Stochoastic Modeling and Simulation

Homework 3.

Due Nov 4, 2021 at the beginning of class

Problem 1. (20 pts)

In Module 14, Example 4 (slide 11) we discussed the problem of determining the number of steps that it would take to flip fair coins and get 4 coins in a row. This problem is a slight variation on that theme.

Given that the initial state is 0, the number of steps required is a random variable N. In R, implement function whose input is a transition probability matrix and whose output is the number of steps needed until you have achieve the goal of 4 heads in a row. The function should return the number of tosses required.

Using the function, generate samples $N_1, N_2, \ldots, N_K$ to accurately estimate the following

1. P($N = 10$),

2. E[N] and

3. Var[N]

Problem 2. (20 pts.)

Part I. Consider the density function

$$f(x|\lambda) = \lambda e^{-\lambda x}.$$

The conjugate prior for f is of the form

$$p(\lambda) \propto \lambda e^{-\lambda \tau},$$

where τ is a hyper-parameter.

1. For a fixed, but arbitrary value of τ determine $c(\tau)$ so that

$$p(\lambda) = c(\tau)\lambda e^{-\lambda \tau},$$

is a probability density function.

2. For an iid sample $x_1, x_2, \ldots, x_n$ from f, determine the posterior modal estimator for λ.

Part 2.

Suppose that we are given the sample $\{.1, .2, .35, .5, .7, .9, 1.0\}$ and that we choose $\tau = 4$.

3. Using R, estimate the posterior median, posterior mean and a 95% credible interval with equally probable tails.

4. Using $\tau = 10$, repeat the computations from part **3.**

5. Compare the results of **3.** and **4.** Specifically, what was the impact of increasing the value of τ? on the location of the posterior median and mean and on the size of the credible interval.

Problem 3. (20 pts.) Work problem 7 on page 217.