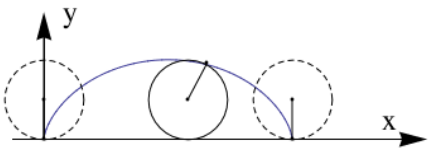
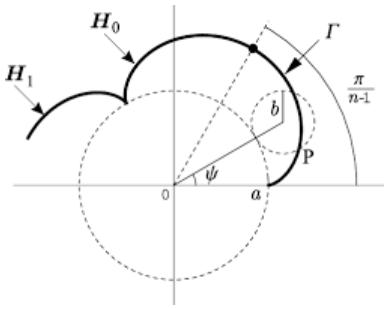
Section 10.2 Calculus of Parametric Curves

1. Parametric curves

   (1) Cycloid



   (2) Epicycloid



2. Derivative of the parametric curves

   (1) First derivative (Tangent)

$$f'(x,y) = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

   (2) Second derivative (Concavity)

$$f''(x,y) = \frac{d(\frac{dy/dt}{dx/dt})}{dx} = \frac{\frac{(d^2y/dt^2)(dx/dt)-(dy/dt)(d^2x/dt^2)}{(dx/dt)^2}}{d^2x/dt^2}$$

3. Integrals of the parametric curves

   (1) Area

$$A = \int_a^b y\,dx = \int_a^b g(t)'f(t)\,dt$$

   (2) Arc length

$$ds = \sqrt{(dx)^2 + (dy)^2}\,dt$$

$$\int_a^b ds = \int_a^b \sqrt{(dx)^2 + (dy)^2}\,dt$$

Example 1

Find the parametric from the spiral with pitch $p = 2$, radius $r = 10$ and height $h = 40$.

Exercise 1

Find the parametric from the cornical spiral with pitch $p = 2$, radius $r = 10$ and height $h = 40$.

Example 2

Find the arc length of the curve $x = \frac{2}{3}t^3$, $y = t^2 - 2$, $0 \le t \le 3$.

Exercise 2

Find the arc length of the curve $x = 3cost - cos3t$, $y = 3sint - sin3t$, $0 \le t \le \pi$.

Example 3

Find the exact area of the surface obtained by rotating $x = tsint$, $y = tcost$, $0 \le t \le \frac{\pi}{2}$ about the x-axis.

Exercise 3

Find the area of the region enclosed by the astroid $x = acos^3\theta$, $y = asin^3\theta$.