Review: Chain Rule and Integrating Methods

In this Review, we'll be mainly focusing on differentiation and integration methods that you'll most likely encounter in either calculus (II) or most importantly, engineering mathematics.

1. Chain Rule

Differentiation of composite functions, we differentiate from the outer function first, then the inner one, one step at a time, afterwards, multiply each differentiation results altogether.

Example 1

Differentiate the following function, $y = \dfrac{cos(2lnx)}{x^3}$

2. Variable Substitution

Upon all four integrating methods, variable substitution should be first considered when facing integrating problems since it is the easiest and most effective method.

Example 2

Evaluate the integral, $y = \int \dfrac{1}{\sqrt{e^x - 1}} dx$

3. Partial Fraction

Patial fraction will be the most used techniques in engineering mathematics, the following example is a problem you'll see in engineering mathematics in the future.

Example 3

Evaluate the integral of $Y(s)$, *where* $Y(s) = \dfrac{-s^3-3s^2+16s+30}{(s^2-2s-3)(s^2-4s)}$

4. Integration By Parts

When seeing the problem of solving two multiplied functions, integration by parts can be applied.

Example 4

Evaluate the integral, $y = \int e^x \sin\pi x \, dx$

5. Trigonometric Substitution

Trigonometric substitution will be the least used method in the future since it is more time-consuming, also, it will not often occur in engineering mathematics either, so that's just take a brief review.

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

Note the domain of θ when making substitutions.

Example 5

Evaluate the integral, $y = \int \frac{\sqrt{x^2 - 9}}{x^3} \, dx$

Also, there's a part of trigonometric integration that is different from the form mentioned above, which involves different powers of sine and cosine functions, in this case, half angle identities of cosine and sine functions will be applied, here's an example.

Example 6

Evaluate the integral, $y = \int \sin^5(2x)\cos^2(2x) \, dx$