Linear Algebra

1 Solving a system of linear Equation.

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 7z = 30$$

find the value of x, y, z.

2 Matrix operation.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ $C = A \times B$

NORMAL OR GAUSSIAN DISTRIBUTON.

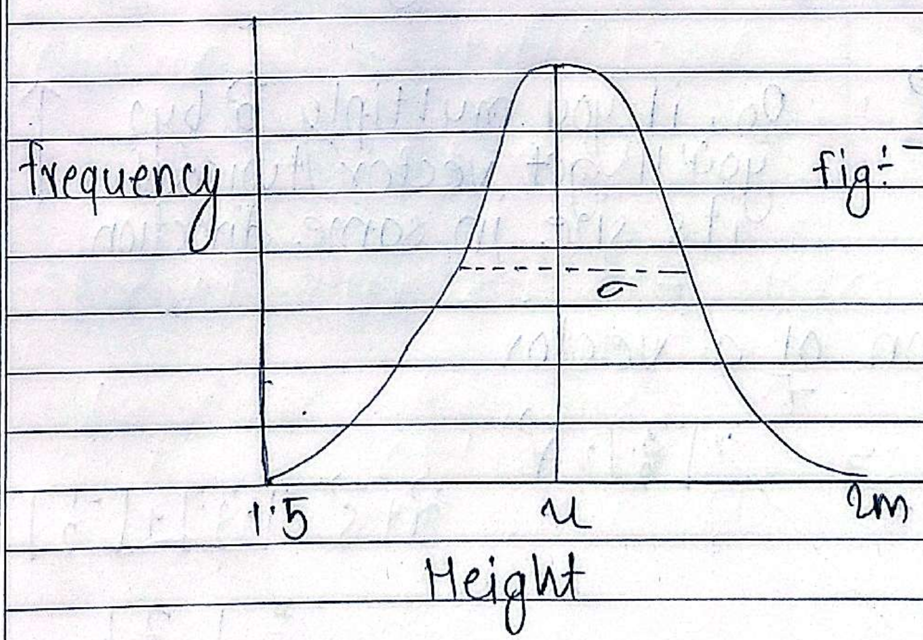
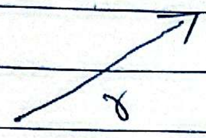


fig: The distribution of Height with population.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-u)^2}{2\sigma^2}\right\}$$

Vector

We can think of vector just as a list.



In computer we think of a vector as a list of attribute of an object.

Suppose for a house, attributes are :- | We generalize the vectors like :-

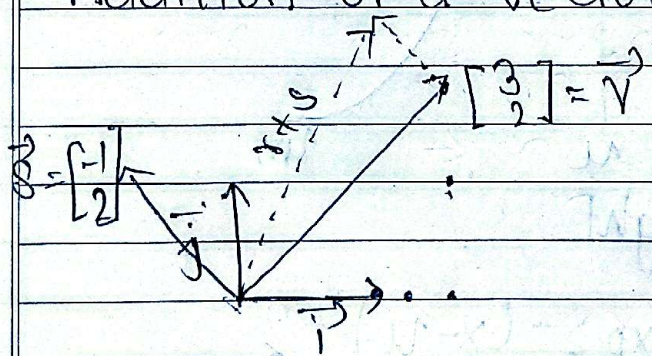
120 sq m
2 bedroom
1 bathroom
$ 150,000

$$\begin{bmatrix} 120 \\ 2 \\ 1 \\ 150,000 \end{bmatrix}$$

- Multiplying a vector by a scaler

$\uparrow \vec{a}$ So, if you multiply $\vec{a}$ by 2 you'll get vector twice of its size in same direction $\uparrow 2\vec{a}$

- Addition of a vector



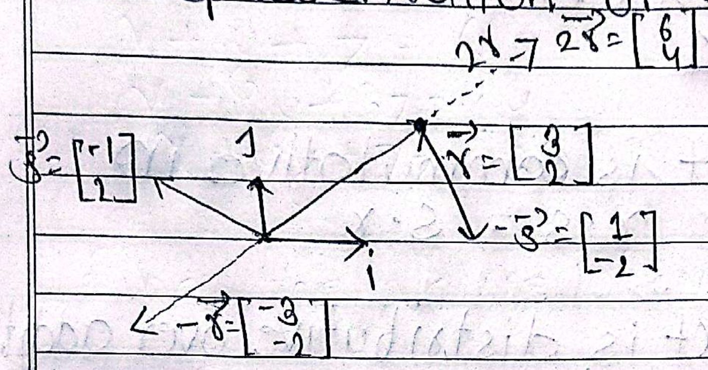
$$\vec{r} + \vec{s} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

- vector addition is associative

$$\vec{r} + \vec{s} = \vec{s} + \vec{r}$$

- Vector substraction is the addition of negative representation of a vector



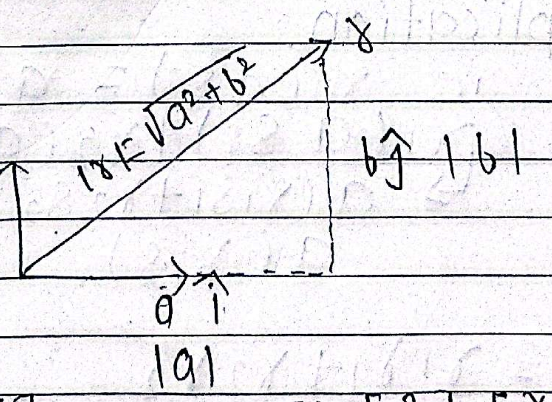
$$\vec{r} - \vec{s} = \begin{bmatrix} 3+1 \\ 2-2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\vec{r} + (-\vec{r}) = \begin{bmatrix} 3-3 \\ 2-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- As we know from the attributes of house

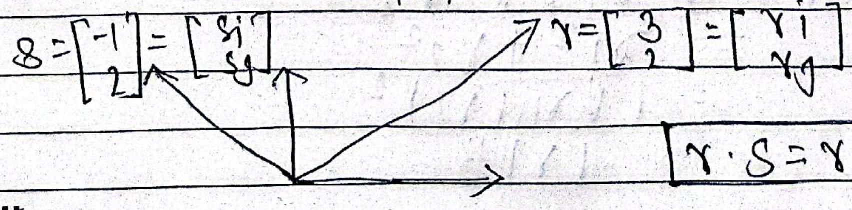
$$⇧ = \begin{bmatrix} 120 \\ 2 \\ 1 \\ 150 \end{bmatrix} \qquad ⇧ ⇧ = \begin{bmatrix} 120 \\ 2 \\ 1 \\ 150 \end{bmatrix} \times 2 = \begin{bmatrix} 240 \\ 4 \\ 2 \\ 300 \end{bmatrix}$$

- Moduls and the dot product of vector



$$r = a\hat{i} + b\hat{j} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$|r| = \sqrt{a^2+b^2}$$



$$\boxed{r \cdot s = r_i s_i + r_j s_j}$$

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$$r \cdot s = r_i s_i + r_j s_j$$
$$= 3 \times (-1) + 2 \times 2$$
$$= 1$$

- Dot product is commutative in nature ie $r \cdot s = s \cdot r$

- Dot product is distributive over addition.

$$r \cdot (s+t) = r \cdot s + r \cdot t$$

proof :-

$$r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} \quad s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} \quad t = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix}$$

$$r \cdot (s+t) = r_1(s_1+t_1) + r_2(s_2+t_2) + \cdots + r_n(s_n+t_n)$$
$$= r_1 s_1 + r_1 t_1 + \cdots + r_n s_n + r_n t_n$$
$$= r \cdot s + r \cdot t$$

- Dot product is associative over scaler multiltiplication.

ie $r \cdot (a s) = a(r \cdot s)$

$$r_1(a s_1) + r_2(a s_2)$$
$$\to a(r_1 s_1 + r_2 s_2)$$
$$a(r \cdot s)$$

$$r \cdot r = r_1 r_1 + r_2 r_2$$
$$= r_1^2 + r_2^2$$
$$= (\sqrt{r_1 + r_2})^2$$
$$= |r|^2$$

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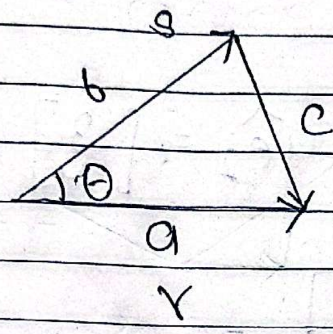
The cosine rule & the dot product.



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$r-s$

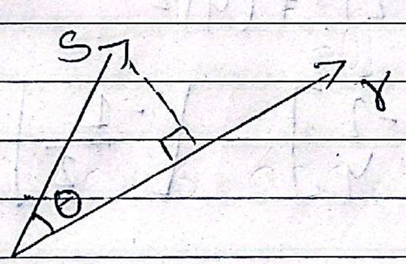
$$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$$

Taking LHS $= |r-s|^2$

$$= (r-s)\cdot(r-s)$$
$$= r\cdot r - r\cdot s - s\cdot r + s\cdot s$$
$$= |r|^2 - 2s\cdot r + |s|^2$$
$$= |r|^2 + |s|^2 - 2|s||r|\cos\theta$$

$$\boxed{r\cdot s = |r||s|\cos\theta}$$

Progection



progection of $\vec{a}$ on $\vec{b}$

$$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \quad \text{(scaler)}$$

vector progection of $\vec{a}$ on $\vec{b}$

$$= \left(\frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\right) \times \vec{b}$$

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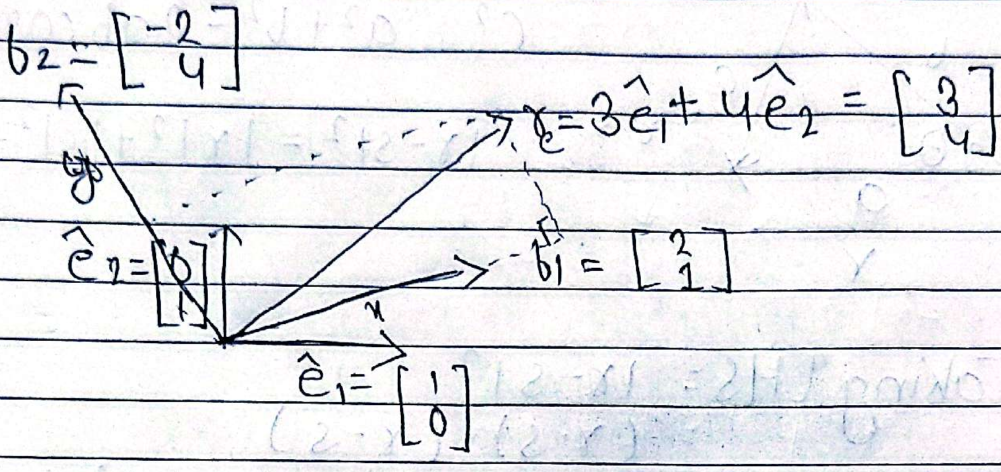
Coordinate system of vector space:



$$x = \frac{r_e \cdot b_1}{|b_1|^2} \times b_1 = \frac{3\times 2 + 4\times 1}{2^2 + 1^2} b_1 = \frac{10}{5} b_1$$

$$= 2 \times b_1 = 2 \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$y = \frac{r_e \cdot b_2}{|b_2|^2} \times b_2 = \frac{3\times -2 + 4\times 4}{(-2)^2 + (4)^2} \times b_2$$

$$= \frac{1}{2} b_2 = \frac{1}{2}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$\hat{i}$ and $\hat{j}$ are the basis vectors of the xy coordinate system.

We could have chosen a different basis vector and got a new coordinate system.

You can reach must of the points in the plane with $a\vec{v} + b\vec{w}$

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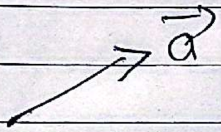
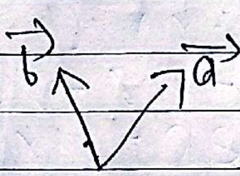
The "span" of $\vec{v}$ and $\vec{w}$ is the ~~combine~~ set of all their linear combination : $a\vec{v} + b\vec{w}$

Basis is a set of n vector that :
i) are not linear combinations of each other (linearly independent)
ii) Span the space ; The space is then n-dimension.



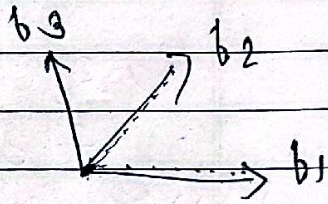
1-D

You can move anywhere in the line



2-D

You can move anywhere in the 2-D space



You can move freely in the 3D space if $b_3 \neq a_1 b_1 + a_2 b_2$