
$$

As usual, the prices of the inputs are equal to their marginal products.

A.3 Goods market clearing

Aggregate the household budget constraint over household and impose asset market clearing $A = B + K$ in the steady-state:

$$C + (p_d + (1-\mu)\varpi\tau_d)D + (B+K) = (1 + (1-\tau^k)r)(B+K) + wL - \int T^y d\Lambda + g.$$

Rewrite the government budget constraint as $g - \int T^y d\Lambda = \tau^k r(K+B) + \tau_d(1-\mu)E - rB$[23], plug it in the aggregated household constraint above and collect terms:

$$C + (p_d + (1-\mu)\varpi\tau_d)D = rK + wL + \tau_d(1-\mu)E.$$

Extend with δK and $(p_d + (1-\mu)\varpi\tau_d)E^p$, recall that $E^p + D$, $K_1 + K_2 = K$, $L_1 + L_2 = L$, and use Euler's theorem to obtain:

$$
\begin{aligned}
C + \delta K = &\underbrace{(r+\delta)K_1 + wL_1 + (p_d + (1-\mu)\varpi\tau_d)E^p}_{Y} \\
&+ \tau_d(1-\mu)E - (p_d + (1-\mu)\varpi\tau_d)(E^p + D) + (r+\delta)K_2 + wL_2
\end{aligned}
$$

Finally, use the first-order conditions of the energy producer and again use Euler's theorem to write

$$
\begin{aligned}
C + \delta K = Y + [p_d - (1-\mu)\tau_d(1-\varpi) - \Psi(\mu)](\underbrace{F_{2,K_2}K_2 + F_{2,L_2}L_2}_{E}) + \tau_d(1-\mu)E - (p_d + (1-\mu)\varpi\tau_d)E \\
C + \delta K + \Psi(\mu)E = Y + \underbrace{[p_d - (1-\mu)\tau_d(1-\varpi)]E + \tau_d(1-\mu)E - (p_d + (1-\mu)\varpi\tau_d)E}_{0} \\
C + \delta K + \Psi(\mu)E = Y
\end{aligned}
$$

Hence, the final good can be used for consumption, investment, and abatement.[24]

[23] To be precise, the government receives carbon tax revenue from the household, $\tau_d(1-\mu)\varpi D$, from the final goods firm, $\tau_d(1-\mu)\varpi E^p$, and from the energy producer $\tau_d(1-\mu)(1-\varpi)E$. These three terms sum up to $\tau_d(1-\mu)E$.

[24] The second term on the left-hand-side equals investment I, since in the steady-state the law of motion of capital $K_{t+1} = (1-\delta)K_t + I_t$ collapses to $\delta K = I$.

B Calibration - Details

B.1 Macroeconomic variables

Capital-output-ratio (K/Y) Current-Cost Net Stock of Fixed Assets (2014) in current dollars divided by Gross Domestic Product (2014) in current dollars; both series from the FRED database with series tags K1TTOTL1ES000 and GDPA, respectively.

Bond-output-ratio (B/Y) Federal Debt Held by the Public (2014) in current dollars divided by Gross Domestic Product (2014) in current dollars; both series from the FRED database with series tags FYGFDPUN and GDPA, respectively.

B.2 Estimation of the productivity process

In the following, I explain how I estimate the labor productivity process which I use in my quantitative model.[25] I follow Flodén and Lindé (2001) and measure productivity as agent's "hourly [pre-tax] wage rate relative to all other agents" (p. 416). The data is taken from the PSID and refers to labor income as described in the main text. The productivity process is estimated on yearly data. To avoid notational clutter, I denote yearly time-steps by τ, compared with a model period (5 years) denoted by t.

As my productivity process, I take the following standard persistent-transitory specification for log wages at year τ

$$\log \hat{\theta}_{i\tau} = f(\mathbf{X}_i, \beta) + \kappa_{i\tau} + \psi_{i\tau} + \nu_{i\tau} \tag{B.24}$$

$$\kappa_{i\tau} = \rho \kappa_{i\tau-1} + \varepsilon^{\kappa}_{i\tau}, \tag{B.25}$$

where $f(\mathbf{X}_i, \beta)$ denotes a set of individual-specific controls, $\kappa_{i\tau}$ is an AR(1) process with persistence ρ and innovation-variance $\sigma^2_{\varepsilon^\kappa}$, $\psi_{i\tau}$ is a transitory component with variance σ^2_{ψ}, and $\nu_{i\tau}$ is measurement error.

Given the short time horizon when estimating the productivity process, instead of estimating the household fixed effect directly, I model the permanent component by controlling for individual-specific characteristics, $\mathbf{X}_i$.

Moreover, the measurement error term cannot be identified from the transitory term. Hence, I follow the literature and set the variance of the measurement error term to 0.02 (French, 2004; Heathcote et al., 2010; Straub, 2019).

The estimation then proceeds along the following steps. First, I residualize log wages using

$$\log \theta_{i\tau} = \log \hat{\theta}_{i\tau} - f(\mathbf{X}_i, \hat{\beta}).$$

Second, I compute empirical variances and covariances from these residuals and stack them in the

[25] The exposition here follows the one in Straub (2019) who uses a similar strategy to estimate a process for log income and from whose description I learned a lot.

Table B.9: Estimated parameters - Productivity process

ρ	$\sigma^2_{\varepsilon^\kappa}$	σ^2_{ψ}
0.9327	0.0426	0.0507
(0.0092)	(0.0060)	(0.0049)

Note. This table shows the estimated parameters of the productivity process. Standard errors are bootstrapped using a non-parametric block bootstrap at the household level with 500 iterations.

vector $\vec{\mathfrak{M}}$. The theoretical variances and covariances, on the other hand, can be computed using

$$\begin{aligned} var(\log \theta_{i\tau}) &= \frac{\sigma^2_{\varepsilon^\kappa}}{1-\rho^2} + \sigma^2_\psi + \sigma^2_\nu \\ cov(\log \theta_{i\tau}, \log \theta_{i\tau-h}) &= \rho^h \frac{\sigma^2_{\varepsilon^\kappa}}{1-\rho^2}. \end{aligned}$$

I denote the stacked theoretical (co)variances by $\vec{\mathfrak{m}}(\rho, \sigma^2_{\varepsilon^\kappa}, \psi_{it})$. This formulation stresses that $\vec{\mathfrak{m}}$ is a function of the parameters that we seek to estimate.

Lastly, I apply a minimum distance estimation (MDE) to minimize the weighted distance, $\mathfrak{U}(\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_\psi) = \vec{\mathfrak{M}} - \vec{\mathfrak{m}}$, between theoretical and empirical moments/covariances:

$$\min_{\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_\psi} \mathfrak{U}(\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_\psi)' \, \mathfrak{W} \, \mathfrak{U}(\rho, \sigma^2_{\varepsilon^\kappa}, \sigma^2_\psi)$$

As is standard in this procedure, I use the identity matrix as weighting matrix $\mathfrak{W}$ which was shown to be more robust to small sample bias (Altonji and Segal, 1996).

Table B.9 shows the result of the MDE. Standard errors are obtained by using a non-parametric block bootstrap (Cameron and Trivdei, 2005, p.362/p.377).

5-year time period To translate these values that were estimated on annual data to their 5-year model counterparts, I proceed in two steps: First, I iterate the persistent component backward such that

$$\kappa_{i\tau} = \rho^5 \kappa_{i\tau-5} + \underbrace{\sum_{s=0}^{4} \rho^s \varepsilon^\kappa_{i\tau-s}}_{\tilde{\varepsilon}^\kappa_{i\tau}}. \tag{B.26}$$

I can compute the variance of $\tilde{\varepsilon}^\kappa_{i\tau}$ given the annual estimate:

$$\sigma^2_{\tilde{\varepsilon}^\kappa} = var(\tilde{\varepsilon}^\kappa_{i\tau}) = \sum_{s=0}^{4} \rho^{2s} var(\varepsilon^\kappa_{i\tau-s}) = \sum_{s=0}^{4} \rho^{2s} \sigma^2_{\varepsilon^\kappa} \tag{B.27}$$

This gives $\tilde{\rho} = 0.7058$ and $\sigma^2_{\tilde{\varepsilon}^\kappa} = 0.1647$.

Second, I set $\sigma^2_{\tilde{\psi}} = \sigma^2_\psi$.

The approach I take here yields similar results as the one proposed by Krueger, Mitman and

Perri (2016), who convert annual to quarterly estimates. They also take $\tilde{\rho} = \rho^5$ and $\sigma^2_{\tilde{\psi}} = \sigma^2_{\psi}$. However, they then choose $\sigma^2_{\tilde{\varepsilon}^\kappa}$ such that

$$\frac{\sigma^2_{\varepsilon^\kappa}}{1-\rho^2} = \frac{\sigma^2_{\tilde{\varepsilon}^\kappa}}{1-\tilde{\rho}^2}. \tag{B.28}$$

This would give $\sigma^2_{\varepsilon^\kappa} = 0.1643$.

C Estimation - Details

Pre-testing and local step The estimation procedure follows the TikTak algorithm in Arnoud et al. (2019). The goal is to find the parameter vector which minimizes the objective function from the main text.

In the pre-test step, I first draw $N = 1000$ quasirandom Sobol starting points in the five dimensional parameter space. Thereby, I specify bounds for all parameters and later make sure that the algorithm finds an interior solution is not constrained by these choices. For each of these starting points I solve the partial equilibrium economy, simulate an artificial panel and construct the moment conditions and evaluate the objective function.

In the next step, the local step, I pick the ten $(= 0.01N)$ points which gave rise to the lowest objective value from the pre-test step. From each of these points, I use the BOBYQA algorithm by Powell (2009) to find a local minimum. The algorithm terminates if either the absolute or the fractional tolerance on the parameter vector is smaller than $1e^{-5}$. The global minimum is then the parameter vector which attains the lowest value of all these ten runs. In my application, the final objective value is $1.97e^{-7}$.

Simulation of the artifical panel I simulate an artificial panel with 25000 agents and 250 time periods. All agents start with zero assets. I then simulate the economy forward using the stationary policy functions and a simulated Markov chain of the labor productivity process. To construct the simulated moments and to run the IV regressions from the main text, I only use the last 5 time periods of the simulated panel to limit dependence on initial conditions. Moreover, the idiosyncratic shocks are always drawn using the same seed to ensure comparability between runs.

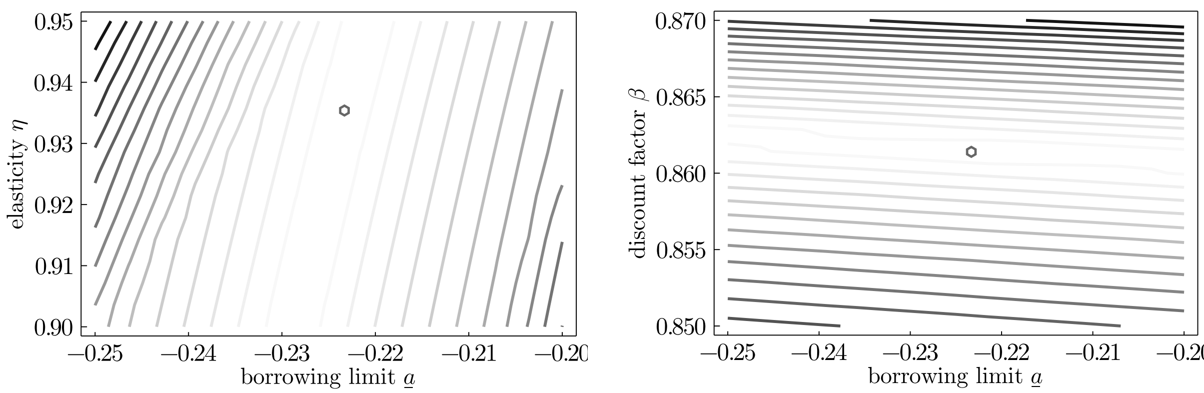
Standard errors Gourieroux et al. (1993) show that show that under no observable exogenous variables that enter the moments, the asymptotic variance-covariance matrix is

$$\mathbb{COV} = \left(1 + \frac{1}{B}\right) \left[\frac{\partial \mathcal{M}^*}{\partial \Theta}^\top W \ \frac{\partial \mathcal{M}^*}{\partial \Theta}\right]^{-1} \frac{\partial \mathcal{M}^*}{\partial \Theta}^\top W \, cov(\mathcal{M}^B) \, W \ \frac{\partial \mathcal{M}^*}{\partial \Theta} \left[\frac{\partial \mathcal{M}^*}{\partial \Theta}^\top W \ \frac{\partial \mathcal{M}^*}{\partial \Theta}\right]^{-1}$$

where B denotes the number of bootstrap repetitions, $\mathcal{M}^* = \mathcal{M}(\Theta^*)$ and $cov(\mathcal{M}^B)$ is the covariance matrix of the bootstrapped moments.

Figure C.6: Numerical identification

(a) Wealth-to-income ratio, 10^{th} (b) Wealth-to-income ratio, Median



Note. Absolute deviation of moments implied by the structural model from their data counterparts when parameters on the respective axis are varying. Other parameters are fixed at their estimated value. Darker regions indicate higher deviation.

In particular, I have 5770 unique households in my sample. I draw $B = 200$ random samples, with replacement, of these households to construct $cov(\mathcal{M}^B)$ from the data. The draws are panel draws (block draws), that is, when I draw a specific household, I keep all household-year observations.

The gradient $\frac{\partial \mathcal{M}^*}{\partial \Theta}$ is a Jacobian, where the elements give partial derivatives from the structural parameters to the model moments:

$$\frac{\partial \mathcal{M}^*}{\partial \Theta} = \begin{bmatrix} \frac{\partial M_1^*}{\partial \eta} & \frac{\partial M_1^*}{\partial \underline{d}} & \frac{\partial M_1^*}{\partial \chi} & \frac{\partial M_1^*}{\partial \beta} & \frac{\partial M_1^*}{\partial \underline{a}} \\ \frac{\partial M_2^*}{\partial \eta} & \frac{\partial M_2^*}{\partial \underline{d}} & \frac{\partial M_2^*}{\partial \chi} & \frac{\partial M_2^*}{\partial \beta} & \frac{\partial M_2^*}{\partial \underline{a}} \\ \frac{\partial M_3^*}{\partial \eta} & \frac{\partial M_3^*}{\partial \underline{d}} & \frac{\partial M_3^*}{\partial \chi} & \frac{\partial M_3^*}{\partial \beta} & \frac{\partial M_3^*}{\partial \underline{a}} \\ \frac{\partial M_4^*}{\partial \eta} & \frac{\partial M_4^*}{\partial \underline{d}} & \frac{\partial M_3^*}{\partial \chi} & \frac{\partial M_4^*}{\partial \beta} & \frac{\partial M_4^*}{\partial \underline{a}} \\ \frac{\partial M_5^*}{\partial \eta} & \frac{\partial M_5^*}{\partial \underline{d}} & \frac{\partial M_3^*}{\partial \chi} & \frac{\partial M_5^*}{\partial \beta} & \frac{\partial M_5^*}{\partial \underline{a}} \end{bmatrix}$$

The partial derivatives are approximated with a numerical two-sided difference. Finally, the weighting matrix W is an identity matrix. The standard errors are then on the diagonal of $\mathbb{COV}$.

Identification II Figure C.6 shows another set of numerical identification plots concerning the two moments of the wealth-to-income distribution. In panel (a), we see that the borrowing limit identifies the 10th percentile of the wealth-to-income distribution. Importantly, this results holds for a given value of β, as otherwise, the discount factor would also affect this particular moment. However, in panel (b), we see that β is pinned down by the median wealth-to-income ratio, even for different values of the borrowing limit.

D Additional tables & figures

Table D.1: Summary statistics

Variable	Mean	Std. dev.	Median
Demographics			
Age	41.52	10.12	41
Sex	0.821	0.383	1
Household size	2.919	1.448	3
Number of children	0.990	1.203	1
Married	0.663	0.473	1
Education			
Elementary or middle school	0.0721	0.259	0
Finished high school	0.244	0.430	0
Some college	0.281	0.449	0
Finished college	0.213	0.410	0
Postgrad. qualification	0.189	0.392	0
Region			
Northeast	0.166	0.372	0
Midwest	0.302	0.459	0
South	0.337	0.473	0
West	0.195	0.397	0
Net income	32,080	19,127	28,132
Net labor income	41,119	29,023	34,637
Wealth	93,804	179,069	30,237
Nondurable consumption	14,919	7,714	13,119
Total consumption	17,136	9,502	14,798

Note. This table shows summary statistics regarding demographic and economic variables of the data which is used in estimation.

E Computational appendix

E.1 Computing the household's optimal decision rules and invariant distribution

I use a variant of the endogenous gridpoint method (EGM) to solve the household's decision problem. Compared to the basic version developed by Carroll (2006), my version accommodates two goods and endogenous labor supply with possibly non-linear taxation.

Grids I represent asset positions by discrete points on a exponentially-spaced grid $\mathcal{A} \subset [\underline{a}, \overline{a}]$, where $\overline{a}$ is chosen large enough such that the upper bound is never binding. I discretize the productivty Markov process with a finite-state Markov chain using Rouwenhorst (1995)'s method. The inputs for this method, such as the persistence parameter ρ, are obtained in Appendix B.2.

Endogenous gridpoint method

Step 1 I start with a guess of the clean consumption policy function defined on the *future* asset and productivity grid, $\boldsymbol{c}(a', \theta')$. Using the intra-temporal first-order condition between clean and dirty consumption, I can express the dirty consumption policy function $\boldsymbol{d}(a', \theta')$ as a function of $\boldsymbol{c}(a', \theta')$:

$$\boldsymbol{d}(a', \theta') = \frac{1 - \eta}{(p_d + \tau_d)\eta} \boldsymbol{c}(a', \theta') + \underline{d}. \tag{E.29}$$

Step 2 Hence, for each pair (a', θ) where the household is not constrained and the Euler equation (EE) holds with equality, I can solve *analytically* for the value $\boldsymbol{c}(a', \theta)$.[26] $\boldsymbol{c}(a', \theta)$ is essentially on the left-hand side of the EE and represents the value of consumption today, which is consistent with having a' assets tomorrow if the productivity shock today is θ:

$$u_c(\boldsymbol{c}(a', \theta)) = \beta(1 + \tilde{r})\mathbb{E}_\theta \left[u_c(\boldsymbol{c}(a', \theta')) \right] \tag{E.30}$$

Note that I write u_c explicitly as a function of $\boldsymbol{c}$ only, as the utility is separable in consumption and labor, and $\boldsymbol{d}$ is implied by Equation (E.29).

Step 3 With $\boldsymbol{c}(a', \theta)$ in hand, I can solve for $\boldsymbol{n}(a', \theta)$ using the intra-temporal FOC between clean consumption and labor. In the following, I assume an interior solution:

$$-u_n(\boldsymbol{n}(a', \theta)) = u_c(\boldsymbol{c}(a', \theta))\mathcal{T}_n, \tag{E.31}$$

where $\mathcal{T}_n$ denotes $\frac{\partial \mathcal{T}}{\partial n}$. Under linearity of $\mathcal{T}$, Equation (E.31) can also be solved analytically for $\boldsymbol{n}(a', \theta)$. Otherwise, a root-finding step has to be implemented at every point in the state space. In the benchmark case, I use a version of Brent's method, modified to take into account multiple roots under the non-linearity of $\mathcal{T}$ as well as the corner solution if $\boldsymbol{n}(a', \theta) = 0$.

[26] Of course, this step depends on the invertability of the utility function. Other functional forms for the consumption composite might not make this feasible.

Step 4 I can then invert the budget constraint to solve for the value of assets today, $a^*(a', \theta)$, which are consistent with the future assets (on grid) and the choices made above.

$$a^* = \frac{1}{1+\tilde{r}}\Big(\boldsymbol{c}(a', \theta) + (p_d + \tau_d)\boldsymbol{d}(a', \theta) + a' - \mathcal{T}\left(w\theta\boldsymbol{n}(a', \theta)\right)\Big), \tag{E.32}$$

implying $\tilde{c}(a^*, \theta) = \boldsymbol{c}(a', \theta)$. Note that these a^* are not on the grid (whence the name) and change each iteration. To obtain a new guess for the clean consumption policy function which is defined on the grid, I linearly interpolate on $(a^*, \tilde{c}(a^*, \theta))$ and apply this mapping to the exogenous grid a'. Use the new guess as a starting point in Step 2 above.

I repeat the above iteration procedure until convergence between two successive clean consumption policy functions is achieved: $||\boldsymbol{c}^{n+1} - \boldsymbol{c}^n|| < 10^{-7}$, where $|| \cdot ||$ denotes the supnorm and n is the iteration counter.

Density discretization With the policy functions in hand, I discretize the invariant density and iterate on it using Young (2010)'s lottery method.