# COMSC 2043
# Induction and Recursion

Student Workbook

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# Chapter 1

# Foundations of Induction and Recursion

This chapter introduces the central ideas of **mathematical induction** and **recursion** as presented in Chapter 5 of Kenneth Rosen's *Discrete Mathematics and Its Applications*.

## 1.1 The Big Picture

Mathematical induction and recursion are two sides of the same elegant coin. Induction is how we *prove* things about a process that repeats. Recursion is how we *define* that process.

We use induction to reason that what works for one step will work for the next. We use recursion to build structures or compute results by defining a problem in terms of smaller instances of itself.

## 1.2 Key Ideas from Rosen's Chapter 5

- **Basis Step:** Prove that a statement is true for an initial value (usually $n = 0$ or $n = 1$).
- **Inductive Step:** Assume it is true for $n = k$ (the *inductive hypothesis*) and prove it for $n = k + 1$.
- **Strong Induction:** Sometimes we assume it's true for *all* previous cases up to $k$ to prove it for $k + 1$.
- **Recursive Definitions:** A way to define sets, sequences, or functions in terms of themselves.

- **Structural Induction:** A generalization used for recursively defined structures like trees or expressions.

## 1.3 Why This Matters

Induction teaches us to trust the domino effect: if one falls and the rule is consistent, they all fall. Recursion lets us *build the dominoes* themselves.

They are the grammar and logic behind everything from factorial functions to sorting algorithms to proofs of algorithmic correctness.

## 1.4 Example: The Factorial Function

The factorial of $n$, written $n!$, is defined recursively:

$$n! = \begin{cases} 1, & n = 0 \\ n \cdot (n-1)!, & n > 0 \end{cases}$$

We can prove by induction that $n! \geq 2^{n-1}$ for all $n \geq 1$.

**Proof (sketch):**

- **Base case:** $n = 1$ $1! = 1 \geq 2^0 = 1$✓
- **Inductive step:** Assume $k! \geq 2^{k-1}$. Then

  $$(k+1)! = (k+1)k! \geq (k+1)2^{k-1} \geq 2^k$$

  for $k \geq 1$.

## 1.5 A Student Challenge

> "Induction is not a leap of faith — it's a method of climbing an infinite ladder, one rung at a time."

**Challenge:** Write your own recursive function in Python that computes $n!$, and then write a proof by induction showing why it works for all $n \geq 0$.

## 1.6 Checkpoint Questions

1. What are the two main steps of a proof by induction?
2. How is recursion related to induction?
3. Give a real-world example of a recursive process.
4. Can every recursive definition be proven correct using induction?