Stochoastic Modeling and Simulation

Homework 4.

Due November 18, 2021

1. (20 pts) In many cases, the general form of the AMISE is

$$\text{AMISE} = g(h) = \frac{1}{nh^p} + bh^4.$$

Show that for $p \geq 1$, g is strictly convex and find the unique global minimum h^* of $g(h)$.

What is the value of $\text{AMISE}^* = g(h^*)$?

Using the tables from lecture 19, use this to determine formulas for the optimal value of h and the AMISE for the B-spline of order $m = 3$. Repeat this for the estimate of the first derivative.

2. (15 pts.) Assuming that the kernel is a symmetric probability density function, prove that

$$\int_{-\infty}^{\infty} (x - \bar{x})^2 \hat{f}_n(x)\, dx = \hat{\sigma}^2 + h^2 \sigma_K^2.$$

3. (10 pts) Given a histogram density estimate

$$\hat{f}_n(x) = \sum_{i=-\infty}^{\infty} \frac{n_i}{hn} B_i^1(x)$$

determine the mean value of the distribution $\hat{f}_n(x)$. That is to say, compute

$$\int_{-\infty}^{\infty} x \hat{f}_n(x)\, dx.$$

Your answer should be a function of h and n_i for each index i.

4. (10 pts.) Given a sample $x_1, \ldots, x_n$, write a two short R programs to compute the mean and variance based on the B-spline density estimate of order $m = 3$. So compute

$$\int_{-\infty}^{\infty} x \hat{f}_n(x)\, dx \quad \text{and} \quad \int_{-\infty}^{\infty} (x - \bar{x})^2 \hat{f}_n(x)\, dx.$$

Compare these values to $\bar{x}$ and $\hat{\sigma}^2$. You will need to generate your own data for this problem.

5. (10 points) Derive the formula for the marginal density functions for the two dimensional B-spline density estimate of order m.