Section 3.0 Differentiation

In this chapter, we'll first be discussing about the concept of derivative graphically, which stands for slope of a tangent line, or rate of change of a function. Later, we'll then be focusing on some rules of differentiation of different functions.

1. Definition of Derivative

   To derive the definition of derivative, we'll start from the slope of tangent line.

   $$\text{Slope of a tangent line at } a = \lim_{x \to a} \tfrac{f(x)-f(a)}{x-a}$$
   $$\text{If let } h = x - a \text{, when } x \to h \text{, then } \lim_{h \to 0} \tfrac{f(a+h)-f(a)}{h}$$

   Knowing that the slope of a tangent line stands for derivative of a function, denoted by $f'(a)$, we can therefore define the derivative at point a as following:

   $$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

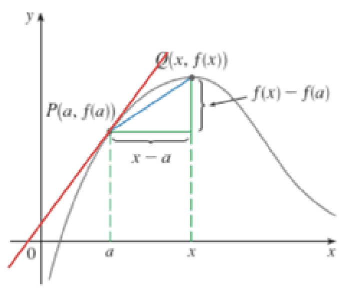


Figure 1

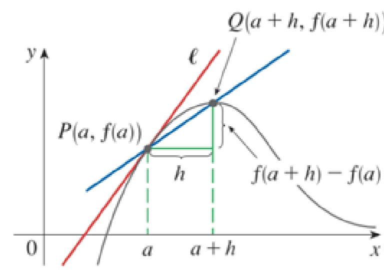


Figure 3

2. Differentiable Failure

   For a differentiable function, it must be <u>continuous</u> and <u>smooth</u>, the following conditions are those which don't meet these criteria.

   (1) Break Point

   (2) Oscillation

   (3) Sharp Point

   (4) Sharp Turn

3. Differentiation of Trigonometric Functions

   Basic Trigonometric Functions:

   i. $sin'(x) = cos(x)$

   ii. $cos'(x) = -sin(x)$

   iii. $tan'(x) = sec^2(x)$

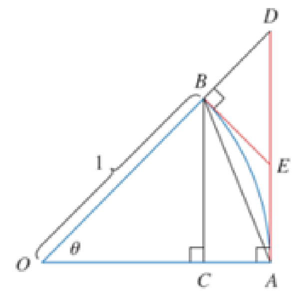
   iv. $sec'(x) = sec(x) \cdot tan(x)$

   v. $csc'(x) = -csc(x) \cdot cot(x)$

   vi. $cot'(x) = -csc^2(x)$

4. Intermediate form for special trigonometric functions

   i. $\lim_{\theta \to 0} \frac{sin(\theta)}{\theta} = 1$
      Proofing method: Geometric argument



   ii. $\lim_{\theta \to 0} \frac{cos(\theta)-1}{\theta} = 0$
      Proofing method: Trigonometric identity

   iii. CH4.4 L'Hospital's Rule (For Refernce)

      L'Hospital's Rule can be applied only when limit encounters indeterminate form, which is often shown as $\frac{\infty}{\infty}$ or $\frac{0}{0}$ in fractional form.

      Example 1

      Solve $\lim_{x \to 0} \frac{sin(x)}{\sqrt{x+1}-\sqrt{1-x}}$

      Example 2

      Solve $\lim_{x \to 0} \frac{x+xcos(x)}{sin(x) \cdot cos(x)}$

Exercise 1

Solve $\lim\limits_{x \to \frac{\pi}{4}} \frac{sin(x) - sin(\pi/4)}{x - \pi/4}$

Exercise 2

Solve $\lim\limits_{x \to 0} cos(x)^{1/x^2}$

5. Chain Rule

In previous chapters, we've already discussed derivatives of non-composite functions, however when dealing with derivatives of composite functions, the situation becomes more complicated, when solving these types of problems, chain rule is applied.

Suppose $F(x)$ is a composite function composed of $f(x)$, $g(x)$, and $h(x)$, then:

$$F(x) = f(g(h(x)))$$

While differentiating composite functions, chain rule states that we differentiate the function from the outer ones to the inner ones respectively and multiply these results altogether. If we take $F(x)$ for example, its differentiation will be:

$$F'(x) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

Example 1

Differentiate $f(x) = (9x^2 - 6x + 2)5^{x^3}$

Example 2

Differentiate $f(x) = (\frac{3-2x}{1+sin(3x)})$

Example 3

Differentiate $f(x) = sec^2(x \cdot tan(sec(x) \cdot tan(x)))$

Exercise 1

Differentiate $f(x) = \sqrt{x\sqrt{x + \sqrt{x}}}$

Exercise 2

Differentiate $f(x) = (e^{-6x} + sec(2 - x))^3$

Exercise 3

Differentiate $f(x) = ln(sin(x)) - (x^4 - 3x)^{10}$