Section 4.4 Curve Sketching and L'hospital's Rule

1. **Curve Sketching**

We've learned several methods to find the characteristics of the graph of a function in the previous sections. Now, we can try to illustrate the graph of the function base on this characteristics.

To illustrate the graph of the function, we might need to find the following infomations:

(1) Domain

(2) Critical point (f'(x))

(3) Inflection point (f"(x))

(4) Concavity (f"(x))

(5) Asymptotes

(6) Particular point

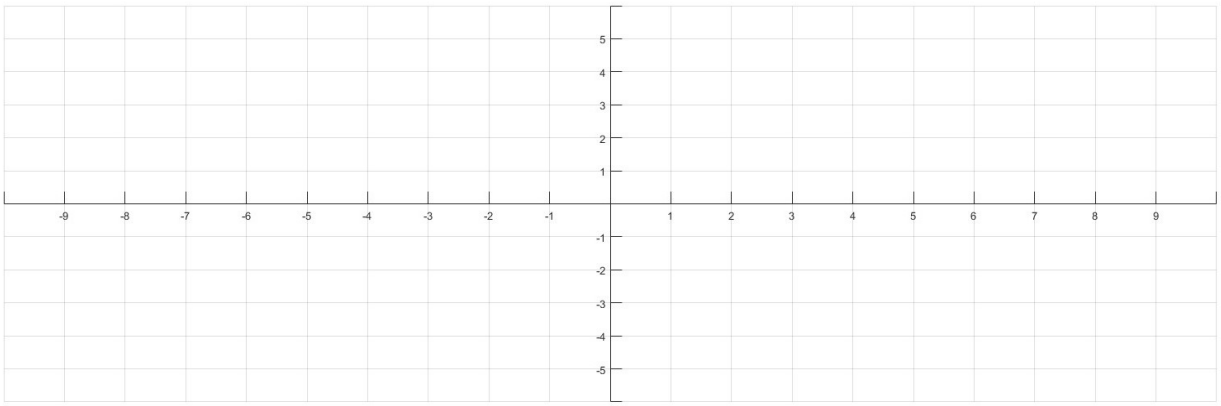
Example 1

Illustrate the graph of the function $y = \frac{x}{\sqrt{x^2-1}}$

(1) Domain:

(2) Asymptotes:

(3) Particular point:



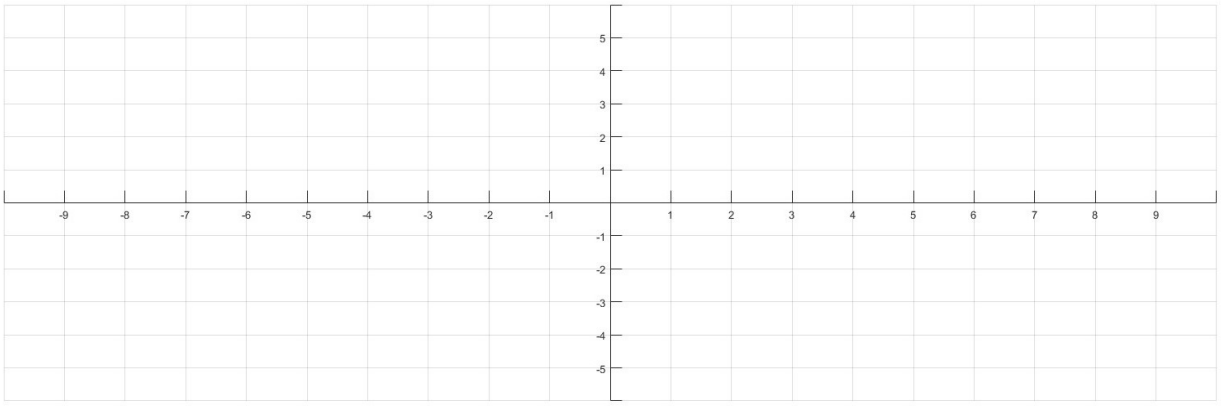
Exercise 1

Illustrate the graph of the function $y = \frac{x-x^2}{2-3x+x^2}$

(1) Domain:

(2) Asymptotes:

(3) Particular point:



2. **L'Hospital's Rule**

We've mentioned that there are 7 types of "Indeterminate form" as follows:

(1) Fraction: $\frac{0}{0}$, $\frac{\infty}{\infty}$ (2) Product: $0 * \infty$ (3) Power: 0^0, 1^∞, ∞^0 (4) Subtraction: $\infty - \infty$

Now, we are going to introduce "L'Hospital's Rule" which can help us to find the limit of the indeterminate form.

$$\text{Suppose} \quad \begin{cases} \lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{0}{0} \ \textit{or} \\ \lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \pm\dfrac{\infty}{\infty} \end{cases} \Rightarrow \begin{array}{l} \lim\limits_{x\to a} \dfrac{f(x)}{g(x)} \ (\text{"}\dfrac{0}{0}\text{"}) \overset{L.H.}{=} \lim\limits_{x\to a} \dfrac{f'(x)}{g'(x)} \\ \lim\limits_{x\to a} \dfrac{f(x)}{g(x)} \ (\text{"}\dfrac{\infty}{\infty}\text{"}) \overset{L.H.}{=} \lim\limits_{x\to a} \dfrac{f'(x)}{g'(x)} \end{array}$$

Goal: Transform all of the undeterminate forms into $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and use L'Hospital Rule to solve the limit.

Example 1

Evaluate $\lim\limits_{x\to\infty}[x - x^2 \ln(\frac{1+x}{x})]$

Example 2

Evaluate $\lim\limits_{x\to 0^+} \frac{x^x-1}{\ln(x)+x-1}$

Exercise 1

Evaluate $\lim_{x\to\infty}(\frac{2x-3}{2x+5})^{2x+1}$

Exercise 2

Evaluate $\lim_{x\to 0^+}(1-cosx)^{sinx}$

Exercise 3

Evaluate $\lim_{x\to 0^+}\frac{1}{x}+\frac{1}{tanx}$

Exercise 4

Evaluate $\lim_{x\to\infty}(1+\frac{a}{x})^{bx}$