Section 1. Basic Functions and Models

1. Mathematic Language

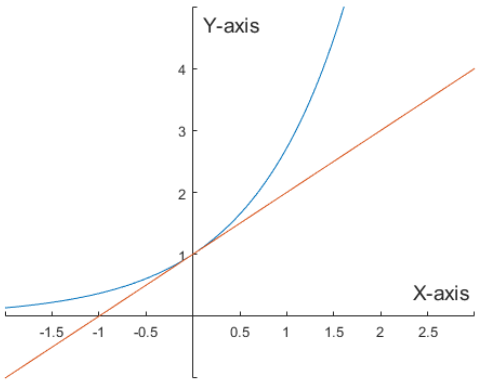
Symbol	Meaning
$\forall$	For all
$\exists$	There exist
$s.t.$	Such that
$\in$	Belong to
$Q.E.D$	Proof complete

2. Genres of Functions

(1) Linear Funtions
(2) Polynomials
(3) Power Functions
(4) Root Functions
(5) Reciprocal Functions
(6) Rational Functions
(7) Algebraic Functions
(8) Trigonometric Functions
(9) Exponential Functions
(10) Logarithmic Functions
(11) Inverse Functions

3. The Number e

Some of the functions in calculus will be greatly simplified if we choose the base "e" of an exponential function ($y = e^x$) so that the the slope of tangent line at (0,1) is exactly 1.



4. Inverse Functions

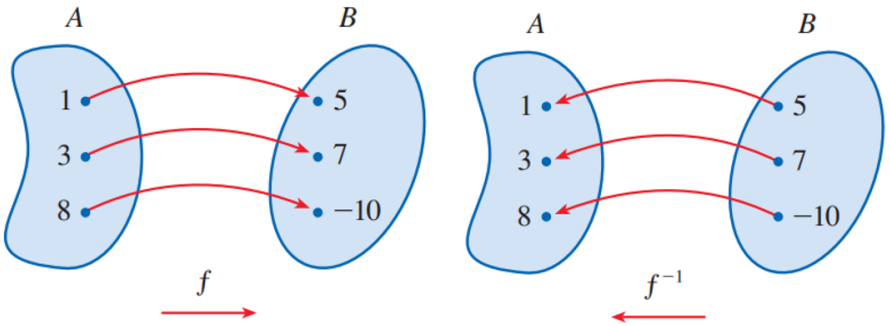
We can see from the graph below, as we input the variables in part A into a function, let's say $f(x)$, we then get the corresponding answer, as shown in part B. Reversely, we can get the original values in part A by inputting the answers we just got in part B into the other function, this function is then termed $f^{-1}(x)$.

When solving these functions, the procedures are given below:

(1). Let $f^{-1}(x) = y$, then $f(y) = x$.

(2). Find the domain and range of the function.

(3). Finalize



5. Inverse Trigonometric Functions

When solving inverse trigonometric functions, remember to note that since trigonometric functions are not one-to-one functions, therefore, we must restrict their domains to make sure that the inverse functions we get later will not an one-to-many functions.

$x = \sin^{-1} y \Leftrightarrow \sin x = y$ *Domain* : ____ $\leq x \leq$ ____ *Range* : ____ $\leq y \leq$ ____

$x = \cos^{-1} y \Leftrightarrow \cos x = y$ *Domain* : ____ $\leq x \leq$ ____ *Range* : ____ $\leq y \leq$ ____

$x = \tan^{-1} y \Leftrightarrow \tan x = y$ *Domain* : _____ *Range* : ____ $\leq y \leq$ ____

To better understand the concept of these functions, let's look at some examples.

(1). Example 1

Find the value of $\cos\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)$

(2). Example 2

Simplify the expression $\cos(\sin(\tan^{-1}(\frac{\pi}{7})))$

6. Application of Calculus

(1). Physics

Assume you drop a ball from the window of TR-615, we've learned how to calculate the displacement and velocity of the ball in ideal condition in high school physics. However, in reality, we have to consider external force such as drag, in this case, we have to use Calculus.

(2). Engineering

i. Automatic Control

Suppose there's a ball on the corner of a rectangle plate, and you're asked to balance the ball on the center of the plate, it might not be difficult to balance it.

However, if you want to control a machine to balance the ball, you can use some controlling methods. For example, you can use PID control to complete it. Before doing so, you have to learn calculus :)))

ii. Diffusion

While manufacturing semiconductors, sometimes we have to dope impurities into the wafer, when doing so, we have to calculate the relationship between impurities concentration and time by Fick's Second Law, to achieve this calculation, calculus is necessary:))))

(3). Computer Science

I believe everyone had used Chat-GPT, which is a Large Language Model(LLM). Have you ever thought how GPT-4 model was trained? The training of a Deep-Learning-Model has involved a large number of Functions, Statistics and more advanced mathematic concepts, and Calculus is the basis of everything.