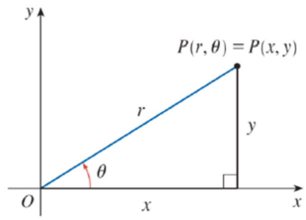
Section 10.3, 10.4 Calculus in Polar Coordinates

1. Polar Coordinates

   (1) Realationship between Cartesian and Polar Coordinates.

   $$x = rcos\theta,\ y = rsin\theta,\ r^2 = x^2 + y^2$$

   (2) Polar Curves

   A. Circle

   $r = 2asin\theta\ or\ r = 2acos\theta$

   B. Coardiod

   $r = a(1 \pm sin\theta)\ or\ r = a(1 \pm cos\theta)$

   C. N-leaved Rose $r = acos(n\theta)\ or\ r = asin(n\theta)$

2. Calculus in Polar Coordinates

   For a curve described in polar coordinates, $r = f(\theta)$, we know that it's parametric equation can be described as:

   $$x = rcos\theta = f(\theta)cos\theta \text{ and } y = rsin\theta = f(\theta)sin\theta$$

Then, the slope can be determined by:

$$m = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)sin\theta + f(\theta)cos\theta}{f'(\theta)cos\theta - f(\theta)sin\theta}$$

(1) Categories of tangents

A. Horizontal tangents: $\frac{dy}{d\theta} = 0,\ \frac{dx}{d\theta} \neq 0$

B. Vertical tangents: $\frac{dx}{d\theta} = 0,\ \frac{dy}{d\theta} \neq 0$

C. Tangent line at pole: $m = tan\theta$

(2) Area

$$A = \int_a^b \frac{1}{2}[f(\theta)]^2 d\theta = \int_a^b \frac{1}{2} r^2 d\theta$$

(3) Arc Length

$$L = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}$$

(4) Surface Area

A. Rotate about the polar axis: $S = \int_a^b 2\pi(rsin\theta)\sqrt{r^2 + (\frac{dr}{d\theta})^2}$

B. Rotate about the line $\theta = \frac{\pi}{2}$: $S = \int_a^b 2\pi(rcos\theta)\sqrt{r^2 + (\frac{dr}{d\theta})^2}$

Example 1

Find the points on the curve $r = 1 + cos\theta$ where the tangent line is horizontal or vertical.

Example 2

Sketch the curve $r = 2 + 2cos\theta$ and find the area that it encloses.

Example 3

Find the exact length of the polar curve $r = \theta^2$, $0 \leq \theta \leq 2\pi$